Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/199-7.4.2-Exponentials-of-inverse-hyperbolic-cotangent-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [935]. This is test number [199].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	97.97 (916)	2.03 (19)
Mathematica	97.75 (914)	2.25 (21)
Fricas	89.95 (841)	10.05 (94)
Maple	83.85 (784)	16.15 (151)
Mupad	55.40 (518)	44.60 (417)
Maxima	54.01 (505)	45.99 (430)
Giac	47.49 (444)	52.51 (491)
Sympy	20.75 (194)	79.25 (741)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

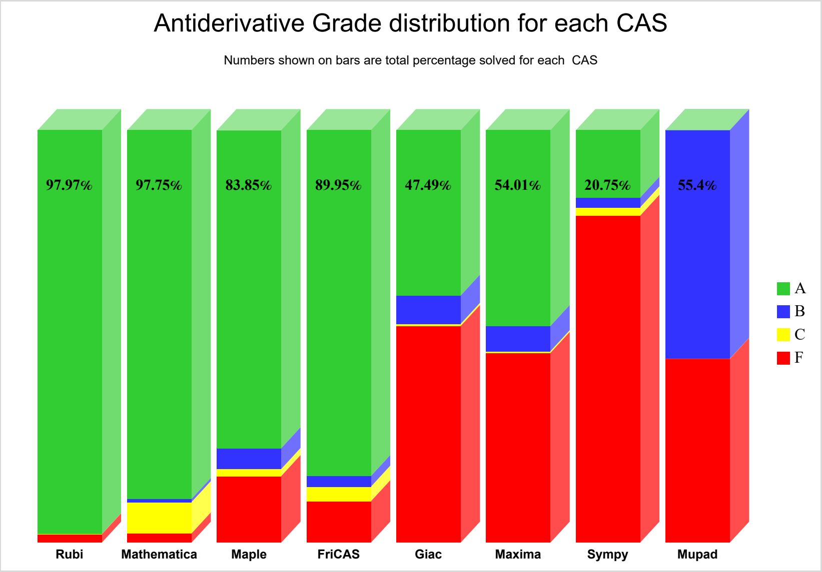
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

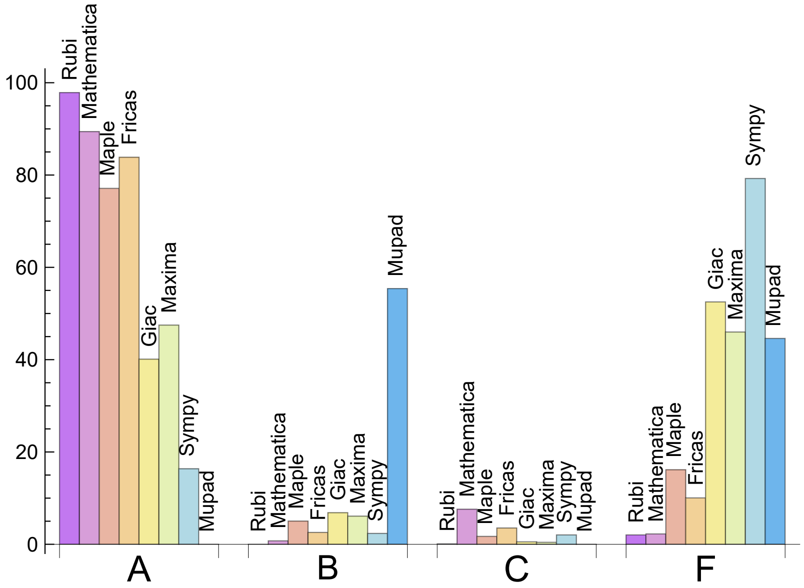
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.861	0.000	0.107	2.032
Mathematica	89.412	0.749	7.594	2.246
Fricas	83.850	2.567	3.529	10.053
Maple	77.112	5.027	1.711	16.150
Maxima	47.487	6.096	0.428	45.989
Giac	40.107	6.845	0.535	52.513
Sympy	16.364	2.353	2.032	79.251
Mupad	0.000	55.401	0.000	44.599

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	19	100.00	0.00	0.00
Mathematica	21	80.95	19.05	0.00
Fricas	94	100.00	0.00	0.00
Maple	151	100.00	0.00	0.00
Mupad	417	0.00	100.00	0.00
Maxima	430	100.00	0.00	0.00
Giac	491	65.78	0.00	34.22
Sympy	741	69.77	29.96	0.27

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.23
Fricas	0.26
Mathematica	0.28
Rubi	0.40
Giac	0.52
Maple	0.63
Mupad	2.32
Sympy	3.49

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	80.65	0.74	71.00	0.67
Mupad	97.81	0.96	81.00	0.90
Rubi	110.75	0.91	94.00	1.00
Maxima	115.90	1.13	97.00	1.03
Maple	117.42	1.04	85.00	0.82
Sympy	120.88	1.56	58.00	1.04
Giac	123.98	1.20	91.00	1.01
Fricas	138.95	1.16	99.00	1.03

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

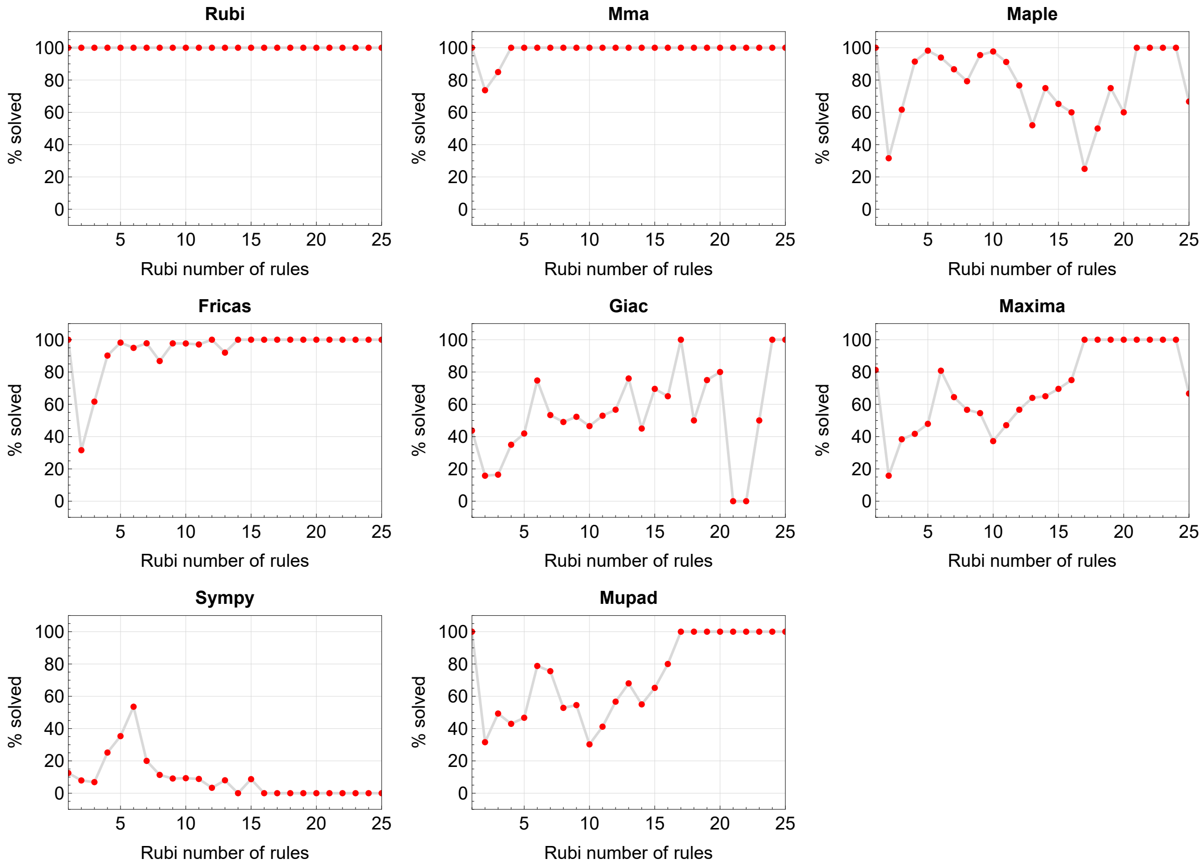


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

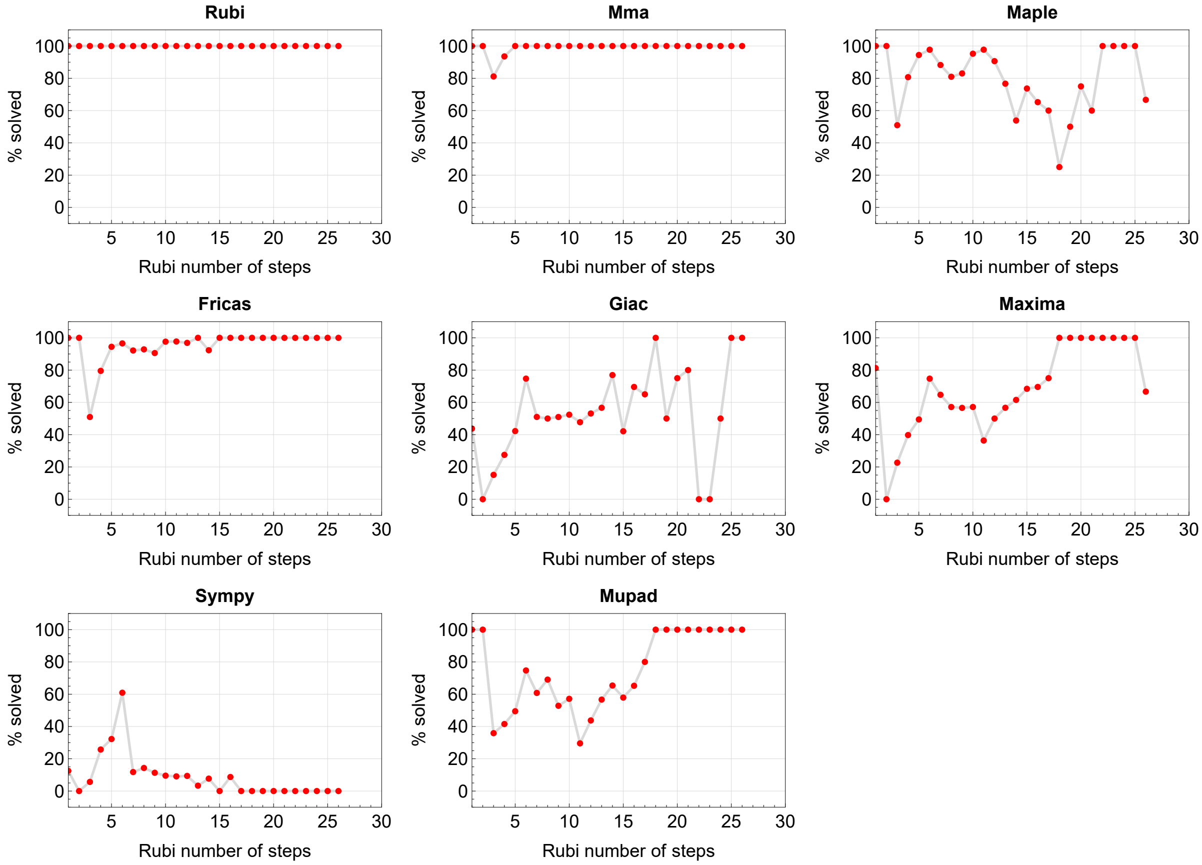


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

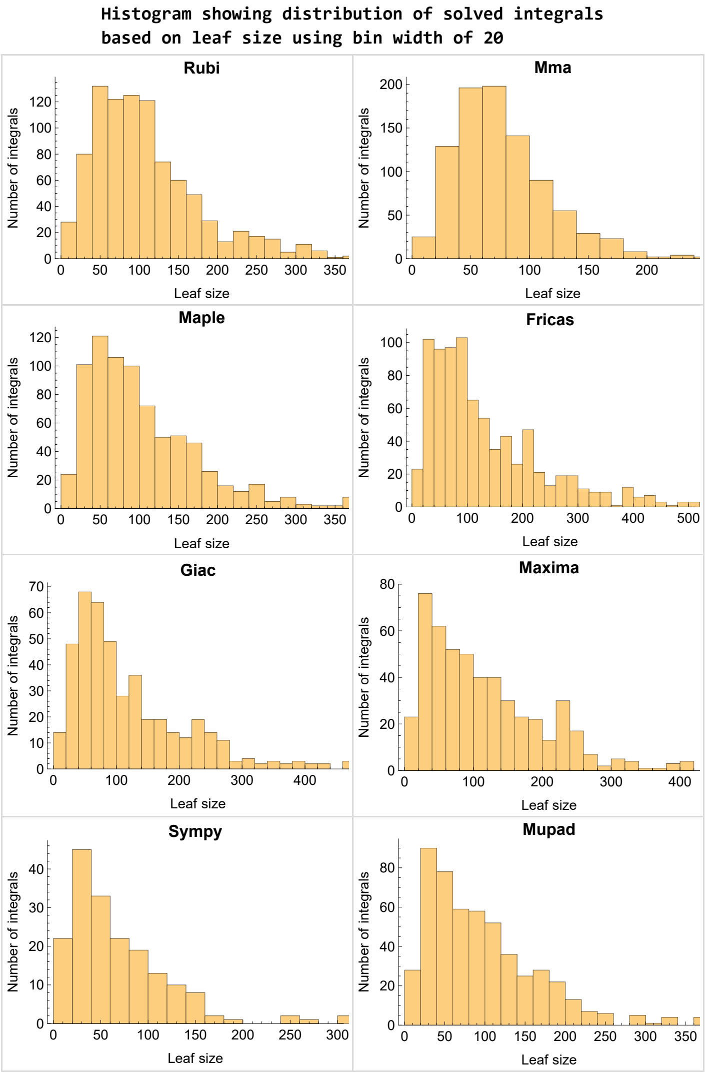
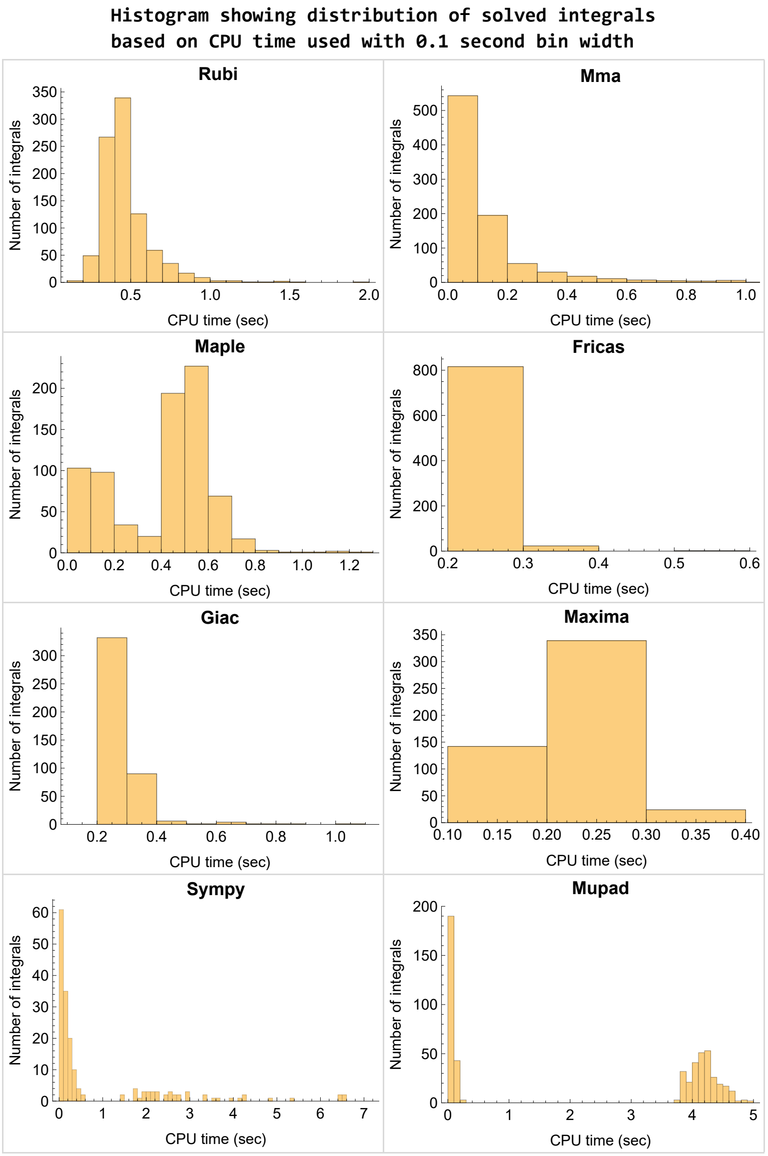


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.



Histogram showing distribution of solved integrals based on CPU time used with 0.1 second bin width

Rubi

Mma

Maple

Fricas

Giac

Maxima

Sympy

Mupad

Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

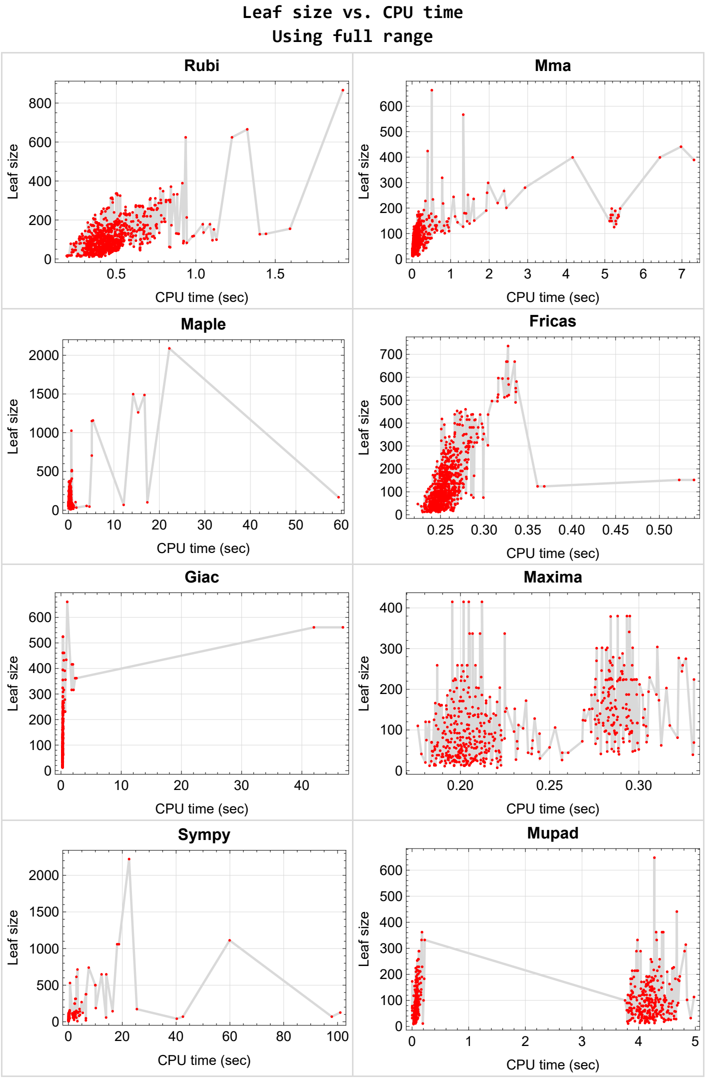


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{64, 65, 66, 67, 73, 74, 75, 76, 82, 83, 84, 85, 91, 92, 93, 94, 100, 101, 102, 103, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 126, 127, 128, 129, 130, 131, 158, 159, 160, 161, 178, 179, 180, 282, 285, 286\}$

Mathematica $\{61, 70, 79, 88, 133, 135, 136, 138, 332, 731, 734\}$

Maple $\{116, 118\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

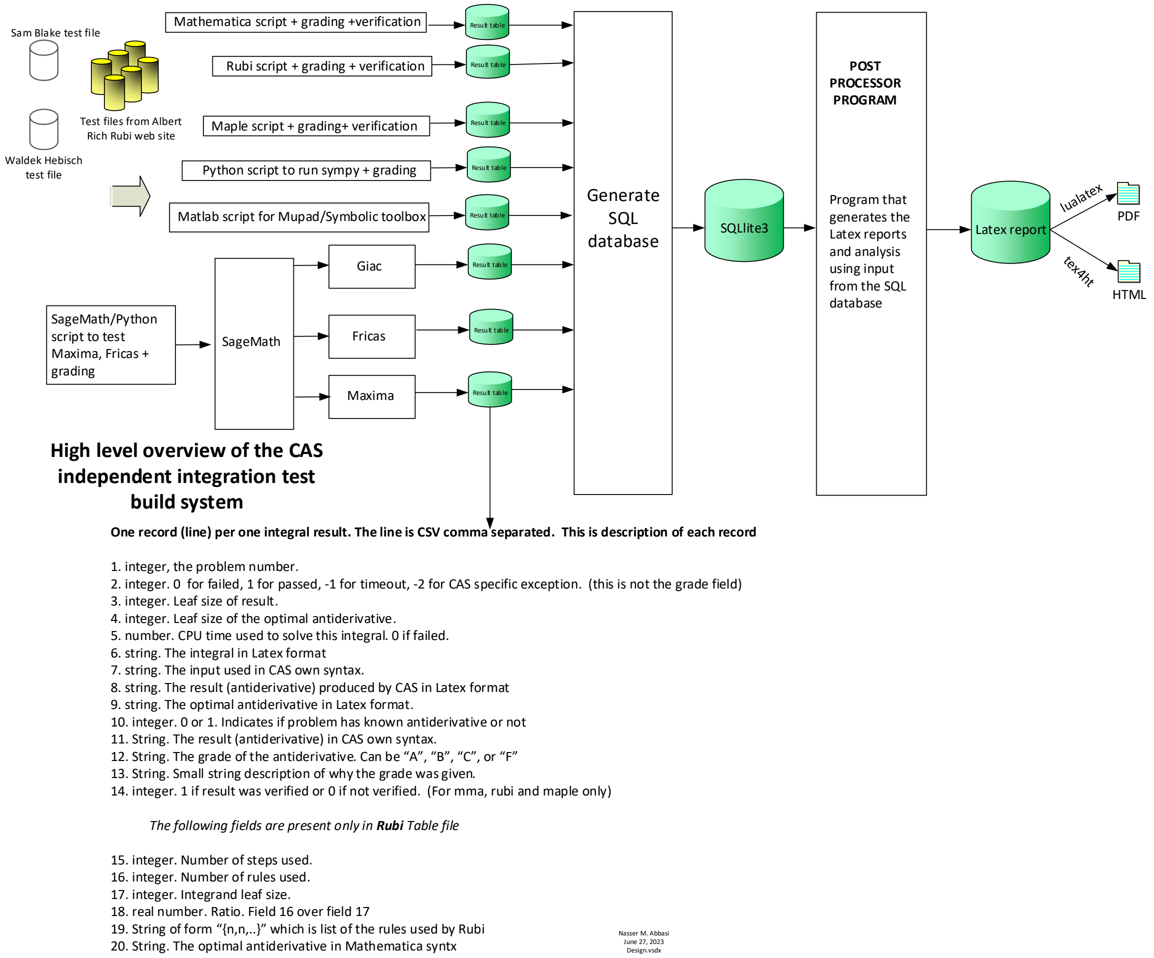
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

B grade { }

C grade { 172 }

F normal fail { 556, 557, 558, 559, 573, 574, 575, 576, 590, 591, 592, 593, 606, 607, 608, 609, 736, 737, 738 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 65, 66, 68, 69, 71, 77, 78, 80, 81, 83, 84, 86, 87, 89, 95, 96, 97, 98, 99, 101, 102, 104, 105, 106, 107, 113, 114, 120, 121, 122, 124, 127, 132, 134, 137, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 484, 489, 490, 491, 492, 493, 494, 495, 496, 497, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 550, 552, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 732, 733, 735, 738, 739, 740, 741, 742, 743, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade { 365, 381, 398, 584, 736, 737, 744 }

C grade { 61, 64, 67, 70, 72, 73, 74, 75, 76, 79, 82, 85, 88, 90, 91, 92, 93, 94, 100, 103, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 123, 125, 126, 128, 129, 130, 131, 133, 135, 136, 138, 172, 267, 268, 269, 429, 430, 431, 432, 451, 452, 453, 454, 458, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 498, 499, 569, 602, 731, 734 }

F normal fail { 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 549, 551, 553, 554, 933, 934, 935 }

F(-1) timedout fail { 545, 546, 547, 548 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 379, 380, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 435, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 476, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade { 4, 5, 6, 7, 20, 21, 22, 36, 37, 38, 39, 53, 54, 55, 161, 162, 182, 201, 220, 381, 382, 383, 398, 399, 416, 418, 432, 433, 434, 436, 437, 450, 451, 452, 453, 454, 473, 474, 477, 478, 479, 501, 502, 527, 528, 529, 677 }

C grade { 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 132, 134, 137 }

F normal fail { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 126, 127, 128, 129, 130, 131, 133, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 86, 87, 88, 89, 90, 95, 96, 97, 98, 99, 104, 105, 106, 107, 108, 113, 114, 115, 116, 120, 121, 122, 123, 124, 125, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 578, 579, 580, 581, 582, 583, 585, 586, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade { 5, 6, 21, 37, 38, 183, 194, 201, 242, 290, 381, 442, 466, 467, 492, 493, 519, 520, 572, 577, 584, 587, 588, 605 }

C grade { 64, 65, 66, 67, 73, 74, 75, 76, 82, 83, 84, 85, 91, 92, 93, 94, 100, 101, 102, 103, 109, 110, 111, 112, 117, 118, 119, 126, 127, 128, 129, 130, 131 }

F normal fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 195, 196, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 279, 280, 283, 284, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 323, 324, 327, 328, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 359, 360, 361, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 673, 674, 675, 676, 677, 712, 713, 714, 715, 716, 732, 733, 740, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 880, 904 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 53, 54, 158, 159, 160, 161, 178, 179, 180, 181, 187, 194, 198, 199, 200, 201, 212, 222, 281, 282, 285, 286, 379, 380, 381, 382, 396, 397, 398, 399, 414, 416, 417, 435, 584, 587, 636, 662 }

C grade { 321, 322, 325, 326 }

F normal fail { 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 277, 278, 295, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 329, 330, 331, 332, 333, 334, 335, 339, 340, 354, 355, 356, 357, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 614, 615, 616, 617, 618, 619, 620, 621, 622, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 30, 31, 32, 34, 35, 36, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 158, 159, 160, 161, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 185, 186, 188, 189, 190, 192, 193, 195, 196, 198, 199, 200, 201, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 222, 226, 228, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 245, 248, 249, 250, 251, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 311, 333, 334, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 382, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 400, 401, 403, 409, 410, 411, 412, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 435, 498, 499, 500, 556, 557, 558, 559, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 581, 582, 585, 587, 588, 589, 590, 591, 592, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 623, 624, 625, 626, 627, 651, 652, 653, 674, 675, 676, 677, 678, 713, 714, 715, 716, 717, 772, 773, 774, 775, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 797, 798, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 838, 839, 840, 863, 864, 865, 886, 887, 888, 890, 892, 910, 911, 912, 914, 916 }

B grade { 5, 6, 7, 8, 9, 29, 37, 39, 41, 174, 183, 184, 191, 194, 212, 235, 236, 237, 294, 301, 302, 379, 380, 381, 396, 398, 399, 404, 405, 406, 407, 408, 413, 414, 450, 451, 452, 453, 454, 501, 530, 531, 532, 577, 583, 584, 586, 610, 629, 655, 679, 680, 681, 718, 719, 720, 799, 800, 891, 893, 894, 915, 917, 918 }

C grade { 321, 322, 325, 326, 328 }

F normal fail { 18, 19, 20, 21, 22, 23, 24, 50, 51, 52, 53, 54, 55, 56, 57, 58, 128, 132, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 162, 167, 187, 197, 202, 206, 217, 218, 219, 220, 221, 223, 224, 225, 295, 335, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 376, 377, 378, 383, 418, 429, 430, 431, 432, 433, 434, 436, 438, 440, 442, 443, 444, 445, 455, 457, 462, 466, 467, 468, 470, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 507, 516, 518, 519, 520, 521, 522, 523, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 560, 561, 562, 563, 579, 580, 594, 595, 596, 597, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 682, 684, 685, 686, 687, 688, 689, 690, 691, 693, 694, 695, 696, 697, 698, 700, 702, 703, 704, 705, 706, 707, 708, 709, 710, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 736, 737, 738, 739, 740, 741, 742, 743, 746, 747, 748, 749, 750, 752, 753, 754, 755, 756, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 776, 777, 778, 779, 795, 796, 810, 811, 812, 813, 828, 829, 830, 831, 832, 833, 834, 846, 847, 848, 849, 850, 851, 852, 855, 856, 857, 858, 871, 872, 873, 874, 875, 880, 881, 882, 883, 884, 885, 895, 896, 897, 898, 900, 902, 904, 905, 906, 907, 909, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 932, 933, 934, 935 }

F(-1) timedout fail { }

F(-2) exception fail { 2, 33, 38, 40, 133, 138, 177, 216, 227, 229, 244, 246, 247, 252, 253, 254, 255, 257, 259, 270, 271, 272, 273, 274, 277, 296, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 327, 329, 330, 331, 332, 336, 350, 351, 352, 353, 354, 355, 356, 357, 358, 372, 373, 384, 386, 402, 419, 437, 439, 441, 446, 447, 448, 449, 456, 458, 459, 460, 461, 463, 464, 465, 469, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 487, 502, 503, 504, 505, 506, 508, 509, 510, 511, 512, 513, 514, 515, 517, 524, 525, 526, 527, 528, 529, 533, 534, 535, 536, 537, 538, 539, 540, 541, 628, 654, 668, 673, 683, 692, 699, 701, 711, 712, 730, 731, 734, 735, 744, 745, 751, 757, 793, 835, 836, 837, 841, 842, 843, 844, 845, 853, 854, 859, 860, 861, 862, 866, 867, 868, 869, 870, 876, 877, 878, 879, 889, 899, 901, 903, 908, 913, 931 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 359, 360, 361, 369, 370, 371, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 494, 495, 496, 497, 503, 504, 505, 506, 521, 522, 523, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 629, 630, 631, 632, 639, 655, 656, 657, 658, 665, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 881, 882, 885, 905, 906, 909 }

C grade { }

F normal fail { }

F(-1) timedout fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 277, 278, 295, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 329, 330, 331, 332, 333, 334, 335, 339, 340, 354, 355, 356, 357, 358, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 498, 499, 500, 501, 502, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 633, 634, 635, 636, 637, 638, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 907, 908, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 169, 170, 171, 172, 173, 174, 175, 188, 189, 190, 191, 192, 193, 195, 196, 207, 208, 209, 210, 211, 212, 213, 214, 215, 235, 236, 237, 238, 239, 240, 241, 242, 261, 262, 263, 264, 265, 266, 267, 268, 269, 288, 292, 301, 302, 303, 304, 320, 324, 330, 332, 333, 334, 341, 342, 343, 344, 345, 387, 388, 389, 390, 391, 392, 393, 394, 395, 404, 405, 406, 407, 408, 409, 410, 411, 412, 421, 422, 423, 424, 425, 427, 428, 564, 565, 566, 567, 568, 569, 570, 571, 572, 585, 588, 589, 598, 599, 600, 601, 602, 603, 604, 605, 780, 781, 782, 783, 784, 785, 786, 787, 788, 797, 798, 799, 800, 801, 802, 803, 804, 805, 814, 815, 816, 817, 818, 819, 820, 821 }

B grade { 134, 167, 168, 176, 187, 194, 305, 426, 502, 528, 581, 582, 583, 584, 586, 587, 623, 624, 625, 626, 651, 652 }

C grade { 137, 327, 328, 369, 370, 446, 447, 448, 449, 552, 740, 767, 770, 838, 839, 840, 863, 864, 865 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 140, 141, 142, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 247, 248, 256, 257, 258, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 293, 294, 296, 297, 298, 299, 300, 306, 307, 308, 309, 310, 311, 312, 313, 314, 319, 322, 323, 325, 326, 329, 331, 336, 337, 338, 339, 340, 346, 347, 348, 349, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 396, 397, 398, 399, 400, 401, 402, 403, 413, 414, 415, 416, 417, 418, 419, 420, 429, 430, 431, 432, 433, 434, 435, 436, 437, 442, 443, 450, 451, 452, 453, 454, 466, 467, 471, 472, 473, 474, 475, 476, 477, 478, 479, 490, 491, 492, 493, 494, 498, 499, 500, 501, 503, 504, 505, 506, 508, 518, 519, 520, 521, 524, 525, 526, 527, 529, 530, 531, 532, 542, 543, 544, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 573, 574, 575, 576, 577, 578, 579, 590, 591, 592, 593, 594, 595, 596, 597, 606, 607, 608, 609, 610, 611, 612, 613, 617, 618, 619, 620, 627, 628, 629, 630, 631, 632, 636, 637, 638, 646, 647, 648, 653, 654, 655, 656, 657, 658, 664, 665, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 691, 692, 693, 694, 695, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 725, 731, 732, 734, 736, 737, 738, 739, 741, 744, 745, 746, 747, 748, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 768, 769, 772, 773, 774, 775, 776, 777, 778, 779, 789, 790, 791, 792, 793, 794, 795, 806, 807, 808, 809, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 833, 841, 842, 843, 844, 845, 859, 866, 867, 868, 869, 870, 882, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 908, 910, 911, 912, 913, 914, 915, 916, 917, 918, 928, 929, 930, 931, 932, 933, 934, 935

}

F(-1) timedout fail { 104, 138, 139, 143, 144, 147, 216, 226, 227, 228, 233, 234, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 259, 260, 270, 271, 272, 273, 274, 275, 276, 277, 278, 295, 315, 316, 317, 318, 321, 335, 350, 351, 352, 353, 354, 355, 356, 357, 358, 371, 438, 439, 440, 441, 444, 445, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 468, 469, 470, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 495, 496, 497, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 522, 523, 533, 534, 535, 536, 537, 538, 539, 540, 541, 545, 580, 614, 615, 616, 621, 622, 633, 634, 635, 639, 640, 641, 642, 643, 644, 645, 649, 650, 659, 660, 661, 662, 663, 666, 667, 687, 688, 689, 690, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 721, 722, 723, 724, 726, 727, 728, 729, 730, 733, 735, 742, 743, 749, 750, 771, 796, 829, 830, 831, 832, 834, 835, 836, 837, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 862, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 885, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 909, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

F(-2) exception fail { 372, 378 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	125	68	117	203	92	0	111	171
N.S.	1	1.10	0.60	1.03	1.78	0.81	0.00	0.97	1.50
time (sec)	N/A	0.346	0.049	0.161	0.210	0.255	0.000	0.278	4.095

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	95	60	109	166	84	0	0	133
N.S.	1	1.06	0.67	1.21	1.84	0.93	0.00	0.00	1.48
time (sec)	N/A	0.304	0.034	0.118	0.214	0.253	0.000	0.000	0.066

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	49	100	128	73	0	77	98
N.S.	1	1.03	0.78	1.59	2.03	1.16	0.00	1.22	1.56
time (sec)	N/A	0.264	0.026	0.111	0.217	0.249	0.000	0.273	0.069

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	90	90	64	0	57	58
N.S.	1	1.00	1.14	2.50	2.50	1.78	0.00	1.58	1.61
time (sec)	N/A	0.216	0.020	0.107	0.217	0.255	0.000	0.281	0.052

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	36	132	69	57	0	63	37
N.S.	1	1.18	1.64	6.00	3.14	2.59	0.00	2.86	1.68
time (sec)	N/A	0.227	0.011	0.118	0.331	0.247	0.000	0.274	4.140

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	27	76	53	46	0	66	55
N.S.	1	1.17	1.12	3.17	2.21	1.92	0.00	2.75	2.29
time (sec)	N/A	0.205	0.015	0.126	0.285	0.253	0.000	0.264	0.091

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	60	42	85	91	60	0	143	81
N.S.	1	1.58	1.11	2.24	2.39	1.58	0.00	3.76	2.13
time (sec)	N/A	0.236	0.031	0.134	0.283	0.255	0.000	0.266	4.423

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	90	51	93	136	68	0	148	105
N.S.	1	1.20	0.68	1.24	1.81	0.91	0.00	1.97	1.40
time (sec)	N/A	0.273	0.059	0.129	0.286	0.253	0.000	0.268	0.070

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	120	59	101	172	76	0	226	129
N.S.	1	1.36	0.67	1.15	1.95	0.86	0.00	2.57	1.47
time (sec)	N/A	0.308	0.062	0.138	0.277	0.254	0.000	0.270	0.088

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	43	42	37	47	38
N.S.	1	1.00	1.00	0.91	1.00	0.98	0.86	1.09	0.88
time (sec)	N/A	0.289	0.016	0.389	0.200	0.242	0.059	0.257	0.039

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	34	33	27	38	30
N.S.	1	1.00	1.00	0.94	1.03	1.00	0.82	1.15	0.91
time (sec)	N/A	0.275	0.012	0.395	0.208	0.234	0.059	0.251	0.040

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	26	25	20	30	23
N.S.	1	1.00	1.00	0.92	1.00	0.96	0.77	1.15	0.88
time (sec)	N/A	0.257	0.010	0.385	0.222	0.240	0.052	0.270	0.043

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	16	10	14	13
N.S.	1	1.00	1.00	1.00	0.93	1.14	0.71	1.00	0.93
time (sec)	N/A	0.232	0.011	0.382	0.186	0.239	0.044	0.261	4.112

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	13	10	15	14
N.S.	1	1.00	1.00	1.00	0.93	0.93	0.71	1.07	1.00
time (sec)	N/A	0.251	0.009	0.387	0.185	0.249	0.071	0.271	0.050

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	22	15	20	14
N.S.	1	1.00	1.00	1.00	0.95	1.16	0.79	1.05	0.74
time (sec)	N/A	0.258	0.010	0.418	0.206	0.251	0.095	0.253	4.183

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	30	35	26	32	23
N.S.	1	1.00	1.00	0.91	0.91	1.06	0.79	0.97	0.70
time (sec)	N/A	0.279	0.012	0.431	0.186	0.253	0.119	0.263	0.047

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	38	43	34	40	30
N.S.	1	1.00	1.00	0.92	0.95	1.08	0.85	1.00	0.75
time (sec)	N/A	0.277	0.012	0.441	0.187	0.244	0.111	0.261	0.046

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	75	151	182	112	0	0	154
N.S.	1	1.00	0.64	1.28	1.54	0.95	0.00	0.00	1.31
time (sec)	N/A	0.618	0.060	0.137	0.208	0.271	0.000	0.000	4.126

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	66	142	145	103	0	0	117
N.S.	1	1.00	0.72	1.54	1.58	1.12	0.00	0.00	1.27
time (sec)	N/A	0.587	0.045	0.138	0.225	0.253	0.000	0.000	0.070

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	133	110	92	0	0	59
N.S.	1	1.00	0.87	2.15	1.77	1.48	0.00	0.00	0.95
time (sec)	N/A	0.547	0.036	0.128	0.176	0.246	0.000	0.000	4.086

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	83	53	363	90	104	0	0	54
N.S.	1	1.80	1.15	7.89	1.96	2.26	0.00	0.00	1.17
time (sec)	N/A	0.613	0.043	0.121	0.284	0.252	0.000	0.000	0.042

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	57	41	116	72	74	0	0	57
N.S.	1	1.12	0.80	2.27	1.41	1.45	0.00	0.00	1.12
time (sec)	N/A	0.275	0.062	0.144	0.268	0.262	0.000	0.000	4.506

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	99	56	125	110	88	0	0	83
N.S.	1	1.09	0.62	1.37	1.21	0.97	0.00	0.00	0.91
time (sec)	N/A	0.723	0.067	0.144	0.293	0.250	0.000	0.000	0.090

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	129	66	135	154	96	0	0	152
N.S.	1	1.39	0.71	1.45	1.66	1.03	0.00	0.00	1.63
time (sec)	N/A	0.919	0.077	0.145	0.285	0.271	0.000	0.000	4.198

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	58	66	49	78	57
N.S.	1	1.00	1.00	0.91	1.02	1.16	0.86	1.37	1.00
time (sec)	N/A	0.301	0.036	0.484	0.209	0.247	0.104	0.281	0.043

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	49	57	39	69	49
N.S.	1	1.00	1.00	0.94	1.04	1.21	0.83	1.47	1.04
time (sec)	N/A	0.293	0.029	0.437	0.222	0.238	0.095	0.266	0.037

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	41	49	31	64	38
N.S.	1	1.00	1.00	0.92	1.05	1.26	0.79	1.64	0.97
time (sec)	N/A	0.275	0.024	0.440	0.178	0.232	0.094	0.277	0.044

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	26	38	19	46	25
N.S.	1	1.00	0.96	0.96	0.96	1.41	0.70	1.70	0.93
time (sec)	N/A	0.236	0.017	0.434	0.203	0.252	0.096	0.264	0.041

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	18	8	57	12
N.S.	1	1.00	1.00	1.00	0.92	1.38	0.62	4.38	0.92
time (sec)	N/A	0.249	0.009	0.457	0.183	0.235	0.093	0.272	0.040

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	55	26	40	28
N.S.	1	1.00	1.00	0.97	1.06	1.72	0.81	1.25	0.88
time (sec)	N/A	0.274	0.020	0.492	0.190	0.240	0.146	0.260	0.059

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	48	73	41	62	41
N.S.	1	1.00	1.00	0.93	1.04	1.59	0.89	1.35	0.89
time (sec)	N/A	0.286	0.028	0.520	0.184	0.260	0.188	0.275	4.194

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	56	81	49	74	49
N.S.	1	1.00	1.00	0.94	1.04	1.50	0.91	1.37	0.91
time (sec)	N/A	0.294	0.047	0.521	0.192	0.260	0.202	0.270	0.067

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	125	68	117	203	91	0	0	172
N.S.	1	1.10	0.60	1.03	1.78	0.80	0.00	0.00	1.51
time (sec)	N/A	0.338	0.050	0.125	0.198	0.255	0.000	0.000	4.212

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	95	60	109	166	83	0	86	134
N.S.	1	1.06	0.67	1.21	1.84	0.92	0.00	0.96	1.49
time (sec)	N/A	0.297	0.043	0.125	0.186	0.253	0.000	0.272	0.061

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	63	49	100	130	73	0	71	97
N.S.	1	0.98	0.77	1.56	2.03	1.14	0.00	1.11	1.52
time (sec)	N/A	0.251	0.033	0.123	0.190	0.242	0.000	0.281	0.061

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	91	90	64	0	52	58
N.S.	1	1.00	1.14	2.46	2.43	1.73	0.00	1.41	1.57
time (sec)	N/A	0.216	0.021	0.112	0.192	0.264	0.000	0.279	4.633

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	24	34	130	70	57	0	59	37
N.S.	1	1.20	1.70	6.50	3.50	2.85	0.00	2.95	1.85
time (sec)	N/A	0.226	0.013	0.112	0.282	0.248	0.000	0.271	0.035

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	26	77	55	47	0	0	55
N.S.	1	1.16	1.04	3.08	2.20	1.88	0.00	0.00	2.20
time (sec)	N/A	0.207	0.018	0.122	0.288	0.248	0.000	0.000	4.187

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	59	41	85	93	60	0	157	82
N.S.	1	1.48	1.02	2.12	2.32	1.50	0.00	3.92	2.05
time (sec)	N/A	0.238	0.032	0.125	0.286	0.252	0.000	0.271	4.179

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	90	52	93	137	68	0	0	105
N.S.	1	1.18	0.68	1.22	1.80	0.89	0.00	0.00	1.38
time (sec)	N/A	0.276	0.056	0.127	0.286	0.270	0.000	0.000	4.380

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	120	59	101	173	77	0	258	129
N.S.	1	1.36	0.67	1.15	1.97	0.88	0.00	2.93	1.47
time (sec)	N/A	0.317	0.065	0.133	0.311	0.257	0.000	0.287	4.079

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	43	42	37	47	38
N.S.	1	1.00	1.00	0.93	1.02	1.00	0.88	1.12	0.90
time (sec)	N/A	0.290	0.016	0.394	0.192	0.243	0.072	0.272	3.998

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	34	33	27	38	31
N.S.	1	1.00	1.00	0.97	1.03	1.00	0.82	1.15	0.94
time (sec)	N/A	0.286	0.012	0.385	0.223	0.239	0.080	0.265	0.040

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	26	25	20	30	23
N.S.	1	1.00	1.00	0.96	1.04	1.00	0.80	1.20	0.92
time (sec)	N/A	0.250	0.010	0.390	0.193	0.244	0.068	0.261	0.041

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	16	10	14	13
N.S.	1	1.00	1.00	1.08	1.00	1.23	0.77	1.08	1.00
time (sec)	N/A	0.225	0.011	0.388	0.202	0.240	0.060	0.258	0.030

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	14
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	1.08
time (sec)	N/A	0.252	0.007	0.395	0.205	0.238	0.072	0.264	0.049

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	23	15	20	14
N.S.	1	1.00	1.00	1.06	1.00	1.28	0.83	1.11	0.78
time (sec)	N/A	0.260	0.009	0.421	0.204	0.262	0.091	0.263	4.022

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	30	30	35	26	32	24
N.S.	1	1.00	1.00	0.94	0.94	1.09	0.81	1.00	0.75
time (sec)	N/A	0.266	0.011	0.432	0.245	0.252	0.103	0.265	0.048

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	38	38	43	34	40	31
N.S.	1	1.00	1.00	0.95	0.95	1.08	0.85	1.00	0.78
time (sec)	N/A	0.274	0.011	0.436	0.188	0.250	0.127	0.264	4.125

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	83	153	223	92	0	0	192
N.S.	1	1.00	0.61	1.12	1.64	0.68	0.00	0.00	1.41
time (sec)	N/A	0.661	0.061	0.141	0.197	0.253	0.000	0.000	0.075

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	75	145	186	84	0	0	156
N.S.	1	1.00	0.65	1.25	1.60	0.72	0.00	0.00	1.34
time (sec)	N/A	0.611	0.062	0.139	0.212	0.256	0.000	0.000	0.063

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	66	136	151	75	0	0	120
N.S.	1	1.00	0.73	1.51	1.68	0.83	0.00	0.00	1.33
time (sec)	N/A	0.574	0.046	0.146	0.192	0.259	0.000	0.000	4.304

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	127	111	66	0	0	78
N.S.	1	1.00	0.90	2.12	1.85	1.10	0.00	0.00	1.30
time (sec)	N/A	0.526	0.038	0.139	0.186	0.244	0.000	0.000	0.061

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	81	55	366	89	74	0	0	54
N.S.	1	1.76	1.20	7.96	1.93	1.61	0.00	0.00	1.17
time (sec)	N/A	0.587	0.042	0.122	0.287	0.256	0.000	0.000	0.049

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	54	41	109	72	49	0	0	59
N.S.	1	1.02	0.77	2.06	1.36	0.92	0.00	0.00	1.11
time (sec)	N/A	0.267	0.047	0.139	0.295	0.258	0.000	0.000	0.087

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	96	56	119	112	61	0	0	118
N.S.	1	1.10	0.64	1.37	1.29	0.70	0.00	0.00	1.36
time (sec)	N/A	0.696	0.095	0.151	0.303	0.246	0.000	0.000	0.114

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	127	66	129	157	69	0	0	153
N.S.	1	1.32	0.69	1.34	1.64	0.72	0.00	0.00	1.59
time (sec)	N/A	0.888	0.074	0.148	0.287	0.256	0.000	0.000	4.235

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	155	75	137	193	77	0	0	190
N.S.	1	1.17	0.56	1.03	1.45	0.58	0.00	0.00	1.43
time (sec)	N/A	0.988	0.039	0.153	0.283	0.251	0.000	0.000	4.162

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	272	173	0	259	119	0	234	229
N.S.	1	1.08	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.387	5.154	0.000	0.292	0.264	0.000	0.351	0.120

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	229	149	0	224	111	0	203	192
N.S.	1	1.06	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.361	5.124	0.000	0.285	0.263	0.000	0.330	4.139

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	179	186	399	0	187	103	0	172	157
N.S.	1	1.04	2.23	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.317	4.165	0.000	0.294	0.269	0.000	0.322	0.087

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	145	66	0	149	93	0	139	120
N.S.	1	1.02	0.46	0.00	1.05	0.65	0.00	0.98	0.85
time (sec)	N/A	0.258	0.115	0.000	0.295	0.261	0.000	0.308	0.082

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	102	51	0	111	84	0	108	78
N.S.	1	1.06	0.53	0.00	1.16	0.88	0.00	1.12	0.81
time (sec)	N/A	0.226	0.062	0.000	0.317	0.266	0.000	0.311	4.294

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	258	30	0	224	173	0	232	101
N.S.	1	0.89	0.10	0.00	0.77	0.59	0.00	0.80	0.35
time (sec)	N/A	0.446	0.031	0.000	0.299	0.244	0.000	0.299	0.093

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	219	148	0	186	185	0	186	87
N.S.	1	0.82	0.55	0.00	0.70	0.69	0.00	0.70	0.33
time (sec)	N/A	0.413	0.170	0.000	0.273	0.254	0.000	0.297	4.291

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	260	173	0	226	202	0	223	132
N.S.	1	0.82	0.54	0.00	0.71	0.63	0.00	0.70	0.41
time (sec)	N/A	0.463	0.137	0.000	0.283	0.262	0.000	0.301	0.087

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	303	93	0	270	217	0	271	168
N.S.	1	0.85	0.26	0.00	0.76	0.61	0.00	0.76	0.47
time (sec)	N/A	0.505	0.079	0.000	0.284	0.262	0.000	0.312	4.431

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	272	173	0	259	119	0	234	229
N.S.	1	1.08	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.410	5.161	0.000	0.296	0.267	0.000	0.414	0.178

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	227	149	0	224	111	0	203	192
N.S.	1	1.05	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.359	5.133	0.000	0.293	0.255	0.000	0.390	4.188

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	179	186	399	0	187	103	0	172	157
N.S.	1	1.04	2.23	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.319	6.428	0.000	0.310	0.255	0.000	0.370	0.093

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	145	70	0	152	95	0	141	120
N.S.	1	1.02	0.49	0.00	1.07	0.67	0.00	0.99	0.85
time (sec)	N/A	0.267	0.133	0.000	0.293	0.261	0.000	0.347	4.200

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	102	56	0	112	86	0	109	79
N.S.	1	1.04	0.57	0.00	1.14	0.88	0.00	1.11	0.81
time (sec)	N/A	0.232	0.050	0.000	0.290	0.264	0.000	0.328	4.128

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	258	30	0	224	173	0	232	101
N.S.	1	0.89	0.10	0.00	0.77	0.59	0.00	0.80	0.35
time (sec)	N/A	0.450	0.037	0.000	0.291	0.254	0.000	0.313	0.059

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	219	46	0	187	184	0	187	88
N.S.	1	0.82	0.17	0.00	0.70	0.69	0.00	0.70	0.33
time (sec)	N/A	0.423	0.052	0.000	0.293	0.249	0.000	0.320	0.090

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	260	76	0	229	209	0	225	132
N.S.	1	0.82	0.24	0.00	0.72	0.66	0.00	0.71	0.41
time (sec)	N/A	0.453	0.063	0.000	0.306	0.262	0.000	0.341	0.095

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	303	93	0	277	217	0	271	168
N.S.	1	0.85	0.26	0.00	0.78	0.61	0.00	0.76	0.47
time (sec)	N/A	0.493	0.085	0.000	0.289	0.263	0.000	0.352	4.164

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	311	198	0	275	152	0	254	248
N.S.	1	1.08	0.69	0.00	0.96	0.53	0.00	0.89	0.86
time (sec)	N/A	0.448	5.179	0.000	0.326	0.261	0.000	0.352	4.233

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	268	161	0	238	144	0	223	211
N.S.	1	1.07	0.64	0.00	0.95	0.58	0.00	0.89	0.84
time (sec)	N/A	0.381	5.154	0.000	0.291	0.263	0.000	0.348	0.139

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	213	225	441	0	203	136	0	192	176
N.S.	1	1.06	2.07	0.00	0.95	0.64	0.00	0.90	0.83
time (sec)	N/A	0.353	6.976	0.000	0.315	0.262	0.000	0.319	0.099

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	179	80	0	166	128	0	161	139
N.S.	1	1.02	0.45	0.00	0.94	0.73	0.00	0.91	0.79
time (sec)	N/A	0.284	0.142	0.000	0.284	0.252	0.000	0.323	4.202

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	136	67	0	131	117	0	141	98
N.S.	1	1.05	0.52	0.00	1.01	0.90	0.00	1.08	0.75
time (sec)	N/A	0.246	0.093	0.000	0.283	0.260	0.000	0.303	0.096

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	320	287	30	0	244	241	0	252	118
N.S.	1	0.90	0.09	0.00	0.76	0.75	0.00	0.79	0.37
time (sec)	N/A	0.477	0.054	0.000	0.324	0.260	0.000	0.302	4.455

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	253	173	0	204	243	0	217	107
N.S.	1	0.85	0.58	0.00	0.68	0.81	0.00	0.73	0.36
time (sec)	N/A	0.450	0.238	0.000	0.284	0.259	0.000	0.295	4.190

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	351	293	186	0	244	261	0	243	152
N.S.	1	0.83	0.53	0.00	0.70	0.74	0.00	0.69	0.43
time (sec)	N/A	0.515	0.169	0.000	0.284	0.270	0.000	0.316	0.090

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	385	332	104	0	288	269	0	291	188
N.S.	1	0.86	0.27	0.00	0.75	0.70	0.00	0.76	0.49
time (sec)	N/A	0.519	0.102	0.000	0.291	0.273	0.000	0.321	4.234

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	272	173	0	259	119	0	234	229
N.S.	1	1.08	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.411	5.202	0.000	0.285	0.276	0.000	0.318	0.095

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	229	149	0	224	111	0	203	193
N.S.	1	1.06	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.360	5.177	0.000	0.296	0.256	0.000	0.316	4.399

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	179	186	389	0	187	102	0	172	157
N.S.	1	1.04	2.17	0.00	1.04	0.57	0.00	0.96	0.88
time (sec)	N/A	0.321	7.312	0.000	0.299	0.260	0.000	0.306	4.144

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	145	66	0	151	93	0	140	121
N.S.	1	1.02	0.46	0.00	1.06	0.65	0.00	0.99	0.85
time (sec)	N/A	0.269	0.189	0.000	0.303	0.264	0.000	0.308	0.072

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	102	33	0	111	84	0	108	79
N.S.	1	1.05	0.34	0.00	1.14	0.87	0.00	1.11	0.81
time (sec)	N/A	0.228	0.059	0.000	0.285	0.263	0.000	0.301	4.049

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	258	30	0	224	173	0	232	101
N.S.	1	0.89	0.10	0.00	0.77	0.59	0.00	0.80	0.35
time (sec)	N/A	0.436	0.054	0.000	0.298	0.266	0.000	0.311	4.485

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	220	33	0	186	177	0	186	88
N.S.	1	0.82	0.12	0.00	0.69	0.66	0.00	0.69	0.33
time (sec)	N/A	0.422	0.055	0.000	0.302	0.258	0.000	0.280	4.182

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	259	56	0	227	201	0	223	132
N.S.	1	0.81	0.18	0.00	0.71	0.63	0.00	0.70	0.41
time (sec)	N/A	0.460	0.079	0.000	0.284	0.264	0.000	0.297	0.068

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	302	93	0	277	216	0	271	169
N.S.	1	0.85	0.26	0.00	0.78	0.61	0.00	0.76	0.47
time (sec)	N/A	0.508	0.135	0.000	0.322	0.256	0.000	0.309	0.077

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	272	173	0	259	119	0	234	229
N.S.	1	1.08	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.429	5.349	0.000	0.325	0.268	0.000	0.357	4.362

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	227	149	0	224	111	0	203	193
N.S.	1	1.05	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.370	5.281	0.000	0.331	0.266	0.000	0.371	4.295

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	186	125	0	187	103	0	172	157
N.S.	1	1.04	0.70	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.323	5.241	0.000	0.283	0.272	0.000	0.335	0.064

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	145	70	0	152	95	0	141	121
N.S.	1	1.02	0.49	0.00	1.07	0.67	0.00	0.99	0.85
time (sec)	N/A	0.263	0.226	0.000	0.282	0.262	0.000	0.324	4.096

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	102	55	0	112	86	0	109	79
N.S.	1	1.04	0.56	0.00	1.14	0.88	0.00	1.11	0.81
time (sec)	N/A	0.228	0.139	0.000	0.285	0.255	0.000	0.280	4.079

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	258	28	0	224	173	0	232	101
N.S.	1	0.89	0.10	0.00	0.77	0.59	0.00	0.80	0.35
time (sec)	N/A	0.438	0.079	0.000	0.297	0.250	0.000	0.305	4.240

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	220	149	0	187	192	0	187	88
N.S.	1	0.82	0.55	0.00	0.70	0.71	0.00	0.70	0.33
time (sec)	N/A	0.413	0.304	0.000	0.275	0.257	0.000	0.292	0.081

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	259	174	0	228	209	0	225	132
N.S.	1	0.81	0.55	0.00	0.71	0.66	0.00	0.71	0.41
time (sec)	N/A	0.459	0.233	0.000	0.289	0.263	0.000	0.314	4.280

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	302	93	0	270	217	0	271	169
N.S.	1	0.85	0.26	0.00	0.76	0.61	0.00	0.76	0.47
time (sec)	N/A	0.490	0.158	0.000	0.276	0.251	0.000	0.328	0.065

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	311	198	0	279	119	0	254	253
N.S.	1	1.08	0.69	0.00	0.97	0.41	0.00	0.89	0.88
time (sec)	N/A	0.459	5.398	0.000	0.286	0.252	0.000	0.320	0.091

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	268	161	0	244	111	0	223	217
N.S.	1	1.07	0.64	0.00	0.98	0.44	0.00	0.89	0.87
time (sec)	N/A	0.389	5.327	0.000	0.291	0.251	0.000	0.329	0.087

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	225	137	0	207	103	0	192	181
N.S.	1	1.06	0.64	0.00	0.97	0.48	0.00	0.90	0.85
time (sec)	N/A	0.349	5.293	0.000	0.285	0.266	0.000	0.308	0.079

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	179	121	0	172	95	0	161	145
N.S.	1	1.02	0.69	0.00	0.98	0.54	0.00	0.91	0.82
time (sec)	N/A	0.288	0.274	0.000	0.280	0.279	0.000	0.305	4.240

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	136	31	0	132	86	0	129	103
N.S.	1	1.05	0.24	0.00	1.02	0.66	0.00	0.99	0.79
time (sec)	N/A	0.250	0.082	0.000	0.288	0.258	0.000	0.283	4.197

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	320	287	28	0	244	190	0	252	118
N.S.	1	0.90	0.09	0.00	0.76	0.59	0.00	0.79	0.37
time (sec)	N/A	0.481	0.105	0.000	0.290	0.249	0.000	0.286	4.005

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	252	31	0	204	185	0	204	106
N.S.	1	0.84	0.10	0.00	0.68	0.62	0.00	0.68	0.35
time (sec)	N/A	0.443	0.100	0.000	0.280	0.261	0.000	0.291	4.057

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	351	292	101	0	247	209	0	243	153
N.S.	1	0.83	0.29	0.00	0.70	0.60	0.00	0.69	0.44
time (sec)	N/A	0.482	0.157	0.000	0.283	0.264	0.000	0.298	0.079

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	385	331	104	0	297	217	0	291	188
N.S.	1	0.86	0.27	0.00	0.77	0.56	0.00	0.76	0.49
time (sec)	N/A	0.539	0.193	0.000	0.282	0.267	0.000	0.307	0.080

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	266	189	705	220	173	0	215	168
N.S.	1	0.93	0.66	2.47	0.77	0.61	0.00	0.75	0.59
time (sec)	N/A	0.402	5.268	5.142	0.288	0.241	0.000	0.302	4.178

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	233	167	1158	194	168	0	191	142
N.S.	1	0.90	0.65	4.49	0.75	0.65	0.00	0.74	0.55
time (sec)	N/A	0.355	0.625	5.403	0.305	0.253	0.000	0.287	4.582

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	201	35	1151	167	160	0	168	115
N.S.	1	0.90	0.16	5.16	0.75	0.72	0.00	0.75	0.52
time (sec)	N/A	0.326	0.051	5.152	0.279	0.255	0.000	0.281	0.114

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	A	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	402	371	26	2088	0	323	0	261	167
N.S.	1	0.92	0.06	5.19	0.00	0.80	0.00	0.65	0.42
time (sec)	N/A	0.526	0.043	22.190	0.000	0.251	0.000	0.295	0.154

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	214	39	1487	152	211	0	152	109
N.S.	1	0.92	0.17	6.38	0.65	0.91	0.00	0.65	0.47
time (sec)	N/A	0.395	0.063	16.722	0.274	0.250	0.000	0.289	4.144

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	260	243	124	1262	178	264	0	175	136
N.S.	1	0.93	0.48	4.85	0.68	1.02	0.00	0.67	0.52
time (sec)	N/A	0.408	0.947	15.330	0.281	0.256	0.000	0.291	0.121

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	276	133	1498	205	269	0	199	161
N.S.	1	0.96	0.46	5.22	0.71	0.94	0.00	0.69	0.56
time (sec)	N/A	0.444	0.234	14.232	0.275	0.253	0.000	0.289	4.139

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	167	189	408	149	100	0	144	171
N.S.	1	1.06	1.20	2.60	0.95	0.64	0.00	0.92	1.09
time (sec)	N/A	0.286	5.272	0.764	0.284	0.250	0.000	0.288	4.049

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	136	165	403	123	95	0	120	145
N.S.	1	1.05	1.27	3.10	0.95	0.73	0.00	0.92	1.12
time (sec)	N/A	0.246	0.603	0.710	0.270	0.249	0.000	0.292	4.039

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	104	85	503	96	87	0	97	118
N.S.	1	1.08	0.89	5.24	1.00	0.91	0.00	1.01	1.23
time (sec)	N/A	0.217	0.188	0.704	0.281	0.249	0.000	0.290	0.050

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	157	26	1026	140	86	0	79	82
N.S.	1	1.01	0.17	6.62	0.90	0.55	0.00	0.51	0.53
time (sec)	N/A	0.254	0.044	0.661	0.278	0.241	0.000	0.291	4.287

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	105	87	517	98	97	0	99	118
N.S.	1	1.06	0.88	5.22	0.99	0.98	0.00	1.00	1.19
time (sec)	N/A	0.227	0.168	0.762	0.282	0.260	0.000	0.283	0.027

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	134	134	506	124	111	0	122	145
N.S.	1	1.03	1.03	3.89	0.95	0.85	0.00	0.94	1.12
time (sec)	N/A	0.240	0.471	0.780	0.269	0.252	0.000	0.278	4.274

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	429	389	167	0	341	266	0	308	227
N.S.	1	0.91	0.39	0.00	0.79	0.62	0.00	0.72	0.53
time (sec)	N/A	0.546	5.359	0.000	0.295	0.251	0.000	0.326	4.620

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	392	348	319	0	304	255	0	288	190
N.S.	1	0.89	0.81	0.00	0.78	0.65	0.00	0.73	0.48
time (sec)	N/A	0.478	0.780	0.000	0.310	0.262	0.000	0.302	0.173

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	352	305	56	0	265	238	0	0	149
N.S.	1	0.87	0.16	0.00	0.75	0.68	0.00	0.00	0.42
time (sec)	N/A	0.434	0.059	0.000	0.284	0.270	0.000	0.000	4.404

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	919	866	30	0	0	418	0	661	648
N.S.	1	0.94	0.03	0.00	0.00	0.45	0.00	0.72	0.71
time (sec)	N/A	1.171	0.050	0.000	0.000	0.252	0.000	1.037	4.278

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	676	624	46	0	0	374	0	432	162
N.S.	1	0.92	0.07	0.00	0.00	0.55	0.00	0.64	0.24
time (sec)	N/A	0.758	0.072	0.000	0.000	0.251	0.000	0.462	4.133

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	731	665	72	0	0	399	0	461	210
N.S.	1	0.91	0.10	0.00	0.00	0.55	0.00	0.63	0.29
time (sec)	N/A	0.818	0.097	0.000	0.000	0.266	0.000	0.503	4.503

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	201	0	0	0	0	0
N.S.	1	1.00	1.04	4.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.029	0.658	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	151	178	228	0	0	0	0	0	0
N.S.	1	1.18	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.667	0.347	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	106	0	0	97	0	0
N.S.	1	1.00	0.74	3.03	0.00	0.00	2.77	0.00	0.00
time (sec)	N/A	0.259	0.013	0.482	0.000	0.000	1.488	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	74	95	128	0	0	0	0	0	0
N.S.	1	1.28	1.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	0.440	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	94	115	0	0	0	0	0	0
N.S.	1	1.25	1.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	93	0	0	116	0	0
N.S.	1	1.00	0.75	2.58	0.00	0.00	3.22	0.00	0.00
time (sec)	N/A	0.257	0.013	0.493	0.000	0.000	1.444	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	150	178	192	0	0	0	0	0	0
N.S.	1	1.19	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.659	0.306	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	178	118	0	0	0	0	0	0
N.S.	1	1.02	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	0.614	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	98	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	0.321	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	142	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	44	0	0	0	0	0	0
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	107	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.362	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	171	132	0	0	0	0	0	0
N.S.	1	1.02	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.636	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	187	148	0	0	0	0	0	0
N.S.	1	1.02	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	0.528	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	176	131	0	0	0	0	0	0
N.S.	1	1.23	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	128	80	128	259	125	0	138	214
N.S.	1	0.97	0.61	0.97	1.96	0.95	0.00	1.05	1.62
time (sec)	N/A	0.426	0.219	0.424	0.201	0.255	0.000	0.286	4.329

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	98	73	120	221	115	0	118	177
N.S.	1	0.93	0.70	1.14	2.10	1.10	0.00	1.12	1.69
time (sec)	N/A	0.312	0.206	0.423	0.196	0.250	0.000	0.284	0.083

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	70	64	112	181	103	0	98	138
N.S.	1	0.90	0.82	1.44	2.32	1.32	0.00	1.26	1.77
time (sec)	N/A	0.271	0.153	0.398	0.205	0.249	0.000	0.292	4.231

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	43	51	92	132	77	0	58	94
N.S.	1	0.91	1.09	1.96	2.81	1.64	0.00	1.23	2.00
time (sec)	N/A	0.232	0.085	0.060	0.205	0.260	0.000	0.291	4.356

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	54	60	247	78	87	0	0	48
N.S.	1	1.06	1.18	4.84	1.53	1.71	0.00	0.00	0.94
time (sec)	N/A	0.275	0.128	0.424	0.195	0.267	0.000	0.000	0.084

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	34	36	23	57	0	49	23
N.S.	1	1.00	1.03	1.09	0.70	1.73	0.00	1.48	0.70
time (sec)	N/A	0.223	0.135	0.397	0.198	0.251	0.000	0.289	4.297

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	64	42	41	39	77	0	85	39
N.S.	1	0.96	0.63	0.61	0.58	1.15	0.00	1.27	0.58
time (sec)	N/A	0.256	0.140	0.404	0.205	0.252	0.000	0.293	4.322

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	128	51	50	55	96	0	105	56
N.S.	1	1.28	0.51	0.50	0.55	0.96	0.00	1.05	0.56
time (sec)	N/A	0.335	0.141	0.415	0.198	0.243	0.000	0.333	0.046

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	192	59	58	71	116	0	125	72
N.S.	1	1.44	0.44	0.44	0.53	0.87	0.00	0.94	0.54
time (sec)	N/A	0.488	0.155	0.396	0.211	0.243	0.000	0.336	0.048

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	47	28	29	49	28	124	0	28
N.S.	1	1.12	0.67	0.69	1.17	0.67	2.95	0.00	0.67
time (sec)	N/A	0.308	0.035	0.487	0.200	0.245	0.388	0.000	4.660

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	36	23	45	60	60	66	60	60
N.S.	1	0.97	0.62	1.22	1.62	1.62	1.78	1.62	1.62
time (sec)	N/A	0.299	0.025	0.465	0.193	0.240	0.055	0.285	0.042

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	36	30	29	37	37	36	37	37
N.S.	1	0.97	0.81	0.78	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.302	0.019	0.486	0.199	0.240	0.040	0.275	0.051

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	36	30	29	38	38	37	38	38
N.S.	1	0.97	0.81	0.78	1.03	1.03	1.00	1.03	1.03
time (sec)	N/A	0.301	0.018	0.485	0.193	0.240	0.048	0.260	0.050

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	17	17	16	18	18	15	18	15
N.S.	1	0.85	0.85	0.80	0.90	0.90	0.75	0.90	0.75
time (sec)	N/A	0.274	0.012	0.464	0.192	0.229	0.031	0.271	0.031

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	26	26	10	12	12	12	12	9
N.S.	1	1.86	1.86	0.71	0.86	0.86	0.86	0.86	0.64
time (sec)	N/A	0.165	0.011	0.415	0.202	0.230	0.026	0.263	0.025

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	30	30	29	30	29	20	31	29
N.S.	1	0.94	0.94	0.91	0.94	0.91	0.62	0.97	0.91
time (sec)	N/A	0.289	0.019	0.520	0.203	0.252	0.079	0.259	4.625

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	25	14	26	26	24	34	13
N.S.	1	1.00	1.79	1.00	1.86	1.86	1.71	2.43	0.93
time (sec)	N/A	0.267	0.012	0.445	0.209	0.255	0.112	0.269	4.280

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	36	23	21	47	47	49	21	46
N.S.	1	0.97	0.62	0.57	1.27	1.27	1.32	0.57	1.24
time (sec)	N/A	0.295	0.018	0.473	0.193	0.238	0.145	0.264	4.315

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	36	23	21	57	57	60	21	56
N.S.	1	0.97	0.62	0.57	1.54	1.54	1.62	0.57	1.51
time (sec)	N/A	0.294	0.019	0.460	0.206	0.254	0.185	0.263	0.098

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	233	138	0	0	0	0	0	0
N.S.	1	1.15	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	101	80	128	259	126	0	138	214
N.S.	1	0.96	0.76	1.22	2.47	1.20	0.00	1.31	2.04
time (sec)	N/A	0.298	0.226	0.474	0.208	0.255	0.000	0.292	0.109

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	76	64	106	221	109	0	84	176
N.S.	1	0.97	0.82	1.36	2.83	1.40	0.00	1.08	2.26
time (sec)	N/A	0.260	0.194	0.416	0.202	0.264	0.000	0.283	4.335

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	77	64	112	181	103	0	98	139
N.S.	1	0.99	0.82	1.44	2.32	1.32	0.00	1.26	1.78
time (sec)	N/A	0.306	0.177	0.407	0.217	0.253	0.000	0.282	4.244

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	70	53	99	135	81	0	74	97
N.S.	1	1.08	0.82	1.52	2.08	1.25	0.00	1.14	1.49
time (sec)	N/A	0.323	0.116	0.119	0.215	0.251	0.000	0.283	4.602

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	81	63	345	95	120	0	35	63
N.S.	1	1.01	0.79	4.31	1.19	1.50	0.00	0.44	0.79
time (sec)	N/A	0.427	0.187	0.393	0.211	0.249	0.000	0.296	0.070

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	36	36	23	77	0	69	23
N.S.	1	1.00	1.09	1.09	0.70	2.33	0.00	2.09	0.70
time (sec)	N/A	0.229	0.158	0.433	0.204	0.257	0.000	0.334	0.038

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	66	41	41	39	95	0	125	39
N.S.	1	0.99	0.61	0.61	0.58	1.42	0.00	1.87	0.58
time (sec)	N/A	0.253	0.169	0.447	0.201	0.256	0.000	0.339	0.042

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	101	50	50	55	116	0	145	56
N.S.	1	1.07	0.53	0.53	0.59	1.23	0.00	1.54	0.60
time (sec)	N/A	0.336	0.169	0.410	0.208	0.246	0.000	0.347	4.182

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	134	58	58	71	134	0	165	72
N.S.	1	1.07	0.46	0.46	0.57	1.07	0.00	1.32	0.58
time (sec)	N/A	0.481	0.184	0.434	0.212	0.244	0.000	0.392	0.048

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	75	50	74	153	81	530	0	57
N.S.	1	1.14	0.76	1.12	2.32	1.23	8.03	0.00	0.86
time (sec)	N/A	0.336	0.115	0.521	0.226	0.259	0.583	0.000	4.377

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	48	31	45	59	59	63	42	59
N.S.	1	0.91	0.58	0.85	1.11	1.11	1.19	0.79	1.11
time (sec)	N/A	0.315	0.027	0.579	0.202	0.238	0.062	0.256	4.162

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	26	26	23	28	28	29	42	24
N.S.	1	0.81	0.81	0.72	0.88	0.88	0.91	1.31	0.75
time (sec)	N/A	0.288	0.019	0.559	0.208	0.234	0.047	0.265	0.045

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	30	29	37	37	37	42	37
N.S.	1	0.94	0.86	0.83	1.06	1.06	1.06	1.20	1.06
time (sec)	N/A	0.290	0.020	0.564	0.209	0.228	0.051	0.257	0.049

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	21	16	25	25	24	40	19
N.S.	1	1.00	1.24	0.94	1.47	1.47	1.41	2.35	1.12
time (sec)	N/A	0.256	0.017	0.557	0.197	0.239	0.036	0.269	0.037

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	26	26	24	24	28	26	50	26
N.S.	1	0.96	0.96	0.89	0.89	1.04	0.96	1.85	0.96
time (sec)	N/A	0.261	0.015	0.537	0.207	0.240	0.077	0.266	4.145

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	43	36	32	44	49	37	57	42
N.S.	1	0.90	0.75	0.67	0.92	1.02	0.77	1.19	0.88
time (sec)	N/A	0.299	0.025	0.553	0.215	0.243	0.160	0.262	0.068

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	51	51	51	50	25
N.S.	1	1.00	1.00	0.96	2.04	2.04	2.04	2.00	1.00
time (sec)	N/A	0.265	0.013	0.557	0.214	0.246	0.178	0.267	4.568

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	47	31	27	65	65	70	42	29
N.S.	1	0.90	0.60	0.52	1.25	1.25	1.35	0.81	0.56
time (sec)	N/A	0.299	0.023	0.540	0.203	0.234	0.201	0.265	4.442

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	48	31	27	77	77	80	42	29
N.S.	1	0.91	0.58	0.51	1.45	1.45	1.51	0.79	0.55
time (sec)	N/A	0.302	0.023	0.542	0.206	0.243	0.258	0.259	4.527

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	76	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	130	72	120	221	114	0	109	176
N.S.	1	1.02	0.57	0.94	1.74	0.90	0.00	0.86	1.39
time (sec)	N/A	0.556	0.240	0.423	0.200	0.244	0.000	0.274	0.091

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	102	64	112	181	104	0	90	140
N.S.	1	1.02	0.64	1.12	1.81	1.04	0.00	0.90	1.40
time (sec)	N/A	0.405	0.166	0.419	0.200	0.251	0.000	0.282	4.089

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	70	53	99	135	81	0	68	96
N.S.	1	1.08	0.82	1.52	2.08	1.25	0.00	1.05	1.48
time (sec)	N/A	0.290	0.114	0.122	0.207	0.249	0.000	0.276	0.067

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	34	76	55	47	0	33	24
N.S.	1	1.00	1.48	3.30	2.39	2.04	0.00	1.43	1.04
time (sec)	N/A	0.223	0.117	0.426	0.209	0.247	0.000	0.282	4.475

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	36	23	39	0	0	23
N.S.	1	1.00	0.96	1.29	0.82	1.39	0.00	0.00	0.82
time (sec)	N/A	0.223	0.132	0.426	0.202	0.241	0.000	0.000	4.055

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	66	34	41	39	57	0	45	38
N.S.	1	1.06	0.55	0.66	0.63	0.92	0.00	0.73	0.61
time (sec)	N/A	0.248	0.145	0.435	0.199	0.241	0.000	0.291	0.036

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	130	43	50	55	77	0	65	55
N.S.	1	1.37	0.45	0.53	0.58	0.81	0.00	0.68	0.58
time (sec)	N/A	0.329	0.144	0.425	0.212	0.254	0.000	0.304	0.045

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	167	51	58	71	95	0	85	71
N.S.	1	1.30	0.40	0.45	0.55	0.74	0.00	0.66	0.55
time (sec)	N/A	0.360	0.157	0.420	0.209	0.246	0.000	0.331	0.048

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	78	56	50	63	68	68	75	63
N.S.	1	0.86	0.62	0.55	0.69	0.75	0.75	0.82	0.69
time (sec)	N/A	0.323	0.025	0.498	0.200	0.237	0.099	0.256	4.192

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	63	48	42	52	57	56	64	52
N.S.	1	0.86	0.66	0.58	0.71	0.78	0.77	0.88	0.71
time (sec)	N/A	0.310	0.022	0.524	0.200	0.238	0.089	0.279	0.040

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	47	39	34	41	45	41	52	41
N.S.	1	0.85	0.71	0.62	0.75	0.82	0.75	0.95	0.75
time (sec)	N/A	0.300	0.020	0.486	0.211	0.256	0.076	0.252	0.051

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	24	35	26
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.92	1.35	1.00
time (sec)	N/A	0.270	0.013	0.421	0.191	0.241	0.075	0.262	0.044

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	15	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.07	1.00
time (sec)	N/A	0.259	0.012	0.475	0.200	0.245	0.030	0.270	0.040

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	24	29	23	20	25	12
N.S.	1	1.00	1.00	2.00	2.42	1.92	1.67	2.08	1.00
time (sec)	N/A	0.261	0.012	0.477	0.208	0.239	0.096	0.266	0.062

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	32	40	48	46	39	46	31
N.S.	1	0.97	0.97	1.21	1.45	1.39	1.18	1.39	0.94
time (sec)	N/A	0.293	0.026	0.498	0.216	0.248	0.153	0.256	0.072

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	47	35	51	63	76	54	51	46
N.S.	1	0.92	0.69	1.00	1.24	1.49	1.06	1.00	0.90
time (sec)	N/A	0.303	0.032	0.500	0.202	0.255	0.182	0.253	0.077

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	62	44	57	84	113	78	89	65
N.S.	1	0.90	0.64	0.83	1.22	1.64	1.13	1.29	0.94
time (sec)	N/A	0.320	0.037	0.490	0.211	0.236	0.258	0.252	4.427

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	96	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	0.079	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	86	157	244	114	0	0	199
N.S.	1	1.00	0.57	1.03	1.61	0.75	0.00	0.00	1.31
time (sec)	N/A	0.695	0.286	0.422	0.202	0.252	0.000	0.000	0.094

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	131	78	148	204	104	0	0	163
N.S.	1	1.02	0.60	1.15	1.58	0.81	0.00	0.00	1.26
time (sec)	N/A	0.556	0.219	0.412	0.205	0.251	0.000	0.000	4.544

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	99	68	136	156	81	0	0	117
N.S.	1	1.08	0.74	1.48	1.70	0.88	0.00	0.00	1.27
time (sec)	N/A	0.409	0.182	0.129	0.207	0.245	0.000	0.000	0.084

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	52	54	248	78	63	0	0	48
N.S.	1	0.98	1.02	4.68	1.47	1.19	0.00	0.00	0.91
time (sec)	N/A	0.274	0.169	0.413	0.219	0.244	0.000	0.000	4.202

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	22	22	0	0	22
N.S.	1	1.00	0.93	0.89	0.79	0.79	0.00	0.00	0.79
time (sec)	N/A	0.222	0.145	0.425	0.197	0.235	0.000	0.000	0.032

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	33	30	48	31	0	22	38
N.S.	1	1.00	1.57	1.43	2.29	1.48	0.00	1.05	1.81
time (sec)	N/A	0.215	0.156	0.424	0.203	0.249	0.000	0.279	4.202

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	45	65	58	0	0	50
N.S.	1	1.00	0.82	0.74	1.07	0.95	0.00	0.00	0.82
time (sec)	N/A	0.259	0.163	0.411	0.206	0.247	0.000	0.000	4.545

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	57	54	82	77	0	0	51
N.S.	1	1.00	0.61	0.57	0.87	0.82	0.00	0.00	0.54
time (sec)	N/A	0.391	0.172	0.408	0.200	0.248	0.000	0.000	4.154

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	129	66	63	97	96	0	0	60
N.S.	1	1.03	0.53	0.50	0.78	0.77	0.00	0.00	0.48
time (sec)	N/A	0.566	0.173	0.451	0.203	0.240	0.000	0.000	0.076

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	195	86	69	99	105	0	147	76
N.S.	1	0.77	0.34	0.27	0.39	0.41	0.00	0.58	0.30
time (sec)	N/A	0.313	0.076	0.411	0.223	0.244	0.000	0.290	4.528

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	157	78	61	83	94	0	0	102
N.S.	1	0.80	0.40	0.31	0.42	0.48	0.00	0.00	0.52
time (sec)	N/A	0.297	0.062	0.418	0.218	0.244	0.000	0.000	4.446

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	119	70	53	67	83	0	97	60
N.S.	1	1.03	0.61	0.46	0.58	0.72	0.00	0.84	0.52
time (sec)	N/A	0.262	0.054	0.409	0.214	0.259	0.000	0.286	4.397

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	87	57	43	45	64	0	0	50
N.S.	1	1.13	0.74	0.56	0.58	0.83	0.00	0.00	0.65
time (sec)	N/A	0.251	0.036	0.415	0.214	0.253	0.000	0.000	4.320

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	26	50	0	49	43
N.S.	1	1.00	1.48	1.21	0.90	1.72	0.00	1.69	1.48
time (sec)	N/A	0.208	0.025	0.407	0.211	0.248	0.000	0.269	4.388

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	105	99	83	0	239	0	94	0
N.S.	1	0.89	0.84	0.70	0.00	2.03	0.00	0.80	0.00
time (sec)	N/A	0.272	0.072	0.445	0.000	0.258	0.000	0.272	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	114	116	118	0	281	0	60	0
N.S.	1	0.89	0.91	0.92	0.00	2.20	0.00	0.47	0.00
time (sec)	N/A	0.272	0.079	0.446	0.000	0.254	0.000	0.295	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	154	123	167	0	337	0	78	0
N.S.	1	0.80	0.64	0.87	0.00	1.75	0.00	0.40	0.00
time (sec)	N/A	0.298	0.174	0.438	0.000	0.255	0.000	0.306	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	190	139	219	0	393	0	105	0
N.S.	1	0.76	0.56	0.88	0.00	1.57	0.00	0.42	0.00
time (sec)	N/A	0.317	0.176	0.426	0.000	0.272	0.000	0.325	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	46	34	21	32	60	58	205	32
N.S.	1	1.15	0.85	0.52	0.80	1.50	1.45	5.12	0.80
time (sec)	N/A	0.321	0.049	0.501	0.198	0.245	2.210	0.257	4.171

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	46	34	21	32	49	58	141	32
N.S.	1	1.15	0.85	0.52	0.80	1.22	1.45	3.52	0.80
time (sec)	N/A	0.319	0.047	0.523	0.194	0.245	2.139	0.261	0.037

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	46	30	21	32	32	58	71	32
N.S.	1	1.15	0.75	0.52	0.80	0.80	1.45	1.78	0.80
time (sec)	N/A	0.319	0.039	0.521	0.195	0.248	2.023	0.260	0.038

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	44	23	20	30	19	56	44	32
N.S.	1	1.16	0.61	0.53	0.79	0.50	1.47	1.16	0.84
time (sec)	N/A	0.309	0.031	0.483	0.206	0.244	2.057	0.263	0.034

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	42	21	20	30	29	54	32	19
N.S.	1	1.17	0.58	0.56	0.83	0.81	1.50	0.89	0.53
time (sec)	N/A	0.312	0.028	0.509	0.206	0.247	1.721	0.265	4.162

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	44	34	21	26	44	56	36	20
N.S.	1	1.16	0.89	0.55	0.68	1.16	1.47	0.95	0.53
time (sec)	N/A	0.325	0.064	0.518	0.197	0.248	1.752	0.259	0.036

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	46	34	21	24	56	58	34	20
N.S.	1	1.15	0.85	0.52	0.60	1.40	1.45	0.85	0.50
time (sec)	N/A	0.316	0.070	0.484	0.213	0.239	1.736	0.265	4.029

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	46	34	21	26	66	58	36	20
N.S.	1	1.15	0.85	0.52	0.65	1.65	1.45	0.90	0.50
time (sec)	N/A	0.316	0.071	0.508	0.196	0.250	1.924	0.270	3.934

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	157	77	64	106	105	0	130	110
N.S.	1	0.80	0.39	0.32	0.54	0.53	0.00	0.66	0.56
time (sec)	N/A	0.289	0.058	0.421	0.253	0.257	0.000	0.290	4.305

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	119	69	56	90	94	0	0	102
N.S.	1	0.87	0.50	0.41	0.66	0.69	0.00	0.00	0.74
time (sec)	N/A	0.272	0.048	0.429	0.216	0.240	0.000	0.000	4.296

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	87	59	48	74	83	0	80	93
N.S.	1	0.98	0.66	0.54	0.83	0.93	0.00	0.90	1.04
time (sec)	N/A	0.248	0.045	0.422	0.240	0.244	0.000	0.284	4.230

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	43	35	41	61	0	0	81
N.S.	1	1.00	1.39	1.13	1.32	1.97	0.00	0.00	2.61
time (sec)	N/A	0.214	0.040	0.418	0.279	0.245	0.000	0.000	4.679

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	138	105	107	0	250	0	0	0
N.S.	1	0.85	0.64	0.66	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.284	0.076	0.452	0.000	0.268	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	148	116	136	0	288	0	75	0
N.S.	1	0.84	0.66	0.77	0.00	1.63	0.00	0.42	0.00
time (sec)	N/A	0.289	0.159	0.452	0.000	0.265	0.000	0.305	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	158	125	174	0	341	0	78	0
N.S.	1	0.84	0.67	0.93	0.00	1.82	0.00	0.42	0.00
time (sec)	N/A	0.299	0.165	0.426	0.000	0.266	0.000	0.303	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	196	142	226	0	393	0	105	0
N.S.	1	0.78	0.57	0.90	0.00	1.57	0.00	0.42	0.00
time (sec)	N/A	0.324	0.160	0.434	0.000	0.254	0.000	0.304	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	234	147	278	0	449	0	129	0
N.S.	1	0.76	0.48	0.91	0.00	1.46	0.00	0.42	0.00
time (sec)	N/A	0.336	0.220	0.442	0.000	0.273	0.000	0.339	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	233	86	77	128	105	0	0	110
N.S.	1	1.20	0.44	0.40	0.66	0.54	0.00	0.00	0.57
time (sec)	N/A	0.323	0.074	0.421	0.242	0.248	0.000	0.000	4.323

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	195	78	69	112	94	0	0	102
N.S.	1	1.21	0.48	0.43	0.70	0.58	0.00	0.00	0.63
time (sec)	N/A	0.305	0.060	0.429	0.222	0.248	0.000	0.000	4.345

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	157	70	61	96	83	0	0	94
N.S.	1	1.23	0.55	0.48	0.75	0.65	0.00	0.00	0.73
time (sec)	N/A	0.288	0.053	0.429	0.230	0.246	0.000	0.000	4.299

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	119	60	53	72	64	0	0	82
N.S.	1	1.25	0.63	0.56	0.76	0.67	0.00	0.00	0.86
time (sec)	N/A	0.266	0.047	0.408	0.232	0.257	0.000	0.000	4.398

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	87	50	42	54	50	0	43	71
N.S.	1	1.40	0.81	0.68	0.87	0.81	0.00	0.69	1.15
time (sec)	N/A	0.244	0.037	0.434	0.216	0.246	0.000	0.274	4.413

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	35	29	44	0	0	34
N.S.	1	1.00	0.97	1.21	1.00	1.52	0.00	0.00	1.17
time (sec)	N/A	0.211	0.028	0.412	0.214	0.253	0.000	0.000	3.990

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	78	0	141	0	64	0
N.S.	1	1.00	1.00	1.03	0.00	1.86	0.00	0.84	0.00
time (sec)	N/A	0.249	0.056	0.451	0.000	0.254	0.000	0.277	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	117	116	123	0	281	0	0	0
N.S.	1	0.86	0.85	0.90	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.275	0.087	0.433	0.000	0.259	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	155	125	172	0	341	0	88	0
N.S.	1	0.80	0.65	0.89	0.00	1.77	0.00	0.46	0.00
time (sec)	N/A	0.295	0.121	0.435	0.000	0.260	0.000	0.301	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	148	88	78	123	204	150	161	112
N.S.	1	1.08	0.64	0.57	0.90	1.49	1.09	1.18	0.82
time (sec)	N/A	0.388	0.089	0.574	0.296	0.248	2.900	0.269	0.094

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	126	80	71	109	182	133	134	95
N.S.	1	1.09	0.69	0.61	0.94	1.57	1.15	1.16	0.82
time (sec)	N/A	0.353	0.070	0.566	0.304	0.268	2.749	0.267	0.072

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	104	71	61	95	146	116	107	78
N.S.	1	1.09	0.75	0.64	1.00	1.54	1.22	1.13	0.82
time (sec)	N/A	0.343	0.052	0.556	0.295	0.266	2.612	0.271	4.072

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	82	61	57	79	119	99	77	61
N.S.	1	1.08	0.80	0.75	1.04	1.57	1.30	1.01	0.80
time (sec)	N/A	0.321	0.038	0.533	0.291	0.255	2.577	0.269	4.517

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	60	58	45	68	118	82	51	47
N.S.	1	1.03	1.00	0.78	1.17	2.03	1.41	0.88	0.81
time (sec)	N/A	0.318	0.031	0.546	0.299	0.248	1.734	0.267	4.021

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	29	52	88	66	36	28
N.S.	1	1.00	1.00	0.78	1.41	2.38	1.78	0.97	0.76
time (sec)	N/A	0.309	0.022	0.494	0.300	0.260	1.839	0.266	0.095

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	62	37	50	71	146	83	54	47
N.S.	1	1.09	0.65	0.88	1.25	2.56	1.46	0.95	0.82
time (sec)	N/A	0.320	0.029	0.513	0.287	0.264	1.970	0.260	4.259

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	91	39	64	81	196	104	73	65
N.S.	1	1.10	0.47	0.77	0.98	2.36	1.25	0.88	0.78
time (sec)	N/A	0.332	0.031	0.532	0.322	0.256	2.241	0.263	0.105

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	120	39	78	101	252	121	93	79
N.S.	1	1.15	0.38	0.75	0.97	2.42	1.16	0.89	0.76
time (sec)	N/A	0.349	0.036	0.550	0.293	0.261	2.466	0.266	0.102

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	267	84	88	152	105	0	0	110
N.S.	1	0.73	0.23	0.24	0.41	0.29	0.00	0.00	0.30
time (sec)	N/A	0.346	0.060	0.463	0.230	0.271	0.000	0.000	4.466

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	229	76	80	136	94	0	0	102
N.S.	1	0.74	0.24	0.26	0.44	0.30	0.00	0.00	0.33
time (sec)	N/A	0.329	0.053	0.448	0.220	0.245	0.000	0.000	4.854

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	191	68	72	120	83	0	0	94
N.S.	1	0.75	0.27	0.28	0.47	0.33	0.00	0.00	0.37
time (sec)	N/A	0.311	0.046	0.467	0.216	0.256	0.000	0.000	4.462

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	153	57	63	93	63	0	0	81
N.S.	1	0.78	0.29	0.32	0.48	0.32	0.00	0.00	0.42
time (sec)	N/A	0.287	0.040	0.482	0.219	0.243	0.000	0.000	4.418

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	115	48	55	75	50	0	0	71
N.S.	1	0.84	0.35	0.40	0.55	0.36	0.00	0.00	0.52
time (sec)	N/A	0.264	0.031	0.464	0.223	0.251	0.000	0.000	4.266

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	83	48	47	48	44	0	42	34
N.S.	1	0.98	0.56	0.55	0.56	0.52	0.00	0.49	0.40
time (sec)	N/A	0.242	0.035	0.460	0.220	0.251	0.000	0.276	4.406

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	41	35	45	43	0	41	32
N.S.	1	1.00	1.41	1.21	1.55	1.48	0.00	1.41	1.10
time (sec)	N/A	0.212	0.033	0.451	0.222	0.257	0.000	0.272	4.234

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	101	122	85	0	235	0	0	0
N.S.	1	0.84	1.02	0.71	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.265	0.096	0.461	0.000	0.265	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	141	130	129	0	285	0	90	0
N.S.	1	0.77	0.71	0.70	0.00	1.55	0.00	0.49	0.00
time (sec)	N/A	0.290	0.315	0.467	0.000	0.282	0.000	0.299	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	112	41	62	110	57	0	59	94
N.S.	1	1.13	0.41	0.63	1.11	0.58	0.00	0.60	0.95
time (sec)	N/A	0.254	0.043	0.127	0.212	0.250	0.000	0.259	0.089

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	80	40	57	87	54	0	48	68
N.S.	1	1.01	0.51	0.72	1.10	0.68	0.00	0.61	0.86
time (sec)	N/A	0.226	0.029	0.098	0.223	0.245	0.000	0.268	0.043

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	21	22	50	24	0	15	18
N.S.	1	1.00	1.17	1.22	2.78	1.33	0.00	0.83	1.00
time (sec)	N/A	0.197	0.025	0.039	0.203	0.239	0.000	0.254	4.448

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	31	39	48	83	51	0	38	63
N.S.	1	0.89	1.11	1.37	2.37	1.46	0.00	1.09	1.80
time (sec)	N/A	0.206	0.023	0.094	0.208	0.243	0.000	0.266	4.011

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	144	52	70	138	66	0	71	118
N.S.	1	1.08	0.39	0.53	1.04	0.50	0.00	0.53	0.89
time (sec)	N/A	0.283	0.048	0.443	0.210	0.248	0.000	0.271	0.049

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	112	47	65	112	61	0	60	94
N.S.	1	1.06	0.44	0.61	1.06	0.58	0.00	0.57	0.89
time (sec)	N/A	0.240	0.035	0.443	0.233	0.250	0.000	0.266	0.049

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	52	70	138	66	0	72	118
N.S.	1	1.04	0.73	0.99	1.94	0.93	0.00	1.01	1.66
time (sec)	N/A	0.269	0.042	0.462	0.214	0.257	0.000	0.265	4.030

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	50	47	60	112	61	0	60	90
N.S.	1	0.94	0.89	1.13	2.11	1.15	0.00	1.13	1.70
time (sec)	N/A	0.238	0.032	0.478	0.205	0.254	0.000	0.270	4.274

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	15	16	26	15	0	14	26
N.S.	1	1.00	0.68	0.73	1.18	0.68	0.00	0.64	1.18
time (sec)	N/A	0.212	0.025	0.430	0.218	0.236	0.000	0.270	0.060

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	35	31	31	26	21	14
N.S.	1	1.00	0.82	1.59	1.41	1.41	1.18	0.95	0.64
time (sec)	N/A	0.200	0.016	0.438	0.199	0.245	2.091	0.264	0.033

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	51	41	64	74	66	0	65	43
N.S.	1	1.09	0.87	1.36	1.57	1.40	0.00	1.38	0.91
time (sec)	N/A	0.261	0.076	0.467	0.208	0.251	0.000	0.272	0.052

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	37	38	52	44	61	0	49	28
N.S.	1	1.12	1.15	1.58	1.33	1.85	0.00	1.48	0.85
time (sec)	N/A	0.233	0.045	0.459	0.257	0.253	0.000	0.267	4.249

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	36	56	44	44	0	44	28
N.S.	1	1.00	0.80	1.24	0.98	0.98	0.00	0.98	0.62
time (sec)	N/A	0.223	0.047	0.440	0.260	0.253	0.000	0.275	0.033

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	12	11	11	8	22	11
N.S.	1	1.00	0.86	0.57	0.52	0.52	0.38	1.05	0.52
time (sec)	N/A	0.187	0.016	0.437	0.213	0.239	3.339	0.275	0.190

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	43	64	56	84	0	79	40
N.S.	1	1.09	0.78	1.16	1.02	1.53	0.00	1.44	0.73
time (sec)	N/A	0.283	0.062	0.482	0.209	0.239	0.000	0.272	4.623

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	13	31	0	46	13
N.S.	1	1.00	1.00	0.92	0.54	1.29	0.00	1.92	0.54
time (sec)	N/A	0.202	0.020	0.464	0.223	0.266	0.000	0.264	0.025

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	118	67	49	55	69	0	0	57
N.S.	1	0.84	0.48	0.35	0.39	0.49	0.00	0.00	0.41
time (sec)	N/A	0.300	0.050	0.444	0.240	0.243	0.000	0.000	4.653

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	86	56	41	41	61	0	81	49
N.S.	1	0.93	0.61	0.45	0.45	0.66	0.00	0.88	0.53
time (sec)	N/A	0.259	0.032	0.454	0.240	0.250	0.000	0.274	4.585

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	26	50	0	49	43
N.S.	1	1.00	1.48	1.21	0.90	1.72	0.00	1.69	1.48
time (sec)	N/A	0.205	0.014	0.432	0.257	0.246	0.000	0.267	0.003

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	78	75	70	0	207	0	88	0
N.S.	1	0.83	0.80	0.74	0.00	2.20	0.00	0.94	0.00
time (sec)	N/A	0.284	0.050	0.472	0.000	0.266	0.000	0.276	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	76	76	78	0	229	0	101	0
N.S.	1	0.78	0.78	0.80	0.00	2.36	0.00	1.04	0.00
time (sec)	N/A	0.284	0.045	0.468	0.000	0.280	0.000	0.289	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	107	48	45	74	44	134	189	83
N.S.	1	1.06	0.48	0.45	0.73	0.44	1.33	1.87	0.82
time (sec)	N/A	0.465	0.080	0.539	0.219	0.242	2.176	0.271	0.047

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	40	37	60	36	109	142	66
N.S.	1	1.08	0.50	0.46	0.75	0.45	1.36	1.78	0.82
time (sec)	N/A	0.447	0.064	0.545	0.214	0.243	2.139	0.265	0.059

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	63	31	28	44	27	80	92	46
N.S.	1	1.11	0.54	0.49	0.77	0.47	1.40	1.61	0.81
time (sec)	N/A	0.376	0.056	0.549	0.212	0.245	2.532	0.252	0.051

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	44	23	20	30	19	56	44	32
N.S.	1	1.16	0.61	0.53	0.79	0.50	1.47	1.16	0.84
time (sec)	N/A	0.309	0.010	0.566	0.220	0.247	1.987	0.261	0.002

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	45	39	32	49	82	80	40	31
N.S.	1	1.15	1.00	0.82	1.26	2.10	2.05	1.03	0.79
time (sec)	N/A	0.400	0.036	0.510	0.298	0.254	3.959	0.273	4.096

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	49	42	43	62	97	0	48	34
N.S.	1	1.17	1.00	1.02	1.48	2.31	0.00	1.14	0.81
time (sec)	N/A	0.397	0.035	0.517	0.313	0.260	0.000	0.257	0.067

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	76	55	52	103	117	0	76	54
N.S.	1	1.12	0.81	0.76	1.51	1.72	0.00	1.12	0.79
time (sec)	N/A	0.406	0.055	0.541	0.294	0.260	0.000	0.267	4.501

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	103	63	62	134	133	0	104	74
N.S.	1	1.16	0.71	0.70	1.51	1.49	0.00	1.17	0.83
time (sec)	N/A	0.419	0.067	0.578	0.294	0.276	0.000	0.265	4.583

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	130	71	70	163	149	0	131	91
N.S.	1	1.18	0.65	0.64	1.48	1.35	0.00	1.19	0.83
time (sec)	N/A	0.439	0.077	0.542	0.293	0.257	0.000	0.256	0.077

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	233	130	146	0	304	0	0	0
N.S.	1	0.75	0.42	0.47	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.404	0.132	0.451	0.000	0.256	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	197	122	138	0	288	0	145	0
N.S.	1	0.75	0.47	0.53	0.00	1.10	0.00	0.56	0.00
time (sec)	N/A	0.401	0.103	0.456	0.000	0.270	0.000	0.294	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	170	114	125	0	272	0	0	0
N.S.	1	0.81	0.54	0.59	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.330	0.084	0.457	0.000	0.261	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	138	105	107	0	250	0	0	0
N.S.	1	0.85	0.64	0.66	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.290	0.047	0.445	0.000	0.262	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	131	120	107	0	352	0	0	0
N.S.	1	0.77	0.71	0.63	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.351	0.073	0.445	0.000	0.266	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	134	120	117	0	390	0	0	0
N.S.	1	0.78	0.70	0.68	0.00	2.27	0.00	0.00	0.00
time (sec)	N/A	0.348	0.065	0.471	0.000	0.275	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	157	132	144	0	428	0	0	0
N.S.	1	0.70	0.59	0.64	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.373	0.175	0.464	0.000	0.274	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	188	140	165	0	444	0	0	0
N.S.	1	0.69	0.51	0.60	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.401	0.229	0.493	0.000	0.272	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	219	148	181	0	460	0	0	0
N.S.	1	0.68	0.46	0.56	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.431	0.294	0.487	0.000	0.279	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	120	46	37	27	33	0	0	48
N.S.	1	0.83	0.32	0.26	0.19	0.23	0.00	0.00	0.33
time (sec)	N/A	0.259	0.035	0.446	0.231	0.237	0.000	0.000	4.348

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	97	41	32	22	28	70	0	38
N.S.	1	0.91	0.38	0.30	0.21	0.26	0.65	0.00	0.36
time (sec)	N/A	0.234	0.025	0.431	0.221	0.249	42.547	0.000	4.268

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	95	48	34	22	45	0	63	35
N.S.	1	0.91	0.46	0.33	0.21	0.43	0.00	0.61	0.34
time (sec)	N/A	0.245	0.033	0.441	0.207	0.252	0.000	0.269	4.251

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	70	43	29	17	40	0	49	30
N.S.	1	1.03	0.63	0.43	0.25	0.59	0.00	0.72	0.44
time (sec)	N/A	0.214	0.025	0.448	0.217	0.256	0.000	0.262	4.242

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	97	39	30	20	26	0	0	38
N.S.	1	0.91	0.36	0.28	0.19	0.24	0.00	0.00	0.36
time (sec)	N/A	0.237	0.023	0.426	0.214	0.234	0.000	0.000	4.105

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	34	25	15	21	56	0	21
N.S.	1	1.00	0.49	0.36	0.21	0.30	0.80	0.00	0.30
time (sec)	N/A	0.212	0.019	0.448	0.203	0.249	2.948	0.000	4.114

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	70	41	29	17	40	0	49	30
N.S.	1	0.99	0.58	0.41	0.24	0.56	0.00	0.69	0.42
time (sec)	N/A	0.226	0.023	0.450	0.211	0.249	0.000	0.272	4.203

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	34	24	12	33	0	33	25
N.S.	1	1.00	1.70	1.20	0.60	1.65	0.00	1.65	1.25
time (sec)	N/A	0.191	0.017	0.457	0.202	0.250	0.000	0.262	4.522

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	70	26	25	13	21	48	0	21
N.S.	1	0.96	0.36	0.34	0.18	0.29	0.66	0.00	0.29
time (sec)	N/A	0.229	0.021	0.443	0.205	0.246	6.502	0.000	4.160

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	30	21	22	7	18	19	23	18
N.S.	1	0.91	0.64	0.67	0.21	0.55	0.58	0.70	0.55
time (sec)	N/A	0.206	0.013	0.444	0.221	0.264	6.478	0.270	4.129

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	98	69	66	0	72	0	0	0
N.S.	1	0.78	0.55	0.52	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.256	0.055	0.460	0.000	0.244	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	78	63	55	0	66	58	0	0
N.S.	1	0.87	0.70	0.61	0.00	0.73	0.64	0.00	0.00
time (sec)	N/A	0.228	0.035	0.455	0.000	0.248	13.994	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	78	65	47	0	46	0	0	0
N.S.	1	0.84	0.70	0.51	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.245	0.048	0.453	0.000	0.250	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	58	55	41	37	0	26	42	0	0
N.S.	1	0.95	0.71	0.64	0.00	0.45	0.72	0.00	0.00
time (sec)	N/A	0.217	0.024	0.442	0.000	0.253	40.258	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	112	75	90	0	84	126	41	0
N.S.	1	0.86	0.58	0.69	0.00	0.65	0.97	0.32	0.00
time (sec)	N/A	0.265	0.098	0.451	0.000	0.249	101.000	0.282	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	85	69	79	0	76	70	32	0
N.S.	1	0.94	0.77	0.88	0.00	0.84	0.78	0.36	0.00
time (sec)	N/A	0.233	0.061	0.468	0.000	0.245	97.850	0.289	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	135	102	0	0	0	0	0	0
N.S.	1	1.03	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	151	67	59	83	69	0	0	88
N.S.	1	1.06	0.47	0.42	0.58	0.49	0.00	0.00	0.62
time (sec)	N/A	0.333	0.052	0.457	0.213	0.241	0.000	0.000	4.539

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	119	58	50	69	60	0	69	57
N.S.	1	1.14	0.56	0.48	0.66	0.58	0.00	0.66	0.55
time (sec)	N/A	0.286	0.045	0.441	0.205	0.257	0.000	0.274	4.510

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	87	50	42	54	50	0	43	71
N.S.	1	1.40	0.81	0.68	0.87	0.81	0.00	0.69	1.15
time (sec)	N/A	0.242	0.026	0.438	0.201	0.246	0.000	0.271	0.003

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	82	74	80	0	206	0	40	0
N.S.	1	0.87	0.79	0.85	0.00	2.19	0.00	0.43	0.00
time (sec)	N/A	0.293	0.055	0.453	0.000	0.259	0.000	0.274	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	82	76	90	0	232	0	48	0
N.S.	1	0.85	0.79	0.94	0.00	2.42	0.00	0.50	0.00
time (sec)	N/A	0.298	0.047	0.464	0.000	0.256	0.000	0.271	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	142	85	75	123	168	175	159	114
N.S.	1	1.02	0.61	0.54	0.88	1.21	1.26	1.14	0.82
time (sec)	N/A	0.511	0.142	0.599	0.283	0.254	4.136	0.269	0.095

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	100	78	68	97	153	134	105	80
N.S.	1	1.03	0.80	0.70	1.00	1.58	1.38	1.08	0.82
time (sec)	N/A	0.486	0.100	0.554	0.296	0.255	3.619	0.260	4.221

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	109	70	59	95	136	122	105	80
N.S.	1	1.12	0.72	0.61	0.98	1.40	1.26	1.08	0.82
time (sec)	N/A	0.394	0.107	0.546	0.273	0.270	3.504	0.276	0.105

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	82	61	57	79	119	99	77	61
N.S.	1	1.08	0.80	0.75	1.04	1.57	1.30	1.01	0.80
time (sec)	N/A	0.327	0.024	0.527	0.277	0.248	2.632	0.263	0.003

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	85	74	58	98	157	122	67	57
N.S.	1	1.15	1.00	0.78	1.32	2.12	1.65	0.91	0.77
time (sec)	N/A	0.447	0.039	0.582	0.286	0.253	5.327	0.272	0.103

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	92	78	70	111	176	0	71	61
N.S.	1	1.18	1.00	0.90	1.42	2.26	0.00	0.91	0.78
time (sec)	N/A	0.463	0.046	0.552	0.280	0.257	0.000	0.274	4.100

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	119	93	80	152	204	0	106	88
N.S.	1	1.12	0.88	0.75	1.43	1.92	0.00	1.00	0.83
time (sec)	N/A	0.496	0.087	0.625	0.287	0.266	0.000	0.267	4.020

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	146	101	89	183	220	0	133	105
N.S.	1	1.15	0.80	0.70	1.44	1.73	0.00	1.05	0.83
time (sec)	N/A	0.547	0.105	0.590	0.283	0.263	0.000	0.276	0.144

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	173	109	97	212	236	0	160	122
N.S.	1	1.17	0.74	0.66	1.43	1.59	0.00	1.08	0.82
time (sec)	N/A	0.559	0.123	0.614	0.300	0.269	0.000	0.275	4.064

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	204	73	80	117	77	0	0	74
N.S.	1	0.73	0.26	0.28	0.42	0.27	0.00	0.00	0.26
time (sec)	N/A	0.379	0.053	0.454	0.211	0.251	0.000	0.000	4.275

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	176	65	72	104	69	0	0	88
N.S.	1	0.76	0.28	0.31	0.45	0.30	0.00	0.00	0.38
time (sec)	N/A	0.356	0.042	0.462	0.235	0.252	0.000	0.000	4.577

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	144	57	64	91	61	0	0	58
N.S.	1	0.79	0.31	0.35	0.50	0.34	0.00	0.00	0.32
time (sec)	N/A	0.313	0.038	0.471	0.244	0.252	0.000	0.000	4.106

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	115	48	55	75	50	0	0	71
N.S.	1	0.84	0.35	0.40	0.55	0.36	0.00	0.00	0.52
time (sec)	N/A	0.267	0.019	0.466	0.200	0.261	0.000	0.000	0.002

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	107	78	80	0	207	0	0	0
N.S.	1	0.76	0.56	0.57	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.317	0.132	0.472	0.000	0.270	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	111	79	86	0	233	0	0	0
N.S.	1	0.79	0.56	0.61	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.328	0.092	0.487	0.000	0.271	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	141	90	103	0	262	0	0	0
N.S.	1	0.74	0.47	0.54	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.332	0.095	0.484	0.000	0.265	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	172	98	114	0	278	0	0	0
N.S.	1	0.72	0.41	0.48	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.341	0.100	0.470	0.000	0.259	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	203	106	125	0	294	0	0	0
N.S.	1	0.71	0.37	0.44	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.367	0.110	0.492	0.000	0.265	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	226	116	104	122	185	0	0	223
N.S.	1	0.81	0.42	0.37	0.44	0.67	0.00	0.00	0.80
time (sec)	N/A	0.372	0.087	1.574	0.216	0.259	0.000	0.000	4.594

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	129	78	61	68	93	0	0	140
N.S.	1	1.02	0.61	0.48	0.54	0.73	0.00	0.00	1.10
time (sec)	N/A	0.290	0.057	1.157	0.216	0.262	0.000	0.000	4.357

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	58	34	37	44	0	0	55
N.S.	1	1.00	1.61	0.94	1.03	1.22	0.00	0.00	1.53
time (sec)	N/A	0.213	0.022	1.142	0.203	0.262	0.000	0.000	4.291

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	96	78	0	0	0	0	0	0
N.S.	1	1.20	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	105	89	0	0	0	0	0	0
N.S.	1	1.19	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	1.923	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	0	0	0	0
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	1.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.586	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	87	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.309	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	33	33	0	58	187	0	32
N.S.	1	1.00	0.69	0.69	0.00	1.21	3.90	0.00	0.67
time (sec)	N/A	0.232	0.285	1.757	0.000	0.254	10.168	0.000	4.919

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	105	64	46	0	128	1112	0	113
N.S.	1	1.01	0.62	0.44	0.00	1.23	10.69	0.00	1.09
time (sec)	N/A	0.275	0.346	4.630	0.000	0.251	59.893	0.000	4.976

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	208	83	68	0	228	0	0	180
N.S.	1	0.93	0.37	0.30	0.00	1.02	0.00	0.00	0.80
time (sec)	N/A	0.368	0.408	12.144	0.000	0.255	0.000	0.000	4.697

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	103	0	0	0	0	0	0
N.S.	1	1.08	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.117	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	101	0	0	0	0	0	0
N.S.	1	1.08	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	106	98	0	0	0	0	0	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	104	96	0	0	0	0	0	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	104	94	0	0	0	0	0	0
N.S.	1	1.08	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	187	117	0	0	0	0	0	0
N.S.	1	1.12	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	249	138	0	0	0	0	0	0
N.S.	1	1.02	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	0.185	0.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	115	175	157	224	156	0	248	183
N.S.	1	1.01	1.54	1.38	1.96	1.37	0.00	2.18	1.61
time (sec)	N/A	0.451	0.217	0.091	0.284	0.252	0.000	0.286	4.266

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	167	140	201	146	0	221	163
N.S.	1	1.00	1.90	1.59	2.28	1.66	0.00	2.51	1.85
time (sec)	N/A	0.334	0.186	0.081	0.286	0.255	0.000	0.289	0.123

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	56	158	131	125	119	0	137	90
N.S.	1	0.90	2.55	2.11	2.02	1.92	0.00	2.21	1.45
time (sec)	N/A	0.294	0.197	0.084	0.288	0.254	0.000	0.279	4.698

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	34	31	63	66	48	0	42	60
N.S.	1	1.26	1.15	2.33	2.44	1.78	0.00	1.56	2.22
time (sec)	N/A	0.221	0.045	0.048	0.276	0.254	0.000	0.270	3.874

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	77	63	143	116	94	0	0	62
N.S.	1	1.10	0.90	2.04	1.66	1.34	0.00	0.00	0.89
time (sec)	N/A	0.322	0.140	0.147	0.201	0.252	0.000	0.000	3.790

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	104	94	185	137	134	0	0	104
N.S.	1	0.99	0.90	1.76	1.30	1.28	0.00	0.00	0.99
time (sec)	N/A	0.474	0.070	0.158	0.208	0.253	0.000	0.000	0.107

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	143	104	225	153	170	0	63	121
N.S.	1	1.04	0.75	1.63	1.11	1.23	0.00	0.46	0.88
time (sec)	N/A	0.629	0.086	0.171	0.197	0.263	0.000	0.295	3.811

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	178	112	265	169	204	0	0	137
N.S.	1	1.04	0.65	1.55	0.99	1.19	0.00	0.00	0.80
time (sec)	N/A	0.774	0.103	0.174	0.196	0.253	0.000	0.000	0.112

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	48	57	47	57	67	63	58	51
N.S.	1	0.79	0.93	0.77	0.93	1.10	1.03	0.95	0.84
time (sec)	N/A	0.426	0.088	0.637	0.197	0.251	0.186	0.288	0.079

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	34	41	32	37	43	39	38	35
N.S.	1	0.85	1.02	0.80	0.92	1.08	0.98	0.95	0.88
time (sec)	N/A	0.409	0.063	0.615	0.184	0.237	0.127	0.258	0.054

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	32	38	31	37	45	37	38	35
N.S.	1	0.82	0.97	0.79	0.95	1.15	0.95	0.97	0.90
time (sec)	N/A	0.402	0.058	0.593	0.192	0.247	0.128	0.261	4.275

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	20	16	17	16	21	15	16	19
N.S.	1	1.25	1.00	1.06	1.00	1.31	0.94	1.00	1.19
time (sec)	N/A	0.386	0.043	0.541	0.185	0.262	0.047	0.259	0.039

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	11
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	1.00
time (sec)	N/A	0.336	0.030	0.502	0.191	0.232	0.051	0.257	3.814

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	36	30	35	35	40	26	36	34
N.S.	1	0.97	0.81	0.95	0.95	1.08	0.70	0.97	0.92
time (sec)	N/A	0.398	0.037	0.494	0.196	0.238	0.106	0.259	0.064

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	52	52	47	55	70	49	42	54
N.S.	1	0.98	0.98	0.89	1.04	1.32	0.92	0.79	1.02
time (sec)	N/A	0.437	0.065	0.490	0.203	0.241	0.163	0.255	3.821

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	68	63	56	75	100	73	50	71
N.S.	1	0.93	0.86	0.77	1.03	1.37	1.00	0.68	0.97
time (sec)	N/A	0.450	0.079	0.489	0.192	0.244	0.235	0.285	3.840

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	81	71	67	93	126	94	58	90
N.S.	1	0.93	0.82	0.77	1.07	1.45	1.08	0.67	1.03
time (sec)	N/A	0.463	0.096	0.489	0.195	0.250	0.279	0.266	0.093

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	102	175	157	223	156	0	248	183
N.S.	1	0.99	1.70	1.52	2.17	1.51	0.00	2.41	1.78
time (sec)	N/A	0.348	0.265	0.092	0.281	0.254	0.000	0.298	0.136

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	63	51	105	151	85	0	75	119
N.S.	1	1.03	0.84	1.72	2.48	1.39	0.00	1.23	1.95
time (sec)	N/A	0.247	0.078	0.088	0.272	0.255	0.000	0.284	3.849

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	67	154	131	125	114	0	138	90
N.S.	1	1.06	2.44	2.08	1.98	1.81	0.00	2.19	1.43
time (sec)	N/A	0.333	0.174	0.087	0.277	0.250	0.000	0.304	3.813

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	55	73	145	114	88	0	91	82
N.S.	1	1.12	1.49	2.96	2.33	1.80	0.00	1.86	1.67
time (sec)	N/A	0.325	0.121	0.125	0.286	0.255	0.000	0.276	0.101

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	109	70	183	133	128	0	63	100
N.S.	1	1.04	0.67	1.74	1.27	1.22	0.00	0.60	0.95
time (sec)	N/A	0.486	0.176	0.162	0.195	0.252	0.000	0.277	3.760

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	143	104	225	153	170	0	63	120
N.S.	1	1.04	0.75	1.63	1.11	1.23	0.00	0.46	0.87
time (sec)	N/A	0.623	0.088	0.178	0.203	0.251	0.000	0.324	0.090

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	158	112	265	169	204	0	0	137
N.S.	1	0.96	0.68	1.61	1.02	1.24	0.00	0.00	0.83
time (sec)	N/A	0.762	0.097	0.180	0.221	0.252	0.000	0.000	4.232

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	213	120	305	185	240	0	63	153
N.S.	1	1.04	0.59	1.50	0.91	1.18	0.00	0.31	0.75
time (sec)	N/A	0.955	0.110	0.171	0.193	0.252	0.000	0.352	3.934

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	51	51	48	59	67	63	123	51
N.S.	1	0.80	0.80	0.75	0.92	1.05	0.98	1.92	0.80
time (sec)	N/A	0.422	0.083	0.755	0.186	0.250	0.214	0.265	3.883

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	28	28	27	31	36	31	59	27
N.S.	1	0.93	0.93	0.90	1.03	1.20	1.03	1.97	0.90
time (sec)	N/A	0.402	0.060	0.701	0.193	0.231	0.096	0.265	0.053

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	35	38	30	34	43	37	98	31
N.S.	1	0.92	1.00	0.79	0.89	1.13	0.97	2.58	0.82
time (sec)	N/A	0.405	0.057	0.673	0.199	0.243	0.110	0.258	3.882

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	23	23	24	27	32	26	94	25
N.S.	1	0.85	0.85	0.89	1.00	1.19	0.96	3.48	0.93
time (sec)	N/A	0.393	0.049	0.613	0.193	0.256	0.086	0.271	3.856

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	22	22	22	24	23	17	55	24
N.S.	1	0.88	0.88	0.88	0.96	0.92	0.68	2.20	0.96
time (sec)	N/A	0.364	0.041	0.604	0.209	0.252	0.154	0.262	0.074

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	51	51	43	49	64	41	74	48
N.S.	1	0.96	0.96	0.81	0.92	1.21	0.77	1.40	0.91
time (sec)	N/A	0.413	0.039	0.575	0.201	0.242	0.153	0.262	0.064

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	63	56	75	100	73	94	71
N.S.	1	0.93	0.89	0.79	1.06	1.41	1.03	1.32	1.00
time (sec)	N/A	0.449	0.072	0.587	0.181	0.248	0.195	0.272	3.860

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	82	71	67	93	126	94	109	90
N.S.	1	0.92	0.80	0.75	1.04	1.42	1.06	1.22	1.01
time (sec)	N/A	0.468	0.087	0.595	0.197	0.240	0.305	0.266	3.793

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	94	79	78	113	154	114	124	109
N.S.	1	0.90	0.75	0.74	1.08	1.47	1.09	1.18	1.04
time (sec)	N/A	0.484	0.100	0.592	0.195	0.244	0.425	0.272	3.859

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	142	175	157	223	156	0	265	185
N.S.	1	1.05	1.30	1.16	1.65	1.16	0.00	1.96	1.37
time (sec)	N/A	0.698	0.176	0.169	0.291	0.260	0.000	0.294	0.131

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	107	167	149	201	143	0	232	163
N.S.	1	1.01	1.58	1.41	1.90	1.35	0.00	2.19	1.54
time (sec)	N/A	0.550	0.147	0.157	0.283	0.256	0.000	0.280	3.862

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	80	55	134	126	113	0	130	90
N.S.	1	1.04	0.71	1.74	1.64	1.47	0.00	1.69	1.17
time (sec)	N/A	0.423	0.220	0.152	0.275	0.260	0.000	0.275	0.083

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	55	73	136	114	88	0	85	82
N.S.	1	1.12	1.49	2.78	2.33	1.80	0.00	1.73	1.67
time (sec)	N/A	0.297	0.126	0.131	0.276	0.246	0.000	0.281	0.074

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	28	44	27	0	24	39
N.S.	1	1.00	1.00	1.47	2.32	1.42	0.00	1.26	2.05
time (sec)	N/A	0.226	0.107	0.187	0.200	0.244	0.000	0.262	0.055

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	69	144	120	97	0	0	62
N.S.	1	1.01	0.95	1.97	1.64	1.33	0.00	0.00	0.85
time (sec)	N/A	0.323	0.044	0.219	0.182	0.256	0.000	0.000	3.814

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	100	94	185	137	134	0	0	105
N.S.	1	0.95	0.90	1.76	1.30	1.28	0.00	0.00	1.00
time (sec)	N/A	0.458	0.076	0.230	0.193	0.258	0.000	0.000	3.865

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	129	104	225	153	170	0	59	121
N.S.	1	0.93	0.75	1.63	1.11	1.23	0.00	0.43	0.88
time (sec)	N/A	0.600	0.088	0.244	0.199	0.244	0.000	0.294	3.867

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	54	54	51	60	71	56	62	61
N.S.	1	0.83	0.83	0.78	0.92	1.09	0.86	0.95	0.94
time (sec)	N/A	0.424	0.085	0.651	0.187	0.274	0.301	0.275	0.102

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	44	46	43	51	62	42	53	51
N.S.	1	0.81	0.85	0.80	0.94	1.15	0.78	0.98	0.94
time (sec)	N/A	0.423	0.074	0.618	0.186	0.249	0.240	0.280	0.085

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	34	34	31	40	43	31	42	40
N.S.	1	0.85	0.85	0.78	1.00	1.08	0.78	1.05	1.00
time (sec)	N/A	0.413	0.061	0.579	0.190	0.239	0.210	0.281	4.252

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	19	23	20	23	22	17	25	23
N.S.	1	0.83	1.00	0.87	1.00	0.96	0.74	1.09	1.00
time (sec)	N/A	0.354	0.038	0.553	0.184	0.248	0.126	0.271	0.075

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	22	22	20	20	19	17	21	19
N.S.	1	1.10	1.10	1.00	1.00	0.95	0.85	1.05	0.95
time (sec)	N/A	0.379	0.023	0.498	0.180	0.243	0.093	0.276	3.801

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	23	18	28	34	27	34	36	17
N.S.	1	1.28	1.00	1.56	1.89	1.50	1.89	2.00	0.94
time (sec)	N/A	0.391	0.046	0.519	0.190	0.242	0.129	0.285	0.073

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	55	57	48	53	59	56	51	52
N.S.	1	0.96	1.00	0.84	0.93	1.04	0.98	0.89	0.91
time (sec)	N/A	0.449	0.075	0.506	0.186	0.242	0.241	0.265	0.096

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	72	72	60	69	93	73	57	68
N.S.	1	0.96	0.96	0.80	0.92	1.24	0.97	0.76	0.91
time (sec)	N/A	0.456	0.095	0.520	0.187	0.242	0.292	0.265	3.879

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	173	567	191	246	157	0	0	211
N.S.	1	1.05	3.46	1.16	1.50	0.96	0.00	0.00	1.29
time (sec)	N/A	0.855	1.328	0.182	0.279	0.258	0.000	0.000	0.144

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	138	663	181	225	146	0	0	190
N.S.	1	1.02	4.91	1.34	1.67	1.08	0.00	0.00	1.41
time (sec)	N/A	0.689	0.513	0.171	0.276	0.272	0.000	0.000	3.914

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	111	424	169	149	120	0	0	117
N.S.	1	1.06	4.04	1.61	1.42	1.14	0.00	0.00	1.11
time (sec)	N/A	0.551	0.404	0.176	0.269	0.276	0.000	0.000	0.110

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	87	234	371	135	92	0	0	107
N.S.	1	1.16	3.12	4.95	1.80	1.23	0.00	0.00	1.43
time (sec)	N/A	0.422	0.541	0.138	0.277	0.263	0.000	0.000	0.099

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	70	61	137	120	69	0	0	87
N.S.	1	0.97	0.85	1.90	1.67	0.96	0.00	0.00	1.21
time (sec)	N/A	0.327	0.146	0.150	0.180	0.257	0.000	0.000	0.062

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	68	69	138	125	67	0	0	90
N.S.	1	0.92	0.93	1.86	1.69	0.91	0.00	0.00	1.22
time (sec)	N/A	0.307	0.045	0.252	0.185	0.258	0.000	0.000	3.840

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	41	92	42	0	42	41
N.S.	1	1.00	0.73	0.91	2.04	0.93	0.00	0.93	0.91
time (sec)	N/A	0.234	0.027	0.203	0.199	0.247	0.000	0.266	0.077

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	105	94	218	160	134	0	0	128
N.S.	1	0.95	0.85	1.96	1.44	1.21	0.00	0.00	1.15
time (sec)	N/A	0.356	0.078	0.243	0.192	0.247	0.000	0.000	3.894

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	132	104	259	176	170	0	0	144
N.S.	1	0.96	0.75	1.88	1.28	1.23	0.00	0.00	1.04
time (sec)	N/A	0.600	0.095	0.242	0.195	0.259	0.000	0.000	0.096

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	183	109	166	0	437	0	0	0
N.S.	1	0.78	0.46	0.71	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.395	0.126	0.092	0.000	0.295	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	150	101	149	0	415	0	0	0
N.S.	1	0.77	0.52	0.76	0.00	2.12	0.00	0.00	0.00
time (sec)	N/A	0.373	0.089	0.082	0.000	0.273	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	115	89	132	0	381	0	0	0
N.S.	1	0.73	0.57	0.84	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	0.336	0.078	0.083	0.000	0.293	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	122	70	105	0	313	0	0	0
N.S.	1	1.04	0.60	0.90	0.00	2.68	0.00	0.00	0.00
time (sec)	N/A	0.398	0.058	0.084	0.000	0.276	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	82	66	87	0	295	0	0	0
N.S.	1	1.05	0.85	1.12	0.00	3.78	0.00	0.00	0.00
time (sec)	N/A	0.325	0.047	0.084	0.000	0.283	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	103	95	151	0	517	0	0	0
N.S.	1	0.68	0.62	0.99	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.340	0.073	0.264	0.000	0.326	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	137	122	259	0	594	0	0	0
N.S.	1	0.64	0.57	1.20	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.377	0.103	0.281	0.000	0.327	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	175	135	328	0	668	0	0	0
N.S.	1	0.63	0.49	1.18	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.405	0.136	0.300	0.000	0.326	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	146	91	155	0	234	2222	0	0
N.S.	1	1.02	0.64	1.08	0.00	1.64	15.54	0.00	0.00
time (sec)	N/A	0.465	0.160	0.546	0.000	0.257	22.535	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	124	83	144	0	212	740	0	0
N.S.	1	1.05	0.70	1.22	0.00	1.80	6.27	0.00	0.00
time (sec)	N/A	0.459	0.128	0.549	0.000	0.258	7.542	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	103	75	108	0	182	144	0	0
N.S.	1	1.08	0.79	1.14	0.00	1.92	1.52	0.00	0.00
time (sec)	N/A	0.433	0.088	0.559	0.000	0.257	16.411	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	80	55	103	0	137	173	0	0
N.S.	1	1.14	0.79	1.47	0.00	1.96	2.47	0.00	0.00
time (sec)	N/A	0.422	0.066	0.541	0.000	0.267	25.431	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	58	50	98	0	124	0	96	0
N.S.	1	1.16	1.00	1.96	0.00	2.48	0.00	1.92	0.00
time (sec)	N/A	0.393	0.037	0.526	0.000	0.257	0.000	0.294	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	83	43	150	0	176	0	170	0
N.S.	1	1.19	0.61	2.14	0.00	2.51	0.00	2.43	0.00
time (sec)	N/A	0.413	0.035	0.538	0.000	0.258	0.000	0.351	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	110	55	201	0	238	0	245	0
N.S.	1	1.16	0.58	2.12	0.00	2.51	0.00	2.58	0.00
time (sec)	N/A	0.436	0.038	0.544	0.000	0.278	0.000	0.410	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	137	58	247	0	294	0	316	0
N.S.	1	1.16	0.49	2.09	0.00	2.49	0.00	2.68	0.00
time (sec)	N/A	0.455	0.037	0.537	0.000	0.261	0.000	0.476	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	164	46	291	0	346	0	391	0
N.S.	1	1.13	0.32	2.01	0.00	2.39	0.00	2.70	0.00
time (sec)	N/A	0.488	0.049	0.540	0.000	0.276	0.000	0.629	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	168	109	178	0	437	0	0	0
N.S.	1	0.63	0.41	0.66	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.401	0.113	0.084	0.000	0.285	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	132	101	161	0	415	0	0	0
N.S.	1	0.56	0.43	0.68	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.355	0.090	0.086	0.000	0.280	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	162	89	144	0	381	0	0	0
N.S.	1	1.04	0.57	0.92	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.439	0.072	0.079	0.000	0.286	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	123	66	118	0	315	0	0	0
N.S.	1	1.04	0.56	1.00	0.00	2.67	0.00	0.00	0.00
time (sec)	N/A	0.387	0.049	0.073	0.000	0.289	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	104	95	160	0	512	0	0	0
N.S.	1	0.68	0.62	1.05	0.00	3.37	0.00	0.00	0.00
time (sec)	N/A	0.348	0.058	0.237	0.000	0.315	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	135	115	259	0	581	0	0	0
N.S.	1	0.63	0.53	1.20	0.00	2.70	0.00	0.00	0.00
time (sec)	N/A	0.369	0.115	0.277	0.000	0.337	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	172	135	326	0	668	0	0	0
N.S.	1	0.63	0.49	1.19	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	0.415	0.141	0.276	0.000	0.335	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	210	143	376	0	736	0	0	0
N.S.	1	0.63	0.43	1.12	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	0.453	0.168	0.282	0.000	0.327	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	148	101	161	0	415	0	0	0
N.S.	1	0.67	0.46	0.73	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.369	0.110	0.154	0.000	0.283	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	113	89	144	0	381	0	0	0
N.S.	1	0.70	0.55	0.89	0.00	2.37	0.00	0.00	0.00
time (sec)	N/A	0.347	0.074	0.161	0.000	0.284	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	93	70	118	0	315	0	0	0
N.S.	1	0.66	0.50	0.84	0.00	2.25	0.00	0.00	0.00
time (sec)	N/A	0.330	0.056	0.151	0.000	0.283	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	87	65	101	0	297	0	0	0
N.S.	1	1.10	0.82	1.28	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.325	0.041	0.138	0.000	0.281	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	84	65	102	0	299	0	0	0
N.S.	1	1.08	0.83	1.31	0.00	3.83	0.00	0.00	0.00
time (sec)	N/A	0.337	0.047	0.142	0.000	0.281	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	112	91	162	0	522	0	0	0
N.S.	1	0.74	0.60	1.07	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.340	0.077	0.312	0.000	0.328	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	140	123	264	0	596	0	0	0
N.S.	1	0.64	0.56	1.21	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.373	0.109	0.303	0.000	0.316	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	175	135	316	0	668	0	0	0
N.S.	1	0.63	0.49	1.14	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.408	0.148	0.334	0.000	0.325	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	172	125	223	0	323	0	0	0
N.S.	1	1.06	0.77	1.37	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.539	0.272	0.539	0.000	0.263	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	150	116	213	0	285	0	0	0
N.S.	1	1.09	0.84	1.54	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.510	0.117	0.517	0.000	0.269	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	128	95	193	0	235	0	0	0
N.S.	1	1.13	0.84	1.71	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.502	0.090	0.505	0.000	0.256	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	108	92	168	0	219	0	0	0
N.S.	1	1.17	1.00	1.83	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.457	0.057	0.501	0.000	0.267	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	105	95	136	0	234	0	0	0
N.S.	1	1.11	1.00	1.43	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.459	0.056	0.500	0.000	0.263	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	116	94	136	0	231	0	0	0
N.S.	1	1.23	1.00	1.45	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.458	0.061	0.536	0.000	0.270	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	132	70	227	0	287	0	0	0
N.S.	1	1.14	0.60	1.96	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.491	0.060	0.496	0.000	0.268	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	162	79	271	0	359	0	0	0
N.S.	1	1.10	0.54	1.84	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.528	0.073	0.517	0.000	0.266	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	192	82	315	0	431	0	0	0
N.S.	1	1.12	0.48	1.83	0.00	2.51	0.00	0.00	0.00
time (sec)	N/A	0.541	0.072	0.487	0.000	0.281	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	204	140	229	0	437	0	0	0
N.S.	1	0.61	0.42	0.68	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.418	0.129	0.165	0.000	0.304	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	177	132	212	0	415	0	0	0
N.S.	1	0.64	0.48	0.77	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.399	0.124	0.158	0.000	0.290	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	144	124	195	0	381	0	0	0
N.S.	1	0.66	0.57	0.89	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.370	0.110	0.161	0.000	0.288	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	109	71	169	0	315	0	0	0
N.S.	1	0.69	0.45	1.07	0.00	1.99	0.00	0.00	0.00
time (sec)	N/A	0.337	0.052	0.148	0.000	0.291	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	92	67	146	0	299	0	0	0
N.S.	1	0.66	0.48	1.04	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.325	0.056	0.147	0.000	0.283	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	125	69	149	0	303	0	0	0
N.S.	1	1.06	0.58	1.26	0.00	2.57	0.00	0.00	0.00
time (sec)	N/A	0.386	0.050	0.143	0.000	0.304	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	121	64	149	0	311	0	0	0
N.S.	1	1.03	0.55	1.27	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.388	0.051	0.139	0.000	0.282	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	121	90	258	0	524	0	0	0
N.S.	1	0.61	0.45	1.30	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.369	0.070	0.319	0.000	0.316	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	160	121	290	0	594	0	0	0
N.S.	1	0.60	0.45	1.09	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.387	0.091	0.330	0.000	0.321	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	174	147	121	0	337	0	0	0
N.S.	1	1.06	0.90	0.74	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.518	0.919	0.066	0.000	0.293	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	129	148	102	0	317	0	0	0
N.S.	1	1.04	1.19	0.82	0.00	2.56	0.00	0.00	0.00
time (sec)	N/A	0.423	1.428	0.066	0.000	0.285	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	82	66	87	0	295	0	0	0
N.S.	1	1.05	0.85	1.12	0.00	3.78	0.00	0.00	0.00
time (sec)	N/A	0.320	0.044	0.055	0.000	0.279	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	78	132	88	0	275	0	0	0
N.S.	1	1.03	1.74	1.16	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.393	0.599	0.069	0.000	0.274	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	41	0	58	0	0	47
N.S.	1	1.00	1.22	1.11	0.00	1.57	0.00	0.00	1.27
time (sec)	N/A	0.316	0.197	0.048	0.000	0.253	0.000	0.000	4.438

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	58	47	0	68	0	0	53
N.S.	1	1.00	0.75	0.61	0.00	0.88	0.00	0.00	0.69
time (sec)	N/A	0.368	0.243	0.047	0.000	0.252	0.000	0.000	4.046

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	122	66	55	0	77	0	0	100
N.S.	1	1.04	0.56	0.47	0.00	0.66	0.00	0.00	0.85
time (sec)	N/A	0.422	0.236	0.048	0.000	0.254	0.000	0.000	4.119

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	175	74	63	0	84	0	0	108
N.S.	1	1.10	0.47	0.40	0.00	0.53	0.00	0.00	0.68
time (sec)	N/A	0.637	0.286	0.049	0.000	0.260	0.000	0.000	4.153

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	157	50	127	0	179	0	142	0
N.S.	1	1.21	0.38	0.98	0.00	1.38	0.00	1.09	0.00
time (sec)	N/A	0.721	0.045	0.497	0.000	0.263	0.000	0.291	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	124	50	119	0	163	0	127	0
N.S.	1	1.18	0.48	1.13	0.00	1.55	0.00	1.21	0.00
time (sec)	N/A	0.691	0.042	0.495	0.000	0.255	0.000	0.300	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	91	77	111	0	147	0	112	0
N.S.	1	1.14	0.96	1.39	0.00	1.84	0.00	1.40	0.00
time (sec)	N/A	0.548	0.082	0.493	0.000	0.264	0.000	0.292	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	58	50	98	0	124	0	96	0
N.S.	1	1.16	1.00	1.96	0.00	2.48	0.00	1.92	0.00
time (sec)	N/A	0.403	0.027	0.489	0.000	0.266	0.000	0.289	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	58	47	98	0	111	95	0	0
N.S.	1	1.23	1.00	2.09	0.00	2.36	2.02	0.00	0.00
time (sec)	N/A	0.652	0.038	0.490	0.000	0.266	4.292	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	55	28	27	0	28	0	0	24
N.S.	1	1.31	0.67	0.64	0.00	0.67	0.00	0.00	0.57
time (sec)	N/A	0.660	0.045	0.484	0.000	0.257	0.000	0.000	4.037

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	78	36	35	0	36	0	0	32
N.S.	1	1.13	0.52	0.51	0.00	0.52	0.00	0.00	0.46
time (sec)	N/A	0.673	0.052	0.478	0.000	0.252	0.000	0.000	3.900

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	105	44	43	0	44	0	0	77
N.S.	1	1.09	0.46	0.45	0.00	0.46	0.00	0.00	0.80
time (sec)	N/A	0.693	0.060	0.482	0.000	0.251	0.000	0.000	4.160

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	130	52	51	0	52	0	0	98
N.S.	1	1.07	0.43	0.42	0.00	0.43	0.00	0.00	0.81
time (sec)	N/A	0.715	0.072	0.478	0.000	0.239	0.000	0.000	4.098

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	188	252	224	0	568	0	0	0
N.S.	1	0.60	0.81	0.72	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.502	1.449	0.257	0.000	0.328	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	160	244	202	0	552	0	0	0
N.S.	1	0.61	0.93	0.77	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.468	1.077	0.247	0.000	0.336	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	132	236	180	0	536	0	0	0
N.S.	1	0.63	1.13	0.86	0.00	2.56	0.00	0.00	0.00
time (sec)	N/A	0.408	1.598	0.243	0.000	0.336	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	104	95	160	0	512	0	0	0
N.S.	1	0.68	0.62	1.05	0.00	3.37	0.00	0.00	0.00
time (sec)	N/A	0.337	0.059	0.233	0.000	0.323	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	100	218	159	0	490	0	0	0
N.S.	1	0.68	1.49	1.09	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.420	0.804	0.256	0.000	0.336	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	138	155	140	0	353	0	0	0
N.S.	1	1.10	1.24	1.12	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.430	0.382	0.247	0.000	0.300	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	177	162	165	0	381	0	0	0
N.S.	1	1.04	0.95	0.97	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.466	0.442	0.259	0.000	0.298	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	221	170	187	0	397	0	0	0
N.S.	1	1.06	0.81	0.89	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.538	0.437	0.261	0.000	0.297	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	276	178	206	0	413	0	0	0
N.S.	1	0.91	0.59	0.68	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.776	0.496	0.262	0.000	0.289	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	117	93	0	0	0	0	0	0
N.S.	1	0.93	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.523	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	177	147	133	0	337	0	0	0
N.S.	1	1.08	0.90	0.81	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.534	0.899	0.141	0.000	0.285	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	132	139	116	0	321	0	0	0
N.S.	1	1.06	1.12	0.94	0.00	2.59	0.00	0.00	0.00
time (sec)	N/A	0.422	1.492	0.137	0.000	0.284	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	87	65	101	0	297	0	0	0
N.S.	1	1.10	0.82	1.28	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.332	0.044	0.124	0.000	0.287	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	83	132	100	0	275	0	0	0
N.S.	1	1.09	1.74	1.32	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.397	0.635	0.135	0.000	0.279	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	78	46	54	0	59	0	0	54
N.S.	1	1.11	0.66	0.77	0.00	0.84	0.00	0.00	0.77
time (sec)	N/A	0.364	0.195	0.124	0.000	0.247	0.000	0.000	4.022

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	120	58	62	0	69	0	0	62
N.S.	1	1.06	0.51	0.55	0.00	0.61	0.00	0.00	0.55
time (sec)	N/A	0.385	0.273	0.116	0.000	0.269	0.000	0.000	3.997

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	169	66	70	0	77	0	0	100
N.S.	1	1.13	0.44	0.47	0.00	0.52	0.00	0.00	0.67
time (sec)	N/A	0.485	0.240	0.135	0.000	0.249	0.000	0.000	4.159

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	207	116	197	0	271	0	0	0
N.S.	1	1.20	0.67	1.15	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.799	0.166	0.490	0.000	0.281	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	174	108	189	0	259	0	0	0
N.S.	1	1.18	0.73	1.29	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.785	0.117	0.490	0.000	0.264	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	141	100	181	0	239	0	0	0
N.S.	1	1.16	0.82	1.48	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.625	0.096	0.484	0.000	0.271	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	108	92	168	0	219	0	0	0
N.S.	1	1.17	1.00	1.83	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.451	0.029	0.481	0.000	0.273	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	104	86	165	0	203	141	0	0
N.S.	1	1.21	1.00	1.92	0.00	2.36	1.64	0.00	0.00
time (sec)	N/A	0.710	0.048	0.523	0.000	0.263	4.276	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	86	69	142	0	161	0	0	0
N.S.	1	1.05	0.84	1.73	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.672	0.075	0.499	0.000	0.258	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	79	152	0	181	0	278	0
N.S.	1	1.00	0.70	1.35	0.00	1.60	0.00	2.46	0.00
time (sec)	N/A	0.696	0.092	0.517	0.000	0.251	0.000	0.638	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	117	87	160	0	201	0	356	0
N.S.	1	1.04	0.77	1.42	0.00	1.78	0.00	3.15	0.00
time (sec)	N/A	0.731	0.146	0.510	0.000	0.255	0.000	0.742	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	167	95	168	0	213	0	434	0
N.S.	1	1.02	0.58	1.03	0.00	1.31	0.00	2.66	0.00
time (sec)	N/A	0.762	0.161	0.505	0.000	0.252	0.000	0.879	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	172	167	197	0	353	0	0	0
N.S.	1	0.57	0.55	0.65	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.449	1.115	0.144	0.000	0.291	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	144	159	180	0	337	0	0	0
N.S.	1	0.57	0.63	0.72	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.435	0.951	0.146	0.000	0.293	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	116	151	163	0	321	0	0	0
N.S.	1	0.58	0.76	0.82	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.374	1.614	0.143	0.000	0.282	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	92	67	146	0	299	0	0	0
N.S.	1	0.66	0.48	1.04	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.325	0.047	0.124	0.000	0.280	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	90	131	151	0	277	0	0	0
N.S.	1	0.67	0.98	1.13	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.407	0.831	0.151	0.000	0.269	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	115	58	62	0	59	0	0	54
N.S.	1	1.06	0.53	0.57	0.00	0.54	0.00	0.00	0.50
time (sec)	N/A	0.408	0.217	0.154	0.000	0.263	0.000	0.000	4.193

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	157	70	70	0	69	0	0	62
N.S.	1	1.05	0.47	0.47	0.00	0.46	0.00	0.00	0.41
time (sec)	N/A	0.432	0.274	0.151	0.000	0.258	0.000	0.000	4.247

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	205	78	78	0	77	0	0	100
N.S.	1	1.09	0.41	0.41	0.00	0.41	0.00	0.00	0.53
time (sec)	N/A	0.497	0.261	0.153	0.000	0.251	0.000	0.000	4.447

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	253	86	86	0	85	0	0	108
N.S.	1	0.88	0.30	0.30	0.00	0.29	0.00	0.00	0.37
time (sec)	N/A	0.756	0.330	0.139	0.000	0.269	0.000	0.000	4.162

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	187	155	0	0	0	0	0	0
N.S.	1	1.01	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.379	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	0.466	0.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	163	113	0	0	0	0	0	0
N.S.	1	0.98	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.369	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	69	46	0	0	0	269	0	0
N.S.	1	1.21	0.81	0.00	0.00	0.00	4.72	0.00	0.00
time (sec)	N/A	0.365	0.033	0.000	0.000	0.000	4.895	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	124	87	0	0	0	0	0	0
N.S.	1	1.09	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.417	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	0	111	160	415	169	0	218	362
N.S.	1	0.00	0.28	0.41	1.06	0.43	0.00	0.55	0.92
time (sec)	N/A	0.000	0.208	0.478	0.202	0.259	0.000	0.307	4.413

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	0	95	144	337	148	0	178	289
N.S.	1	0.00	0.30	0.46	1.08	0.47	0.00	0.57	0.92
time (sec)	N/A	0.000	0.203	0.489	0.207	0.248	0.000	0.293	3.960

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	0	79	128	259	125	0	137	214
N.S.	1	0.00	0.34	0.55	1.11	0.54	0.00	0.59	0.92
time (sec)	N/A	0.000	0.136	0.489	0.204	0.260	0.000	0.287	3.885

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	0	61	108	171	91	0	90	131
N.S.	1	0.00	0.42	0.74	1.18	0.63	0.00	0.62	0.90
time (sec)	N/A	0.000	0.110	0.062	0.201	0.248	0.000	0.278	0.078

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	23	22	34	0	0	22
N.S.	1	1.00	1.00	1.77	1.69	2.62	0.00	0.00	1.69
time (sec)	N/A	0.192	0.153	0.496	0.213	0.247	0.000	0.000	3.834

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	47	65	58	0	0	50
N.S.	1	1.00	0.98	0.92	1.27	1.14	0.00	0.00	0.98
time (sec)	N/A	0.306	0.301	0.474	0.208	0.247	0.000	0.000	0.081

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	87	66	65	99	86	0	0	60
N.S.	1	1.02	0.78	0.76	1.16	1.01	0.00	0.00	0.71
time (sec)	N/A	0.433	0.377	0.482	0.197	0.262	0.000	0.000	4.298

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	123	82	81	132	134	0	0	142
N.S.	1	1.03	0.69	0.68	1.11	1.13	0.00	0.00	1.19
time (sec)	N/A	0.566	0.456	0.485	0.210	0.241	0.000	0.000	0.071

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	74	47	85	113	113	119	113	113
N.S.	1	0.88	0.56	1.01	1.35	1.35	1.42	1.35	1.35
time (sec)	N/A	0.365	0.045	0.647	0.204	0.244	0.042	0.281	3.900

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	62	39	61	82	82	87	82	82
N.S.	1	0.90	0.57	0.88	1.19	1.19	1.26	1.19	1.19
time (sec)	N/A	0.342	0.036	0.625	0.201	0.232	0.038	0.260	0.041

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	48	31	53	70	70	70	70	70
N.S.	1	0.92	0.60	1.02	1.35	1.35	1.35	1.35	1.35
time (sec)	N/A	0.324	0.029	0.634	0.195	0.236	0.033	0.284	0.037

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	34	23	29	38	38	36	38	38
N.S.	1	0.97	0.66	0.83	1.09	1.09	1.03	1.09	1.09
time (sec)	N/A	0.308	0.020	0.592	0.194	0.243	0.028	0.284	0.053

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	14	21	21	20	21	17
N.S.	1	1.00	1.33	0.93	1.40	1.40	1.33	1.40	1.13
time (sec)	N/A	0.245	0.016	0.490	0.199	0.238	0.025	0.268	0.037

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	13	13	13	10	14	14
N.S.	1	1.00	1.12	0.81	0.81	0.81	0.62	0.88	0.88
time (sec)	N/A	0.279	0.061	0.574	0.196	0.235	0.058	0.280	0.039

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	47	35	51	63	76	54	51	46
N.S.	1	0.92	0.69	1.00	1.24	1.49	1.06	1.00	0.90
time (sec)	N/A	0.323	0.032	0.574	0.187	0.233	0.147	0.282	3.895

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	76	63	76	91	121	85	74	73
N.S.	1	0.88	0.73	0.88	1.06	1.41	0.99	0.86	0.85
time (sec)	N/A	0.351	0.045	0.612	0.214	0.244	0.221	0.267	0.094

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	105	82	92	140	217	141	91	121
N.S.	1	0.87	0.68	0.76	1.16	1.79	1.17	0.75	1.00
time (sec)	N/A	0.383	0.071	0.618	0.204	0.252	0.334	0.267	4.141

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	0	111	160	415	170	0	216	362
N.S.	1	0.00	0.28	0.41	1.06	0.43	0.00	0.55	0.92
time (sec)	N/A	0.000	0.207	0.504	0.212	0.257	0.000	0.320	4.308

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	0	95	144	337	147	0	177	289
N.S.	1	0.00	0.30	0.46	1.08	0.47	0.00	0.57	0.92
time (sec)	N/A	0.000	0.172	0.492	0.225	0.243	0.000	0.298	4.033

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	0	79	128	259	126	0	138	214
N.S.	1	0.00	0.34	0.55	1.11	0.54	0.00	0.59	0.92
time (sec)	N/A	0.000	0.137	0.489	0.187	0.247	0.000	0.289	3.985

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	0	61	108	171	91	0	90	133
N.S.	1	0.00	0.42	0.74	1.18	0.63	0.00	0.62	0.92
time (sec)	N/A	0.000	0.103	0.126	0.202	0.248	0.000	0.279	0.073

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	51	0	49	23
N.S.	1	1.00	1.00	1.33	1.28	2.83	0.00	2.72	1.28
time (sec)	N/A	0.195	0.177	0.513	0.189	0.240	0.000	0.285	0.037

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	43	49	55	77	0	65	55
N.S.	1	1.00	0.78	0.89	1.00	1.40	0.00	1.18	1.00
time (sec)	N/A	0.313	0.342	0.499	0.194	0.245	0.000	0.311	4.370

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	93	66	63	97	96	0	0	60
N.S.	1	1.02	0.73	0.69	1.07	1.05	0.00	0.00	0.66
time (sec)	N/A	0.434	0.410	0.504	0.191	0.244	0.000	0.000	4.310

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	82	81	131	124	0	0	76
N.S.	1	1.03	0.65	0.64	1.03	0.98	0.00	0.00	0.60
time (sec)	N/A	0.592	0.489	0.489	0.192	0.250	0.000	0.000	3.981

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	58	39	75	101	101	109	102	101
N.S.	1	0.88	0.59	1.14	1.53	1.53	1.65	1.55	1.53
time (sec)	N/A	0.345	0.039	0.689	0.197	0.235	0.049	0.265	3.968

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	47	31	69	92	92	100	90	92
N.S.	1	0.90	0.60	1.33	1.77	1.77	1.92	1.73	1.77
time (sec)	N/A	0.319	0.032	0.655	0.190	0.242	0.043	0.268	0.047

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	23	45	59	59	63	78	59
N.S.	1	0.94	0.66	1.29	1.69	1.69	1.80	2.23	1.69
time (sec)	N/A	0.305	0.025	0.654	0.191	0.234	0.036	0.264	0.035

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	16	47	47	48	64	47
N.S.	1	1.00	2.18	0.94	2.76	2.76	2.82	3.76	2.76
time (sec)	N/A	0.289	0.024	0.624	0.185	0.224	0.034	0.270	0.033

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	44	38	32	33	37	36	60	33
N.S.	1	0.96	0.83	0.70	0.72	0.80	0.78	1.30	0.72
time (sec)	N/A	0.290	0.020	0.595	0.193	0.249	0.066	0.259	0.053

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	13	19	19	17	27	12
N.S.	1	1.00	1.92	1.00	1.46	1.46	1.31	2.08	0.92
time (sec)	N/A	0.279	0.014	0.647	0.194	0.239	0.092	0.275	0.055

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	41	41	42	15	40
N.S.	1	1.00	1.00	0.89	2.28	2.28	2.33	0.83	2.22
time (sec)	N/A	0.274	0.088	0.625	0.190	0.232	0.107	0.263	0.068

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	76	52	65	102	147	99	91	83
N.S.	1	0.87	0.60	0.75	1.17	1.69	1.14	1.05	0.95
time (sec)	N/A	0.342	0.042	0.621	0.188	0.252	0.253	0.276	0.105

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	105	80	92	130	191	129	127	111
N.S.	1	0.86	0.66	0.75	1.07	1.57	1.06	1.04	0.91
time (sec)	N/A	0.375	0.066	0.632	0.195	0.250	0.294	0.267	3.960

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	0	111	160	415	170	0	196	362
N.S.	1	0.00	0.28	0.41	1.06	0.43	0.00	0.50	0.92
time (sec)	N/A	0.000	0.219	0.487	0.195	0.249	0.000	0.303	4.435

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	0	95	144	337	147	0	161	289
N.S.	1	0.00	0.30	0.46	1.08	0.47	0.00	0.51	0.92
time (sec)	N/A	0.000	0.207	0.493	0.205	0.257	0.000	0.289	0.124

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	0	79	128	259	126	0	126	214
N.S.	1	0.00	0.34	0.55	1.11	0.54	0.00	0.54	0.92
time (sec)	N/A	0.000	0.134	0.489	0.204	0.255	0.000	0.287	3.967

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	0	61	108	171	91	0	82	132
N.S.	1	0.00	0.42	0.74	1.18	0.63	0.00	0.57	0.91
time (sec)	N/A	0.000	0.116	0.066	0.200	0.243	0.000	0.287	3.893

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	23	0	0	23
N.S.	1	1.00	1.00	1.50	1.44	1.44	0.00	0.00	1.44
time (sec)	N/A	0.195	0.165	0.497	0.202	0.256	0.000	0.000	0.033

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	67	50	0	0	55
N.S.	1	1.00	0.87	0.89	1.22	0.91	0.00	0.00	1.00
time (sec)	N/A	0.309	0.304	0.500	0.202	0.238	0.000	0.000	0.058

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	93	64	65	102	76	0	0	109
N.S.	1	1.02	0.70	0.71	1.12	0.84	0.00	0.00	1.20
time (sec)	N/A	0.425	0.386	0.504	0.204	0.256	0.000	0.000	0.065

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	80	81	135	104	0	0	148
N.S.	1	1.03	0.63	0.64	1.06	0.82	0.00	0.00	1.17
time (sec)	N/A	0.573	0.471	0.507	0.197	0.262	0.000	0.000	4.235

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	66	39	61	80	80	87	80	80
N.S.	1	0.90	0.53	0.84	1.10	1.10	1.19	1.10	1.10
time (sec)	N/A	0.342	0.037	0.620	0.204	0.237	0.041	0.259	0.083

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	51	31	53	70	70	70	70	70
N.S.	1	0.93	0.56	0.96	1.27	1.27	1.27	1.27	1.27
time (sec)	N/A	0.332	0.030	0.631	0.193	0.238	0.037	0.272	0.057

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	36	30	29	37	37	36	37	37
N.S.	1	0.97	0.81	0.78	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.300	0.022	0.627	0.197	0.236	0.029	0.267	0.055

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	14	20	20	19	20	17
N.S.	1	1.00	1.31	0.88	1.25	1.25	1.19	1.25	1.06
time (sec)	N/A	0.253	0.017	0.516	0.189	0.238	0.029	0.264	0.038

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	14	12	12	10	14	12
N.S.	1	1.00	1.29	1.00	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.278	0.058	0.592	0.196	0.247	0.070	0.259	0.054

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	45	33	51	63	76	54	51	46
N.S.	1	0.92	0.67	1.04	1.29	1.55	1.10	1.04	0.94
time (sec)	N/A	0.319	0.034	0.589	0.199	0.242	0.160	0.276	0.066

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	74	61	76	91	121	85	74	73
N.S.	1	0.88	0.73	0.90	1.08	1.44	1.01	0.88	0.87
time (sec)	N/A	0.352	0.048	0.610	0.205	0.271	0.240	0.274	4.001

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	103	80	92	140	217	141	91	121
N.S.	1	0.87	0.67	0.77	1.18	1.82	1.18	0.76	1.02
time (sec)	N/A	0.381	0.068	0.602	0.205	0.271	0.339	0.284	4.101

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	0	111	160	415	169	0	198	362
N.S.	1	0.00	0.28	0.41	1.06	0.43	0.00	0.50	0.92
time (sec)	N/A	0.000	0.229	0.513	0.205	0.261	0.000	0.305	0.175

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	0	95	144	337	148	0	162	289
N.S.	1	0.00	0.30	0.46	1.08	0.47	0.00	0.52	0.92
time (sec)	N/A	0.000	0.174	0.513	0.211	0.247	0.000	0.278	0.120

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	0	79	128	259	125	0	126	214
N.S.	1	0.00	0.34	0.55	1.11	0.54	0.00	0.54	0.92
time (sec)	N/A	0.000	0.145	0.510	0.198	0.247	0.000	0.282	3.932

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	0	61	108	171	92	0	82	133
N.S.	1	0.00	0.42	0.74	1.18	0.63	0.00	0.57	0.92
time (sec)	N/A	0.000	0.126	0.132	0.200	0.251	0.000	0.282	0.067

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	34	0	49	23
N.S.	1	1.00	1.00	1.33	1.28	1.89	0.00	2.72	1.28
time (sec)	N/A	0.197	0.192	0.518	0.195	0.249	0.000	0.309	4.274

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	43	47	60	58	0	65	60
N.S.	1	1.00	0.78	0.85	1.09	1.05	0.00	1.18	1.09
time (sec)	N/A	0.305	0.312	0.510	0.218	0.239	0.000	0.313	3.852

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	93	66	65	103	86	0	0	116
N.S.	1	1.02	0.73	0.71	1.13	0.95	0.00	0.00	1.27
time (sec)	N/A	0.442	0.415	0.507	0.205	0.244	0.000	0.000	3.847

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	82	81	136	134	0	0	155
N.S.	1	1.03	0.65	0.64	1.07	1.06	0.00	0.00	1.22
time (sec)	N/A	0.583	0.505	0.515	0.214	0.249	0.000	0.000	0.043

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	105	79	113	0	117	0	0	0
N.S.	1	0.46	0.34	0.49	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.458	0.088	0.510	0.000	0.250	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	92	71	97	0	95	0	0	0
N.S.	1	0.50	0.39	0.53	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.460	0.068	0.500	0.000	0.241	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	76	63	81	0	73	0	0	0
N.S.	1	0.56	0.46	0.60	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.444	0.052	0.498	0.000	0.250	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	66	53	63	0	43	0	0	0
N.S.	1	0.71	0.57	0.68	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.433	0.038	0.497	0.000	0.249	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	46	41	44	0	22	0	0	0
N.S.	1	0.68	0.60	0.65	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.358	0.023	0.497	0.000	0.243	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	51	0	22	0	0	0
N.S.	1	1.00	1.00	1.34	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.390	0.022	0.478	0.000	0.254	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	63	56	84	0	86	0	0	0
N.S.	1	0.69	0.62	0.92	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.440	0.053	0.484	0.000	0.285	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	93	83	169	0	136	0	0	0
N.S.	1	0.51	0.45	0.92	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.465	0.069	0.514	0.000	0.248	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	121	101	241	0	191	0	0	0
N.S.	1	0.44	0.36	0.87	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.484	0.107	0.542	0.000	0.273	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	198	167	138	192	329	714	164	0
N.S.	1	1.12	0.95	0.78	1.09	1.87	4.06	0.93	0.00
time (sec)	N/A	0.435	0.203	0.661	0.280	0.298	3.360	0.306	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	172	151	122	173	286	614	141	0
N.S.	1	1.12	0.99	0.80	1.13	1.87	4.01	0.92	0.00
time (sec)	N/A	0.408	0.166	0.592	0.287	0.267	2.966	0.315	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	146	135	106	154	241	314	116	0
N.S.	1	1.12	1.04	0.82	1.18	1.85	2.42	0.89	0.00
time (sec)	N/A	0.389	0.145	0.589	0.283	0.279	2.530	0.284	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	120	117	90	131	180	250	85	0
N.S.	1	1.12	1.09	0.84	1.22	1.68	2.34	0.79	0.00
time (sec)	N/A	0.378	0.111	0.596	0.293	0.271	2.240	0.301	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	95	76	69	47	134	0	62	0
N.S.	1	1.10	0.88	0.80	0.55	1.56	0.00	0.72	0.00
time (sec)	N/A	0.355	0.063	0.597	0.282	0.256	0.000	0.282	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	66	82	79	40	153	0	0	0
N.S.	1	1.12	1.39	1.34	0.68	2.59	0.00	0.00	0.00
time (sec)	N/A	0.339	0.049	0.579	0.289	0.258	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	57	64	31	61	47	0	148	33
N.S.	1	1.12	1.25	0.61	1.20	0.92	0.00	2.90	0.65
time (sec)	N/A	0.329	0.047	0.589	0.199	0.267	0.000	0.326	4.055

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	88	53	47	80	75	0	0	56
N.S.	1	1.19	0.72	0.64	1.08	1.01	0.00	0.00	0.76
time (sec)	N/A	0.345	0.050	0.614	0.198	0.288	0.000	0.000	4.024

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	119	96	64	99	124	0	0	134
N.S.	1	1.23	0.99	0.66	1.02	1.28	0.00	0.00	1.38
time (sec)	N/A	0.373	0.061	0.626	0.202	0.361	0.000	0.000	4.067

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	150	112	80	118	152	0	0	177
N.S.	1	1.25	0.93	0.67	0.98	1.27	0.00	0.00	1.48
time (sec)	N/A	0.382	0.073	0.612	0.191	0.540	0.000	0.000	4.144

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	94	71	100	204	95	0	0	0
N.S.	1	0.51	0.38	0.54	1.10	0.51	0.00	0.00	0.00
time (sec)	N/A	0.454	0.069	0.517	0.209	0.265	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	79	63	100	172	95	0	0	0
N.S.	1	0.57	0.45	0.72	1.24	0.68	0.00	0.00	0.00
time (sec)	N/A	0.457	0.058	0.513	0.214	0.240	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	66	55	84	140	73	0	0	0
N.S.	1	0.71	0.59	0.90	1.51	0.78	0.00	0.00	0.00
time (sec)	N/A	0.430	0.049	0.510	0.219	0.259	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	58	48	97	42	0	0	0
N.S.	1	1.00	1.26	1.04	2.11	0.91	0.00	0.00	0.00
time (sec)	N/A	0.399	0.038	0.514	0.214	0.259	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	61	59	67	0	33	0	0	0
N.S.	1	0.54	0.52	0.59	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.384	0.031	0.513	0.000	0.253	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	57	49	64	0	39	0	0	0
N.S.	1	0.72	0.62	0.81	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.420	0.038	0.506	0.000	0.252	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	51	39	0	39	0	0	90
N.S.	1	1.00	1.09	0.83	0.00	0.83	0.00	0.00	1.91
time (sec)	N/A	0.399	0.091	0.515	0.000	0.244	0.000	0.000	4.189

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	93	71	169	0	139	0	0	0
N.S.	1	0.50	0.38	0.91	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.465	0.074	0.521	0.000	0.264	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	123	99	241	0	190	0	0	0
N.S.	1	0.44	0.36	0.87	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.489	0.094	0.565	0.000	0.244	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	111	79	113	0	117	0	0	0
N.S.	1	0.47	0.34	0.48	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.472	0.072	0.507	0.000	0.245	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	95	71	97	0	95	0	0	0
N.S.	1	0.51	0.38	0.52	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.459	0.059	0.510	0.000	0.259	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	81	63	81	0	73	0	0	0
N.S.	1	0.58	0.45	0.58	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.447	0.049	0.556	0.000	0.254	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	67	53	63	0	43	0	0	0
N.S.	1	0.71	0.56	0.66	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.438	0.041	0.514	0.000	0.252	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	47	41	44	0	22	0	0	0
N.S.	1	0.68	0.59	0.64	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.352	0.025	0.523	0.000	0.248	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	0	22	0	0	0
N.S.	1	1.00	1.00	1.38	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.385	0.025	0.521	0.000	0.243	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	63	54	84	0	83	0	0	0
N.S.	1	0.70	0.60	0.93	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.433	0.056	0.520	0.000	0.258	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	91	81	169	0	137	0	0	0
N.S.	1	0.50	0.44	0.92	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.458	0.080	0.515	0.000	0.269	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	121	99	241	0	186	0	0	0
N.S.	1	0.44	0.36	0.87	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.483	0.099	0.523	0.000	0.258	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	147	136	106	154	241	314	117	0
N.S.	1	1.12	1.04	0.81	1.18	1.84	2.40	0.89	0.00
time (sec)	N/A	0.397	0.174	0.638	0.291	0.282	2.742	0.294	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	121	117	90	130	180	248	85	0
N.S.	1	1.12	1.08	0.83	1.20	1.67	2.30	0.79	0.00
time (sec)	N/A	0.380	0.115	0.629	0.302	0.261	2.410	0.283	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	95	100	69	47	134	0	62	0
N.S.	1	1.09	1.15	0.79	0.54	1.54	0.00	0.71	0.00
time (sec)	N/A	0.355	0.062	0.639	0.286	0.256	0.000	0.284	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	67	100	73	39	151	0	0	0
N.S.	1	1.12	1.67	1.22	0.65	2.52	0.00	0.00	0.00
time (sec)	N/A	0.345	0.075	0.620	0.330	0.255	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	58	63	31	60	47	0	148	33
N.S.	1	1.12	1.21	0.60	1.15	0.90	0.00	2.85	0.63
time (sec)	N/A	0.331	0.048	0.621	0.205	0.262	0.000	0.314	3.998

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	89	79	47	79	75	0	0	56
N.S.	1	1.19	1.05	0.63	1.05	1.00	0.00	0.00	0.75
time (sec)	N/A	0.343	0.059	0.625	0.225	0.299	0.000	0.000	4.066

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	120	96	64	98	124	0	0	134
N.S.	1	1.22	0.98	0.65	1.00	1.27	0.00	0.00	1.37
time (sec)	N/A	0.369	0.075	0.622	0.211	0.369	0.000	0.000	4.120

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	151	112	80	117	152	0	0	177
N.S.	1	1.25	0.93	0.66	0.97	1.26	0.00	0.00	1.46
time (sec)	N/A	0.385	0.097	0.622	0.213	0.523	0.000	0.000	4.261

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	97	71	100	204	95	0	0	0
N.S.	1	0.51	0.38	0.53	1.08	0.50	0.00	0.00	0.00
time (sec)	N/A	0.448	0.065	0.520	0.222	0.256	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	83	63	100	172	95	0	0	0
N.S.	1	0.58	0.44	0.70	1.21	0.67	0.00	0.00	0.00
time (sec)	N/A	0.452	0.059	0.524	0.237	0.240	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	67	55	84	140	73	0	0	0
N.S.	1	0.71	0.58	0.88	1.47	0.77	0.00	0.00	0.00
time (sec)	N/A	0.432	0.048	0.523	0.212	0.239	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	58	48	97	42	0	0	0
N.S.	1	1.00	1.23	1.02	2.06	0.89	0.00	0.00	0.00
time (sec)	N/A	0.397	0.038	0.519	0.207	0.242	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	59	58	67	0	33	0	0	0
N.S.	1	0.53	0.52	0.60	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.388	0.032	0.529	0.000	0.249	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	57	47	62	0	38	0	0	0
N.S.	1	0.74	0.61	0.81	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.421	0.045	0.532	0.000	0.245	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	39	0	39	0	0	58
N.S.	1	1.00	1.11	0.85	0.00	0.85	0.00	0.00	1.26
time (sec)	N/A	0.395	0.101	0.523	0.000	0.252	0.000	0.000	4.394

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	91	71	169	0	136	0	0	0
N.S.	1	0.50	0.39	0.93	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.463	0.073	0.522	0.000	0.257	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	119	99	241	0	193	0	0	0
N.S.	1	0.43	0.36	0.88	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.491	0.103	0.521	0.000	0.250	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	52	45	47	0	25	0	0	0
N.S.	1	0.68	0.59	0.62	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.453	0.028	0.520	0.000	0.252	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	52	43	47	0	25	0	0	0
N.S.	1	0.70	0.58	0.64	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.421	0.025	0.518	0.000	0.263	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	46	41	44	0	22	0	0	0
N.S.	1	0.68	0.60	0.65	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.346	0.011	0.518	0.000	0.262	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	42	41	44	0	18	0	0	0
N.S.	1	0.61	0.59	0.64	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.375	0.025	0.529	0.000	0.254	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	46	44	48	0	22	0	0	0
N.S.	1	0.63	0.60	0.66	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.435	0.026	0.530	0.000	0.244	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	192	96	97	117	184	0	0	0
N.S.	1	1.40	0.70	0.71	0.85	1.34	0.00	0.00	0.00
time (sec)	N/A	0.602	0.201	0.664	0.296	0.259	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	161	88	89	93	168	0	84	0
N.S.	1	1.44	0.79	0.79	0.83	1.50	0.00	0.75	0.00
time (sec)	N/A	0.566	0.153	0.654	0.288	0.257	0.000	0.292	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	120	79	80	70	150	0	72	0
N.S.	1	1.41	0.93	0.94	0.82	1.76	0.00	0.85	0.00
time (sec)	N/A	0.472	0.133	0.651	0.291	0.258	0.000	0.290	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	95	76	69	47	134	0	62	0
N.S.	1	1.10	0.88	0.80	0.55	1.56	0.00	0.72	0.00
time (sec)	N/A	0.359	0.045	0.628	0.289	0.249	0.000	0.269	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	84	97	129	90	191	0	95	0
N.S.	1	1.12	1.29	1.72	1.20	2.55	0.00	1.27	0.00
time (sec)	N/A	0.519	0.133	0.633	0.295	0.273	0.000	0.295	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	89	104	102	0	209	0	134	0
N.S.	1	1.09	1.27	1.24	0.00	2.55	0.00	1.63	0.00
time (sec)	N/A	0.533	0.151	0.660	0.000	0.260	0.000	0.288	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	88	76	78	0	148	0	200	0
N.S.	1	1.13	0.97	1.00	0.00	1.90	0.00	2.56	0.00
time (sec)	N/A	0.493	0.171	0.674	0.000	0.250	0.000	0.277	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	114	82	86	0	164	0	250	0
N.S.	1	1.15	0.83	0.87	0.00	1.66	0.00	2.53	0.00
time (sec)	N/A	0.529	0.181	0.689	0.000	0.261	0.000	0.293	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	150	95	95	0	180	0	324	0
N.S.	1	1.15	0.73	0.73	0.00	1.38	0.00	2.49	0.00
time (sec)	N/A	0.594	0.198	0.733	0.000	0.262	0.000	0.290	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	89	87	92	0	58	0	0	0
N.S.	1	0.39	0.38	0.40	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.509	0.064	0.572	0.000	0.242	0.000	0.000	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	77	74	83	0	49	0	0	0
N.S.	1	0.41	0.40	0.45	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.498	0.049	0.531	0.000	0.270	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	71	66	76	0	42	0	0	0
N.S.	1	0.47	0.43	0.50	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.462	0.044	0.526	0.000	0.239	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	61	59	67	0	33	0	0	0
N.S.	1	0.54	0.52	0.59	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.399	0.017	0.528	0.000	0.247	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	53	54	59	0	28	0	0	0
N.S.	1	0.46	0.47	0.52	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.411	0.034	0.523	0.000	0.264	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	58	54	65	0	33	0	0	0
N.S.	1	0.51	0.47	0.57	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.483	0.041	0.523	0.000	0.259	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	70	64	77	0	90	0	0	0
N.S.	1	0.46	0.42	0.50	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.484	0.055	0.535	0.000	0.259	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	80	76	85	0	98	0	0	0
N.S.	1	0.41	0.39	0.44	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.492	0.062	0.547	0.000	0.255	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	86	80	93	0	106	0	0	0
N.S.	1	0.38	0.35	0.41	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.494	0.067	0.553	0.000	0.250	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	94	83	106	0	76	0	0	0
N.S.	1	0.45	0.39	0.50	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.481	0.106	0.548	0.000	0.260	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	84	74	98	0	69	0	0	0
N.S.	1	0.49	0.43	0.57	0.00	0.40	0.00	0.00	0.00
time (sec)	N/A	0.444	0.070	0.549	0.000	0.263	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	79	62	86	0	56	0	0	0
N.S.	1	0.61	0.48	0.66	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.503	0.060	0.549	0.000	0.260	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	63	52	84	0	86	0	0	0
N.S.	1	0.72	0.60	0.97	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.463	0.055	0.542	0.000	0.265	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	63	56	84	0	86	0	0	0
N.S.	1	0.69	0.62	0.92	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.424	0.028	0.559	0.000	0.246	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	72	68	93	0	63	0	0	0
N.S.	1	0.41	0.38	0.53	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.495	0.069	0.566	0.000	0.254	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	82	79	118	0	92	0	0	0
N.S.	1	0.38	0.37	0.55	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.505	0.071	0.548	0.000	0.255	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	99	94	138	0	113	0	0	0
N.S.	1	0.39	0.37	0.55	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.518	0.089	0.546	0.000	0.255	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	113	100	185	0	138	0	0	0
N.S.	1	0.43	0.38	0.71	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.480	0.126	0.549	0.000	0.273	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	109	84	169	0	122	0	0	0
N.S.	1	0.50	0.39	0.78	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.540	0.107	0.546	0.000	0.258	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	93	72	169	0	136	0	0	0
N.S.	1	0.53	0.41	0.96	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.532	0.101	0.549	0.000	0.244	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	93	85	164	0	134	0	0	0
N.S.	1	0.51	0.46	0.89	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.528	0.070	0.553	0.000	0.259	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	78	62	164	0	134	0	0	0
N.S.	1	0.57	0.45	1.20	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.480	0.074	0.544	0.000	0.246	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	93	83	169	0	136	0	0	0
N.S.	1	0.51	0.45	0.92	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.462	0.033	0.560	0.000	0.246	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	96	88	196	0	145	0	0	0
N.S.	1	0.35	0.32	0.72	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.516	0.117	0.534	0.000	0.265	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	108	99	225	0	174	0	0	0
N.S.	1	0.35	0.32	0.73	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.540	0.112	0.537	0.000	0.256	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	53	45	47	0	25	0	0	0
N.S.	1	0.70	0.59	0.62	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.469	0.031	0.540	0.000	0.238	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	53	43	47	0	25	0	0	0
N.S.	1	0.72	0.58	0.64	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.441	0.029	0.543	0.000	0.247	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	47	41	44	0	22	0	0	0
N.S.	1	0.68	0.59	0.64	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.355	0.014	0.549	0.000	0.242	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	44	42	46	0	20	0	0	0
N.S.	1	0.63	0.60	0.66	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.405	0.026	0.542	0.000	0.240	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	48	43	48	0	22	0	0	0
N.S.	1	0.67	0.60	0.67	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.451	0.028	0.550	0.000	0.237	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	193	96	97	117	184	0	0	0
N.S.	1	1.41	0.70	0.71	0.85	1.34	0.00	0.00	0.00
time (sec)	N/A	0.605	0.209	0.712	0.299	0.261	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	161	88	89	93	168	0	84	0
N.S.	1	1.44	0.79	0.79	0.83	1.50	0.00	0.75	0.00
time (sec)	N/A	0.575	0.161	0.685	0.296	0.268	0.000	0.298	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	121	79	79	70	150	0	73	0
N.S.	1	1.44	0.94	0.94	0.83	1.79	0.00	0.87	0.00
time (sec)	N/A	0.464	0.135	0.672	0.304	0.265	0.000	0.284	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	95	100	69	47	134	0	62	0
N.S.	1	1.09	1.15	0.79	0.54	1.54	0.00	0.71	0.00
time (sec)	N/A	0.361	0.049	0.652	0.297	0.271	0.000	0.287	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	84	97	121	86	191	0	95	0
N.S.	1	1.12	1.29	1.61	1.15	2.55	0.00	1.27	0.00
time (sec)	N/A	0.512	0.155	0.657	0.305	0.275	0.000	0.289	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	91	104	102	0	210	0	134	0
N.S.	1	1.11	1.27	1.24	0.00	2.56	0.00	1.63	0.00
time (sec)	N/A	0.521	0.150	0.691	0.000	0.261	0.000	0.294	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	88	76	79	0	149	0	200	0
N.S.	1	1.13	0.97	1.01	0.00	1.91	0.00	2.56	0.00
time (sec)	N/A	0.512	0.186	0.710	0.000	0.273	0.000	0.283	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	115	82	87	0	165	0	250	0
N.S.	1	1.14	0.81	0.86	0.00	1.63	0.00	2.48	0.00
time (sec)	N/A	0.536	0.176	0.722	0.000	0.270	0.000	0.282	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	150	95	95	0	181	0	324	0
N.S.	1	1.15	0.73	0.73	0.00	1.39	0.00	2.49	0.00
time (sec)	N/A	0.573	0.199	0.750	0.000	0.269	0.000	0.276	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	87	86	92	0	58	0	0	0
N.S.	1	0.38	0.38	0.41	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.497	0.075	0.550	0.000	0.243	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	75	74	83	0	49	0	0	0
N.S.	1	0.40	0.40	0.45	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.492	0.051	0.546	0.000	0.246	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	69	65	76	0	42	0	0	0
N.S.	1	0.46	0.43	0.50	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.455	0.045	0.537	0.000	0.254	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	59	58	67	0	33	0	0	0
N.S.	1	0.53	0.52	0.60	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.387	0.019	0.540	0.000	0.240	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	50	50	61	0	26	0	0	0
N.S.	1	0.45	0.45	0.54	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.401	0.032	0.546	0.000	0.253	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	56	54	64	0	33	0	0	0
N.S.	1	0.49	0.47	0.56	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.472	0.040	0.546	0.000	0.263	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	68	63	77	0	88	0	0	0
N.S.	1	0.45	0.41	0.51	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.470	0.053	0.542	0.000	0.261	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	78	75	85	0	98	0	0	0
N.S.	1	0.40	0.39	0.44	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.484	0.069	0.544	0.000	0.250	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	83	79	93	0	104	0	0	0
N.S.	1	0.37	0.35	0.41	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.490	0.056	0.545	0.000	0.246	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	86	74	0	0	0	0	0	0
N.S.	1	0.63	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	172	173	129	0	0	0	0	0	0
N.S.	1	1.01	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	0.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	60	56	62	54	74	0	0	93
N.S.	1	0.73	0.68	0.76	0.66	0.90	0.00	0.00	1.13
time (sec)	N/A	0.464	0.049	0.608	0.232	0.261	0.000	0.000	4.237

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	62	60	64	57	75	0	0	94
N.S.	1	0.75	0.72	0.77	0.69	0.90	0.00	0.00	1.13
time (sec)	N/A	0.479	0.039	0.559	0.250	0.258	0.000	0.000	4.107

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	172	173	110	0	0	0	0	0	0
N.S.	1	1.01	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.563	0.264	0.000	0.000	0.000	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	85	75	0	0	0	0	0	0
N.S.	1	0.62	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.501	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	267	0	0	0	0	0	0
N.S.	1	0.00	3.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	2.386	0.000	0.000	0.000	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	179	0	0	0	0	0	0
N.S.	1	0.00	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.400	0.000	0.000	0.000	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	111	0	0	0	0	0	0
N.S.	1	0.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.796	0.000	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	30	27	49	0	39
N.S.	1	1.00	1.00	1.00	1.67	1.50	2.72	0.00	2.17
time (sec)	N/A	0.217	0.179	1.076	0.213	0.253	0.509	0.000	4.059

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	55	0	79	0	0	106
N.S.	1	1.00	0.76	0.76	0.00	1.10	0.00	0.00	1.47
time (sec)	N/A	0.354	0.325	4.000	0.000	0.249	0.000	0.000	4.223

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	124	97	101	0	174	0	0	192
N.S.	1	0.98	0.76	0.80	0.00	1.37	0.00	0.00	1.51
time (sec)	N/A	0.518	0.411	17.351	0.000	0.259	0.000	0.000	4.726

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	176	152	167	0	309	0	0	314
N.S.	1	0.89	0.77	0.85	0.00	1.57	0.00	0.00	1.59
time (sec)	N/A	0.698	0.523	59.376	0.000	0.261	0.000	0.000	4.832

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	280	0	0	0	0	0	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	2.930	0.000	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	101	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.848	0.000	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	81	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	0.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	80	0	0	78
N.S.	1	1.00	0.93	1.07	0.00	1.74	0.00	0.00	1.70
time (sec)	N/A	0.242	0.323	0.527	0.000	0.259	0.000	0.000	4.079

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	165	0	0	173
N.S.	1	1.00	1.08	0.82	0.00	1.62	0.00	0.00	1.70
time (sec)	N/A	0.395	0.774	0.523	0.000	0.279	0.000	0.000	4.220

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	163	299	140	0	291	0	0	289
N.S.	1	0.98	1.80	0.84	0.00	1.75	0.00	0.00	1.74
time (sec)	N/A	0.586	1.974	0.519	0.000	0.256	0.000	0.000	4.810

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	224	260	218	0	453	0	0	441
N.S.	1	0.94	1.09	0.91	0.00	1.90	0.00	0.00	1.85
time (sec)	N/A	0.793	1.946	0.523	0.000	0.269	0.000	0.000	4.674

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	359	275	133	0	0	0	0	0	0
N.S.	1	0.77	0.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	0.917	0.000	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	127	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.676	0.623	0.000	0.000	0.000	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	82	0	0	81
N.S.	1	1.00	0.93	1.07	0.00	1.78	0.00	0.00	1.76
time (sec)	N/A	0.278	0.369	0.536	0.000	0.255	0.000	0.000	4.129

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	80	0	0	78
N.S.	1	1.00	0.93	1.07	0.00	1.74	0.00	0.00	1.70
time (sec)	N/A	0.241	0.036	0.533	0.000	0.251	0.000	0.000	0.002

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	224	127	0	0	0	0	0	0
N.S.	1	0.81	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	1.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	463	362	201	0	0	0	0	0	0
N.S.	1	0.78	0.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	2.448	0.000	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	239	110	93	0	175	0	0	175
N.S.	1	0.72	0.33	0.28	0.00	0.53	0.00	0.00	0.53
time (sec)	N/A	0.615	0.959	0.543	0.000	0.273	0.000	0.000	4.217

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	108	109	96	0	180	0	0	175
N.S.	1	1.06	1.07	0.94	0.00	1.76	0.00	0.00	1.72
time (sec)	N/A	0.471	0.959	0.585	0.000	0.269	0.000	0.000	4.717

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	103	108	86	0	171	0	0	176
N.S.	1	1.06	1.11	0.89	0.00	1.76	0.00	0.00	1.81
time (sec)	N/A	0.428	0.756	0.548	0.000	0.271	0.000	0.000	4.226

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	165	0	0	173
N.S.	1	1.00	1.08	0.82	0.00	1.62	0.00	0.00	1.70
time (sec)	N/A	0.377	0.173	0.539	0.000	0.262	0.000	0.000	0.003

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	944	624	220	0	0	0	0	0	0
N.S.	1	0.66	0.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.966	2.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	83	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.206	0.000	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	36	38	34	42	0	0	59
N.S.	1	1.00	0.71	0.75	0.67	0.82	0.00	0.00	1.16
time (sec)	N/A	0.372	0.098	1.209	0.199	0.247	0.000	0.000	3.965

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	40	36	44	0	0	59
N.S.	1	1.00	0.69	0.77	0.69	0.85	0.00	0.00	1.13
time (sec)	N/A	0.366	0.076	0.866	0.202	0.247	0.000	0.000	4.010

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	72	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.401	0.294	0.000	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	67	0	0	0	648	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	12.00	0.00	0.00
time (sec)	N/A	0.314	0.025	0.000	0.000	0.000	14.066	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	0.259	0.000	0.000	0.000	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.272	0.000	0.000	0.000	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	73	0	0	0	648	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	11.78	0.00	0.00
time (sec)	N/A	0.319	0.033	0.000	0.000	0.000	12.314	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	119	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	0.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	336	120	188	380	201	0	461	332
N.S.	1	0.98	0.35	0.55	1.11	0.59	0.00	1.35	0.97
time (sec)	N/A	0.507	0.324	0.109	0.288	0.268	0.000	0.302	0.224

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	262	104	172	302	179	0	355	258
N.S.	1	0.98	0.39	0.64	1.13	0.67	0.00	1.32	0.96
time (sec)	N/A	0.450	0.264	0.095	0.282	0.267	0.000	0.313	4.215

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	188	94	156	223	157	0	249	183
N.S.	1	0.97	0.48	0.80	1.15	0.81	0.00	1.28	0.94
time (sec)	N/A	0.381	0.177	0.085	0.282	0.280	0.000	0.295	4.647

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	113	53	127	117	104	0	130	84
N.S.	1	1.06	0.50	1.19	1.09	0.97	0.00	1.21	0.79
time (sec)	N/A	0.314	0.152	0.079	0.280	0.271	0.000	0.292	0.075

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	101	56	144	116	93	0	0	62
N.S.	1	0.97	0.54	1.38	1.12	0.89	0.00	0.00	0.60
time (sec)	N/A	0.300	0.137	0.192	0.197	0.252	0.000	0.000	0.081

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	170	83	218	160	134	0	0	128
N.S.	1	0.94	0.46	1.21	0.89	0.74	0.00	0.00	0.71
time (sec)	N/A	0.361	0.340	0.224	0.189	0.251	0.000	0.000	0.071

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	232	99	292	194	178	0	0	171
N.S.	1	0.91	0.39	1.15	0.76	0.70	0.00	0.00	0.67
time (sec)	N/A	0.413	0.408	0.243	0.196	0.252	0.000	0.000	0.096

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	307	115	366	230	274	0	0	210
N.S.	1	0.94	0.35	1.12	0.70	0.84	0.00	0.00	0.64
time (sec)	N/A	0.473	0.515	0.256	0.212	0.259	0.000	0.000	0.069

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	99	127	88	114	122	124	115	89
N.S.	1	0.78	1.00	0.69	0.90	0.96	0.98	0.91	0.70
time (sec)	N/A	0.487	0.049	0.848	0.198	0.238	0.371	0.259	0.111

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	73	90	64	81	89	88	82	65
N.S.	1	0.81	1.00	0.71	0.90	0.99	0.98	0.91	0.72
time (sec)	N/A	0.455	0.038	0.784	0.205	0.240	0.238	0.277	3.968

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	60	76	55	70	78	76	71	56
N.S.	1	0.79	1.00	0.72	0.92	1.03	1.00	0.93	0.74
time (sec)	N/A	0.448	0.033	0.710	0.197	0.254	0.172	0.276	4.013

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	35	39	31	35	43	39	36	32
N.S.	1	0.90	1.00	0.79	0.90	1.10	1.00	0.92	0.82
time (sec)	N/A	0.412	0.024	0.641	0.194	0.267	0.092	0.273	0.057

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	23
N.S.	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.10
time (sec)	N/A	0.359	0.019	0.573	0.204	0.245	0.051	0.282	0.047

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	37	28	36	35	40	36	36	33
N.S.	1	1.03	0.78	1.00	0.97	1.11	1.00	1.00	0.92
time (sec)	N/A	0.427	0.038	0.531	0.198	0.241	0.073	0.281	0.057

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	72	75	60	69	93	73	57	68
N.S.	1	0.96	1.00	0.80	0.92	1.24	0.97	0.76	0.91
time (sec)	N/A	0.458	0.056	0.563	0.211	0.244	0.202	0.266	0.099

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	99	82	84	97	137	102	80	94
N.S.	1	0.90	0.75	0.76	0.88	1.25	0.93	0.73	0.85
time (sec)	N/A	0.495	0.084	0.546	0.207	0.254	0.333	0.284	4.189

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	130	98	108	145	233	156	96	142
N.S.	1	0.90	0.68	0.74	1.00	1.61	1.08	0.66	0.98
time (sec)	N/A	0.532	0.120	0.539	0.201	0.260	0.469	0.279	4.214

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	336	126	180	380	201	0	461	332
N.S.	1	0.98	0.37	0.52	1.11	0.59	0.00	1.34	0.97
time (sec)	N/A	0.493	0.291	0.104	0.295	0.258	0.000	0.311	4.318

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	263	110	173	302	179	0	355	258
N.S.	1	0.98	0.41	0.64	1.12	0.67	0.00	1.32	0.96
time (sec)	N/A	0.455	0.236	0.093	0.297	0.263	0.000	0.301	4.371

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	187	94	157	223	156	0	248	183
N.S.	1	0.96	0.48	0.81	1.14	0.80	0.00	1.27	0.94
time (sec)	N/A	0.386	0.198	0.081	0.289	0.255	0.000	0.297	0.215

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	79	57	129	118	106	0	130	84
N.S.	1	1.04	0.75	1.70	1.55	1.39	0.00	1.71	1.11
time (sec)	N/A	0.286	0.159	0.142	0.286	0.267	0.000	0.300	0.138

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	140	69	185	133	128	0	0	100
N.S.	1	0.97	0.48	1.28	0.92	0.89	0.00	0.00	0.69
time (sec)	N/A	0.322	0.162	0.149	0.199	0.251	0.000	0.000	0.203

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	173	78	225	153	170	0	63	121
N.S.	1	0.96	0.43	1.24	0.85	0.94	0.00	0.35	0.67
time (sec)	N/A	0.344	0.361	0.214	0.210	0.288	0.000	0.314	0.157

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	244	101	299	192	204	0	0	160
N.S.	1	0.96	0.40	1.17	0.75	0.80	0.00	0.00	0.63
time (sec)	N/A	0.415	0.435	0.227	0.204	0.254	0.000	0.000	4.164

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	313	117	373	226	248	0	0	203
N.S.	1	0.95	0.36	1.13	0.69	0.75	0.00	0.00	0.62
time (sec)	N/A	0.479	0.529	0.247	0.197	0.258	0.000	0.000	0.115

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	93	116	80	103	111	112	184	81
N.S.	1	0.80	1.00	0.69	0.89	0.96	0.97	1.59	0.70
time (sec)	N/A	0.469	0.047	0.940	0.209	0.237	0.341	0.280	0.090

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	78	100	71	92	100	100	160	72
N.S.	1	0.78	1.00	0.71	0.92	1.00	1.00	1.60	0.72
time (sec)	N/A	0.461	0.038	0.837	0.198	0.234	0.257	0.275	4.241

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	53	63	47	59	67	65	136	48
N.S.	1	0.84	1.00	0.75	0.94	1.06	1.03	2.16	0.76
time (sec)	N/A	0.427	0.030	0.730	0.201	0.254	0.157	0.269	0.119

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	41	51	40	46	56	53	112	43
N.S.	1	0.80	1.00	0.78	0.90	1.10	1.04	2.20	0.84
time (sec)	N/A	0.418	0.024	0.697	0.199	0.234	0.105	0.263	0.126

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	32	35	26	66	32
N.S.	1	1.00	1.00	0.88	0.97	1.06	0.79	2.00	0.97
time (sec)	N/A	0.373	0.025	0.645	0.200	0.257	0.146	0.285	0.078

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	52	53	43	49	64	41	74	48
N.S.	1	0.98	1.00	0.81	0.92	1.21	0.77	1.40	0.91
time (sec)	N/A	0.450	0.042	0.629	0.189	0.244	0.128	0.263	3.879

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	63	56	75	100	83	93	71
N.S.	1	0.93	0.89	0.79	1.06	1.41	1.17	1.31	1.00
time (sec)	N/A	0.453	0.047	0.601	0.193	0.254	0.171	0.270	3.919

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	102	89	82	107	163	114	130	104
N.S.	1	0.92	0.80	0.74	0.96	1.47	1.03	1.17	0.94
time (sec)	N/A	0.501	0.073	0.593	0.193	0.243	0.336	0.267	0.121

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	129	98	108	135	207	144	170	131
N.S.	1	0.88	0.67	0.74	0.92	1.42	0.99	1.16	0.90
time (sec)	N/A	0.525	0.111	0.603	0.195	0.252	0.479	0.270	0.155

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	328	120	189	380	201	0	524	332
N.S.	1	0.96	0.35	0.55	1.11	0.59	0.00	1.53	0.97
time (sec)	N/A	0.513	0.347	0.111	0.293	0.278	0.000	0.320	3.979

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	258	110	173	302	179	0	394	258
N.S.	1	0.96	0.41	0.64	1.12	0.67	0.00	1.46	0.96
time (sec)	N/A	0.450	0.247	0.095	0.279	0.257	0.000	0.308	0.109

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	184	94	157	223	156	0	264	183
N.S.	1	0.94	0.48	0.81	1.14	0.80	0.00	1.35	0.94
time (sec)	N/A	0.378	0.198	0.087	0.296	0.257	0.000	0.300	3.881

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	112	55	127	117	107	0	121	84
N.S.	1	1.04	0.51	1.18	1.08	0.99	0.00	1.12	0.78
time (sec)	N/A	0.302	0.147	0.076	0.299	0.259	0.000	0.282	3.872

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	102	57	138	121	67	0	0	86
N.S.	1	0.97	0.54	1.31	1.15	0.64	0.00	0.00	0.82
time (sec)	N/A	0.297	0.139	0.199	0.197	0.252	0.000	0.000	0.060

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	171	85	213	163	119	0	0	137
N.S.	1	0.96	0.47	1.19	0.91	0.66	0.00	0.00	0.77
time (sec)	N/A	0.348	0.344	0.229	0.188	0.247	0.000	0.000	3.875

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	241	101	287	197	161	0	0	178
N.S.	1	0.95	0.40	1.13	0.77	0.63	0.00	0.00	0.70
time (sec)	N/A	0.409	0.437	0.238	0.194	0.249	0.000	0.000	0.063

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	312	117	361	231	205	0	0	217
N.S.	1	0.95	0.36	1.10	0.70	0.62	0.00	0.00	0.66
time (sec)	N/A	0.462	0.534	0.244	0.201	0.254	0.000	0.000	4.266

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	73	90	64	81	89	88	82	67
N.S.	1	0.81	1.00	0.71	0.90	0.99	0.98	0.91	0.74
time (sec)	N/A	0.457	0.043	0.778	0.190	0.247	0.251	0.275	4.029

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	60	76	55	70	78	76	71	56
N.S.	1	0.79	1.00	0.72	0.92	1.03	1.00	0.93	0.74
time (sec)	N/A	0.437	0.032	0.729	0.184	0.240	0.170	0.278	0.106

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	34	40	32	37	43	39	38	35
N.S.	1	0.85	1.00	0.80	0.92	1.08	0.98	0.95	0.88
time (sec)	N/A	0.416	0.025	0.663	0.194	0.233	0.093	0.274	0.078

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	25
N.S.	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.19
time (sec)	N/A	0.363	0.019	0.625	0.198	0.232	0.055	0.272	0.065

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	36	28	36	34	38	36	36	33
N.S.	1	1.03	0.80	1.03	0.97	1.09	1.03	1.03	0.94
time (sec)	N/A	0.427	0.039	0.543	0.200	0.244	0.075	0.273	0.103

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	70	70	60	69	92	75	57	68
N.S.	1	0.96	0.96	0.82	0.95	1.26	1.03	0.78	0.93
time (sec)	N/A	0.466	0.059	0.555	0.184	0.244	0.201	0.266	4.024

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	97	104	84	97	137	102	80	94
N.S.	1	0.90	0.96	0.78	0.90	1.27	0.94	0.74	0.87
time (sec)	N/A	0.493	0.088	0.543	0.194	0.257	0.341	0.263	0.121

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	128	124	108	145	233	156	96	142
N.S.	1	0.90	0.87	0.76	1.01	1.63	1.09	0.67	0.99
time (sec)	N/A	0.527	0.107	0.566	0.199	0.246	0.474	0.266	0.163

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	325	126	180	379	201	0	525	332
N.S.	1	0.95	0.37	0.52	1.10	0.59	0.00	1.53	0.97
time (sec)	N/A	0.510	0.306	0.105	0.284	0.255	0.000	0.331	0.168

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	257	110	173	301	179	0	395	258
N.S.	1	0.96	0.41	0.64	1.12	0.67	0.00	1.47	0.96
time (sec)	N/A	0.434	0.250	0.103	0.276	0.259	0.000	0.302	0.114

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	189	94	157	224	156	0	264	183
N.S.	1	0.97	0.48	0.81	1.15	0.80	0.00	1.35	0.94
time (sec)	N/A	0.386	0.197	0.087	0.289	0.274	0.000	0.289	0.093

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	57	130	118	103	0	122	84
N.S.	1	1.07	0.75	1.71	1.55	1.36	0.00	1.61	1.11
time (sec)	N/A	0.272	0.169	0.143	0.278	0.260	0.000	0.282	0.064

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	141	69	173	140	96	0	59	114
N.S.	1	0.98	0.48	1.20	0.97	0.67	0.00	0.41	0.79
time (sec)	N/A	0.321	0.158	0.146	0.186	0.246	0.000	0.292	3.920

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	173	78	207	161	135	0	59	141
N.S.	1	0.96	0.43	1.14	0.89	0.75	0.00	0.33	0.78
time (sec)	N/A	0.349	0.360	0.243	0.189	0.265	0.000	0.311	0.051

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	245	101	281	199	179	0	0	183
N.S.	1	0.97	0.40	1.11	0.79	0.71	0.00	0.00	0.72
time (sec)	N/A	0.414	0.438	0.231	0.202	0.246	0.000	0.000	3.857

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	314	117	355	231	275	0	0	224
N.S.	1	0.96	0.36	1.09	0.71	0.84	0.00	0.00	0.69
time (sec)	N/A	0.475	0.532	0.251	0.204	0.254	0.000	0.000	0.065

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	101	94	112	0	96	0	0	0
N.S.	1	0.31	0.29	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.410	0.092	0.095	0.000	0.242	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	78	73	96	0	74	0	0	0
N.S.	1	0.33	0.31	0.41	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.389	0.067	0.058	0.000	0.251	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	62	62	80	0	42	0	0	0
N.S.	1	0.42	0.42	0.55	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.374	0.052	0.052	0.000	0.249	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	39	38	50	0	17	0	0	0
N.S.	1	0.58	0.57	0.75	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.342	0.030	0.045	0.000	0.265	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	51	43	57	0	24	0	0	0
N.S.	1	0.71	0.60	0.79	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.361	0.036	0.043	0.000	0.265	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	84	71	102	0	68	0	0	0
N.S.	1	0.49	0.41	0.59	0.00	0.39	0.00	0.00	0.00
time (sec)	N/A	0.402	0.083	0.057	0.000	0.259	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	113	97	175	0	137	0	0	0
N.S.	1	0.43	0.37	0.67	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.432	0.131	0.056	0.000	0.267	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	142	119	247	0	205	0	0	0
N.S.	1	0.40	0.33	0.69	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.458	0.184	0.059	0.000	0.258	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	182	150	250	0	438	1059	561	0
N.S.	1	0.49	0.40	0.67	0.00	1.18	2.85	1.51	0.00
time (sec)	N/A	0.562	0.248	0.606	0.000	0.287	18.755	41.994	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	147	134	234	0	394	500	416	0
N.S.	1	0.50	0.46	0.80	0.00	1.34	1.70	1.41	0.00
time (sec)	N/A	0.516	0.199	0.622	0.000	0.272	10.045	1.810	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	112	115	202	0	317	376	266	0
N.S.	1	0.53	0.54	0.95	0.00	1.49	1.77	1.25	0.00
time (sec)	N/A	0.474	0.172	0.550	0.000	0.257	6.572	0.424	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	72	80	197	0	267	0	0	0
N.S.	1	0.62	0.69	1.70	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.420	0.134	0.526	0.000	0.255	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	93	68	160	0	216	0	0	0
N.S.	1	0.84	0.61	1.44	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.398	0.169	0.553	0.000	0.267	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	126	95	214	0	280	0	0	0
N.S.	1	1.02	0.77	1.74	0.00	2.28	0.00	0.00	0.00
time (sec)	N/A	0.527	0.141	0.579	0.000	0.262	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	158	105	298	0	352	0	0	0
N.S.	1	0.78	0.52	1.47	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.642	0.173	0.562	0.000	0.279	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	190	133	382	0	496	0	0	0
N.S.	1	0.67	0.47	1.35	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.786	0.197	0.572	0.000	0.309	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	102	95	112	0	96	0	0	0
N.S.	1	0.32	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.418	0.093	0.059	0.000	0.248	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	102	97	112	0	96	0	0	0
N.S.	1	0.31	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.410	0.082	0.050	0.000	0.267	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	80	78	96	0	72	0	0	0
N.S.	1	0.34	0.33	0.41	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.398	0.064	0.054	0.000	0.250	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	60	57	69	0	44	0	0	0
N.S.	1	0.41	0.39	0.47	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.375	0.055	0.049	0.000	0.254	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	50	51	65	0	27	0	0	0
N.S.	1	0.46	0.47	0.60	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.365	0.042	0.049	0.000	0.250	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	62	57	85	0	49	0	0	0
N.S.	1	0.54	0.50	0.74	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.377	0.068	0.050	0.000	0.241	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	84	64	102	0	81	0	0	0
N.S.	1	0.49	0.37	0.60	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.404	0.096	0.051	0.000	0.243	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	114	96	175	0	138	0	0	0
N.S.	1	0.43	0.36	0.66	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.433	0.129	0.058	0.000	0.265	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	145	116	247	0	207	0	0	0
N.S.	1	0.40	0.32	0.69	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.483	0.189	0.056	0.000	0.280	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	100	95	112	0	96	0	0	0
N.S.	1	0.31	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.422	0.083	0.052	0.000	0.252	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	80	75	96	0	74	0	0	0
N.S.	1	0.34	0.32	0.40	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.399	0.068	0.051	0.000	0.247	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	59	63	80	0	44	0	0	0
N.S.	1	0.40	0.43	0.54	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.374	0.048	0.051	0.000	0.240	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	41	39	52	0	19	0	0	0
N.S.	1	0.60	0.57	0.76	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.350	0.028	0.050	0.000	0.249	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	48	43	59	0	26	0	0	0
N.S.	1	0.67	0.60	0.82	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.357	0.037	0.046	0.000	0.244	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	85	70	103	0	66	0	0	0
N.S.	1	0.49	0.41	0.60	0.00	0.38	0.00	0.00	0.00
time (sec)	N/A	0.404	0.079	0.052	0.000	0.266	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	110	95	175	0	135	0	0	0
N.S.	1	0.42	0.36	0.67	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.429	0.171	0.056	0.000	0.256	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	143	119	247	0	201	0	0	0
N.S.	1	0.40	0.33	0.69	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.473	0.183	0.057	0.000	0.280	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	182	150	250	0	438	1059	561	0
N.S.	1	0.49	0.40	0.67	0.00	1.17	2.82	1.50	0.00
time (sec)	N/A	0.557	0.260	0.575	0.000	0.266	18.111	46.812	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	147	134	234	0	394	500	416	0
N.S.	1	0.50	0.46	0.80	0.00	1.34	1.71	1.42	0.00
time (sec)	N/A	0.502	0.201	0.569	0.000	0.269	9.966	1.980	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	110	115	202	0	316	376	266	0
N.S.	1	0.52	0.54	0.95	0.00	1.48	1.77	1.25	0.00
time (sec)	N/A	0.466	0.168	0.540	0.000	0.265	6.474	0.447	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	72	80	197	0	267	0	0	0
N.S.	1	0.62	0.69	1.70	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.438	0.132	0.511	0.000	0.255	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	93	68	154	0	212	0	0	0
N.S.	1	0.83	0.61	1.38	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	0.398	0.172	0.529	0.000	0.270	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	127	95	202	0	279	0	0	0
N.S.	1	1.02	0.77	1.63	0.00	2.25	0.00	0.00	0.00
time (sec)	N/A	0.555	0.133	0.536	0.000	0.267	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	159	105	286	0	351	0	0	0
N.S.	1	0.82	0.54	1.47	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	0.675	0.159	0.546	0.000	0.277	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	191	131	370	0	495	0	0	0
N.S.	1	0.71	0.49	1.37	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.819	0.194	0.552	0.000	0.315	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	100	95	112	0	96	0	0	0
N.S.	1	0.31	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.424	0.100	0.054	0.000	0.256	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	103	97	112	0	96	0	0	0
N.S.	1	0.32	0.30	0.35	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.427	0.088	0.053	0.000	0.256	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	77	79	96	0	74	0	0	0
N.S.	1	0.33	0.34	0.41	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.395	0.072	0.053	0.000	0.254	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	62	57	69	0	42	0	0	0
N.S.	1	0.42	0.39	0.47	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.371	0.057	0.052	0.000	0.250	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	47	47	67	0	25	0	0	0
N.S.	1	0.44	0.44	0.63	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.364	0.039	0.051	0.000	0.257	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	62	55	87	0	47	0	0	0
N.S.	1	0.55	0.49	0.77	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.377	0.059	0.053	0.000	0.258	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	79	63	102	0	81	0	0	0
N.S.	1	0.47	0.38	0.61	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.400	0.093	0.053	0.000	0.248	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	113	95	175	0	137	0	0	0
N.S.	1	0.43	0.36	0.66	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.431	0.133	0.059	0.000	0.275	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	140	108	247	0	208	0	0	0
N.S.	1	0.39	0.30	0.69	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.468	0.236	0.060	0.000	0.246	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	53	52	63	44	72	0	0	0
N.S.	1	0.66	0.65	0.79	0.55	0.90	0.00	0.00	0.00
time (sec)	N/A	0.500	0.044	0.039	0.238	0.262	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	49	45	53	0	24	0	0	46
N.S.	1	0.64	0.59	0.70	0.00	0.32	0.00	0.00	0.61
time (sec)	N/A	0.483	0.032	0.046	0.000	0.241	0.000	0.000	4.050

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	43	43	52	0	21	0	0	45
N.S.	1	0.61	0.61	0.73	0.00	0.30	0.00	0.00	0.63
time (sec)	N/A	0.401	0.035	0.059	0.000	0.246	0.000	0.000	4.063

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	39	38	50	0	17	0	0	0
N.S.	1	0.58	0.57	0.75	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.343	0.016	0.043	0.000	0.247	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	43	41	50	0	21	0	0	0
N.S.	1	0.61	0.59	0.71	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.479	0.032	0.049	0.000	0.239	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	53	0	21	0	0	63
N.S.	1	1.00	1.00	1.15	0.00	0.46	0.00	0.00	1.37
time (sec)	N/A	0.462	0.029	0.051	0.000	0.254	0.000	0.000	4.319

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	149	93	134	0	222	0	128	0
N.S.	1	0.93	0.58	0.84	0.00	1.39	0.00	0.80	0.00
time (sec)	N/A	0.623	0.221	0.579	0.000	0.251	0.000	0.291	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	112	84	124	0	204	0	116	0
N.S.	1	0.91	0.68	1.01	0.00	1.66	0.00	0.94	0.00
time (sec)	N/A	0.591	0.173	0.585	0.000	0.267	0.000	0.287	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	91	77	114	0	188	0	106	0
N.S.	1	0.93	0.79	1.16	0.00	1.92	0.00	1.08	0.00
time (sec)	N/A	0.482	0.077	0.576	0.000	0.252	0.000	0.295	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	72	80	197	0	267	0	0	0
N.S.	1	0.62	0.69	1.70	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.418	0.037	0.579	0.000	0.259	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	76	82	145	0	252	0	127	0
N.S.	1	0.65	0.70	1.24	0.00	2.15	0.00	1.09	0.00
time (sec)	N/A	0.579	0.193	0.575	0.000	0.256	0.000	0.361	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	97	78	142	0	177	0	194	0
N.S.	1	0.87	0.70	1.28	0.00	1.59	0.00	1.75	0.00
time (sec)	N/A	0.599	0.136	0.608	0.000	0.249	0.000	0.376	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	119	86	151	0	201	0	231	0
N.S.	1	0.87	0.63	1.10	0.00	1.47	0.00	1.69	0.00
time (sec)	N/A	0.613	0.141	0.581	0.000	0.252	0.000	0.690	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	151	94	159	0	217	0	316	0
N.S.	1	0.97	0.60	1.02	0.00	1.39	0.00	2.03	0.00
time (sec)	N/A	0.664	0.144	0.586	0.000	0.263	0.000	2.078	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	173	102	167	0	233	0	362	0
N.S.	1	0.96	0.56	0.92	0.00	1.29	0.00	2.00	0.00
time (sec)	N/A	0.679	0.168	0.575	0.000	0.265	0.000	2.364	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	74	71	89	0	48	0	0	0
N.S.	1	0.40	0.38	0.48	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.538	0.058	0.090	0.000	0.251	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	68	63	82	0	41	0	0	0
N.S.	1	0.45	0.41	0.54	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.531	0.050	0.050	0.000	0.249	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	58	56	73	0	32	0	0	0
N.S.	1	0.51	0.50	0.65	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.445	0.036	0.049	0.000	0.246	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	50	51	65	0	27	0	0	0
N.S.	1	0.46	0.47	0.60	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.363	0.020	0.049	0.000	0.251	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	55	51	67	0	32	0	0	0
N.S.	1	0.51	0.47	0.62	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.514	0.045	0.052	0.000	0.253	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	67	61	82	0	85	0	0	0
N.S.	1	0.46	0.41	0.56	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.524	0.059	0.051	0.000	0.258	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	77	73	90	0	93	0	0	0
N.S.	1	0.41	0.39	0.48	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.534	0.072	0.059	0.000	0.251	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	83	77	98	0	101	0	0	0
N.S.	1	0.37	0.35	0.44	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.526	0.079	0.054	0.000	0.246	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	95	91	106	0	109	0	0	0
N.S.	1	0.36	0.34	0.40	0.00	0.41	0.00	0.00	0.00
time (sec)	N/A	0.533	0.072	0.056	0.000	0.259	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	55	49	65	46	73	0	0	0
N.S.	1	0.68	0.60	0.80	0.57	0.90	0.00	0.00	0.00
time (sec)	N/A	0.503	0.051	0.039	0.244	0.243	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	50	45	53	0	24	0	0	46
N.S.	1	0.66	0.59	0.70	0.00	0.32	0.00	0.00	0.61
time (sec)	N/A	0.504	0.035	0.046	0.000	0.254	0.000	0.000	3.942

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	44	44	52	0	21	0	0	45
N.S.	1	0.61	0.61	0.72	0.00	0.29	0.00	0.00	0.62
time (sec)	N/A	0.407	0.031	0.043	0.000	0.241	0.000	0.000	4.012

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	41	39	52	0	19	0	0	0
N.S.	1	0.60	0.57	0.76	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.357	0.016	0.046	0.000	0.240	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	45	40	50	0	21	0	0	0
N.S.	1	0.65	0.58	0.72	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.496	0.030	0.048	0.000	0.244	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	46	53	0	21	0	0	63
N.S.	1	1.00	0.98	1.13	0.00	0.45	0.00	0.00	1.34
time (sec)	N/A	0.476	0.029	0.046	0.000	0.244	0.000	0.000	4.109

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	149	93	134	0	222	0	128	0
N.S.	1	0.91	0.57	0.82	0.00	1.36	0.00	0.79	0.00
time (sec)	N/A	0.643	0.213	0.577	0.000	0.264	0.000	0.299	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	116	84	125	0	204	0	117	0
N.S.	1	0.94	0.68	1.01	0.00	1.65	0.00	0.94	0.00
time (sec)	N/A	0.575	0.161	0.574	0.000	0.256	0.000	0.297	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	92	100	114	0	188	0	106	0
N.S.	1	0.93	1.01	1.15	0.00	1.90	0.00	1.07	0.00
time (sec)	N/A	0.483	0.076	0.578	0.000	0.255	0.000	0.282	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	72	80	197	0	267	0	0	0
N.S.	1	0.62	0.69	1.70	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.439	0.037	0.546	0.000	0.255	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	79	82	145	0	252	0	127	0
N.S.	1	0.68	0.70	1.24	0.00	2.15	0.00	1.09	0.00
time (sec)	N/A	0.598	0.187	0.633	0.000	0.253	0.000	0.369	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	97	78	143	0	176	0	194	0
N.S.	1	0.87	0.70	1.28	0.00	1.57	0.00	1.73	0.00
time (sec)	N/A	0.599	0.131	0.589	0.000	0.255	0.000	0.373	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	120	86	150	0	200	0	231	0
N.S.	1	0.86	0.61	1.07	0.00	1.43	0.00	1.65	0.00
time (sec)	N/A	0.654	0.135	0.592	0.000	0.249	0.000	0.671	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	151	94	159	0	216	0	316	0
N.S.	1	0.97	0.60	1.02	0.00	1.38	0.00	2.03	0.00
time (sec)	N/A	0.669	0.139	0.601	0.000	0.255	0.000	1.773	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	174	102	167	0	232	0	362	0
N.S.	1	0.96	0.56	0.92	0.00	1.28	0.00	2.00	0.00
time (sec)	N/A	0.703	0.150	0.610	0.000	0.264	0.000	2.566	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	72	71	89	0	48	0	0	0
N.S.	1	0.39	0.38	0.48	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.543	0.062	0.095	0.000	0.241	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	66	62	82	0	41	0	0	0
N.S.	1	0.44	0.41	0.54	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.529	0.053	0.054	0.000	0.244	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	56	55	73	0	32	0	0	0
N.S.	1	0.50	0.49	0.65	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.472	0.038	0.052	0.000	0.241	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	47	47	67	0	25	0	0	0
N.S.	1	0.44	0.44	0.63	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.363	0.019	0.049	0.000	0.247	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	53	51	66	0	32	0	0	0
N.S.	1	0.49	0.47	0.61	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.516	0.047	0.057	0.000	0.259	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	65	60	82	0	83	0	0	0
N.S.	1	0.45	0.41	0.56	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.516	0.057	0.060	0.000	0.250	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	75	72	90	0	91	0	0	0
N.S.	1	0.40	0.39	0.48	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.526	0.069	0.055	0.000	0.243	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	80	76	98	0	99	0	0	0
N.S.	1	0.36	0.34	0.44	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.532	0.061	0.056	0.000	0.242	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	93	90	106	0	107	0	0	0
N.S.	1	0.35	0.34	0.40	0.00	0.41	0.00	0.00	0.00
time (sec)	N/A	0.527	0.072	0.055	0.000	0.252	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	258	123	0	0	0	0	0	0
N.S.	1	1.68	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	0.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	147	94	0	0	0	0	0	0
N.S.	1	0.98	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	0.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	290	180	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	1.356	0.000	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	239	146	0	0	0	0	0	0
N.S.	1	0.81	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	0.710	0.000	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	157	112	0	0	0	0	0	0
N.S.	1	0.86	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.433	0.647	0.000	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [129] had the largest ratio of [1.785709999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	14	1.10	10	1.400
2	A	12	11	1.06	10	1.100
3	A	9	8	1.03	8	1.000
4	A	6	5	1.00	6	0.833
5	A	7	6	1.18	10	0.600
6	A	4	3	1.17	10	0.300
7	A	6	5	1.58	10	0.500
8	A	8	7	1.20	10	0.700
9	A	10	9	1.36	10	0.900
10	A	4	4	1.00	12	0.333
11	A	4	4	1.00	12	0.333
12	A	4	4	1.00	10	0.400
13	A	4	4	1.00	8	0.500
14	A	4	4	1.00	12	0.333
15	A	4	4	1.00	12	0.333
16	A	4	4	1.00	12	0.333
17	A	4	4	1.00	12	0.333
18	A	4	3	1.00	12	0.250
19	A	4	3	1.00	10	0.300
20	A	4	3	1.00	8	0.375
21	A	12	11	1.80	12	0.917
22	A	7	6	1.12	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	15	14	1.09	12	1.167
24	A	17	16	1.39	12	1.333
25	A	4	4	1.00	12	0.333
26	A	4	4	1.00	12	0.333
27	A	4	4	1.00	10	0.400
28	A	4	4	1.00	8	0.500
29	A	4	4	1.00	12	0.333
30	A	4	4	1.00	12	0.333
31	A	4	4	1.00	12	0.333
32	A	4	4	1.00	12	0.333
33	A	12	11	1.10	12	0.917
34	A	10	9	1.06	12	0.750
35	A	8	7	0.98	10	0.700
36	A	6	5	1.00	8	0.625
37	A	7	6	1.20	12	0.500
38	A	4	3	1.16	12	0.250
39	A	7	6	1.48	12	0.500
40	A	10	9	1.18	12	0.750
41	A	13	12	1.36	12	1.000
42	A	4	4	1.00	12	0.333
43	A	4	4	1.00	12	0.333
44	A	4	4	1.00	10	0.400
45	A	4	4	1.00	8	0.500
46	A	4	4	1.00	12	0.333
47	A	4	4	1.00	12	0.333
48	A	4	4	1.00	12	0.333
49	A	4	4	1.00	12	0.333
50	A	4	3	1.00	12	0.250
51	A	4	3	1.00	12	0.250
52	A	4	3	1.00	10	0.300
53	A	4	3	1.00	8	0.375
54	A	12	11	1.76	12	0.917
55	A	7	6	1.02	12	0.500
56	A	13	12	1.10	12	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	15	14	1.32	12	1.167
58	A	17	16	1.17	12	1.333
59	A	16	15	1.08	14	1.071
60	A	14	13	1.06	14	0.929
61	A	12	11	1.04	14	0.786
62	A	8	7	1.02	12	0.583
63	A	7	6	1.06	10	0.600
64	A	17	16	0.89	14	1.143
65	A	13	12	0.82	14	0.857
66	A	14	13	0.82	14	0.929
67	A	16	15	0.85	14	1.071
68	A	17	16	1.08	14	1.143
69	A	15	14	1.05	14	1.000
70	A	13	12	1.04	14	0.857
71	A	9	8	1.02	12	0.667
72	A	8	7	1.04	10	0.700
73	A	18	17	0.89	14	1.214
74	A	13	12	0.82	14	0.857
75	A	14	13	0.82	14	0.929
76	A	16	15	0.85	14	1.071
77	A	18	17	1.08	14	1.214
78	A	16	15	1.07	14	1.071
79	A	14	13	1.06	14	0.929
80	A	9	8	1.02	12	0.667
81	A	8	7	1.05	10	0.700
82	A	20	19	0.90	14	1.357
83	A	14	13	0.85	14	0.929
84	A	15	14	0.83	14	1.000
85	A	17	16	0.86	14	1.143
86	A	17	16	1.08	14	1.143
87	A	15	14	1.06	14	1.000
88	A	13	12	1.04	14	0.857
89	A	9	8	1.02	12	0.667
90	A	8	7	1.05	10	0.700

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	18	17	0.89	14	1.214
92	A	13	12	0.82	14	0.857
93	A	14	13	0.81	14	0.929
94	A	16	15	0.85	14	1.071
95	A	16	15	1.08	14	1.071
96	A	14	13	1.05	14	0.929
97	A	12	11	1.04	14	0.786
98	A	8	7	1.02	12	0.583
99	A	7	6	1.04	10	0.600
100	A	17	16	0.89	14	1.143
101	A	13	12	0.82	14	0.857
102	A	14	13	0.81	14	0.929
103	A	16	15	0.85	14	1.071
104	A	19	18	1.08	14	1.286
105	A	17	16	1.07	14	1.143
106	A	15	14	1.06	14	1.000
107	A	10	9	1.02	12	0.750
108	A	9	8	1.05	10	0.800
109	A	21	20	0.90	14	1.429
110	A	14	13	0.84	14	0.929
111	A	15	14	0.83	14	1.000
112	A	17	16	0.86	14	1.143
113	A	17	16	0.93	12	1.333
114	A	13	12	0.90	10	1.200
115	A	12	11	0.90	8	1.375
116	A	17	16	0.92	12	1.333
117	A	13	12	0.92	12	1.000
118	A	14	13	0.93	12	1.083
119	A	16	15	0.96	12	1.250
120	A	9	8	1.06	12	0.667
121	A	5	4	1.05	10	0.400
122	A	4	3	1.08	8	0.375
123	A	5	4	1.01	12	0.333
124	A	4	3	1.06	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	5	4	1.03	12	0.333
126	A	21	20	0.91	14	1.429
127	A	17	16	0.89	12	1.333
128	A	16	15	0.87	10	1.500
129	A	26	25	0.94	14	1.786
130	A	13	12	0.92	14	0.857
131	A	14	13	0.91	14	0.929
132	A	6	6	1.00	12	0.500
133	A	9	8	1.18	12	0.667
134	A	4	4	1.00	12	0.333
135	A	5	4	1.28	10	0.400
136	A	5	4	1.25	12	0.333
137	A	4	4	1.00	12	0.333
138	A	9	8	1.19	12	0.667
139	A	3	2	1.00	14	0.143
140	A	3	2	1.00	14	0.143
141	A	3	2	1.00	14	0.143
142	A	3	2	1.00	14	0.143
143	A	3	2	1.00	14	0.143
144	A	3	2	1.00	14	0.143
145	A	3	2	1.00	12	0.167
146	A	3	2	1.00	12	0.167
147	A	3	2	1.00	14	0.143
148	A	3	2	1.00	12	0.167
149	A	9	8	1.02	12	0.667
150	A	4	3	1.00	10	0.300
151	A	3	2	1.00	8	0.250
152	A	5	4	1.00	12	0.333
153	A	3	2	1.00	12	0.167
154	A	4	3	1.00	12	0.250
155	A	7	6	1.02	12	0.500
156	A	7	6	1.02	12	0.500
157	A	4	3	1.23	16	0.188
158	A	11	10	0.97	16	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	9	8	0.93	16	0.500
160	A	9	8	0.90	16	0.500
161	A	6	5	0.91	14	0.357
162	A	7	6	1.06	16	0.375
163	A	5	4	1.00	16	0.250
164	A	5	4	0.96	16	0.250
165	A	10	9	1.28	16	0.562
166	A	11	10	1.44	16	0.625
167	A	5	5	1.12	18	0.278
168	A	5	5	0.97	18	0.278
169	A	5	5	0.97	18	0.278
170	A	5	5	0.97	18	0.278
171	A	5	5	0.85	18	0.278
172	C	1	1	1.86	16	0.062
173	A	5	5	0.94	18	0.278
174	A	4	4	1.00	18	0.222
175	A	5	5	0.97	18	0.278
176	A	5	5	0.97	18	0.278
177	A	5	4	1.15	18	0.222
178	A	10	9	0.96	18	0.500
179	A	7	6	0.97	18	0.333
180	A	10	9	0.99	18	0.500
181	A	10	9	1.08	16	0.562
182	A	11	10	1.01	18	0.556
183	A	5	4	1.00	18	0.222
184	A	5	4	0.99	18	0.222
185	A	9	8	1.07	18	0.444
186	A	9	8	1.07	18	0.444
187	A	5	5	1.14	18	0.278
188	A	5	5	0.91	18	0.278
189	A	6	6	0.81	18	0.333
190	A	5	5	0.94	18	0.278
191	A	4	4	1.00	18	0.222
192	A	5	5	0.96	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	5	5	0.90	18	0.278
194	A	4	4	1.00	18	0.222
195	A	5	5	0.90	18	0.278
196	A	5	5	0.91	18	0.278
197	A	3	2	1.00	18	0.111
198	A	11	10	1.02	18	0.556
199	A	10	9	1.02	18	0.500
200	A	8	7	1.08	16	0.438
201	A	5	4	1.00	18	0.222
202	A	5	4	1.00	18	0.222
203	A	5	4	1.06	18	0.222
204	A	10	9	1.37	18	0.500
205	A	10	9	1.30	18	0.500
206	A	4	4	1.00	18	0.222
207	A	5	5	0.86	18	0.278
208	A	5	5	0.86	18	0.278
209	A	5	5	0.85	18	0.278
210	A	5	5	1.00	16	0.312
211	A	4	4	1.00	18	0.222
212	A	5	5	1.00	18	0.278
213	A	5	5	0.97	18	0.278
214	A	5	5	0.92	18	0.278
215	A	5	5	0.90	18	0.278
216	A	3	2	1.00	18	0.111
217	A	13	12	1.00	18	0.667
218	A	13	12	1.02	18	0.667
219	A	9	8	1.08	16	0.500
220	A	7	6	0.98	18	0.333
221	A	5	4	1.00	18	0.222
222	A	3	2	1.00	18	0.111
223	A	6	5	1.00	18	0.278
224	A	8	7	1.00	18	0.389
225	A	10	9	1.03	18	0.500
226	A	9	8	0.77	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	8	7	0.80	18	0.389
228	A	7	6	1.03	18	0.333
229	A	5	4	1.13	18	0.222
230	A	1	1	1.00	18	0.056
231	A	6	5	0.89	18	0.278
232	A	6	5	0.89	18	0.278
233	A	7	6	0.80	18	0.333
234	A	8	7	0.76	18	0.389
235	A	5	5	1.15	20	0.250
236	A	5	5	1.15	20	0.250
237	A	5	5	1.15	20	0.250
238	A	5	5	1.16	20	0.250
239	A	5	5	1.17	20	0.250
240	A	5	5	1.16	20	0.250
241	A	5	5	1.15	20	0.250
242	A	5	5	1.15	20	0.250
243	A	8	7	0.80	20	0.350
244	A	7	6	0.87	20	0.300
245	A	5	4	0.98	20	0.200
246	A	1	1	1.00	20	0.050
247	A	7	6	0.85	20	0.300
248	A	7	6	0.84	20	0.300
249	A	7	6	0.84	20	0.300
250	A	8	7	0.78	20	0.350
251	A	9	8	0.76	20	0.400
252	A	10	9	1.20	20	0.450
253	A	9	8	1.21	20	0.400
254	A	8	7	1.23	20	0.350
255	A	7	6	1.25	20	0.300
256	A	5	4	1.40	20	0.200
257	A	1	1	1.00	20	0.050
258	A	5	4	1.00	20	0.200
259	A	6	5	0.86	20	0.250
260	A	7	6	0.80	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	11	10	1.08	20	0.500
262	A	10	9	1.09	20	0.450
263	A	9	8	1.09	20	0.400
264	A	8	7	1.08	20	0.350
265	A	7	6	1.03	20	0.300
266	A	6	5	1.00	20	0.250
267	A	7	6	1.09	20	0.300
268	A	8	7	1.10	20	0.350
269	A	9	8	1.15	20	0.400
270	A	11	10	0.73	20	0.500
271	A	10	9	0.74	20	0.450
272	A	9	8	0.75	20	0.400
273	A	8	7	0.78	20	0.350
274	A	7	6	0.84	20	0.300
275	A	5	4	0.98	20	0.200
276	A	1	1	1.00	20	0.050
277	A	6	5	0.84	20	0.250
278	A	7	6	0.77	20	0.300
279	A	7	6	1.13	9	0.667
280	A	6	5	1.01	8	0.625
281	A	3	2	1.00	11	0.182
282	A	6	5	0.89	10	0.500
283	A	8	7	1.08	11	0.636
284	A	7	6	1.06	10	0.600
285	A	9	8	1.04	13	0.615
286	A	8	7	0.94	12	0.583
287	A	3	2	1.00	11	0.182
288	A	4	3	1.00	10	0.300
289	A	7	6	1.09	13	0.462
290	A	6	5	1.12	12	0.417
291	A	5	4	1.00	11	0.364
292	A	3	2	1.00	10	0.200
293	A	11	10	1.09	13	0.769
294	A	4	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	3	2	1.00	21	0.095
296	A	5	4	0.84	21	0.190
297	A	4	3	0.93	19	0.158
298	A	1	1	1.00	18	0.056
299	A	5	4	0.83	21	0.190
300	A	5	4	0.78	21	0.190
301	A	5	5	1.06	23	0.217
302	A	5	5	1.08	23	0.217
303	A	5	5	1.11	21	0.238
304	A	5	5	1.16	20	0.250
305	A	7	6	1.15	23	0.261
306	A	7	6	1.17	23	0.261
307	A	8	7	1.12	23	0.304
308	A	9	8	1.16	23	0.348
309	A	10	9	1.18	23	0.391
310	A	15	14	0.75	23	0.609
311	A	13	12	0.75	23	0.522
312	A	8	7	0.81	21	0.333
313	A	7	6	0.85	20	0.300
314	A	10	9	0.77	23	0.391
315	A	10	9	0.78	23	0.391
316	A	12	11	0.70	23	0.478
317	A	14	13	0.69	23	0.565
318	A	16	15	0.68	23	0.652
319	A	7	6	0.83	13	0.462
320	A	6	5	0.91	12	0.417
321	A	5	4	0.91	15	0.267
322	A	4	3	1.03	14	0.214
323	A	5	4	0.91	13	0.308
324	A	4	3	1.00	12	0.250
325	A	4	3	0.99	15	0.200
326	A	1	1	1.00	14	0.071
327	A	4	3	0.96	13	0.231
328	A	3	2	0.91	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	6	5	0.78	15	0.333
330	A	5	4	0.87	14	0.286
331	A	5	4	0.84	13	0.308
332	A	4	3	0.95	12	0.250
333	A	6	5	0.86	15	0.333
334	A	5	4	0.94	14	0.286
335	A	5	4	1.03	23	0.174
336	A	7	6	1.06	23	0.261
337	A	6	5	1.14	21	0.238
338	A	5	4	1.40	20	0.200
339	A	6	5	0.87	23	0.217
340	A	6	5	0.85	23	0.217
341	A	5	5	1.02	23	0.217
342	A	5	5	1.03	23	0.217
343	A	9	8	1.12	21	0.381
344	A	8	7	1.08	20	0.350
345	A	10	9	1.15	23	0.391
346	A	10	9	1.18	23	0.391
347	A	12	11	1.12	23	0.478
348	A	14	13	1.15	23	0.565
349	A	16	15	1.17	23	0.652
350	A	10	9	0.73	23	0.391
351	A	9	8	0.76	23	0.348
352	A	8	7	0.79	21	0.333
353	A	7	6	0.84	20	0.300
354	A	8	7	0.76	23	0.304
355	A	8	7	0.79	23	0.304
356	A	9	8	0.74	23	0.348
357	A	10	9	0.72	23	0.391
358	A	11	10	0.71	23	0.435
359	A	8	7	0.81	24	0.292
360	A	5	4	1.02	24	0.167
361	A	1	1	1.00	22	0.045
362	A	4	3	1.20	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	4	3	1.19	24	0.125
364	A	3	2	1.00	18	0.111
365	A	3	2	1.00	18	0.111
366	A	3	2	1.00	18	0.111
367	A	3	2	1.00	16	0.125
368	A	3	2	1.00	18	0.111
369	A	3	2	1.00	18	0.111
370	A	4	3	1.01	18	0.167
371	A	8	7	0.93	18	0.389
372	A	3	2	1.08	20	0.100
373	A	3	2	1.08	20	0.100
374	A	3	2	1.08	20	0.100
375	A	3	2	1.08	20	0.100
376	A	3	2	1.08	20	0.100
377	A	4	3	1.12	20	0.150
378	A	5	4	1.02	20	0.200
379	A	12	11	1.01	20	0.550
380	A	10	9	1.00	20	0.450
381	A	9	8	0.90	20	0.400
382	A	4	3	1.26	18	0.167
383	A	9	8	1.10	20	0.400
384	A	12	11	0.99	20	0.550
385	A	14	13	1.04	20	0.650
386	A	16	15	1.04	20	0.750
387	A	6	6	0.79	22	0.273
388	A	6	6	0.85	22	0.273
389	A	6	6	0.82	22	0.273
390	A	7	7	1.25	22	0.318
391	A	6	6	1.00	20	0.300
392	A	6	6	0.97	22	0.273
393	A	6	6	0.98	22	0.273
394	A	6	6	0.93	22	0.273
395	A	6	6	0.93	22	0.273
396	A	12	11	0.99	22	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	5	4	1.03	22	0.182
398	A	10	9	1.06	22	0.409
399	A	11	10	1.12	20	0.500
400	A	12	11	1.04	22	0.500
401	A	14	13	1.04	22	0.591
402	A	16	15	0.96	22	0.682
403	A	18	17	1.04	22	0.773
404	A	6	6	0.80	22	0.273
405	A	7	7	0.93	22	0.318
406	A	6	6	0.92	22	0.273
407	A	6	6	0.85	22	0.273
408	A	6	6	0.88	20	0.300
409	A	6	6	0.96	22	0.273
410	A	6	6	0.93	22	0.273
411	A	6	6	0.92	22	0.273
412	A	6	6	0.90	22	0.273
413	A	15	14	1.05	22	0.636
414	A	14	13	1.01	22	0.591
415	A	11	10	1.04	22	0.455
416	A	9	8	1.12	20	0.400
417	A	3	2	1.00	22	0.091
418	A	10	9	1.01	22	0.409
419	A	12	11	0.95	22	0.500
420	A	14	13	0.93	22	0.591
421	A	6	6	0.83	22	0.273
422	A	6	6	0.81	22	0.273
423	A	6	6	0.85	22	0.273
424	A	6	6	0.83	20	0.300
425	A	6	6	1.10	22	0.273
426	A	7	7	1.28	22	0.318
427	A	6	6	0.96	22	0.273
428	A	6	6	0.96	22	0.273
429	A	16	15	1.05	22	0.682
430	A	15	14	1.02	22	0.636

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	12	11	1.06	22	0.500
432	A	10	9	1.16	20	0.450
433	A	9	8	0.97	22	0.364
434	A	9	8	0.92	22	0.364
435	A	4	3	1.00	22	0.136
436	A	11	10	0.95	22	0.455
437	A	14	13	0.96	22	0.591
438	A	13	12	0.78	22	0.545
439	A	11	10	0.77	22	0.455
440	A	10	9	0.73	22	0.409
441	A	6	5	1.04	22	0.227
442	A	5	4	1.05	22	0.182
443	A	9	8	0.68	22	0.364
444	A	11	10	0.64	22	0.455
445	A	13	12	0.63	22	0.545
446	A	13	12	1.02	24	0.500
447	A	12	11	1.05	24	0.458
448	A	11	10	1.08	24	0.417
449	A	10	9	1.14	24	0.375
450	A	9	8	1.16	24	0.333
451	A	10	9	1.19	24	0.375
452	A	11	10	1.16	24	0.417
453	A	12	11	1.16	24	0.458
454	A	13	12	1.13	24	0.500
455	A	12	11	0.63	24	0.458
456	A	11	10	0.56	24	0.417
457	A	7	6	1.04	24	0.250
458	A	6	5	1.04	24	0.208
459	A	9	8	0.68	24	0.333
460	A	12	11	0.63	24	0.458
461	A	14	13	0.63	24	0.542
462	A	16	15	0.63	24	0.625
463	A	11	10	0.67	24	0.417
464	A	9	8	0.70	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
465	A	9	8	0.66	24	0.333
466	A	5	4	1.10	24	0.167
467	A	5	4	1.08	24	0.167
468	A	10	9	0.74	24	0.375
469	A	11	10	0.64	24	0.417
470	A	13	12	0.63	24	0.500
471	A	17	16	1.06	24	0.667
472	A	15	14	1.09	24	0.583
473	A	13	12	1.13	24	0.500
474	A	11	10	1.17	24	0.417
475	A	11	10	1.11	24	0.417
476	A	11	10	1.23	24	0.417
477	A	14	13	1.14	24	0.542
478	A	15	14	1.10	24	0.583
479	A	17	16	1.12	24	0.667
480	A	15	14	0.61	24	0.583
481	A	13	12	0.64	24	0.500
482	A	11	10	0.66	24	0.417
483	A	9	8	0.69	24	0.333
484	A	9	8	0.66	24	0.333
485	A	6	5	1.06	24	0.208
486	A	6	5	1.03	24	0.208
487	A	10	9	0.61	24	0.375
488	A	13	12	0.60	24	0.500
489	A	4	3	1.00	25	0.120
490	A	7	6	1.06	25	0.240
491	A	6	5	1.04	23	0.217
492	A	5	4	1.05	22	0.182
493	A	5	4	1.03	25	0.160
494	A	3	2	1.00	25	0.080
495	A	4	3	1.00	25	0.120
496	A	6	5	1.04	25	0.200
497	A	8	7	1.10	25	0.280
498	A	12	11	1.21	27	0.407

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	11	10	1.18	27	0.370
500	A	10	9	1.14	25	0.360
501	A	9	8	1.16	24	0.333
502	A	9	8	1.23	27	0.296
503	A	8	7	1.31	27	0.259
504	A	8	7	1.13	27	0.259
505	A	8	7	1.09	27	0.259
506	A	8	7	1.07	27	0.259
507	A	15	14	0.60	27	0.519
508	A	13	12	0.61	27	0.444
509	A	11	10	0.63	25	0.400
510	A	9	8	0.68	24	0.333
511	A	10	9	0.68	27	0.333
512	A	6	5	1.10	27	0.185
513	A	7	6	1.04	27	0.222
514	A	9	8	1.06	27	0.296
515	A	11	10	0.91	27	0.370
516	A	6	5	0.93	27	0.185
517	A	7	6	1.08	27	0.222
518	A	6	5	1.06	25	0.200
519	A	5	4	1.10	24	0.167
520	A	5	4	1.09	27	0.148
521	A	4	3	1.11	27	0.111
522	A	5	4	1.06	27	0.148
523	A	7	6	1.13	27	0.222
524	A	17	16	1.20	27	0.593
525	A	15	14	1.18	27	0.519
526	A	13	12	1.16	25	0.480
527	A	11	10	1.17	24	0.417
528	A	11	10	1.21	27	0.370
529	A	10	9	1.05	27	0.333
530	A	11	10	1.00	27	0.370
531	A	8	7	1.04	27	0.259
532	A	8	7	1.02	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	12	11	0.57	27	0.407
534	A	11	10	0.57	27	0.370
535	A	10	9	0.58	25	0.360
536	A	9	8	0.66	24	0.333
537	A	6	5	0.67	27	0.185
538	A	5	4	1.06	27	0.148
539	A	6	5	1.05	27	0.185
540	A	8	7	1.09	27	0.259
541	A	10	9	0.88	27	0.333
542	A	6	5	1.01	20	0.250
543	A	4	3	1.00	22	0.136
544	A	9	8	0.98	22	0.364
545	A	4	3	1.00	24	0.125
546	A	4	3	1.00	24	0.125
547	A	4	3	1.00	24	0.125
548	A	4	3	1.00	24	0.125
549	A	4	3	1.00	22	0.136
550	A	4	3	1.00	23	0.130
551	A	4	3	1.00	23	0.130
552	A	8	7	1.21	22	0.318
553	A	4	3	1.00	20	0.150
554	A	4	3	1.00	22	0.136
555	A	11	10	1.09	22	0.455
556	F	0	0	N/A	0.000	N/A
557	F	0	0	N/A	0.000	N/A
558	F	0	0	N/A	0.000	N/A
559	F	0	0	N/A	0.000	N/A
560	A	1	1	1.00	20	0.050
561	A	3	3	1.00	20	0.150
562	A	4	4	1.02	20	0.200
563	A	5	5	1.03	20	0.250
564	A	5	5	0.88	22	0.227
565	A	5	5	0.90	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	5	5	0.92	22	0.227
567	A	5	5	0.97	22	0.227
568	A	4	4	1.00	20	0.200
569	A	4	4	1.00	22	0.182
570	A	5	5	0.92	22	0.227
571	A	5	5	0.88	22	0.227
572	A	5	5	0.87	22	0.227
573	F	0	0	N/A	0.000	N/A
574	F	0	0	N/A	0.000	N/A
575	F	0	0	N/A	0.000	N/A
576	F	0	0	N/A	0.000	N/A
577	A	1	1	1.00	22	0.045
578	A	3	3	1.00	22	0.136
579	A	4	4	1.02	22	0.182
580	A	5	5	1.03	22	0.227
581	A	5	5	0.88	22	0.227
582	A	5	5	0.90	22	0.227
583	A	5	5	0.94	22	0.227
584	A	4	4	1.00	22	0.182
585	A	5	5	0.96	20	0.250
586	A	4	4	1.00	22	0.182
587	A	4	4	1.00	22	0.182
588	A	5	5	0.87	22	0.227
589	A	5	5	0.86	22	0.227
590	F	0	0	N/A	0.000	N/A
591	F	0	0	N/A	0.000	N/A
592	F	0	0	N/A	0.000	N/A
593	F	0	0	N/A	0.000	N/A
594	A	1	1	1.00	22	0.045
595	A	3	3	1.00	22	0.136
596	A	4	4	1.02	22	0.182
597	A	5	5	1.03	22	0.227
598	A	5	5	0.90	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
599	A	5	5	0.93	22	0.227
600	A	5	5	0.97	22	0.227
601	A	4	4	1.00	20	0.200
602	A	4	4	1.00	22	0.182
603	A	5	5	0.92	22	0.227
604	A	5	5	0.88	22	0.227
605	A	5	5	0.87	22	0.227
606	F	0	0	N/A	0.000	N/A
607	F	0	0	N/A	0.000	N/A
608	F	0	0	N/A	0.000	N/A
609	F	0	0	N/A	0.000	N/A
610	A	1	1	1.00	22	0.045
611	A	3	3	1.00	22	0.136
612	A	4	4	1.02	22	0.182
613	A	5	5	1.03	22	0.227
614	A	4	4	0.46	22	0.182
615	A	5	5	0.50	22	0.227
616	A	4	4	0.56	22	0.182
617	A	5	5	0.71	22	0.227
618	A	3	3	0.68	22	0.136
619	A	3	3	1.00	22	0.136
620	A	4	4	0.69	22	0.182
621	A	5	5	0.51	22	0.227
622	A	4	4	0.44	22	0.182
623	A	11	10	1.12	24	0.417
624	A	10	9	1.12	24	0.375
625	A	9	8	1.12	24	0.333
626	A	8	7	1.12	24	0.292
627	A	7	6	1.10	24	0.250
628	A	6	5	1.12	24	0.208
629	A	4	4	1.12	24	0.167
630	A	5	5	1.19	24	0.208
631	A	6	6	1.23	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
632	A	7	7	1.25	24	0.292
633	A	5	5	0.51	24	0.208
634	A	4	4	0.57	24	0.167
635	A	5	5	0.71	24	0.208
636	A	3	3	1.00	24	0.125
637	A	5	5	0.54	24	0.208
638	A	4	4	0.72	24	0.167
639	A	3	3	1.00	24	0.125
640	A	4	4	0.50	24	0.167
641	A	5	5	0.44	24	0.208
642	A	5	5	0.47	24	0.208
643	A	4	4	0.51	24	0.167
644	A	5	5	0.58	24	0.208
645	A	4	4	0.71	24	0.167
646	A	3	3	0.68	24	0.125
647	A	3	3	1.00	24	0.125
648	A	5	5	0.70	24	0.208
649	A	4	4	0.50	24	0.167
650	A	5	5	0.44	24	0.208
651	A	9	8	1.12	24	0.333
652	A	8	7	1.12	24	0.292
653	A	7	6	1.09	24	0.250
654	A	6	5	1.12	24	0.208
655	A	4	4	1.12	24	0.167
656	A	5	5	1.19	24	0.208
657	A	6	6	1.22	24	0.250
658	A	7	7	1.25	24	0.292
659	A	4	4	0.51	24	0.167
660	A	5	5	0.58	24	0.208
661	A	4	4	0.71	24	0.167
662	A	3	3	1.00	24	0.125
663	A	4	4	0.53	24	0.167
664	A	5	5	0.74	24	0.208
665	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
666	A	5	5	0.50	24	0.208
667	A	4	4	0.43	24	0.167
668	A	4	4	0.68	25	0.160
669	A	4	4	0.70	23	0.174
670	A	3	3	0.68	22	0.136
671	A	4	4	0.61	25	0.160
672	A	4	4	0.63	25	0.160
673	A	15	14	1.40	27	0.519
674	A	13	12	1.44	27	0.444
675	A	11	10	1.41	25	0.400
676	A	7	6	1.10	24	0.250
677	A	12	11	1.12	27	0.407
678	A	12	11	1.09	27	0.407
679	A	10	9	1.13	27	0.333
680	A	12	11	1.15	27	0.407
681	A	16	15	1.15	27	0.556
682	A	5	5	0.39	27	0.185
683	A	5	5	0.41	27	0.185
684	A	5	5	0.47	25	0.200
685	A	5	5	0.54	24	0.208
686	A	5	5	0.46	27	0.185
687	A	5	5	0.51	27	0.185
688	A	5	5	0.46	27	0.185
689	A	5	5	0.41	27	0.185
690	A	5	5	0.38	27	0.185
691	A	4	4	0.45	25	0.160
692	A	4	4	0.49	25	0.160
693	A	4	4	0.61	25	0.160
694	A	4	4	0.72	23	0.174
695	A	4	4	0.69	22	0.182
696	A	4	4	0.41	25	0.160
697	A	4	4	0.38	25	0.160
698	A	4	4	0.39	25	0.160
699	A	5	5	0.43	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
700	A	5	5	0.50	25	0.200
701	A	5	5	0.53	25	0.200
702	A	5	5	0.51	25	0.200
703	A	5	5	0.57	23	0.217
704	A	5	5	0.51	22	0.227
705	A	5	5	0.35	25	0.200
706	A	5	5	0.35	25	0.200
707	A	5	5	0.70	27	0.185
708	A	5	5	0.72	25	0.200
709	A	3	3	0.68	24	0.125
710	A	5	5	0.63	27	0.185
711	A	5	5	0.67	27	0.185
712	A	16	15	1.41	27	0.556
713	A	15	14	1.44	27	0.519
714	A	11	10	1.44	25	0.400
715	A	7	6	1.09	24	0.250
716	A	12	11	1.12	27	0.407
717	A	11	10	1.11	27	0.370
718	A	9	8	1.13	27	0.296
719	A	11	10	1.14	27	0.370
720	A	13	12	1.15	27	0.444
721	A	4	4	0.38	27	0.148
722	A	4	4	0.40	27	0.148
723	A	4	4	0.46	25	0.160
724	A	4	4	0.53	24	0.167
725	A	4	4	0.45	27	0.148
726	A	4	4	0.49	27	0.148
727	A	4	4	0.45	27	0.148
728	A	4	4	0.40	27	0.148
729	A	4	4	0.37	27	0.148
730	A	5	5	0.63	27	0.185
731	A	8	8	1.01	27	0.296
732	A	4	4	0.73	25	0.160
733	A	5	5	0.75	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
734	A	8	8	1.01	27	0.296
735	A	4	4	0.62	27	0.148
736	F	0	0	N/A	0.000	N/A
737	F	0	0	N/A	0.000	N/A
738	F	0	0	N/A	0.000	N/A
739	A	3	2	1.00	8	0.250
740	A	1	1	1.00	22	0.045
741	A	3	3	1.00	22	0.136
742	A	4	4	0.98	22	0.182
743	A	5	5	0.89	22	0.227
744	A	4	3	1.00	24	0.125
745	A	4	3	1.00	24	0.125
746	A	4	3	1.00	24	0.125
747	A	1	1	1.00	24	0.042
748	A	2	2	1.00	24	0.083
749	A	3	3	0.98	24	0.125
750	A	4	4	0.94	24	0.167
751	A	12	11	0.77	27	0.407
752	A	5	4	1.00	27	0.148
753	A	1	1	1.00	25	0.040
754	A	1	1	1.00	24	0.042
755	A	7	6	0.81	27	0.222
756	A	14	13	0.78	27	0.481
757	A	7	6	0.72	27	0.222
758	A	2	2	1.06	27	0.074
759	A	2	2	1.06	25	0.080
760	A	2	2	1.00	24	0.083
761	A	5	4	0.66	27	0.148
762	A	4	3	1.00	22	0.136
763	A	4	3	1.00	23	0.130
764	A	4	3	1.00	23	0.130
765	A	4	4	1.00	22	0.182
766	A	4	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
767	A	4	4	1.00	22	0.182
768	A	4	3	1.00	20	0.150
769	A	4	3	1.00	22	0.136
770	A	4	4	1.00	22	0.182
771	A	4	3	1.00	22	0.136
772	A	24	23	0.98	20	1.150
773	A	20	19	0.98	20	0.950
774	A	16	15	0.97	20	0.750
775	A	13	12	1.06	18	0.667
776	A	9	8	0.97	20	0.400
777	A	15	14	0.94	20	0.700
778	A	19	18	0.91	20	0.900
779	A	23	22	0.94	20	1.100
780	A	6	6	0.78	22	0.273
781	A	6	6	0.81	22	0.273
782	A	6	6	0.79	22	0.273
783	A	6	6	0.90	22	0.273
784	A	6	6	1.00	20	0.300
785	A	6	6	1.03	22	0.273
786	A	6	6	0.96	22	0.273
787	A	6	6	0.90	22	0.273
788	A	6	6	0.90	22	0.273
789	A	25	24	0.98	22	1.091
790	A	21	20	0.98	22	0.909
791	A	17	16	0.96	22	0.727
792	A	9	8	1.04	20	0.400
793	A	11	10	0.97	22	0.455
794	A	13	12	0.96	22	0.545
795	A	20	19	0.96	22	0.864
796	A	24	23	0.95	22	1.045
797	A	6	6	0.80	22	0.273
798	A	6	6	0.78	22	0.273
799	A	6	6	0.84	22	0.273
800	A	6	6	0.80	22	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
801	A	6	6	1.00	20	0.300
802	A	6	6	0.98	22	0.273
803	A	6	6	0.93	22	0.273
804	A	6	6	0.92	22	0.273
805	A	6	6	0.88	22	0.273
806	A	26	25	0.96	22	1.136
807	A	21	20	0.96	22	0.909
808	A	17	16	0.94	22	0.727
809	A	11	10	1.04	20	0.500
810	A	8	7	0.97	22	0.318
811	A	13	12	0.96	22	0.545
812	A	17	16	0.95	22	0.727
813	A	22	21	0.95	22	0.955
814	A	6	6	0.81	22	0.273
815	A	6	6	0.79	22	0.273
816	A	6	6	0.85	22	0.273
817	A	6	6	1.00	20	0.300
818	A	6	6	1.03	22	0.273
819	A	6	6	0.96	22	0.273
820	A	6	6	0.90	22	0.273
821	A	6	6	0.90	22	0.273
822	A	26	25	0.95	22	1.136
823	A	20	19	0.96	22	0.864
824	A	16	15	0.97	22	0.682
825	A	9	8	1.07	20	0.400
826	A	11	10	0.98	22	0.455
827	A	13	12	0.96	22	0.545
828	A	17	16	0.97	22	0.727
829	A	21	20	0.96	22	0.909
830	A	5	5	0.31	22	0.227
831	A	4	4	0.33	22	0.182
832	A	5	5	0.42	22	0.227
833	A	4	4	0.58	22	0.182
834	A	5	5	0.71	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
835	A	4	4	0.49	22	0.182
836	A	5	5	0.43	22	0.227
837	A	4	4	0.40	22	0.182
838	A	16	15	0.49	24	0.625
839	A	14	13	0.50	24	0.542
840	A	12	11	0.53	24	0.458
841	A	11	10	0.62	24	0.417
842	A	5	5	0.84	24	0.208
843	A	7	7	1.02	24	0.292
844	A	8	8	0.78	24	0.333
845	A	9	9	0.67	24	0.375
846	A	5	5	0.32	24	0.208
847	A	4	4	0.31	24	0.167
848	A	5	5	0.34	24	0.208
849	A	4	4	0.41	24	0.167
850	A	5	5	0.46	24	0.208
851	A	4	4	0.54	24	0.167
852	A	5	5	0.49	24	0.208
853	A	4	4	0.43	24	0.167
854	A	5	5	0.40	24	0.208
855	A	4	4	0.31	24	0.167
856	A	5	5	0.34	24	0.208
857	A	4	4	0.40	24	0.167
858	A	5	5	0.60	24	0.208
859	A	4	4	0.67	24	0.167
860	A	5	5	0.49	24	0.208
861	A	4	4	0.42	24	0.167
862	A	5	5	0.40	24	0.208
863	A	16	15	0.49	24	0.625
864	A	14	13	0.50	24	0.542
865	A	12	11	0.52	24	0.458
866	A	12	11	0.62	24	0.458
867	A	5	5	0.83	24	0.208
868	A	9	9	1.02	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
869	A	10	10	0.82	24	0.417
870	A	11	11	0.71	24	0.458
871	A	4	4	0.31	24	0.167
872	A	5	5	0.32	24	0.208
873	A	4	4	0.33	24	0.167
874	A	5	5	0.42	24	0.208
875	A	4	4	0.44	24	0.167
876	A	5	5	0.55	24	0.208
877	A	4	4	0.47	24	0.167
878	A	5	5	0.43	24	0.208
879	A	4	4	0.39	24	0.167
880	A	4	4	0.66	25	0.160
881	A	4	4	0.64	25	0.160
882	A	3	3	0.61	23	0.130
883	A	4	4	0.58	22	0.182
884	A	4	4	0.61	25	0.160
885	A	3	3	1.00	25	0.120
886	A	11	11	0.93	27	0.407
887	A	9	9	0.91	27	0.333
888	A	5	5	0.93	25	0.200
889	A	11	10	0.62	24	0.417
890	A	11	10	0.65	27	0.370
891	A	10	9	0.87	27	0.333
892	A	12	11	0.87	27	0.407
893	A	16	15	0.97	27	0.556
894	A	17	16	0.96	27	0.593
895	A	5	5	0.40	27	0.185
896	A	5	5	0.45	27	0.185
897	A	5	5	0.51	25	0.200
898	A	5	5	0.46	24	0.208
899	A	5	5	0.51	27	0.185
900	A	5	5	0.46	27	0.185
901	A	5	5	0.41	27	0.185
902	A	5	5	0.37	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
903	A	5	5	0.36	27	0.185
904	A	5	5	0.68	27	0.185
905	A	5	5	0.66	27	0.185
906	A	3	3	0.61	25	0.120
907	A	5	5	0.60	24	0.208
908	A	5	5	0.65	27	0.185
909	A	4	4	1.00	27	0.148
910	A	14	14	0.91	27	0.519
911	A	6	6	0.94	27	0.222
912	A	5	5	0.93	25	0.200
913	A	12	11	0.62	24	0.458
914	A	11	10	0.68	27	0.370
915	A	10	9	0.87	27	0.333
916	A	12	11	0.86	27	0.407
917	A	14	13	0.97	27	0.481
918	A	16	15	0.96	27	0.556
919	A	4	4	0.39	27	0.148
920	A	4	4	0.44	27	0.148
921	A	4	4	0.50	25	0.160
922	A	4	4	0.44	24	0.167
923	A	4	4	0.49	27	0.148
924	A	4	4	0.45	27	0.148
925	A	4	4	0.40	27	0.148
926	A	4	4	0.36	27	0.148
927	A	4	4	0.35	27	0.148
928	A	10	9	1.68	20	0.450
929	A	9	8	0.98	22	0.364
930	A	14	13	1.00	22	0.591
931	A	7	6	0.81	24	0.250
932	A	5	4	0.86	24	0.167
933	A	4	3	1.00	22	0.136
934	A	4	3	1.00	23	0.130
935	A	4	3	1.00	23	0.130

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int e^{\coth^{-1}(ax)}x^3\,dx$

3.1.1 Optimal result

Integrand size = 10, antiderivative size = 114

$$\int e^{\coth^{-1}(ax)}x^3\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

output
```
3/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4+2/3*x*(1-1/a^2/x^2)^(1/2)/a^3+3/8*x^2
*(1-1/a^2/x^2)^(1/2)/a^2+1/3*x^3*(1-1/a^2/x^2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^
2)^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}x^3\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(16+9ax+8a^2x^2+6a^3x^3)+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

input
```
Integrate[E^ArcCoth[a*x]*x^3,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
 + Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

3.1.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {6719, 539, 25, 27, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \color{blue}{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \color{blue}{539}$$

$$\frac{1}{4}\int -\frac{\left(4a+\frac{3}{x}\right)x^4}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{4}\int \frac{\left(4a+\frac{3}{x}\right)x^4}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a^2}$$

$$\Big\downarrow \color{blue}{539}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{-\frac{1}{3}\int -\frac{\left(9a+\frac{8}{x}\right)x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{1}{3}\int \frac{\left(9a+\frac{8}{x}\right)x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\int \frac{\left(9a+\frac{8}{x}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 539$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{-\frac{1}{2}\int -\frac{\left(16a+\frac{9}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{1}{2}\int \frac{\left(16a+\frac{9}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{\int \frac{\left(16a+\frac{9}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 534$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{9\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{\frac{9}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{-9a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{-9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

3.1. $\int e^{\coth^{-1}(ax)}x^3\,dx$

input
```
Int[E^ArcCoth[a*x]*x^3,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 - ((-4*a*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ((-9
*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (-16*a*Sqrt[1 - 1/(a^2*x^2)]*x - 9*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a))/(3*a))/(4*a^2)
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.1.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(6a^3x^3+8a^2x^2+9ax+16)(ax-1)}{24a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{8a^3\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$-\frac{(ax-1)\Big(-6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax-15\sqrt{a^2x^2-1}\sqrt{a^2}\,ax-8((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a-24a\ln\Big(a^2x+\sqrt{a^2}}{24\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}\,a^4\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3+8*a^2*x^2+9*a*x+16)*(a*x-1)/a^4/((a*x-1)/(a*x+1))^(1/2)+3/
8/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int e^{\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{(6\,a^4x^4+14\,a^3x^3+17\,a^2x^2+25\,ax+16)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="fricas")
```

output
```
1/24*((6*a^4*x^4 + 14*a^3*x^3 + 17*a^2*x^2 + 25*a*x + 16)*sqrt((a*x - 1)/(
a*x + 1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a
*x + 1)) - 1))/a^4
```

3.1.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3,x)
```

output
```
Integral(x**3/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(94) = 188$.

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \frac{1}{24}\,a\Bigg(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-49\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-39\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}-\frac{9\log\Big(\sqrt{\frac{ax-1}{ax+1}}-1}{a^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="maxima")
```

output
```
1/24*a*(2*(9*((a*x - 1)/(a*x + 1))^(7/2) - 49*((a*x - 1)/(a*x + 1))^(5/2)
+ 31*((a*x - 1)/(a*x + 1))^(3/2) - 39*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x -
 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x
 + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 9*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^5 - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2x\left(\frac{3x}{a\operatorname{sgn}(ax+1)}+\frac{4}{a^2\operatorname{sgn}(ax+1)}\right)+\frac{9}{a^3\operatorname{sgn}(ax+1)}\right)x+\frac{16}{a^4\operatorname{sgn}(ax+1)}\right) - \frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8a^3|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="giac")
```

output
```
1/24*sqrt(a^2*x^2 - 1)*((2*x*(3*x/(a*sgn(a*x + 1)) + 4/(a^2*sgn(a*x + 1)))
 + 9/(a^3*sgn(a*x + 1)))*x + 16/(a^4*sgn(a*x + 1))) - 3/8*log(abs(-x*abs(a
) + sqrt(a^2*x^2 - 1)))/(a^3*abs(a)*sgn(a*x + 1))
```

3.1.9 Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.50

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \frac{\frac{13\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12}+\frac{49\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{12}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}$$

input
```
int(x^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((13*((a*x - 1)/(a*x + 1))^(1/2))/4 - (31*((a*x - 1)/(a*x + 1))^(3/2))/12
+ (49*((a*x - 1)/(a*x + 1))^(5/2))/12 - (3*((a*x - 1)/(a*x + 1))^(7/2))/4)
/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3
+ (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (3*atanh(
((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4)
```

3.2 $\int e^{\coth^{-1}(ax)}x^2\,dx$

3.2.1 Optimal result

Integrand size = 10, antiderivative size = 90

$$\int e^{\coth^{-1}(ax)}x^2\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

output
```
1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+2/3*x*(1-1/a^2/x^2)^(1/2)/a^2+1/2*x^2
*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/x^2)^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int e^{\coth^{-1}(ax)}x^2\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(4+3ax+2a^2x^2)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

input
```
Integrate[E^ArcCoth[a*x]*x^2,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + 3*a*x + 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 - 1
/(a^2*x^2)])*x])/(6*a^3)
```

3.2.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6719, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 539}$$

$$\frac{1}{3}\int -\frac{\left(3a+\frac{2}{x}\right)x^3}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{3}\int \frac{\left(3a+\frac{2}{x}\right)x^3}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{\left(3a+\frac{2}{x}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a^2}$$

$$\Big\downarrow \text{ 539}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{-\frac{1}{2}\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{1}{2}\int \frac{\left(4a+\frac{3}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow 534$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow 243$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow 73$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow 221$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

input
```
Int[E^ArcCoth[a*x]*x^2,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ((-3*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (-4*
a*Sqrt[1 - 1/(a^2*x^2)]*x - 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a))/(3*a^
2)
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.2. $\int e^{\coth^{-1}(ax)} x^2 \, dx$

3.2.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(2a^2x^2+3ax+4)(ax-1)}{6a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{2a^2\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+6\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+6a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-}}{\sqrt{a^2}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^3\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2+3*a*x+4)*(a*x-1)/a^3/((a*x-1)/(a*x+1))^(1/2)+1/2/a^2*ln(a^2
*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*
x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int e^{\coth^{-1}(ax)}x^2\,dx$$
$$=\frac{(2\,a^3x^3+5\,a^2x^2+7\,ax+4)\sqrt{\frac{ax-1}{ax+1}}+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="fricas")
```

output
```
1/6*((2*a^3*x^3 + 5*a^2*x^2 + 7*a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) + 3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^
3
```

3.2.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^2\,dx=\int\frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2,x)
```

output
```
Integral(x**2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(74) = 148.

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.84

$$\int e^{\coth^{-1}(ax)}x^2\,dx=$$
$$-\frac{1}{6}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="maxima")
```

output
```
-1/6*a*(2*(3*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
 9*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a
^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 3*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^4 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

3.2.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\,dx=\text{Exception raised: TypeError}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}x^2\,dx = \frac{3\sqrt{\frac{ax-1}{ax+1}} - \frac{4\left(\frac{ax-1}{ax+1}\right)^{3/2}}{3} + \left(\frac{ax-1}{ax+1}\right)^{5/2}}{a^3 + \frac{3a^3(ax-1)^2}{(ax+1)^2} - \frac{a^3(ax-1)^3}{(ax+1)^3} - \frac{3a^3(ax-1)}{ax+1}} + \frac{\operatorname{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^3}$$

input

```
int(x^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
(3*((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + ((a*x
 - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x
 - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*
x + 1))^(1/2))/a^3
```

3.3 $\int e^{\coth^{-1}(ax)}x\,dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 63

$$\int e^{\coth^{-1}(ax)}x\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2+x*(1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/
a^2/x^2)^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int e^{\coth^{-1}(ax)}x\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(2+ax)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[E^ArcCoth[a*x]*x,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])
/(2*a^2)
```

3.3.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6719, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{2}\int -\frac{\left(2a+\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{2}\int \frac{\left(2a+\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{\left(2a+\frac{1}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a^2}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^2\left(-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{-\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}$$

input
```
Int[E^ArcCoth[a*x]*x,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (-2*a*Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(2*a^2)
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.3.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.59

method	result	size
risch	$\frac{(ax+2)(ax-1)}{2a^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)\sqrt{(ax-1)(ax+1)}}{2a\sqrt{a^{2}}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	100
default	$\frac{(ax-1)\left(\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax-\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a+2\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}+2a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}\sqrt{a^{2}}}$	152

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+2)*(a*x-1)/a^2/((a*x-1)/(a*x+1))^(1/2)+1/2/a*ln(a^2*x/(a^2)^(1/2)
+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^
(1/2)/(a*x+1)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int e^{\coth^{-1}(ax)}x\,dx=\frac{(a^2x^2+3\,ax+2)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 + 3*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

3.3.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\,dx=\int\frac{x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x,x)
```

output
```
Integral(x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(53) = 106.

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int e^{\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{2}\,a\left(\frac{2\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="maxima")
```

output
```
1/2*a*(2*(((a*x - 1)/(a*x + 1))^(3/2) - 3*sqrt((a*x - 1)/(a*x + 1)))/(2*(a
*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^3 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x}{a\mathrm{sgn}\left(ax+1\right)}+\frac{2}{a^2\mathrm{sgn}\left(ax+1\right)}\right) - \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,a|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="giac")
```

output
```
1/2*sqrt(a^2*x^2 - 1)*(x/(a*sgn(a*x + 1)) + 2/(a^2*sgn(a*x + 1))) - 1/2*lo
g(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(a*abs(a)*sgn(a*x + 1))
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{3\sqrt{\frac{a\,x-1}{a\,x+1}}-\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(3*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*
(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)
/(a*x + 1))^(1/2))/a^2
```

3.4 $\int e^{\coth^{-1}(ax)}\,dx$

3.4.1 Optimal result

Integrand size = 6, antiderivative size = 36

$$\int e^{\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
arctanh((1-1/a^2/x^2)^(1/2))/a+x*(1-1/a^2/x^2)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int e^{\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x+\frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^ArcCoth[a*x],x]
```

output
```
Sqrt[1 - 1/(a^2*x^2)]*x + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6718, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6718}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{534}$$

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{243}$$

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a}$$

$$\Big\downarrow \text{73}$$

$$a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}+x\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[E^ArcCoth[a*x],x]
```

output
```
Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

3.4.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6718

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

3.4.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(32) = 64$.

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

method	result	size
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	90
default	$\frac{(ax-1)\left(a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^{2}}}$	97

input
```
int(1/((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/((a*x-1)/(a*x+1))^(1/2)+ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2)
)/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}\,dx=\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*x + 1)) + 1)
- log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

3.4.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\,dx=\int\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(1/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int e^{\coth^{-1}(ax)}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - log(sqrt
((a*x - 1)/(a*x + 1)) + 1)/a^2 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int e^{\coth^{-1}(ax)}\,dx = -\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2-1}}{a\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*
x^2 - 1)/(a*sgn(a*x + 1))
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int e^{\coth^{-1}(ax)}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}} + \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (2*atanh((
(a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.5 $\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx$

3.5.1 Optimal result

Integrand size = 10, antiderivative size = 22

$$\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx = -\csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right)$$

output
```
-arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx = -\arcsin\left(\frac{1}{ax}\right) + \log\left(x\left(1 + \sqrt{\frac{-1 + a^2x^2}{a^2x^2}}\right)\right)$$

input
```
Integrate[E^ArcCoth[a*x]/x,x]
```

output
```
-ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

3.5.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6719, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 223}$$

$$-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\arcsin\left(\frac{1}{ax}\right)$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-\arcsin\left(\frac{1}{ax}\right)$$

$$\Big\downarrow \text{ 73}$$

$$a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-\arcsin\left(\frac{1}{ax}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\arcsin\left(\frac{1}{ax}\right)$$

input

```
Int[E^ArcCoth[a*x]/x,x]
```

output
```
-ArcSin[1/(a*x)] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

3.5.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 6.00

method	result	size
default	$-\frac{(ax-1)\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)-\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	132

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^
(1/2)-a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(
1/2)/(a^2)^(1/2)
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = 2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

output
```
2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

3.5.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(1/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.5.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(20) = 40$.

Time $= 0.33$ (sec) , antiderivative size $= 69$, normalized size of antiderivative $= 3.14$

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

output
```
a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(20) = 40$.

Time $= 0.27$ (sec) , antiderivative size $= 63$, normalized size of antiderivative $= 2.86$

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = \frac{2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{\operatorname{sgn}(ax+1)} - \frac{a\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)}{|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

output
```
2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - a*log(abs(-x*abs(a)
 + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

input

```
int(1/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output

```
2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
```

3.6 $\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx = a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

output
```
-a*arccsc(a*x)+a*(1-1/a^2/x^2)^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx = a\left(\sqrt{1-\frac{1}{a^2x^2}} - \arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^ArcCoth[a*x]/x^2,x]
```

output
```
a*(Sqrt[1 - 1/(a^2*x^2)] - ArcSin[1/(a*x)])
```

3.6.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6719, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1+\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{455}$$

$$a\sqrt{1-\frac{1}{a^2x^2}}-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{223}$$

$$a\sqrt{1-\frac{1}{a^2x^2}}-a\arcsin\left(\frac{1}{ax}\right)$$

input

```
Int[E^ArcCoth[a*x]/x^2,x]
```

output

```
a*Sqrt[1 - 1/(a^2*x^2)] - a*ArcSin[1/(a*x)]
```

3.6.3.1 Defintions of rubi rules used

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455

```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.17

method	result
risch	$\frac{ax-1}{x\sqrt{\frac{ax-1}{ax+1}}}-\frac{a\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+(a^{2}x^{2}-1)^{\frac{3}{2}}\sqrt{a^{2}}+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax+\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{2}x+ax\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)-\right.}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,x\sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/x/((a*x-1)/(a*x+1))^(1/2)-a*arctan(1/(a^2*x^2-1)^(1/2))/((a*x-1)/(
a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx=\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*x + 1)*sqrt((a*x - 1)/(a*x +
 1)))/x
```

3.6.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(1/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.6.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} + \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) + arctan(sqrt((a*
x - 1)/(a*x + 1))))
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,a\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{\operatorname{sgn}\left(ax+1\right)} + \frac{2\,|a|}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)\operatorname{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*a*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) + 2*abs(a)/(((x*abs
(a) - sqrt(a^2*x^2 - 1))^2 + 1)*sgn(a*x + 1))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

input

```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output

```
2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (2*a*((a*x - 1)/(a*x + 1))^(1/2))/
((a*x - 1)/(a*x + 1) + 1)
```

3.7 $\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx$

3.7.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx=\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

output
```
-1/2*a^2*arccsc(a*x)+1/2*a*(2*a+1/x)*(1-1/a^2/x^2)^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx=\frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(1+2ax)-ax\arcsin\left(\frac{1}{ax}\right)\right)}{2x}$$

input
```
Integrate[E^ArcCoth[a*x]/x^3,x]
```

output
```
(a*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a*x) - a*x*ArcSin[1/(a*x)]))/(2*x)
```

3.7.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6719, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1+\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int \frac{a+\frac{2}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}\int \frac{a+\frac{2}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(2a^2\sqrt{1-\frac{1}{a^2x^2}}-a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(2a^2\sqrt{1-\frac{1}{a^2x^2}}-a^2\arcsin\left(\frac{1}{ax}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

input
```
Int[E^ArcCoth[a*x]/x^3,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (2*a^2*Sqrt[1 - 1/(a^2*x^2)] - a^2*ArcSi
n[1/(a*x)])/2
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

method	result
risch	$\frac{(ax-1)(2ax+1)}{2x^{2}\sqrt{\frac{ax-1}{ax+1}}}-\frac{a^{2}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{(ax-1)(ax+1)}}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\Big(-2\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{3}x^{3}+2(a^{2}x^{2}-1)^{\frac{3}{2}}\sqrt{a^{2}}\,ax+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+a^{2}x^{2}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+2\ln\Big(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}}{\sqrt{a^{2}}}}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,x^{2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-1)*(2*a*x+1)/x^2/((a*x-1)/(a*x+1))^(1/2)-1/2*a^2*arctan(1/(a^2*x^
2-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + (2\,a^2x^2+3\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (2*a^2*x^2 + 3*a*x + 1)
*sqrt((a*x - 1)/(a*x + 1)))/x^2
```

3.7.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

output
```
Integral(1/(x**3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.39

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
(a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*((a*x - 1)/(a*x + 1))^(3/2) + 3*
a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1
)^2 + 1))*a
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(32) = 64$.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.76

$$
\begin{aligned}
&\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx \\
&= \frac{a^2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{\operatorname{sgn}(ax+1)} \\
&\quad - \frac{\left(x|a| - \sqrt{a^2x^2-1}\right)^3 a^2 - 2\left(x|a| - \sqrt{a^2x^2-1}\right)^2 a|a| - \left(x|a| - \sqrt{a^2x^2-1}\right)a^2 - 2\,a|a|}{\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)^2 \operatorname{sgn}(ax+1)}
\end{aligned}
$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

output
```
a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - ((x*abs(a) - sqrt
(a^2*x^2 - 1))^3*a^2 - 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*abs(a) - (x*ab
s(a) - sqrt(a^2*x^2 - 1))*a^2 - 2*a*abs(a))/(((x*abs(a) - sqrt(a^2*x^2 - 1
))^2 + 1)^2*sgn(a*x + 1))
```

3.7.9 Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$
\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = a^2\sqrt{\frac{a\,x-1}{a\,x+1}} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2} + a^2\operatorname{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{3\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}
$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
a^2*((a*x - 1)/(a*x + 1))^(1/2) + ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) + a^
2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (3*a*((a*x - 1)/(a*x + 1))^(1/2))/(2
*x)
```

3.7. $\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx$

3.8 $\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx$

3.8.1 Optimal result

Integrand size = 10, antiderivative size = 75

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = a^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

output
```
-1/3*a^3*(1-1/a^2/x^2)^(3/2)-1/2*a^3*arccsc(a*x)+a^3*(1-1/a^2/x^2)^(1/2)+1
/2*a^2*(1-1/a^2/x^2)^(1/2)/x
```

3.8.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(2+3ax+4a^2x^2)}{x^2}-3a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^ArcCoth[a*x]/x^4,x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x + 4*a^2*x^2))/x^2 - 3*a^2*ArcSin[1/(
a*x)]))/6
```

3.8.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6719, 533, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1+\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a^2\int \frac{2a+\frac{3}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}\int \frac{2a+\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int \frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\int \frac{3a+\frac{4}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(3a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(3a^2\arcsin\left(\frac{1}{ax}\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

input
```
Int[E^ArcCoth[a*x]/x^4,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) + ((3*a^2*Sqrt[1 - 1/(a^2*x^2)])/(2*x) -
 (a*(-4*a^2*Sqrt[1 - 1/(a^2*x^2)] + 3*a^2*ArcSin[1/(a*x)]))/2)/3
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.8.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

method	result
risch	$\frac{(ax-1)\left(4a^2x^2+3ax+2\right)}{6x^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln\left(\frac{a^2x+\sqrt{a^2}}{\sqrt{}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x-1)*(4*a^2*x^2+3*a*x+2)/x^3/((a*x-1)/(a*x+1))^(1/2)-1/2*a^3*arctan
(1/(a^2*x^2-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x
+1)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx=\frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(4\,a^3x^3+7\,a^2x^2+5\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (4*a^3*x^3 + 7*a^2*x^2
+ 5*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

3.8.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx=\int \frac{1}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

output
```
Integral(1/(x**4*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(63) = 126$.

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.81

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx$$
$$=\frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{3\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (3*a^2*((a*x - 1)/(a*x + 1)
)^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 9*a^2*sqrt((a*x - 1)/(a*x +
1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x
 + 1)^3 + 1))*a
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(63) = 126$.

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx=\frac{a^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{\operatorname{sgn}(ax+1)}$$
$$-\frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5a^3-12\left(x|a|-\sqrt{a^2x^2-1}\right)^2a^2|a|-3\left(x|a|-\sqrt{a^2x^2-1}\right)a^3-4\,a^2|a|}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

output
```
a^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - 1/3*(3*(x*abs(a)
- sqrt(a^2*x^2 - 1))^5*a^3 - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^2*abs(a
) - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*a^3 - 4*a^2*abs(a))/(((x*abs(a) - sqr
t(a^2*x^2 - 1))^2 + 1)^3*sgn(a*x + 1))
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{2\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^3} + a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{7\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x} + \frac{5\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^2}$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3
) + a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (7*a^2*((a*x - 1)/(a*x + 1))^(
1/2))/(6*x) + (5*a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

3.9 $\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx$

3.9.1 Optimal result

Integrand size = 10, antiderivative size = 88

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx=\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{3}{8}a^4\csc^{-1}(ax)$$

output
```
-3/8*a^4*arccsc(a*x)+1/24*a^3*(16*a+9/x)*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^
2/x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

3.9.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx=\frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(6+8ax+9a^2x^2+16a^3x^3)}{x^3}-9a^3\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^ArcCoth[a*x]/x^5,x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(6 + 8*a*x + 9*a^2*x^2 + 16*a^3*x^3))/x^3 - 9*a
^3*ArcSin[1/(a*x)]))/24
```

3.9.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6719, 533, 27, 533, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{1+\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{1}{4}a^2\int \frac{3a+\frac{4}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{1}{4}\int \frac{3a+\frac{4}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a^2\int \frac{8a+\frac{9}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\int \frac{8a+\frac{9}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a^2\int \frac{9a+\frac{16}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a\int\frac{9a+\frac{16}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 455$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a\left(9a\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 223$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a\left(9a^2\arcsin\left(\frac{1}{ax}\right)-16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

input
```
Int[E^ArcCoth[a*x]/x^5,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)])/(4*x^3) + ((4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2)
 - (a*((-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (a*(-16*a^2*Sqrt[1 - 1/(a^2*
x^2)] + 9*a^2*ArcSin[1/(a*x)]))/2))/3)/4
```

3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.9.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(ax-1)(16a^3x^3+9a^2x^2+8ax+6)}{24x^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{3a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{8\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-24\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+9a^4x^4\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+24\ln\left(\frac{a^2x+\sqrt{}}{}\right.\right.}{24}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(a*x-1)*(16*a^3*x^3+9*a^2*x^2+8*a*x+6)/x^4/((a*x-1)/(a*x+1))^(1/2)-3/
8*a^4*arctan(1/(a^2*x^2-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1)
)^(1/2)/(a*x+1)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int\frac{e^{\coth^{-1}(ax)}}{x^5}\,dx=\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(16\,a^4x^4+25\,a^3x^3+17\,a^2x^2+14\,ax+6)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) + (16*a^4*x^4 + 25*a^3*
x^3 + 17*a^2*x^2 + 14*a*x + 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

3.9.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

output
```
Integral(1/(x**5*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.95

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx$$
$$= \frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{9\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 39\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1} + \frac{6\,(ax-1)^2}{(ax+1)^2} + \frac{4\,(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (9*a^3*((a*x - 1)/(a*x + 1
))^(7/2) + 49*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 31*a^3*((a*x - 1)/(a*x + 1
))^(3/2) + 39*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a
*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^
4 + 1))*a
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{3\,a^4 \arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{4\,\mathrm{sgn}\left(ax+1\right)}$$
$$-\frac{9\left(x|a| - \sqrt{a^2x^2-1}\right)^7 a^4 + 33\left(x|a| - \sqrt{a^2x^2-1}\right)^5 a^4 - 48\left(x|a| - \sqrt{a^2x^2-1}\right)^4 a^3|a| - 33\left(x|a| - \sqrt{a^2x}\right.}{12\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2+1\right)^4 \mathrm{sg}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

output
```
3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - 1/12*(9*(x*ab
s(a) - sqrt(a^2*x^2 - 1))^7*a^4 + 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*a^4
- 48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a) - 33*(x*abs(a) - sqrt(a^2
*x^2 - 1))^3*a^4 - 64*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a) - 9*(x*a
bs(a) - sqrt(a^2*x^2 - 1))*a^4 - 16*a^3*abs(a))/(((x*abs(a) - sqrt(a^2*x^2
 - 1))^2 + 1)^4*sgn(a*x + 1))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.47

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{2\,a^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4} + \frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}$$
$$+ \frac{17\,a^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2} + \frac{25\,a^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x} + \frac{7\,a\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

input
```
int(1/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4
) + (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 + (17*a^2*((a*x - 1)/(a*x
+ 1))^(1/2))/(24*x^2) + (25*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (7*a
*((a*x - 1)/(a*x + 1))^(1/2))/(12*x^3)
```

3.10 $\int e^{2\coth^{-1}(ax)}x^3\,dx$

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 43

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

output
```
2*x/a^3+x^2/a^2+2/3*x^3/a+1/4*x^4+2*ln(-a*x+1)/a^4
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^3,x]
```

output
```
(2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

3.10.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^3dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^3(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x^3-\frac{2x^2}{a}-\frac{2x}{a^2}-\frac{2}{a^3(ax-1)}-\frac{2}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(1-ax)}{a^4}+\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

input
```
Int[E^(2*ArcCoth[a*x])*x^3,x]
```

output
```
(2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

3.10.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.10.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{x^2}{a^2}+\frac{x^4}{4}+\frac{2x}{a^3}+\frac{2x^3}{3a}+\frac{2\ln(ax-1)}{a^4}$	39
risch	$\frac{x^2}{a^2}+\frac{x^4}{4}+\frac{2x}{a^3}+\frac{2x^3}{3a}+\frac{2\ln(ax-1)}{a^4}$	39
default	$\frac{\frac{1}{4}a^3x^4+\frac{2}{3}a^2x^3+a\,x^2+2x}{a^3}+\frac{2\ln(ax-1)}{a^4}$	42
parallelrisch	$\frac{3a^4x^4+8a^3x^3+12a^2x^2+24ax+24\ln(ax-1)}{12a^4}$	43
meijerg	$\frac{\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln(-ax+1)}{a^4}-\frac{-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)}{a^4}$	73

input
```
int(1/(a*x-1)*(a*x+1)*x^3,x,method=_RETURNVERBOSE)
```

output
```
x^2/a^2+1/4*x^4+2*x/a^3+2/3*x^3/a+2/a^4*ln(a*x-1)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{3\,a^4x^4+8\,a^3x^3+12\,a^2x^2+24\,ax+24\,\log\left(ax-1\right)}{12\,a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="fricas")
```

output
```
1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x + 24*log(a*x - 1))/a^4
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{x^4}{4}+\frac{2x^3}{3a}+\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\log{(ax-1)}}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**3,x)
```

output
```
x**4/4 + 2*x**3/(3*a) + x**2/a**2 + 2*x/a**3 + 2*log(a*x - 1)/a**4
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{3\,a^3x^4+8\,a^2x^3+12\,ax^2+24\,x}{12\,a^3}+\frac{2\,\log\left(ax-1\right)}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="maxima")
```

output
```
1/12*(3*a^3*x^4 + 8*a^2*x^3 + 12*a*x^2 + 24*x)/a^3 + 2*log(a*x - 1)/a^4
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{3\,a^4x^4+8\,a^3x^3+12\,a^2x^2+24\,ax}{12\,a^4}+\frac{2\,\log\left(|ax-1|\right)}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="giac")
```

output
```
1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x)/a^4 + 2*log(abs(a*x - 1
))/a^4
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{2\,\ln\left(a\,x-1\right)}{a^4}+\frac{2\,x}{a^3}+\frac{x^4}{4}+\frac{2\,x^3}{3\,a}+\frac{x^2}{a^2}$$

input

```
int((x^3*(a*x + 1))/(a*x - 1),x)
```

output

```
(2*log(a*x - 1))/a^4 + (2*x)/a^3 + x^4/4 + (2*x^3)/(3*a) + x^2/a^2
```

3.11 $\int e^{2\coth^{-1}(ax)}x^2\,dx$

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int e^{2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

output
```
2*x/a^2+x^2/a+1/3*x^3+2*ln(-a*x+1)/a^3
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^2,x]
```

output
```
(2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

3.11.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^2dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^2(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x^2-\frac{2x}{a}-\frac{2}{a^2(ax-1)}-\frac{2}{a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(1-ax)}{a^3}+\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}$$

input
```
Int[E^(2*ArcCoth[a*x])*x^2,x]
```

output
```
(2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

3.11.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.11.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
norman	$\frac{x^{2}}{a}+\frac{x^{3}}{3}+\frac{2x}{a^{2}}+\frac{2\ln\left(ax-1\right)}{a^{3}}$	31
risch	$\frac{x^{2}}{a}+\frac{x^{3}}{3}+\frac{2x}{a^{2}}+\frac{2\ln\left(ax-1\right)}{a^{3}}$	31
default	$\frac{\frac{1}{3}a^{2}x^{3}+a\,x^{2}+2x}{a^{2}}+\frac{2\ln\left(ax-1\right)}{a^{3}}$	34
parallelrisch	$\frac{a^{3}x^{3}+3a^{2}x^{2}+6ax+6\ln\left(ax-1\right)}{3a^{3}}$	34
meijerg	$-\frac{-\frac{ax\left(4a^{2}x^{2}+6ax+12\right)}{12}-\ln\left(-ax+1\right)}{a^{3}}+\frac{\frac{ax\left(3ax+6\right)}{6}+\ln\left(-ax+1\right)}{a^{3}}$	57

input
```
int(1/(a*x-1)*(a*x+1)*x^2,x,method=_RETURNVERBOSE)
```

output
```
x^2/a+1/3*x^3+2*x/a^2+2/a^3*ln(a*x-1)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^{2}\,dx=\frac{a^{3}x^{3}+3\,a^{2}x^{2}+6\,ax+6\,\log\left(ax-1\right)}{3\,a^{3}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^3
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{x^3}{3} + \frac{x^2}{a} + \frac{2x}{a^2} + \frac{2\log(ax-1)}{a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**2,x)
```

output
```
x**3/3 + x**2/a + 2*x/a**2 + 2*log(a*x - 1)/a**3
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{a^2x^3 + 3\,ax^2 + 6\,x}{3\,a^2} + \frac{2\,\log(ax-1)}{a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="maxima")
```

output
```
1/3*(a^2*x^3 + 3*a*x^2 + 6*x)/a^2 + 2*log(a*x - 1)/a^3
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 + 3\,a^2x^2 + 6\,ax}{3\,a^3} + \frac{2\,\log(|ax-1|)}{a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="giac")
```

output
```
1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x)/a^3 + 2*log(abs(a*x - 1))/a^3
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{2\,\ln\left(a\,x-1\right)}{a^3}+\frac{2\,x}{a^2}+\frac{x^3}{3}+\frac{x^2}{a}$$

input
```
int((x^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(2*log(a*x - 1))/a^3 + (2*x)/a^2 + x^3/3 + x^2/a
```

3.12 $\int e^{2\coth^{-1}(ax)}x\,dx$

3.12.1 Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

output
```
2*x/a+1/2*x^2+2*ln(-a*x+1)/a^2
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x,x]
```

output
```
(2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

3.12.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}xdx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x-\frac{2}{a}-\frac{2}{a(ax-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(1-ax)}{a^2}+\frac{2x}{a}+\frac{x^2}{2}$$

input
```
Int[E^(2*ArcCoth[a*x])*x,x]
```

output
```
(2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

3.12.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.12.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
norman	$\frac{x^2}{2}+\frac{2x}{a}+\frac{2\ln(ax-1)}{a^2}$	24
risch	$\frac{x^2}{2}+\frac{2x}{a}+\frac{2\ln(ax-1)}{a^2}$	24
parallelrisch	$\frac{a^2x^2+4ax+4\ln(ax-1)}{2a^2}$	26
default	$\frac{\frac{1}{2}a\,x^2+2x}{a}+\frac{2\ln(ax-1)}{a^2}$	27
meijerg	$\frac{\frac{ax(3ax+6)}{6}+\ln(-ax+1)}{a^2}-\frac{-ax-\ln(-ax+1)}{a^2}$	43

input
```
int(1/(a*x-1)*(a*x+1)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+2*x/a+2/a^2*ln(a*x-1)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{a^2x^2+4\,ax+4\,\log(ax-1)}{2\,a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2 + 4*a*x + 4*log(a*x - 1))/a^2
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{x^2}{2}+\frac{2x}{a}+\frac{2\log{(ax-1)}}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x,x)
```

output
```
x**2/2 + 2*x/a + 2*log(a*x - 1)/a**2
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{ax^2+4\,x}{2\,a}+\frac{2\,\log\left(ax-1\right)}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="maxima")
```

output
```
1/2*(a*x^2 + 4*x)/a + 2*log(a*x - 1)/a^2
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{a^2x^2+4\,ax}{2\,a^2}+\frac{2\,\log\left(|ax-1|\right)}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="giac")
```

output
```
1/2*(a^2*x^2 + 4*a*x)/a^2 + 2*log(abs(a*x - 1))/a^2
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{2\,\ln\left(a\,x-1\right)}{a^2}+\frac{2\,x}{a}+\frac{x^2}{2}$$

input

```
int((x*(a*x + 1))/(a*x - 1),x)
```

output

```
(2*log(a*x - 1))/a^2 + (2*x)/a + x^2/2
```

3.13 $\int e^{2\coth^{-1}(ax)}\,dx$

3.13.1 Optimal result

Integrand size = 8, antiderivative size = 14

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(1-ax)}{a}$$

output

```
x+2*ln(-a*x+1)/a
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(1-ax)}{a}$$

input

```
Integrate[E^(2*ArcCoth[a*x]),x]
```

output

```
x + (2*Log[1 - a*x])/a
```

3.13.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6717, 6675, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6675}$$

$$-\int \frac{ax+1}{1-ax}dx$$

$$\Big\downarrow \text{49}$$

$$-\int \left(-1-\frac{2}{ax-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(1-ax)}{a}+x$$

input
```
Int[E^(2*ArcCoth[a*x]),x]
```

output
```
x + (2*Log[1 - a*x])/a
```

3.13.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6675
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.13.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$x+\frac{2\ln(ax-1)}{a}$	14
norman	$x+\frac{2\ln(ax-1)}{a}$	14
risch	$x+\frac{2\ln(ax-1)}{a}$	14
parallelrisch	$\frac{ax+2\ln(ax-1)}{a}$	17
meijerg	$\frac{\ln(-ax+1)}{a}-\frac{-ax-\ln(-ax+1)}{a}$	32

input
```
int((a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
x+2/a*ln(a*x-1)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int e^{2\coth^{-1}(ax)}\,dx=\frac{ax+2\,\log(ax-1)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1),x, algorithm="fricas")
```

output
```
(a*x + 2*log(a*x - 1))/a
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(ax-1)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1),x)
```

output
```
x + 2*log(a*x - 1)/a
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\,\log(ax-1)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1),x, algorithm="maxima")
```

output
```
x + 2*log(a*x - 1)/a
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\,\log(|ax-1|)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1),x, algorithm="giac")
```

output
```
x + 2*log(abs(a*x - 1))/a
```

3.13.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\,\ln\left(a\,x-1\right)}{a}$$

input

```
int((a*x + 1)/(a*x - 1),x)
```

output

```
x + (2*log(a*x - 1))/a
```

3.14 $\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx$

3.14.1 Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = -\log(x) + 2\log(1-ax)$$

output
```
-ln(x)+2*ln(-a*x+1)
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = -\log(x) + 2\log(1-ax)$$

input
```
Integrate[E^(2*ArcCoth[a*x])/x,x]
```

output
```
-Log[x] + 2*Log[1 - a*x]
```

3.14.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{x}dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{ax+1}{x(1-ax)}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(\frac{1}{x}-\frac{2a}{ax-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(1-ax)-\log(x)$$

input
```
Int[E^(2*ArcCoth[a*x])/x,x]
```

output
```
-Log[x] + 2*Log[1 - a*x]
```

3.14.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.14.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$-\ln \left(x \right)+2 \ln \left(a x -1\right)$	14
norman	$-\ln \left(x \right)+2 \ln \left(a x -1\right)$	14
parallelrisch	$-\ln \left(x \right)+2 \ln \left(a x -1\right)$	14
risch	$-\ln \left(x \right)+2 \ln \left(-a x +1\right)$	15
meijerg	$2 \ln \left(-a x +1\right)-\ln \left(x \right)-\ln \left(-a \right)$	21

input
```
int(1/(a*x-1)*(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+2*ln(a*x-1)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2 \coth ^{-1}(a x)}}{x} \, dx = 2 \, \log \left(a x - 1\right) - \log \left(x\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="fricas")
```

output
```
2*log(a*x - 1) - log(x)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = -\log(x) + 2\log\left(x - \frac{1}{a}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x,x)
```

output
```
-log(x) + 2*log(x - 1/a)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\log(ax - 1) - \log(x)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="maxima")
```

output
```
2*log(a*x - 1) - log(x)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\log(|ax - 1|) - \log(|x|)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="giac")
```

output
```
2*log(abs(a*x - 1)) - log(abs(x))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\ln\left(3-3\,a\,x\right)-\ln\left(x\right)$$

input
```
int((a*x + 1)/(x*(a*x - 1)),x)
```

output
```
2*log(3 - 3*a*x) - log(x)
```

3.15 $\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx$

3.15.1 Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} - 2a\log(x) + 2a\log(1-ax)$$

output
```
1/x-2*a*ln(x)+2*a*ln(-a*x+1)
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} - 2a\log(x) + 2a\log(1-ax)$$

input
```
Integrate[E^(2*ArcCoth[a*x])/x^2,x]
```

output
```
x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

3.15.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}dx$$
$$\Big\downarrow \text{6717}$$
$$-\int \frac{e^{2\text{arctanh}(ax)}}{x^2}dx$$
$$\Big\downarrow \text{6676}$$
$$-\int \frac{ax+1}{x^2(1-ax)}dx$$
$$\Big\downarrow \text{86}$$
$$-\int \left(-\frac{2a^2}{ax-1}+\frac{2a}{x}+\frac{1}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-2a\log(x)+2a\log(1-ax)+\frac{1}{x}$$

input
```
Int[E^(2*ArcCoth[a*x])/x^2,x]
```

output
```
x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

3.15.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.15.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{1}{x}-2a\ln(x)+2a\ln(ax-1)$	19
norman	$\frac{1}{x}-2a\ln(x)+2a\ln(ax-1)$	19
risch	$\frac{1}{x}-2a\ln(x)+2a\ln(-ax+1)$	20
parallelrisch	$-\frac{2a\ln(x)x-2a\ln(ax-1)x-1}{x}$	24
meijerg	$-a(-\ln(-ax+1)+\ln(x)+\ln(-a))+a\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)$	48

input
```
int(1/(a*x-1)*(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x-2*a*ln(x)+2*a*ln(a*x-1)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx=\frac{2\,ax\log(ax-1)-2\,ax\log(x)+1}{x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
(2*a*x*log(a*x - 1) - 2*a*x*log(x) + 1)/x
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = 2a\left(-\log{(x)} + \log{\left(x - \frac{1}{a}\right)}\right) + \frac{1}{x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x**2,x)
```

output
```
2*a*(-log(x) + log(x - 1/a)) + 1/x
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\log\left(ax - 1\right) - 2\,a\log\left(x\right) + \frac{1}{x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
2*a*log(a*x - 1) - 2*a*log(x) + 1/x
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\log\left(|ax - 1|\right) - 2\,a\log\left(|x|\right) + \frac{1}{x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="giac")
```

output
```
2*a*log(abs(a*x - 1)) - 2*a*log(abs(x)) + 1/x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} - 4\,a\,\operatorname{atanh}(2\,a\,x - 1)$$

input
```
int((a*x + 1)/(x^2*(a*x - 1)),x)
```

output
```
1/x - 4*a*atanh(2*a*x - 1)
```

3.16 $\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx$

3.16.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2x^2} + \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1-ax)$$

output

```
1/2/x^2+2*a/x-2*a^2*ln(x)+2*a^2*ln(-a*x+1)
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2x^2} + \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1-ax)$$

input

```
Integrate[E^(2*ArcCoth[a*x])/x^3,x]
```

output

```
1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

3.16.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{ax+1}{x^3(1-ax)}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-\frac{2a^3}{ax-1}+\frac{2a^2}{x}+\frac{2a}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2a^2\log(x)+2a^2\log(1-ax)+\frac{2a}{x}+\frac{1}{2x^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/x^3,x]
```

output
```
1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

3.16.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.16.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result
norman	$\frac{\frac{1}{2}+2ax}{x^{2}}-2a^{2}\ln\left(x\right)+2a^{2}\ln\left(ax-1\right)$
default	$\frac{1}{2x^{2}}+\frac{2a}{x}-2a^{2}\ln\left(x\right)+2a^{2}\ln\left(ax-1\right)$
risch	$\frac{\frac{1}{2}+2ax}{x^{2}}+2a^{2}\ln\left(-ax+1\right)-2a^{2}\ln\left(x\right)$
parallelrisch	$-\frac{4a^{2}\ln(x)x^{2}-4a^{2}\ln(ax-1)x^{2}-1-4ax}{2x^{2}}$
meijerg	$a^{2}\left(\ln\left(-ax+1\right)-\ln\left(x\right)-\ln\left(-a\right)+\frac{1}{ax}\right)-a^{2}\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)-\frac{1}{2a^{2}x^{2}}-\right.$

input
```
int(1/(a*x-1)*(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2+2*a*x)/x^2-2*a^2*ln(x)+2*a^2*ln(a*x-1)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int\frac{e^{2\coth^{-1}(ax)}}{x^{3}}\,dx=\frac{4\,a^{2}x^{2}\log\left(ax-1\right)-4\,a^{2}x^{2}\log\left(x\right)+4\,ax+1}{2\,x^{2}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*x^2*log(a*x - 1) - 4*a^2*x^2*log(x) + 4*a*x + 1)/x^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = 2a^2\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{4ax+1}{2x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x**3,x)
```

output
```
2*a**2*(-log(x) + log(x - 1/a)) + (4*a*x + 1)/(2*x**2)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log\left(ax-1\right)-2\,a^2\log\left(x\right)+\frac{4\,ax+1}{2\,x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
2*a^2*log(a*x - 1) - 2*a^2*log(x) + 1/2*(4*a*x + 1)/x^2
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log\left(|ax-1|\right)-2\,a^2\log\left(|x|\right)+\frac{4\,ax+1}{2\,x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="giac")
```

output
```
2*a^2*log(abs(a*x - 1)) - 2*a^2*log(abs(x)) + 1/2*(4*a*x + 1)/x^2
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{2\,a\,x+\frac{1}{2}}{x^2} - 4\,a^2\,\mathrm{atanh}(2\,a\,x-1)$$

input
```
int((a*x + 1)/(x^3*(a*x - 1)),x)
```

output
```
(2*a*x + 1/2)/x^2 - 4*a^2*atanh(2*a*x - 1)
```

3.17 $\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx=\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

output
```
1/3/x^3+a/x^2+2*a^2/x-2*a^3*ln(x)+2*a^3*ln(-a*x+1)
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx=\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

input
```
Integrate[E^(2*ArcCoth[a*x])/x^4,x]
```

output
```
1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

3.17.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6676}$$

$$-\int \frac{ax+1}{x^4(1-ax)}dx$$

$$\Big\downarrow \text{ 86}$$

$$-\int \left(-\frac{2a^4}{ax-1}+\frac{2a^3}{x}+\frac{2a^2}{x^2}+\frac{2a}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-2a^3\log(x)+2a^3\log(1-ax)+\frac{2a^2}{x}+\frac{a}{x^2}+\frac{1}{3x^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/x^4,x]
```

output
```
1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

3.17.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.17.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{1}{3}+2a^2x^2+ax}{x^3}-2a^3\ln(x)+2a^3\ln(ax-1)$
default	$\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\ln(x)+2a^3\ln(ax-1)$
risch	$\frac{\frac{1}{3}+2a^2x^2+ax}{x^3}-2a^3\ln(x)+2a^3\ln(-ax+1)$
parallelrisch	$-\frac{6a^3\ln(x)x^3-6a^3\ln(ax-1)x^3-1-6a^2x^2-3ax}{3x^3}$
meijerg	$-a^3\big(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\big)+a^3\big(\ln(-ax+1)-\ln(x)-\ln(-a)$

input
```
int(1/(a*x-1)*(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(1/3+2*a^2*x^2+a*x)/x^3-2*a^3*ln(x)+2*a^3*ln(a*x-1)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int\frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx=\frac{6\,a^3x^3\log(ax-1)-6\,a^3x^3\log(x)+6\,a^2x^2+3\,ax+1}{3\,x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
1/3*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 1)/x^
3
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = 2a^3\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{6a^2x^2+3ax+1}{3x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x**4,x)
```

output
```
2*a**3*(-log(x) + log(x - 1/a)) + (6*a**2*x**2 + 3*a*x + 1)/(3*x**3)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = 2\,a^3\log\left(ax-1\right)-2\,a^3\log\left(x\right)+\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
2*a^3*log(a*x - 1) - 2*a^3*log(x) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x^3
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = 2\,a^3\log\left(|ax-1|\right)-2\,a^3\log\left(|x|\right)+\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="giac")
```

output
```
2*a^3*log(abs(a*x - 1)) - 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/
x^3
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = \frac{2\,a^2\,x^2+a\,x+\frac{1}{3}}{x^3}-4\,a^3\,\mathrm{atanh}(2\,a\,x-1)$$

input
```
int((a*x + 1)/(x^4*(a*x - 1)),x)
```

output
```
(a*x + 2*a^2*x^2 + 1/3)/x^3 - 4*a^3*atanh(2*a*x - 1)
```

3.18 $\int e^{3\coth^{-1}(ax)}x^2\,dx$

3.18.1 Optimal result

Integrand size = 12, antiderivative size = 118

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{11\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

output
```
11/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3-4*(1-1/a^2/x^2)^(1/2)/a^2/(a-1/x)+14
/3*x*(1-1/a^2/x^2)^(1/2)/a^2+3/2*x^2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^
2/x^2)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-52+19ax+7a^2x^2+2a^3x^3\right)}{-1+ax}+33\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^2,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-52 + 19*a*x + 7*a^2*x^2 + 2*a^3*x^3))/(-1 +
a*x) + 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

3.18.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2 x^4}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^4}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^2,x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*
x)/(3*a^2) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*
x^3)/3 + (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.18.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

method	result
risch	$\frac{(2a^2x^2+9ax+28)(ax-1)}{6a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a^2\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^4\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-4\sqrt{a^2}\,((ax-1)(ax+1}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2+9*a*x+28)*(a*x-1)/a^3/((a*x-1)/(a*x+1))^(1/2)+(11/2/a^2*ln(
a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^4/(x-1/a)*((x-1/a)^2*
a^2+2*(x-1/a)*a)^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^
(1/2)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = \frac{33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2\,a^4x^4+9\,a^3x^3+26\,a^2x^2-33\,ax-52)}{6\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="fricas")
```

output
```
1/6*(33*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 33*(a*x - 1)*log(sq
rt((a*x - 1)/(a*x + 1)) - 1) + (2*a^4*x^4 + 9*a^3*x^3 + 26*a^2*x^2 - 33*a*
x - 52)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)
```

3.18.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.54

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=$$

$$-\frac{1}{6}a\left(\frac{2\left(\frac{75\,(ax-1)}{ax+1}-\frac{88\,(ax-1)^2}{(ax+1)^2}+\frac{33\,(ax-1)^3}{(ax+1)^3}-12\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{a^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="maxima")
```

output
```
-1/6*a*(2*(75*(a*x - 1)/(a*x + 1) - 88*(a*x - 1)^2/(a*x + 1)^2 + 33*(a*x -
 1)^3/(a*x + 1)^3 - 12)/(a^4*((a*x - 1)/(a*x + 1))^(7/2) - 3*a^4*((a*x - 1
)/(a*x + 1))^(5/2) + 3*a^4*((a*x - 1)/(a*x + 1))^(3/2) - a^4*sqrt((a*x - 1
)/(a*x + 1))) - 33*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 + 33*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^4)
```

3.18.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="giac")
```

output
```
undef
```

3.18.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.31

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = \frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3} - \frac{\frac{88\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{11\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{25\,(a\,x-1)}{a\,x+1}+4}{a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}+3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input

```
int(x^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output

```
(11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3 - ((88*(a*x - 1)^2)/(3*(a*x +
1)^2) - (11*(a*x - 1)^3)/(a*x + 1)^3 - (25*(a*x - 1))/(a*x + 1) + 4)/(a^3*
((a*x - 1)/(a*x + 1))^(1/2) - 3*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^3*((
a*x - 1)/(a*x + 1))^(5/2) - a^3*((a*x - 1)/(a*x + 1))^(7/2))
```

3.19 $\int e^{3\coth^{-1}(ax)}x\,dx$

3.19.1 Optimal result

Integrand size = 10, antiderivative size = 92

$$\int e^{3\coth^{-1}(ax)}x\,dx = -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{9\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-4*(1-1/a^2/x^2)^(1/2)/a/(a-1/x)+3*x*(
1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int e^{3\coth^{-1}(ax)}x\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-14+5ax+a^2x^2\right)}{-1+ax}+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 + 5*a*x + a^2*x^2))/(-1 + a*x) + 9*Log[(1
 + Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

3.19.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow 6719$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2x^3}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow 2353$$

$$-\int \left(\frac{x^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow 2009$$

$$\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}$$

input
```
Int[E^(3*ArcCoth[a*x])*x,x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) + (3*Sqrt[1 - 1/(a^2*x^2)]*x)/
a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*
a^2)
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.19.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.54

method	result
risch	$\frac{(ax+6)(ax-1)}{2a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^3\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+10\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+10\ln\Big(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)}}{\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+6)*(a*x-1)/a^2/((a*x-1)/(a*x+1))^(1/2)+(9/2/a*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^3/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)
^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int e^{3\coth^{-1}(ax)}x\,dx$$
$$=\frac{9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^3x^3+6\,a^2x^2-9\,ax-14)\sqrt{\frac{ax-1}{ax+1}}}{2\,(a^3x-a^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="fricas")
```

output
```
1/2*(9*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x - 1)*log(sqrt
((a*x - 1)/(a*x + 1)) - 1) + (a^3*x^3 + 6*a^2*x^2 - 9*a*x - 14)*sqrt((a*x
- 1)/(a*x + 1)))/(a^3*x - a^2)
```

3.19.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\,dx=\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x,x)
```

output

```
Integral(x/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.58

$$\int e^{3\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{2}\,a\left(\frac{2\left(\frac{15\,(ax-1)}{ax+1}-\frac{9\,(ax-1)^2}{(ax+1)^2}-4\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-2\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="maxima")
```

output

```
1/2*a*(2*(15*(a*x - 1)/(a*x + 1) - 9*(a*x - 1)^2/(a*x + 1)^2 - 4)/(a^3*((a
*x - 1)/(a*x + 1))^(5/2) - 2*a^3*((a*x - 1)/(a*x + 1))^(3/2) + a^3*sqrt((a
*x - 1)/(a*x + 1))) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - 9*log(sqr
t((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

3.19.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}x\,dx=\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="giac")
```

output

```
undef
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int e^{3\coth^{-1}(ax)}x\,dx=\frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{\frac{9\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{15\,(a\,x-1)}{a\,x+1}+4}{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-2\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}+a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input

```
int(x/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output

```
(9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - ((9*(a*x - 1)^2)/(a*x + 1)^2
- (15*(a*x - 1))/(a*x + 1) + 4)/(a^2*((a*x - 1)/(a*x + 1))^(1/2) - 2*a^2*(
(a*x - 1)/(a*x + 1))^(3/2) + a^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.20 $\int e^{3\coth^{-1}(ax)}\,dx$

3.20.1 Optimal result

Integrand size = 8, antiderivative size = 62

$$\int e^{3\coth^{-1}(ax)}\,dx=-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
3*arctanh((1-1/a^2/x^2)^(1/2))/a-4*(1-1/a^2/x^2)^(1/2)/(a-1/x)+x*(1-1/a^2/
x^2)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{3\coth^{-1}(ax)}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-5+ax)}{-1+ax}+\frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + a*x))/(-1 + a*x) + (3*Log[a*(1 + Sqrt[1 - 1
/(a^2*x^2)])*x])/a
```

3.20.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6718, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6718}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2 x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}$$

input
```
Int[E^(3*ArcCoth[a*x]),x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x + (3*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6718

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.15

method	result
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^2\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{3\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2-6\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^2x-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-}{a\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$

input

```
int(1/((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/a*(a*x-1)/((a*x-1)/(a*x+1))^(1/2)+(3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1
/2))/(a^2)^(1/2)-4/a^2/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))/(a*x+1)/
((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int e^{3\coth^{-1}(ax)}\,dx = \frac{3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-4\,ax-5)\sqrt{\frac{ax-1}{ax+1}}}{a^2x-a}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output

```
(3*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a*x - 1)*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 4*a*x - 5)*sqrt((a*x - 1)/(a*x + 1)))/
(a^2*x - a)
```

3.20.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-3/2), x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int e^{3\coth^{-1}(ax)}\,dx = -a\left(\frac{2\left(\frac{3(ax-1)}{ax+1}-2\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-a*(2*(3*(a*x - 1)/(a*x + 1) - 2)/(a^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*s
qrt((a*x - 1)/(a*x + 1))) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*l
og(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.20.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.20.9 Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int e^{3\coth^{-1}(ax)}\,dx=\frac{2\,a\,x+12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-10}{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input

```
int(1/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output

```
(2*a*x + 12*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
 - 10)/(2*a*((a*x - 1)/(a*x + 1))^(1/2))
```

3.21 $\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx$

3.21.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx=-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\csc^{-1}(ax)+\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

output
```
arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))-4*a*(1-1/a^2/x^2)^(1/2)/(a-1/x)
```

3.21.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx=-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}\,x}{-1+ax}+\arcsin\left(\frac{1}{ax}\right)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)$$

input
```
Integrate[E^(3*ArcCoth[a*x])/x,x]
```

output
```
(-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]
```

3.21.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6719, 2351, 27, 564, 25, 27, 243, 73, 221, 671, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2 x}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2351}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-\int \frac{ax}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{564}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\int -\frac{x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\int \frac{x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}\right)$$

$$\Big\downarrow \text{243}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}\right)$$

$$\Big\downarrow \textbf{73}$$

$$-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \textbf{221}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)$$

$$\Big\downarrow \textbf{671}$$

$$\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\left(a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)\right)-\frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}$$

$$\Big\downarrow \textbf{223}$$

$$-\left(a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)\right)-\frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])/x,x]
```

output
```
(-3*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + ArcSin[1/(a*x)] - a*(Sqrt[1 -
1/(a^2*x^2)]/(a - x^(-1)) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a)
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

3.21. $\int \frac{e^{3 \coth^{-1}(ax)}}{x} \, dx$

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(42) = 84$.

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 7.89

method	result
default	$\frac{\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+a^{2}x^{2}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x^{2}-2\sqrt{a^{2}x^{2}-1}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x
^2-1)^(1/2))+ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a
^3*x^2+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2-2*(a^2*x^2-1)^(1/2)*(a^
2)^(1/2)*a*x-2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-2*ln((a^2*x+(a^
2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x-2*((a*x-1)*(a*x+1))^(
3/2)*(a^2)^(1/2)-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+(a^2*x^2-1)^(1/
2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+a*ln((a^2*x+(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2))/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.26

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -\frac{2\,(ax-1)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+4\,(ax+1)}{ax-1}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

output
```
-(2*(a*x - 1)*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x - 1)*log(sqrt((a*x
- 1)/(a*x + 1)) + 1) + (a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + 4*(a
*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)
```

3.21.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx=\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}+\frac{4}{a\sqrt{\frac{ax-1}{ax+1}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

output
```
-a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a + 4/(a*sqrt((a*x - 1)/(a*x +
 1))))
```

3.21.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

output

```
undef
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{4}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input

```
int(1/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output

```
2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
 - 4/((a*x - 1)/(a*x + 1))^(1/2)
```

3.22 $\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx$

3.22.1 Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -3a\sqrt{1-\frac{1}{a^2x^2}} - \frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3a\csc^{-1}(ax)$$

output
```
3*a*arccsc(a*x)-2*(a+1/x)^2/a/(1-1/a^2/x^2)^(1/2)-3*a*(1-1/a^2/x^2)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}(1-5ax)}{-1+ax} + 3a\arcsin\left(\frac{1}{ax}\right)$$

input
```
Integrate[E^(3*ArcCoth[a*x])/x^2,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x))/(-1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

3.22.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6719, 711, 25, 27, 671, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2} \, dx$$

$$\Big\downarrow \; 6719$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \; 711$$

$$a^4 \int -\frac{a+\frac{3}{x}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}-a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \; 25$$

$$a^4\left(-\int \frac{a+\frac{3}{x}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$a\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{a+\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \; 671$$

$$3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-a\sqrt{1-\frac{1}{a^2x^2}}+3a\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])/x^2,x]
```

output
```
-(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) +
3*a*ArcSin[1/(a*x)]
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 711
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)
/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))
Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)
^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - 2*e*g^n*(m + p + n)*(d + e*x)^(n
 - 2)*(a*e - c*d*x), x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && Eq
Q[c*d^2 + a*e^2, 0] && IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(47) = 94$.

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{ax-1}{x\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+5\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)/x/((a*x-1)/(a*x+1))^(1/2)+(3*a*arctan(1/(a^2*x^2-1)^(1/2))-4/(x-1
/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a
*x-1)*(a*x+1))^(1/2)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -\frac{6\left(a^2x^2-ax\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(5\,a^2x^2+4\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(6*(a^2*x^2 - a*x)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (5*a^2*x^2 + 4*a*x
 - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)
```

3.22.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\left(\frac{\frac{3\,(ax-1)}{ax+1}+2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}+3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-2*a*((3*(a*x - 1)/(a*x + 1) + 2)/(((a*x - 1)/(a*x + 1))^(3/2) + sqrt((a*x
 - 1)/(a*x + 1))) + 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

3.22.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

3.22.9 Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x\sqrt{\frac{a\,x-1}{a\,x+1}}} - 6\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{5\,a}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input

```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output

```
1/(x*((a*x - 1)/(a*x + 1))^(1/2)) - 6*a*atan(((a*x - 1)/(a*x + 1))^(1/2))
- (5*a)/((a*x - 1)/(a*x + 1))^(1/2)
```

3.23 $\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx$

3.23.1 Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx=-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}+\frac{9}{2}a^2\csc^{-1}(ax)$$

output
```
-a^5*(1-1/a^2/x^2)^(5/2)/(a-1/x)^3-3/2*a^3*(1-1/a^2/x^2)^(3/2)/(a-1/x)+9/2
*a^2*arccsc(a*x)-9/2*a^2*(1-1/a^2/x^2)^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx=\frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1+5ax-14a^2x^2\right)}{x(-1+ax)}+9a\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^(3*ArcCoth[a*x])/x^3,x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x - 14*a^2*x^2))/(x*(-1 + a*x)) + 9*a*
ArcSin[1/(a*x)]))/2
```

3.23.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {6719, 2164, 25, 27, 2027, 2164, 25, 27, 563, 25, 2346, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)x}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$\frac{\int -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}+\frac{1}{x^2}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}+\frac{1}{x^2}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}+\frac{1}{x^2}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2027}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{\left(a-\frac{1}{x}\right)^2x}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$a^2\int -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3x}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-a^2\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3x}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3 x}d\frac{1}{x}$$

$$\Big\downarrow \text{563}$$

$$-a^3\left(\frac{\int -\frac{4a^2+\frac{3a}{x}+\frac{1}{x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}\right)$$

$$\Big\downarrow \text{25}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\int \frac{4a^2+\frac{3a}{x}+\frac{1}{x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}\right)$$

$$\Big\downarrow \text{2346}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{-\frac{1}{2}a^2\int -\frac{3\left(3a+\frac{2}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

$$\Big\downarrow \text{27}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{3}{2}a\int \frac{3a+\frac{2}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

$$\Big\downarrow \text{455}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{3}{2}a\left(3a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

$$\Big\downarrow \text{223}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{3}{2}a\left(3a^2\arcsin\left(\frac{1}{ax}\right)-2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

3.23. $\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx$

input
```
Int[E^(3*ArcCoth[a*x])/x^3,x]
```

output
```
-(a^3*((4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) - (-1/2*(a^2*Sqrt[1 - 1/(a^2
*x^2)])/x + (3*a*(-2*a^2*Sqrt[1 - 1/(a^2*x^2)] + 3*a^2*ArcSin[1/(a*x)]))/2
)/a^4))
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.23.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{(ax-1)(6ax+1)}{2x^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{9a^{2}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}-\frac{4a\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{-6\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{5}x^{5}+6\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{3}x^{3}+21\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{4}x^{4}+9a^{4}x^{4}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+6\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-1)*(6*a*x+1)/x^2/((a*x-1)/(a*x+1))^(1/2)+(9/2*a^2*arctan(1/(a^2*
x^2-1)^(1/2))-4*a/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))/(a*x+1)/((a*x
-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\frac{18\left(a^3x^3-a^2x^2\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(14\,a^3x^3+9\,a^2x^2-6\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^3-x^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(18*(a^3*x^3 - a^2*x^2)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (14*a^3*x
^3 + 9*a^2*x^2 - 6*a*x - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)
```

3.23.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{15\,(ax-1)a}{ax+1}+\frac{9\,(ax-1)^2a}{(ax+1)^2}+4\,a}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (15*(a*x - 1)*a/(a*x + 1) + 9*(a
*x - 1)^2*a/(a*x + 1)^2 + 4*a)/(((a*x - 1)/(a*x + 1))^(5/2) + 2*((a*x - 1)
/(a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

3.23.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.91

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2\,x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}} - \frac{7\,a^2}{\sqrt{\frac{a\,x-1}{a\,x+1}}} - 9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{5\,a}{2\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
1/(2*x^2*((a*x - 1)/(a*x + 1))^(1/2)) - (7*a^2)/((a*x - 1)/(a*x + 1))^(1/2
) - 9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (5*a)/(2*x*((a*x - 1)/(a*x +
 1))^(1/2))
```

3.24 $\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$

3.24.1 Optimal result

Integrand size = 12, antiderivative size = 93

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{11}{2}a^3\csc^{-1}(ax)$$

output
```
11/2*a^3*arccsc(a*x)-(a+1/x)^3/(1-1/a^2/x^2)^(1/2)-1/3*a*(3*a+1/x)^2*(1-1/
a^2/x^2)^(1/2)-1/6*a^2*(28*a+3/x)*(1-1/a^2/x^2)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(2+7ax+19a^2x^2-52a^3x^3\right)}{x^2(-1+ax)} + 33a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^(3*ArcCoth[a*x])/x^4,x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 7*a*x + 19*a^2*x^2 - 52*a^3*x^3))/(x^2*(-1
 + a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

3.24.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6719, 2164, 25, 27, 2027, 2164, 25, 27, 563, 2346, 25, 2346, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$\frac{\int -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}+\frac{1}{x^3}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}+\frac{1}{x^3}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}+\frac{1}{x^3}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2027}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{\left(a-\frac{1}{x}\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$a^2\int -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-a^2\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^3 \int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3 x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{563}$$

$$-a^3 \left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\int \frac{4a^3+\frac{4a^2}{x}+\frac{3a}{x^2}+\frac{1}{x^3}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}\right)$$

$$\Big\downarrow \text{2346}$$

$$-a^3 \left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{-\frac{1}{3}a^2\int -\frac{12a+\frac{14}{x}+\frac{9}{x^2a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \text{25}$$

$$-a^3 \left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\int \frac{12a+\frac{14}{x}+\frac{9}{x^2a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \text{2346}$$

$$-a^3 \left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(-\frac{1}{2}a^2\int -\frac{33a+\frac{28}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \text{25}$$

$$-a^3 \left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}a^2\int \frac{33a+\frac{28}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \text{27}$$

3.24. $\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\int \frac{33a+\frac{28}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow 455$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\left(33a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-28a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow 223$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\left(33a^2\arcsin\left(\frac{1}{ax}\right)-28a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])/x^4,x]
```

output
```
-(a^3*((4*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) - (-1/3*(a^2*Sqrt[1 - 1/(a
^2*x^2)])/x^2 + (a^2*((-9*a*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (-28*a^2*Sqrt[1
 - 1/(a^2*x^2)] + 33*a^2*ArcSin[1/(a*x)])/2))/3)/a^4))
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.24. $\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$

3.24.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{(ax-1)(28a^2x^2+9ax+2)}{6x^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{11a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{4a^2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-30\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+93\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+33\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^5x^5+30\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x-1)*(28*a^2*x^2+9*a*x+2)/x^3/((a*x-1)/(a*x+1))^(1/2)+(11/2*a^3*ar
ctan(1/(a^2*x^2-1)^(1/2))-4*a^2/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))
/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{66\,(a^4x^4-a^3x^3)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(52\,a^4x^4+33\,a^3x^3-26\,a^2x^2-9\,ax-2)\sqrt{\frac{ax-1}{ax+1}}}{6\,(ax^4-x^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(66*(a^4*x^4 - a^3*x^3)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (52*a^4*x
^4 + 33*a^3*x^3 - 26*a^2*x^2 - 9*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^
4 - x^3)
```

3.24.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.66

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$$
$$= -\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{\frac{75\,(ax-1)a^2}{ax+1} + \frac{88\,(ax-1)^2a^2}{(ax+1)^2} + \frac{33\,(ax-1)^3a^2}{(ax+1)^3} + 12\,a^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 3\,\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 3\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + \sqrt{\frac{ax-1}{ax+1}}}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (75*(a*x - 1)*a^2/(a*x +
1) + 88*(a*x - 1)^2*a^2/(a*x + 1)^2 + 33*(a*x - 1)^3*a^2/(a*x + 1)^3 + 12*
a^2)/(((a*x - 1)/(a*x + 1))^(7/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + 3*((a*
x - 1)/(a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

3.24.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

output
```
undef
```

3.24.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.63

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{4\,a^3+\frac{88\,a^3\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}+\frac{11\,a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{25\,a^3\,\left(a\,x-1\right)}{a\,x+1}}{\sqrt{\frac{a\,x-1}{a\,x+1}}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}-11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

input

```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output

```
- (4*a^3 + (88*a^3*(a*x - 1)^2)/(3*(a*x + 1)^2) + (11*a^3*(a*x - 1)^3)/(a*
x + 1)^3 + (25*a^3*(a*x - 1))/(a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/2) + 3*
((a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + ((a*x - 1)/(
a*x + 1))^(7/2)) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

3.25 $\int e^{4\coth^{-1}(ax)}x^3\,dx$

3.25.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{12x}{a^3} + \frac{4x^2}{a^2} + \frac{4x^3}{3a} + \frac{x^4}{4} + \frac{4}{a^4(1-ax)} + \frac{16\log(1-ax)}{a^4}$$

output
```
12*x/a^3+4*x^2/a^2+4/3*x^3/a+1/4*x^4+4/a^4/(-a*x+1)+16*ln(-a*x+1)/a^4
```

3.25.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{12x}{a^3} + \frac{4x^2}{a^2} + \frac{4x^3}{3a} + \frac{x^4}{4} + \frac{4}{a^4(1-ax)} + \frac{16\log(1-ax)}{a^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*x^3,x]
```

output
```
(12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16
*Log[1 - a*x])/a^4
```

3.25.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int x^3 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 6676}$$

$$\int \frac{x^3(ax+1)^2}{(1-ax)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{16}{a^3(ax-1)}+\frac{4}{a^3(ax-1)^2}+\frac{12}{a^3}+\frac{8x}{a^2}+\frac{4x^2}{a}+x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

input
```
Int[E^(4*ArcCoth[a*x])*x^3,x]
```

output
```
(12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16
*Log[1 - a*x])/a^4
```

3.25.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.25.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x^{4}}{4}+\frac{4x^{3}}{3a}+\frac{4x^{2}}{a^{2}}+\frac{12x}{a^{3}}-\frac{4}{a^{4}(ax-1)}+\frac{16\ln(ax-1)}{a^{4}}$
norman	$\frac{\frac{13x^{4}}{12}+\frac{a\,x^{5}}{4}+\frac{8x^{2}}{a^{2}}+\frac{8x^{3}}{3a}-\frac{16}{a^{4}}}{ax-1}+\frac{16\ln(ax-1)}{a^{4}}$
default	$\frac{\frac{1}{4}a^{3}x^{4}+\frac{4}{3}a^{2}x^{3}+4a\,x^{2}+12x}{a^{3}}-\frac{4}{a^{4}(ax-1)}+\frac{16\ln(ax-1)}{a^{4}}$
parallelrisch	$\frac{3a^{5}x^{5}+13a^{4}x^{4}+32a^{3}x^{3}-192+96a^{2}x^{2}+192a\ln(ax-1)x-192\ln(ax-1)}{12a^{4}(ax-1)}$
meijerg	$\frac{\frac{ax\left(-3a^{4}x^{4}-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{-12ax+12}+5\ln(-ax+1)}{a^{4}}-\frac{2\left(-\frac{ax\left(-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a^{4}}+$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+4/3*x^3/a+4*x^2/a^2+12*x/a^3-4/a^4/(a*x-1)+16/a^4*ln(a*x-1)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int e^{4\coth^{-1}(ax)}x^{3}\,dx = \frac{3\,a^{5}x^{5}+13\,a^{4}x^{4}+32\,a^{3}x^{3}+96\,a^{2}x^{2}-144\,ax+192\,(ax-1)\log(ax-1)-48}{12\,(a^{5}x-a^{4})}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="fricas")
```

output
```
1/12*(3*a^5*x^5 + 13*a^4*x^4 + 32*a^3*x^3 + 96*a^2*x^2 - 144*a*x + 192*(a*
x - 1)*log(a*x - 1) - 48)/(a^5*x - a^4)
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{x^4}{4} - \frac{4}{a^5x - a^4} + \frac{4x^3}{3a} + \frac{4x^2}{a^2} + \frac{12x}{a^3} + \frac{16\log{(ax-1)}}{a^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*x**3,x)
```

output
```
x**4/4 - 4/(a**5*x - a**4) + 4*x**3/(3*a) + 4*x**2/a**2 + 12*x/a**3 + 16*l
og(a*x - 1)/a**4
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = -\frac{4}{a^5x - a^4} + \frac{3\,a^3x^4 + 16\,a^2x^3 + 48\,ax^2 + 144\,x}{12\,a^3} + \frac{16\,\log{(ax-1)}}{a^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="maxima")
```

output
```
-4/(a^5*x - a^4) + 1/12*(3*a^3*x^4 + 16*a^2*x^3 + 48*a*x^2 + 144*x)/a^3 +
16*log(a*x - 1)/a^4
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{(ax-1)^4\left(\frac{28}{ax-1}+\frac{114}{(ax-1)^2}+\frac{300}{(ax-1)^3}+3\right)}{12\,a^4} - \frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^4} - \frac{4}{(ax-1)a^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="giac")
```

output
```
1/12*(a*x - 1)^4*(28/(a*x - 1) + 114/(a*x - 1)^2 + 300/(a*x - 1)^3 + 3)/a^
4 - 16*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a^4 - 4/((a*x - 1)*a^4)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{16\,\ln\left(a\,x-1\right)}{a^4} - \frac{4}{a\,\left(a^4\,x-a^3\right)} + \frac{12\,x}{a^3} + \frac{x^4}{4} + \frac{4\,x^3}{3\,a} + \frac{4\,x^2}{a^2}$$

input
```
int((x^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(16*log(a*x - 1))/a^4 - 4/(a*(a^4*x - a^3)) + (12*x)/a^3 + x^4/4 + (4*x^3)
/(3*a) + (4*x^2)/a^2
```

3.26 $\int e^{4\coth^{-1}(ax)}x^2\,dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 47

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}$$

output
```
8*x/a^2+2*x^2/a+1/3*x^3+4/a^3/(-a*x+1)+12*ln(-a*x+1)/a^3
```

3.26.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*x^2,x]
```

output
```
(8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

3.26.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int x^2 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{x^2(ax+1)^2}{(1-ax)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{12}{a^2(ax-1)}+\frac{4}{a^2(ax-1)^2}+\frac{8}{a^2}+\frac{4x}{a}+x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}$$

input

```
Int[E^(4*ArcCoth[a*x])*x^2,x]
```

output

```
(8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

3.26.3.1 Defintions of rubi rules used

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.26.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^3}{3}+\frac{2x^2}{a}+\frac{8x}{a^2}-\frac{4}{a^3(ax-1)}+\frac{12\ln(ax-1)}{a^3}$
norman	$\frac{\frac{5x^3}{3}+\frac{a\,x^4}{3}+\frac{6x^2}{a}-\frac{12}{a^3}}{ax-1}+\frac{12\ln(ax-1)}{a^3}$
default	$\frac{\frac{1}{3}a^2x^3+2a\,x^2+8x}{a^2}-\frac{4}{a^3(ax-1)}+\frac{12\ln(ax-1)}{a^3}$
parallelrisch	$\frac{a^4x^4+5a^3x^3-36+18a^2x^2+36a\ln(ax-1)x-36\ln(ax-1)}{3a^3(ax-1)}$
meijerg	$-\frac{-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)}{a^3}+\frac{\frac{2ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+6\ln(-ax+1)}{a^3}-\frac{-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)}{a^3}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+2*x^2/a+8*x/a^2-4/a^3/(a*x-1)+12/a^3*ln(a*x-1)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{a^4x^4+5\,a^3x^3+18\,a^2x^2-24\,ax+36\,(ax-1)\log{(ax-1)}-12}{3\,(a^4x-a^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="fricas")
```

output
```
1/3*(a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 24*a*x + 36*(a*x - 1)*log(a*x - 1)
 - 12)/(a^4*x - a^3)
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \frac{x^3}{3} - \frac{4}{a^4x - a^3} + \frac{2x^2}{a} + \frac{8x}{a^2} + \frac{12\log(ax-1)}{a^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*x**2,x)
```

output
```
x**3/3 - 4/(a**4*x - a**3) + 2*x**2/a + 8*x/a**2 + 12*log(a*x - 1)/a**3
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = -\frac{4}{a^4x - a^3} + \frac{a^2x^3 + 6\,ax^2 + 24\,x}{3\,a^2} + \frac{12\,\log(ax-1)}{a^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="maxima")
```

output
```
-4/(a^4*x - a^3) + 1/3*(a^2*x^3 + 6*a*x^2 + 24*x)/a^2 + 12*log(a*x - 1)/a^
3
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \frac{(ax-1)^3\left(\frac{9}{ax-1} + \frac{39}{(ax-1)^2} + 1\right)}{3\,a^3} - \frac{12\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^3} - \frac{4}{(ax-1)a^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="giac")
```

output
```
1/3*(a*x - 1)^3*(9/(a*x - 1) + 39/(a*x - 1)^2 + 1)/a^3 - 12*log(abs(a*x -
1)/((a*x - 1)^2*abs(a)))/a^3 - 4/((a*x - 1)*a^3)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{12\ln\left(a\,x-1\right)}{a^3}-\frac{4}{a\,\left(a^3\,x-a^2\right)}+\frac{8\,x}{a^2}+\frac{x^3}{3}+\frac{2\,x^2}{a}$$

input

```
int((x^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output

```
(12*log(a*x - 1))/a^3 - 4/(a*(a^3*x - a^2)) + (8*x)/a^2 + x^3/3 + (2*x^2)/
a
```

3.27 $\int e^{4\coth^{-1}(ax)}x\,dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 39

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}$$

output
```
4*x/a+1/2*x^2+4/a^2/(-a*x+1)+8*ln(-a*x+1)/a^2
```

3.27.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*x,x]
```

output
```
(4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

3.27.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int xe^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{x(ax+1)^2}{(1-ax)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{8}{a(ax-1)}+\frac{4}{a(ax-1)^2}+\frac{4}{a}+x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

input
```
Int[E^(4*ArcCoth[a*x])*x,x]
```

output
```
(4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

3.27.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.27.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{x^{2}}{2}+\frac{4 x}{a}-\frac{4}{a^{2} \left(a x -1\right)}+\frac{8 \ln \left(a x -1\right)}{a^{2}}$	36
default	$\frac{\frac{1}{2} a \,x^{2}+4 x}{a}-\frac{4}{a^{2} \left(a x -1\right)}+\frac{8 \ln \left(a x -1\right)}{a^{2}}$	39
norman	$\frac{\frac{7 x^{2}}{2}+\frac{a \,x^{3}}{2}-\frac{8 x}{a}}{a x -1}+\frac{8 \ln \left(a x -1\right)}{a^{2}}$	39
parallelrisch	$\frac{a^{3} x^{3}+7 a^{2} x^{2}+16 a \ln \left(a x -1\right) x -16 a x -16 \ln \left(a x -1\right)}{2 \left(a x -1\right) a^{2}}$	51
meijerg	$\frac{\frac{a x \left(-2 a^{2} x^{2}-6 a x +12\right)}{-4 a x +4}+3 \ln \left(-a x +1\right)}{a^{2}}-\frac{2 \left(-\frac{a x \left(-3 a x +6\right)}{3 \left(-a x +1\right)}-2 \ln \left(-a x +1\right)\right)}{a^{2}}+\frac{\frac{a x}{-a x +1}+\ln \left(-a x +1\right)}{a^{2}}$	98

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+4*x/a-4/a^2/(a*x-1)+8/a^2*ln(a*x-1)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{a^{3}x^{3}+7\,a^{2}x^{2}-8\,ax+16\,(ax-1)\log\left(ax-1\right)-8}{2\,\left(a^{3}x-a^{2}\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="fricas")
```

output
```
1/2*(a^3*x^3 + 7*a^2*x^2 - 8*a*x + 16*(a*x - 1)*log(a*x - 1) - 8)/(a^3*x -
 a^2)
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{x^2}{2}-\frac{4}{a^3x-a^2}+\frac{4x}{a}+\frac{8\log{(ax-1)}}{a^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*x,x)
```

output
```
x**2/2 - 4/(a**3*x - a**2) + 4*x/a + 8*log(a*x - 1)/a**2
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{ax^2+8\,x}{2\,a}-\frac{4}{a^3x-a^2}+\frac{8\,\log{(ax-1)}}{a^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="maxima")
```

output
```
1/2*(a*x^2 + 8*x)/a - 4/(a^3*x - a^2) + 8*log(a*x - 1)/a^2
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int e^{4\coth^{-1}(ax)}x\,dx=\frac{\frac{(ax-1)^2\left(\frac{10}{ax-1}+1\right)}{a}-\frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{8}{(ax-1)a}}{2\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="giac")
```

output
```
1/2*((a*x - 1)^2*(10/(a*x - 1) + 1)/a - 16*log(abs(a*x - 1)/((a*x - 1)^2*a
bs(a)))/a - 8/((a*x - 1)*a))/a
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{8\,\ln\left(a\,x-1\right)}{a^2}+\frac{4\,x}{a}+\frac{x^2}{2}+\frac{4}{a\,\left(a-a^2\,x\right)}$$

input
```
int((x*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(8*log(a*x - 1))/a^2 + (4*x)/a + x^2/2 + 4/(a*(a - a^2*x))
```

3.28 $\int e^{4\coth^{-1}(ax)}\,dx$

3.28.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int e^{4\coth^{-1}(ax)}\,dx=x+\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}$$

output
```
x+4/a/(-a*x+1)+4*ln(-a*x+1)/a
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\,dx=x-\frac{4}{a(-1+ax)}+\frac{4\log(1-ax)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x]),x]
```

output
```
x - 4/(a*(-1 + a*x)) + (4*Log[1 - a*x])/a
```

3.28.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6717, 6675, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 6675}$$

$$\int \frac{(ax+1)^2}{(1-ax)^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{4}{ax-1}+\frac{4}{(ax-1)^2}+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

input
```
Int[E^(4*ArcCoth[a*x]),x]
```

output
```
x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

3.28.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6675
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.28.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$x-\frac{4}{a(ax-1)}+\frac{4\ln(ax-1)}{a}$	26
risch	$x-\frac{4}{a(ax-1)}+\frac{4\ln(ax-1)}{a}$	26
norman	$\frac{a\,x^{2}-5x}{ax-1}+\frac{4\ln(ax-1)}{a}$	30
parallelrisch	$\frac{a^{2}x^{2}+4a\ln(ax-1)x-5-4\ln(ax-1)}{(ax-1)a}$	39
meijerg	$-\frac{-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)}{a}+\frac{\frac{2ax}{-ax+1}+2\ln(-ax+1)}{a}+\frac{x}{-ax+1}$	69

input
```
int(1/(a*x-1)^2*(a*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x-4/a/(a*x-1)+4/a*ln(a*x-1)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int e^{4\coth^{-1}(ax)}\,dx=\frac{a^{2}x^{2}-ax+4\,(ax-1)\log\left(ax-1\right)-4}{a^{2}x-a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="fricas")
```

output
```
(a^2*x^2 - a*x + 4*(a*x - 1)*log(a*x - 1) - 4)/(a^2*x - a)
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{4\coth^{-1}(ax)}\,dx = x - \frac{4}{a^2x - a} + \frac{4\log(ax-1)}{a}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2,x)
```

output
```
x - 4/(a**2*x - a) + 4*log(a*x - 1)/a
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\,dx = x + \frac{4\log(ax-1)}{a} - \frac{4}{a^2x - a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="maxima")
```

output
```
x + 4*log(a*x - 1)/a - 4/(a^2*x - a)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int e^{4\coth^{-1}(ax)}\,dx = \frac{ax-1}{a} - \frac{4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{4}{(ax-1)a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="giac")
```

output
```
(a*x - 1)/a - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4/((a*x - 1)*a)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\,dx = x - \frac{4}{a\,(a\,x-1)} + \frac{4\,\ln(a\,x-1)}{a}$$

input

```
int((a*x + 1)^2/(a*x - 1)^2,x)
```

output

```
x - 4/(a*(a*x - 1)) + (4*log(a*x - 1))/a
```

3.29 $\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx$

3.29.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \frac{4}{1-ax} + \log(x)$$

output
```
4/(-a*x+1)+ln(x)
```

3.29.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \frac{4}{1-ax} + \log(x)$$

input
```
Integrate[E^(4*ArcCoth[a*x])/x,x]
```

output
```
4/(1 - a*x) + Log[x]
```

3.29.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{x}dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{(ax+1)^2}{x(1-ax)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{4a}{(ax-1)^2}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{1-ax}+\log(x)$$

input
```
Int[E^(4*ArcCoth[a*x])/x,x]
```

output
```
4/(1 - a*x) + Log[x]
```

3.29.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.29.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
default	$\ln\left(x\right)-\frac{4}{a x -1}$	13
norman	$-\frac{4 a x}{a x -1}+\ln\left(x\right)$	15
risch	$-\frac{4}{a x -1}+\ln\left(-x\right)$	15
parallelrisch	$\frac{a \ln\left(x\right) x -4 a x -\ln\left(x\right)}{a x -1}$	23
meijerg	$\frac{3 a x}{-a x +1}+\frac{2 a x}{-2 a x +2}+1+\ln\left(x\right)+\ln\left(-a\right)$	33

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)-4/(a*x-1)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx=\frac{(ax-1)\log\left(x\right)-4}{ax-1}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="fricas")
```

output
```
((a*x - 1)*log(x) - 4)/(a*x - 1)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \log(x) - \frac{4}{ax-1}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/x,x)
```

output
```
log(x) - 4/(a*x - 1)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = -\frac{4}{ax-1} + \log(x)$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="maxima")
```

output
```
-4/(a*x - 1) + log(x)
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a} + \frac{4}{(ax-1)a}\right)$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="giac")
```

output
```
-a*(log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - log(abs(-1/(a*x - 1) - 1))/
a + 4/((a*x - 1)*a))
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \ln(x) - \frac{4}{a\,x-1}$$

input

```
int((a*x + 1)^2/(x*(a*x - 1)^2),x)
```

output

```
log(x) - 4/(a*x - 1)
```

3.30 $\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx$

3.30.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx=-\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

output
```
-1/x+4*a/(-a*x+1)+4*a*ln(x)-4*a*ln(-a*x+1)
```

3.30.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx=-\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

input
```
Integrate[E^(4*ArcCoth[a*x])/x^2,x]
```

output
```
-x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

3.30.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{(ax+1)^2}{x^2(1-ax)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(-\frac{4a^2}{ax-1}+\frac{4a^2}{(ax-1)^2}+\frac{4a}{x}+\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

input
```
Int[E^(4*ArcCoth[a*x])/x^2,x]
```

output
```
-x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

3.30.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.30.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result
default	$-\frac{1}{x}+4a\ln\left(x\right)-\frac{4a}{ax-1}-4a\ln\left(ax-1\right)$
risch	$\frac{-5ax+1}{(ax-1)x}+4a\ln\left(-x\right)-4a\ln\left(ax-1\right)$
norman	$\frac{-5a^2x^2+1}{(ax-1)x}+4a\ln\left(x\right)-4a\ln\left(ax-1\right)$
parallelrisch	$\frac{4a^2\ln(x)x^2-4a^2\ln(ax-1)x^2-5a^2x^2+1-4a\ln(x)x+4a\ln(ax-1)x}{(ax-1)x}$
meijerg	$\frac{a^2x}{-ax+1}+2a\left(\frac{2ax}{-2ax+2}-\ln\left(-ax+1\right)+1+\ln\left(x\right)+\ln\left(-a\right)\right)-a\left(-\frac{3ax}{-3ax+3}+2\ln\left(-ax+1\right)$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+4*a*ln(x)-4*a/(a*x-1)-4*a*ln(a*x-1)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx=-\frac{5\,ax+4\,(a^2x^2-ax)\log\left(ax-1\right)-4\,(a^2x^2-ax)\log\left(x\right)-1}{ax^2-x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="fricas")
```

output
```
-(5*a*x + 4*(a^2*x^2 - a*x)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 1)/(
a*x^2 - x)
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 4a\left(\log(x) - \log\left(x - \frac{1}{a}\right)\right) + \frac{-5ax+1}{ax^2-x}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/x**2,x)
```

output
```
4*a*(log(x) - log(x - 1/a)) + (-5*a*x + 1)/(a*x**2 - x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -4\,a\log(ax-1) + 4\,a\log(x) - \frac{5\,ax-1}{ax^2-x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="maxima")
```

output
```
-4*a*log(a*x - 1) + 4*a*log(x) - (5*a*x - 1)/(a*x^2 - x)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 4\,a\log\left(\left|-\frac{1}{ax-1} - 1\right|\right) - \frac{4\,a}{ax-1} + \frac{a}{\frac{1}{ax-1}+1}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="giac")
```

output
```
4*a*log(abs(-1/(a*x - 1) - 1)) - 4*a/(a*x - 1) + a/(1/(a*x - 1) + 1)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 8\,a\,\mathrm{atanh}(2\,a\,x-1)+\frac{5\,a\,x-1}{x-a\,x^2}$$

input
```
int((a*x + 1)^2/(x^2*(a*x - 1)^2),x)
```

output
```
8*a*atanh(2*a*x - 1) + (5*a*x - 1)/(x - a*x^2)
```

3.31 $\int \frac{e^{4 \coth^{-1}(ax)}}{x^3} \, dx$

3.31.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{4 \coth^{-1}(ax)}}{x^3} \, dx = -\frac{1}{2x^2} - \frac{4a}{x} + \frac{4a^2}{1-ax} + 8a^2 \log(x) - 8a^2 \log(1-ax)$$

output
```
-1/2/x^2-4*a/x+4*a^2/(-a*x+1)+8*a^2*ln(x)-8*a^2*ln(-a*x+1)
```

3.31.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{e^{4 \coth^{-1}(ax)}}{x^3} \, dx = -\frac{1}{2x^2} - \frac{4a}{x} + \frac{4a^2}{1-ax} + 8a^2 \log(x) - 8a^2 \log(1-ax)$$

input
```
Integrate[E^(4*ArcCoth[a*x])/x^3,x]
```

output
```
-1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*
x]
```

3.31.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3} \, dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{(ax+1)^2}{x^3(1-ax)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(-\frac{8a^3}{ax-1}+\frac{4a^3}{(ax-1)^2}+\frac{8a^2}{x}+\frac{4a}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

input
```
Int[E^(4*ArcCoth[a*x])/x^3,x]
```

output
```
-1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*
x]
```

3.31.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.31.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result
default	$-\frac{1}{2x^2}-\frac{4a}{x}+8a^2\ln(x)-\frac{4a^2}{ax-1}-8a^2\ln(ax-1)$
norman	$\frac{\frac{1}{2}-8a^3x^3+\frac{7}{2}ax}{(ax-1)x^2}+8a^2\ln(x)-8a^2\ln(ax-1)$
risch	$\frac{-8a^2x^2+\frac{7}{2}ax+\frac{1}{2}}{x^2(ax-1)}-8a^2\ln(ax-1)+8a^2\ln(-x)$
parallelrisch	$\frac{16a^3\ln(x)x^3-16a^3\ln(ax-1)x^3-16a^3x^3-16a^2\ln(x)x^2+16a^2\ln(ax-1)x^2+1+7ax}{2x^2(ax-1)}$
meijerg	$a^2\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)-2a^2\bigl(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-4*a/x+8*a^2*ln(x)-4*a^2/(a*x-1)-8*a^2*ln(a*x-1)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = -\frac{16\,a^2x^2-7\,ax+16\,(a^3x^3-a^2x^2)\log(ax-1)-16\,(a^3x^3-a^2x^2)\log(x)-1}{2\,(ax^3-x^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="fricas")
```

output

```
-1/2*(16*a^2*x^2 - 7*a*x + 16*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 16*(a^3*x
^3 - a^2*x^2)*log(x) - 1)/(a*x^3 - x^2)
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = 8a^2\left(\log{(x)} - \log{\left(x - \frac{1}{a}\right)}\right) + \frac{-16a^2x^2 + 7ax + 1}{2ax^3 - 2x^2}$$

input

```
integrate(1/(a*x-1)**2*(a*x+1)**2/x**3,x)
```

output

```
8*a**2*(log(x) - log(x - 1/a)) + (-16*a**2*x**2 + 7*a*x + 1)/(2*a*x**3 - 2
*x**2)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = -8\,a^2\log\left(ax - 1\right) + 8\,a^2\log\left(x\right) - \frac{16\,a^2x^2 - 7\,ax - 1}{2\,(ax^3 - x^2)}$$

input

```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="maxima")
```

output

```
-8*a^2*log(a*x - 1) + 8*a^2*log(x) - 1/2*(16*a^2*x^2 - 7*a*x - 1)/(a*x^3 -
 x^2)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = 8\,a^2\log\left(\left|-\frac{1}{ax-1} - 1\right|\right) - \frac{4\,a^2}{ax-1} + \frac{9\,a^2 + \frac{10\,a^2}{ax-1}}{2\,\left(\frac{1}{ax-1} + 1\right)^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="giac")
```

output
```
8*a^2*log(abs(-1/(a*x - 1) - 1)) - 4*a^2/(a*x - 1) + 1/2*(9*a^2 + 10*a^2/(
a*x - 1))/(1/(a*x - 1) + 1)^2
```

3.31.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = 16\,a^2\,\mathrm{atanh}(2\,a\,x-1)+\frac{-8\,a^2\,x^2+\frac{7\,a\,x}{2}+\frac{1}{2}}{a\,x^3-x^2}$$

input
```
int((a*x + 1)^2/(x^3*(a*x - 1)^2),x)
```

output
```
16*a^2*atanh(2*a*x - 1) + ((7*a*x)/2 - 8*a^2*x^2 + 1/2)/(a*x^3 - x^2)
```

3.32 $\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx$

3.32.1 Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)$$

output
```
-1/3/x^3-2*a/x^2-8*a^2/x+4*a^3/(-a*x+1)+12*a^3*ln(x)-12*a^3*ln(-a*x+1)
```

3.32.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)$$

input
```
Integrate[E^(4*ArcCoth[a*x])/x^4,x]
```

output
```
-1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 1
2*a^3*Log[1 - a*x]
```

3.32.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6676}$$

$$\int \frac{(ax+1)^2}{x^4(1-ax)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(-\frac{12a^4}{ax-1}+\frac{4a^4}{(ax-1)^2}+\frac{12a^3}{x}+\frac{8a^2}{x^2}+\frac{4a}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)-\frac{8a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}$$

input

```
Int[E^(4*ArcCoth[a*x])/x^4,x]
```

output

```
-1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 1
2*a^3*Log[1 - a*x]
```

3.32.3.1 Defintions of rubi rules used

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.32.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result
default	$-\frac{1}{3x^{3}}-\frac{2a}{x^{2}}-\frac{8a^{2}}{x}+12a^{3}\ln\left(x\right)-\frac{4a^{3}}{ax-1}-12a^{3}\ln\left(ax-1\right)$
norman	$\frac{\frac{1}{3}-12a^{4}x^{4}+\frac{5}{3}ax+6a^{2}x^{2}}{(ax-1)x^{3}}+12a^{3}\ln\left(x\right)-12a^{3}\ln\left(ax-1\right)$
risch	$\frac{-12a^{3}x^{3}+6a^{2}x^{2}+\frac{5}{3}ax+\frac{1}{3}}{x^{3}(ax-1)}+12a^{3}\ln\left(-x\right)-12a^{3}\ln\left(ax-1\right)$
parallelrisch	$\frac{36\ln(x)x^{4}a^{4}-36\ln(ax-1)x^{4}a^{4}-36a^{4}x^{4}-36a^{3}\ln(x)x^{3}+36a^{3}\ln(ax-1)x^{3}+1+18a^{2}x^{2}+5ax}{3x^{3}(ax-1)}$
meijerg	$-a^{3}\left(-\frac{3ax}{-3ax+3}+2\ln\left(-ax+1\right)-1-2\ln\left(x\right)-2\ln\left(-a\right)+\frac{1}{ax}\right)+2a^{3}\big(\frac{4ax}{-4ax+4}-3\ln\big(-ax$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3-2*a/x^2-8*a^2/x+12*a^3*ln(x)-4*a^3/(a*x-1)-12*a^3*ln(a*x-1)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -\frac{36\,a^{3}x^{3}-18\,a^{2}x^{2}-5\,ax+36\,(a^{4}x^{4}-a^{3}x^{3})\log\left(ax-1\right)-36\,(a^{4}x^{4}-a^{3}x^{3})\log\left(x\right)-1}{3\,(ax^{4}-x^{3})}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="fricas")
```

output
```
-1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x + 36*(a^4*x^4 - a^3*x^3)*log(a*x - 1
) - 36*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*x^4 - x^3)
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = 12a^3\left(\log\left(x\right) - \log\left(x - \frac{1}{a}\right)\right) + \frac{-36a^3x^3 + 18a^2x^2 + 5ax + 1}{3ax^4 - 3x^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/x**4,x)
```

output
```
12*a**3*(log(x) - log(x - 1/a)) + (-36*a**3*x**3 + 18*a**2*x**2 + 5*a*x +
1)/(3*a*x**4 - 3*x**3)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -12\,a^3\log\left(ax - 1\right) + 12\,a^3\log\left(x\right) - \frac{36\,a^3x^3 - 18\,a^2x^2 - 5\,ax - 1}{3\left(ax^4 - x^3\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="maxima")
```

output
```
-12*a^3*log(a*x - 1) + 12*a^3*log(x) - 1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*
x - 1)/(a*x^4 - x^3)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = 12\,a^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^3}{ax-1}+\frac{31\,a^3+\frac{69\,a^3}{ax-1}+\frac{39\,a^3}{(ax-1)^2}}{3\left(\frac{1}{ax-1}+1\right)^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="giac")
```

output
```
12*a^3*log(abs(-1/(a*x - 1) - 1)) - 4*a^3/(a*x - 1) + 1/3*(31*a^3 + 69*a^3
/(a*x - 1) + 39*a^3/(a*x - 1)^2)/(1/(a*x - 1) + 1)^3
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = 24\,a^3\,\mathrm{atanh}(2\,a\,x-1)+\frac{-12\,a^3\,x^3+6\,a^2\,x^2+\frac{5\,a\,x}{3}+\frac{1}{3}}{a\,x^4-x^3}$$

input
```
int((a*x + 1)^2/(x^4*(a*x - 1)^2),x)
```

output
```
24*a^3*atanh(2*a*x - 1) + ((5*a*x)/3 + 6*a^2*x^2 - 12*a^3*x^3 + 1/3)/(a*x^
4 - x^3)
```

3.33 $\int e^{-\coth^{-1}(ax)}x^3\,dx$

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 114

$$\int e^{-\coth^{-1}(ax)}x^3\,dx=-\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

output

```
3/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4-2/3*x*(1-1/a^2/x^2)^(1/2)/a^3+3/8*x^2
*(1-1/a^2/x^2)^(1/2)/a^2-1/3*x^3*(1-1/a^2/x^2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^
2)^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int e^{-\coth^{-1}(ax)}x^3\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-16+9ax-8a^2x^2+6a^3x^3)+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

input

```
Integrate[x^3/E^ArcCoth[a*x],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-16 + 9*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(
1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

3.33.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6719, 539, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}\int \frac{\left(4a-\frac{3}{x}\right)x^4}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(4a-\frac{3}{x}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$\frac{-\frac{1}{3}\int \frac{\left(9a-\frac{8}{x}\right)x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(9a-\frac{8}{x}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{-\frac{-\frac{1}{2}\int \frac{\left(16a-\frac{9}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{-\frac{-\frac{\int \frac{\left(16a-\frac{9}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \textbf{534}$$

$$-\frac{-\frac{-\frac{-9\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \textbf{243}$$

$$-\frac{-\frac{-\frac{-\frac{9}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{-\frac{-\frac{9a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{-\frac{-\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[x^3/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + ((-4*a*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ((-9
*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (-16*a*Sqrt[1 - 1/(a^2*x^2)]*x + 9*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a))/(3*a))/(4*a^2)
```

3.33. $\int e^{-\coth^{-1}(ax)}x^3\,dx$

3.33.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.33.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
risch	$\frac{\left(6a^{3}x^{3}-8a^{2}x^{2}+9ax-16\right)\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{24a^{4}}+\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\left(ax-1\right)\left(ax+1\right)}}{8a^{3}\sqrt{a^{2}}\,\left(ax-1\right)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)\left(6\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,ax+15\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax-8\left(\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{a^{2}}-24\sqrt{a^{2}}\,\sqrt{\left(ax-1\right)\left(ax+1\right)}-15\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}}}{\sqrt{a^{2}}}\right.\right.}{24\sqrt{\left(ax-1\right)\left(ax+1\right)}\,a^{4}\sqrt{a^{2}}}$

input
```
int(x^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3-8*a^2*x^2+9*a*x-16)*(a*x+1)/a^4*((a*x-1)/(a*x+1))^(1/2)+3/
8/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.80

$$\int e^{-\coth^{-1}(ax)}x^{3}\,dx$$
$$=\frac{\left(6\,a^{4}x^{4}-2\,a^{3}x^{3}+a^{2}x^{2}-7\,ax-16\right)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^{4}}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/24*((6*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 7*a*x - 16)*sqrt((a*x - 1)/(a*x +
 1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x +
1)) - 1))/a^4
```

3.33.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = \int x^3\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**3*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(94) = 188.

Time = 0.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.78

$$\int e^{-\coth^{-1}(ax)}x^3\,dx =$$
$$-\frac{1}{24}a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-31\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{a^5}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/24*a*(2*(39*((a*x - 1)/(a*x + 1))^(7/2) - 31*((a*x - 1)/(a*x + 1))^(5/2
) + 49*((a*x - 1)/(a*x + 1))^(3/2) - 9*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x
- 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*
x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 9*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^5 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

3.33.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.33.9 Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.51

$$\int e^{-\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}-\frac{\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{49\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{31\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{13\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((3*((a*x - 1)/(a*x + 1))
^(1/2))/4 - (49*((a*x - 1)/(a*x + 1))^(3/2))/12 + (31*((a*x - 1)/(a*x + 1)
)^(5/2))/12 - (13*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1)^
2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x
+ 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1))
```

3.34 $\int e^{-\coth^{-1}(ax)}x^2\,dx$

3.34.1 Optimal result

Integrand size = 12, antiderivative size = 90

$$\int e^{-\coth^{-1}(ax)}x^2\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

output
```
-1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+2/3*x*(1-1/a^2/x^2)^(1/2)/a^2-1/2*x^
2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/x^2)^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int e^{-\coth^{-1}(ax)}x^2\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(4-3ax+2a^2x^2)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

input
```
Integrate[x^2/E^ArcCoth[a*x],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 - 3*a*x + 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 - 1
/(a^2*x^2)])*x])/(6*a^3)
```

3.34.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6719, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{3}\int \frac{\left(3a-\frac{2}{x}\right)x^3}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(3a-\frac{2}{x}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$\frac{-\frac{1}{2}\int \frac{\left(4a-\frac{3}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(4a-\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{-\frac{-3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{-\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow 73$$

$$\frac{-\frac{3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow 221$$

$$\frac{-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[x^2/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ((-3*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (-4*
a*Sqrt[1 - 1/(a^2*x^2)]*x + 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a))/(3*a^
2)
```

3.34.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.34.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(2a^2x^2-3ax+4)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{6a^3}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2a^2\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\Bigg(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-6\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+6a\ln\Bigg(}{6\sqrt{(ax-1)(ax+1)}\,a^3\sqrt{a^2}}$

input
```
int(x^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-3*a*x+4)*(a*x+1)/a^3*((a*x-1)/(a*x+1))^(1/2)-1/2/a^2*ln(a^2
*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*
x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}x^2\,dx$$
$$=\frac{(2\,a^3x^3-a^2x^2+ax+4)\sqrt{\frac{ax-1}{ax+1}}-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/6*((2*a^3*x^3 - a^2*x^2 + a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqr
t((a*x - 1)/(a*x + 1)) + 1) + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

3.34.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\,dx=\int x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(74) = 148.

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.84

$$\int e^{-\coth^{-1}(ax)}x^2\,dx=$$
$$-\frac{1}{6}\,a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*a*(2*(9*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
 3*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a
^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 3*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^4 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{1}{6}\sqrt{a^2x^2-1}\left(x\left(\frac{2\,x\mathrm{sgn}(ax+1)}{a}-\frac{3\,\mathrm{sgn}(ax+1)}{a^2}\right)+\frac{4\,\mathrm{sgn}(ax+1)}{a^3}\right) + \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a^2|a|}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(a^2*x^2 - 1)*(x*(2*x*sgn(a*x + 1)/a - 3*sgn(a*x + 1)/a^2) + 4*sgn
(a*x + 1)/a^3) + 1/2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/
(a^2*abs(a))
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + 3*((a*x
 - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x
 - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - atanh(((a*x - 1)/(a*
x + 1))^(1/2))/a^3
```

3.35 $\int e^{-\coth^{-1}(ax)}x\,dx$

3.35.1 Optimal result

Integrand size = 10, antiderivative size = 64

$$\int e^{-\coth^{-1}(ax)}x\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-x*(1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/
a^2/x^2)^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int e^{-\coth^{-1}(ax)}x\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-2+ax)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[x/E^ArcCoth[a*x],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]
)/(2*a^2)
```

3.35.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6719, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{2}\int \frac{\left(2a-\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(2a-\frac{1}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[x/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (-2*a*Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(2*a^2)
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.35.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.56

method	result	siz
risch	$\frac{(ax-2)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2a^{2}}+\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2a\sqrt{a^{2}}\,(ax-1)}$	10
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax-\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a-2\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}+2a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)\right)}{2\sqrt{(ax-1)(ax+1)}\,a^{2}\sqrt{a^{2}}}$	15

input
```
int(x*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-2)*(a*x+1)/a^2*((a*x-1)/(a*x+1))^(1/2)+1/2/a*ln(a^2*x/(a^2)^(1/2)
+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^
(1/2)/(a*x-1)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}x\,dx=\frac{\left(a^{2}x^{2}-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^{2}}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a
*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

3.35.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\,dx=\int x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(54)=108$.

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.03

$$\begin{aligned}&\int e^{-\coth^{-1}(ax)}x\,dx\\&=-\frac{1}{2}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)\end{aligned}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*a*(2*(3*((a*x - 1)/(a*x + 1))^(3/2) - sqrt((a*x - 1)/(a*x + 1)))/(2*(
a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/a^3 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}x\,dx=&\,\frac{1}{2}\,\sqrt{a^2x^2-1}\left(\frac{x\mathrm{sgn}(ax+1)}{a}-\frac{2\,\mathrm{sgn}(ax+1)}{a^2}\right)\\&-\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a|a|}\end{aligned}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a^2*x^2 - 1)*(x*sgn(a*x + 1)/a - 2*sgn(a*x + 1)/a^2) - 1/2*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a*abs(a))
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int e^{-\coth^{-1}(ax)}x\,dx=\frac{\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
atanh(((a*x - 1)/(a*x + 1))^(1/2))/a^2 - ((((a*x - 1)/(a*x + 1))^(1/2) - 3*
((a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2
*(a*x - 1))/(a*x + 1))
```

3.36 $\int e^{-\coth^{-1}(ax)}\,dx$

3.36.1 Optimal result

Integrand size = 8, antiderivative size = 37

$$\int e^{-\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-arctanh((1-1/a^2/x^2)^(1/2))/a+x*(1-1/a^2/x^2)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x-\frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^(-ArcCoth[a*x]),x]
```

output
```
Sqrt[1 - 1/(a^2*x^2)]*x - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

3.36.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6718, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6718}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+x\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a}+x\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$x\sqrt{1-\frac{1}{a^2x^2}}-a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{ 221}$$

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

input

```
Int[E^(-ArcCoth[a*x]),x]
```

output

```
Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

3.36.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6718

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(33) = 66$.

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.46

method	result	size
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a}-\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}\,(ax-1)}$	91
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)-\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^{2}}}$	99

input
```
int(((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)*((a*x-1)/(a*x+1))^(1/2)-ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2)
)/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int e^{-\coth^{-1}(ax)}\,dx=\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)
+ log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

3.36.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1)), x)
```

3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(33) = 66$.

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int e^{-\coth^{-1}(ax)}\,dx=-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + log(sqrt
((a*x - 1)/(a*x + 1)) + 1)/a^2 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2
 - 1)*sgn(a*x + 1)/a
```

3.36.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*atanh((
(a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.37 $\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx$

3.37.1 Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx = \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right)$$

output
```
arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

3.37.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx = \arcsin\left(\frac{1}{ax}\right) + \log\left(x\left(1 + \sqrt{\frac{-1 + a^2x^2}{a^2x^2}}\right)\right)$$

input
```
Integrate[1/(E^ArcCoth[a*x]*x),x]
```

output
```
ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

3.37.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6719, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right) x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{538}$$

$$\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{223}$$

$$\arcsin\left(\frac{1}{ax}\right)-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{243}$$

$$\arcsin\left(\frac{1}{ax}\right)-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{73}$$

$$a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}+\arcsin\left(\frac{1}{ax}\right)$$

$$\Big\downarrow \text{221}$$

$$\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[1/(E^ArcCoth[a*x]*x),x]
```

output
```
ArcSin[1/(a*x)] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

3.37.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.37.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 6.50

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}+\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{a^{2}}+a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)-\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}$	130

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a
^2*x^2-1)^(1/2))*(a^2)^(1/2)+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2))/(a^2)^(1/2))-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)*(a*x+1))^(1
/2)/(a^2)^(1/2)
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1)
- log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

3.37.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.37.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

output
```
-a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.95

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1) - \frac{a\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

output
```
-2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) - a*log(abs(-x*abs(a
) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/x,x)
```

output
```
2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

3.38 $\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx$

3.38.1 Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

output

```
-a*arccsc(a*x)-a*(1-1/a^2/x^2)^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -a\left(\sqrt{1-\frac{1}{a^2x^2}} + \arcsin\left(\frac{1}{ax}\right)\right)$$

input

```
Integrate[1/(E^ArcCoth[a*x]*x^2),x]
```

output

```
-(a*(Sqrt[1 - 1/(a^2*x^2)] + ArcSin[1/(a*x)]))
```

3.38.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6719, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{1-\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 455}$$

$$a\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 223}$$

$$a\left(-\arcsin\left(\frac{1}{ax}\right)\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*x^2),x]
```

output
```
-(a*Sqrt[1 - 1/(a^2*x^2)]) - a*ArcSin[1/(a*x)]
```

3.38.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(23) = 46$.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.08

method	result
risch	$-\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}-\frac{a\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+(a^{2}x^{2}-1)^{\frac{3}{2}}\sqrt{a^{2}}-\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax+\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{2}x-ax\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}}}\right.\right.}{\sqrt{(ax-1)(ax+1)}\,x\sqrt{a^{2}}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x+1)/x*((a*x-1)/(a*x+1))^(1/2)-a*arctan(1/(a^2*x^2-1)^(1/2))*((a*x-1)/
(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x + 1)*sqrt((a*x - 1)/(a*x +
 1)))/x
```

3.38.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x**2, x)
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(23) = 46$.

Time $= 0.29$ (sec) , antiderivative size $= 55$, normalized size of antiderivative $= 2.20$

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} - \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) - arctan(sqrt((a
*x - 1)/(a*x + 1))))
```

3.38.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.38.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/x^2,x)
```

output
```
2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (2*a*((a*x - 1)/(a*x + 1))^(1/2))/
((a*x - 1)/(a*x + 1) + 1)
```

3.39 $\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx$

3.39.1 Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx=\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

output
```
1/2*a^2*arccsc(a*x)+1/2*a*(2*a-1/x)*(1-1/a^2/x^2)^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx=\frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+2ax)+ax\arcsin\left(\frac{1}{ax}\right)\right)}{2x}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*x^3),x]
```

output
```
(a*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + 2*a*x) + a*x*ArcSin[1/(a*x)]))/(2*x)
```

3.39.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6719, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{1-\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 533}$$

$$-\frac{1}{2}a^2\int -\frac{a-\frac{2}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}a^2\int \frac{a-\frac{2}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{a-\frac{2}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{2}\left(a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{2}\left(a^2\arcsin\left(\frac{1}{ax}\right)+2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

input
```
Int[1/(E^ArcCoth[a*x]*x^3),x]
```

output
```
-1/2*(a*Sqrt[1 - 1/(a^2*x^2)])/x + (2*a^2*Sqrt[1 - 1/(a^2*x^2)] + a^2*ArcS
in[1/(a*x)])/2
```

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.39. $\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx$

3.39.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

method	result
risch	$\frac{(ax+1)(2ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2x^2}+\frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2ax-2}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+2\ln\left(\frac{a^2x+}{}\right.\right.}{2\sqrt{(ax-1)(ax+1)}\,x^2}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+1)*(2*a*x-1)/x^2*((a*x-1)/(a*x+1))^(1/2)+1/2*a^2*arctan(1/(a^2*x^
2-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx=-\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(2\,a^2x^2+ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^2*x^2 + a*x - 1)*
sqrt((a*x - 1)/(a*x + 1)))/x^2
```

3.39.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x**3, x)
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.32

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-(a*arctan(sqrt((a*x - 1)/(a*x + 1))) - (3*a*((a*x - 1)/(a*x + 1))^(3/2) +
 a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x +
1)^2 + 1))*a
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.92

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -a^2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1) + \frac{\left(x|a| - \sqrt{a^2x^2-1}\right)^3 a^2\mathrm{sgn}(ax+1) + 2\left(x|a| - \sqrt{a^2x^2-1}\right)^2 a|a|\mathrm{sgn}(ax+1) - \left(x|a| - \sqrt{a^2x^2-1}\right)a^2\mathrm{sg}}{\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + ((x*abs(a) - sqr
t(a^2*x^2 - 1))^3*a^2*sgn(a*x + 1) + 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*
abs(a)*sgn(a*x + 1) - (x*abs(a) - sqrt(a^2*x^2 - 1))*a^2*sgn(a*x + 1) + 2*
a*abs(a)*sgn(a*x + 1))/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2
```

3.39.9 Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = a^2\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2} - a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}$$

input

```
int(((a*x - 1)/(a*x + 1))^(1/2)/x^3,x)
```

output

```
a^2*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) - a^
2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(2*x
)
```

3.40 $\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx$

3.40.1 Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = -a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

output
```
1/3*a^3*(1-1/a^2/x^2)^(3/2)-1/2*a^3*arccsc(a*x)-a^3*(1-1/a^2/x^2)^(1/2)+1/
2*a^2*(1-1/a^2/x^2)^(1/2)/x
```

3.40.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,(2-3ax+4a^2x^2)}{6x^2}-\frac{1}{2}a^3\arcsin\left(\frac{1}{ax}\right)$$

input
```
Integrate[1/(E^ArcCoth[a*x]*x^4),x]
```

output
```
-1/6*(a*Sqrt[1 - 1/(a^2*x^2)]*(2 - 3*a*x + 4*a^2*x^2))/x^2 - (a^3*ArcSin[1
/(a*x)])/2
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6719, 533, 25, 27, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1-\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$-\frac{1}{3}a^2\int -\frac{2a-\frac{3}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}a^2\int \frac{2a-\frac{3}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{2a-\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{3}\left(\frac{1}{2}a^2\int -\frac{3a-\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int \frac{3a-\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\int \frac{3a-\frac{4}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(3a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(3a^2\arcsin\left(\frac{1}{ax}\right)+4a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

input
```
Int[1/(E^ArcCoth[a*x]*x^4),x]
```

output
```
-1/3*(a*Sqrt[1 - 1/(a^2*x^2)])/x^2 + ((3*a^2*Sqrt[1 - 1/(a^2*x^2)])/(2*x)
- (a*(4*a^2*Sqrt[1 - 1/(a^2*x^2)] + 3*a^2*ArcSin[1/(a*x)]))/2)/3
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.40.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(ax+1)(4a^2x^2-3ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6x^3}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-3a^3x^3\sqrt{a}\right.}{6\sqrt{(ax-1)(}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x+1)*(4*a^2*x^2-3*a*x+2)/x^3*((a*x-1)/(a*x+1))^(1/2)-1/2*a^3*arcta
n(1/(a^2*x^2-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*
x-1)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int\frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx=\frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(4\,a^3x^3+a^2x^2-ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (4*a^3*x^3 + a^2*x^2 -
a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

3.40.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x**4, x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(64) = 128$.

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.80

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1} + \frac{3\,(ax-1)^2}{(ax+1)^2} + \frac{(ax-1)^3}{(ax+1)^3} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (9*a^2*((a*x - 1)/(a*x + 1)
)^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^2*sqrt((a*x - 1)/(a*x +
1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x
 + 1)^3 + 1))*a
```

3.40.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.40.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^3} - \frac{2\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} - \frac{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x} + \frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/x^4,x)
```

output
```
a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) - ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3
) - (2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 - (a^2*((a*x - 1)/(a*x + 1))^(1/
2))/(6*x) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

3.41 $\int \frac{e^{-\coth^{-1}(ax)}}{x^5} \, dx$

3.41.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} \, dx = \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{3}{8}a^4\csc^{-1}(ax)$$

output
```
3/8*a^4*arccsc(a*x)+1/24*a^3*(16*a-9/x)*(1-1/a^2/x^2)^(1/2)-1/4*a*(1-1/a^2
/x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

3.41.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} \, dx = \frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(-6+8ax-9a^2x^2+16a^3x^3)}{x^3}+9a^3\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[1/(E^ArcCoth[a*x]*x^5),x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(-6 + 8*a*x - 9*a^2*x^2 + 16*a^3*x^3))/x^3 + 9*
a^3*ArcSin[1/(a*x)]))/24
```

3.41.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6719, 533, 25, 27, 533, 25, 27, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1-\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$-\frac{1}{4}a^2\int -\frac{3a-\frac{4}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}a^2\int \frac{3a-\frac{4}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{3a-\frac{4}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{4}\left(\frac{1}{3}a^2\int -\frac{8a-\frac{9}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a^2\int \frac{8a-\frac{9}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\int\frac{8a-\frac{9}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a^2\int-\frac{9a-\frac{16}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int\frac{9a-\frac{16}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\int\frac{9a-\frac{16}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(9a\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(9a^2\arcsin\left(\frac{1}{ax}\right)+16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

input

```
Int[1/(E^ArcCoth[a*x]*x^5),x]
```

output

```
-1/4*(a*Sqrt[1 - 1/(a^2*x^2)])/x^3 + ((4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2
) - (a*((9*a^2*Sqrt[1 - 1/(a^2*x^2)])/(2*x) - (a*(16*a^2*Sqrt[1 - 1/(a^2*x
^2)] + 9*a^2*ArcSin[1/(a*x)]))/2))/3)/4
```

3.41. $\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx$

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.41.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(ax+1)\left(16a^3x^3-9a^2x^2+8ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{24x^4}+\frac{3a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{8(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\Bigg(-24\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4-9a^4x^4\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+24\ln}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(a*x+1)*(16*a^3*x^3-9*a^2*x^2+8*a*x-6)/x^4*((a*x-1)/(a*x+1))^(1/2)+3/
8*a^4*arctan(1/(a^2*x^2-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1)
)^(1/2)/(a*x-1)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = -\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (16\,a^4x^4 + 7\,a^3x^3 - a^2x^2 + 2\,ax - 6)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (16*a^4*x^4 + 7*a^3*
x^3 - a^2*x^2 + 2*a*x - 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

3.41.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^5}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x**5, x)
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(74) = 148.

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.97

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = -\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{39\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 9\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1} + \frac{6\,(ax-1)^2}{(ax+1)^2} + \frac{4\,(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (39*a^3*((a*x - 1)/(a*x +
 1))^(7/2) + 31*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 49*a^3*((a*x - 1)/(a*x +
 1))^(3/2) + 9*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(
a*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)
^4 + 1))*a
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.93

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = -\frac{3}{4}\,a^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)$$
$$+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^{7}a^4\operatorname{sgn}(ax+1)+33\left(x|a|-\sqrt{a^2x^2-1}\right)^{5}a^4\operatorname{sgn}(ax+1)+48\left(x|a|-\sqrt{a^2x^2-1}\right)^{4}}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

output
```
-3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + 1/12*(9*(x*a
bs(a) - sqrt(a^2*x^2 - 1))^7*a^4*sgn(a*x + 1) + 33*(x*abs(a) - sqrt(a^2*x^
2 - 1))^5*a^4*sgn(a*x + 1) + 48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a
)*sgn(a*x + 1) - 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*a^4*sgn(a*x + 1) + 64
*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a)*sgn(a*x + 1) - 9*(x*abs(a) -
sqrt(a^2*x^2 - 1))*a^4*sgn(a*x + 1) + 16*a^3*abs(a)*sgn(a*x + 1))/((x*abs(
a) - sqrt(a^2*x^2 - 1))^2 + 1)^4
```

3.41.9 Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.47

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = \frac{2\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4}-\frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}$$
$$-\frac{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2}+\frac{7\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x}+\frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

input

```
int(((a*x - 1)/(a*x + 1))^(1/2)/x^5,x)
```

output

```
(2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 - ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4
) - (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - (a^2*((a*x - 1)/(a*x + 1
))^(1/2))/(24*x^2) + (7*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (a*((a*x
 - 1)/(a*x + 1))^(1/2))/(12*x^3)
```

3.41. $\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx$

3.42 $\int e^{-2\coth^{-1}(ax)}x^3\,dx$

3.42.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

output
```
-2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4
```

3.42.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

input
```
Integrate[x^3/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

3.42.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^3dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^3(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x^3+\frac{2x^2}{a}-\frac{2x}{a^2}-\frac{2}{a^3(ax+1)}+\frac{2}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(ax+1)}{a^4}-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

input
```
Int[x^3/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

3.42.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] && !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.42.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

method	result	size
norman	$-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\ln(ax+1)}{a^4}$	39
risch	$-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\ln(ax+1)}{a^4}$	39
default	$\frac{2\ln(ax+1)}{a^4}+\frac{\frac{1}{4}a^3x^4-\frac{2}{3}a^2x^3+a\,x^2-2x}{a^3}$	42
parallelrisch	$\frac{3a^4x^4-8a^3x^3+12a^2x^2-24ax+24\ln(ax+1)}{12a^4}$	43
meijerg	$\frac{-\frac{ax\left(-15a^3x^3+20a^2x^2-30ax+60\right)}{60}+\ln(ax+1)}{a^4}-\frac{\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)}{a^4}$	71

input
```
int(x^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=\frac{3\,a^4x^4-8\,a^3x^3+12\,a^2x^2-24\,ax+24\,\log\left(ax+1\right)}{12\,a^4}$$

input
```
integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x + 24*log(a*x + 1))/a^4
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{x^4}{4} - \frac{2x^3}{3a} + \frac{x^2}{a^2} - \frac{2x}{a^3} + \frac{2\log(ax+1)}{a^4}$$

input
```
integrate(x**3*(a*x-1)/(a*x+1),x)
```

output
```
x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**3 + 2*log(a*x + 1)/a**4
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^3x^4 - 8\,a^2x^3 + 12\,ax^2 - 24\,x}{12\,a^3} + \frac{2\,\log(ax+1)}{a^4}$$

input
```
integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/12*(3*a^3*x^4 - 8*a^2*x^3 + 12*a*x^2 - 24*x)/a^3 + 2*log(a*x + 1)/a^4
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax}{12\,a^4} + \frac{2\,\log(|ax+1|)}{a^4}$$

input
```
integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x)/a^4 + 2*log(abs(a*x + 1
))/a^4
```

3.42.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=\frac{2\,\ln\left(a\,x+1\right)}{a^4}-\frac{2\,x}{a^3}+\frac{x^4}{4}-\frac{2\,x^3}{3\,a}+\frac{x^2}{a^2}$$

input

```
int((x^3*(a*x - 1))/(a*x + 1),x)
```

output

```
(2*log(a*x + 1))/a^4 - (2*x)/a^3 + x^4/4 - (2*x^3)/(3*a) + x^2/a^2
```

3.43 $\int e^{-2\coth^{-1}(ax)}x^2\,dx$

3.43.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}$$

output
```
2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

3.43.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}$$

input
```
Integrate[x^2/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

3.43.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^2dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^2(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x^2+\frac{2x}{a}+\frac{2}{a^2(ax+1)}-\frac{2}{a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\log(ax+1)}{a^3}+\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}$$

input
```
Int[x^2/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

3.43.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.43.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\ln(ax+1)}{a^3}$	32
risch	$\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\ln(ax+1)}{a^3}$	32
default	$-\frac{2\ln(ax+1)}{a^3}+\frac{\frac{1}{3}a^2x^3-a\,x^2+2x}{a^2}$	35
parallelrisch	$-\frac{-a^3x^3+3a^2x^2-6ax+6\ln(ax+1)}{3a^3}$	35
meijerg	$\frac{\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)}{a^3}-\frac{-\frac{ax(-3ax+6)}{6}+\ln(ax+1)}{a^3}$	55

input
```
int(x^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx=\frac{a^3x^3-3\,a^2x^2+6\,ax-6\,\log(ax+1)}{3\,a^3}$$

input
```
integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x - 6*log(a*x + 1))/a^3
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{x^3}{3} - \frac{x^2}{a} + \frac{2x}{a^2} - \frac{2\log(ax+1)}{a^3}$$

input
```
integrate(x**2*(a*x-1)/(a*x+1),x)
```

output
```
x**3/3 - x**2/a + 2*x/a**2 - 2*log(a*x + 1)/a**3
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{a^2x^3 - 3\,ax^2 + 6\,x}{3\,a^2} - \frac{2\,\log(ax+1)}{a^3}$$

input
```
integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/3*(a^2*x^3 - 3*a*x^2 + 6*x)/a^2 - 2*log(a*x + 1)/a^3
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 - 3\,a^2x^2 + 6\,ax}{3\,a^3} - \frac{2\,\log(|ax+1|)}{a^3}$$

input
```
integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x)/a^3 - 2*log(abs(a*x + 1))/a^3
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{2\,x}{a^2} - \frac{2\,\ln\left(a\,x+1\right)}{a^3} + \frac{x^3}{3} - \frac{x^2}{a}$$

input
```
int((x^2*(a*x - 1))/(a*x + 1),x)
```

output
```
(2*x)/a^2 - (2*log(a*x + 1))/a^3 + x^3/3 - x^2/a
```

3.44 $\int e^{-2\coth^{-1}(ax)}x\,dx$

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int e^{-2\coth^{-1}(ax)}x\,dx=-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}$$

output
```
-2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

3.44.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x\,dx=-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}$$

input
```
Integrate[x/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

3.44.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6717$$

$$-\int e^{-2\text{arctanh}(ax)}xdx$$

$$\Big\downarrow \ 6676$$

$$-\int \frac{x(1-ax)}{ax+1}dx$$

$$\Big\downarrow \ 86$$

$$-\int \left(-x+\frac{2}{a}-\frac{2}{a(ax+1)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

input
```
Int[x/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

3.44.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.44.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
norman	$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\ln(ax+1)}{a^2}$	24
risch	$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\ln(ax+1)}{a^2}$	24
parallelrisch	$\frac{a^2x^2-4ax+4\ln(ax+1)}{2a^2}$	26
default	$\frac{2\ln(ax+1)}{a^2}+\frac{\frac{1}{2}a\,x^2-2x}{a}$	27
meijerg	$\frac{-\frac{ax(-3ax+6)}{6}+\ln(ax+1)}{a^2}-\frac{ax-\ln(ax+1)}{a^2}$	40

input
```
int(x*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x\,dx=\frac{a^2x^2-4\,ax+4\,\log\left(ax+1\right)}{2\,a^2}$$

input
```
integrate(x*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2 - 4*a*x + 4*log(a*x + 1))/a^2
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}x\,dx=\frac{x^2}{2}-\frac{2x}{a}+\frac{2\log{(ax+1)}}{a^2}$$

input
```
integrate(x*(a*x-1)/(a*x+1),x)
```

output
```
x**2/2 - 2*x/a + 2*log(a*x + 1)/a**2
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}x\,dx=\frac{ax^2-4\,x}{2\,a}+\frac{2\,\log{(ax+1)}}{a^2}$$

input
```
integrate(x*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/2*(a*x^2 - 4*x)/a + 2*log(a*x + 1)/a^2
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int e^{-2\coth^{-1}(ax)}x\,dx=\frac{a^2x^2-4\,ax}{2\,a^2}+\frac{2\,\log{(|ax+1|)}}{a^2}$$

input
```
integrate(x*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/2*(a^2*x^2 - 4*a*x)/a^2 + 2*log(abs(a*x + 1))/a^2
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{2\,\ln\left(a\,x+1\right)}{a^2}-\frac{2\,x}{a}+\frac{x^2}{2}$$

input

```
int((x*(a*x - 1))/(a*x + 1),x)
```

output

```
(2*log(a*x + 1))/a^2 - (2*x)/a + x^2/2
```

3.45 $\int e^{-2\coth^{-1}(ax)}\,dx$

3.45.1 Optimal result

Integrand size = 8, antiderivative size = 13

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(1+ax)}{a}$$

output

```
x-2*ln(a*x+1)/a
```

3.45.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(1+ax)}{a}$$

input

```
Integrate[E^(-2*ArcCoth[a*x]),x]
```

output

```
x - (2*Log[1 + a*x])/a
```

3.45.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6717, 6675, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6675}$$

$$-\int \frac{1-ax}{ax+1}dx$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{2}{ax+1}-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x-\frac{2\log(ax+1)}{a}$$

input
```
Int[E^(-2*ArcCoth[a*x]),x]
```

output
```
x - (2*Log[1 + a*x])/a
```

3.45.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6675
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.45.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$x-\frac{2\ln\left(ax+1\right)}{a}$	14
norman	$x-\frac{2\ln\left(ax+1\right)}{a}$	14
risch	$x-\frac{2\ln\left(ax+1\right)}{a}$	14
parallelrisch	$-\frac{-ax+2\ln\left(ax+1\right)}{a}$	19
meijerg	$\frac{ax-\ln\left(ax+1\right)}{a}-\frac{\ln\left(ax+1\right)}{a}$	29

input
```
int((a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
x-2*ln(a*x+1)/a
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int e^{-2\coth^{-1}(ax)}\,dx=\frac{ax-2\,\log\left(ax+1\right)}{a}$$

input
```
integrate((a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
(a*x - 2*log(a*x + 1))/a
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(ax+1)}{a}$$

input
```
integrate((a*x-1)/(a*x+1),x)
```

output
```
x - 2*log(a*x + 1)/a
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\,\log(ax+1)}{a}$$

input
```
integrate((a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
x - 2*log(a*x + 1)/a
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\,\log(|ax+1|)}{a}$$

input
```
integrate((a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
x - 2*log(abs(a*x + 1))/a
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\,\ln\left(a\,x+1\right)}{a}$$

input

```
int((a*x - 1)/(a*x + 1),x)
```

output

```
x - (2*log(a*x + 1))/a
```

3.46 $\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx$

3.46.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = -\log(x)+2\log(1+ax)$$

output
```
-ln(x)+2*ln(a*x+1)
```

3.46.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = -\log(x)+2\log(1+ax)$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-Log[x] + 2*Log[1 + a*x]
```

3.46.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{x}dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{1-ax}{x(ax+1)}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(\frac{1}{x}-\frac{2a}{ax+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(ax+1)-\log(x)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-Log[x] + 2*Log[1 + a*x]
```

3.46.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.46.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\ln(x)+2\ln(ax+1)$	14
norman	$-\ln(x)+2\ln(ax+1)$	14
parallelrisch	$-\ln(x)+2\ln(ax+1)$	14
risch	$2\ln(-ax-1)-\ln(x)$	15
meijerg	$2\ln(ax+1)-\ln(x)-\ln(a)$	18

input
```
int((a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+2*ln(a*x+1)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\,\log(ax+1) - \log(x)$$

input
```
integrate((a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
2*log(a*x + 1) - log(x)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = -\log(x) + 2\log\left(x + \frac{1}{a}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/x,x)
```

output
```
-log(x) + 2*log(x + 1/a)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\,\log(ax+1) - \log(x)$$

input
```
integrate((a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
2*log(a*x + 1) - log(x)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\,\log(|ax+1|) - \log(|x|)$$

input
```
integrate((a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
2*log(abs(a*x + 1)) - log(abs(x))
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\,\ln\left(-3\,a\,x-3\right)-\ln\left(x\right)$$

input
```
int((a*x - 1)/(x*(a*x + 1)),x)
```

output
```
2*log(- 3*a*x - 3) - log(x)
```

3.47 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx$

3.47.1 Optimal result

Integrand size = 12, antiderivative size = 18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} + 2a\log(x) - 2a\log(1+ax)$$

output

```
1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

3.47.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} + 2a\log(x) - 2a\log(1+ax)$$

input

```
Integrate[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

output

```
x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

3.47.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6676}$$

$$-\int \frac{1-ax}{x^2(ax+1)}dx$$

$$\Big\downarrow \text{ 86}$$

$$-\int \left(\frac{2a^2}{ax+1}-\frac{2a}{x}+\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2a\log(x)-2a\log(ax+1)+\frac{1}{x}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

3.47.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.47.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{1}{x}+2a\ln\left(x\right)-2a\ln\left(ax+1\right)$	19
norman	$\frac{1}{x}+2a\ln\left(x\right)-2a\ln\left(ax+1\right)$	19
risch	$\frac{1}{x}+2a\ln\left(-x\right)-2a\ln\left(ax+1\right)$	21
parallelrisch	$\frac{2a\ln(x)x-2a\ln(ax+1)x+1}{x}$	23
meijerg	$a(-\ln\left(ax+1\right)+\ln\left(x\right)+\ln\left(a\right))-a\left(\ln\left(ax+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{ax}\right)$	43

input
```
int((a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int\frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx=-\frac{2\,ax\log\left(ax+1\right)-2\,ax\log\left(x\right)-1}{x}$$

input
```
integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
-(2*a*x*log(a*x + 1) - 2*a*x*log(x) - 1)/x
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = 2a\left(\log{(x)} - \log{\left(x + \frac{1}{a}\right)}\right) + \frac{1}{x}$$

input
```
integrate((a*x-1)/(a*x+1)/x**2,x)
```

output
```
2*a*(log(x) - log(x + 1/a)) + 1/x
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\log\left(ax+1\right) + 2\,a\log\left(x\right) + \frac{1}{x}$$

input
```
integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
-2*a*log(a*x + 1) + 2*a*log(x) + 1/x
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\log\left(|ax+1|\right) + 2\,a\log\left(|x|\right) + \frac{1}{x}$$

input
```
integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
-2*a*log(abs(a*x + 1)) + 2*a*log(abs(x)) + 1/x
```

3.47.9 Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} - 4\,a\,\operatorname{atanh}(2\,a\,x + 1)$$

input

```
int((a*x - 1)/(x^2*(a*x + 1)),x)
```

output

```
1/x - 4*a*atanh(2*a*x + 1)
```

3.48 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx$

3.48.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2x^2} - \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1+ax)$$

output
```
1/2/x^2-2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

3.48.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2x^2} - \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1+ax)$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

3.48.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{1-ax}{x^3(ax+1)}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-\frac{2a^3}{ax+1}+\frac{2a^2}{x}-\frac{2a}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2a^2\log(x)+2a^2\log(ax+1)-\frac{2a}{x}+\frac{1}{2x^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

3.48.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.48.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	s
norman	$\frac{\frac{1}{2}-2ax}{x^{2}}-2a^{2}\ln\left(x\right)+2a^{2}\ln\left(ax+1\right)$	3
default	$\frac{1}{2x^{2}}-\frac{2a}{x}-2a^{2}\ln\left(x\right)+2a^{2}\ln\left(ax+1\right)$	3
risch	$\frac{\frac{1}{2}-2ax}{x^{2}}+2a^{2}\ln\left(-ax-1\right)-2a^{2}\ln\left(x\right)$	3
parallelrisch	$-\frac{4a^{2}\ln(x)x^{2}-4a^{2}\ln(ax+1)x^{2}-1+4ax}{2x^{2}}$	3
meijerg	$a^{2}\left(\ln\left(ax+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{ax}\right)-a^{2}\left(-\ln\left(ax+1\right)+\ln\left(x\right)+\ln\left(a\right)-\frac{1}{2a^{2}x^{2}}+\frac{1}{ax}\right)$	6

input
```
int((a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2-2*a*x)/x^2-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^{3}}\,dx=\frac{4\,a^{2}x^{2}\log\left(ax+1\right)-4\,a^{2}x^{2}\log\left(x\right)-4\,ax+1}{2\,x^{2}}$$

input
```
integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*x^2*log(a*x + 1) - 4*a^2*x^2*log(x) - 4*a*x + 1)/x^2
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2a^2\left(-\log(x)+\log\left(x+\frac{1}{a}\right)\right)+\frac{-4ax+1}{2x^2}$$

input
```
integrate((a*x-1)/(a*x+1)/x**3,x)
```

output
```
2*a**2*(-log(x) + log(x + 1/a)) + (-4*a*x + 1)/(2*x**2)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log(ax+1)-2\,a^2\log(x)-\frac{4\,ax-1}{2\,x^2}$$

input
```
integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
2*a^2*log(a*x + 1) - 2*a^2*log(x) - 1/2*(4*a*x - 1)/x^2
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log(|ax+1|)-2\,a^2\log(|x|)-\frac{4\,ax-1}{2\,x^2}$$

input
```
integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
2*a^2*log(abs(a*x + 1)) - 2*a^2*log(abs(x)) - 1/2*(4*a*x - 1)/x^2
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 4\,a^2\,\mathrm{atanh}(2\,a\,x+1) - \frac{2\,a\,x-\frac{1}{2}}{x^2}$$

input

```
int((a*x - 1)/(x^3*(a*x + 1)),x)
```

output

```
4*a^2*atanh(2*a*x + 1) - (2*a*x - 1/2)/x^2
```

3.49 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx$

3.49.1 Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx=\frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1+ax)$$

output
```
1/3/x^3-a/x^2+2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

3.49.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx=\frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1+ax)$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

3.49.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \; 6717$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{x^4}dx$$

$$\Big\downarrow \; 6676$$

$$-\int \frac{1-ax}{x^4(ax+1)}dx$$

$$\Big\downarrow \; 86$$

$$-\int \left(\frac{2a^4}{ax+1}-\frac{2a^3}{x}+\frac{2a^2}{x^2}-\frac{2a}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$2a^3\log(x)-2a^3\log(ax+1)+\frac{2a^2}{x}-\frac{a}{x^2}+\frac{1}{3x^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

3.49.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.49.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\frac{1}{3}+2a^2x^2-ax}{x^3}+2a^3\ln\left(x\right)-2a^3\ln\left(ax+1\right)$
default	$\frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\ln\left(x\right)-2a^3\ln\left(ax+1\right)$
risch	$\frac{\frac{1}{3}+2a^2x^2-ax}{x^3}+2a^3\ln\left(-x\right)-2a^3\ln\left(ax+1\right)$
parallelrisch	$\frac{6a^3\ln(x)x^3-6a^3\ln(ax+1)x^3+1+6a^2x^2-3ax}{3x^3}$
meijerg	$a^3\left(-\ln\left(ax+1\right)+\ln\left(x\right)+\ln\left(a\right)-\frac{1}{2a^2x^2}+\frac{1}{ax}\right)-a^3\left(\ln\left(ax+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{3x^3a^3}+\right.$

input
```
int((a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(1/3+2*a^2*x^2-a*x)/x^3+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int\frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx=-\frac{6\,a^3x^3\log\left(ax+1\right)-6\,a^3x^3\log\left(x\right)-6\,a^2x^2+3\,ax-1}{3\,x^3}$$

input
```
integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(6*a^3*x^3*log(a*x + 1) - 6*a^3*x^3*log(x) - 6*a^2*x^2 + 3*a*x - 1)/x
^3
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = 2a^3\left(\log\left(x\right) - \log\left(x + \frac{1}{a}\right)\right) + \frac{6a^2x^2 - 3ax + 1}{3x^3}$$

input
```
integrate((a*x-1)/(a*x+1)/x**4,x)
```

output
```
2*a**3*(log(x) - log(x + 1/a)) + (6*a**2*x**2 - 3*a*x + 1)/(3*x**3)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -2\,a^3\log\left(ax+1\right) + 2\,a^3\log\left(x\right) + \frac{6\,a^2x^2 - 3\,ax + 1}{3\,x^3}$$

input
```
integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
-2*a^3*log(a*x + 1) + 2*a^3*log(x) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/x^3
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -2\,a^3\log\left(|ax+1|\right) + 2\,a^3\log\left(|x|\right) + \frac{6\,a^2x^2 - 3\,ax + 1}{3\,x^3}$$

input
```
integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
-2*a^3*log(abs(a*x + 1)) + 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*a*x + 1)
/x^3
```

3.49.9 Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = \frac{2\,a^2\,x^2-a\,x+\frac{1}{3}}{x^3}-4\,a^3\,\mathrm{atanh}(2\,a\,x+1)$$

input
```
int((a*x - 1)/(x^4*(a*x + 1)),x)
```

output
```
(2*a^2*x^2 - a*x + 1/3)/x^3 - 4*a^3*atanh(2*a*x + 1)
```

3.50 $\int e^{-3\coth^{-1}(ax)}x^3\,dx$

3.50.1 Optimal result

Integrand size = 12, antiderivative size = 136

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}-\frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3}+\frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{51\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

output

```
51/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4-4*(1-1/a^2/x^2)^(1/2)/a^3/(a+1/x)-6*
x*(1-1/a^2/x^2)^(1/2)/a^3+19/8*x^2*(1-1/a^2/x^2)^(1/2)/a^2-x^3*(1-1/a^2/x^
2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.61

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-80-29ax+11a^2x^2-6a^3x^3+2a^4x^4)}{1+ax}+51\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{8a^4}$$

input

```
Integrate[x^3/E^(3*ArcCoth[a*x]),x]
```

output

```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-80 - 29*a*x + 11*a^2*x^2 - 6*a^3*x^3 + 2*a^4
*x^4))/(1 + a*x) + 51*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(8*a^4)
```

3.50.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x^5}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^5}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^4}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^3}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x^2}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{19x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{51\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}-\frac{6x\sqrt{1-\frac{1}{a^2x^2}}}{a^3}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}$$

input
```
Int[x^3/E^(3*ArcCoth[a*x]),x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a^3*(a + x^(-1))) - (6*Sqrt[1 - 1/(a^2*x^2)]*x
)/a^3 + (19*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^
3)/a + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (51*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
/(8*a^4)
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.50.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(2a^3x^3-8a^2x^2+19ax-48)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{8a^4}+\frac{\left(\frac{51\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{8a^3\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^5\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\Big(2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+4(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+21\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-8\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+42$

input
```
int(x^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*a^3*x^3-8*a^2*x^2+19*a*x-48)*(a*x+1)/a^4*((a*x-1)/(a*x+1))^(1/2)+(5
1/8/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^5/(x+1/a)*
(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1
)*(a*x+1))^(1/2)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{\left(2\,a^4x^4-6\,a^3x^3+11\,a^2x^2-29\,ax-80\right)\sqrt{\frac{ax-1}{ax+1}}+51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{8\,a^4}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/8*((2*a^4*x^4 - 6*a^3*x^3 + 11*a^2*x^2 - 29*a*x - 80)*sqrt((a*x - 1)/(a*
x + 1)) + 51*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*log(sqrt((a*x - 1)/(a
*x + 1)) - 1))/a^4
```

3.50.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(x**3*((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.64

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=$$
$$-\frac{1}{8}\,a\left(\frac{2\left(77\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-149\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-35\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{51\,\log\left(\sqrt{\right.}{a^5}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/8*a*(2*(77*((a*x - 1)/(a*x + 1))^(7/2) - 149*((a*x - 1)/(a*x + 1))^(5/2
) + 123*((a*x - 1)/(a*x + 1))^(3/2) - 35*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*
x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(
a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 51*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^5 + 51*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5 + 32*sqrt((a
*x - 1)/(a*x + 1))/a^5)
```

3.50.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.41

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \frac{51\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4} - \frac{\frac{35\sqrt{\frac{a\,x-1}{a\,x+1}}}{4} - \frac{123\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4} + \frac{149\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{4} - \frac{77\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a^4 + \frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{4\,a^4\,(a\,x-1)}{a\,x+1}} - \frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^4}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(51*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((35*((a*x - 1)/(a*x + 1
))^(1/2))/4 - (123*((a*x - 1)/(a*x + 1))^(3/2))/4 + (149*((a*x - 1)/(a*x +
 1))^(5/2))/4 - (77*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1
)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*
x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (4*((a*x - 1)/(a*x + 1))^(1/2))/
a^4
```

3.51 $\int e^{-3\coth^{-1}(ax)}x^2\,dx$

3.51.1 Optimal result

Integrand size = 12, antiderivative size = 116

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

output

```
-11/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+4*(1-1/a^2/x^2)^(1/2)/a^2/(a+1/x)+1
4/3*x*(1-1/a^2/x^2)^(1/2)/a^2-3/2*x^2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a
^2/x^2)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(52+19ax-7a^2x^2+2a^3x^3)}{1+ax}-33\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

input

```
Integrate[x^2/E^(3*ArcCoth[a*x]),x]
```

output

```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(52 + 19*a*x - 7*a^2*x^2 + 2*a^3*x^3))/(1 + a*
x) - 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

3.51.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x^4}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^4}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

input
```
Int[x^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a + x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x
)/(3*a^2) - (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x
^3)/3 - (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.51.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.25

method	result
risch	$\frac{\left(2a^{2}x^{2}-9ax+28\right)\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{6a^{3}}+\frac{\left(-\frac{11\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{2a^{2}\sqrt{a^{2}}}+\frac{4\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{a^{4}\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\left(ax-1\right)\left(ax+1\right)}}{ax-1}$
default	$-\frac{\left(9\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{3}x^{3}-2\sqrt{a^{2}}\,\left(\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}a^{2}x^{2}+18\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}-9\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{3}x^{2}-4\sqrt{a^{2}}\,\left(\left(ax-1\right)\left(a\right.\right.\right.}{}$

input
```
int(x^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-9*a*x+28)*(a*x+1)/a^3*((a*x-1)/(a*x+1))^(1/2)+(-11/2/a^2*ln
(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+4/a^4/(x+1/a)*(a^2*(x+1/
a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))
^(1/2)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int e^{-3\coth^{-1}(ax)}x^{2}\,dx$$
$$=\frac{\left(2\,a^{3}x^{3}-7\,a^{2}x^{2}+19\,ax+52\right)\sqrt{\frac{ax-1}{ax+1}}-33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^{3}}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/6*((2*a^3*x^3 - 7*a^2*x^2 + 19*a*x + 52)*sqrt((a*x - 1)/(a*x + 1)) - 33*
log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
)/a^3
```

3.51.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(x**2*((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.60

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=$$

$$-\frac{1}{6}a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-52\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}-\frac{24\,\sqrt{}}{}\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/6*a*(2*(39*((a*x - 1)/(a*x + 1))^(5/2) - 52*((a*x - 1)/(a*x + 1))^(3/2)
 + 21*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^
2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^4 - 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4 - 24*
sqrt((a*x - 1)/(a*x + 1))/a^4)
```

3.51.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.34

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \frac{7\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{52\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+13\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}} + \frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^3}-\frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(7*((a*x - 1)/(a*x + 1))^(1/2) - (52*((a*x - 1)/(a*x + 1))^(3/2))/3 + 13*(
(a*x - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*
(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (4*((a*x - 1)/(a
*x + 1))^(1/2))/a^3 - (11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3
```

3.52 $\int e^{-3\coth^{-1}(ax)}x\,dx$

3.52.1 Optimal result

Integrand size = 10, antiderivative size = 90

$$\int e^{-3\coth^{-1}(ax)}x\,dx = -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{9\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-4*(1-1/a^2/x^2)^(1/2)/a/(a+1/x)-3*x*(
1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int e^{-3\coth^{-1}(ax)}x\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-14-5ax+a^2x^2)}{1+ax}+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[x/E^(3*ArcCoth[a*x]),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 - 5*a*x + a^2*x^2))/(1 + a*x) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

3.52.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x^3}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^3}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4}{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}$$

input
```
Int[x/E^(3*ArcCoth[a*x]),x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a + x^(-1))) - (3*Sqrt[1 - 1/(a^2*x^2)]*x)/
a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*
a^2)
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.52.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.51

method	result
risch	$\frac{(ax-6)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2a^2}+\frac{\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^3\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\left(\sqrt{a^2x^2-1}\sqrt{a^2}\,a^3x^3+2\sqrt{a^2x^2-1}\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-10\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\,a^2x^2+10\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax-}{\sqrt{a^2}}\right.\right.$

input
```
int(x*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-6)*(a*x+1)/a^2*((a*x-1)/(a*x+1))^(1/2)+(9/2/a*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))
^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\frac{(a^2x^2-5\,ax-14)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 - 5*a*x - 14)*sqrt((a*x - 1)/(a*x + 1)) + 9*log(sqrt((a*x -
1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

3.52.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(x*((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.68

$$\int e^{-3\coth^{-1}(ax)}x\,dx=-\frac{1}{2}a\left(\frac{2\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}+\frac{8\sqrt{\frac{ax-1}{ax+1}}}{a^3}\right)$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/2*a*(2*(7*((a*x - 1)/(a*x + 1))^(3/2) - 5*sqrt((a*x - 1)/(a*x + 1)))/(2
*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 9*log(sqrt
((a*x - 1)/(a*x + 1)) + 1)/a^3 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3
+ 8*sqrt((a*x - 1)/(a*x + 1))/a^3)
```

3.52.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.52.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^2}-\frac{5\,\sqrt{\frac{a\,x-1}{a\,x+1}}-7\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - (4*((a*x - 1)/(a*x + 1))^(1/2
))/a^2 - (5*((a*x - 1)/(a*x + 1))^(1/2) - 7*((a*x - 1)/(a*x + 1))^(3/2))/(
a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1))
```

3.53 $\int e^{-3\coth^{-1}(ax)}\,dx$

3.53.1 Optimal result

Integrand size = 8, antiderivative size = 60

$$\int e^{-3\coth^{-1}(ax)}\,dx=\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-3*arctanh((1-1/a^2/x^2)^(1/2))/a+4*(1-1/a^2/x^2)^(1/2)/(a+1/x)+x*(1-1/a^2
/x^2)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int e^{-3\coth^{-1}(ax)}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(5+ax)}{1+ax}-\frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^(-3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(5 + a*x))/(1 + a*x) - (3*Log[a*(1 + Sqrt[1 - 1/(
a^2*x^2)])*x])/a
```

3.53.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6718, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6718}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2353}$$

$$-\int \left(\frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}$$

input
```
Int[E^(-3*ArcCoth[a*x]),x]
```

output
```
(4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x - (3*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6718
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

3.53.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(54) = 108$.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.12

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(3\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+6\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^2x+2((ax-1)(ax+1))^{\frac{3}{2}}\right.}{a\sqrt{a^2}\,(ax-1)\sqrt{(ax-1)(ax+}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)*((a*x-1)/(a*x+1))^(1/2)+(-3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(
1/2))/(a^2)^(1/2)+4/a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)
*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int e^{-3\coth^{-1}(ax)}\,dx=\frac{(ax+5)\sqrt{\frac{ax-1}{ax+1}}-3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
((a*x + 5)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

3.53.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\,dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(54) = 108$.

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.85

$$\begin{aligned}&\int e^{-3\coth^{-1}(ax)}\,dx\\&= -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)\end{aligned}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + 3*log(sq
rt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 - 4*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

3.53.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\,dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int e^{-3\coth^{-1}(ax)}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (4*((a*x -
 1)/(a*x + 1))^(1/2))/a - (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.54 $\int \frac{e^{-3\coth^{-1}(ax)}}{x} \, dx$

3.54.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x} \, dx = -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

output
```
-arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))-4*a*(1-1/a^2/x^2)^(1/2)/(a+1/x)
```

3.54.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x} \, dx = -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}x}{1+ax} - \arcsin\left(\frac{1}{ax}\right) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - ArcSin[1/(a*x)] + Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]
```

3.54.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6719, 2351, 27, 564, 25, 27, 243, 73, 221, 671, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2351}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-\int \frac{ax}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 564}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\int -\frac{x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\int \frac{x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{ 243}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow 73$$

$$-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow 221$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)$$

$$\Big\downarrow 671$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\left(a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)\right)-\frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}$$

$$\Big\downarrow 223$$

$$-\left(a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)\right)-\frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(-3*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ArcSin[1/(a*x)] - a*(Sqrt[1 -
1/(a^2*x^2)]/(a + x^(-1)) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a)
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

3.54. $\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx$

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.54.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(42) = 84$.

Time = 0.12 (sec) , antiderivative size = 366, normalized size of antiderivative = 7.96

method	result
default	$-\left(\sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}\, a^{2} x^{2}+a^{2} x^{2} \sqrt{a^{2}}\, \arctan\left(\frac{1}{\sqrt{a^{2} x^{2}-1}}\right)-\ln\left(\frac{a^{2} x+\sqrt{a^{2}}\, \sqrt{(a x-1)(a x+1)}}{\sqrt{a^{2}}}\right) a^{3} x^{2}+\sqrt{a^{2}}\, \sqrt{(a x-1)(a x+1)}\, a^{2} x^{2}+2\sqrt{a^{2} x^{2}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*
x^2-1)^(1/2))-ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*
a^3*x^2+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2+2*(a^2*x^2-1)^(1/2)*(a
^2)^(1/2)*a*x+2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-2*ln((a^2*x+(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x-2*((a*x-1)*(a*x+1))^
(3/2)*(a^2)^(1/2)+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+(a^2*x^2-1)^(1
/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-a*ln((a^2*x+(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2))*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = -4\sqrt{\frac{ax-1}{ax+1}}+2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

output
```
-4*sqrt((a*x - 1)/(a*x + 1)) + 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

3.54.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x, x)
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(42) = 84$.

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a} - \frac{4\sqrt{\frac{ax-1}{ax+1}}}{a}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

output
```
a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a - 4*sqrt((a*x - 1)/(a*x + 1))
/a)
```

3.54.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

output
```
undef
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 4\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x,x)
```

output
```
2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
 - 4*((a*x - 1)/(a*x + 1))^(1/2)
```

3.55 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx$

3.55.1 Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = 3a\sqrt{1-\frac{1}{a^2x^2}}+\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\csc^{-1}(ax)$$

output
```
3*a*arccsc(a*x)+2*(a-1/x)^2/a/(1-1/a^2/x^2)^(1/2)+3*a*(1-1/a^2/x^2)^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}(1+5ax)}{1+ax}+3a\arcsin\left(\frac{1}{ax}\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x))/(1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

3.55.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6719, 711, 25, 27, 671, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 711}$$

$$a^4\int -\frac{a-\frac{3}{x}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}+a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$a\sqrt{1-\frac{1}{a^2x^2}}-a^4\int \frac{a-\frac{3}{x}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$a\sqrt{1-\frac{1}{a^2x^2}}-\int \frac{a-\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 671}$$

$$3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+a\sqrt{1-\frac{1}{a^2x^2}}+3a\arcsin\left(\frac{1}{ax}\right)$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
a*Sqrt[1 - 1/(a^2*x^2)] + (4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + 3*a
*ArcSin[1/(a*x)]
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 711
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)
/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))
Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)
^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - 2*e*g^n*(m + p + n)*(d + e*x)^(n
 - 2)*(a*e - c*d*x), x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && Eq
Q[c*d^2 + a*e^2, 0] && IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.55. $\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx$

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(49) = 98.

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.06

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}+\frac{\left(3a\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+\frac{4\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(-\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{4}x^{4}+\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{2}x^{2}-5\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{3}x^{3}+\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{4}x^{3}-3a^{3}x^{3}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}}}\right.\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(a*x+1)/x*((a*x-1)/(a*x+1))^(1/2)+(3*a*arctan(1/(a^2*x^2-1)^(1/2))+4/(x+1/
a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*
x-1)*(a*x+1))^(1/2)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^{2}}\,dx=-\frac{6\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(6*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (5*a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/x
```

3.55.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**2, x)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\left(2\sqrt{\frac{ax-1}{ax+1}} + \frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} - 3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
2*a*(2*sqrt((a*x - 1)/(a*x + 1)) + sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a
*x + 1) + 1) - 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

3.55.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}} + 5\,a\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}} - 6\,a\,x\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{x}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x^2,x)
```

output
```
(((a*x - 1)/(a*x + 1))^(1/2) + 5*a*x*((a*x - 1)/(a*x + 1))^(1/2) - 6*a*x*a
tan(((a*x - 1)/(a*x + 1))^(1/2)))/x
```

3.56 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx$

3.56.1 Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx=-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}-\frac{9}{2}a^2\csc^{-1}(ax)$$

output
```
-a^5*(1-1/a^2/x^2)^(5/2)/(a+1/x)^3-3/2*a^3*(1-1/a^2/x^2)^(3/2)/(a+1/x)-9/2
*a^2*arccsc(a*x)-9/2*a^2*(1-1/a^2/x^2)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx=\frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(1-5ax-14a^2x^2)}{x(1+ax)}-9a\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x - 14*a^2*x^2))/(x*(1 + a*x)) - 9*a*A
rcSin[1/(a*x)]))/2
```

3.56.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6719, 2164, 27, 2027, 2164, 27, 563, 25, 2346, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2164}$$

$$-\frac{\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}-\frac{1}{x^2}\right)}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 27}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}-\frac{1}{x^2}\right)}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2027}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{\left(a+\frac{1}{x}\right)^2x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2164}$$

$$-a^2\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 563}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\int -\frac{4a^2-\frac{3a}{x}+\frac{1}{x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}\right)$$

$$\Big\downarrow \text{25}$$

$$-a^3\left(\frac{\int \frac{4a^2-\frac{3a}{x}+\frac{1}{x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{2346}$$

$$-a^3\left(\frac{-\frac{1}{2}a^2\int -\frac{3\left(3a-\frac{2}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$-a^3\left(\frac{\frac{3}{2}a\int \frac{3a-\frac{2}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{455}$$

$$-a^3\left(\frac{\frac{3}{2}a\left(3a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{223}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\frac{\frac{3}{2}a\left(3a^2\arcsin\left(\frac{1}{ax}\right)+2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

3.56. $\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx$

output
```
-(a^3*((4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + (-1/2*(a^2*Sqrt[1 - 1/(a^2
*x^2)])/x + (3*a*(2*a^2*Sqrt[1 - 1/(a^2*x^2)] + 3*a^2*ArcSin[1/(a*x)]))/2)
/a^4))
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.56.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{(ax+1)(6ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2x^{2}}+\frac{\left(-\frac{9a^{2}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}-\frac{4a\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\left(-6\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{5}x^{5}+6\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{5}x^{4}+6\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{3}x^{3}-21\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{4}x^{4}-9a^{4}x^{4}\sqrt{a^{2}}\,\arctan\Bigg(\frac{}{\sqrt{a^{2}}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+1)*(6*a*x-1)/x^2*((a*x-1)/(a*x+1))^(1/2)+(-9/2*a^2*arctan(1/(a^2
*x^2-1)^(1/2))-4*a/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/(a*x-1)*((a*
x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^{3}}\,dx=\frac{18\,a^{2}x^{2}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(14\,a^{2}x^{2}+5\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^{2}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(18*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (14*a^2*x^2 + 5*a*x -
1)*sqrt((a*x - 1)/(a*x + 1)))/x^2
```

3.56.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**3, x)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 4\,a\sqrt{\frac{ax-1}{ax+1}} - \frac{7\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 5\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) - 4*a*sqrt((a*x - 1)/(a*x + 1)) - (
7*a*((a*x - 1)/(a*x + 1))^(3/2) + 5*a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

3.56.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = 9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{5\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+7\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1} - 4\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x^3,x)
```

output
```
9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (5*a^2*((a*x - 1)/(a*x + 1))^(1/
2) + 7*a^2*((a*x - 1)/(a*x + 1))^(3/2))/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x
 - 1))/(a*x + 1) + 1) - 4*a^2*((a*x - 1)/(a*x + 1))^(1/2)
```

3.57 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 96

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{11}{2}a^3\csc^{-1}(ax)$$

output
```
11/2*a^3*arccsc(a*x)+(a-1/x)^3/(1-1/a^2/x^2)^(1/2)+1/6*a^2*(28*a-3/x)*(1-1
/a^2/x^2)^(1/2)+1/3*a*(3*a-1/x)^2*(1-1/a^2/x^2)^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(2-7ax+19a^2x^2+52a^3x^3)}{x^2(1+ax)}+33a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(2 - 7*a*x + 19*a^2*x^2 + 52*a^3*x^3))/(x^2*(1
+ a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

3.57.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {6719, 2164, 27, 2027, 2164, 27, 563, 2346, 25, 2346, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$-\frac{\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}-\frac{1}{x^3}\right)}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}-\frac{1}{x^3}\right)}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2027}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{\left(a+\frac{1}{x}\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$-a^2\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{563}$$

$$-a^3\left(-\frac{\int \frac{4a^3-\frac{4a^2}{x}+\frac{3a}{x^2}-\frac{1}{x^3}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{ 2346}$$

$$-a^3\left(-\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a^2\int -\frac{12a-\frac{14}{x}+\frac{9}{x^2a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-a^3\left(-\frac{\frac{1}{3}a^2\int \frac{12a-\frac{14}{x}+\frac{9}{x^2a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{ 2346}$$

$$-a^3\left(-\frac{\frac{1}{3}a^2\left(-\frac{1}{2}a^2\int -\frac{33a-\frac{28}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-a^3\left(-\frac{\frac{1}{3}a^2\left(\frac{1}{2}a^2\int \frac{33a-\frac{28}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-a^3\left(-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\int \frac{33a-\frac{28}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{ 455}$$

3.57. $\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$

$$-a^3\left(-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\left(33a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+28a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow 223$$

$$-a^3\left(-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\left(33a^2\arcsin\left(\frac{1}{ax}\right)+28a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
-(a^3*((-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ((a^2*Sqrt[1 - 1/(a^2*x
^2)])/(3*x^2) + (a^2*((-9*a*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (28*a^2*Sqrt[1
- 1/(a^2*x^2)] + 33*a^2*ArcSin[1/(a*x)])/2))/3)/a^4))
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.57.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.34

method	result
risch	$\frac{(ax+1)\left(28a^{2}x^{2}-9ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6x^{3}}+\frac{\left(\frac{11a^{3}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}+\frac{4a^{2}\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(-30\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{6}x^{6}+30\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{4}x^{4}-93\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{5}x^{5}-33\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{a^{2}}\,a^{5}x^{5}+30\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}}}{\sqrt{a^{2}}}\right.\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)*(28*a^2*x^2-9*a*x+2)/x^3*((a*x-1)/(a*x+1))^(1/2)+(11/2*a^3*arc
tan(1/(a^2*x^2-1)^(1/2))+4*a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/
(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{66\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \left(52\,a^3x^3 + 19\,a^2x^2 - 7\,ax + 2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(66*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (52*a^3*x^3 + 19*a^2*
x^2 - 7*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

3.57.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**4, x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 12\,a^2\sqrt{\frac{ax-1}{ax+1}} - \frac{39\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 52\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 21\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1} + \frac{3\,(ax-1)^2}{(ax+1)^2} + \frac{(ax-1)^3}{(ax+1)^3} + 1}\right)a$$

input

```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

output

```
-1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 12*a^2*sqrt((a*x - 1)/(a*
x + 1)) - (39*a^2*((a*x - 1)/(a*x + 1))^(5/2) + 52*a^2*((a*x - 1)/(a*x + 1
))^(3/2) + 21*a^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a
*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

3.57.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input

```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

output

```
undef
```

3.57.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.59

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \frac{7\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}} + \frac{52\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3} + 13\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2} + \frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3} + \frac{3\,\left(a\,x-1\right)}{a\,x+1} + 1} + 4\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}} - 11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x^4,x)
```

output
```
(7*a^3*((a*x - 1)/(a*x + 1))^(1/2) + (52*a^3*((a*x - 1)/(a*x + 1))^(3/2))/
3 + 13*a^3*((a*x - 1)/(a*x + 1))^(5/2))/((3*(a*x - 1)^2)/(a*x + 1)^2 + (a*
x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 4*a^3*((a*x - 1)/(a*
x + 1))^(1/2) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

3.57. $\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$

3.58 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

3.58.1 Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}} - \frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right) - \frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} - \frac{51}{8}a^4\csc^{-1}(ax)$$

output
```
-51/8*a^4*arccsc(a*x)-a*(a-1/x)^3/(1-1/a^2/x^2)^(1/2)-27/4*a^4*(1-1/a^2/x^
2)^(1/2)-9/8*a^3*(2*a-3/x)*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^2/x^2)^(1/2)/x
^3-a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

3.58.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(-2+6ax-11a^2x^2+29a^3x^3+80a^4x^4\right)}{8x^3(1+ax)} - \frac{51}{8}a^4\arcsin\left(\frac{1}{ax}\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

output

```
-1/8*(a*Sqrt[1 - 1/(a^2*x^2)]*(-2 + 6*a*x - 11*a^2*x^2 + 29*a^3*x^3 + 80*a
^4*x^4))/(x^3*(1 + a*x)) - (51*a^4*ArcSin[1/(a*x)])/8
```

3.58.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6719, 2164, 27, 2027, 2164, 27, 563, 25, 2346, 25, 2346, 27, 2346, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$-\frac{\int \frac{a^2\left(\frac{a}{x^3}-\frac{1}{x^4}\right)\sqrt{1-\frac{1}{a^2x^2}}}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$-a\int \frac{\left(\frac{a}{x^3}-\frac{1}{x^4}\right)\sqrt{1-\frac{1}{a^2x^2}}}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2027}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{\left(a+\frac{1}{x}\right)^2x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$-a^2\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{563}$$

$$-a^3\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\int -\frac{4a^4-\frac{4a^3}{x}+\frac{4a^2}{x^2}-\frac{3a}{x^3}+\frac{1}{x^4}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}\right)$$

$$\Big\downarrow \text{25}$$

$$-a^3\left(\frac{\int \frac{4a^4-\frac{4a^3}{x}+\frac{4a^2}{x^2}-\frac{3a}{x^3}+\frac{1}{x^4}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{2346}$$

$$-a^3\left(\frac{-\frac{1}{4}a^2\int -\frac{16a^2-\frac{16a}{x}+\frac{19}{x^2}-\frac{12}{x^3a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{25}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\int \frac{16a^2-\frac{16a}{x}+\frac{19}{x^2}-\frac{12}{x^3a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{2346}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{1}{3}a^2\int -\frac{3\left(16-\frac{24}{ax}+\frac{19}{a^2x^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(a^2\int \frac{16-\frac{24}{ax}+\frac{19}{a^2x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{2346}$$

3.58. $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(a^2\left(-\frac{1}{2}a^2\int -\frac{3\left(17a-\frac{16}{x}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow 27$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(a^2\left(\frac{3\int \frac{17a-\frac{16}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow 455$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(a^2\left(\frac{3\left(17a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow 223$$

$$-a^3\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\frac{\frac{1}{4}a^2\left(a^2\left(\frac{3\left(17a^2\arcsin\left(\frac{1}{ax}\right)+16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}\right)$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

output

```
-(a^3*((4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + (-1/4*(a^2*Sqrt[1 - 1/
(a^2*x^2)])/x^3 + (a^2*((4*a*Sqrt[1 - 1/(a^2*x^2)])/x^2 + a^2*((-19*Sqrt[1
 - 1/(a^2*x^2)])/(2*x) + (3*(16*a^2*Sqrt[1 - 1/(a^2*x^2)] + 17*a^2*ArcSin[
1/(a*x)]))/(2*a))))/4)/a^4))
```

3.58. $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.58. $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

3.58.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{(ax+1)(48a^3x^3-19a^2x^2+8ax-2)\sqrt{\frac{ax-1}{ax+1}}}{8x^4}+\frac{\left(-\frac{51a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}-\frac{4a^3\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1}}{ax-1}$
default	$\frac{(-56\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^7x^7+56(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5-163\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6-51\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^6x^6+56\ln(\frac{a^2x+\sqrt{a^2x^2-}{\sqrt{a^2}}}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*x+1)*(48*a^3*x^3-19*a^2*x^2+8*a*x-2)/x^4*((a*x-1)/(a*x+1))^(1/2)+(
-51/8*a^4*arctan(1/(a^2*x^2-1)^(1/2))-4*a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+
1/a))^(1/2))/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{102\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(80\,a^4x^4+29\,a^3x^3-11\,a^2x^2+6\,ax-2)\sqrt{\frac{ax-1}{ax+1}}}{8\,x^4}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/8*(102*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (80*a^4*x^4 + 29*a^3*
x^3 - 11*a^2*x^2 + 6*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

3.58.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**5, x)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$$

$$= \frac{1}{4}\left(51\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 16\,a^3\sqrt{\frac{ax-1}{ax+1}} - \frac{77\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 149\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 123\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 35\,a}{\frac{4\,(ax-1)}{ax+1} + \frac{6\,(ax-1)^2}{(ax+1)^2} + \frac{4\,(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/4*(51*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 16*a^3*sqrt((a*x - 1)/(a*x
 + 1)) - (77*a^3*((a*x - 1)/(a*x + 1))^(7/2) + 149*a^3*((a*x - 1)/(a*x + 1
))^(5/2) + 123*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 35*a^3*sqrt((a*x - 1)/(a*
x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^
3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + 1))*a
```

3.58.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

output
```
undef
```

3.58.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.43

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{51\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4} - 4\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{\frac{35\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4} + \frac{123\,a^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4} + \frac{149\,a^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{4} + \frac{77\,a^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{\frac{6\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{(a\,x-1)^4}{(a\,x+1)^4} + \frac{4\,(a\,x-1)}{a\,x+1} + 1}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x^5,x)
```

output
```
(51*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - 4*a^4*((a*x - 1)/(a*x + 1))
^(1/2) - ((35*a^4*((a*x - 1)/(a*x + 1))^(1/2))/4 + (123*a^4*((a*x - 1)/(a*
x + 1))^(3/2))/4 + (149*a^4*((a*x - 1)/(a*x + 1))^(5/2))/4 + (77*a^4*((a*x
 - 1)/(a*x + 1))^(7/2))/4)/((6*(a*x - 1)^2)/(a*x + 1)^2 + (4*(a*x - 1)^3)/
(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + (4*(a*x - 1))/(a*x + 1) + 1)
```

3.59 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

3.59.1 Optimal result

Integrand size = 14, antiderivative size = 253

$$\begin{aligned}
&\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx \\
&= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4}+\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} \\
&\quad+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} \\
&\quad+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5+\frac{31\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}
\end{aligned}$$

output
```
611/1920*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4+269/960*(1-1/a/x)^(3/4)*(1+
1/a/x)^(1/4)*x^2/a^3+11/48*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2+9/40*(1
-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^
5+31/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+31/128*arctanh((1+1/a
/x)^(1/4)/(1-1/a/x)^(1/4))/a^5
```

3.59.2 Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{\frac{24576e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}+\frac{62976e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{64640e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{34000e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{9620e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+930\arctan}{3840a^5}$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x^4,x]
```

output
```
((24576*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (62976*E^(ArcCot
h[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^(ArcCoth[a*x]/2))/(-1 +
E^(2*ArcCoth[a*x]))^3 + (34000*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]
))^2 + (9620*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 930*ArcTan[E^
(ArcCoth[a*x]/2)] - 465*Log[1 - E^(ArcCoth[a*x]/2)] + 465*Log[1 + E^(ArcCo
th[a*x]/2)])/(3840*a^5)
```

3.59.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^6}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$
$$\Big\downarrow \text{110}$$
$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{5}\int \frac{\left(9a+\frac{8}{x}\right)x^5}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\int \frac{\left(9a+\frac{8}{x}\right)x^5}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{10a^2}$$

$$\Big\downarrow 168$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{-\frac{1}{4}\int -\frac{\left(55a+\frac{54}{x}\right)x^4}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\int \frac{\left(55a+\frac{54}{x}\right)x^4}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow 168$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{-\frac{1}{3}\int -\frac{\left(269a+\frac{220}{x}\right)x^3}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\int \frac{\left(269a+\frac{220}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow 168$$

3.59. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{-\frac{1}{2}\int -\frac{\left(611a+\frac{538}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{\int \frac{\left(611a+\frac{538}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{-\int -\frac{465x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4(1-}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{\frac{465}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4(1-}{10a^2}$$

$$\Big\downarrow \; 104$$

3.59. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{930\int\frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)}{10a^2}$$

$$\Big\downarrow 756$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{930\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}}{8a}}{10a^2}$$

$$\Big\downarrow 216$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{930\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}}{8a}}{10a^2}$$

$$\Big\downarrow 219$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{930\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)}{8a}}{10a^2}$$

3.59. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

input

```
Int[E^(ArcCoth[a*x]/2)*x^4,x]
```

output

```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 - ((-9*a*(1 - 1/(a*x))^(3/
4)*(1 + 1/(a*x))^(1/4)*x^4)/4 + ((-55*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 + ((-269*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/2 + (
-611*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + 930*(-1/2*ArcTan[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a
*x))^(1/4)]/2))/(4*a))/(6*a))/(8*a))/(10*a^2)
```

3.59.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.59.4 Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{2\left(384\,a^5x^5 + 816\,a^4x^4 + 872\,a^3x^3 + 978\,a^2x^2 + 1149\,ax + 611\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 465\,\mathrm{lo}}{3840\,a^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="fricas")
```

output
```
1/3840*(2*(384*a^5*x^5 + 816*a^4*x^4 + 872*a^3*x^3 + 978*a^2*x^2 + 1149*a*
x + 611)*((a*x - 1)/(a*x + 1))^(3/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1
/4)) + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 465*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a^5
```

3.59.6 Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx=\int\frac{x^4}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**4,x)
```

output
```
Integral(x**4/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx=$$

$$-\frac{1}{3840}a\left(\frac{4\left(465\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+2405\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\,\mathrm{arct}}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="maxima")
```

output
```
-1/3840*a*(4*(465*((a*x - 1)/(a*x + 1))^(19/4) - 696*((a*x - 1)/(a*x + 1))
^(15/4) + 5090*((a*x - 1)/(a*x + 1))^(11/4) - 1120*((a*x - 1)/(a*x + 1))^(
7/4) + 2405*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(
a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^
4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6
+ 465*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}a\left(\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{465\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{1120\,(ax-1)\left(\right.}{ax+1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="giac")
```

output
```
-1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 + 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 - 4*(1120*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5090*(a*
x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 696*(a*x - 1)^3*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 465*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(
3/4)/(a*x + 1)^4 - 2405*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x
+ 1) - 1)^5))
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{481\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{192}-\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{6}+\frac{509\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{96}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{40}+\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{19/4}}{64}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}} - \frac{31\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+\frac{31\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

input
```
int(x^4/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output

```
((481*((a*x - 1)/(a*x + 1))^(3/4))/192 - (7*((a*x - 1)/(a*x + 1))^(7/4))/6
 + (509*((a*x - 1)/(a*x + 1))^(11/4))/96 - (29*((a*x - 1)/(a*x + 1))^(15/4
))/40 + (31*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/
(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x
+ 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (3
1*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (31*atanh(((a*x - 1)/(a*x
 + 1))^(1/4)))/(128*a^5)
```

3.59. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

3.60 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

3.60.1 Optimal result

Integrand size = 14, antiderivative size = 216

$$\begin{aligned}\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx &= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3}+\frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2}+\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}\\&\quad+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4+\frac{11\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}\end{aligned}$$

output
```
83/192*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+29/96*(1-1/a/x)^(3/4)*(1+1/a/
x)^(1/4)*x^2/a^2+7/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^
(3/4)*(1+1/a/x)^(1/4)*x^4+11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^
4+11/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

3.60.2 Mathematica [A] (verified)

Time = 5.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{\frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{3200e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{2512e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{980e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+66\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-33\,\mathrm{lo}}{384a^4}$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x^3,x]
```

output
```
((1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (3200*E^(ArcCoth[
a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^(ArcCoth[a*x]/2))/(-1 + E^(
2*ArcCoth[a*x]))^2 + (980*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) +
66*ArcTan[E^(ArcCoth[a*x]/2)] - 33*Log[1 - E^(ArcCoth[a*x]/2)] + 33*Log[1
+ E^(ArcCoth[a*x]/2)])/(384*a^4)
```

3.60.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{4}\int \frac{\left(7a+\frac{6}{x}\right)x^4}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\int \frac{\left(7a+\frac{6}{x}\right)x^4}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a^2}$$

$$\Big\downarrow \ 168$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{-\frac{1}{3}\int -\frac{\left(29a+\frac{28}{x}\right)x^3}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\int \frac{\left(29a+\frac{28}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \ 168$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{-\frac{1}{2}\int -\frac{\left(83a+\frac{58}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\int \frac{\left(83a+\frac{58}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \ 168$$

3.60. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{-\int -\frac{33x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{33}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow 104$$

$$\frac{\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{66\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow 756$$

$$\frac{\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{66\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}}{8a^2}$$

$$\Big\downarrow 216$$

3.60. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{66\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}}{8a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{66\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)}{8a^2}$$

input
```
Int[E^(ArcCoth[a*x]/2)*x^3,x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 - ((-7*a*(1 - 1/(a*x))^(3/
4)*(1 + 1/(a*x))^(1/4)*x^3)/3 + ((-29*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^2)/2 + (-83*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + 66*(-1/2
*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(
1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a))/(8*a^2)
```

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.60. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

3.60.4 Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\,(48\,a^4x^4+104\,a^3x^3+114\,a^2x^2+141\,ax+83)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{384\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="fricas")
```

output
```
1/384*(2*(48*a^4*x^4 + 104*a^3*x^3 + 114*a^2*x^2 + 141*a*x + 83)*((a*x - 1
)/(a*x + 1))^(3/4) - 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x
 - 1)/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

3.60.6 Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**3,x)
```

output
```
Integral(x**3/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{384}a\left(\frac{4\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-279\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+107\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-245\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\log\Big(}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="maxima")
```

output
```
1/384*a*(4*(33*((a*x - 1)/(a*x + 1))^(15/4) - 279*((a*x - 1)/(a*x + 1))^(1
1/4) + 107*((a*x - 1)/(a*x + 1))^(7/4) - 245*((a*x - 1)/(a*x + 1))^(3/4))/
(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3
*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 3
3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx=$$
$$-\frac{1}{384}a\left(\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{33\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{107(ax-1)\left(\frac{ax-1}{ax+1}\right)}{ax+1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="giac")
```

output
```
-1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^5 + 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
5 + 4*(107*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 279*(a*x - 1)
^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 33*(a*x - 1)^3*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1)^3 - 245*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x -
1)/(a*x + 1) - 1)^4))
```

3.60.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx=\frac{\frac{245\left(\frac{ax-1}{ax+1}\right)^{3/4}}{96}-\frac{107\left(\frac{ax-1}{ax+1}\right)^{7/4}}{96}+\frac{93\left(\frac{ax-1}{ax+1}\right)^{11/4}}{32}-\frac{11\left(\frac{ax-1}{ax+1}\right)^{15/4}}{32}}{a^4+\frac{6a^4(ax-1)^2}{(ax+1)^2}-\frac{4a^4(ax-1)^3}{(ax+1)^3}+\frac{a^4(ax-1)^4}{(ax+1)^4}-\frac{4a^4(ax-1)}{ax+1}}-\frac{11\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{64\,a^4}+\frac{11\,\mathrm{atanh}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{64\,a^4}$$

input
```
int(x^3/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((245*((a*x - 1)/(a*x + 1))^(3/4))/96 - (107*((a*x - 1)/(a*x + 1))^(7/4))/
96 + (93*((a*x - 1)/(a*x + 1))^(11/4))/32 - (11*((a*x - 1)/(a*x + 1))^(15/
4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x
+ 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (1
1*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (11*atanh(((a*x - 1)/(a*x
+ 1))^(1/4)))/(64*a^4)
```

3.61 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

3.61.1 Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2}+\frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3+\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
11/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2+5/12*(1-1/a/x)^(3/4)*(1+1/a/x)
^(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3+3/8*arctan((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^3+3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^
3
```

3.61.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.16 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.23

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{e^{-\frac{7}{2}\coth^{-1}(ax)}\left(-1070609085-946471617e^{2\coth^{-1}(ax)}+369641285e^{4\coth^{-1}(ax)}+351173641e^{6\coth^{-1}(ax)}-\right.}{}$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x^2,x]
```

output
```
-1/1909440*(-1070609085 - 946471617*E^(2*ArcCoth[a*x]) + 369641285*E^(4*Ar
cCoth[a*x]) + 351173641*E^(6*ArcCoth[a*x]) - 23818496*E^(8*ArcCoth[a*x]) +
 1070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 732349800
*E^(2*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] - 6
35067810*E^(4*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*
x])] - 384831720*E^(6*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*Ar
cCoth[a*x])] + 60913125*E^(8*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4,
E^(2*ArcCoth[a*x])] + 1280*E^(8*ArcCoth[a*x])*(821 + 1346*E^(2*ArcCoth[a*x
]) + 557*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4
}, E^(2*ArcCoth[a*x])] + 10240*E^(8*ArcCoth[a*x])*(23 + 42*E^(2*ArcCoth[a*
x]) + 19*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1
, 21/4}, E^(2*ArcCoth[a*x])] + 20480*E^(8*ArcCoth[a*x])*HypergeometricPFQ[
{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])] + 40960*E^(1
0*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}
, E^(2*ArcCoth[a*x])] + 20480*E^(12*ArcCoth[a*x])*HypergeometricPFQ[{2, 2,
 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((7*ArcCot
h[a*x])/2))
```

3.61.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow\ 6721$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow\ 110$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{3}\int \frac{\left(5a+\frac{4}{x}\right)x^3}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\int \frac{\left(5a+\frac{4}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a^2}$$

$$\Big\downarrow\ 168$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{-\frac{1}{2}\int -\frac{\left(11a+\frac{10}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\int \frac{\left(11a+\frac{10}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow\ 168$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{-\int -\frac{9x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow\ 27$$

3.61. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{9}{2}\int\frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow\ 104$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{18\int\frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow\ 756$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{18\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow\ 216$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{18\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow\ 219$$

3.61. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{18\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

input
```
Int[E^(ArcCoth[a*x]/2)*x^2,x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^3)/3 - ((-5*a*(1 - 1/(a*x))^(3/
4)*(1 + 1/(a*x))^(1/4)*x^2)/2 + (-11*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(
1/4)*x + 18*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a^2)
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.61.4 Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{2\,(8\,a^3x^3+18\,a^2x^2+21\,ax+11)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\,\arctan\Big(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\Big)+9\,\log\Big(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\Big)-9\,\log\Big(\Big(\frac{ax-1}{ax+1}}{48\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="fricas")
```

output
```
1/48*(2*(8*a^3*x^3 + 18*a^2*x^2 + 21*a*x + 11)*((a*x - 1)/(a*x + 1))^(3/4)
 - 18*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1
/4) + 1) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

3.61.6 Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=\int\frac{x^2}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=$$
$$-\frac{1}{48}\,a\Bigg(\frac{4\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+29\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\,\arctan\Big(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\Big)}{a^4}-\frac{9\,\log\Big(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\Big)}{a^4}+\frac{9\,\mathrm{lo}}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="maxima")
```

output
```
-1/48*a*(4*(9*((a*x - 1)/(a*x + 1))^(11/4) - 6*((a*x - 1)/(a*x + 1))^(7/4)
 + 29*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1
)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x
 - 1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 +
 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-9\right.}{a^4(}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="giac")
```

output
```
-1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 -
 4*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 9*(a*x - 1)^2*((a*
x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 29*((a*x - 1)/(a*x + 1))^(3/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=\frac{\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}}-\frac{3\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8\,a^3}+\frac{3\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8\,a^3}$$

input

```
int(x^2/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output

```
((29*((a*x - 1)/(a*x + 1))^(3/4))/12 - ((a*x - 1)/(a*x + 1))^(7/4)/2 + (3*
((a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 -
(a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a
*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4))
)/(8*a^3)
```

3.61. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

3.62 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx$

3.62.1 Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 + \frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output
```
1/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4
)*x^2+1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/x
)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

3.62.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+5e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x,x]
```

output
```
((2*E^(ArcCoth[a*x]/2)*(-1 + 5*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
]))^2 + ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

3.62.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6721, 107, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \color{blue}{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \color{blue}{107}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^2}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}}{4a}$$

$$\Big\downarrow \color{blue}{105}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

$$\Big\downarrow \color{blue}{104}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{2\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{2\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

input

```
Int[E^(ArcCoth[a*x]/2)*x,x]
```

output

```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4)*x^2)/2 - (-((1 - 1/(a*x))^(3/4)*(
1 + 1/(a*x))^(1/4)*x) + (2*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^
(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/a)/(4*a)
```

3.62. $\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx$

3.62.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.62.4 Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{2\left(2a^2x^2+5ax+3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 + 5*a*x + 3)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*
x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

3.62.6 Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{8}\,a\left(\frac{4\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="maxima")
```

output
```
1/8*a*(4*(((a*x - 1)/(a*x + 1))^(7/4) - 5*((a*x - 1)/(a*x + 1))^(3/4))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((
a*x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3
- log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.98

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx =$$
$$-\frac{1}{8}\,a\Biggl(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\Biggl(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-5\left(\frac{ax-1}{ax+1}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="giac")
```

output
```
-1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^3 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*((a
*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5*((a*x - 1)/(a*x + 1))^(3
/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx=\frac{\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((5*((a*x - 1)/(a*x + 1))^(3/4))/2 - ((a*x - 1)/(a*x + 1))^(7/4)/2)/(a^2 +
 (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - atan(((a*x
 - 1)/(a*x + 1))^(1/4))/(4*a^2) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^
2)
```

3.63 $\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$

3.63.1 Optimal result

Integrand size = 10, antiderivative size = 96

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/
a+arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

3.63.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^(ArcCoth[a*x]/2),x]
```

output
```
((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + ArcTan[E^(ArcCoth[a*x]
/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

3.63.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6720, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^2}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{104}$$

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{2\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}$$

$$\Big\downarrow \text{756}$$

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$\Big\downarrow \text{216}$$

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}$$

$$\Big\downarrow 219$$

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{2\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}$$

input
```
Int[E^(ArcCoth[a*x]/2),x]
```

output
```
(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - (2*(-1/2*ArcTan[(1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/2))/a
```

3.63.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]
```

3.63.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x - 1)/(a*x +
1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a
```

3.63.6 Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-1/4), x)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
-1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a
*x - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.63.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
 - 1)/(a*x + 1))^(1/4))/a + atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

3.64 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

3.64.1 Optimal result

Integrand size = 14, antiderivative size = 291

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

output
```
2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))+1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2
)/(1+1/a/x)^(1/2))*2^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4
)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1/
2)/(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/
4))*2^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx=\frac{8}{5}e^{\frac{5}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{5}{8},1,\frac{13}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/2)/x,x]
```

output
```
(8*E^((5*ArcCoth[a*x])/2)*Hypergeometric2F1[5/8, 1, 13/8, E^(4*ArcCoth[a*x
])])/5
```

3.64.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.89, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 73$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 104$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 756$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$\Big\downarrow \; 216$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \; 219$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \; 854$$

$$4\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \; 826$$

3.64. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}dx$

$$4\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\big\downarrow$ 1476

$$4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\big\downarrow$ 1082

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\big\downarrow$ 217

3.64. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1479}$$

$$4\Bigg(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\Bigg(\frac{\arctan\Bigg(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{\sqrt{2}}$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{25}$$

3.64. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow 27$$

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow 1103$$

3.64. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\Bigg(\frac{1}{2}\Bigg(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log}{}$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

input
```
Int[E^(ArcCoth[a*x]/2)/x,x]
```

output
```
-4*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2) + 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.64. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.64.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

3.64.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

output
```
(1/2*I - 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
 - (1/2*I + 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)) + (1/2*I + 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4)) - (1/2*I - 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x +
 1))^(1/4)) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

3.64.6 Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(1/4)/x,x)
```

output

```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.77

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\Bigg(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\Big(\sqrt{2}\Big(}{a}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

output

```
1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1)/a)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$=\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

output
```
1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
sqrt((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a
 + 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx=-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)$$
$$+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(1-\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(1+1\mathrm{i})$$

input
```
int(1/(x*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

output
```
2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 - 1i) -
2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)
*2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 +
1i)
```

3.65 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

3.65.1 Optimal result

Integrand size = 14, antiderivative size = 267

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

output
```
a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^
(1/4))*2^(1/2)+1/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-1/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a
/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}+\frac{\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/2)/x^2,x]
```

output
```
a*((2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + ArcTan[1 - Sqrt[2]*E^
(ArcCoth[a*x]/2)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)]/Sqrt[2]
 + Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]) - Log[
1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]))
```

3.65.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{2}\int\frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \ 73$$

$$2a\int\frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 854$$

$$2a\int\frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 826$$

$$2a\left(\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 1476$$

$$2a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 1082$$

3.65. $\int\frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}dx$

$$2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 217$$

$$2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 1479$$

$$2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

3.65. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$\Big\downarrow \text{25}$$

$$2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{1103}$$

3.65. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}dx$

$$2a\Bigg(\frac{1}{2}\Bigg(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\ \text{lo}$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[E^(ArcCoth[a*x]/2)/x^2,x]
```

output
```
a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) + 2*a*((-(ArcTan[1 - (Sqrt[2]*(1
 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.65. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.65.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^2}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

3.65.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.69

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{(-a^4)^{\frac{1}{4}}\,x\log\left(a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+(-a^4)^{\frac{3}{4}}\right)-i\,(-a^4)^{\frac{1}{4}}\,x\log\left(a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+i\,(-a^4)^{\frac{3}{4}}\right)+i\,(-a^4)^{\frac{1}{4}}\,x\log\Bigl(a^3\left(\frac{ax-1}{ax+1}\right)}{2\,x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

output
```
1/2*((-a^4)^(1/4)*x*log(a^3*((a*x - 1)/(a*x + 1))^(1/4) + (-a^4)^(3/4)) -
I*(-a^4)^(1/4)*x*log(a^3*((a*x - 1)/(a*x + 1))^(1/4) + I*(-a^4)^(3/4)) + I
*(-a^4)^(1/4)*x*log(a^3*((a*x - 1)/(a*x + 1))^(1/4) - I*(-a^4)^(3/4)) - (-
a^4)^(1/4)*x*log(a^3*((a*x - 1)/(a*x + 1))^(1/4) - (-a^4)^(3/4)) + 2*(a*x
+ 1)*((a*x - 1)/(a*x + 1))^(3/4))/x
```

3.65.6 Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\Bigg(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1
) + 1))*a
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$=\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1
) + 1))*a
```

3.65.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.33

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$=(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)+\frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

output
```
(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (-1)^(1/4)*a*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) + (2*a*((a*x - 1)/(a*x + 1))^
(3/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.66 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.66.1 Optimal result

Integrand size = 14, antiderivative size = 319

$$\begin{aligned}\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = {} & \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} \\ & +\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} \\ & +\frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}+\frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} \\ & -\frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\end{aligned}$$

output

```
1/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^
(5/4)+1/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1
/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/16*a^2*
ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/
2))*2^(1/2)-1/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}a^2\left(-\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{40e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}+2\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)-\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

input

```
Integrate[E^(ArcCoth[a*x]/2)/x^3,x]
```

output

```
(a^2*((-32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (40*E^(ArcCoth
[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCo
th[a*x]/2)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] + Sqrt[2]*L
og[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] - Sqrt[2]*Log[1 + Sqrt
[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

3.66.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.82, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}\,x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{1}{4}a\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{1}{4}a\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$
$$\frac{1}{4}a\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 854}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{1}{4}a\left(-2a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 826}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$
$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 1476}$$

3.66. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\Bigg(-2a\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\Bigg)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}$$

$$\Big\downarrow 1082$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\Bigg(-2a\Bigg(\frac{1}{2}\Bigg(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\Bigg)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\Bigg)-$$

$$\Big\downarrow 217$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\Bigg(-2a\Bigg(\frac{1}{2}\Bigg(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\Bigg)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\Bigg)-a\Big(1-$$

$$\Big\downarrow 1479$$

3.66. $\int\frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{2}}\right.}{\sqrt{}}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

3.66. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}}{\right.}}{}\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right.}{2\sqrt{2}}\right.\right.\right.$$

input
```
Int[E^(ArcCoth[a*x]/2)/x^3,x]
```

output
```
(a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/2 - (a*(-(a*(1 - 1/(a*x))^(3
/4)*(1 + 1/(a*x))^(1/4)) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4)
)/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4
```

3.66. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.66.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^{3}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

3.66.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\left(-a^8\right)^{\frac{1}{4}} x^2 \log\left(a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + \left(-a^8\right)^{\frac{3}{4}}\right) - i\left(-a^8\right)^{\frac{1}{4}} x^2 \log\left(a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + i\left(-a^8\right)^{\frac{3}{4}}\right) + i\left(-a^8\right)^{\frac{1}{4}} x^2 \log\left(a^6\left(\frac{ax-}{ax+}\right.\right.}{8\,x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

output
```
1/8*((-a^8)^(1/4)*x^2*log(a^6*((a*x - 1)/(a*x + 1))^(1/4) + (-a^8)^(3/4))
- I*(-a^8)^(1/4)*x^2*log(a^6*((a*x - 1)/(a*x + 1))^(1/4) + I*(-a^8)^(3/4))
 + I*(-a^8)^(1/4)*x^2*log(a^6*((a*x - 1)/(a*x + 1))^(1/4) - I*(-a^8)^(3/4)
) - (-a^8)^(1/4)*x^2*log(a^6*((a*x - 1)/(a*x + 1))^(1/4) - (-a^8)^(3/4)) +
 2*(3*a^2*x^2 + 5*a*x + 2)*((a*x - 1)/(a*x + 1))^(3/4))/x^2
```

3.66.6 Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.71

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$= \frac{1}{16}\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

output
```
1/16*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1))*a + 8*(a*((a*x - 1)/(a*x + 1))^(7/4) + 5*a*((a*x
 - 1)/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
 1))*a
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$= \frac{1}{16}\Bigg(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

output
```
1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1) + sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/
4)/(a*x + 1) + 5*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^
2)*a
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{5\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}+\frac{a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4} - \frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

output
```
((5*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (a^2*((a*x - 1)/(a*x + 1))^(7/4))
/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*
a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - ((-1)^(1/4)*a^2*atan
h((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

3.67 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

3.67.1 Optimal result

Integrand size = 14, antiderivative size = 356

$$\begin{aligned}\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx ={}& \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\\ &+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x}\\ &-\frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{3a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\\ &+\frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}\\ &-\frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}\end{aligned}$$

output

```
3/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/12*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)
^(5/4)+1/3*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4)/x+3/16*a^3*arctan(-1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+(1-1/a/x)^(1/
4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-3/32*a^3*ln(1+(1-
1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1
/2)
```

3.67.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx=\frac{1}{96}a^3\left(\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(9+6e^{2\coth^{-1}(ax)}+29e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3}+9\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

input

```
Integrate[E^(ArcCoth[a*x]/2)/x^4,x]
```

output

```
(a^3*((8*E^(ArcCoth[a*x]/2)*(9 + 6*E^(2*ArcCoth[a*x]) + 29*E^(4*ArcCoth[a*
x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x] -
2*Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ]))/96
```

3.67.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 101, 27, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 101}$$

$$\frac{1}{3}a^2\int -\frac{\left(2a+\frac{1}{x}\right)\sqrt[4]{1+\frac{1}{ax}}}{2a\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\int \frac{\left(2a+\frac{1}{x}\right)\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{9}{4}a\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)$$

$$\Big\downarrow \text{ 854}$$

3.67. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\int\frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)$$

$$\Big\downarrow \ 826$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right.\right.$$

$$\Big\downarrow \ 1476$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int\right.\right.\right.$$

$$\Big\downarrow \ 1082$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.\right.\right.$$

$$\Big\downarrow \ 217$$

3.67. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\right.\right.\right.$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\arctan\left(\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 25}$$

3.67. $\int\frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\arctan\left(\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\arctan\right.\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

3.67. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}\right.}{2\sqrt{2}}\right.\right.\right.\right.$$

input
```
Int[E^(ArcCoth[a*x]/2)/x^4,x]
```

output
```
(a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/(3*x) - (a*(-1/2*(a^2*(1 - 1
/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4)) + (9*a*(-(a*(1 - 1/(a*x))^(3/4)*(1 + 1/
(a*x))^(1/4)) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-
4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4)
)^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^
(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x
^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4))/6
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.67. $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.67.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^4}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

3.67.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{9\left(-a^{12}\right)^{\frac{1}{4}} x^3 \log\left(27\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+27\left(-a^{12}\right)^{\frac{3}{4}}\right)-9i\left(-a^{12}\right)^{\frac{1}{4}} x^3 \log\left(27\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+27i\left(-a^{12}\right)^{\frac{3}{4}}\right)+9i\,(-}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

output
```
1/48*(9*(-a^12)^(1/4)*x^3*log(27*a^9*((a*x - 1)/(a*x + 1))^(1/4) + 27*(-a^
12)^(3/4)) - 9*I*(-a^12)^(1/4)*x^3*log(27*a^9*((a*x - 1)/(a*x + 1))^(1/4)
+ 27*I*(-a^12)^(3/4)) + 9*I*(-a^12)^(1/4)*x^3*log(27*a^9*((a*x - 1)/(a*x +
 1))^(1/4) - 27*I*(-a^12)^(3/4)) - 9*(-a^12)^(1/4)*x^3*log(27*a^9*((a*x -
1)/(a*x + 1))^(1/4) - 27*(-a^12)^(3/4)) + 2*(11*a^3*x^3 + 21*a^2*x^2 + 18*
a*x + 8)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

3.67.6 Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**4,x)
```

output
```
Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\Bigg(9\Bigg(2\sqrt{2}\arctan\Bigg(\frac{1}{2}\sqrt{2}\Bigg(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\Bigg)\Bigg)+2\sqrt{2}\arctan\Bigg(-\frac{1}{2}\sqrt{2}\Bigg(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\Bigg)\Bigg)-$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

output
```
1/96*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1))*a^2 + 8*(9*a^2*((a*x - 1)/(a*x + 1))^(11/4) +
6*a^2*((a*x - 1)/(a*x + 1))^(7/4) + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3
*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3
 + 1))*a
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\Bigg(18\sqrt{2}a^2\arctan\Bigg(\frac{1}{2}\sqrt{2}\Bigg(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\Bigg)\Bigg)+18\sqrt{2}a^2\arctan\Bigg(-\frac{1}{2}\sqrt{2}\Bigg(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\Bigg)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

output
```
1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) - 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(6*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1) + 9*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
3/4)/(a*x + 1)^2 + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1
) + 1)^3)*a
```

3.67.9 Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.47

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{29\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}+\frac{a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1} + \frac{3\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8} - \frac{3\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8}$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

output
```
((29*a^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (a^3*((a*x - 1)/(a*x + 1))^(7/4
))/2 + (3*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^
2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + (3*(-1)^(1/4)
*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (3*(-1)^(1/4)*a^3*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

3.68 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

3.68.1 Optimal result

Integrand size = 14, antiderivative size = 253

$$\begin{aligned}\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx &= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4}+\frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3}\\&\quad+\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}\\&\quad+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{237\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}\end{aligned}$$

output
```
557/640*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4+157/320*(1-1/a/x)^(1/4)*(1+1
/a/x)^(3/4)*x^2/a^3+5/16*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2+11/40*(1-
1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^5
-237/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+237/128*arctanh((1+1/
a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5
```

3.68.2 Mathematica [A] (verified)

Time = 5.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{8192e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}+\frac{22016e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{23936e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{14032e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{5500e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-2370\arctan}{1280a^5}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

output
```
((8192*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (22016*E^((3*
ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((3*ArcCoth[a*x])
/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (14032*E^((3*ArcCoth[a*x])/2))/(-1 + E^
(2*ArcCoth[a*x]))^2 + (5500*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x
])) - 2370*ArcTan[E^(ArcCoth[a*x]/2)] - 1185*Log[1 - E^(ArcCoth[a*x]/2)] +
 1185*Log[1 + E^(ArcCoth[a*x]/2)])/(1280*a^5)
```

3.68.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^6}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{5}\int \frac{\left(11a+\frac{8}{x}\right)x^5}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\int \frac{\left(11a+\frac{8}{x}\right)x^5}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{10a^2}$$

$$\Big\downarrow \ 168$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{-\frac{1}{4}\int -\frac{3\left(25a+\frac{22}{x}\right)x^4}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\int \frac{\left(25a+\frac{22}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \ 168$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\left(-\frac{1}{3}\int -\frac{\left(157a+\frac{100}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\left(\frac{\int \frac{\left(157a+\frac{100}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \ 168$$

3.68. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{-\frac{1}{2}\int -\frac{\left(557a+\frac{314}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{\int \frac{\left(557a+\frac{314}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow 168$$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{-\int -\frac{1185x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax}{10a^2}$$

$$\Big\downarrow 27$$

3.68. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{\frac{1185}{2}\int\frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax}{10a^2}$$

$$\Big\downarrow \text{104}$$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{2370\int-\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax}{10a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{-2370\int\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax}{10a^2}$$

$$\Big\downarrow \text{827}$$

3.68. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\left(\frac{\frac{2370\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\right.}{8a}}{10a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\left(\frac{\frac{2370\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\right.}{8a}}{10a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\left(\frac{\frac{2370\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-}\right.}{8a}}{10a^2}$$

3.68. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

input
```
Int[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 - ((-11*a*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 + (3*((-25*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*
x))^(3/4)*x^3)/3 + ((-157*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/2
 + (-557*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + 2370*(ArcTan[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/
(a*x))^(1/4)]/2))/(4*a))/(6*a)))/(8*a))/(10*a^2)
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.68.4 Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{2\left(128\,a^5x^5+304\,a^4x^4+376\,a^3x^3+514\,a^2x^2+871\,ax+557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1185}{1280\,a^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="fricas")
```

output
```
1/1280*(2*(128*a^5*x^5 + 304*a^4*x^4 + 376*a^3*x^3 + 514*a^2*x^2 + 871*a*x
 + 557)*((a*x - 1)/(a*x + 1))^(1/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1
/4)) + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1185*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1))/a^5
```

3.68.6 Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx=\int\frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**4,x)
```

output
```
Integral(x**4/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx=$$

$$-\frac{1}{1280}\,a\left(\frac{4\left(395\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+1375\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\,\mathrm{ar}}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="maxima")
```

output
```
-1/1280*a*(4*(395*((a*x - 1)/(a*x + 1))^(17/4) - 1440*((a*x - 1)/(a*x + 1)
)^(13/4) + 3710*((a*x - 1)/(a*x + 1))^(9/4) - 1992*((a*x - 1)/(a*x + 1))^(
5/4) + 1375*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(
a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^
4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x
 - 1)/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^
6 + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{1}{1280}a\left(\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6} + \frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6} - \frac{1185\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6} + \frac{4\left(\frac{1992\,(ax-1)}{ax+}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="giac")
```

output
```
1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x -
1)/(a*x + 1))^(1/4) + 1)/a^6 - 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) -
1))/a^6 + 4*(1992*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 3710*(
a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1440*(a*x - 1)^3*((a*
x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 395*(a*x - 1)^4*((a*x - 1)/(a*x + 1)
)^(1/4)/(a*x + 1)^4 - 1375*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a
*x + 1) - 1)^5))
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{\frac{275\left(\frac{ax-1}{ax+1}\right)^{1/4}}{64}-\frac{249\left(\frac{ax-1}{ax+1}\right)^{5/4}}{40}+\frac{371\left(\frac{ax-1}{ax+1}\right)^{9/4}}{32}-\frac{9\left(\frac{ax-1}{ax+1}\right)^{13/4}}{2}+\frac{79\left(\frac{ax-1}{ax+1}\right)^{17/4}}{64}}{a^5+\frac{10a^5(ax-1)^2}{(ax+1)^2}-\frac{10a^5(ax-1)^3}{(ax+1)^3}+\frac{5a^5(ax-1)^4}{(ax+1)^4}-\frac{a^5(ax-1)^5}{(ax+1)^5}-\frac{5a^5(ax-1)}{ax+1}} + \frac{237\operatorname{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128a^5}+\frac{237\operatorname{atanh}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128a^5}$$

input

```
int(x^4/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output

```
((275*((a*x - 1)/(a*x + 1))^(1/4))/64 - (249*((a*x - 1)/(a*x + 1))^(5/4))/
40 + (371*((a*x - 1)/(a*x + 1))^(9/4))/32 - (9*((a*x - 1)/(a*x + 1))^(13/4
))/2 + (79*((a*x - 1)/(a*x + 1))^(17/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(
a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x +
 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (23
7*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (237*atanh(((a*x - 1)/(a*
x + 1))^(1/4)))/(128*a^5)
```

3.69 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

3.69.1 Optimal result

Integrand size = 14, antiderivative size = 216

$$\begin{aligned}\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx &= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3}+\frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2}+\frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}\\&\quad+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{123\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}\end{aligned}$$

output
```
63/64*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+15/32*(1-1/a/x)^(1/4)*(1+1/a/x
)^(3/4)*x^2/a^2+3/8*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)^(1
/4)*(1+1/a/x)^(3/4)*x^4-123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
+123/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

3.69.2 Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{\frac{512e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{1152e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{1008e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{532e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-246\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-12}{128a^4}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

output
```
((512*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (1152*E^((3*Ar
cCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((3*ArcCoth[a*x])/2)
)/(-1 + E^(2*ArcCoth[a*x]))^2 + (532*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*Ar
cCoth[a*x])) - 246*ArcTan[E^(ArcCoth[a*x]/2)] - 123*Log[1 - E^(ArcCoth[a*x
]/2)] + 123*Log[1 + E^(ArcCoth[a*x]/2)])/(128*a^4)
```

3.69.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^5}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{4}\int \frac{3\left(3a+\frac{2}{x}\right)x^4}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\int \frac{\left(3a+\frac{2}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{8a^2}$$

$$\Big\downarrow\ 168$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(-\frac{1}{3}\int -\frac{3\left(5a+\frac{4}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\int \frac{\left(5a+\frac{4}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow\ 168$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{-\frac{1}{2}\int -\frac{\left(21a+\frac{10}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{\int \frac{\left(21a+\frac{10}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow\ 168$$

3.69. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{-\int -\frac{41x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{\frac{41}{2}\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow \ 104$$

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{82\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow \ 25$$

3.69. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{-82\int\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow 827$$

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{82\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\Big(}{8a^2}$$

$$\Big\downarrow 216$$

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{82\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\Big(}{8a^2}$$

$$\Big\downarrow 219$$

3.69. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{82\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\right.}{8a^2}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - (3*(-(a*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(3/4)*x^3) + ((-5*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3
/4)*x^2)/2 + (-21*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + 82*(ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/
(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(2*a)))/(8*a^2)
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.69. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

3.69.4 Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\left(16\,a^4x^4+40\,a^3x^3+54\,a^2x^2+93\,ax+63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) -}{128\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="fricas")
```

output
```
1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 54*a^2*x^2 + 93*a*x + 63)*((a*x - 1)/(
a*x + 1))^(1/4) + 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

3.69.6 Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**3,x)
```

output
```
Integral(x**3/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$

$$= \frac{1}{128}a\left(\frac{4\left(41\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}} - 183\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}} + 147\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} - 133\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1} - \frac{6(ax-1)^2a^5}{(ax+1)^2} + \frac{4(ax-1)^3a^5}{(ax+1)^3} - \frac{(ax-1)^4a^5}{(ax+1)^4} - a^5} + \frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} + 123\log\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="maxima")
```

output
```
1/128*a*(4*(41*((a*x - 1)/(a*x + 1))^(13/4) - 183*((a*x - 1)/(a*x + 1))^(9
/4) + 147*((a*x - 1)/(a*x + 1))^(5/4) - 133*((a*x - 1)/(a*x + 1))^(1/4))/(
4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*
a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 -
123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$

$$= \frac{1}{128}a\left(\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} + \frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^5} - \frac{123\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right|\right)}{a^5} - 4\left(\frac{147(ax-1)\left(\frac{ax-1}{ax+1}\right.}{ax+1}\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="giac")
```

output
```
1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^5 - 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^5 - 4*(147*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 183*(a*x -
1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 41*(a*x - 1)^3*((a*x - 1)/(
a*x + 1))^(1/4)/(a*x + 1)^3 - 133*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x
- 1)/(a*x + 1) - 1)^4))
```

3.69.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{\frac{133\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{147\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{183\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{32}-\frac{41\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{32}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}} + \frac{123\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}+\frac{123\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

input
```
int(x^3/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((133*((a*x - 1)/(a*x + 1))^(1/4))/32 - (147*((a*x - 1)/(a*x + 1))^(5/4))/
32 + (183*((a*x - 1)/(a*x + 1))^(9/4))/32 - (41*((a*x - 1)/(a*x + 1))^(13/
4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x
+ 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (1
23*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (123*atanh(((a*x - 1)/(a*
x + 1))^(1/4)))/(64*a^4)
```

3.70 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

3.70.1 Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx=\frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2}+\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a}+\frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3-\frac{17\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{17\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
23/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2+7/12*(1-1/a/x)^(1/4)*(1+1/a/x)
^(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3-17/8*arctan((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^3+17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/
a^3
```

3.70.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.43 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.23

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\left(-1357846875-1400453615e^{2\coth^{-1}(ax)}+276606275e^{4\coth^{-1}(ax)}+438715415e^{6\coth^{-1}(ax)}\right.}{}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

output
```
-1/4213440*(-1357846875 - 1400453615*E^(2*ArcCoth[a*x]) + 276606275*E^(4*A
rcCoth[a*x]) + 438715415*E^(6*ArcCoth[a*x]) - 12962560*E^(8*ArcCoth[a*x])
+ 1357846875*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 81851924
0*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] -
997722110*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a
*x])] - 501106760*E^(6*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*A
rcCoth[a*x])] + 137997475*E^(8*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4
, E^(2*ArcCoth[a*x])] + 1792*E^(8*ArcCoth[a*x])*(965 + 1618*E^(2*ArcCoth[a
*x]) + 685*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 11/4}, {1, 1, 2
3/4}, E^(2*ArcCoth[a*x])] + 14336*E^(8*ArcCoth[a*x])*(25 + 46*E^(2*ArcCoth
[a*x]) + 21*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 11/4}, {1,
1, 1, 23/4}, E^(2*ArcCoth[a*x])] + 28672*E^(8*ArcCoth[a*x])*Hypergeometric
PFQ[{2, 2, 2, 2, 2, 11/4}, {1, 1, 1, 1, 23/4}, E^(2*ArcCoth[a*x])] + 57344
*E^(10*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 11/4}, {1, 1, 1, 1,
 23/4}, E^(2*ArcCoth[a*x])] + 28672*E^(12*ArcCoth[a*x])*HypergeometricPFQ[
{2, 2, 2, 2, 2, 11/4}, {1, 1, 1, 1, 23/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((5
*ArcCoth[a*x])/2))
```

3.70.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4} x^4}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{3}\int \frac{\left(7a+\frac{4}{x}\right)x^3}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\int \frac{\left(7a+\frac{4}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{-\frac{1}{2}\int -\frac{\left(23a+\frac{14}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{\int \frac{\left(23a+\frac{14}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{-\int -\frac{51x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{27}$$

3.70. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{\frac{51}{2}\int\frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \ 104$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{102\int-\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{-102\int\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \ 827$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{102\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \ 216$$

3.70. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{102\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow 219$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{102\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

input

```
Int[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

output

```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^3)/3 - ((-7*a*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4)*x^2)/2 + (-23*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(
3/4)*x + 102*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[
(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a^2)
```

3.70.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.70. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

3.70.4 Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\left(8a^3x^3+22a^2x^2+37ax+23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-51\log\Bigg(\Bigg(}{48a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="fricas")
```

output
```
1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 37*a*x + 23)*((a*x - 1)/(a*x + 1))^(1/4)
 + 102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 51*log(((a*x - 1)/(a*x + 1))^
(1/4) + 1) - 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

3.70.6 Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}a\left(\frac{4\left(17\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+45\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="maxima")
```

output
```
-1/48*a*(4*(17*((a*x - 1)/(a*x + 1))^(9/4) - 30*((a*x - 1)/(a*x + 1))^(5/4
) + 45*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x -
1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a
*x - 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^
4 + 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\right.}{a}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="giac")
```

output
```
1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^4 - 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
 + 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 17*(a*x - 1)^2*
((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 45*((a*x - 1)/(a*x + 1))^(1/4))/
(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}} + \frac{17\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8\,a^3} + \frac{17\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8\,a^3}$$

input

```
int(x^2/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output

```
((15*((a*x - 1)/(a*x + 1))^(1/4))/4 - (5*((a*x - 1)/(a*x + 1))^(5/4))/2 +
(17*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^
2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) + (17*atanh(((a*x - 1)/(a*x + 1))^
(1/4)))/(8*a^3)
```

3.71 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$

3.71.1 Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx=\frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a}+\frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2-\frac{9\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output
```
3/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4
)*x^2-9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/x
)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

3.71.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx=\frac{\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(-3+7e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-9\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+9\operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x,x]
```

output
```
((2*E^((3*ArcCoth[a*x])/2)*(-3 + 7*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth
[a*x]))^2 - 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/
(4*a^2)
```

3.71.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6721, 107, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{3}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^3}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^2}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(\frac{3\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(\frac{6\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

$\downarrow$ 25

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}\right)}{4a}$$

$\downarrow$ 827

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(\frac{6\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

$\downarrow$ 216

3.71. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$

$$\frac{\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-3\left(\frac{6\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-3\left(\frac{6\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

input

```
Int[E^((3*ArcCoth[a*x])/2)*x,x]
```

output

```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4)*x^2)/2 - (3*(-((1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(3/4)*x) + (6*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1
/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/a))/(4*a)
```

3.71.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.71. $\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$

3.71.4 Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2+7\,ax+5)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 + 7*a*x + 5)*((a*x - 1)/(a*x + 1))^(1/4) + 18*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

3.71.6 Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$$

$$=\frac{1}{8}a\left(\frac{4\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="maxima")
```

output
```
1/8*a*(4*(3*((a*x - 1)/(a*x + 1))^(5/4) - 7*((a*x - 1)/(a*x + 1))^(1/4))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan
(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$$

$$=\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{3(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-7\left(\frac{ax}{ax}\right.\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="giac")
```

output
```
1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4
*(3*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 7*((a*x - 1)/(a*x +
1))^(1/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.71.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx=\frac{\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}+\frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((7*((a*x - 1)/(a*x + 1))^(1/4))/2 - (3*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a
^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + (9*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) + (9*atanh(((a*x - 1)/(a*x + 1))^(
1/4)))/(4*a^2)
```

3.72 $\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$

3.72.1 Optimal result

Integrand size = 10, antiderivative size = 98

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx=\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x-\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)/a+3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

3.72.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx=\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(1+\left(-1+e^{2\coth^{-1}(ax)}\right)\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},e^{2\coth^{-1}(ax)}\right)\right)}{a\left(-1+e^{2\coth^{-1}(ax)}\right)}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2),x]
```

output
```
(8*E^((3*ArcCoth[a*x])/2)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1
[3/4, 2, 7/4, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

3.72.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6720, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^2}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{104}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{827}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$\Big\downarrow \text{216}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$\Big\downarrow \text{219}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}$$

input

```
Int[E^((3*ArcCoth[a*x])/2),x]
```

output

```
(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - (6*(ArcTan[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/2))/a
```

3.72.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.72.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) + 6*arctan(((a*x - 1)/(a*x +
1))^(1/4)) + 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 3*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1))/a
```

3.72.6 Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-3/4), x)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{4\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2} - \frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2} - \frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2} + \frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1)/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{1}{2}\,a\left(\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1)/a^2 - 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*
((a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.72.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((
a*x - 1)/(a*x + 1))^(1/4)))/a + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.73 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

3.73.1 Optimal result

Integrand size = 14, antiderivative size = 291

$$\begin{aligned}\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = {}&-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\ &-2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\\ &-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\ &+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\end{aligned}$$

output
```
-2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/
a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/
4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1
/2)/(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1
/4))*2^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \frac{8}{7}e^{\frac{7}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)/x,x]
```

output
```
(8*E^((7*ArcCoth[a*x])/2)*Hypergeometric2F1[7/8, 1, 15/8, E^(4*ArcCoth[a*x
])])/7
```

3.73.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.89, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {6721, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4} x}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow 73$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow 104$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 25$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 770$$

$$4\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 755$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 827$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

3.73. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}dx$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1476}$$

$$4\Bigg(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\Bigg(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1082}$$

3.73. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1479}$$

3.73. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\downarrow 25$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\downarrow 27$$

3.73. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \ 1103$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log\left(\right.}{}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

input
```
Int[E^((3*ArcCoth[a*x])/2)/x,x]
```

3.73. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

output
```
-4*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2) + 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^
(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.73. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.73.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

3.73.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = {} & \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & -\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & +\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & -\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & +2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & +\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)\end{aligned}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

output
```
(1/2*I + 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
 - (1/2*I - 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)) + (1/2*I - 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4)) - (1/2*I + 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x +
 1))^(1/4)) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

3.73.6 Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.77

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.}{a}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

output
```
1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1)/a)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

output
```
1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
sqrt((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a
 + 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(1+1\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(1-\mathrm{i})$$

input

```
int(1/(x*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

output

```
2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)
*2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 +
1i) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 -
1i)
```

3.74 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

3.74.1 Optimal result

Integrand size = 14, antiderivative size = 268

$$\begin{aligned}\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx &= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\&\quad+\frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\&\quad-\frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}\\&\quad+\frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}\end{aligned}$$

output

```
a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+3/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^
(1/4))*2^(1/2)-3/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+3/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a
/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.74.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -8ae^{\frac{3}{2}\coth^{-1}(ax)}\left(-\frac{1}{1+e^{2\coth^{-1}(ax)}} + \text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

input

```
Integrate[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

output

```
-8*a*E^((3*ArcCoth[a*x])/2)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometr
ic2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])])
```

3.74.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 73$$

$$6a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 770$$

$$6a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 755$$

$$6a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 1476$$

$$6a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 1082$$

3.74. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$6a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 217$$

$$6a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)+$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 1479$$

$$6a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

3.74. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$\Big\downarrow 25$$

$$6a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\right.}{\sqrt{2}}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 27$$

$$6a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 1103$$

3.74. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log}{}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

output
```
a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) + 6*a*((-(ArcTan[1 - (Sqrt[2]*(1
 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(
1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2
]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.74. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.74.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^{2}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

3.74.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.69

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{3\left(-a^{4}\right)^{\frac{1}{4}} x \log \left(3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+3\left(-a^{4}\right)^{\frac{1}{4}}\right)+3i\left(-a^{4}\right)^{\frac{1}{4}} x \log \left(3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+3i\left(-a^{4}\right)^{\frac{1}{4}}\right)-3i\left(-a^{4}\right)^{\frac{1}{4}} x \log \left(3\right.}{2\,x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

output
```
1/2*(3*(-a^4)^(1/4)*x*log(3*a*((a*x - 1)/(a*x + 1))^(1/4) + 3*(-a^4)^(1/4)
) + 3*I*(-a^4)^(1/4)*x*log(3*a*((a*x - 1)/(a*x + 1))^(1/4) + 3*I*(-a^4)^(1
/4)) - 3*I*(-a^4)^(1/4)*x*log(3*a*((a*x - 1)/(a*x + 1))^(1/4) - 3*I*(-a^4)
^(1/4)) - 3*(-a^4)^(1/4)*x*log(3*a*((a*x - 1)/(a*x + 1))^(1/4) - 3*(-a^4)^
(1/4)) + 2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

3.74.6 Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

output
```
1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x
 + 1) + 1))*a
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

output
```
1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x
 + 1) + 1))*a
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.33

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1} - (-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,3\mathrm{i} - (-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,3\mathrm{i}$$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

output
```
(2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1) - (-1)^(1/4)*a
*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i - (-1)^(1/4)*a*atan((-1)
^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i
```

3.75 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.75.1 Optimal result

Integrand size = 14, antiderivative size = 319

$$\begin{aligned}\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx &= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}\\
&-\frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}+\frac{9a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}\\
&-\frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\\
&+\frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\end{aligned}$$

output
```
3/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^
(7/4)+9/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+9
/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/16*a^2*
ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/
2))*2^(1/2)+9/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.75.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.24

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right.$$
$$-3\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)$$
$$\left.+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

output
```
(-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCo
th[a*x])] - 3*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])] + 2*Hype
rgeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

3.75.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.82, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}dx$$
$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}x}d\frac{1}{x}$$

$$\Big\downarrow \; 90$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3}{4}a\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3}{4}a\left(\frac{3}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3}{4}a\left(-6a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \; 770$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3}{4}a\left(-6a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \; 755$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \; 1476$$

3.75. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\right.\right.\right.$$

$$\Big\downarrow\ 1082$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\right.$$

$$\Big\downarrow\ 217$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-a\sqrt[4]{1-}\right.$$

$$\Big\downarrow\ 1479$$

3.75. $\int\frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$

$$\frac{3}{4}a\Bigg(-6a\Bigg(\frac{1}{2}\Bigg(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\arctan\Bigg(\frac{\sqrt{2}}{\sqrt[4]{}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$

$$\frac{3}{4}a\Bigg(-6a\Bigg(\frac{1}{2}\Bigg(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\Bigg(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-}}}{\sqrt{2}}$$

$$\Big\downarrow \text{27}$$

3.75. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$

$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{\sqrt{}}\right.\right.\right.$$

$$\Big\downarrow \, 1103$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$

$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}\right.\right.\right.$$

input

```
Int[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

output

```
(a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/2 - (3*a*(-(a*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)) - 6*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4)
)/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4
```

3.75. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.75. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.75.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^{3}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

3.75.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.66

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{9\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+9\left(-a^8\right)^{\frac{1}{4}}\right)+9i\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+9i\left(-a^8\right)^{\frac{1}{4}}\right)-9i\left(-a^8\right)^{\frac{1}{4}}x^2\,\mathrm{lo}}{8}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

output
```
1/8*(9*(-a^8)^(1/4)*x^2*log(9*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 9*(-a^8)^(
1/4)) + 9*I*(-a^8)^(1/4)*x^2*log(9*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 9*I*(
-a^8)^(1/4)) - 9*I*(-a^8)^(1/4)*x^2*log(9*a^2*((a*x - 1)/(a*x + 1))^(1/4)
- 9*I*(-a^8)^(1/4)) - 9*(-a^8)^(1/4)*x^2*log(9*a^2*((a*x - 1)/(a*x + 1))^(
1/4) - 9*(-a^8)^(1/4)) + 2*(5*a^2*x^2 + 7*a*x + 2)*((a*x - 1)/(a*x + 1))^(
1/4))/x^2
```

3.75.6 Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.72

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$=\frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

output
```
1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*a*((a*x - 1)/(a*x + 1))^(5/4)
 + 7*a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(
a*x + 1)^2 + 1))*a
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.71

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$=\frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

output

```
1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*(a*x - 1)*a*((a*x - 1)/(a*x +
 1))^(1/4)/(a*x + 1) + 7*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x +
1) + 1)^2)*a
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{7\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{3\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}-\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,9\mathrm{i}}{4} - \frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,9\mathrm{i}}{4}$$

input

```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

output

```
((7*a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (3*a^2*((a*x - 1)/(a*x + 1))^(5/4
))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4
)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4 - ((-1)^(1/4)*a^2
*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4
```

3.76 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

3.76.1 Optimal result

Integrand size = 14, antiderivative size = 356

$$\begin{aligned}
\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx &= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} \\
&\quad + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} \\
&\quad - \frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{17a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} \\
&\quad - \frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} \\
&\quad + \frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}
\end{aligned}$$

output
```
17/24*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/4*a^3*(1-1/a/x)^(1/4)*(1+1/a/x
)^(7/4)+1/3*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)/x+17/16*a^3*arctan(-1+(1-1
/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+(1-1/a/x)^
(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-17/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1
/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+17/32*a^3*ln(
1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))
*2^(1/2)
```

3.76.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(17+30e^{2\coth^{-1}(ax)}+45e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 51\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]\right)$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

output
```
(a^3*((8*E^((3*ArcCoth[a*x])/2)*(17 + 30*E^(2*ArcCoth[a*x]) + 45*E^(4*ArcC
oth[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[
a*x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1 & ]))/96
```

3.76.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6721, 101, 27, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}x^2}d\frac{1}{x}$$

$$\Big\downarrow \ 101$$

$$\frac{1}{3}a^2\int -\frac{\left(2a+\frac{3}{x}\right)\left(1+\frac{1}{ax}\right)^{3/4}}{2a\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}$$

$$\Big\downarrow \ 27$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{1}{6}a\int \frac{\left(2a+\frac{3}{x}\right)\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \ 90$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(\frac{3}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}\right)$$

$$\Big\downarrow \ 770$$

3.76. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\int\frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{17}{4}a\Bigg(-6a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\Bigg)-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{a}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{17}{4}a\Bigg(-6a\Bigg(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\Bigg(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{17}{4}a\Bigg(-6a\Bigg(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{2-\frac{1}{x^4}}}$$

$$\Big\downarrow \text{217}$$

3.76. $\int\frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{17}{4}a\Bigg(-6a\Bigg(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\Bigg(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\Bigg)\Bigg)\Bigg)-$$

$$\Big\downarrow \text{1479}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{17}{4}a\Bigg(-6a\Bigg(\frac{1}{2}\Bigg(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\text{arctan}$$

$$\Big\downarrow \text{25}$$

3.76. $\int\frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{2}\right.\right.\right.\right.\right.$$

$$\Big\downarrow 27$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\arctan\right.\right.\right.\right.$$

$$\Big\downarrow 1103$$

3.76. $\int\frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{17}{4}a\Bigg(-6a\Bigg(\frac{1}{2}\Bigg(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\log\Bigg(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}}{2\sqrt{2}}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/(3*x) - (a*((-3*a^2*(1 - 1/(
a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/2 + (17*a*(-(a*(1 - 1/(a*x))^(1/4)*(1 + 1
/(a*x))^(3/4)) - 6*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(
-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4
))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^
(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 -
 x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4))/6
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

3.76. $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.76.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^{4}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

3.76.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{51\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(17\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+17\left(-a^{12}\right)^{\frac{1}{4}}\right)+51i\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(17\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+17i\left(-a^{12}\right)^{\frac{1}{4}}\right)-51i}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

output
```
1/48*(51*(-a^12)^(1/4)*x^3*log(17*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 17*(-a
^12)^(1/4)) + 51*I*(-a^12)^(1/4)*x^3*log(17*a^3*((a*x - 1)/(a*x + 1))^(1/4
) + 17*I*(-a^12)^(1/4)) - 51*I*(-a^12)^(1/4)*x^3*log(17*a^3*((a*x - 1)/(a*
x + 1))^(1/4) - 17*I*(-a^12)^(1/4)) - 51*(-a^12)^(1/4)*x^3*log(17*a^3*((a*
x - 1)/(a*x + 1))^(1/4) - 17*(-a^12)^(1/4)) + 2*(23*a^3*x^3 + 37*a^2*x^2 +
 22*a*x + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

3.76.6 Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

output
```
Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.78

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$= \frac{1}{96}\Bigg(102\sqrt{2}a^2\arctan\bigg(\frac{1}{2}\sqrt{2}\bigg(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\bigg)\bigg) + 102\sqrt{2}a^2\arctan\bigg(-\frac{1}{2}\sqrt{2}\bigg(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

output
```
1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a
*x + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(17*a^2*((a*x - 1)/(
a*x + 1))^(9/4) + 30*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 45*a^2*((a*x - 1)/(
a*x + 1))^(1/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x
 - 1)^3/(a*x + 1)^3 + 1))*a
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$= \frac{1}{96}\Bigg(102\sqrt{2}a^2\arctan\bigg(\frac{1}{2}\sqrt{2}\bigg(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\bigg)\bigg) + 102\sqrt{2}a^2\arctan\bigg(-\frac{1}{2}\sqrt{2}\bigg(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

output
```
1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a
*x + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(30*(a*x - 1)*a^2*((
a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 17*(a*x - 1)^2*a^2*((a*x - 1)/(a*x +
 1))^(1/4)/(a*x + 1)^2 + 45*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a
*x + 1) + 1)^3)*a
```

3.76.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.47

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{15\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{5\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1} - \frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,17{}\mathrm{i}}{8} - \frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,17{}\mathrm{i}}{8}$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

output
```
((15*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (5*a^3*((a*x - 1)/(a*x + 1))^(5/
4))/2 + (17*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1
)^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)
*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8 - ((-1)^(1/4)*a^3
*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8
```

3.77 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

3.77.1 Optimal result

Integrand size = 14, antiderivative size = 287

$$\begin{aligned}\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx={}&-\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{5533\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}}\\&+\frac{181\sqrt[4]{1+\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{21\sqrt[4]{1+\frac{1}{ax}}x^4}{40a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{5\sqrt[4]{1-\frac{1}{ax}}}\\&+\frac{1003\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{1003\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}\end{aligned}$$

output
```
-26111/1920*(1+1/a/x)^(1/4)/a^5/(1-1/a/x)^(1/4)+5533/1920*(1+1/a/x)^(1/4)*
x/a^4/(1-1/a/x)^(1/4)+1189/960*(1+1/a/x)^(1/4)*x^2/a^3/(1-1/a/x)^(1/4)+181
/240*(1+1/a/x)^(1/4)*x^3/a^2/(1-1/a/x)^(1/4)+21/40*(1+1/a/x)^(1/4)*x^4/a/(
1-1/a/x)^(1/4)+1/5*(1+1/a/x)^(1/4)*x^5/(1-1/a/x)^(1/4)+1003/128*arctan((1+
1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/
x)^(1/4))/a^5
```

3.77.2 Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.69

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{-8e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{2\coth^{-1}(ax)}\right)^5} + \frac{122e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{233e^{\frac{1}{2}\coth^{-1}(ax)}}{6\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{1661e^{\frac{1}{2}\coth^{-1}(ax)}}{48\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{4117e}{192\Big(-1-}}{a^5}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

output
```
(-8*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x
]))^5) + (122*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]))^4) + (233*E
^(ArcCoth[a*x]/2))/(6*(-1 + E^(2*ArcCoth[a*x]))^3) + (1661*E^(ArcCoth[a*x]
/2))/(48*(-1 + E^(2*ArcCoth[a*x]))^2) + (4117*E^(ArcCoth[a*x]/2))/(192*(-1
 + E^(2*ArcCoth[a*x]))) + (1003*ArcTan[E^(ArcCoth[a*x]/2)])/128 - (1003*Lo
g[1 - E^(ArcCoth[a*x]/2)])/256 + (1003*Log[1 + E^(ArcCoth[a*x]/2)])/256)/a
^5
```

3.77.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {6721, 109, 27, 168, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^6}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{5}\int -\frac{\left(21a+\frac{20}{x}\right)x^5}{2a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\int\frac{\left(21a+\frac{20}{x}\right)x^5}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{10a^2}$$

$$\Big\downarrow 168$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{-\frac{1}{4}\int-\frac{\left(181a+\frac{168}{x}\right)x^4}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow 27$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\int\frac{\left(181a+\frac{168}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow 168$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{-\frac{1}{3}\int-\frac{\left(1189a+\frac{1086}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow 27$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\int\frac{\left(1189a+\frac{1086}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow 168$$

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$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{-\frac{1}{2}\int -\frac{\left(5533a+\frac{4756}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \, 27$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\int \frac{\left(5533a+\frac{4756}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \, 168$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{-\int -\frac{\left(15045a+\frac{11066}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \, 27$$

3.77. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{\int\frac{\left(15045a+\frac{11066}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \; 172$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-2a\int-\frac{15045x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \; 27$$

3.77. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{15045a\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \; 104$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{60180a\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \; 756$$

3.77. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{60180a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}}{10a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{60180a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}}{10a^2}$$

$$\Big\downarrow \text{219}$$

3.77. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{60180a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{}}{3\sqrt[4]{1-}}}{8a}}{10a^2}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

output
```
((1 + 1/(a*x))^(1/4)*x^5)/(5*(1 - 1/(a*x))^(1/4)) - ((-21*a*(1 + 1/(a*x))^
(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) + ((-181*a*(1 + 1/(a*x))^(1/4)*x^3)/(3*
(1 - 1/(a*x))^(1/4)) + ((-1189*a*(1 + 1/(a*x))^(1/4)*x^2)/(2*(1 - 1/(a*x))
^(1/4)) + ((-5533*a*(1 + 1/(a*x))^(1/4)*x)/(1 - 1/(a*x))^(1/4) + ((52222*a
*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + 60180*a*(-1/2*ArcTan[(1 + 1/(a
*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x)
)^(1/4)]/2))/(2*a))/(4*a))/(6*a))/(8*a))/(10*a^2)
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.77. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.77.4 Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.53

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{30090\,(ax-1)\arctan\Big(\big(\frac{ax-1}{ax+1}\big)^{\frac{1}{4}}\Big)-15045\,(ax-1)\log\Big(\big(\frac{ax-1}{ax+1}\big)^{\frac{1}{4}}+1\Big)+15045\,(ax-1)\log\Big(\big(\frac{ax-1}{ax+1}\big)^{\frac{1}{4}}-}{3840\,(a^6x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="fricas")
```

output
```
-1/3840*(30090*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 15045*(a*x
- 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*(a*x - 1)*log(((a*x - 1)
/(a*x + 1))^(1/4) - 1) - 2*(384*a^6*x^6 + 1392*a^5*x^5 + 2456*a^4*x^4 + 38
26*a^3*x^3 + 7911*a^2*x^2 - 20578*a*x - 26111)*((a*x - 1)/(a*x + 1))^(3/4)
)/(a^6*x - a^5)
```

3.77.6 Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx=\int\frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**4,x)
```

output

```
Integral(x**4/((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.96

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx=$$

$$-\frac{1}{3840}\,a\left(\frac{4\left(\frac{58985\,(ax-1)}{ax+1}-\frac{125920\,(ax-1)^2}{(ax+1)^2}+\frac{137930\,(ax-1)^3}{(ax+1)^3}-\frac{72216\,(ax-1)^4}{(ax+1)^4}+\frac{15045\,(ax-1)^5}{(ax+1)^5}-7680\right)}{a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{21}{4}}-5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}+10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\right.$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="maxima")
```

output

```
-1/3840*a*(4*(58985*(a*x - 1)/(a*x + 1) - 125920*(a*x - 1)^2/(a*x + 1)^2 +
 137930*(a*x - 1)^3/(a*x + 1)^3 - 72216*(a*x - 1)^4/(a*x + 1)^4 + 15045*(a
*x - 1)^5/(a*x + 1)^5 - 7680)/(a^6*((a*x - 1)/(a*x + 1))^(21/4) - 5*a^6*((
a*x - 1)/(a*x + 1))^(17/4) + 10*a^6*((a*x - 1)/(a*x + 1))^(13/4) - 10*a^6*
((a*x - 1)/(a*x + 1))^(9/4) + 5*a^6*((a*x - 1)/(a*x + 1))^(5/4) - a^6*((a*
x - 1)/(a*x + 1))^(1/4)) + 30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 -
 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1)/a^6)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}\,a\left(\frac{30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{15045\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{3072}{a^6\left(\frac{ax-}{ax+}\right.}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="giac")
```

output
```
-1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045*log(((a*x
 - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4
) - 1))/a^6 + 30720/(a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(49120*(a*x - 1)
*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x
 + 1))^(3/4)/(a*x + 1)^2 + 33816*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(
a*x + 1)^3 - 7365*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^4 - 20
585*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

3.77.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \frac{1003\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$
$$-\frac{\frac{787\,{\left(a\,x-1\right)}^2}{6\,{\left(a\,x+1\right)}^2}-\frac{13793\,{\left(a\,x-1\right)}^3}{96\,{\left(a\,x+1\right)}^3}+\frac{3009\,{\left(a\,x-1\right)}^4}{40\,{\left(a\,x+1\right)}^4}-\frac{1003\,{\left(a\,x-1\right)}^5}{64\,{\left(a\,x+1\right)}^5}-\frac{11797\,\left(a\,x-1\right)}{192\,\left(a\,x+1\right)}+8}{a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-5\,a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+10\,a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}-10\,a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}+5\,a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/4}-a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{21/4}}$$

input
```
int(x^4/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output

```
(1003*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (1003*atan(((a*x - 1
)/(a*x + 1))^(1/4)))/(128*a^5) - ((787*(a*x - 1)^2)/(6*(a*x + 1)^2) - (137
93*(a*x - 1)^3)/(96*(a*x + 1)^3) + (3009*(a*x - 1)^4)/(40*(a*x + 1)^4) - (
1003*(a*x - 1)^5)/(64*(a*x + 1)^5) - (11797*(a*x - 1))/(192*(a*x + 1)) + 8
)/(a^5*((a*x - 1)/(a*x + 1))^(1/4) - 5*a^5*((a*x - 1)/(a*x + 1))^(5/4) + 1
0*a^5*((a*x - 1)/(a*x + 1))^(9/4) - 10*a^5*((a*x - 1)/(a*x + 1))^(13/4) +
5*a^5*((a*x - 1)/(a*x + 1))^(17/4) - a^5*((a*x - 1)/(a*x + 1))^(21/4))
```

3.77. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

3.78 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

3.78.1 Optimal result

Integrand size = 14, antiderivative size = 250

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx=-\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}}+\frac{475\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output
```
-2467/192*(1+1/a/x)^(1/4)/a^4/(1-1/a/x)^(1/4)+521/192*(1+1/a/x)^(1/4)*x/a^
3/(1-1/a/x)^(1/4)+113/96*(1+1/a/x)^(1/4)*x^2/a^2/(1-1/a/x)^(1/4)+17/24*(1+
1/a/x)^(1/4)*x^3/a/(1-1/a/x)^(1/4)+1/4*(1+1/a/x)^(1/4)*x^4/(1-1/a/x)^(1/4)
+475/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

3.78.2 Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{-3072e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{5248e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{7376e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{6292e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 2850\,\mathrm{arcta}}{384a^4}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

output
```
(-3072*E^(ArcCoth[a*x]/2) + (1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a
*x]))^4 + (5248*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (7376*E^
(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (6292*E^(ArcCoth[a*x]/2))/
(-1 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(ArcCoth[a*x]/2)] - 1425*Log[1 -
 E^(ArcCoth[a*x]/2)] + 1425*Log[1 + E^(ArcCoth[a*x]/2)])/(384*a^4)
```

3.78.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 109, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^5}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{4}\int -\frac{\left(17a+\frac{16}{x}\right)x^4}{2a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\int \frac{\left(17a+\frac{16}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{-\frac{1}{3}\int -\frac{\left(113a+\frac{102}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\int \frac{\left(113a+\frac{102}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{-\frac{1}{2}\int -\frac{\left(521a+\frac{452}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\int \frac{\left(521a+\frac{452}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \; 168$$

3.78. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{-\int -\frac{\left(1425a+\frac{1042}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\int \frac{\left(1425a+\frac{1042}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \ 172$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-2a\int -\frac{1425x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \ 27$$

3.78. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{1425a\int\frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow 104$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{5700a\int\frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow 756$$

3.78. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{5700a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{5700a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \text{219}$$

3.78. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{5700a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}}}{3\sqrt[4]{1-}}}{8a^2}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

output
```
((1 + 1/(a*x))^(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) - ((-17*a*(1 + 1/(a*x))^
(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) + ((-113*a*(1 + 1/(a*x))^(1/4)*x^2)/(2*
(1 - 1/(a*x))^(1/4)) + ((-521*a*(1 + 1/(a*x))^(1/4)*x)/(1 - 1/(a*x))^(1/4)
 + ((4934*a*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + 5700*a*(-1/2*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1
 - 1/(a*x))^(1/4)]/2))/(2*a))/(4*a))/(6*a))/(8*a^2)
```

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.78. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.78.4 Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx =$$

$$-\frac{2850\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{384\,(a^5x-a^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="fricas")
```

output
```
-1/384*(2850*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 1425*(a*x - 1
)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1425*(a*x - 1)*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1) - 2*(48*a^5*x^5 + 184*a^4*x^4 + 362*a^3*x^3 + 747*a^2*x
^2 - 1946*a*x - 2467)*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*x - a^4)
```

3.78.6 Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx=\int\frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**3,x)
```

output
```
Integral(x**3/((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.95

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{384}a\left(\frac{4\left(\frac{4645\,(ax-1)}{ax+1}-\frac{7483\,(ax-1)^2}{(ax+1)^2}+\frac{5415\,(ax-1)^3}{(ax+1)^3}-\frac{1425\,(ax-1)^4}{(ax+1)^4}-768\right)}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+6\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="maxima")
```

output
```
1/384*a*(4*(4645*(a*x - 1)/(a*x + 1) - 7483*(a*x - 1)^2/(a*x + 1)^2 + 5415
*(a*x - 1)^3/(a*x + 1)^3 - 1425*(a*x - 1)^4/(a*x + 1)^4 - 768)/(a^5*((a*x
- 1)/(a*x + 1))^(17/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 6*a^5*((a*x
- 1)/(a*x + 1))^(9/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(5/4) + a^5*((a*x - 1)
/(a*x + 1))^(1/4)) - 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*l
og(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(
1/4) - 1)/a^5)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx =$$

$$-\frac{1}{384}\,a\left(\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{1425\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{3072}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="giac")
```

output
```
-1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x -
1)/(a*x + 1))^(1/4) + 1)/a^5 + 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) -
1))/a^5 + 3072/(a^5*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(2875*(a*x - 1)*((a*x
 - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 2343*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^
(3/4)/(a*x + 1)^2 + 657*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^
3 - 1573*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.84

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$

$$=\frac{475\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

$$-\frac{\frac{7483\,{\left(a\,x-1\right)}^2}{96\,{\left(a\,x+1\right)}^2}-\frac{1805\,{\left(a\,x-1\right)}^3}{32\,{\left(a\,x+1\right)}^3}+\frac{475\,{\left(a\,x-1\right)}^4}{32\,{\left(a\,x+1\right)}^4}-\frac{4645\,\left(a\,x-1\right)}{96\,\left(a\,x+1\right)}+8}{a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-4\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+6\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}-4\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}+a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/4}}$$

input
```
int(x^3/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output

```
(475*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (475*atan(((a*x - 1)/(
a*x + 1))^(1/4)))/(64*a^4) - ((7483*(a*x - 1)^2)/(96*(a*x + 1)^2) - (1805*
(a*x - 1)^3)/(32*(a*x + 1)^3) + (475*(a*x - 1)^4)/(32*(a*x + 1)^4) - (4645
*(a*x - 1))/(96*(a*x + 1)) + 8)/(a^4*((a*x - 1)/(a*x + 1))^(1/4) - 4*a^4*(
(a*x - 1)/(a*x + 1))^(5/4) + 6*a^4*((a*x - 1)/(a*x + 1))^(9/4) - 4*a^4*((a
*x - 1)/(a*x + 1))^(13/4) + a^4*((a*x - 1)/(a*x + 1))^(17/4))
```

3.78. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

3.79 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

3.79.1 Optimal result

Integrand size = 14, antiderivative size = 213

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \frac{55\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output

```
-287/24*(1+1/a/x)^(1/4)/a^3/(1-1/a/x)^(1/4)+61/24*(1+1/a/x)^(1/4)*x/a^2/(1
-1/a/x)^(1/4)+13/12*(1+1/a/x)^(1/4)*x^2/a/(1-1/a/x)^(1/4)+1/3*(1+1/a/x)^(1
/4)*x^3/(1-1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3+5
5/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

3.79.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.98 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.07

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{8e^{\frac{9}{2}\coth^{-1}(ax)}\left(-\frac{27653}{195}-\frac{899079}{512}e^{-8\coth^{-1}(ax)}-\frac{3309759e^{-6\coth^{-1}(ax)}}{2560}+\frac{8521937e^{-4\coth^{-1}(ax)}}{7680}+\frac{69571361e^{-2\coth^{-1}(ax)}}{99840}\right.}{}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

output
```
(-8*E^((9*ArcCoth[a*x])/2)*(-27653/195 - 899079/(512*E^(8*ArcCoth[a*x])) -
 3309759/(2560*E^(6*ArcCoth[a*x])) + 8521937/(7680*E^(4*ArcCoth[a*x])) + 6
9571361/(99840*E^(2*ArcCoth[a*x])) - (653*E^(2*ArcCoth[a*x]))/390 + (13340
7*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/512 + (899079*Hyperg
eometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(512*E^(8*ArcCoth[a*x])) + (
60267*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(64*E^(6*ArcCoth
[a*x])) - (382227*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(256
*E^(4*ArcCoth[a*x])) - (40827*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[
a*x])])/(64*E^(2*ArcCoth[a*x])) + (E^(2*ArcCoth[a*x])*(1117 + 1906*E^(2*Ar
cCoth[a*x]) + 821*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 13/4}, {
1, 1, 25/4}, E^(2*ArcCoth[a*x])])/3094 + (4*E^(2*ArcCoth[a*x])*(27 + 50*E^
(2*ArcCoth[a*x]) + 23*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 1
3/4}, {1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547 + (8*E^(2*ArcCoth[a*x])*H
ypergeometricPFQ[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[a
*x])])/1547 + (16*E^(4*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 13/
4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547 + (8*E^(6*ArcCoth[a*x])*
HypergeometricPFQ[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[
a*x])])/1547))/(9*a^3)
```

3.79.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 109, 27, 168, 27, 168, 27, 172, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4} x^4}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\int -\frac{\left(13a+\frac{12}{x}\right)x^3}{2a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\int \frac{\left(13a+\frac{12}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{-\frac{1}{2}\int -\frac{\left(61a+\frac{52}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\int \frac{\left(61a+\frac{52}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \text{168}$$

3.79. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{-\int -\frac{\left(165a+\frac{122}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\int \frac{\left(165a+\frac{122}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \text{172}$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-2a\int -\frac{165x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{165a\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \text{104}$$

3.79. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{660a\int\frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \; 756$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{660a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \; 216$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{660a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \; 219$$

3.79. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{660a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

output
```
((1 + 1/(a*x))^(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) - ((-13*a*(1 + 1/(a*x))^
(1/4)*x^2)/(2*(1 - 1/(a*x))^(1/4)) + ((-61*a*(1 + 1/(a*x))^(1/4)*x)/(1 - 1
/(a*x))^(1/4) + ((574*a*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + 660*a*(
-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(2*a))/(4*a))/(6*a^2)
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.79. $\int e^{\frac{5}{2}\coth^{-1}(ax)} x^2 \, dx$

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.79.4 Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{330\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-}{48\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="fricas")
```

output
```
-1/48*(330*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*(a*x - 1)*l
og(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 165*(a*x - 1)*log(((a*x - 1)/(a*x +
1))^(1/4) - 1) - 2*(8*a^4*x^4 + 34*a^3*x^3 + 87*a^2*x^2 - 226*a*x - 287)*(
(a*x - 1)/(a*x + 1))^(3/4))/(a^4*x - a^3)
```

3.79.6 Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=\int\frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.95

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=$$
$$-\frac{1}{48}\,a\left(\frac{4\left(\frac{425\,(ax-1)}{ax+1}-\frac{462\,(ax-1)^2}{(ax+1)^2}+\frac{165\,(ax-1)^3}{(ax+1)^3}-96\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\log\left(\left(\right.\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="maxima")
```

output
```
-1/48*a*(4*(425*(a*x - 1)/(a*x + 1) - 462*(a*x - 1)^2/(a*x + 1)^2 + 165*(a
*x - 1)^3/(a*x + 1)^3 - 96)/(a^4*((a*x - 1)/(a*x + 1))^(13/4) - 3*a^4*((a*
x - 1)/(a*x + 1))^(9/4) + 3*a^4*((a*x - 1)/(a*x + 1))^(5/4) - a^4*((a*x -
1)/(a*x + 1))^(1/4)) + 330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*l
og(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1)/a^4)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=$$
$$-\frac{1}{48}\,a\left(\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{165\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{384}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{4}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="giac")
```

output
```
-1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^4 + 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^4 + 384/(a^4*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(174*(a*x - 1)*((a*x - 1)/
(a*x + 1))^(3/4)/(a*x + 1) - 69*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a
*x + 1)^2 - 137*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a*x - 1)/(a*x + 1) - 1
)^3))
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{55\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3} - \frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3} - \frac{\frac{77\,(a\,x-1)^2}{2\,(a\,x+1)^2} - \frac{55\,(a\,x-1)^3}{4\,(a\,x+1)^3} - \frac{425\,(a\,x-1)}{12\,(a\,x+1)} + 8}{a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - 3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4} + 3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4} - a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}$$

input
```
int(x^2/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(55*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (55*atan(((a*x - 1)/(a*x
 + 1))^(1/4)))/(8*a^3) - ((77*(a*x - 1)^2)/(2*(a*x + 1)^2) - (55*(a*x - 1)
^3)/(4*(a*x + 1)^3) - (425*(a*x - 1))/(12*(a*x + 1)) + 8)/(a^3*((a*x - 1)/
(a*x + 1))^(1/4) - 3*a^3*((a*x - 1)/(a*x + 1))^(5/4) + 3*a^3*((a*x - 1)/(a
*x + 1))^(9/4) - a^3*((a*x - 1)/(a*x + 1))^(13/4))
```

3.80 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$

3.80.1 Optimal result

Integrand size = 12, antiderivative size = 176

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx=-\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output
```
-25/2*(1+1/a/x)^(1/4)/a^2/(1-1/a/x)^(1/4)+5/4*(1+1/a/x)^(5/4)*x/a/(1-1/a/x
)^(1/4)+1/2*(1+1/a/x)^(9/4)*x^2/(1-1/a/x)^(1/4)+25/4*arctan((1+1/a/x)^(1/4
)/(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

3.80.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx=\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(25-45e^{2\coth^{-1}(ax)}+16e^{4\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+25\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+25\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x,x]
```

output
```
((-2*E^(ArcCoth[a*x]/2)*(25 - 45*E^(2*ArcCoth[a*x]) + 16*E^(4*ArcCoth[a*x]
)))/(-1 + E^(2*ArcCoth[a*x]))^2 + 25*ArcTan[E^(ArcCoth[a*x]/2)] + 25*ArcTa
nh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

3.80.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6721, 107, 105, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{5}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \textcolor{blue}{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^3}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \textcolor{blue}{107}$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^2}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}$$

$$\Big\downarrow \textcolor{blue}{105}$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\int \frac{\sqrt[4]{1+\frac{1}{ax}}x}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$$\Big\downarrow \textcolor{blue}{105}$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\left(\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$$\Big\downarrow \; 104$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\left(4\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$$\Big\downarrow \; 756$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\left(4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$$\Big\downarrow \; 216$$

3.80. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\left(4\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$$\Big\downarrow 219$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\left(4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

input

```
Int[E^((5*ArcCoth[a*x])/2)*x,x]
```

output

```
((1 + 1/(a*x))^(9/4)*x^2)/(2*(1 - 1/(a*x))^(1/4)) - (5*(-(((1 + 1/(a*x))^(
5/4)*x)/(1 - 1/(a*x))^(1/4)) + (5*((4*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(
1/4) + 4*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(
1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2)))/(2*a)))/(4*a)
```

3.80.3.1 Defintions of rubi rules used

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.80. $\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$

3.80.4 Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.73

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{50\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\,(2}{8\,(a^3x-a^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="fricas")
```

output
```
-1/8*(50*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 25*(a*x - 1)*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1) + 25*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^
(1/4) - 1) - 2*(2*a^3*x^3 + 11*a^2*x^2 - 34*a*x - 43)*((a*x - 1)/(a*x + 1)
)^(3/4))/(a^3*x - a^2)
```

3.80.6 Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{8}a\left(\frac{4\left(\frac{45\,(ax-1)}{ax+1}-\frac{25\,(ax-1)^2}{(ax+1)^2}-16\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-2\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-25\log\bigl(\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="maxima")
```

output
```
1/8*a*(4*(45*(a*x - 1)/(a*x + 1) - 25*(a*x - 1)^2/(a*x + 1)^2 - 16)/(a^3*(
(a*x - 1)/(a*x + 1))^(9/4) - 2*a^3*((a*x - 1)/(a*x + 1))^(5/4) + a^3*((a*x
 - 1)/(a*x + 1))^(1/4)) - 50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1)/a^3)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx=$$
$$-\frac{1}{8}a\left(\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{64}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+4\left(\frac{9\,(a}{}\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="giac")
```

output
```
-1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^3 + 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
+ 64/(a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(9*(a*x - 1)*((a*x - 1)/(a*x +
1))^(3/4)/(a*x + 1) - 13*((a*x - 1)/(a*x + 1))^(3/4))/(a^3*((a*x - 1)/(a*x
 + 1) - 1)^2))
```

3.80.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{25\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{25\,(a\,x-1)^2}{2\,(a\,x+1)^2}-\frac{45\,(a\,x-1)}{2\,(a\,x+1)}+8}{a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-2\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}$$

input

```
int(x/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output

```
(25*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (25*atan(((a*x - 1)/(a*x
 + 1))^(1/4)))/(4*a^2) - ((25*(a*x - 1)^2)/(2*(a*x + 1)^2) - (45*(a*x - 1)
)/(2*(a*x + 1)) + 8)/(a^2*((a*x - 1)/(a*x + 1))^(1/4) - 2*a^2*((a*x - 1)/(
a*x + 1))^(5/4) + a^2*((a*x - 1)/(a*x + 1))^(9/4))
```

3.81 $\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$

3.81.1 Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
-10*(1+1/a/x)^(1/4)/a/(1-1/a/x)^(1/4)+(1+1/a/x)^(5/4)*x/(1-1/a/x)^(1/4)+5*
arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a+5*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))/a
```

3.81.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-5+4e^{2\coth^{-1}(ax)}\right)}{-1+e^{2\coth^{-1}(ax)}} + 5\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 5\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2),x]
```

output
```
((-2*E^(ArcCoth[a*x]/2)*(-5 + 4*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*
x])) + 5*ArcTan[E^(ArcCoth[a*x]/2)] + 5*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

3.81.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6720, 105, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^2}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\int \frac{\sqrt[4]{1+\frac{1}{ax}}x}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{105}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

$$\Big\downarrow \text{104}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(4\int\frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

$$\Big\downarrow \ 756$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(4\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

$$\Big\downarrow \ 216$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(4\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

$$\Big\downarrow \ 219$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

input

```
Int[E^((5*ArcCoth[a*x])/2),x]
```

output

```
((1 + 1/(a*x))^(5/4)*x)/(1 - 1/(a*x))^(1/4) - (5*((4*(1 + 1/(a*x))^(1/4))/
(1 - 1/(a*x))^(1/4) + 4*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2)))/(2*a)
```

3.81. $\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$

3.81.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.81.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{10\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\,(a^2x}{2\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
-1/2*(10*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 5*(a*x - 1)*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1) - 2*(a^2*x^2 - 8*a*x - 9)*((a*x - 1)/(a*x + 1))^(3/4))/(a^2*x - a
)
```

3.81.6 Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-5/4), x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{5\,(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)/(a*x + 1))^(5/4) - a
^2*((a*x - 1)/(a*x + 1))^(1/4)) + 10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a
^2 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^2 + 5*log(((a*x - 1)/(a*x +
1))^(1/4) - 1)/a^2)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{5\,(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax}{ax}\right.\right.}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
-1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 5*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^2 + 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 +
4*(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/
(a*x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))))
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx=\frac{\frac{10\,(a\,x-1)}{a\,x+1}-8}{a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}-\frac{5\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{a}+\frac{5\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
((10*(a*x - 1))/(a*x + 1) - 8)/(a*((a*x - 1)/(a*x + 1))^(1/4) - a*((a*x -
1)/(a*x + 1))^(5/4)) - (5*atan(((a*x - 1)/(a*x + 1))^(1/4)))/a + (5*atanh(
((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.82 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

3.82.1 Optimal result

Integrand size = 14, antiderivative size = 320

$$\begin{aligned}\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx =& -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\ &-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\\ &+2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\ &+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\end{aligned}$$

output
```
-8*(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4
))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(
1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-
1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1
/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1
-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

3.82.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.09

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = 8e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{4\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)/x,x]
```

output
```
8*E^(ArcCoth[a*x]/2)*(-1 + Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x
])])
```

3.82.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.90, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.357$, Rules used = {6721, 109, 27, 35, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4} x}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$4a\int -\frac{\left(a-\frac{1}{x}\right)x}{4a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{a}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 35$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 140$$

$$\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 73$$

$$-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 104$$

$$-4\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 756$$

3.82. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}dx$

$$-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 216}$$

$$-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 219}$$

$$-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 854}$$

$$-4\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 826}$$

3.82. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 1476$$

$$-4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1}}{\sqrt[4]{2}}\right.$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 1082$$

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 217$$

3.82. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 1479$$

$$-4\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 25$$

3.82. $\int\frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1103}$$

3.82. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log}{}\right.\right.$$

$$\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)/x,x]
```

output
```
(-8*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) - 4*(-1/2*ArcTan[(1 + 1/(a*x)
)^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/2) - 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/
4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)
]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) +
 x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^
(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.82. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.82. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.82.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x}\,dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

3.82.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.75

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{\sqrt{2}(-(i-1)\ ax+i-1)\log\left((i+1)\ \sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\sqrt{2}((i+1)\ ax-i-1)\log\left(-(i-1)\ \sqrt{2}+2\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*(-(I - 1)*a*x + I - 1)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*
x + 1))^(1/4)) + sqrt(2)*((I + 1)*a*x - I - 1)*log(-(I - 1)*sqrt(2) + 2*((
a*x - 1)/(a*x + 1))^(1/4)) + sqrt(2)*(-(I + 1)*a*x + I + 1)*log((I - 1)*sq
rt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)) + sqrt(2)*((I - 1)*a*x - I + 1)*log
(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(a*x - 1)*arctan(((
a*x - 1)/(a*x + 1))^(1/4)) + 2*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) +
 1) - 2*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) - 1) - 16*(a*x + 1)*((a*
x - 1)/(a*x + 1))^(3/4))/(a*x - 1)
```

3.82.6 Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx =$$
$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\right.}{a}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a -
2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1)/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.79

$$\int\frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx=$$

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

output
```
-1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/
a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

3.82.9 Mupad [B] (verification not implemented)

Time = 4.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.37

$$\int\frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx=-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\frac{8}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}$$

$$-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)2\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(-1+1\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right.\right.$$

input
```
int(1/(x*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

output
```
- atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(
1/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1
- 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1
+ 1i) - 8/((a*x - 1)/(a*x + 1))^(1/4)
```

3.83 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

3.83.1 Optimal result

Integrand size = 14, antiderivative size = 299

$$\begin{aligned}\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = {} & -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\\ & +\frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{5a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\ & -\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}\\ & +\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}\end{aligned}$$

output
```
-5*a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-4*a*(1+1/a/x)^(5/4)/(1-1/a/x)^(1/4)-5
/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arct
an(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/4*a*ln(1-(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+5/
4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)
^(1/2))*2^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.58

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(-\frac{10e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}-\frac{8e^{\frac{5}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}-\frac{5\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}+\frac{5\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

output
```
a*((-10*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - (8*E^((5*ArcCoth[a*
x])/2))/(1 + E^(2*ArcCoth[a*x])) - (5*ArcTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2
)])/Sqrt[2] + (5*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (5*Log[
1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]) + (5*Log[1 +
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

3.83.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 57, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{57}$$

$$5\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$5\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$5\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{854}$$

$$5\left(-2a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{826}$$

$$5\left(-2a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 1476$$

$$5\Bigg(-2a\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\Bigg)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 1082$$

$$5\Bigg(-2a\Bigg(\frac{1}{2}\Bigg(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\Bigg)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\Bigg)-a\Big($$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 217$$

3.83. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$5\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1479}$$

$$5\left(-2a\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{25}$$

3.83. $\int\frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$5\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$5\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{}\right.\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 1103$$

3.83. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$5\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}\right.\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

output
```
(-4*a*(1 + 1/(a*x))^(5/4))/(1 - 1/(a*x))^(1/4) + 5*(-(a*(1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(1/4)) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 -
 x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2))
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.83. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.83.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^{2}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

3.83.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.81

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx =$$

$$-\frac{5\left(-a^{4}\right)^{\frac{1}{4}}\left(ax^{2}-x\right)\log\left(125\,a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+125\left(-a^{4}\right)^{\frac{3}{4}}\right)-5\left(-a^{4}\right)^{\frac{1}{4}}\left(i\,ax^{2}-i\,x\right)\log\left(125\,a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+12\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(5*(-a^4)^(1/4)*(a*x^2 - x)*log(125*a^3*((a*x - 1)/(a*x + 1))^(1/4) +
 125*(-a^4)^(3/4)) - 5*(-a^4)^(1/4)*(I*a*x^2 - I*x)*log(125*a^3*((a*x - 1)
/(a*x + 1))^(1/4) + 125*I*(-a^4)^(3/4)) - 5*(-a^4)^(1/4)*(-I*a*x^2 + I*x)*
log(125*a^3*((a*x - 1)/(a*x + 1))^(1/4) - 125*I*(-a^4)^(3/4)) - 5*(-a^4)^(
1/4)*(a*x^2 - x)*log(125*a^3*((a*x - 1)/(a*x + 1))^(1/4) - 125*(-a^4)^(3/4
)) + 2*(9*a^2*x^2 + 8*a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x^2 - x)
```

3.83.6 Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.68

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx =$$
$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

output
```
-1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/(((a*x - 1)/
(a*x + 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.73

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx =$$

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

output
```
-1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/((a*x - 1)*(
(a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

3.83.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.36

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$=5\,(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-5\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\frac{8\,a+\frac{10\,a\,(a\,x-}{a\,x+1}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}+\left(\frac{a\,x-}{a\,x-}\right.}$$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

output
```
5*(-1)^(1/4)*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 5*(-1)^(1/4
)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (8*a + (10*a*(a*x - 1))
/(a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/4) + ((a*x - 1)/(a*x + 1))^(5/4))
```

3.84 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.84.1 Optimal result

Integrand size = 14, antiderivative size = 351

$$\begin{aligned}\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx ={}& -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} \\ & -\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} \\ & +\frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}-\frac{25a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} \\ & -\frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} \\ & +\frac{25a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\end{aligned}$$

output
```
-25/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-5/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x
)^(5/4)-2*a^2*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-25/8*a^2*arctan(-1+(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/8*a^2*arctan(1+(1-1/a/x)^(1/4)*
2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/16*a^2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1
+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+25/16*a^2*ln(1+(1-1
/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/
2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.53

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx=\frac{1}{16}a^2\left(-128e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{104e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}\right.$$
$$-50\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)+50\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)$$
$$-25\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)$$
$$\left.+25\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

output
```
(a^2*(-128*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[
a*x]))^2 - (104*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 50*Sqrt[2]*
ArcTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)] + 50*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(
ArcCoth[a*x]/2)] - 25*Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCo
th[a*x]] + 25*Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]
))/16
```

3.84.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 87, 60, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\left(1-\frac{1}{ax}\right)^{5/4}x}d\frac{1}{x}$$

$$\Big\downarrow \; 87$$

$$5a\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 60$$

$$5a\left(\frac{5}{4}\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 60$$

$$5a\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$5a\left(\frac{5}{4}\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 854$$

3.84. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$5a\left(\frac{5}{4}\left(-2a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{826}$$

$$5a\Bigg(\frac{5}{4}\Bigg(-2a\Bigg(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\Bigg)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\Bigg)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/}$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{1476}$$

$$5a\Bigg(\frac{5}{4}\Bigg(-2a\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\Bigg)-\frac{1}{2}\int \frac{1-}{1+}$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{1082}$$

3.84. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 217$$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 1479$$

3.84. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\arctan\left(\right.\right.\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 25$$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\arctan\left(\right.\right.\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

3.84. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\text{arctan}\left(\right.\right.\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1103}$$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+\right.}{2\sqrt{2}}\right.\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

input

```
Int[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

3.84. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

output
```
(-2*a^2*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1/4) + 5*a*(-1/2*(a*(1 - 1/(a*
x))^(3/4)*(1 + 1/(a*x))^(5/4)) + (5*(-(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))
^(1/4)) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1
/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4
)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)
+ x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))
^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4)
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.84. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.84.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^3}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

3.84.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.74

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\,25\left(-a^8\right)^{\frac{1}{4}}\left(ax^3-x^2\right)\log\left(15625\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+15625\left(-a^8\right)^{\frac{3}{4}}\right)-25\left(-a^8\right)^{\frac{1}{4}}\left(i\,ax^3-i\,x^2\right)\log\left(15625\,a^6\right($$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

output
```
-1/8*(25*(-a^8)^(1/4)*(a*x^3 - x^2)*log(15625*a^6*((a*x - 1)/(a*x + 1))^(1
/4) + 15625*(-a^8)^(3/4)) - 25*(-a^8)^(1/4)*(I*a*x^3 - I*x^2)*log(15625*a^
6*((a*x - 1)/(a*x + 1))^(1/4) + 15625*I*(-a^8)^(3/4)) - 25*(-a^8)^(1/4)*(-
I*a*x^3 + I*x^2)*log(15625*a^6*((a*x - 1)/(a*x + 1))^(1/4) - 15625*I*(-a^8
)^(3/4)) - 25*(-a^8)^(1/4)*(a*x^3 - x^2)*log(15625*a^6*((a*x - 1)/(a*x + 1
))^(1/4) - 15625*(-a^8)^(3/4)) + 2*(43*a^3*x^3 + 34*a^2*x^2 - 11*a*x - 2)*
((a*x - 1)/(a*x + 1))^(3/4))/(a*x^3 - x^2)
```

3.84.6 Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\Bigg(25\Bigg(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\Bigg(-\frac{1}{2}\sqrt{2}\Bigg(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\Bigg)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

output
```
-1/16*(25*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1))*a + 8*(45*(a*x - 1)*a/(a*x + 1) + 25*(a*x -
1)^2*a/(a*x + 1)^2 + 16*a)/(((a*x - 1)/(a*x + 1))^(9/4) + 2*((a*x - 1)/(a*
x + 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.69

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

output
```
-1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
1))^(1/4))) - 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 128*a/((a*x - 1)/(a*x + 1))^(1/
4) + 8*(9*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 13*a*((a*x -
 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.43

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4} - \frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4} - \frac{8\,a^2+\frac{25\,a^2\,{(a\,x-1)}^2}{2\,{(a\,x+1)}^2}+\frac{45\,a^2\,(a\,x-1)}{2\,(a\,x+1)}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

output
```
(25*(-1)^(1/4)*a^2*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (25*
(-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (8*a^2 +
(25*a^2*(a*x - 1)^2)/(2*(a*x + 1)^2) + (45*a^2*(a*x - 1))/(2*(a*x + 1)))/(
((a*x - 1)/(a*x + 1))^(1/4) + 2*((a*x - 1)/(a*x + 1))^(5/4) + ((a*x - 1)/(
a*x + 1))^(9/4))
```

3.84. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.85 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

3.85.1 Optimal result

Integrand size = 14, antiderivative size = 385

$$\begin{aligned}\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx &= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}\\&\quad-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4}+\frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}-\frac{55a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}-\end{aligned}$$

output
```
-55/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-11/4*a^3*(1-1/a/x)^(3/4)*(1+1/a/
x)^(5/4)-2*a^3*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-1/3*a^3*(1-1/a/x)^(3/4)*(1+
1/a/x)^(9/4)-55/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*
2^(1/2)-55/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2
)-55/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(
1+1/a/x)^(1/2))*2^(1/2)+55/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(
1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.85.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.27

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= a^3\left(-\frac{e^{\frac{1}{2}\coth^{-1}(ax)}\left(165+462e^{2\coth^{-1}(ax)}+425e^{4\coth^{-1}(ax)}+96e^{6\coth^{-1}(ax)}\right)}{12\left(1+e^{2\coth^{-1}(ax)}\right)^3}\right.$$
$$\left.-\frac{55}{32}\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

output
```
a^3*(-1/12*(E^(ArcCoth[a*x]/2)*(165 + 462*E^(2*ArcCoth[a*x]) + 425*E^(4*Ar
cCoth[a*x]) + 96*E^(6*ArcCoth[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 - (55*Roo
tSum[1 + #1^4 & , (ArcCoth[a*x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ]
)/32)
```

3.85.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.86, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 100, 27, 90, 60, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\left(1-\frac{1}{ax}\right)^{5/4}x^2}d\frac{1}{x}$$
$$\Big\downarrow \text{100}$$

$$2a^3\int \frac{\left(5a+\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{5/4}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$a\int \frac{\left(5a+\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 90$$

$$a\left(\frac{11}{2}a\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{3}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}\right)-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{1}{3}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}\right)-$$
$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{1}{3}a$$
$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 73$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-$$
$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

3.85. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\Big\downarrow \text{854}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{1}{3}\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{826}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1476}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1082}$$

3.85. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{217}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1479}$$

3.85. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arctan}}{}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 25$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arctan}}{}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

3.85. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\text{arc}}{}\right.\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1103}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\right.}{2\sqrt{2}}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

input

```
Int[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

3.85. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

output

```
(-2*a^3*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1/4) + a*(-1/3*(a^2*(1 - 1/(a*
x))^(3/4)*(1 + 1/(a*x))^(9/4)) + (11*a*(-1/2*(a*(1 - 1/(a*x))^(3/4)*(1 + 1
/(a*x))^(5/4)) + (5*(-(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) - 2*a*((
-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) +
ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 +
(Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqr
t[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]
/(2*Sqrt[2]))/2))/4))/2)
```

3.85.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

3.85. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

rule 100

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.85. $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.85.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^4} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

3.85.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4} dx = -\frac{165 \left(-a^{12}\right)^{\frac{1}{4}} \left(ax^4 - x^3\right) \log \left(166375 \, a^9 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 166375 \left(-a^{12}\right)^{\frac{3}{4}}\right) + 165 \left(-a^{12}\right)^{\frac{1}{4}} \left(-i \, ax^4 + i \, x^3\right) \log \Big(1}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

output
```
-1/48*(165*(-a^12)^(1/4)*(a*x^4 - x^3)*log(166375*a^9*((a*x - 1)/(a*x + 1)
)^(1/4) + 166375*(-a^12)^(3/4)) + 165*(-a^12)^(1/4)*(-I*a*x^4 + I*x^3)*log
(166375*a^9*((a*x - 1)/(a*x + 1))^(1/4) + 166375*I*(-a^12)^(3/4)) + 165*(-
a^12)^(1/4)*(I*a*x^4 - I*x^3)*log(166375*a^9*((a*x - 1)/(a*x + 1))^(1/4) -
 166375*I*(-a^12)^(3/4)) - 165*(-a^12)^(1/4)*(a*x^4 - x^3)*log(166375*a^9*
((a*x - 1)/(a*x + 1))^(1/4) - 166375*(-a^12)^(3/4)) + 2*(287*a^4*x^4 + 226
*a^3*x^3 - 87*a^2*x^2 - 34*a*x - 8)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x^4 -
x^3)
```

3.85.6 Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

output
```
Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.75

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$
$$-\frac{1}{96}\left(165\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

output
```
-1/96*(165*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1))*a^2 + 8*(425*(a*x - 1)*a^2/(a*x + 1) + 462*
(a*x - 1)^2*a^2/(a*x + 1)^2 + 165*(a*x - 1)^3*a^2/(a*x + 1)^3 + 96*a^2)/((
(a*x - 1)/(a*x + 1))^(13/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + 3*((a*x - 1)
/(a*x + 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(330\,\sqrt{2}a^2\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\,\sqrt{2}a^2\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right.\right.\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

output
```
-1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(
a*x + 1))^(1/4))) - 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 768*a^2/((a*x - 1)/
(a*x + 1))^(1/4) + 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1) + 69*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 137*a^2
*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

3.85.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{55\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{55\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{8\,a^3+\frac{77\,a^3\,{(a\,x-1)}^2}{2\,{(a\,x+1)}^2}+\frac{55\,a^3\,{(a\,x-1)}^3}{4\,{(a\,x+1)}^3}+\frac{425\,a^3\,(a\,x-1)}{12\,(a\,x+1)}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

output
```
(55*(-1)^(1/4)*a^3*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (55*
(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (8*a^3 +
(77*a^3*(a*x - 1)^2)/(2*(a*x + 1)^2) + (55*a^3*(a*x - 1)^3)/(4*(a*x + 1)^3
) + (425*a^3*(a*x - 1))/(12*(a*x + 1)))/(((a*x - 1)/(a*x + 1))^(1/4) + 3*(
(a*x - 1)/(a*x + 1))^(5/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + ((a*x - 1)/(a
*x + 1))^(13/4))
```

3.86 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

3.86.1 Optimal result

Integrand size = 14, antiderivative size = 253

$$\begin{aligned}\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx &= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4}-\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3}\\ &\quad+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2}-\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}\\ &\quad+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5+\frac{31\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}\end{aligned}$$

output
```
611/1920*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4-269/960*(1-1/a/x)^(1/4)*(1+
1/a/x)^(3/4)*x^2/a^3+11/48*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2-9/40*(1
-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^
5+31/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-31/128*arctanh((1+1/a
/x)^(1/4)/(1-1/a/x)^(1/4))/a^5
```

3.86.2 Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{\frac{24576e^{\frac{19}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}-\frac{62976e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{64640e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{34000e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{9620e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-930\arctan}{3840a^5}$$

input

```
Integrate[x^4/E^(ArcCoth[a*x]/2),x]
```

output

```
((24576*E^((19*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (62976*E^((
15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^((11*ArcCoth[a
*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (34000*E^((7*ArcCoth[a*x])/2))/(-1
+ E^(2*ArcCoth[a*x]))^2 + (9620*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth
[a*x])) - 930*ArcTan[E^(-1/2*ArcCoth[a*x])] + 465*Log[1 - E^(-1/2*ArcCoth[
a*x])] - 465*Log[1 + E^(-1/2*ArcCoth[a*x])])/(3840*a^5)
```

3.86.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^6}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{5}\int -\frac{\left(9a-\frac{8}{x}\right)x^5}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(9a-\frac{8}{x}\right)x^5}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 168$$

$$\frac{-\frac{1}{4}\int \frac{\left(55a-\frac{54}{x}\right)x^4}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{\left(55a-\frac{54}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 168$$

$$\frac{-\frac{-\frac{1}{3}\int \frac{\left(269a-\frac{220}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{-\frac{\int \frac{\left(269a-\frac{220}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 168$$

3.86. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{-\frac{-\frac{-\frac{1}{2}\int \frac{\left(611a-\frac{538}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+$$
$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 27

$$\frac{-\frac{-\frac{-\frac{\int \frac{\left(611a-\frac{538}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+$$
$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 168

$$\frac{-\frac{-\frac{-\frac{-\int \frac{465x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4}{10a^2}$$
$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 27

$$\frac{-\frac{-\frac{-\frac{-\frac{465}{2}\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax}{10a^2}$$
$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 104

3.86. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

$$-\frac{-\frac{-\frac{-\frac{-930\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{-\frac{-\frac{930\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 827$$

$$-\frac{-\frac{-\frac{-\frac{-930\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-}}{8a}}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 216$$

3.86. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{-\frac{-\frac{-\frac{-930\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-}}{8a}}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 219$$

$$\frac{-\frac{-\frac{-\frac{-930\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1}}{8a}}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[x^4/E^(ArcCoth[a*x]/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 + ((-9*a*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - ((-55*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 - ((-269*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/2 - (
-611*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - 930*(ArcTan[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x)
)^(1/4)]/2))/(4*a))/(6*a))/(8*a))/(10*a^2)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.86. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.86.4 Maple [F]

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

input
```
int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{2\left(384\,a^5x^5-48\,a^4x^4+8\,a^3x^3-98\,a^2x^2+73\,ax+611\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-465\log\left(\left(\right.\right.}{3840\,a^5}$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/3840*(2*(384*a^5*x^5 - 48*a^4*x^4 + 8*a^3*x^3 - 98*a^2*x^2 + 73*a*x + 61
1)*((a*x - 1)/(a*x + 1))^(1/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4)) -
 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 465*log(((a*x - 1)/(a*x + 1))^
(1/4) - 1))/a^5
```

3.86.6 Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx=\int x^4\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**4*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x**4*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx=$$

$$-\frac{1}{3840}a\left(\frac{4\left(2405\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+465\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\ \mathrm{arcta}}{}\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/3840*a*(4*(2405*((a*x - 1)/(a*x + 1))^(17/4) - 1120*((a*x - 1)/(a*x + 1
))^(13/4) + 5090*((a*x - 1)/(a*x + 1))^(9/4) - 696*((a*x - 1)/(a*x + 1))^(
5/4) + 465*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a
*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4
*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x -
 1)/(a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 -
 465*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx=$$

$$-\frac{1}{3840}a\left(\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{465\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{696(ax-1)\left(\right.}{ax+1}\right.}{}\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
-1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 - 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 - 4*(696*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 5090*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1120*(a*x - 1)^3*((a*x -
 1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 2405*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1)^4 - 465*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x
+ 1) - 1)^5))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{31\left(\frac{ax-1}{ax+1}\right)^{1/4}}{64}-\frac{29\left(\frac{ax-1}{ax+1}\right)^{5/4}}{40}+\frac{509\left(\frac{ax-1}{ax+1}\right)^{9/4}}{96}-\frac{7\left(\frac{ax-1}{ax+1}\right)^{13/4}}{6}+\frac{481\left(\frac{ax-1}{ax+1}\right)^{17/4}}{192}}{a^5+\frac{10a^5(ax-1)^2}{(ax+1)^2}-\frac{10a^5(ax-1)^3}{(ax+1)^3}+\frac{5a^5(ax-1)^4}{(ax+1)^4}-\frac{a^5(ax-1)^5}{(ax+1)^5}-\frac{5a^5(ax-1)}{ax+1}} - \frac{31\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128\,a^5} - \frac{31\,\mathrm{atanh}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128\,a^5}$$

input
```
int(x^4*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((31*((a*x - 1)/(a*x + 1))^(1/4))/64 - (29*((a*x - 1)/(a*x + 1))^(5/4))/40
 + (509*((a*x - 1)/(a*x + 1))^(9/4))/96 - (7*((a*x - 1)/(a*x + 1))^(13/4))
/6 + (481*((a*x - 1)/(a*x + 1))^(17/4))/192)/(a^5 + (10*a^5*(a*x - 1)^2)/(
a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x +
 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (31
*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (31*atanh(((a*x - 1)/(a*x
+ 1))^(1/4)))/(128*a^5)
```

3.87 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

3.87.1 Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}$$
$$+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{11\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\mathrm{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output

```
-83/192*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+29/96*(1-1/a/x)^(1/4)*(1+1/a
/x)^(3/4)*x^2/a^2-7/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)
^(1/4)*(1+1/a/x)^(3/4)*x^4-11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
^4+11/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

3.87.2 Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{3200e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{2512e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{980e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+66\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-33}{384a^4}$$

input
```
Integrate[x^3/E^(ArcCoth[a*x]/2),x]
```

output
```
((1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (3200*E^((11
*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^((7*ArcCoth[a*x])
/2))/(-1 + E^(2*ArcCoth[a*x]))^2 - (980*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2
*ArcCoth[a*x])) + 66*ArcTan[E^(-1/2*ArcCoth[a*x])] - 33*Log[1 - E^(-1/2*Ar
cCoth[a*x])] + 33*Log[1 + E^(-1/2*ArcCoth[a*x])])/(384*a^4)
```

3.87.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{4}\int -\frac{\left(7a-\frac{6}{x}\right)x^4}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(7a-\frac{6}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 168$$

$$\frac{-\frac{1}{3}\int \frac{\left(29a-\frac{28}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 27$$

$$\frac{-\frac{\int \frac{\left(29a-\frac{28}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 168$$

$$\frac{-\frac{-\frac{1}{2}\int \frac{\left(83a-\frac{58}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 27$$

$$\frac{-\frac{-\frac{\int \frac{\left(83a-\frac{58}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 168$$

3.87. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{-\frac{-\frac{-\int \frac{33x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{-\frac{-\frac{33}{2}\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 104$$

$$\frac{-\frac{-\frac{-66\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 25$$

$$\frac{-\frac{-\frac{66\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 827$$

3.87. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

$$-\frac{-\frac{-\frac{-66\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}}{8a^2}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 216$$

$$-\frac{-\frac{-\frac{-66\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}}{8a^2}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow 219$$

$$-\frac{-\frac{-\frac{-66\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-}}{8a^2}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[x^3/E^(ArcCoth[a*x]/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 + ((-7*a*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4)*x^3)/3 - ((-29*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^2)/2 - (-83*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - 66*(ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4
)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a))/(8*a^2)
```

3.87. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.87.4 Maple [F]

$$\int x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\,(48\,a^4x^4 - 8\,a^3x^3 + 2\,a^2x^2 - 25\,ax - 83)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - 33}{384\,a^4}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/384*(2*(48*a^4*x^4 - 8*a^3*x^3 + 2*a^2*x^2 - 25*a*x - 83)*((a*x - 1)/(a*
x + 1))^(1/4) + 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

3.87.6 Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx=\int x^3\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x**3*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx=$$

$$-\frac{1}{384}a\left(\frac{4\left(245\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-107\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+279\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-33\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\log}{}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/384*a*(4*(245*((a*x - 1)/(a*x + 1))^(13/4) - 107*((a*x - 1)/(a*x + 1))^
(9/4) + 279*((a*x - 1)/(a*x + 1))^(5/4) - 33*((a*x - 1)/(a*x + 1))^(1/4))/
(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3
*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 3
3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$

$$=\frac{1}{384}a\left(\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{33\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{279(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\right.}{}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^5 - 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5
 + 4*(279*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 107*(a*x - 1)^
2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 245*(a*x - 1)^3*((a*x - 1)/(a*
x + 1))^(1/4)/(a*x + 1)^3 - 33*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1
)/(a*x + 1) - 1)^4))
```

3.87.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{11\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{\frac{11\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{93\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{107\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{245\,\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}} + \frac{11\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
(11*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - ((11*((a*x - 1)/(a*x + 1
))^(1/4))/32 - (93*((a*x - 1)/(a*x + 1))^(5/4))/32 + (107*((a*x - 1)/(a*x
+ 1))^(9/4))/96 - (245*((a*x - 1)/(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*
x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4
)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (11*atanh(((a*x - 1)/(a*x +
 1))^(1/4)))/(64*a^4)
```

3.88 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

3.88.1 Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2}-\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a}+\frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3+\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
11/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2-5/12*(1-1/a/x)^(1/4)*(1+1/a/x)
^(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3+3/8*arctan((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^3-3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^
3
```

3.88.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.31 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.17

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\Big(-22034705-26688365e^{2\coth^{-1}(ax)}-3731255e^{4\coth^{-1}(ax)}+3122405e^{6\coth^{-1}(ax)}+2203470}{}$$

input
```
Integrate[x^2/E^(ArcCoth[a*x]/2),x]
```

output
```
-1/221760*(-22034705 - 26688365*E^(2*ArcCoth[a*x]) - 3731255*E^(4*ArcCoth[
a*x]) + 3122405*E^(6*ArcCoth[a*x]) + 22034705*Hypergeometric2F1[3/4, 1, 7/
4, E^(2*ArcCoth[a*x])] + 17244920*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4
, 1, 7/4, E^(2*ArcCoth[a*x])] - 9077530*E^(4*ArcCoth[a*x])*Hypergeometric2
F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] - 7043960*E^(6*ArcCoth[a*x])*Hypergeom
etric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 446985*E^(8*ArcCoth[a*x])*Hype
rgeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 256*E^(6*ArcCoth[a*x])*(6
85 + 1090*E^(2*ArcCoth[a*x]) + 437*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{
7/4, 2, 2, 2}, {1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 2048*E^(6*ArcCoth[a*x])
*(21 + 38*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7
/4, 2, 2, 2, 2}, {1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(6*ArcCoth[
a*x])*HypergeometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*Arc
Coth[a*x])] + 8192*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{7/4, 2, 2, 2, 2,
2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(10*ArcCoth[a*x])*Hyp
ergeometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x]
)])/(a^3*E^((5*ArcCoth[a*x])/2))
```

3.88.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{3}\int -\frac{\left(5a-\frac{4}{x}\right)x^3}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(5a-\frac{4}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{1}{2}\int \frac{\left(11a-\frac{10}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(11a-\frac{10}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{-\int \frac{9x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{27}$$

3.88. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{-\frac{-\frac{9}{2}\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{104}$$

$$\frac{-\frac{-18\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{18\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{827}$$

$$\frac{-\frac{-18\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{216}$$

3.88. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{-\frac{-18\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{-18\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[x^2/E^(ArcCoth[a*x]/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^3)/3 + ((-5*a*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4)*x^2)/2 - (-11*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(
3/4)*x - 18*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(
1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a^2)
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.88. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.88.4 Maple [F]

$$\int x^{2} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.57

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{2\left(8\,a^{3}x^{3}-2\,a^{2}x^{2}+ax+11\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\right.}{48\,a^{3}}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/48*(2*(8*a^3*x^3 - 2*a^2*x^2 + a*x + 11)*((a*x - 1)/(a*x + 1))^(1/4) - 1
8*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 9*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

3.88.6 Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \int x^{2}\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x**2*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}\,a\Bigg(\frac{4\left(29\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-9\ \mathrm{lo}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/48*a*(4*(29*((a*x - 1)/(a*x + 1))^(9/4) - 6*((a*x - 1)/(a*x + 1))^(5/4)
 + 9*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)
^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 -
9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}\,a\Bigg(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\Bigg(\frac{6\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-}{a^4\Bigg(}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
-1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 -
 4*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 29*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.88.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=\frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((3*((a*x - 1)/(a*x + 1))^(1/4))/4 - ((a*x - 1)/(a*x + 1))^(5/4)/2 + (29*(
(a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (
a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a*
x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))
/(8*a^3)
```

3.89 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx$

3.89.1 Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx=-\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2-\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output

```
-1/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/
4)*x^2-1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

3.89.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx=\frac{-\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(-5+e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[x/E^(ArcCoth[a*x]/2),x]
```

output
```
((-2*E^((3*ArcCoth[a*x])/2)*(-5 + E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[
a*x]))^2 - ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^
2)
```

3.89.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6721, 107, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{1}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \ 6721$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \ 107$$

$$\frac{\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^2}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 105$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 104$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 25$$

$$\frac{\frac{2\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 827$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}}{4a}+$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 216$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}}{4a}+$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow\ 219$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}}{4a}+$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

3.89. $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx$

input
```
Int[x/E^(ArcCoth[a*x]/2),x]
```

output
```
((1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)*x^2)/2 + (-((1 - 1/(a*x))^(1/4)*(
1 + 1/(a*x))^(3/4)*x) - (2*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/a)/(4*a)
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.89.4 Maple [F]

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

input
```
int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2-ax-3)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 - a*x - 3)*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x
- 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x
- 1)/(a*x + 1))^(1/4) - 1))/a^2
```

3.89.6 Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx=\int x\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx=$$

$$-\frac{1}{8}\,a\left(\frac{4\left(5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/8*a*(4*(5*((a*x - 1)/(a*x + 1))^(5/4) - ((a*x - 1)/(a*x + 1))^(1/4))/(2
*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan((
(a*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3
 + log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx$$

$$=\frac{1}{8}\,a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\left(\frac{5\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right.$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^3 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*(5*(
a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - ((a*x - 1)/(a*x + 1))^(1/
4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx=\frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}+\frac{\operatorname{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
atan(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2) - (((a*x - 1)/(a*x + 1))^(1/4)/2
 - (5*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2
 - (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^
2)
```

3.90 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$

3.90.1 Optimal result

Integrand size = 10, antiderivative size = 97

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$=\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/
a-arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

3.90.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx=-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},e^{2\coth^{-1}(ax)}\right)}{3a}$$

input
```
Integrate[E^(-1/2*ArcCoth[a*x]),x]
```

output
```
(-8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, E^(2*ArcCoth[a*x
])])/(3*a)
```

3.90.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6720, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^2}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{104}$$

$$\frac{2\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{25}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[E^(-1/2*ArcCoth[a*x]),x]
```

output
```
(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + (2*(ArcTan[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/2))/a
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.90.4 Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4),x)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) - 2*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + log(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a
```

3.90.6 Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = \int \sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2} + \frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2} + \frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2} - \frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
-1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a
*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.90.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}-\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
 - 1)/(a*x + 1))^(1/4))/a - atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

3.91 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

3.91.1 Optimal result

Integrand size = 14, antiderivative size = 291

$$\begin{aligned}\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx &= \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\ &\quad-2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\\ &\quad+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\ &\quad-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\end{aligned}$$

output
```
-2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/
a/x)^(1/4))+1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/
4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1
/2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1
/4))*2^(1/2)
```

3.91.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \frac{8}{3}e^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{8},1,\frac{11}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^(ArcCoth[a*x]/2)*x),x]
```

output
```
(8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/8, 1, 11/8, E^(4*ArcCoth[a*x
])])/3
```

3.91.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.89, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {6721, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$\frac{\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow 73$$

$$-4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow 104$$

$$-4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow 25$$

$$4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}$$

$$\Big\downarrow 770$$

$$4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}$$

$$\Big\downarrow 755$$

$$4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)$$

$$\Big\downarrow 827$$

$$-4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)$$

3.91. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

$$\Big\downarrow \text{216}$$

$$-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-4\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-4\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1476}$$

$$-4\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1082}$$

3.91. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \ 217$$

$$-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \ 1479$$

3.91. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow 25$$

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow 27$$

3.91. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \; 1103$$

$$-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log\left(\right.}{}\right.\right.$$

input
```
Int[1/(E^(ArcCoth[a*x]/2)*x),x]
```

3.91. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

output
```
-4*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2) - 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^
(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.91. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.91.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x}dx$$

input

```
int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

output

```
int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

3.91.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = & -\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\ & +\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\ & -\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\ & +\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\ & +2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\ & +\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)\end{aligned}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

output
```
-(1/2*I + 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
) + (1/2*I - 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4)) - (1/2*I - 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4)) + (1/2*I + 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x
+ 1))^(1/4)) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

3.91.6 Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4)/x, x)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\right.}{a}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a -
2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1)/a)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\right.}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

output
```
-1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/
a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(-1-\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(-1+1\mathrm{i})$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4)/x,x)
```

output
```
2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)
*2i - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 +
1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 -
1i)
```

3.92 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

3.92.1 Optimal result

Integrand size = 14, antiderivative size = 268

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

output
```
-a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)
/(1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)
^(1/4))*2^(1/2)-1/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/
x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/
a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.12

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx=-\frac{8}{3}ae^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

output
```
(-8*a*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[
a*x])])/3
```

3.92.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6721, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$a\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 770$$

$$2a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 755$$

$$2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 1476$$

$$2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 1082$$

$$2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 217$$

3.92. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{ 1479}$$

$$2a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-\frac{1}{x}}}\right.}{\sqrt{2}}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{ 25}$$

3.92. $\int\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$\frac{2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\right.}{\sqrt{2}}\right.\right.}{a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$\big\downarrow$ 27

$$\frac{2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.}{a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$\big\downarrow$ 1103

3.92. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log}{}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

output
```
-(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)) + 2*a*((-(ArcTan[1 - (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(
1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2
]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqr
t[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.92. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.92.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^2}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

3.92.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{(-a^4)^{\frac{1}{4}}\,x\log\left(a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+(-a^4)^{\frac{1}{4}}\right)+i\,(-a^4)^{\frac{1}{4}}\,x\log\left(a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+i\,(-a^4)^{\frac{1}{4}}\right)-i\,(-a^4)^{\frac{1}{4}}\,x\log\left(a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\right.}{2\,x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

output
```
1/2*((-a^4)^(1/4)*x*log(a*((a*x - 1)/(a*x + 1))^(1/4) + (-a^4)^(1/4)) + I*
(-a^4)^(1/4)*x*log(a*((a*x - 1)/(a*x + 1))^(1/4) + I*(-a^4)^(1/4)) - I*(-a
^4)^(1/4)*x*log(a*((a*x - 1)/(a*x + 1))^(1/4) - I*(-a^4)^(1/4)) - (-a^4)^(
1/4)*x*log(a*((a*x - 1)/(a*x + 1))^(1/4) - (-a^4)^(1/4)) - 2*(a*x + 1)*((a
*x - 1)/(a*x + 1))^(1/4))/x
```

3.92.6 Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**2, x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.69

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1
) + 1))*a
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.69

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1
) + 1))*a
```

3.92.9 Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1} - (-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i} - (-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4)/x^2,x)
```

output
```
- (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (-1)^(1/4
)*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (2*a*((a*x - 1)/(a*
x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.93 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.93.1 Optimal result

Integrand size = 14, antiderivative size = 319

$$\begin{aligned}\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx =\ & \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}\\ &+\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}-\frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}\\ &+\frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\\ &-\frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\end{aligned}$$

output
```
1/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^
(3/4)-1/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-1
/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/16*a^2*
ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/
2))*2^(1/2)-1/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.93.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.18

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right.$$
$$\left.-2\,\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

output
```
(-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCo
th[a*x])] - 2*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

3.93.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.81, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6721, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{4}a\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}a\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{770}$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+$$
$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{1476}$$

3.93. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\downarrow 1082$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\downarrow 217$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{\sqrt{2}}\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\downarrow 1479$$

3.93. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}}{2\sqrt{2}}\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 27$$

3.93. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\right.}{}\right.\right.\right.$$

input
```
Int[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

output
```
(a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/2 + (a*(a*(1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(3/4) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
- x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4
```

3.93. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.93. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.93.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^3}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

3.93.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+(-a^8)^{\frac{1}{4}}\right)+i\,(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+i\,(-a^8)^{\frac{1}{4}}\right)-i\,(-a^8)^{\frac{1}{4}}\,x^2\log\left(a^2\left(\right.\right.}{8\,x^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

output
```
-1/8*((-a^8)^(1/4)*x^2*log(a^2*((a*x - 1)/(a*x + 1))^(1/4) + (-a^8)^(1/4))
 + I*(-a^8)^(1/4)*x^2*log(a^2*((a*x - 1)/(a*x + 1))^(1/4) + I*(-a^8)^(1/4)
) - I*(-a^8)^(1/4)*x^2*log(a^2*((a*x - 1)/(a*x + 1))^(1/4) - I*(-a^8)^(1/4
)) - (-a^8)^(1/4)*x^2*log(a^2*((a*x - 1)/(a*x + 1))^(1/4) - (-a^8)^(1/4))
- 2*(3*a^2*x^2 + a*x - 2)*((a*x - 1)/(a*x + 1))^(1/4))/x^2
```

3.93.6 Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**3, x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input

```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

output

```
-1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*a*((a*x - 1)/(a*x + 1))^(5/4) + a*
((a*x - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1
)^2 + 1))*a
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input

```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

output
```
-1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1) + a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
^2)*a
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{5\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}+\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}}{4} + \frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}}{4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4)/x^3,x)
```

output
```
((a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (5*a^2*((a*x - 1)/(a*x + 1))^(5/4))
/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*
a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4 + ((-1)^(1/4)*a^2*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4
```

3.94 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

3.94.1 Optimal result

Integrand size = 14, antiderivative size = 356

$$\begin{aligned}\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = {} & -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} \\ & -\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} \\ & -\frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{3a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} \\ & -\frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}} \\ & +\frac{3a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}\end{aligned}$$

output
```
-3/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-1/12*a^3*(1-1/a/x)^(5/4)*(1+1/a/x
)^(3/4)+1/3*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)/x+3/16*a^3*arctan(-1+(1-1/
a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+(1-1/a/x)^(1
/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-3/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)
/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+3/32*a^3*ln(1+(1
-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(
1/2)
```

3.94.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx=\frac{1}{96}a^3\left(-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(29+6e^{2\coth^{-1}(ax)}+9e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3}+9\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

input
```
Integrate[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

output
```
(a^3*((-8*E^((3*ArcCoth[a*x])/2)*(29 + 6*E^(2*ArcCoth[a*x]) + 9*E^(4*ArcCo
th[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*
x] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ]))/96
```

3.94.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 101, 27, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \ 6721$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \ 101$$

$$\frac{1}{3}a^2\int -\frac{\left(2a-\frac{1}{x}\right)\sqrt[4]{1-\frac{1}{ax}}}{2a\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}$$

$$\Big\downarrow \ 27$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\int \frac{\left(2a-\frac{1}{x}\right)\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \ 90$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \ 770$$

3.94. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int\frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$\downarrow$ 755

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5\right.$$

$\downarrow$ 1476

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\right.\right.\right.\right.$$

$\downarrow$ 1082

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$\downarrow$ 217

3.94. $\int\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\arctan\left(1\right.\right.\right.\right.\right.$$

$$\Big\downarrow 1479$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d}{2\sqrt{2}}\right.\right.\right.\right.$$

$$\Big\downarrow 25$$

3.94. $\int\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{9}{4}a\Bigg(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\Bigg(\frac{1}{2}\Bigg(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}}{2\sqrt{2}}$$

$$\Big\downarrow 27$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{9}{4}a\Bigg(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\Bigg(\frac{1}{2}\Bigg(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d$$

$$\Big\downarrow 1103$$

3.94. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{9}{4}a\Bigg(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\Bigg(\frac{1}{2}\Bigg(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-$$

input
```
Int[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

output
```
(a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/(3*x) - (a*((a^2*(1 - 1/(a*x
))^(5/4)*(1 + 1/(a*x))^(3/4))/2 + (9*a*(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x)
)^(3/4) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1
/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4
)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(
1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4)
)^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4))/6
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

3.94. $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.94.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^4}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

3.94.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$= \frac{9\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(3\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+3\left(-a^{12}\right)^{\frac{1}{4}}\right)+9i\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(3\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+3i\left(-a^{12}\right)^{\frac{1}{4}}\right)-9i\left(-a^{12}\right)^{\frac{1}{4}}}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

output
```
1/48*(9*(-a^12)^(1/4)*x^3*log(3*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 3*(-a^12
)^(1/4)) + 9*I*(-a^12)^(1/4)*x^3*log(3*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 3
*I*(-a^12)^(1/4)) - 9*I*(-a^12)^(1/4)*x^3*log(3*a^3*((a*x - 1)/(a*x + 1))^
(1/4) - 3*I*(-a^12)^(1/4)) - 9*(-a^12)^(1/4)*x^3*log(3*a^3*((a*x - 1)/(a*x
 + 1))^(1/4) - 3*(-a^12)^(1/4)) - 2*(11*a^3*x^3 + a^2*x^2 - 2*a*x + 8)*((a
*x - 1)/(a*x + 1))^(1/4))/x^3
```

3.94.6 Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4)/x**4,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**4, x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.78

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$=\frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

output
```
1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(29*a^2*((a*x - 1)/(a*x
+ 1))^(9/4) + 6*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 9*a^2*((a*x - 1)/(a*x +
1))^(1/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^
3/(a*x + 1)^3 + 1))*a
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$=\frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

output
```
1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(6*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(1/4)/(a*x + 1) + 29*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1)^2 + 9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1
) + 1)^3)*a
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.47

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{\frac{3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{29\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1} - \frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}}{8} - \frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}}{8}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4)/x^4,x)
```

output
```
- ((3*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (a^3*((a*x - 1)/(a*x + 1))^(5/4
))/2 + (29*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1)
^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*
a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8 - ((-1)^(1/4)*a^3*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8
```

3.95 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

3.95.1 Optimal result

Integrand size = 14, antiderivative size = 253

$$\begin{aligned}\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx &= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{640a^4}-\frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{320a^3}\\&\quad+\frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{16a^2}-\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}\\&\quad+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5-\frac{237\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}\end{aligned}$$

output
```
557/640*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4-157/320*(1-1/a/x)^(3/4)*(1+1
/a/x)^(1/4)*x^2/a^3+5/16*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2-11/40*(1-
1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5
-237/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-237/128*arctanh((1+1/
a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5
```

3.95.2 Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$
$$=\frac{\frac{8192e^{\frac{17}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}-\frac{22016e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{23936e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{14032e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{5500e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+2370\,\mathrm{arcta}}{1280a^5}$$

input
```
Integrate[x^4/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((8192*E^((17*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (22016*E^((1
3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((9*ArcCoth[a*x
])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (14032*E^((5*ArcCoth[a*x])/2))/(-1 +
E^(2*ArcCoth[a*x]))^2 + (5500*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])
) + 2370*ArcTan[E^(-1/2*ArcCoth[a*x])] + 1185*Log[1 - E^(-1/2*ArcCoth[a*x]
)] - 1185*Log[1 + E^(-1/2*ArcCoth[a*x])])/(1280*a^5)
```

3.95.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4e^{-\frac{3}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^6}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{5}\int -\frac{\left(11a-\frac{8}{x}\right)x^5}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(11a-\frac{8}{x}\right)x^5}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 168$$

$$\frac{-\frac{1}{4}\int \frac{3\left(25a-\frac{22}{x}\right)x^4}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{3\int \frac{\left(25a-\frac{22}{x}\right)x^4}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 168$$

$$\frac{-\frac{3\left(-\frac{1}{3}\int \frac{\left(157a-\frac{100}{x}\right)x^3}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{3\left(-\frac{\int \frac{\left(157a-\frac{100}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 168$$

3.95. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{-\frac{3\left(-\frac{-\frac{1}{2}\int \frac{\left(557a-\frac{314}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{3\left(-\frac{-\frac{\int \frac{\left(557a-\frac{314}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 168$$

$$\frac{-\frac{3\left(-\frac{-\frac{-\int \frac{1185x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 27$$

3.95. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{-\frac{3\left(-\frac{-\frac{-\frac{1185}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{-\frac{3\left(-\frac{-\frac{-2370\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{-\frac{3\left(-\frac{-\frac{-2370\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3(1-\right.}{8a}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

3.95. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

$$\Big\downarrow \textcolor{blue}{216}$$

$$\frac{-\frac{3\left(-\frac{-\frac{-2370\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3(1-}{8a}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{-\frac{3\left(-\frac{-\frac{-2370\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax}{8a}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[x^4/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 + ((-11*a*(1 - 1/(a*x))^(3
/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 - (3*((-25*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*
x))^(1/4)*x^3)/3 - ((-157*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/2
 - (-557*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - 2370*(-1/2*ArcTan[(
1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/2))/(4*a))/(6*a)))/(8*a))/(10*a^2)
```

3.95. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

3.95.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.95.4 Maple [F]

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

input

```
int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

output

```
int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$
$$=\frac{2\left(128\,a^5x^5-48\,a^4x^4+24\,a^3x^3-114\,a^2x^2+243\,ax+557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1185\log}{1280\,a^5}$$

input

```
integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output

```
1/1280*(2*(128*a^5*x^5 - 48*a^4*x^4 + 24*a^3*x^3 - 114*a^2*x^2 + 243*a*x +
 557)*((a*x - 1)/(a*x + 1))^(3/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1185*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a^5
```

3.95.6 Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

input
```
integrate(x**4*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(x**4*((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\frac{1}{1280}a\Bigg(\frac{4\left(1375\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+395\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6} - 2370$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/1280*a*(4*(1375*((a*x - 1)/(a*x + 1))^(19/4) - 1992*((a*x - 1)/(a*x + 1
))^(15/4) + 3710*((a*x - 1)/(a*x + 1))^(11/4) - 1440*((a*x - 1)/(a*x + 1))
^(7/4) + 395*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*
(a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)
^4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^6 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{1}{1280}a\Bigg(\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{1185\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+$$

input

```
integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output

```
1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x -
1)/(a*x + 1))^(1/4) + 1)/a^6 + 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) -
1))/a^6 + 4*(1440*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3710*(
a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 1992*(a*x - 1)^3*((a*
x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1375*(a*x - 1)^4*((a*x - 1)/(a*x + 1
))^(3/4)/(a*x + 1)^4 - 395*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a
*x + 1) - 1)^5))
```

3.95.9 Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{79\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{64}-\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{371\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32}-\frac{249\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{40}+\frac{275\left(\frac{a\,x-1}{a\,x+1}\right)^{19/4}}{64}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}} + \frac{237\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{237\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

input

```
int(x^4*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output

```
((79*((a*x - 1)/(a*x + 1))^(3/4))/64 - (9*((a*x - 1)/(a*x + 1))^(7/4))/2 +
 (371*((a*x - 1)/(a*x + 1))^(11/4))/32 - (249*((a*x - 1)/(a*x + 1))^(15/4)
)/40 + (275*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/
(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x
+ 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (2
37*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (237*atanh(((a*x - 1)/(a
*x + 1))^(1/4)))/(128*a^5)
```

3.96 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

3.96.1 Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4+\frac{123\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output
```
-63/64*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+15/32*(1-1/a/x)^(3/4)*(1+1/a/
x)^(1/4)*x^2/a^2-3/8*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^(
3/4)*(1+1/a/x)^(1/4)*x^4+123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^
4+123/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

3.96.2 Mathematica [A] (verified)

Time = 5.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{\frac{512e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{1152e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{1008e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{532e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-246\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-1}{128a^4}$$

input
```
Integrate[x^3/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((512*E^((13*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (1152*E^((9*A
rcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((5*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^2 - (532*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCo
th[a*x])) - 246*ArcTan[E^(-1/2*ArcCoth[a*x])] - 123*Log[1 - E^(-1/2*ArcCot
h[a*x])] + 123*Log[1 + E^(-1/2*ArcCoth[a*x])])/(128*a^4)
```

3.96.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{-\frac{3}{2}\coth^{-1}(ax)}dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^5}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$
$$\Big\downarrow \text{110}$$
$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{4}\int -\frac{3\left(3a-\frac{2}{x}\right)x^4}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$
$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\left(3a-\frac{2}{x}\right)x^4}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 168$$

$$\frac{3\left(-\frac{1}{3}\int \frac{3\left(5a-\frac{4}{x}\right)x^3}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 27$$

$$\frac{3\left(-\frac{\int \frac{\left(5a-\frac{4}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 168$$

$$\frac{3\left(-\frac{-\frac{1}{2}\int \frac{\left(21a-\frac{10}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 27$$

$$\frac{3\left(-\frac{-\frac{\int \frac{\left(21a-\frac{10}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 168$$

3.96. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{3\left(-\frac{-\frac{-\int \frac{41x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(-\frac{-\frac{-\frac{41}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 104$$

$$\frac{3\left(-\frac{-\frac{-82\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 756$$

3.96. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

$$3\left(-\frac{-\frac{-82\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3(1-$$

$$8a^2$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{216}$$

$$3\left(-\frac{-\frac{-82\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3(1-$$

$$8a^2$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{219}$$

$$3\left(-\frac{-\frac{-82\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3(1-$$

$$8a^2$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

3.96. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

input

```
Int[x^3/E^((3*ArcCoth[a*x])/2),x]
```

output

```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 + (3*(-(a*(1 - 1/(a*x))^(3
/4)*(1 + 1/(a*x))^(1/4)*x^3) - ((-5*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1
/4)*x^2)/2 - (-21*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - 82*(-1/2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/
4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(2*a)))/(8*a^2)
```

3.96.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.96.4 Maple [F]

$$\int x^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{4}}d x$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{2\left(16\,a^{4}x^{4}-8\,a^{3}x^{3}+6\,a^{2}x^{2}-33\,ax-63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-1}{128\,a^{4}}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/128*(2*(16*a^4*x^4 - 8*a^3*x^3 + 6*a^2*x^2 - 33*a*x - 63)*((a*x - 1)/(a*
x + 1))^(3/4) - 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x -
1)/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

3.96.6 Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(x**3*((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx =$$
$$-\frac{1}{128}a\left(\frac{4\left(133\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-147\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+183\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-41\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123}{}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/128*a*(4*(133*((a*x - 1)/(a*x + 1))^(15/4) - 147*((a*x - 1)/(a*x + 1))^
(11/4) + 183*((a*x - 1)/(a*x + 1))^(7/4) - 41*((a*x - 1)/(a*x + 1))^(3/4))
/(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^
3*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5
+ 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx =$$

$$-\frac{1}{128}a\left(\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{123\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{183\,(ax-1)\left(\frac{ax}{ax}\right.}{ax+1}\right.}{}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
-1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^5 + 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))
/a^5 - 4*(183*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 147*(a*x -
 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 133*(a*x - 1)^3*((a*x - 1)
/(a*x + 1))^(3/4)/(a*x + 1)^3 - 41*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x
 - 1)/(a*x + 1) - 1)^4))
```

3.96.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{123\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{123\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{41\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{32}-\frac{183\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{32}+\frac{147\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32}-\frac{133\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{32}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
(123*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (123*atan(((a*x - 1)/(
a*x + 1))^(1/4)))/(64*a^4) - ((41*((a*x - 1)/(a*x + 1))^(3/4))/32 - (183*(
(a*x - 1)/(a*x + 1))^(7/4))/32 + (147*((a*x - 1)/(a*x + 1))^(11/4))/32 - (
133*((a*x - 1)/(a*x + 1))^(15/4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)
^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*
a^4*(a*x - 1))/(a*x + 1))
```

3.97 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

3.97.1 Optimal result

Integrand size = 14, antiderivative size = 179

$$\begin{aligned}\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx &= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2}-\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a}\\ &\quad+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3\\ &\quad-\frac{17\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{17\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}\end{aligned}$$

output
```
23/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2-7/12*(1-1/a/x)^(3/4)*(1+1/a/x)
^(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3-17/8*arctan((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^3-17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/
a^3
```

3.97.2 Mathematica [A] (verified)

Time = 5.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{\frac{128e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{240e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{180e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+102\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+51\log\left(1-e^{-\frac{1}{2}\coth^{-1}}\right.}{48a^3}$$

input
```
Integrate[x^2/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((128*E^((9*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (240*E^((5*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (180*E^(ArcCoth[a*x]/2))/(-1
+ E^(2*ArcCoth[a*x])) + 102*ArcTan[E^(-1/2*ArcCoth[a*x])] + 51*Log[1 - E^(
-1/2*ArcCoth[a*x])] - 51*Log[1 + E^(-1/2*ArcCoth[a*x])])/(48*a^3)
```

3.97.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow 6721$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^4}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow 110$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{3}\int -\frac{\left(7a-\frac{4}{x}\right)x^3}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\left(7a-\frac{4}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{1}{2}\int \frac{\left(23a-\frac{14}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(23a-\frac{14}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{-\int \frac{51x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{51}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{104}$$

$$\frac{-\frac{-102\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

3.97. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

$$\Big\downarrow 756$$

$$\frac{-\frac{-102\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 216$$

$$\frac{-\frac{-102\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 219$$

$$\frac{-\frac{-102\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[x^2/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^3)/3 + ((-7*a*(1 - 1/(a*x))^(3/
4)*(1 + 1/(a*x))^(1/4)*x^2)/2 - (-23*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(
1/4)*x - 102*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a^2)
```

3.97. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.97.4 Maple [F]

$$\int x^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{4}}d x$$

input

```
int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

output

```
int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\,(8\,a^{3}x^{3}-6\,a^{2}x^{2}+9\,ax+23)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+51\,\log\bigg(\Big(\frac{ax-1}{ax+1}}{48\,a^{3}}$$

input

```
integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output

```
1/48*(2*(8*a^3*x^3 - 6*a^2*x^2 + 9*a*x + 23)*((a*x - 1)/(a*x + 1))^(3/4) +
 102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 51*log(((a*x - 1)/(a*x + 1))^(1
/4) + 1) + 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

3.97.6 Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(x**2*((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx=$$
$$-\frac{1}{48}a\left(\frac{4\left(45\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+17\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/48*a*(4*(45*((a*x - 1)/(a*x + 1))^(11/4) - 30*((a*x - 1)/(a*x + 1))^(7/
4) + 17*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x -
 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((
a*x - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^4 - 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$$
$$=\frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\right.}{a}\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^4 + 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
 + 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 45*(a*x - 1)^2*
((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 17*((a*x - 1)/(a*x + 1))^(3/4))/
(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} + \frac{17\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{17\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((17*((a*x - 1)/(a*x + 1))^(3/4))/12 - (5*((a*x - 1)/(a*x + 1))^(7/4))/2 +
 (15*((a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)
^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*at
an(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (17*atanh(((a*x - 1)/(a*x + 1))
^(1/4)))/(8*a^3)
```

3.98 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx$

3.98.1 Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 +\frac{9\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output
```
-3/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/
4)*x^2+9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

3.98.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-7+3e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+9\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+9\operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[x/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((-2*E^(ArcCoth[a*x]/2)*(-7 + 3*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*
x]))^2 + 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4*
a^2)
```

3.98.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6721, 107, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{3}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^3}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 107}$$

$$\frac{3\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^2}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{3\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{6\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; \text{756}$$

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{6\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; \text{216}$$

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{6\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; \text{219}$$

3.98. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx$

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{6\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

input

```
Int[x/E^((3*ArcCoth[a*x])/2),x]
```

output

```
((1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4)*x^2)/2 + (3*(-((1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(1/4)*x) - (6*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x
))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/a))/(4*a)
```

3.98.3.1 Defintions of rubi rules used

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.98.4 Maple [F]

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

input
```
int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{2\left(2a^2x^2-3ax-5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 - 3*a*x - 5)*((a*x - 1)/(a*x + 1))^(3/4) - 18*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

3.98.6 Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(x*((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{1}{8}a\left(\frac{4\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right.}{a^3}\right.$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/8*a*(4*(7*((a*x - 1)/(a*x + 1))^(7/4) - 3*((a*x - 1)/(a*x + 1))^(3/4))/
(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arcta
n(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1
)/a^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{1}{8}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{7\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-3\,\left(\right.\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right.$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
-1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^3 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 -
4*(7*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3*((a*x - 1)/(a*x +
 1))^(3/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.98.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx=\frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}-\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
(9*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (9*atan(((a*x - 1)/(a*x +
 1))^(1/4)))/(4*a^2) - ((3*((a*x - 1)/(a*x + 1))^(3/4))/2 - (7*((a*x - 1)/
(a*x + 1))^(7/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x -
1))/(a*x + 1))
```

3.98. $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx$

3.99 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$

3.99.1 Optimal result

Integrand size = 10, antiderivative size = 98

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x-\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)/a-3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

3.99.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-3\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-3\operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^((-3*ArcCoth[a*x])/2),x]
```

output
```
((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) - 3*ArcTan[E^(ArcCoth[a*
x]/2)] - 3*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

3.99.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6720, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \ 6720$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^2}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \ 105$$

$$\frac{3\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 104$$

$$\frac{6\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 756$$

$$\frac{6\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 216$$

$$\frac{6\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 219$$

$$\frac{6\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input

```
Int[E^((-3*ArcCoth[a*x])/2),x]
```

output

```
(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + (6*(-1/2*ArcTan[(1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/2))/a
```

3.99.3.1 Defintions of rubi rules used

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.99.4 Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4),x)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} + 6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) + 6*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 3*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1))/a
```

3.99.6 Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=\int\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1)/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=\frac{1}{2}\,a\left(\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1)/a^2 + 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*
((a*x - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

3.99.9 Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}-\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((
a*x - 1)/(a*x + 1))^(1/4)))/a - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.100 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

3.100.1 Optimal result

Integrand size = 14, antiderivative size = 291

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx &= \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\
&\quad+2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\\
&\quad-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\
&\quad+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\end{aligned}$$

output

```
2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2
)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4
)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1/
2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/
4))*2^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.10

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = 8e^{\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{4\coth^{-1}(ax)}\right)$$

input

```
Integrate[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

output

```
8*E^(ArcCoth[a*x]/2)*Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x])]
```

3.100.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.89, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4} x}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 73$$

$$-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 104$$

$$-4\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}$$

$$\Big\downarrow \; 756$$

$$-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}$$

$$\Big\downarrow \; 216$$

$$-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}$$

$$\Big\downarrow \; 219$$

$$-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \; 854$$

$$-4\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \; 826$$

3.100. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1476}$$

$$-4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1}}{\sqrt[4]{2}}\right.$$

$$\left.4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)\right)$$

$$\Big\downarrow \text{1082}$$

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{217}$$

3.100. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow 1479$$

$$-4\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow 25$$

3.100. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\Bigg(\frac{1}{2}\Bigg(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\Bigg(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{\sqrt{2}}$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)\Bigg)$$

$$\Big\downarrow \text{ 27}$$

$$-4\Bigg(\frac{1}{2}\Bigg(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\Bigg(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-}}}{\sqrt{2}}$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)\Bigg)$$

$$\Big\downarrow \text{ 1103}$$

3.100. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

$$-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log}{}\right.\right.$$

input
```
Int[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

output
```
-4*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2) - 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.100. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.100.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

3.100.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = &-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\&+\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\&-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\&+\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\&-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\&+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)\end{aligned}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

output
```
-(1/2*I - 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
) + (1/2*I + 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4)) - (1/2*I + 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4)) + (1/2*I - 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x
+ 1))^(1/4)) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

3.100.6 Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4)/x, x)
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\right.}{a}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a -
2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1)/a)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\sqrt{2}\log\left(\sqrt{2}\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

output
```
-1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/
a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(-1+1\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)(-1-\mathrm{i})$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4)/x,x)
```

output
```
- atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(
1/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1
- 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1
+ 1i)
```

3.101 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

3.101.1 Optimal result

Integrand size = 14, antiderivative size = 269

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx &= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\ &\quad+\frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\ &\quad+\frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}\\ &\quad-\frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}\end{aligned}$$

output
```
-a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+3/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)
/(1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)
^(1/4))*2^(1/2)+3/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/
x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-3/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/
a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}+\frac{3\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}-\frac{3\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

input
```
Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

output
```
a*((-2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + (3*ArcTan[1 - Sqrt[2
]*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)]
)/Sqrt[2] + (3*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sq
rt[2]) - (3*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[
2]))
```

3.101.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6721, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 60$$

$$a\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{3}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 73$$

$$6a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 854$$

$$6a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 826$$

$$6a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 1476$$

$$\frac{6a\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{6a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{217}$$

$$\frac{6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1479}$$

3.101. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$\frac{6a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 25$$

$$\frac{6a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 27$$

3.101. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$6a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-x}}\right.}{\sqrt{2}}\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{1103}$$

$$6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

output
```
-(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) + 6*a*((-(ArcTan[1 - (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(
1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.101. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.101.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^2}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

3.101.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{3\left(-a^4\right)^{\frac{1}{4}} x\log\left(27\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+27\left(-a^4\right)^{\frac{3}{4}}\right)-3i\left(-a^4\right)^{\frac{1}{4}} x\log\left(27\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+27i\left(-a^4\right)^{\frac{3}{4}}\right)+3i\left(-a^4\right)^{\frac{1}{4}} x}{2\,x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

output
```
1/2*(3*(-a^4)^(1/4)*x*log(27*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 27*(-a^4)^(
3/4)) - 3*I*(-a^4)^(1/4)*x*log(27*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 27*I*(
-a^4)^(3/4)) + 3*I*(-a^4)^(1/4)*x*log(27*a^3*((a*x - 1)/(a*x + 1))^(1/4) -
 27*I*(-a^4)^(3/4)) - 3*(-a^4)^(1/4)*x*log(27*a^3*((a*x - 1)/(a*x + 1))^(1
/4) - 27*(-a^4)^(3/4)) - 2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/x
```

3.101.6 Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**2, x)
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{\vphantom{2}}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

output
```
1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x
 + 1) + 1))*a
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$=\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

output
```
1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x
 + 1) + 1))*a
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$=3\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-3\,(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4)/x^2,x)
```

output
```
3*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 3*(-1)^(1/4)
*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (2*a*((a*x - 1)/(a*x +
1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.101. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

3.102 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.102.1 Optimal result

Integrand size = 14, antiderivative size = 319

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

output
```
3/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^
(1/4)-9/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9
/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/16*a^2*
ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/
2))*2^(1/2)+9/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}a^2\left(\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{24e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} - 18\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 18\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 9\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right) + 9\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

output
```
(a^2*((32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (24*E^(ArcCoth[
a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 18*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCo
th[a*x]/2)] + 18*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] - 9*Sqrt[2
]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] + 9*Sqrt[2]*Log[1 +
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

3.102.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.81, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6721, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \ 6721$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}x}d\frac{1}{x}$$

$$\Big\downarrow \ 90$$

$$\frac{3}{4}a\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 60$$

$$\frac{3}{4}a\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 73$$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 854$$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 826$$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+$$
$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\right.\right.\right.}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\right.\right.}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-}{1+}\right.\right.}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 1479$$

3.102. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}}{2\sqrt{2}}\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{2-\frac{1}{x}}}}{2\sqrt{2}}\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 27$$

3.102. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow 1103$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+$$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\right.}{}\right.\right.\right.$$

input
```
Int[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

output
```
(a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))/2 + (3*a*(a*(1 - 1/(a*x))^(3
/4)*(1 + 1/(a*x))^(1/4) - 6*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))
/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
- x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4
```

3.102. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.102. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.102.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

3.102.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{9\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(729\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+729\left(-a^8\right)^{\frac{3}{4}}\right)-9i\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(729\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+729i\left(-a^8\right)^{\frac{3}{4}}\right)+9i}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

output
```
-1/8*(9*(-a^8)^(1/4)*x^2*log(729*a^6*((a*x - 1)/(a*x + 1))^(1/4) + 729*(-a
^8)^(3/4)) - 9*I*(-a^8)^(1/4)*x^2*log(729*a^6*((a*x - 1)/(a*x + 1))^(1/4)
+ 729*I*(-a^8)^(3/4)) + 9*I*(-a^8)^(1/4)*x^2*log(729*a^6*((a*x - 1)/(a*x +
 1))^(1/4) - 729*I*(-a^8)^(3/4)) - 9*(-a^8)^(1/4)*x^2*log(729*a^6*((a*x -
1)/(a*x + 1))^(1/4) - 729*(-a^8)^(3/4)) - 2*(5*a^2*x^2 + 3*a*x - 2)*((a*x
- 1)/(a*x + 1))^(3/4))/x^2
```

3.102.6 Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**3, x)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

output
```
-1/16*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1))*a - 8*(7*a*((a*x - 1)/(a*x + 1))^(7/4) + 3*a*
((a*x - 1)/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1
)^2 + 1))*a
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

output
```
-1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
1))^(1/4))) - 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^
(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(7*(a*x - 1)*a*((a*x - 1)/(a*x
+ 1))^(3/4)/(a*x + 1) + 3*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x +
 1) + 1)^2)*a
```

3.102.9 Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{3\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{2}+\frac{7\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}-\frac{9\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}+\frac{9\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4)/x^3,x)
```

output
```
((3*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (7*a^2*((a*x - 1)/(a*x + 1))^(7/4
))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - (9*(-1)^(1
/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 + (9*(-1)^(1/4)*a^
2*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

3.103 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

3.103.1 Optimal result

Integrand size = 14, antiderivative size = 356

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = &-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\\
&-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}\\
&-\frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{17a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}\\
&+\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}\\
&-\frac{17a^3\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{16\sqrt{2}}\end{aligned}$$

output

```
-17/24*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-1/4*a^3*(1-1/a/x)^(7/4)*(1+1/a/
x)^(1/4)+1/3*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4)/x+17/16*a^3*arctan(-1+(1-
1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(
1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-17/32*a^3*ln
(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
)*2^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(-\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(45+30e^{2\coth^{-1}(ax)}+17e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 51\text{RootSum}\left[1 + \#1^4\&, \frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]\right)$$

input

```
Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

output

```
(a^3*((-8*E^(ArcCoth[a*x]/2)*(45 + 30*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth
[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*x
] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1 & ]))/96
```

3.103.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 101, 27, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow 6721$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}x^2}d\frac{1}{x}$$

$$\Big\downarrow 101$$

$$\frac{1}{3}a^2\int -\frac{\left(2a-\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{3/4}}{2a\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}$$

$$\Big\downarrow 27$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\int \frac{\left(2a-\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow 90$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow 60$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow 73$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow 854$$

3.103. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\int\frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{826}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)\right.$$

$$\Big\downarrow \text{1476}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

3.103. $\int\frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2(1-\frac{1}{ax})^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\right.\right.\right.$$

$$\Big\downarrow \text{1479}$$

$$\frac{a^2(1-\frac{1}{ax})^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d}{2\sqrt{2}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

3.103. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2(1-\frac{1}{ax})^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{17}{4}a\Bigg(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\Bigg(\frac{1}{2}\Bigg(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d}{2\sqrt{2}}$$

$$\Big\downarrow 27$$

$$\frac{a^2(1-\frac{1}{ax})^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-$$

$$\frac{1}{6}a\Bigg(\frac{17}{4}a\Bigg(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\Bigg(\frac{1}{2}\Bigg(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}$$

$$\Big\downarrow 1103$$

3.103. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-$$

$$\frac{1}{6}a\left(\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\text{arcta}}{}\right.\right.\right.\right.$$

input

```
Int[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

output

```
(a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))/(3*x) - (a*((3*a^2*(1 - 1/(a
*x))^(7/4)*(1 + 1/(a*x))^(1/4))/2 + (17*a*(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a
*x))^(1/4) - 6*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))
^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(
1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/
4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-
4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4))/6
```

3.103.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.103. $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.103.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

3.103.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$= \frac{51\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(4913\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4913\left(-a^{12}\right)^{\frac{3}{4}}\right)-51i\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(4913\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4913i\left(-a^{12}\right)\right.}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

output
```
1/48*(51*(-a^12)^(1/4)*x^3*log(4913*a^9*((a*x - 1)/(a*x + 1))^(1/4) + 4913
*(-a^12)^(3/4)) - 51*I*(-a^12)^(1/4)*x^3*log(4913*a^9*((a*x - 1)/(a*x + 1)
)^(1/4) + 4913*I*(-a^12)^(3/4)) + 51*I*(-a^12)^(1/4)*x^3*log(4913*a^9*((a*
x - 1)/(a*x + 1))^(1/4) - 4913*I*(-a^12)^(3/4)) - 51*(-a^12)^(1/4)*x^3*log
(4913*a^9*((a*x - 1)/(a*x + 1))^(1/4) - 4913*(-a^12)^(3/4)) - 2*(23*a^3*x^
3 + 9*a^2*x^2 - 6*a*x + 8)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

3.103.6 Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**4, x)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$=\frac{1}{96}\left(51\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

output
```
1/96*(51*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1))*a^2 - 8*(45*a^2*((a*x - 1)/(a*x + 1))^(11/4)
+ 30*a^2*((a*x - 1)/(a*x + 1))^(7/4) + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))
/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1
)^3 + 1))*a
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$=\frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

output
```
1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a
*x + 1))^(1/4))) - 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) + 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(30*(a*x - 1)*a^2*((
a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 45*(a*x - 1)^2*a^2*((a*x - 1)/(a*x +
 1))^(3/4)/(a*x + 1)^2 + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a
*x + 1) + 1)^3)*a
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.47

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{17\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{\frac{17\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}+\frac{5\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} - \frac{17\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4)/x^4,x)
```

output
```
(17*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - ((17*
a^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (5*a^3*((a*x - 1)/(a*x + 1))^(7/4))/
2 + (15*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^2
+ (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - (17*(-1)^(1/4)*
a^3*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

3.104 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

3.104.1 Optimal result

Integrand size = 14, antiderivative size = 287

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx=\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}}+\frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}}+\frac{1003\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{1003\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

output
```
26111/1920*(1-1/a/x)^(1/4)/a^5/(1+1/a/x)^(1/4)+5533/1920*(1-1/a/x)^(1/4)*x
/a^4/(1+1/a/x)^(1/4)-1189/960*(1-1/a/x)^(1/4)*x^2/a^3/(1+1/a/x)^(1/4)+181/
240*(1-1/a/x)^(1/4)*x^3/a^2/(1+1/a/x)^(1/4)-21/40*(1-1/a/x)^(1/4)*x^4/a/(1
+1/a/x)^(1/4)+1/5*(1-1/a/x)^(1/4)*x^5/(1+1/a/x)^(1/4)+1003/128*arctan((1+1
/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x
)^(1/4))/a^5
```

3.104.2 Mathematica [A] (verified)

Time = 5.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.69

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= 8e^{-\frac{1}{2}\coth^{-1}(ax)} - \frac{32e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{-2\coth^{-1}(ax)}\right)^5} - \frac{122e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{-2\coth^{-1}(ax)}\right)^4} - \frac{233e^{-\frac{1}{2}\coth^{-1}(ax)}}{6\left(-1+e^{-2\coth^{-1}(ax)}\right)^3} - \frac{1661e^{-\frac{1}{2}\coth^{-1}(ax)}}{48\left(-1+e^{-2\coth^{-1}(ax)}\right)^2} -$$

input

```
Integrate[x^4/E^((5*ArcCoth[a*x])/2),x]
```

output

```
(8/E^(ArcCoth[a*x]/2) - 32/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x])
)^5) - 122/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^4) - 233/(6*E^
(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^3) - 1661/(48*E^(ArcCoth[a*x]/
2)*(-1 + E^(-2*ArcCoth[a*x]))^2) - 4117/(192*E^(ArcCoth[a*x]/2)*(-1 + E^(-
2*ArcCoth[a*x]))) - (1003*ArcTan[E^(-1/2*ArcCoth[a*x])])/128 + (1003*Log[1
 - E^(-1/2*ArcCoth[a*x])])/256 - (1003*Log[1 + E^(-1/2*ArcCoth[a*x])])/256
)/a^5
```

3.104.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {6721, 109, 27, 168, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^6}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{5}\int \frac{\left(21a-\frac{20}{x}\right)x^5}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \, 27$$

$$\frac{\int \frac{\left(21a-\frac{20}{x}\right)x^5}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \, 168$$

$$\frac{-\frac{1}{4}\int \frac{\left(181a-\frac{168}{x}\right)x^4}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \, 27$$

$$\frac{-\frac{\int \frac{\left(181a-\frac{168}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \, 168$$

$$\frac{-\frac{-\frac{1}{3}\int \frac{\left(1189a-\frac{1086}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \, 27$$

$$\frac{-\frac{-\frac{\int \frac{\left(1189a-\frac{1086}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \, 168$$

3.104. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{-\frac{-\frac{-\frac{1}{2}\int \frac{\left(5533a-\frac{4756}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{\left(5533a-\frac{4756}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{-\frac{-\frac{-\int \frac{\left(15045a-\frac{11066}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

3.104. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{-\frac{-\frac{-\frac{-\frac{\int \frac{\left(15045a-\frac{11066}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 172$$

$$\frac{-\frac{-\frac{-\frac{-\frac{2a\int \frac{15045x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-}}{4\sqrt[4]{\frac{1}{ax}}}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

3.104. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{-\frac{-\frac{-\frac{-\frac{15045a\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1}}{4\sqrt[4]{\frac{1}{ax}}}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 104$$

$$\frac{-\frac{-\frac{-\frac{-\frac{60180a\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1}}{4\sqrt[4]{\frac{1}{ax}}}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 25$$

3.104. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$-\frac{-\frac{-\frac{-\frac{\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-60180a\int\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1}}{4\sqrt[4]{\frac{1}{ax}}}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow\ 827$$

$$-\frac{-\frac{-\frac{-\frac{60180a\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{}}{3\sqrt[4]{\frac{1}{a}}}}{8a}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow\ 216$$

3.104. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

$$\frac{-\frac{-\frac{-\frac{-\frac{60180a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{}}{3\sqrt[4]{\frac{1}{a}}}}{8a}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{-\frac{-\frac{-\frac{-\frac{60180a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax}{3\sqrt[4]{}}}{8a}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[x^4/E^((5*ArcCoth[a*x])/2),x]
```

3.104. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

output

```
((1 - 1/(a*x))^(1/4)*x^5)/(5*(1 + 1/(a*x))^(1/4)) + ((-21*a*(1 - 1/(a*x))^
(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) - ((-181*a*(1 - 1/(a*x))^(1/4)*x^3)/(3*
(1 + 1/(a*x))^(1/4)) - ((-1189*a*(1 - 1/(a*x))^(1/4)*x^2)/(2*(1 + 1/(a*x))
^(1/4)) - ((-5533*a*(1 - 1/(a*x))^(1/4)*x)/(1 + 1/(a*x))^(1/4) - ((52222*a
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) + 60180*a*(ArcTan[(1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/2))/(2*a))/(4*a))/(6*a))/(8*a))/(10*a^2)
```

3.104.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.104.4 Maple [F]

$$\int x^{4} \left(\frac{a x - 1}{a x + 1}\right)^{\frac{5}{4}}d x$$

input
```
int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.41

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{2\,(384\,a^5x^5-1008\,a^4x^4+1448\,a^3x^3-2378\,a^2x^2+5533\,ax+26111)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{3840\,a^5}$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/3840*(2*(384*a^5*x^5 - 1008*a^4*x^4 + 1448*a^3*x^3 - 2378*a^2*x^2 + 5533
*a*x + 26111)*((a*x - 1)/(a*x + 1))^(1/4) - 30090*arctan(((a*x - 1)/(a*x +
 1))^(1/4)) - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*log(((a*x
 - 1)/(a*x + 1))^(1/4) - 1))/a^5
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \text{Timed out}$$

input
```
integrate(x**4*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Timed out
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.97

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$
$$-\frac{1}{3840}\,a\left(\frac{4\left(20585\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-49120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+61130\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-33816\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+7365\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/3840*a*(4*(20585*((a*x - 1)/(a*x + 1))^(17/4) - 49120*((a*x - 1)/(a*x +
 1))^(13/4) + 61130*((a*x - 1)/(a*x + 1))^(9/4) - 33816*((a*x - 1)/(a*x +
1))^(5/4) + 7365*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) -
 10*(a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x
- 1)^4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 30090*arctan
(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^6 - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6 - 30720*((a*x -
1)/(a*x + 1))^(1/4)/a^6)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}a\left(\frac{30090\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{15045\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{15045\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-30720\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
-1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x
 - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4
) - 1))/a^6 - 30720*((a*x - 1)/(a*x + 1))^(1/4)/a^6 - 4*(33816*(a*x - 1)*(
(a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x +
 1))^(1/4)/(a*x + 1)^2 + 49120*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*
x + 1)^3 - 20585*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^4 - 736
5*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

3.104.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.88

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{8\left(\frac{ax-1}{ax+1}\right)^{1/4}}{a^5} + \frac{\frac{491\left(\frac{ax-1}{ax+1}\right)^{1/4}}{64} - \frac{1409\left(\frac{ax-1}{ax+1}\right)^{5/4}}{40} + \frac{6113\left(\frac{ax-1}{ax+1}\right)^{9/4}}{96} - \frac{307\left(\frac{ax-1}{ax+1}\right)^{13/4}}{6} + \frac{4117\left(\frac{ax-1}{ax+1}\right)^{17/4}}{192}}{a^5 + \frac{10a^5(ax-1)^2}{(ax+1)^2} - \frac{10a^5(ax-1)^3}{(ax+1)^3} + \frac{5a^5(ax-1)^4}{(ax+1)^4} - \frac{a^5(ax-1)^5}{(ax+1)^5} - \frac{5a^5(ax-1)}{ax+1}} - \frac{1003\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128\,a^5} + \frac{\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}1\mathrm{i}\right)\,1003\mathrm{i}}{128\,a^5}$$

input
```
int(x^4*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*1003i)/(128*a^5) + (8*((a*x - 1)/(a*
x + 1))^(1/4))/a^5 + ((491*((a*x - 1)/(a*x + 1))^(1/4))/64 - (1409*((a*x -
 1)/(a*x + 1))^(5/4))/40 + (6113*((a*x - 1)/(a*x + 1))^(9/4))/96 - (307*((
a*x - 1)/(a*x + 1))^(13/4))/6 + (4117*((a*x - 1)/(a*x + 1))^(17/4))/192)/(
a^5 + (10*a^5*(a*x - 1)^2)/(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3
+ (5*a^5*(a*x - 1)^4)/(a*x + 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5
*(a*x - 1))/(a*x + 1)) - (1003*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5
)
```

3.105 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

3.105.1 Optimal result

Integrand size = 14, antiderivative size = 250

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{521\sqrt[4]{1-\frac{1}{ax}}x}{192a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{113\sqrt[4]{1-\frac{1}{ax}}x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{17\sqrt[4]{1-\frac{1}{ax}}x^3}{24a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{4\sqrt[4]{1+\frac{1}{ax}}}-\frac{475\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output
```
-2467/192*(1-1/a/x)^(1/4)/a^4/(1+1/a/x)^(1/4)-521/192*(1-1/a/x)^(1/4)*x/a^
3/(1+1/a/x)^(1/4)+113/96*(1-1/a/x)^(1/4)*x^2/a^2/(1+1/a/x)^(1/4)-17/24*(1-
1/a/x)^(1/4)*x^3/a/(1+1/a/x)^(1/4)+1/4*(1-1/a/x)^(1/4)*x^4/(1+1/a/x)^(1/4)
-475/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

3.105.2 Mathematica [A] (verified)

Time = 5.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{-3072e^{-\frac{1}{2}\coth^{-1}(ax)}+\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{5248e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{7376e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{6292e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+2850\,\mathrm{arc}}{384a^4}$$

input
```
Integrate[x^3/E^((5*ArcCoth[a*x])/2),x]
```

output
```
(-3072/E^(ArcCoth[a*x]/2) + (1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcC
oth[a*x]))^4 - (5248*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3
+ (7376*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 - (6292*E^((3*
ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(-1/2*ArcCoth[
a*x])] - 1425*Log[1 - E^(-1/2*ArcCoth[a*x])] + 1425*Log[1 + E^(-1/2*ArcCot
h[a*x])])/(384*a^4)
```

3.105.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {6721, 109, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^5}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{4}\int \frac{\left(17a-\frac{16}{x}\right)x^4}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\left(17a-\frac{16}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 168$$

$$\frac{-\frac{1}{3}\int \frac{\left(113a-\frac{102}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{\int \frac{\left(113a-\frac{102}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 168$$

$$\frac{-\frac{-\frac{1}{2}\int \frac{\left(521a-\frac{452}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{-\frac{\int \frac{\left(521a-\frac{452}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 168$$

3.105. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{-\frac{-\frac{-\int \frac{\left(1425a-\frac{1042}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{\left(1425a-\frac{1042}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 172$$

$$\frac{-\frac{-\frac{-\frac{2a\int \frac{1425x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 27$$

3.105. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{-\frac{-\frac{-\frac{1425a\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 104$$

$$\frac{-\frac{-\frac{-\frac{5700a\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 25$$

3.105. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

$$\frac{-\frac{-\frac{-\frac{\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-5700a\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 827$$

$$\frac{-\frac{-\frac{-\frac{5700a\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-}}{3\sqrt[4]{\frac{1}{ax}+}}}{8a^2}$$

$$\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 216$$

3.105. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

$$-\frac{-\frac{-\frac{-\frac{5700a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-}}{3\sqrt[4]{\frac{1}{ax}+}}}{8a^2}$$

$$\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \color{blue}{219}$$

$$-\frac{-\frac{-\frac{-\frac{5700a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{}}{3\sqrt[4]{\frac{1}{a}}}}{8a^2}$$

$$\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

input

```
Int[x^3/E^((5*ArcCoth[a*x])/2),x]
```

output

```
((1 - 1/(a*x))^(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) + ((-17*a*(1 - 1/(a*x))^
(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) - ((-113*a*(1 - 1/(a*x))^(1/4)*x^2)/(2*
(1 + 1/(a*x))^(1/4)) - ((-521*a*(1 - 1/(a*x))^(1/4)*x)/(1 + 1/(a*x))^(1/4)
 - ((4934*a*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) + 5700*a*(ArcTan[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)]/2))/(2*a))/(4*a))/(6*a))/(8*a^2)
```

3.105. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.105. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.105.4 Maple [F]

$$\int x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.44

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{2\left(48\,a^4x^4-136\,a^3x^3+226\,a^2x^2-521\,ax-2467\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)\right.}{384\,a^4}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/384*(2*(48*a^4*x^4 - 136*a^3*x^3 + 226*a^2*x^2 - 521*a*x - 2467)*((a*x -
 1)/(a*x + 1))^(1/4) + 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 1425*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1))/a^4
```

3.105.6 Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(x**3*((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx =$$
$$-\frac{1}{384}\,a\left(\frac{4\left(1573\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-2875\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+2343\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-657\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/384*a*(4*(1573*((a*x - 1)/(a*x + 1))^(13/4) - 2875*((a*x - 1)/(a*x + 1)
)^(9/4) + 2343*((a*x - 1)/(a*x + 1))^(5/4) - 657*((a*x - 1)/(a*x + 1))^(1/
4))/(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x -
1)^3*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 2850*arctan(((
a*x - 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^5 + 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5 + 3072*((a*x - 1)/(a*
x + 1))^(1/4)/a^5)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$

$$= \frac{1}{384}a\left(\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} + \frac{1425\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5} - \frac{1425\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5} - \frac{3072\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^5}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1
))/a^5 - 3072*((a*x - 1)/(a*x + 1))^(1/4)/a^5 + 4*(2343*(a*x - 1)*((a*x -
1)/(a*x + 1))^(1/4)/(a*x + 1) - 2875*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/
4)/(a*x + 1)^2 + 1573*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^3
- 657*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

3.105.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.87

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{475\,\mathrm{atan}\Big(\big(\frac{a\,x-1}{a\,x+1}\big)^{1/4}\Big)}{64\,a^4} - \frac{\frac{219\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32} - \frac{781\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32} + \frac{2875\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96} - \frac{1573\,\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4 + \frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{4\,a^4\,(a\,x-1)}{a\,x+1}} - \frac{8\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^4} - \frac{\mathrm{atan}\Big(\big(\frac{a\,x-1}{a\,x+1}\big)^{1/4}\,1\mathrm{i}\Big)\,475\mathrm{i}}{64\,a^4}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(475*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (8*((a*x - 1)/(a*x + 1)
)^(1/4))/a^4 - ((219*((a*x - 1)/(a*x + 1))^(1/4))/32 - (781*((a*x - 1)/(a*
x + 1))^(5/4))/32 + (2875*((a*x - 1)/(a*x + 1))^(9/4))/96 - (1573*((a*x -
1)/(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*
(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1
))/(a*x + 1)) - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*475i)/(64*a^4)
```

3.106 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

3.106.1 Optimal result

Integrand size = 14, antiderivative size = 213

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}}+\frac{55\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{55\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
287/24*(1-1/a/x)^(1/4)/a^3/(1+1/a/x)^(1/4)+61/24*(1-1/a/x)^(1/4)*x/a^2/(1+
1/a/x)^(1/4)-13/12*(1-1/a/x)^(1/4)*x^2/a/(1+1/a/x)^(1/4)+1/3*(1-1/a/x)^(1/
4)*x^3/(1+1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3-55
/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

3.106.2 Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.64

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{384e^{-\frac{1}{2}\coth^{-1}(ax)}+\frac{128e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{400e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{548e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-330\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+16}{48a^3}$$

input
```
Integrate[x^2/E^((5*ArcCoth[a*x])/2),x]
```

output
```
(384/E^(ArcCoth[a*x]/2) + (128*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth
[a*x]))^3 - (400*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (54
8*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) - 330*ArcTan[E^(-1/2*A
rcCoth[a*x])] + 165*Log[1 - E^(-1/2*ArcCoth[a*x])] - 165*Log[1 + E^(-1/2*A
rcCoth[a*x])])/(48*a^3)
```

3.106.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 109, 27, 168, 27, 168, 27, 172, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^4}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\int \frac{\left(13a-\frac{12}{x}\right)x^3}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(13a-\frac{12}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 168$$

$$\frac{-\frac{1}{2}\int \frac{\left(61a-\frac{52}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\left(61a-\frac{52}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 168$$

$$\frac{-\frac{-\int \frac{\left(165a-\frac{122}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{\int \frac{\left(165a-\frac{122}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 172$$

3.106. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{-\frac{-\frac{2a\int \frac{165x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{165a\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 104$$

$$\frac{-\frac{-\frac{660a\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 25$$

3.106. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{-\frac{-\frac{\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-660a\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 827$$

$$\frac{-\frac{-\frac{660a\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 216$$

$$\frac{-\frac{-\frac{660a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 219$$

3.106. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

$$\frac{-\frac{-\frac{660a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

input

```
Int[x^2/E^((5*ArcCoth[a*x])/2),x]
```

output

```
((1 - 1/(a*x))^(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) + ((-13*a*(1 - 1/(a*x))^
(1/4)*x^2)/(2*(1 + 1/(a*x))^(1/4)) - ((-61*a*(1 - 1/(a*x))^(1/4)*x)/(1 + 1
/(a*x))^(1/4) - ((574*a*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) + 660*a*(
ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(2*a))/(4*a))/(6*a^2)
```

3.106.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.106. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

rule 827

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.106.4 Maple [F]

$$\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

input

```
int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

output

```
int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.48

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\left(8a^3x^3-26a^2x^2+61ax+287\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\log\bigg(}{48a^3}$$

input

```
integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output

```
1/48*(2*(8*a^3*x^3 - 26*a^2*x^2 + 61*a*x + 287)*((a*x - 1)/(a*x + 1))^(1/4
) - 330*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*log(((a*x - 1)/(a*x + 1)
)^(1/4) + 1) + 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

3.106.6 Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(x**2*((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=$$

$$-\frac{1}{48}\,a\Bigg(\frac{4\left(137\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-174\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+69\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\log\Big(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1}{a^4}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/48*a*(4*(137*((a*x - 1)/(a*x + 1))^(9/4) - 174*((a*x - 1)/(a*x + 1))^(5
/4) + 69*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x
- 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 330*arctan((
(a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^4 - 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4 - 384*((a*x - 1)/(a*x
+ 1))^(1/4)/a^4)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=$$

$$-\frac{1}{48}\,a\left(\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{165\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{384\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^4}-\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
-1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^4 - 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^4 - 384*((a*x - 1)/(a*x + 1))^(1/4)/a^4 - 4*(174*(a*x - 1)*((a*x - 1)/(a
*x + 1))^(1/4)/(a*x + 1) - 137*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*
x + 1)^2 - 69*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^
3))
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{23\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{137\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^3} - \frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,55\mathrm{i}}{8\,a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
((23*((a*x - 1)/(a*x + 1))^(1/4))/4 - (29*((a*x - 1)/(a*x + 1))^(5/4))/2 +
 (137*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1
)^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (atan
(((a*x - 1)/(a*x + 1))^(1/4)*1i)*55i)/(8*a^3) + (8*((a*x - 1)/(a*x + 1))^(
1/4))/a^3 - (55*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3)
```

3.107 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$

3.107.1 Optimal result

Integrand size = 12, antiderivative size = 176

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx=-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}-\frac{25\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output
```
-25/2*(1-1/a/x)^(1/4)/a^2/(1+1/a/x)^(1/4)-5/4*(1-1/a/x)^(5/4)*x/a/(1+1/a/x
)^(1/4)+1/2*(1-1/a/x)^(9/4)*x^2/(1+1/a/x)^(1/4)-25/4*arctan((1+1/a/x)^(1/4
)/(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

3.107.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx=$$

$$-\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(32-90e^{2\coth^{-1}(ax)}+50e^{4\coth^{-1}(ax)}+25e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+e^{2\coth^{-1}(ax)}\right)^2\arctan\left(e^{\frac{1}{2}\coth^{-1}}\right.\right.}{4a^2\left(-1+e^{2\coth^{-1}(ax)}\right)^2}$$

input
```
Integrate[x/E^((5*ArcCoth[a*x])/2),x]
```

output
```
-1/4*(32 - 90*E^(2*ArcCoth[a*x]) + 50*E^(4*ArcCoth[a*x]) + 25*E^(ArcCoth[a
*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTan[E^(ArcCoth[a*x]/2)] - 25*E^(ArcC
oth[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTanh[E^(ArcCoth[a*x]/2)])/(a^2*
E^(ArcCoth[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2)
```

3.107.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6721, 107, 105, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x e^{-\frac{5}{2}\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4} x^3}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{5\int \frac{\left(1-\frac{1}{ax}\right)^{5/4} x^2}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}+\frac{x^2 \left(1-\frac{1}{ax}\right)^{9/4}}{2 \sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{105}$$

$$\frac{5 \left(-\frac{5\int \frac{\sqrt[4]{1-\frac{1}{ax}} x}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{x \left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2 \left(1-\frac{1}{ax}\right)^{9/4}}{2 \sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{105}$$

$$\frac{5\left(-\frac{5\left(\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 104$$

$$\frac{5\left(-\frac{5\left(4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 25$$

$$\frac{5\left(-\frac{5\left(\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 827$$

3.107. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$

$$\frac{5\left(-\frac{5\left(4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 216$$

$$\frac{5\left(-\frac{5\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 219$$

$$\frac{5\left(-\frac{5\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

input

```
Int[x/E^((5*ArcCoth[a*x])/2),x]
```

3.107. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$

output
```
((1 - 1/(a*x))^(9/4)*x^2)/(2*(1 + 1/(a*x))^(1/4)) + (5*(-(((1 - 1/(a*x))^(
5/4)*x)/(1 + 1/(a*x))^(1/4)) - (5*((4*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4) + 4*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2)))/(2*a)))/(4*a)
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.107.4 Maple [F]

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

input
```
int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.54

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2-9\,ax-43)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 - 9*a*x - 43)*((a*x - 1)/(a*x + 1))^(1/4) + 50*arctan(((
a*x - 1)/(a*x + 1))^(1/4)) + 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 25*
log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

3.107.6 Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(x*((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx=$$
$$-\frac{1}{8}\,a\left(\frac{4\left(13\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a^3}\right.$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/8*a*(4*(13*((a*x - 1)/(a*x + 1))^(5/4) - 9*((a*x - 1)/(a*x + 1))^(1/4))
/(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 50*arct
an(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4) +
 1)/a^3 + 25*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3 + 64*((a*x - 1)/(a*x
 + 1))^(1/4)/a^3)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{8}\,a\left(\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{64\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^3}+\frac{4\left(\frac{13\,(ax-}{}\right.}{}\right.$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^3 - 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 -
 64*((a*x - 1)/(a*x + 1))^(1/4)/a^3 + 4*(13*(a*x - 1)*((a*x - 1)/(a*x + 1)
)^(1/4)/(a*x + 1) - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^3*((a*x - 1)/(a*x +
1) - 1)^2))
```

3.107.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^2} - \frac{\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2} - \frac{13\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}} - \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,25\mathrm{i}}{4\,a^2}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(25*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - ((9*((a*x - 1)/(a*x + 1))
^(1/4))/2 - (13*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(
a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (8*((a*x - 1)/(a*x + 1))^(1/4)
)/a^2 - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*25i)/(4*a^2)
```

3.108 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$

3.108.1 Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx=\frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{5\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{5\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
10*(1-1/a/x)^(1/4)/a/(1+1/a/x)^(1/4)+(1-1/a/x)^(5/4)*x/(1+1/a/x)^(1/4)+5*a
rctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a-5*arctanh((1+1/a/x)^(1/4)/(1-1/a/
x)^(1/4))/a
```

3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.24

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx=\frac{8e^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},2,\frac{3}{4},e^{2\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^((-5*ArcCoth[a*x])/2),x]
```

output
```
(8*Hypergeometric2F1[-1/4, 2, 3/4, E^(2*ArcCoth[a*x])])/(a*E^(ArcCoth[a*x]
/2))
```

3.108.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6720, 105, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^2}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{5\int \frac{\sqrt[4]{1-\frac{1}{ax}}x}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{105}$$

$$\frac{5\left(\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{104}$$

$$\frac{5\left(4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 25$$

$$\frac{5\left(\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 827$$

$$\frac{5\left(4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 216$$

$$\frac{5\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 219$$

$$\frac{5\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[E^((-5*ArcCoth[a*x])/2),x]
```

3.108. $\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$

output
```
((1 - 1/(a*x))^(5/4)*x)/(1 + 1/(a*x))^(1/4) + (5*((4*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4) + 4*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2
 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2)))/(2*a)
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.108.4 Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4),x)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+9)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 9)*((a*x - 1)/(a*x + 1))^(1/4) - 10*arctan(((a*x - 1)/(a*x +
 1))^(1/4)) - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*log(((a*x - 1)/(a
*x + 1))^(1/4) - 1))/a
```

3.108.6 Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx =$$

$$-\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right.}{a^2}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 10
*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x + 1))^(1/
4) + 1)/a^2 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2 - 16*((a*x - 1)/(
a*x + 1))^(1/4)/a^2)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx =$$

$$-\frac{1}{2}a\left(\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right.}{a^2\left(\frac{ax-1}{ax+1}-\right.}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
-1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^2 - 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 -
16*((a*x - 1)/(a*x + 1))^(1/4)/a^2 + 4*((a*x - 1)/(a*x + 1))^(1/4)/(a^2*((
a*x - 1)/(a*x + 1) - 1)))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx=\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a}-\frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,5\mathrm{i}}{a}$$

input

```
int(((a*x - 1)/(a*x + 1))^(5/4),x)
```

output

```
(2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (atan(((a*
x - 1)/(a*x + 1))^(1/4)*1i)*5i)/a + (8*((a*x - 1)/(a*x + 1))^(1/4))/a - (5
*atan(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.109 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

3.109.1 Optimal result

Integrand size = 14, antiderivative size = 320

$$\begin{aligned}\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx &= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\ &\quad+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\\ &\quad+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\\ &\quad+\frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}\end{aligned}$$

output
```
-8*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4
))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(
1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-
1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1
/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1
-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

3.109.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.09

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -8e^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},1,\frac{7}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```

output
```
(-8*Hypergeometric2F1[-1/8, 1, 7/8, E^(4*ArcCoth[a*x])])/E^(ArcCoth[a*x]/2
)
```

3.109.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.90, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {6721, 109, 27, 35, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 109$$

$$-4a\int \frac{\left(a+\frac{1}{x}\right)x}{4a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 35$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 140$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 73$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 104$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 25$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 770$$

3.109. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}dx$

$$4\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 755$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 827$$

$$-4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+$$
$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 216$$

$$-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+$$
$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 219$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 1476$$

3.109. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{2-x}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\downarrow \text{1082}$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\downarrow \text{217}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\downarrow \text{1479}$$

3.109. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{ 25}$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{ 27}$$

3.109. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1103}$$

$$-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log\left(\right.}{}\right.\right.$$

$$\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

input

```
Int[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```

3.109. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

output
```
(-8*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) - 4*(ArcTan[(1 + 1/(a*x))^(1/
4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4
)]/2) + 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]
/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/S
qrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)
 + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1
/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.109. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.109. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

3.109.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

3.109.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = {} & \left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & -\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & +\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & -\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & -8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) \\ & +\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)\end{aligned}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

output
```
(1/2*I + 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
 - (1/2*I - 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)) + (1/2*I - 1/2)*sqrt(2)*log((I - 1)*sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4)) - (1/2*I + 1/2)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*((a*x - 1)/(a*x +
 1))^(1/4)) - 8*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1
))^(1/4) - 1)
```

3.109.6 Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4)/x, x)
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$=\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.}{a}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

output
```
1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1)/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$=\frac{1}{2}a\Bigg(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\sqrt{2}\log\Big(\sqrt{2}\Big($$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

output
```
1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
sqrt((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a
 + 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.37

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}$$

$$-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(1+1\mathrm{i})+\sqrt{2}\,\mathrm{atan}\Bigg(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4)/x,x)
```

output
```
2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)
*2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 +
1i) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 -
1i) - 8*((a*x - 1)/(a*x + 1))^(1/4)
```

3.110 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

3.110.1 Optimal result

Integrand size = 14, antiderivative size = 299

$$\begin{aligned}\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx =\ & \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}+5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} \\ & +\frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{5a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} \\ & +\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} \\ & -\frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}\end{aligned}$$

output
```
4*a*(1-1/a/x)^(5/4)/(1+1/a/x)^(1/4)+5*a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-5/
2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arcta
n(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+5/4*a*ln(1-(1-1/a/x)^
(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-5/4
*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^
(1/2))*2^(1/2)
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.10

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = 8ae^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},2,\frac{3}{4},-e^{2\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

output
```
(8*a*Hypergeometric2F1[-1/4, 2, 3/4, -E^(2*ArcCoth[a*x])])/E^(ArcCoth[a*x]
/2)
```

3.110.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 57, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{57}$$

$$5\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 60$$

$$5\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 73$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 770$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 755$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 1476$$

3.110. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\right.\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1082}$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{217}$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}}\right.\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1479}$$

3.110. $\int\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}}{2\sqrt{2}}\right.\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 25$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 27$$

3.110. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-}}\right.\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1103}$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\log\left(\right.\right.\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

3.110. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

output
```
(4*a*(1 - 1/(a*x))^(5/4))/(1 + 1/(a*x))^(1/4) + 5*(a*(1 - 1/(a*x))^(1/4)*(
1 + 1/(a*x))^(3/4) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 -
 x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x
^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
- x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2))
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.110. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.110.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

3.110.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.62

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{5\left(-a^4\right)^{\frac{1}{4}}x\log\left(5\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+5\left(-a^4\right)^{\frac{1}{4}}\right)+5i\left(-a^4\right)^{\frac{1}{4}}x\log\left(5\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+5i\left(-a^4\right)^{\frac{1}{4}}\right)-5i\left(-a^4\right)^{\frac{1}{4}}x\log}{2\,x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(5*(-a^4)^(1/4)*x*log(5*a*((a*x - 1)/(a*x + 1))^(1/4) + 5*(-a^4)^(1/4
)) + 5*I*(-a^4)^(1/4)*x*log(5*a*((a*x - 1)/(a*x + 1))^(1/4) + 5*I*(-a^4)^(
1/4)) - 5*I*(-a^4)^(1/4)*x*log(5*a*((a*x - 1)/(a*x + 1))^(1/4) - 5*I*(-a^4
)^(1/4)) - 5*(-a^4)^(1/4)*x*log(5*a*((a*x - 1)/(a*x + 1))^(1/4) - 5*(-a^4)
^(1/4)) - 2*(9*a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

3.110.6 Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}\,dx$$

input

```
integrate(((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

output

```
Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**2, x)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\right.$$

input

```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

output

```
-1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x -
 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

output
```
-1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x -
 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

3.110.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.35

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = 8\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} + 5\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} 1\mathrm{i}\right) + \frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1} + (-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,5$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4)/x^2,x)
```

output
```
8*a*((a*x - 1)/(a*x + 1))^(1/4) + (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/
(a*x + 1))^(1/4))*5i + 5*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1)
)^(1/4)*1i) + (2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.111 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.111.1 Optimal result

Integrand size = 14, antiderivative size = 351

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{25a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} -$$

output
```
-2*a^2*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)-25/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^
(3/4)-5/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+25/8*a^2*arctan(-1+(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+25/8*a^2*arctan(1+(1-1/a/x)^(1/4)*
2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/16*a^2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1
+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+25/16*a^2*ln(1+(1-1
/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/
2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.29

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{-\frac{1}{2}\coth^{-1}(ax)}\bigg(3 + e^{2\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) + e^{2\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) + 2e^{2\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\bigg)$$

input
```
Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

output
```
(-8*a^2*(3 + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCo
th[a*x])] + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCot
h[a*x])] + 2*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCo
th[a*x])]))/(3*E^(ArcCoth[a*x]/2))
```

3.111.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 87, 60, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\left(1+\frac{1}{ax}\right)^{5/4}x}d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$-5a\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\big\downarrow \; 60$$

$$-5a\left(\frac{5}{4}\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\big\downarrow \; 60$$

$$-5a\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\big\downarrow \; 73$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\big\downarrow \; 770$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\big\downarrow \; 755$$

3.111. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/}$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 1476$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 1082$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 217$$

3.111. $\int\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1\right.}{}\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 1479$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d}{2\sqrt{2}}\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 25$$

3.111. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}}{2\sqrt{2}}\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 1103$$

3.111. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

$$-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-$$

$$5a\Bigg(\frac{5}{4}\Bigg(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\Bigg(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\Bigg($$

input

```
Int[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

output

```
(-2*a^2*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) - 5*a*((a*(1 - 1/(a*x))^(
5/4)*(1 + 1/(a*x))^(3/4))/2 + (5*(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4
) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/S
qrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqr
t[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) +
 x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4
) + x^(-2)]/(2*Sqrt[2]))/2)))/4)
```

3.111.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.111. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.111.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

3.111.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{25\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(25\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+25\left(-a^8\right)^{\frac{1}{4}}\right)+25i\left(-a^8\right)^{\frac{1}{4}}x^2\log\left(25\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+25i\left(-a^8\right)^{\frac{1}{4}}\right)-25i\,(-}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

output
```
1/8*(25*(-a^8)^(1/4)*x^2*log(25*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 25*(-a^8
)^(1/4)) + 25*I*(-a^8)^(1/4)*x^2*log(25*a^2*((a*x - 1)/(a*x + 1))^(1/4) +
25*I*(-a^8)^(1/4)) - 25*I*(-a^8)^(1/4)*x^2*log(25*a^2*((a*x - 1)/(a*x + 1)
)^(1/4) - 25*I*(-a^8)^(1/4)) - 25*(-a^8)^(1/4)*x^2*log(25*a^2*((a*x - 1)/(
a*x + 1))^(1/4) - 25*(-a^8)^(1/4)) - 2*(43*a^2*x^2 + 9*a*x - 2)*((a*x - 1)
/(a*x + 1))^(1/4))/x^2
```

3.111.6 Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**3, x)
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

output
```
1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4
) - 8*(13*a*((a*x - 1)/(a*x + 1))^(5/4) + 9*a*((a*x - 1)/(a*x + 1))^(1/4))
/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.69

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$= \frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

output
```
1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4
) - 8*(13*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 9*a*((a*x -
1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -8\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - \frac{\frac{9\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{13\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}$$
$$-\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,25\mathrm{i}}{4}$$
$$-\frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)}{4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4)/x^3,x)
```

output

```
- 8*a^2*((a*x - 1)/(a*x + 1))^(1/4) - ((9*a^2*((a*x - 1)/(a*x + 1))^(1/4))
/2 + (13*a^2*((a*x - 1)/(a*x + 1))^(5/4))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2
*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a
*x + 1))^(1/4))*25i)/4 - (25*(-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*
x + 1))^(1/4)*1i))/4
```

3.111. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

3.112 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

3.112.1 Optimal result

Integrand size = 14, antiderivative size = 385

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}-$$

output
```
2*a^3*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)+55/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(
3/4)+11/4*a^3*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+1/3*a^3*(1-1/a/x)^(9/4)*(1+1
/a/x)^(3/4)-55/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2
^(1/2)-55/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
+55/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2))*2^(1/2)-55/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1
/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.27

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= a^3\left(\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(96+425e^{2\coth^{-1}(ax)}+462e^{4\coth^{-1}(ax)}+165e^{6\coth^{-1}(ax)}\right)}{12\left(1+e^{2\coth^{-1}(ax)}\right)^3}\right.$$
$$\left.-\frac{55}{32}\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

input
```
Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

output
```
a^3*((96 + 425*E^(2*ArcCoth[a*x]) + 462*E^(4*ArcCoth[a*x]) + 165*E^(6*ArcC
oth[a*x]))/(12*E^(ArcCoth[a*x]/2)*(1 + E^(2*ArcCoth[a*x]))^3) - (55*RootSu
m[1 + #1^4 & , (ArcCoth[a*x] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ]
)/32)
```

3.112.3 Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.86, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 100, 27, 90, 60, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\left(1+\frac{1}{ax}\right)^{5/4}x^2}d\frac{1}{x}$$
$$\Big\downarrow \text{100}$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-2a^3\int -\frac{\left(5a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{5/4}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow\ 27$$

$$a\int \frac{\left(5a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow\ 90$$

$$a\left(\frac{11}{2}a\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{3}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{9/4}\right)+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow\ 60$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)+\frac{1}{3}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{9/4}\right)+$$
$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow\ 60$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)+\frac{1}{3}a$$
$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow\ 73$$

3.112. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}dx$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int\frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)+\frac{1}{3}a^2\left(\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 770$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int\frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)+\frac{1}{3}a^2\left(\frac{1}{ax}\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 755$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 1476$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 1082$$

3.112. $\int\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{217}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan}{}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1479}$$

3.112. $\int\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$a\Bigg(\frac{11}{2}a\Bigg(\frac{5}{4}\Bigg(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\Bigg(\frac{1}{2}\Bigg(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}}{2\sqrt{2}}$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{ 25}$$

$$a\Bigg(\frac{11}{2}a\Bigg(\frac{5}{4}\Bigg(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\Bigg(\frac{1}{2}\Bigg(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d}{2\sqrt{2}}$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{ 27}$$

3.112. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 1103$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}+$$

$$a\left(\frac{1}{3}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{9/4}+\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\right.\right.\right.\right.\right.$$

input
```
Int[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

3.112. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

output
```
(2*a^3*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) + a*((a^2*(1 - 1/(a*x))^(9
/4)*(1 + 1/(a*x))^(3/4))/3 + (11*a*((a*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(
3/4))/2 + (5*(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) - 2*a*((-(ArcTan[1
 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 +
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[
1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + L
og[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[
2]))/2)))/4))/2)
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.112. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.112.4 Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

3.112.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.56

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$
$$-\frac{165\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(55\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+55\left(-a^{12}\right)^{\frac{1}{4}}\right)+165i\left(-a^{12}\right)^{\frac{1}{4}}x^3\log\left(55\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+55i\left(-a^{12}\right)^{\frac{1}{4}}\right)-}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

output
```
-1/48*(165*(-a^12)^(1/4)*x^3*log(55*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 55*(
-a^12)^(1/4)) + 165*I*(-a^12)^(1/4)*x^3*log(55*a^3*((a*x - 1)/(a*x + 1))^(
1/4) + 55*I*(-a^12)^(1/4)) - 165*I*(-a^12)^(1/4)*x^3*log(55*a^3*((a*x - 1)
/(a*x + 1))^(1/4) - 55*I*(-a^12)^(1/4)) - 165*(-a^12)^(1/4)*x^3*log(55*a^3
*((a*x - 1)/(a*x + 1))^(1/4) - 55*(-a^12)^(1/4)) - 2*(287*a^3*x^3 + 61*a^2
*x^2 - 26*a*x + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

3.112.6 Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**4, x)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)\right.\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

output
```
-1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(
a*x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/
(a*x + 1))^(1/4) - 8*(137*a^2*((a*x - 1)/(a*x + 1))^(9/4) + 174*a^2*((a*x
- 1)/(a*x + 1))^(5/4) + 69*a^2*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)/(
a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\Bigg(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\Bigg(-\frac{1}{2}\sqrt{2}\Bigg(\sqrt{2}-2\Bigg(\frac{ax-1}{ax+1}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

output
```
-1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(
a*x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/
(a*x + 1))^(1/4) - 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x +
 1) + 137*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 69*a^2
*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{23\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{29\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{137\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1} + 8\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+\frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,55{}\mathrm{i}}{8} + \frac{55\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)}{8}$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4)/x^4,x)
```

output
```
((23*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (29*a^3*((a*x - 1)/(a*x + 1))^(5
/4))/2 + (137*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x +
 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 8*a^3*((a
*x - 1)/(a*x + 1))^(1/4) + ((-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x
 + 1))^(1/4))*55i)/8 + (55*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x
+ 1))^(1/4)*1i))/8
```

3.112. $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

3.113 $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

3.113.1 Optimal result

Integrand size = 12, antiderivative size = 285

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x+\frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^2 +\frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3-\frac{19\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}} +\frac{19}{81}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)-\frac{19}{324}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{19}{324}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}+\frac{\sqrt[6]{1+}}{\sqrt[6]{\frac{-1+}{x}}}\right.$$

output
```
11/27*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+7/18*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)
*x^2+1/3*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x^3+19/81*arctanh((1+1/x)^(1/6)/((
-1+x)/x)^(1/6))-19/324*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(1+1/x)^(1/6)/(
(-1+x)/x)^(1/6))+19/324*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+1/x)^(1/6)/
((-1+x)/x)^(1/6))-19/162*arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3
^(1/2))*3^(1/2)+19/162*arctan(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(
1/2))*3^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.66

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx=\frac{1}{324}\left(\frac{864e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^3}+\frac{1368e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{732e^{\frac{1}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+38\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)+38\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-38\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)+38\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)-19\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+19\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)*x^2,x]
```

output
```
((864*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^3 + (1368*E^(ArcCoth[x]/3)
)/(-1 + E^(2*ArcCoth[x]))^2 + (732*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]
)) + 38*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] + 38*Sqrt[3]*Arc
Tan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 38*Log[1 - E^(ArcCoth[x]/3)] + 38*
Log[1 + E^(ArcCoth[x]/3)] - 19*Log[1 - E^(ArcCoth[x]/3) + E^((2*ArcCoth[x]
)/3)] + 19*Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/324
```

3.113.3 Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{\frac{1}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}x^4}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3-\frac{1}{3}\int \frac{\left(7+\frac{6}{x}\right)x^3}{3\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3-\frac{1}{9}\int \frac{\left(7+\frac{6}{x}\right)x^3}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{9}\left(\frac{1}{2}\int -\frac{\left(22+\frac{21}{x}\right)x^2}{3\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}+\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2\right)+\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2-\frac{1}{6}\int \frac{\left(22+\frac{21}{x}\right)x^2}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}\right)+\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{9}\left(\frac{1}{6}\left(\int -\frac{19x}{3\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}+22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x\right)+\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2\right)+\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-\frac{19}{3}\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}\right)+\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2\right)+\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow \text{104}$$

3.113. $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\int\frac{1}{\frac{1}{x^6}-1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2\right)+$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow 754$$

$$\frac{1}{9}\Bigg(\frac{1}{6}\Bigg(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\Bigg(-\frac{1}{3}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{2\left(-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow 27$$

$$\frac{1}{9}\Bigg(\frac{1}{6}\Bigg(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\Bigg(-\frac{1}{3}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow 219$$

3.113. $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\Big\downarrow$ 1142

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(\frac{1}{2}\int-\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{3}{2}\int\frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\right.\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\Big\downarrow$ 25

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\right.\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\Big\downarrow$ 1083

3.113. $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

$$\frac{\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(3\int\frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-1\right)-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right.\right.\right.\right.\right.}{\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right.\right.\right.\right.}{\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3}$$

$$\Big\downarrow \textcolor{blue}{1103}$$

$$\frac{\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{1}{x^2}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\right.\right.\right.\right.}{\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3}$$

input
```
Int[E^(ArcCoth[x]/3)*x^2,x]
```

3.113. $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

output

```
((1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6)*x^3)/3 + ((7*(1 - x^(-1))^(5/6)*(1
+ x^(-1))^(1/6)*x^2)/2 + (22*(1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6)*x - 38*
(-1/3*ArcTanh[(1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6)] + (-(Sqrt[3]*ArcTan[(
-1 + (2*(1 + x^(-1))^(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]]) + Log[1 - (1 + x
^(-1))^(1/6)/(1 - x^(-1))^(1/6) + x^(-2)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2
*(1 + x^(-1))^(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]]) - Log[1 + (1 + x^(-1))^
(1/6)/(1 - x^(-1))^(1/6) + x^(-2)]/2)/6))/6)/9
```

3.113.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.113. $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

3.113.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.14 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.47

method	result
trager	$\frac{(1+x)(18x^{2}+21x+22)\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}}{54} - \frac{19\ln\left(3\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}x+9\,\mathrm{RootOf}\left(9\_Z^{2}+3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+3\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}+9\,\mathrm{RootOf}\left(9\_\right.\right.}{}$
risch	Expression too large to display

input
```
int(1/((x-1)/(1+x))^(1/6)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/54*(1+x)*(18*x^2+21*x+22)*(-(1-x)/(1+x))^(5/6)-19/162*ln(3*(-(1-x)/(1+x)
)^(5/6)*x+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^
(5/6)+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/3)*
x-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x-3*(-(1-x)/(1+x))^(2/3)-1
8*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1
-x)/(1+x))^(1/3)*x+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)
/(1+x))^(1/3)*x-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(
1+x))^(1/3)-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^
(1/6)*x-3*(-(1-x)/(1+x))^(1/6)+3*RootOf(9*_Z^2+3*_Z+1)+2)+19/54*RootOf(9*_
Z^2+3*_Z+1)*ln(3*(-(1-x)/(1+x))^(5/6)*x-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(
1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x
))^(2/3)-3*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(2/3)-6*(-(1-x)/(1+x))^
(1/2)*x+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-6*(-(1-x)/(1+x))^(
1/2)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/3)*
x+3*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^(1/6)-6
*RootOf(9*_Z^2+3*_Z+1)-1)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.61

$$
\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx &= \frac{1}{54}\left(18\,x^3+39\,x^2+43\,x+22\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} \\
&\quad-\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\,\sqrt{3}\right) \\
&\quad-\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\,\sqrt{3}\right) \\
&\quad+\frac{19}{324}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
&\quad-\frac{19}{324}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
&\quad+\frac{19}{162}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{162}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)
\end{aligned}
$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="fricas")
```

output
```
1/54*(18*x^3 + 39*x^2 + 43*x + 22)*((x - 1)/(x + 1))^(5/6) - 19/162*sqrt(3
)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 19/162*sqrt(
3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 19/324*log(
((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x -
1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x
+ 1))^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

3.113.6 Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx=\int\frac{x^2}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/6)*x**2,x)
```

output
```
Integral(x**2/((x - 1)/(x + 1))**(1/6), x)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.77

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = &-\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)\\ &-\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)\\ &-\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}-8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{3\,(x-1)}{x+1}-\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}\\ &+\frac{19}{324}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &-\frac{19}{324}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &+\frac{19}{162}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{162}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="maxima")
```

output
```
-19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/1
62*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/27*(19*
((x - 1)/(x + 1))^(17/6) - 8*((x - 1)/(x + 1))^(11/6) + 61*((x - 1)/(x + 1
))^(5/6))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3
 - 1) + 19/324*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1)
- 19/324*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/1
62*log(((x - 1)/(x + 1))^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) -
 1)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.75

$$
\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx =& -\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) \\
&- \frac{19}{162}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) \\
&+ \frac{\frac{8\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1} - \frac{19\,(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^2} - 61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{x-1}{x+1}-1\right)^3} \\
&+ \frac{19}{324}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
&- \frac{19}{324}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
&+ \frac{19}{162}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{19}{162}\,\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)
\end{aligned}
$$

input

```
integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="giac")
```

output

```
-19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/1
62*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/27*(8*(
x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) - 19*(x - 1)^2*((x - 1)/(x + 1))^(5
/6)/(x + 1)^2 - 61*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1) - 1)^3 + 19/3
24*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log
(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x -
 1)/(x + 1))^(1/6) + 1) - 19/162*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

3.113.9 Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.59

$$
\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx =& -\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\,1\mathrm{i}\right)\,19\mathrm{i}}{81} - \frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27} - \frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27} + \frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3\,(x-1)}{x+1} - \frac{3\,(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1} \\
&- \mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}\,4952198\mathrm{i}}{14348907\left(-\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19\,\sqrt{3}}{162}-\frac{19}{162}\mathrm{i}\right) - \mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}\,4952198\mathrm{i}}{14348907\left(\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right.}\right.
\end{aligned}
$$

input
```
int(x^2/((x - 1)/(x + 1))^(1/6),x)
```

output
```
- (atan(((x - 1)/(x + 1))^(1/6)*1i)*19i)/81 - ((61*((x - 1)/(x + 1))^(5/6)
)/27 - (8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)
/((3*(x - 1))/(x + 1) - (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
 - atan((((x - 1)/(x + 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/1
4348907 - 2476099/14348907)))*((19*3^(1/2))/162 - 19i/162) - atan((((x - 1
)/(x + 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/14348907 + 247609
9/14348907)))*((19*3^(1/2))/162 + 19i/162)
```

3.113. $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

3.114 $\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx$

3.114.1 Optimal result

Integrand size = 10, antiderivative size = 258

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) - \frac{1}{36}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{36}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

output
```
1/6*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+1/2*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)*x^
2+1/9*arctanh((1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/36*ln(1+(1+1/x)^(1/3)/((-1
+x)/x)^(1/3)-(1+1/x)^(1/6)/((-1+x)/x)^(1/6))+1/36*ln(1+(1+1/x)^(1/3)/((-1+
x)/x)^(1/3)+(1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/18*arctan(1/3*(1-2*(1+1/x)^(
1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)+1/18*arctan(1/3*(1+2*(1+1/x)^(1/6)
/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=\frac{1}{36}\left(\frac{72e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{84e^{\frac{1}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)+2\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)-\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)*x,x]
```

output
```
((72*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^2 + (84*E^(ArcCoth[x]/3))/(
-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3
]] + 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E^(Arc
Coth[x]/3)] + 2*Log[1 + E^(ArcCoth[x]/3)] - Log[1 - E^(ArcCoth[x]/3) + E^(
(2*ArcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/36
```

3.114.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {6721, 107, 105, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}x^3}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2-\frac{1}{6}\int \frac{\sqrt[6]{1+\frac{1}{x}}x^2}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow 105$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-\frac{1}{3}\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow 104$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\int \frac{1}{\frac{1}{x^6}-1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow 754$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{2\left(-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow 27$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow 219$$

3.114. $\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{ar}\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$\downarrow$ 1142

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(\frac{1}{2}\int-\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{3}{2}\int\frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$\downarrow$ 25

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$\downarrow$ 1083

3.114. $\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx$

$$\frac{\frac{1}{6}\Bigg(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\Bigg(\frac{1}{6}\left(3\int\frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-1\right)-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{6}}{\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{6}\Bigg(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\Bigg(\frac{1}{6}\Bigg(-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\Bigg)\Bigg)+\frac{1}{6}\Bigg(}{\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{1}{6}\Bigg(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\Bigg(\frac{1}{6}\Bigg(\frac{1}{2}\log\left(\frac{1}{x^2}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\Bigg)\Bigg)+\frac{1}{6}\Bigg(-\sqrt{3}\,\mathrm{arc}}{\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2}$$

input

```
Int[E^(ArcCoth[x]/3)*x,x]
```

3.114. $\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx$

output

```
((1 - x^(-1))^(5/6)*(1 + x^(-1))^(7/6)*x^2)/2 + ((1 - x^(-1))^(5/6)*(1 + x
^(-1))^(1/6)*x - 2*(-1/3*ArcTanh[(1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6)] +
(-(Sqrt[3]*ArcTan[(-1 + (2*(1 + x^(-1))^(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]
]) + Log[1 - (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6) + x^(-2)]/2)/6 + (-(Sqr
t[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]]) - Lo
g[1 + (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6) + x^(-2)]/2)/6))/6
```

3.114.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.114. $\int e^{\frac{1}{3}\coth^{-1}(x)} x \, dx$

3.114.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.40 (sec) , antiderivative size = 1158, normalized size of antiderivative = 4.49

method	result	size
trager	Expression too large to display	1158
risch	Expression too large to display	1702

input

```
int(1/((x-1)/(1+x))^(1/6)*x,x,method=_RETURNVERBOSE)
```

output

```
1/6*(1+x)*(4+3*x)*(-(1-x)/(1+x))^(5/6)+1/18*ln(3*(-(1-x)/(1+x))^(5/6)*x+9*
RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+9*Root
Of(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(
9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_
Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(
1/3)*x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3
)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)+
9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-
(1-x)/(1+x))^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)-1/6*ln(3*(-(1-x)/(1+x))^(5/6
)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+
9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x+18*R
ootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18*Root
Of(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1
+x))^(1/3)*x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x)
)^(1/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^
(1/3)+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*
x-3*(-(1-x)/(1+x))^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)*RootOf(9*_Z^2-3*_Z+1)+
1/6*RootOf(9*_Z^2-3*_Z+1)*ln(3*(-(1-x)/(1+x))^(5/6)*x-9*RootOf(9*_Z^2-3*_Z
+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)-9*RootOf(9*_Z^2-3*_Z+1)*
(-(1-x)/(1+x))^(2/3)+6*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(9*_Z^2-3*_Z+1)*...
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.65

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = {} & \frac{1}{6}\left(3\,x^{2}+7\,x+4\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}-\frac{1}{18}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\,\sqrt{3}\right)\\ & -\frac{1}{18}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\,\sqrt{3}\right)\\ & +\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ & -\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ & +\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="fricas")
```

output
```
1/6*(3*x^2 + 7*x + 4)*((x - 1)/(x + 1))^(5/6) - 1/18*sqrt(3)*arctan(2/3*sq
rt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 1/18*sqrt(3)*arctan(2/3*sqr
t(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 1/36*log(((x - 1)/(x + 1))^(
1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - (
(x - 1)/(x + 1))^(1/6) + 1) + 1/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18
*log(((x - 1)/(x + 1))^(1/6) - 1)
```

3.114.6 Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=\int\frac{x}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/6)*x,x)
```

output
```
Integral(x/((x - 1)/(x + 1))**(1/6), x)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.75

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx =& -\frac{1}{18}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)\\ &-\frac{1}{18}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)\\ &+\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2\,(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &-\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &+\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="maxima")
```

output
```
-1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*s
qrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/3*(((x - 1)
/(x + 1))^(11/6) - 7*((x - 1)/(x + 1))^(5/6))/(2*(x - 1)/(x + 1) - (x - 1)
^2/(x + 1)^2 - 1) + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(
1/6) + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1
) + 1/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(((x - 1)/(x + 1))^(1/
6) - 1)
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.74

$$
\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = {} & -\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) \\
& -\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) \\
& -\frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}-1\right)^{2}}+\frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
& -\frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
& +\frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)
\end{aligned}
$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="giac")
```

output
```
-1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*s
qrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/3*((x - 1)*
((x - 1)/(x + 1))^(5/6)/(x + 1) - 7*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x +
 1) - 1)^2 + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) +
1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1/1
8*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(abs(((x - 1)/(x + 1))^(1/6)
- 1))
```

3.114.9 Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.55

$$
\begin{aligned}
& \int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx \\
& = \frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3}-\frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2\,(x-1)}{x+1}+1}-\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\,1\mathrm{i}\right)\,1\mathrm{i}}{9} \\
& \quad -\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}\,2\mathrm{i}}{243\,\left(-\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\,\left(\frac{\sqrt{3}}{18}-\frac{1}{18}\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}\,2\mathrm{i}}{243\,\left(\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\,\left(\frac{\sqrt{3}}{18}+\frac{1}{18}\mathrm{i}\right)
\end{aligned}
$$

input

```
int(x/((x - 1)/(x + 1))^(1/6),x)
```

output

```
((7*((x - 1)/(x + 1))^(5/6))/3 - ((x - 1)/(x + 1))^(11/6)/3)/((x - 1)^2/(x
 + 1)^2 - (2*(x - 1))/(x + 1) + 1) - (atan(((x - 1)/(x + 1))^(1/6)*1i)*1i)
/9 - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)/243 - 1/243)))*(
3^(1/2)/18 - 1i/18) - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)
/243 + 1/243)))*(3^(1/2)/18 + 1i/18)
```

3.114. $\int e^{\frac{1}{3}\coth^{-1}(x)} x \, dx$

3.115 $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

3.115.1 Optimal result

Integrand size = 8, antiderivative size = 223

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx=\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x-\frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)-\frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)+\frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

output
```
(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+2/3*arctanh((1+1/x)^(1/6)/((-1+x)/x)^(1/6
))-1/6*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(1+1/x)^(1/6)/((-1+x)/x)^(1/6))
+1/6*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1
/3*arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)+1/3*ar
ctan(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)
```

3.115.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.16

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx=2e^{\frac{1}{3}\coth^{-1}(x)}\left(\frac{1}{-1+e^{2\coth^{-1}(x)}}+\text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},e^{2\coth^{-1}(x)}\right)\right)$$

input

```
Integrate[E^(ArcCoth[x]/3),x]
```

output

```
2*E^(ArcCoth[x]/3)*((-1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6,
1, 7/6, E^(2*ArcCoth[x])])
```

3.115.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.375$, Rules used = {6720, 105, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}x^2}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-\frac{1}{3}\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \text{104}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\int \frac{1}{\frac{1}{x^6}-1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}$$

$$\Big\downarrow \textcolor{blue}{754}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{2\left(-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{2\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right.$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\Big\downarrow \textcolor{blue}{1142}$$

3.115. $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-$$

$$2\Bigg(\frac{1}{6}\Bigg(\frac{1}{2}\int -\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\Bigg)+\frac{1}{6}\Bigg(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}}$$

$$\Big\downarrow \ 25$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-$$

$$2\Bigg(\frac{1}{6}\Bigg(-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\Bigg)+\frac{1}{6}\Bigg(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}}$$

$$\Big\downarrow \ 1083$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-$$

$$2\Bigg(\frac{1}{6}\Bigg(3\int \frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\Bigg)+\frac{1}{6}\Bigg(3\int \frac{1}{-3-\frac{1}{x^2}}d\Bigg(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}$$

$$\Big\downarrow \ 217$$

3.115. $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-$$
$$2\Bigg(\frac{1}{6}\Bigg(-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\sqrt{3}\arctan\Bigg(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\Bigg)\Bigg)+\frac{1}{6}\Bigg(-\frac{1}{2}\int\frac{\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d$$

$$\Big\downarrow \text{1103}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-$$
$$2\Bigg(\frac{1}{6}\Bigg(\frac{1}{2}\log\Bigg(\frac{1}{x^2}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1\Bigg)-\sqrt{3}\arctan\Bigg(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\Bigg)\Bigg)+\frac{1}{6}\Bigg(-\sqrt{3}\arctan\Bigg(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1}{\sqrt{3}}\Bigg)-\frac{1}{2}\mathrm{lo}$$

input
```
Int[E^(ArcCoth[x]/3),x]
```

output
```
(1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6)*x - 2*(-1/3*ArcTanh[(1 + x^(-1))^(1/
6)/(1 - x^(-1))^(1/6)] + (-(Sqrt[3]*ArcTan[(-1 + (2*(1 + x^(-1))^(1/6))/(1
 - x^(-1))^(1/6))/Sqrt[3]]) + Log[1 - (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6
) + x^(-2)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/(1 - x^(-
1))^(1/6))/Sqrt[3]]) - Log[1 + (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6) + x^(
-2)]/2)/6)
```

3.115.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.115. $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.115. $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.115.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.15 (sec) , antiderivative size = 1151, normalized size of antiderivative = 5.16

method	result	size
trager	Expression too large to display	1151
risch	Expression too large to display	1761

input
```
int(1/((x-1)/(1+x))^(1/6),x,method=_RETURNVERBOSE)
```

output
```
(1+x)*(-(1-x)/(1+x))^(5/6)+RootOf(9*_Z^2-3*_Z+1)*ln(3*(-(1-x)/(1+x))^(5/6)
*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)-9
*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+6*(-(1-x)/(1+x))^(2/3)*x-18*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+6*(-(1-x)/(1+x))^(2/3)-18*RootO
f(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+6*(-(1-x)/(1+x))^(1/2)*x-18*RootOf(9
*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)-18*RootOf(9*_Z
^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(9*_Z^2-3
*_Z+1)*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^(1/3)-9*RootOf(9*_Z^2-3*_Z+
1)*(-(1-x)/(1+x))^(1/6)-3*RootOf(9*_Z^2-3*_Z+1)-1)+1/3*ln(3*(-(1-x)/(1+x))
^(5/6)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(
5/6)+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x
+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18
*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-
x)/(1+x))^(1/3)*x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/
(1+x))^(1/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1
+x))^(1/3)+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(
1/6)*x-3*(-(1-x)/(1+x))^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)-ln(3*(-(1-x)/(1+x
))^(5/6)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))
^(5/6)+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)
*x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/...
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.72

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx &= (x+1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} - \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \frac{1}{3}\sqrt{3}\right)\\ &\quad - \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right)\\ &\quad + \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right)\\ &\quad - \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right)\\ &\quad + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - 1\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="fricas")
```

output
```
(x + 1)*((x - 1)/(x + 1))^(5/6) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/
(x + 1))^(1/6) + 1/3*sqrt(3)) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x
 + 1))^(1/6) - 1/3*sqrt(3)) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(
x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(
1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1
))^(1/6) - 1)
```

3.115.6 Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = \int \frac{1}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/6),x)
```

output
```
Integral(((x - 1)/(x + 1))**(-1/6), x)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$
\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = & -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) \\
& -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) \\
& -\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
& -\frac{1}{6}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
& +\frac{1}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)
\end{aligned}
$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqr
t(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x +
 1))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x -
 1)/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x +
1))^(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(
x + 1))^(1/6) - 1)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx =& -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)\\ &-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)\\ &-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &-\frac{1}{6}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &+\frac{1}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{3}\,\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqr
t(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x +
 1))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x -
 1)/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x +
1))^(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(abs(((x -
1)/(x + 1))^(1/6) - 1))
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx =& -\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\,1\mathrm{i}\right)\,2\mathrm{i}}{3}-\frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}-1}\\ &-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}64\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}64\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)\end{aligned}$$

input
```
int(1/((x - 1)/(x + 1))^(1/6),x)
```

output

```
- (atan(((x - 1)/(x + 1))^(1/6)*1i)*2i)/3 - (2*((x - 1)/(x + 1))^(5/6))/((
x - 1)/(x + 1) - 1) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i - 32
))*(3^(1/2)/3 - 1i/3) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i +
32))*(3^(1/2)/3 + 1i/3)
```

3.115. $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

3.116 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

3.116.1 Optimal result

Integrand size = 12, antiderivative size = 402

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = &-\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)+\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)\\
&-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)\\
&+2\arctan\left(\frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+2\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)\\
&-\frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)\\
&+\frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{\frac{-1+x}{x}}}+\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)\\
&+\frac{1}{2}\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)\\
&-\frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)\end{aligned}$$

output

```
2*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+arctan(2*((-1+x)/x)^(1/6)/(1+1/x)
^(1/6)-3^(1/2))+arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+2*arctanh
((1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/2*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(
1+1/x)^(1/6)/((-1+x)/x)^(1/6))+1/2*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+
1/x)^(1/6)/((-1+x)/x)^(1/6))-arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6
))*3^(1/2))*3^(1/2)+arctan(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2
))*3^(1/2)+1/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2
)/(1+1/x)^(1/6))*3^(1/2)-1/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x
)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

3.116.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx=\frac{12}{7}e^{\frac{7}{3}\coth^{-1}(x)}\,\text{Hypergeometric2F1}\left(\frac{7}{12},1,\frac{19}{12},e^{4\coth^{-1}(x)}\right)$$

input

```
Integrate[E^(ArcCoth[x]/3)/x,x]
```

output

```
(12*E^((7*ArcCoth[x])/3)*Hypergeometric2F1[7/12, 1, 19/12, E^(4*ArcCoth[x]
)])/7
```

3.116.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.92, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6721, 140, 73, 104, 754, 27, 219, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}x}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow 140$$

$$-\int \frac{1}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}-\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow 73$$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow 104$$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-6\int \frac{1}{\frac{1}{x^6}-1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}$$

$$\Big\downarrow 754$$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-$$

$$6\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{2\left(-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{2\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right.$$

$$\Big\downarrow 27$$

3.116. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-$$

$$6\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)$$

$$\Big\downarrow 219$$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\Big\downarrow 854$$

$$6\int \frac{1}{\left(1+\frac{1}{x^6}\right)x^4}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\Big\downarrow 824$$

3.116. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

$$6\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{2\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{2\left(\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\right.$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$6\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)-$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\Big\downarrow \text{216}$$

3.116. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

$$6\Bigg(-\frac{1}{6}\int\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int\frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\Bigg)-$$

$$6\left(-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$6\Bigg(\frac{1}{6}\Bigg(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{2}\sqrt{3}\int-\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\Bigg)+\frac{1}{6}\Bigg(\frac{1}{2}\int$$

$$6\Bigg(\frac{1}{6}\Bigg(\frac{1}{2}\int-\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{3}{2}\int\frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\Bigg)+\frac{1}{6}\Bigg(-\frac{3}{2}\int$$

$$\Big\downarrow \text{25}$$

3.116. $\int\frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

$$6\Bigg(\frac{1}{6}\Bigg(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\Bigg)+\frac{1}{6}\Bigg(\frac{1}{2}\int$$

$$6\Bigg(\frac{1}{6}\Bigg(-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\Bigg)+\frac{1}{6}\Bigg(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}}$$

$$\Big\downarrow 1083$$

$$6\Bigg(\frac{1}{6}\Bigg(-\int \frac{1}{-1-\frac{1}{x^2}}d\Bigg(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\sqrt{3}\Bigg)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\Bigg)+\frac{1}{6}\Bigg(-\int \frac{1}{-1-\frac{1}{x^2}}d\Bigg($$

$$6\Bigg(\frac{1}{6}\Bigg(3\int \frac{1}{-3-\frac{1}{x^2}}d\Bigg(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-1\Bigg)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\Bigg)+\frac{1}{6}\Bigg(3\int \frac{1}{-3-\frac{1}{x^2}}d\Bigg(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}$$

$$\Big\downarrow 217$$

3.116. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

$$6\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\arctan\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\sqrt{\right.\right.\right.$$

$$6\left(\frac{1}{6}\left(-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int \frac{\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\right.$$

$$\Big\downarrow \text{1103}$$

$$6\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\mathrm{arcta}\right.\right.$$

$$6\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{1}{x^2}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1}{\sqrt{3}}\right)-\frac{1}{2}\mathrm{lo}\right.\right.$$

input
```
Int[E^(ArcCoth[x]/3)/x,x]
```

3.116. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

output
```
6*(ArcTan[(1 - x^(-1))^(1/6)/(2 - x^(-6))^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2
*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)] + (Sqrt[3]*Log[1 - (Sqrt[3]*(1 -
x^(-1))^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6 + (ArcTan[Sqrt[3] + (2*(
1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)] - (Sqrt[3]*Log[1 + (Sqrt[3]*(1 - x^
(-1))^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6) - 6*(-1/3*ArcTanh[(1 + x^
(-1))^(1/6)/(1 - x^(-1))^(1/6)] + (-(Sqrt[3]*ArcTan[(-1 + (2*(1 + x^(-1))^
(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]]) + Log[1 - (1 + x^(-1))^(1/6)/(1 - x^(
-1))^(1/6) + x^(-2)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/
(1 - x^(-1))^(1/6))/Sqrt[3]]) - Log[1 + (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1
/6) + x^(-2)]/2)/6)
```

3.116.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.116. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.116.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 22.19 (sec) , antiderivative size = 2088, normalized size of antiderivative = 5.19

method	result	size
trager	Expression too large to display	2088

input
```
int(1/((x-1)/(1+x))^(1/6)/x,x,method=_RETURNVERBOSE)
```

output

```
-3*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*ln(-18*RootOf(3*_Z*
RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(3
*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)
/(1+x))^(5/6)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(
1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(5/6)+3*(-(1-x)/(1+x))^(2/3)*x+18*Roo
tOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+3*(-
(1-x)/(1+x))^(2/3)+6*(-(1-x)/(1+x))^(1/2)*x+6*(-(1-x)/(1+x))^(1/2)-3*(-(1-
x)/(1+x))^(1/3)*x-6*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)-3*
(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/6)+1)-3*
ln(-(18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1
-x)/(1+x))^(5/6)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(3*_Z*
RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(5/6)+3*Roo
tOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(
-(1-x)/(1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1
+x))^(1/3)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+
6*(-(1-x)/(1+x))^(1/2)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+6*RootOf(3*_Z
*RootOf(_Z^2+1)+9*_Z^2-1)*x+3*(-(1-x)/(1+x))^(1/6)*x+RootOf(_Z^2+1)*x+3*(-
(1-x)/(1+x))^(1/6))/x)*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)-RootOf(_Z^2+1)
*ln((9*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x-18*(-(1
-x)/(1+x))^(1/2)*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*x-...
```

3.116. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

3.116.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = & -\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right) \\ & -\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right) \\ & +\frac{1}{2}\sqrt{2\sqrt{-3}+2}\log\left(\sqrt{2\sqrt{-3}+2}(\sqrt{-3}-1)+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) \\ & -\frac{1}{2}\sqrt{2\sqrt{-3}+2}\log\left(-\sqrt{2\sqrt{-3}+2}(\sqrt{-3}-1)+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) \\ & -\frac{1}{2}\sqrt{-2\sqrt{-3}+2}\log\left((\sqrt{-3}+1)\sqrt{-2\sqrt{-3}+2}+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) \\ & +\frac{1}{2}\sqrt{-2\sqrt{-3}+2}\log\left(-(\sqrt{-3}+1)\sqrt{-2\sqrt{-3}+2}+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) \\ & +2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\ & -\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\ & +\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="fricas")
```

output
```
-sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - sqrt(
3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 1/2*sqrt(2*
sqrt(-3) + 2)*log(sqrt(2*sqrt(-3) + 2)*(sqrt(-3) - 1) + 4*((x - 1)/(x + 1)
)^(1/6)) - 1/2*sqrt(2*sqrt(-3) + 2)*log(-sqrt(2*sqrt(-3) + 2)*(sqrt(-3) -
1) + 4*((x - 1)/(x + 1))^(1/6)) - 1/2*sqrt(-2*sqrt(-3) + 2)*log((sqrt(-3)
+ 1)*sqrt(-2*sqrt(-3) + 2) + 4*((x - 1)/(x + 1))^(1/6)) + 1/2*sqrt(-2*sqrt
(-3) + 2)*log(-(sqrt(-3) + 1)*sqrt(-2*sqrt(-3) + 2) + 4*((x - 1)/(x + 1))^
(1/6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1)/(x + 1))^(1/
3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1))^(1/3) - ((x
- 1)/(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1) - log(((x - 1)
/(x + 1))^(1/6) - 1)
```

3.116.6 Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{1}{x\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/6)/x,x)
```

output
```
Integral(1/(x*((x - 1)/(x + 1))**(1/6)), x)
```

3.116.7 Maxima [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{1}{x\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="maxima")
```

output
```
integrate(1/(x*((x - 1)/(x + 1))^(1/6)), x)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.65

$$
\begin{aligned}
\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = & -\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) \\
& -\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) \\
& -\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) \\
& +\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) \\
& +\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) \\
& +2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
& -\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
& +\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)
\end{aligned}
$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="giac")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - sqrt(3)*arc
tan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/2*sqrt(3)*log(sqrt(3)
*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*sqrt(3)*log(
-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + arctan(s
qrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + arctan(-sqrt(3) + 2*((x - 1)/(x + 1)
)^(1/6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1)/(x + 1))^(
1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1))^(1/3) - ((
x - 1)/(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1) - log(abs(((
x - 1)/(x + 1))^(1/6) - 1))
```

3.116.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.42

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right) - \mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)2\mathrm{i}$$
$$-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}1486016741376\mathrm{i}}{-743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)\left(\sqrt{3}-\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}1486016741376\mathrm{i}}{743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)$$

input
```
int(1/(x*((x - 1)/(x + 1))^(1/6)),x)
```

output
```
2*atan(((x - 1)/(x + 1))^(1/6)) - atan(((x - 1)/(x + 1))^(1/6)*1i)*2i - at
an((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(3^(1/2)*743008370688i - 74300
8370688))*(3^(1/2) - 1i) - atan((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(
3^(1/2)*743008370688i + 743008370688))*(3^(1/2) + 1i) - atan((148601674137
6*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*743008370688i - 743008370688))*(3^(1/2
)*1i + 1) - atan((1486016741376*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*74300837
0688i + 743008370688))*(3^(1/2)*1i - 1)
```

3.117 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

3.117.1 Optimal result

Integrand size = 12, antiderivative size = 233

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx &= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} \\ &\quad -\frac{1}{3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{3}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) \\ &\quad +\frac{2}{3}\arctan\left(\frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}} \\ &\quad -\frac{\log\left(1+\frac{\sqrt{3}\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}}\end{aligned}$$

output
```
(1+1/x)^(1/6)*((-1+x)/x)^(5/6)+2/3*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+
1/3*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)-3^(1/2))+1/3*arctan(2*((-1+x)/
x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+1/6*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((
-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)-1/6*ln(1+((-1+x)/x)^(1/3)/(1
+1/x)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

3.117.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.17

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -2e^{\frac{1}{3}\coth^{-1}(x)}\left(-\frac{1}{1+e^{2\coth^{-1}(x)}} + \text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-e^{2\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)/x^2,x]
```

output
```
-2*E^(ArcCoth[x]/3)*(-(1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6,
 1, 7/6, -E^(2*ArcCoth[x])])
```

3.117.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6721, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \ 60$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}-\frac{1}{3}\int \frac{1}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \ 73$$

$$2\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}+\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \ 854$$

$$2\int \frac{1}{\left(1+\frac{1}{x^6}\right)x^4}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \ 824$$

$$2\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{2\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{2\left(\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\right.$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \ 216$$

3.117. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

$$2\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \text{ 1142}$$

$$2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \text{ 25}$$

$$2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \text{ 1083}$$

3.117. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

$$2\Bigg(\frac{1}{6}\Bigg(-\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\Bigg)+\frac{1}{6}\Bigg(-\int\frac{1}{-1-\frac{1}{x^2}}d\Bigg($$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow {\color{blue}217}$$

$$2\Bigg(\frac{1}{6}\Bigg(-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\Bigg)+\frac{1}{6}\Bigg(\arctan\Bigg(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\sqrt{}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow {\color{blue}1103}$$

$$2\Bigg(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\Bigg(\text{arcta}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

input
```
Int[E^(ArcCoth[x]/3)/x^2,x]
```

output
```
(1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6) + 2*(ArcTan[(1 - x^(-1))^(1/6)/(2 -
x^(-6))^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2*(1 - x^(-1))^(1/6))/(2 - x^(-6))^
(1/6)] + (Sqrt[3]*Log[1 - (Sqrt[3]*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)
+ x^(-2)])/2)/6 + (ArcTan[Sqrt[3] + (2*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1
/6)] - (Sqrt[3]*Log[1 + (Sqrt[3]*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6) +
x^(-2)])/2)/6)
```

3.117. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

3.117.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

3.117. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.117.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 16.72 (sec) , antiderivative size = 1487, normalized size of antiderivative = 6.38

method	result	size
trager	Expression too large to display	1487
risch	Expression too large to display	2991

input
```
int(1/((x-1)/(1+x))^(1/6)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(1+x)*(-(1-x)/(1+x))^(5/6)/x-9*RootOf(81*_Z^4-9*_Z^2+1)^3*ln((-27*RootOf(8
1*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-
(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-
x)/(1+x))^(2/3)*x+18*x*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*
RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)-3*R
ootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2
*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2
*(-(1-x)/(1+x))^(1/2)*x+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-
9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x+18*RootOf(81*_Z^4-9*_Z
^2+1)^3*x-2*(-(1-x)/(1+x))^(1/2)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))
^(1/3)-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-RootOf(81*_Z^4-9*
_Z^2+1)*x)/x)+RootOf(81*_Z^4-9*_Z^2+1)*ln((-27*RootOf(81*_Z^4-9*_Z^2+1)^3*
(-(1-x)/(1+x))^(2/3)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+
(-(1-x)/(1+x))^(5/6)*x-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+1
8*x*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z
^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)-3*RootOf(81*_Z^4-9*_Z^
2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/
2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2*(-(1-x)/(1+x))^(1/
2)*x+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(81*_Z^4-9*
_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^3*x-2*(-(1...
```

3.117.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.91

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$$

$$= \frac{x\sqrt{2i\sqrt{3}+2}\log\left(\sqrt{2i\sqrt{3}+2}(i\sqrt{3}-1)+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-x\sqrt{2i\sqrt{3}+2}\log\left(\sqrt{2i\sqrt{3}+2}(-i\sqrt{3}+1)+4\right.}{}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="fricas")
```

output
```
1/6*(x*sqrt(2*I*sqrt(3) + 2)*log(sqrt(2*I*sqrt(3) + 2)*(I*sqrt(3) - 1) + 4
*((x - 1)/(x + 1))^(1/6)) - x*sqrt(2*I*sqrt(3) + 2)*log(sqrt(2*I*sqrt(3) +
 2)*(-I*sqrt(3) + 1) + 4*((x - 1)/(x + 1))^(1/6)) - x*sqrt(-2*I*sqrt(3) +
2)*log((I*sqrt(3) + 1)*sqrt(-2*I*sqrt(3) + 2) + 4*((x - 1)/(x + 1))^(1/6))
 + x*sqrt(-2*I*sqrt(3) + 2)*log((-I*sqrt(3) - 1)*sqrt(-2*I*sqrt(3) + 2) +
4*((x - 1)/(x + 1))^(1/6)) + 4*x*arctan(((x - 1)/(x + 1))^(1/6)) + 6*(x +
1)*((x - 1)/(x + 1))^(5/6))/x
```

3.117.6 Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx=\int \frac{1}{x^2\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/6)/x**2,x)
```

output
```
Integral(1/(x**2*((x - 1)/(x + 1))**(1/6)), x)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.65

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx=&-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\&+\frac{1}{3}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{2}{3}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arct
an(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)
/(x + 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.65

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = &-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\ &+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\ &+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\ &+\frac{1}{3}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{2}{3}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arct
an(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)
/(x + 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

3.117.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\begin{aligned}&\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx\\ &=\frac{2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{3}+\frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}+1}\\ &\quad-\mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)-\mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)\end{aligned}$$

input
```
int(1/(x^2*((x - 1)/(x + 1))^(1/6)),x)
```

output
```
(2*atan(((x - 1)/(x + 1))^(1/6)))/3 + (2*((x - 1)/(x + 1))^(5/6))/((x - 1)
/(x + 1) + 1) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i - 32))*((3^
(1/2)*1i)/3 + 1/3) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i + 32))
*((3^(1/2)*1i)/3 - 1/3)
```

3.117. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

3.118 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

3.118.1 Optimal result

Integrand size = 12, antiderivative size = 260

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$-\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{18}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{9}\arctan\left(\frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{\log\left(1-\right.}{}$$

output
```
1/6*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)+1/2*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)+1/9*
arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+1/18*arctan(2*((-1+x)/x)^(1/6)/(1+1
/x)^(1/6)-3^(1/2))+1/18*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+1
/36*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(
1/6))*3^(1/2)-1/36*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x)^(1/6)*3^
(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

3.118.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.48

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$$
$$=\frac{1}{54}\left(\frac{18e^{\frac{1}{3}\coth^{-1}(x)}\left(1+7e^{2\coth^{-1}(x)}\right)}{\left(1+e^{2\coth^{-1}(x)}\right)^2}-6\arctan\left(e^{\frac{1}{3}\coth^{-1}(x)}\right)+\text{RootSum}\left[1-\#1^2\right.\right.$$
$$\left.\left.+\#1^4\&,\frac{2\coth^{-1}(x)-6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)-\coth^{-1}(x)\#1^2+3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-\#1+2\#1^3}\&\right]\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)/x^3,x]
```

output
```
((18*E^(ArcCoth[x]/3)*(1 + 7*E^(2*ArcCoth[x])))/(1 + E^(2*ArcCoth[x]))^2 -
 6*ArcTan[E^(ArcCoth[x]/3)] + RootSum[1 - #1^2 + #1^4 & , (2*ArcCoth[x] -
6*Log[E^(ArcCoth[x]/3) - #1] - ArcCoth[x]*#1^2 + 3*Log[E^(ArcCoth[x]/3) -
#1]*#1^2)/(-#1 + 2*#1^3) & ])/54
```

3.118.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6721, 90, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}x}d\frac{1}{x}$$
$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{1}{6}\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \ 60$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}-\frac{1}{3}\int \frac{1}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{6}\left(2\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}+\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 854$$

$$\frac{1}{6}\left(2\int \frac{1}{\left(1+\frac{1}{x^6}\right)x^4}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 824$$

$$\frac{1}{6}\left(2\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{2\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{2\left(\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right.}\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 27$$

3.118. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

$$\frac{1}{6}\left(2\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{6}\left(2\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{6}\left(2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 25$$

3.118. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

$$\frac{\frac{1}{6}\left(2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.\right.}{\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}$$

$\downarrow$ 1083

$$\frac{\frac{1}{6}\left(2\left(\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x^2}}\right.\right.\right.}{\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}$$

$\downarrow$ 217

$$\frac{\frac{1}{6}\left(2\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\arctan\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right.\right.\right.\right.}{\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}$$

$\downarrow$ 1103

3.118. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

$$\frac{1}{6}\left(2\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\mathrm{ar}\right.\right.\right.$$
$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

input
```
Int[E^(ArcCoth[x]/3)/x^3,x]
```

output
```
((1 - x^(-1))^(5/6)*(1 + x^(-1))^(7/6))/2 + ((1 - x^(-1))^(5/6)*(1 + x^(-1
))^(1/6) + 2*(ArcTan[(1 - x^(-1))^(1/6)/(2 - x^(-6))^(1/6)]/3 + (-ArcTan[S
qrt[3] - (2*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)] + (Sqrt[3]*Log[1 - (Sq
rt[3]*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6 + (ArcTan[Sqr
t[3] + (2*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)] - (Sqrt[3]*Log[1 + (Sqrt
[3]*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6))/6
```

3.118.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.118. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.118.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 15.33 (sec) , antiderivative size = 1262, normalized size of antiderivative = 4.85

method	result	size
trager	Expression too large to display	1262
risch	Expression too large to display	3478

input
```
int(1/((x-1)/(1+x))^(1/6)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(1+x)*(4*x+3)/x^2*(-(1-x)/(1+x))^(5/6)-1/18*RootOf(_Z^2+1)*ln(-(9*Root
Of(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x-18*(-(1-x)/(1+x))^
(1/2)*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*x-3*(-(1-x)/(1+x
))^(5/6)*x+6*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(3*_Z*RootOf(_Z
^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)-18*(-(1-x)/(1+x))^(1/2)*RootOf(3*_Z*R
ootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)-3*(-(1-x)/(1+x))^(5/6)+6*RootOf(_Z^
2+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/
(1+x))^(1/3)*x+9*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-
x)/(1+x))^(1/6)*x+6*(-(1-x)/(1+x))^(1/2)*x-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))
^(1/3)*x-18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+9*Ro
otOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/6)+6*(
-(1-x)/(1+x))^(1/2)-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+3*RootOf(3*_Z*Ro
otOf(_Z^2+1)+9*_Z^2-1)*x-RootOf(_Z^2+1)*x)/x)-1/18*ln((18*RootOf(3*_Z*Root
Of(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+3*Roo
tOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)
*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(5/6)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x
))^(2/3)+18*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)*x+6*
(-(1-x)/(1+x))^(1/2)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+18*RootOf(3
*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+6*(-(1-x)/(1+x))^(1/2)+3
*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+6*RootOf(3*_Z*RootOf(_Z^2+1)+9*_Z^...
```

3.118.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.02

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$$

$$= \sqrt{2}x^2\sqrt{i\sqrt{3}+1}\log\Big(\big(i\sqrt{3}\sqrt{2}-\sqrt{2}\big)\sqrt{i\sqrt{3}+1}+4\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\Big)-\sqrt{2}x^2\sqrt{i\sqrt{3}+1}\log\Big(\big(-i\sqrt{3}\sqrt{2}+\sqrt{2}\big)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="fricas")
```

output
```
1/36*(sqrt(2)*x^2*sqrt(I*sqrt(3) + 1)*log((I*sqrt(3)*sqrt(2) - sqrt(2))*sq
rt(I*sqrt(3) + 1) + 4*((x - 1)/(x + 1))^(1/6)) - sqrt(2)*x^2*sqrt(I*sqrt(3
) + 1)*log((-I*sqrt(3)*sqrt(2) + sqrt(2))*sqrt(I*sqrt(3) + 1) + 4*((x - 1)
/(x + 1))^(1/6)) - sqrt(2)*x^2*sqrt(-I*sqrt(3) + 1)*log((I*sqrt(3)*sqrt(2)
 + sqrt(2))*sqrt(-I*sqrt(3) + 1) + 4*((x - 1)/(x + 1))^(1/6)) + sqrt(2)*x^
2*sqrt(-I*sqrt(3) + 1)*log((-I*sqrt(3)*sqrt(2) - sqrt(2))*sqrt(-I*sqrt(3)
+ 1) + 4*((x - 1)/(x + 1))^(1/6)) + 4*x^2*arctan(((x - 1)/(x + 1))^(1/6))
+ 6*(4*x^2 + 7*x + 3)*((x - 1)/(x + 1))^(5/6))/x^2
```

3.118.6 Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/6)/x**3,x)
```

output
```
Integral(1/(x**3*((x - 1)/(x + 1))**(1/6)), x)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.68

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = & -\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) \\ & + \frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) \\ & + \frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}} + 7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1} + \frac{(x-1)^2}{(x+1)^2} + 1\right)} + \frac{1}{18}\arctan\left(\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) \\ & + \frac{1}{18}\arctan\left(-\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{9}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="maxima")
```

output
```
-1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3
) + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x +
 1))^(1/3) + 1) + 1/3*(((x - 1)/(x + 1))^(11/6) + 7*((x - 1)/(x + 1))^(5/6
))/(2*(x - 1)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) + 1/18*arctan(sqrt(3) + 2
*((x - 1)/(x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/
6)) + 1/9*arctan(((x - 1)/(x + 1))^(1/6))
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = &-\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}+7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}+1\right)^2}+\frac{1}{18}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\&+\frac{1}{18}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{9}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="giac")
```

output
```
-1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3
) + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x +
 1))^(1/3) + 1) + 1/3*((x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 7*((x - 1
)/(x + 1))^(5/6))/((x - 1)/(x + 1) + 1)^2 + 1/18*arctan(sqrt(3) + 2*((x -
1)/(x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1
/9*arctan(((x - 1)/(x + 1))^(1/6))
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.52

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{9} + \frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3} + \frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{2(x-1)}{x+1} + \frac{(x-1)^2}{(x+1)^2} + 1}$$
$$-\operatorname{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\left(-\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\operatorname{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\left(\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)$$

input
```
int(1/(x^3*((x - 1)/(x + 1))^(1/6)),x)
```

output
```
atan(((x - 1)/(x + 1))^(1/6))/9 + ((7*((x - 1)/(x + 1))^(5/6))/3 + ((x - 1
)/(x + 1))^(11/6)/3)/((2*(x - 1))/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - ata
n((2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 - 1/243)))*((3^(1/2)*
1i)/18 + 1/18) - atan((2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 +
 1/243)))*((3^(1/2)*1i)/18 - 1/18)
```

3.119 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

3.119.1 Optimal result

Integrand size = 12, antiderivative size = 287

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x}$$

$$-\frac{19}{162}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19}{162}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19}{81}\arctan\left(\frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19\log}{}$$

output
```
19/54*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)+1/18*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)+1
/3*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)/x+19/81*arctan(((-1+x)/x)^(1/6)/(1+1/x)^
(1/6))+19/162*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)-3^(1/2))+19/162*arct
an(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+19/324*ln(1+((-1+x)/x)^(1/3)/
(1+1/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)-19/324*ln(1+
((-1+x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(
1/2)
```

3.119.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.46

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \frac{1}{486}\left(\frac{18e^{\frac{1}{3}\coth^{-1}(x)}\left(19+8e^{2\coth^{-1}(x)}+61e^{4\coth^{-1}(x)}\right)}{\left(1+e^{2\coth^{-1}(x)}\right)^3} - 114\arctan\left(e^{\frac{1}{3}\coth^{-1}(x)}\right) - 19\text{RootSum}\left[1-\#1^2 +\#1^4\&, \frac{-2\coth^{-1}(x)+6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+\coth^{-1}(x)\#1^2-3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-\#1+2\#1^3}\&\right]\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)/x^4,x]
```

output
```
((18*E^(ArcCoth[x]/3)*(19 + 8*E^(2*ArcCoth[x]) + 61*E^(4*ArcCoth[x])))/(1
+ E^(2*ArcCoth[x]))^3 - 114*ArcTan[E^(ArcCoth[x]/3)] - 19*RootSum[1 - #1^2
 + #1^4 & , (-2*ArcCoth[x] + 6*Log[E^(ArcCoth[x]/3) - #1] + ArcCoth[x]*#1^
2 - 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-#1 + 2*#1^3) & ])/486
```

3.119.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {6721, 101, 27, 90, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{3}\int -\frac{\sqrt[6]{1+\frac{1}{x}}\left(3+\frac{1}{x}\right)}{3\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}-\frac{1}{9}\int \frac{\sqrt[6]{1+\frac{1}{x}}\left(3+\frac{1}{x}\right)}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}\right)+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(\frac{1}{3}\int \frac{1}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}-\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)\right)+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)\right)+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\int \frac{1}{\left(1+\frac{1}{x^6}\right)x^4}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)\right)+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \text{824}$$

3.119. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}dx$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{3}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int-\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{2\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1-}}{\sqrt[6]{2-}}\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{3}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \color{blue}{216}$$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(-\frac{1}{6}\int\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int\frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \color{blue}{1142}$$

3.119. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{2}\sqrt{3}\int-\frac{\sqrt{3}-\frac{2\sqrt[6]{1}}{\sqrt[6]{2-}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}\right.\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-}}{\sqrt[6]{2-}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+}\right.\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{6}\left(-\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+}\right.\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \ 217$$

3.119. $\int\frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

$$\frac{1}{9}\Bigg(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\Bigg(-2\Bigg(\frac{1}{6}\Bigg(-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\arctan\Bigg(\sqrt{3}-\frac{2\sqrt[6]{1}}{\sqrt[6]{2}}$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$$\Big\downarrow \color{blue}{1103}$$

$$\frac{1}{9}\Bigg(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\Bigg(-2\Bigg(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\Bigg(\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)-\mathrm{a}$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

input
```
Int[E^(ArcCoth[x]/3)/x^4,x]
```

output
```
((1 - x^(-1))^(5/6)*(1 + x^(-1))^(7/6))/(3*x) + (((1 - x^(-1))^(5/6)*(1 +
x^(-1))^(7/6))/2 - (19*(-((1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6)) - 2*(ArcT
an[(1 - x^(-1))^(1/6)/(2 - x^(-6))^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2*(1 - x
^(-1))^(1/6))/(2 - x^(-6))^(1/6)] + (Sqrt[3]*Log[1 - (Sqrt[3]*(1 - x^(-1))
^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6 + (ArcTan[Sqrt[3] + (2*(1 - x^(
-1))^(1/6))/(2 - x^(-6))^(1/6)] - (Sqrt[3]*Log[1 + (Sqrt[3]*(1 - x^(-1))^(
1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6)))/6)/9
```

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.119. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.119. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

3.119.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 14.23 (sec) , antiderivative size = 1498, normalized size of antiderivative = 5.22

method	result	size
trager	Expression too large to display	1498
risch	Expression too large to display	3004

input

```
int(1/((x-1)/(1+x))^(1/6)/x^4,x,method=_RETURNVERBOSE)
```

output

```
1/54*(1+x)*(22*x^2+21*x+18)/x^3*(-(1-x)/(1+x))^(5/6)+19/54*RootOf(81*_Z^4-
9*_Z^2+1)*ln((54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+54*Root
Of(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x-3*RootO
f(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-18*x*RootOf(81*_Z^4-9*_Z^2+1)^2
*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x
+(-(1-x)/(1+x))^(5/6)-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)-18*R
ootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)
^3*(-(1-x)/(1+x))^(1/3)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+
9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-9*RootOf(81*_Z^4-9*_Z^
2+1)^3*x+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(81*_Z^4-
9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-(-(1-x)/(1+x))^(1/6)*x-RootOf(81*_Z^4-9*_
Z^2+1)*x-(-(1-x)/(1+x))^(1/6))/x)+19/6*RootOf(81*_Z^4-9*_Z^2+1)^3*ln((27*R
ootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+27*RootOf(81*_Z^4-9*_Z^2+
1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x+3*RootOf(81*_Z^4-9*_Z^2+1
)*(-(1-x)/(1+x))^(2/3)*x+18*x*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1
/2)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5
/6)+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z
^2+1)^2*(-(1-x)/(1+x))^(1/2)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^
(1/3)-2*(-(1-x)/(1+x))^(1/2)*x-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(
1/3)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-18*RootOf(81...
```

3.119.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.94

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$$
$$= \frac{\sqrt{2}x^3\sqrt{361i\sqrt{3}+361}\log\left(\left(i\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{361i\sqrt{3}+361}+76\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-\sqrt{2}x^3\sqrt{361i\sqrt{3}+361}\log\Big(}{}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="fricas")
```

output
```
1/324*(sqrt(2)*x^3*sqrt(361*I*sqrt(3) + 361)*log((I*sqrt(3)*sqrt(2) - sqrt
(2))*sqrt(361*I*sqrt(3) + 361) + 76*((x - 1)/(x + 1))^(1/6)) - sqrt(2)*x^3
*sqrt(361*I*sqrt(3) + 361)*log((-I*sqrt(3)*sqrt(2) + sqrt(2))*sqrt(361*I*s
qrt(3) + 361) + 76*((x - 1)/(x + 1))^(1/6)) - sqrt(2)*x^3*sqrt(-361*I*sqrt
(3) + 361)*log((I*sqrt(3)*sqrt(2) + sqrt(2))*sqrt(-361*I*sqrt(3) + 361) +
76*((x - 1)/(x + 1))^(1/6)) + sqrt(2)*x^3*sqrt(-361*I*sqrt(3) + 361)*log((
-I*sqrt(3)*sqrt(2) - sqrt(2))*sqrt(-361*I*sqrt(3) + 361) + 76*((x - 1)/(x
+ 1))^(1/6)) + 76*x^3*arctan(((x - 1)/(x + 1))^(1/6)) + 6*(22*x^3 + 43*x^2
 + 39*x + 18)*((x - 1)/(x + 1))^(5/6))/x^3
```

3.119.6 Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/6)/x**4,x)
```

output
```
Integral(1/(x**4*((x - 1)/(x + 1))**(1/6)), x)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.71

$$
\begin{aligned}
\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = &-\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\
&+\frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\
&+\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}+8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{3(x-1)}{x+1}+\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}+1\right)}\\
&+\frac{19}{162}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\
&+\frac{19}{162}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{19}{81}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)
\end{aligned}
$$

input

```
integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="maxima")
```

output

```
-19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1
/3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/
(x + 1))^(1/3) + 1) + 1/27*(19*((x - 1)/(x + 1))^(17/6) + 8*((x - 1)/(x +
1))^(11/6) + 61*((x - 1)/(x + 1))^(5/6))/(3*(x - 1)/(x + 1) + 3*(x - 1)^2/
(x + 1)^2 + (x - 1)^3/(x + 1)^3 + 1) + 19/162*arctan(sqrt(3) + 2*((x - 1)/
(x + 1))^(1/6)) + 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19
/81*arctan(((x - 1)/(x + 1))^(1/6))
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.69

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = &-\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\ &+\frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\ &+\frac{\frac{8\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}+\frac{19\,(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^2}+61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{x-1}{x+1}+1\right)^3}\\ &+\frac{19}{162}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\ &+\frac{19}{162}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{19}{81}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="giac")
```

output
```
-19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1
/3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/
(x + 1))^(1/3) + 1) + 1/27*(8*(x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 19
*(x - 1)^2*((x - 1)/(x + 1))^(5/6)/(x + 1)^2 + 61*((x - 1)/(x + 1))^(5/6))
/((x - 1)/(x + 1) + 1)^3 + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(1/
6)) + 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*arctan((
(x - 1)/(x + 1))^(1/6))
```

3.119.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.56

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = &\frac{19\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{81}+\frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27}+\frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27}+\frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3\,(x-1)}{x+1}+\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}+1}\\ &-\mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(-\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19}{162}+\frac{\sqrt{3}\,19\mathrm{i}}{162}\right)-\mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right.}\right.\end{aligned}$$

input
```
int(1/(x^4*((x - 1)/(x + 1))^(1/6)),x)
```

output
```
(19*atan(((x - 1)/(x + 1))^(1/6)))/81 + ((61*((x - 1)/(x + 1))^(5/6))/27 +
 (8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)/((3*(
x - 1))/(x + 1) + (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 + 1) - ata
n((4952198*((x - 1)/(x + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907
 - 2476099/14348907)))*((3^(1/2)*19i)/162 + 19/162) - atan((4952198*((x -
1)/(x + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907 + 2476099/143489
07)))*((3^(1/2)*19i)/162 - 19/162)
```

3.119. $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

3.120 $\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx$

3.120.1 Optimal result

Integrand size = 12, antiderivative size = 157

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx=\frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x+\frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^2+\frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3-\frac{22\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{27\sqrt{3}}-\frac{11}{27}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right)-\frac{11\log(x)}{81}$$

output
```
14/27*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x+4/9*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*
x^2+1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x^3-11/27*ln((1+1/x)^(1/3)-((-1+x)/
x)^(1/3))-11/81*ln(x)-22/81*arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x
)^(1/3)*3^(1/2))*3^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx=\frac{1}{81}\left(\frac{216e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^3}+\frac{360e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{210e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+22\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-22\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-22\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)-22\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)+11\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+11\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3)*x^2,x]
```

output
```
((216*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^3 + (360*E^((2*ArcCoth
[x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (210*E^((2*ArcCoth[x])/3))/(-1 + E^(2
*ArcCoth[x])) + 22*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 22*
Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 22*Log[1 - E^(ArcCoth[x
]/3)] - 22*Log[1 + E^(ArcCoth[x]/3)] + 11*Log[1 - E^(ArcCoth[x]/3) + E^((2
*ArcCoth[x])/3)] + 11*Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/81
```

3.120.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6721, 110, 27, 168, 27, 168, 27, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{\frac{2}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \ 6721$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}x^4}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \ 110$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-\frac{1}{3}\int \frac{2\left(4+\frac{3}{x}\right)x^3}{3\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-\frac{2}{9}\int \frac{\left(4+\frac{3}{x}\right)x^3}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \ 168$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-\frac{2}{9}\left(-\frac{1}{2}\int -\frac{2\left(7+\frac{6}{x}\right)x^2}{3\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-2\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^2\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-\frac{2}{9}\left(\frac{1}{3}\int \frac{\left(7+\frac{6}{x}\right)x^2}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-2\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^2\right)$$

$$\Big\downarrow \ 168$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(-\int -\frac{11x}{3\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-7\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x\right)-2\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^2\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{11}{3}\int \frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-7\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x\right)-2\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^2\right)$$

$$\Big\downarrow \ 102$$

3.120. $\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-$$

$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{11}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{\frac{1}{x}+1}\right)-\frac{1}{2}\log\left(\frac{1}{x}\right)\right)-7\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}}$$

input
```
Int[E^((2*ArcCoth[x])/3)*x^2,x]
```

output
```
((1 - x^(-1))^(2/3)*(1 + x^(-1))^(1/3)*x^3)/3 - (2*(-2*(1 - x^(-1))^(2/3)*
(1 + x^(-1))^(1/3)*x^2 + (-7*(1 - x^(-1))^(2/3)*(1 + x^(-1))^(1/3)*x + (11
*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 + x^(-1))^
(1/3))] + (3*Log[(1 - x^(-1))^(1/3) - (1 + x^(-1))^(1/3)])/2 - Log[x^(-1)]
/2))/3)/3))/9
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.120.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.60

method	result
trager	$\frac{(1+x)(9x^2+12x+14)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{27}-\frac{22\ln(-9\,\mathrm{RootOf}(9\_Z^2-3\_Z+1)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-9\,\mathrm{RootOf}(9\_Z^2-3\_Z+1)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}+9}{\;}$
risch	$\frac{(9x^2+12x+14)(x-1)}{27\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{(\frac{22\,\mathrm{RootOf}(\_Z^2-\_Z+1)\ln(-\frac{-2\,\mathrm{RootOf}(\_Z^2-\_Z+1)^2x^2+3\,\mathrm{RootOf}(\_Z^2-\_Z+1)(x^3+x^2-x-1)}{\;}}{\;}}{\;}$

input
```
int(1/((x-1)/(1+x))^(1/3)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/27*(1+x)*(9*x^2+12*x+14)*(-(1-x)/(1+x))^(2/3)-22/81*ln(-9*RootOf(9*_Z^2-
3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3
)+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-36*RootOf(9*_Z^2-3*_Z+1)^
2*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x+
12*RootOf(9*_Z^2-3*_Z+1)*x-3*(-(1-x)/(1+x))^(1/3)-6*RootOf(9*_Z^2-3*_Z+1)-
x+1)+22/27*RootOf(9*_Z^2-3*_Z+1)*ln(9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))
^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/
3)*x+18*RootOf(9*_Z^2-3*_Z+1)^2*x-3*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^
(1/3)*x-15*RootOf(9*_Z^2-3*_Z+1)*x+3*(-(1-x)/(1+x))^(1/3)-3*RootOf(9*_Z^2-
3*_Z+1)+2*x+2)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$
\begin{aligned}
\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx &= \frac{1}{27}\left(9\,x^3+21\,x^2+26\,x+14\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} \\
&\quad -\frac{22}{81}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right) \\
&\quad +\frac{11}{81}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{22}{81}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)
\end{aligned}
$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="fricas")
```

output
```
1/27*(9*x^3 + 21*x^2 + 26*x + 14)*((x - 1)/(x + 1))^(2/3) - 22/81*sqrt(3)*
arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 11/81*log(((x
- 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(((x - 1)/(x
 + 1))^(1/3) - 1)
```

3.120.6 Sympy [F]

$$
\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx=\int\frac{x^2}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx
$$

input
```
integrate(1/((-1+x)/(1+x))**(1/3)*x**2,x)
```

output
```
Integral(x**2/((x - 1)/(x + 1))**(1/3), x)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\begin{aligned}\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = &-\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)\\ &-\frac{2\left(11\left(\frac{x-1}{x+1}\right)^{\frac{8}{3}}-10\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}\\ &+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{22}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="maxima")
```

output
```
-22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/27*
(11*((x - 1)/(x + 1))^(8/3) - 10*((x - 1)/(x + 1))^(5/3) + 35*((x - 1)/(x
+ 1))^(2/3))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1
)^3 - 1) + 11/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1
) - 22/81*log(((x - 1)/(x + 1))^(1/3) - 1)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\begin{aligned}\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = &-\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)\\ &+\frac{2\left(\frac{10(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-\frac{11(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{(x+1)^2}-35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{x-1}{x+1}-1\right)^3}\\ &+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{22}{81}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="giac")
```

output
```
-22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/27*
(10*(x - 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 11*(x - 1)^2*((x - 1)/(x + 1
))^(2/3)/(x + 1)^2 - 35*((x - 1)/(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)^3 +
 11/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*
log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

3.120.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx=-\frac{22\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-\frac{484}{729}\right)}{81}-\frac{\frac{70\left(\frac{x-1}{x+1}\right)^{2/3}}{27}-\frac{20\left(\frac{x-1}{x+1}\right)^{5/3}}{27}+\frac{22\left(\frac{x-1}{x+1}\right)^{8/3}}{27}}{\frac{3\,(x-1)}{x+1}-\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$
$$-\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(-\frac{11}{81}+\frac{\sqrt{3}\,11\mathrm{i}}{81}\right)^2\right)\left(-\frac{11}{81}+\frac{\sqrt{3}\,11\mathrm{i}}{81}\right)+\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(\frac{11}{81}+\frac{\sqrt{3}\,11\mathrm{i}}{81}\right)^2\right.$$

input
```
int(x^2/((x - 1)/(x + 1))^(1/3),x)
```

output
```
log((484*((x - 1)/(x + 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 + 11/81)^2)*((
3^(1/2)*11i)/81 + 11/81) - ((70*((x - 1)/(x + 1))^(2/3))/27 - (20*((x - 1)
/(x + 1))^(5/3))/27 + (22*((x - 1)/(x + 1))^(8/3))/27)/((3*(x - 1))/(x + 1
) - (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1) - log((484*((x - 1)
/(x + 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 - 11/81)^2)*((3^(1/2)*11i)/81 -
 11/81) - (22*log((484*((x - 1)/(x + 1))^(1/3))/729 - 484/729))/81
```

3.121 $\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx$

3.121.1 Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right) - \frac{\log(x)}{9}$$

output
```
1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x+1/2*(1+1/x)^(4/3)*((-1+x)/x)^(2/3)*x^
2-1/3*ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-1/9*ln(x)-2/9*arctan(1/3*3^(1/2)+
2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=\frac{1}{9}\left(\frac{18e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{24e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)-2\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)+\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

input

```
Integrate[E^((2*ArcCoth[x])/3)*x,x]
```

output

```
((18*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (24*E^((2*ArcCoth[x
])/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3)
)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1
- E^(ArcCoth[x]/3)] - 2*Log[1 + E^(ArcCoth[x]/3)] + Log[1 - E^(ArcCoth[x]/
3) + E^((2*ArcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/
3)])/9
```

3.121.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6721, 107, 105, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{2}{3}\coth^{-1}(x)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}x^3}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\downarrow 107$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}x^2-\frac{1}{3}\int\frac{\sqrt[3]{1+\frac{1}{x}}x^2}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\downarrow 105$$

$$\frac{1}{3}\left(\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x-\frac{2}{3}\int\frac{x}{\sqrt[3]{1-\frac{1}{x}}\,(1+\frac{1}{x})^{2/3}}d\frac{1}{x}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}x^2$$

$$\downarrow 102$$

$$\frac{1}{3}\left(\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x-\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{\frac{1}{x}+1}\right)-\frac{1}{2}\log\left(\frac{1}{x}\right)\right)\right)$$
$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}x^2$$

input
```
Int[E^((2*ArcCoth[x])/3)*x,x]
```

output
```
((1 - x^(-1))^(2/3)*(1 + x^(-1))^(4/3)*x^2)/2 + ((1 - x^(-1))^(2/3)*(1 + x
^(-1))^(1/3)*x - (2*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - x^(-1))^(1/3))/(Sq
rt[3]*(1 + x^(-1))^(1/3))] + (3*Log[(1 - x^(-1))^(1/3) - (1 + x^(-1))^(1/3
)])/2 - Log[x^(-1)]/2))/3)/3
```

3.121.3.1 Defintions of rubi rules used

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.121.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.10

method	result
risch	$\frac{(5+3x)(x-1)}{6\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\Bigg(-\frac{2\ln\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-3}{}\right.}{}}{}$
trager	$\frac{(1+x)(5+3x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{6}+\frac{2\ln\left(9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+18\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x+9\,\mathrm{RootOf}\left(9\_Z^2-\right.\right.}{}$

input
```
int(1/((x-1)/(1+x))^(1/3)*x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(5+3*x)*(x-1)/((x-1)/(1+x))^(1/3)+(-2/9*ln(-(4*RootOf(_Z^2-_Z+1)^2*x^2
+4*RootOf(_Z^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)-3*RootOf(
_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x-4*RootOf(_Z^2-_Z+1)*x^2-3*RootOf(_Z^2-_Z+
1)*(x^3+x^2-x-1)^(1/3)-2*RootOf(_Z^2-_Z+1)*x+3*(x^3+x^2-x-1)^(1/3)*x+x^2+2
*RootOf(_Z^2-_Z+1)+3*(x^3+x^2-x-1)^(1/3)-1)/(1+x))+2/9*RootOf(_Z^2-_Z+1)*l
n((2*RootOf(_Z^2-_Z+1)^2*x^2+2*RootOf(_Z^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*(
x^3+x^2-x-1)^(2/3)-5*RootOf(_Z^2-_Z+1)*x^2-6*RootOf(_Z^2-_Z+1)*x-3*(x^3+x^
2-x-1)^(2/3)+3*(x^3+x^2-x-1)^(1/3)*x+2*x^2-RootOf(_Z^2-_Z+1)+3*(x^3+x^2-x-
1)^(1/3)+4*x+2)/(1+x)))/((x-1)/(1+x))^(1/3)*((1+x)^2*(x-1))^(1/3)/(1+x)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = \frac{1}{6}\left(3x^2+8x+5\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \frac{2}{9}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="fricas")
```

output
```
1/6*(3*x^2 + 8*x + 5)*((x - 1)/(x + 1))^(2/3) - 2/9*sqrt(3)*arctan(2/3*sqr
t(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 1/9*log(((x - 1)/(x + 1))^(2
/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

3.121.6 Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = \int \frac{x}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/3)*x,x)
```

output
```
Integral(x/((x - 1)/(x + 1))**(1/3), x)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

input

```
integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="maxima")
```

output

```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/3*(((
x - 1)/(x + 1))^(5/3) - 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) - (x
 - 1)^2/(x + 1)^2 - 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1
))^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) - \frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}-1\right)^2} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

input

```
integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="giac")
```

output

```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/3*((x
 - 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 4*((x - 1)/(x + 1))^(2/3))/((x - 1
)/(x + 1) - 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/
3) + 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

3.121.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}-\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2\,(x-1)}{x+1}+1}-\frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-\frac{4}{9}\right)}{9}$$
$$-\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\right)\left(\frac{1}{9}+\frac{\sqrt{3}}{9}\right.$$

input
```
int(x/((x - 1)/(x + 1))^(1/3),x)
```

output
```
((8*((x - 1)/(x + 1))^(2/3))/3 - (2*((x - 1)/(x + 1))^(5/3))/3)/((x - 1)^2
/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))
/9 - 4/9))/9 - log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1/2)*1i)/9 - 1/9
)^2)*((3^(1/2)*1i)/9 - 1/9) + log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1
/2)*1i)/9 + 1/9)^2)*((3^(1/2)*1i)/9 + 1/9)
```

3.122 $\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx$

3.122.1 Optimal result

Integrand size = 8, antiderivative size = 96

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x-\frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right)-\frac{\log(x)}{3}$$

output
```
(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x-ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-1/3*ln
(x)-2/3*arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(
1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=\frac{1}{3}\left(\frac{6e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{2}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3),x]
```

output
```
((6*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(1 +
2*E^((2*ArcCoth[x])/3))/Sqrt[3]] - 2*Log[1 - E^((2*ArcCoth[x])/3)] + Log[1
 + E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)])/3
```

3.122.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6720, 105, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{2}{3}\coth^{-1}(x)}dx$$

$$\Big\downarrow \text{ 6720}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}x^2}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 105}$$

$$\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x-\frac{2}{3}\int \frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 102}$$

$$\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x-\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{\frac{1}{x}+1}\right)-\frac{1}{2}\log\left(\frac{1}{x}\right)\right)$$

input
```
Int[E^((2*ArcCoth[x])/3),x]
```

output
```
(1 - x^(-1))^(2/3)*(1 + x^(-1))^(1/3)*x - (2*(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))] + (3*Log[(1 - x^(-1))^
(1/3) - (1 + x^(-1))^(1/3)])/2 - Log[x^(-1)]/2))/3
```

3.122.3.1 Defintions of rubi rules used

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.122.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 503, normalized size of antiderivative = 5.24

method	result
trager	$(1+x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}-\frac{2\ln\left(9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-36\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x+9\,\mathrm{RootOf}\left(9\_Z^2-3\_\right.\right.}{}$
risch	$\frac{x-1}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}+3\,\mathrm{RootOf}\left(\_\right.}{}\right.}{}\right.}{}$

input
```
int(1/((x-1)/(1+x))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(1+x)*(-(1-x)/(1+x))^(2/3)-2/3*ln(9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(
2/3)*x-36*RootOf(9*_Z^2-3*_Z+1)^2*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))
^(2/3)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(2/
3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+12*RootOf(9*_Z^2-3*_Z+1)
*x-3*(-(1-x)/(1+x))^(2/3)+6*RootOf(9*_Z^2-3*_Z+1)-x-1)+2/3*ln(-9*RootOf(9*
_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2-3*_Z+1)^2*x-9*RootOf(
9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*RootOf(9*_Z^2-3*_Z+1)*x+3*(-(1-x)/(1
+x))^(1/3)*x+3*RootOf(9*_Z^2-3*_Z+1)+3*(-(1-x)/(1+x))^(1/3)-x+1)-2*ln(-9*R
ootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2-3*_Z+1)^2*x-9
*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*RootOf(9*_Z^2-3*_Z+1)*x+3*(-
(1-x)/(1+x))^(1/3)*x+3*RootOf(9*_Z^2-3*_Z+1)+3*(-(1-x)/(1+x))^(1/3)-x+1)*R
ootOf(9*_Z^2-3*_Z+1)
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="fricas")
```

output
```
(x + 1)*((x - 1)/(x + 1))^(2/3) - 2/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/
(x + 1))^(1/3) + 1/3*sqrt(3)) + 1/3*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)
/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

3.122.6 Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=\int\frac{1}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/3),x)
```

output
```
Integral(((x - 1)/(x + 1))**(-1/3), x)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=-\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
 1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3)
 + ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) - \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
 1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3)
 + ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) - 1)
)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-4\right)}{3} - \ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) + \ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) - \frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}-1}$$

input
```
int(1/((x - 1)/(x + 1))^(1/3),x)
```

output
```
log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 + 1/3)^2)*((3^(1/2)*1i)/
3 + 1/3) - log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 - 1/3)^2)*((3
^(1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) - 4))/3 - (2*((x -
1)/(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)
```

3.123 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx$

3.123.1 Optimal result

Integrand size = 12, antiderivative size = 155

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = &-\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)\\ &-\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)-\frac{3}{2}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right)\\ &-\frac{3}{2}\log\left(1+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)-\frac{1}{2}\log\left(1+\frac{1}{x}\right)-\frac{\log(x)}{2}\end{aligned}$$

output
```
-3/2*ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-3/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^
(1/3))-1/2*ln(1+1/x)-1/2*ln(x)+arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1
+1/x)^(1/3)*3^(1/2))*3^(1/2)-arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/
x)^(1/3)*3^(1/2))*3^(1/2)
```

3.123.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.17

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx=\frac{3}{2}e^{\frac{8}{3}\coth^{-1}(x)}\operatorname{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},e^{4\coth^{-1}(x)}\right)$$

input

```
Integrate[E^((2*ArcCoth[x])/3)/x,x]
```

output

```
(3*E^((8*ArcCoth[x])/3)*Hypergeometric2F1[2/3, 1, 5/3, E^(4*ArcCoth[x])])/
2
```

3.123.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6721, 140, 72, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}dx$$

$$\Big\downarrow \ 6721$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}x}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \ 140$$

$$-\int \frac{1}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-\int \frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \ 72$$

$$-\int \frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)$$

$$\Big\downarrow 102$$

$$-\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{\frac{1}{x}+1}\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)+\frac{1}{2}\log\left(\frac{1}{x}\right)$$

input
```
Int[E^((2*ArcCoth[x])/3)/x,x]
```

output
```
-(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 + x^(-1))^
(1/3))]) - Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 +
 x^(-1))^(1/3))] - (3*Log[1 + (1 - x^(-1))^(1/3)/(1 + x^(-1))^(1/3)])/2 -
(3*Log[(1 - x^(-1))^(1/3) - (1 + x^(-1))^(1/3)])/2 - Log[1 + x^(-1)]/2 + L
og[x^(-1)]/2
```

3.123.3.1 Defintions of rubi rules used

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.123.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 1026, normalized size of antiderivative = 6.62

method	result	size
trager	Expression too large to display	1026

input
```
int(1/((x-1)/(1+x))^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-3*ln((945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1890*RootOf(9*_Z
^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-168*(-(1-x)/(1+x))^(2/3)*x^2-504*(-(1-x)
/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+72*RootOf(9*_Z^2-3*_Z+1)^2*x^2+945
*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-336*(-(1-x)/(1+x))^(2/3)*x-147
*(-(1-x)/(1+x))^(1/3)*x^2-180*RootOf(9*_Z^2-3*_Z+1)^2*x-465*RootOf(9*_Z^2-
3*_Z+1)*x^2-168*(-(1-x)/(1+x))^(2/3)+504*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+
x))^(1/3)+72*RootOf(9*_Z^2-3*_Z+1)^2+1026*RootOf(9*_Z^2-3*_Z+1)*x+323*x^2+
147*(-(1-x)/(1+x))^(1/3)-465*RootOf(9*_Z^2-3*_Z+1)-34*x+323)/x)*RootOf(9*_
Z^2-3*_Z+1)-ln(-(945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1890*R
ootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-147*(-(1-x)/(1+x))^(2/3)*x^2-4
41*(-(1-x)/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2-1152*RootOf(9*_Z^2-3*_Z+
1)^2*x^2+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-294*(-(1-x)/(1+x))
^(2/3)*x-168*(-(1-x)/(1+x))^(1/3)*x^2+2880*RootOf(9*_Z^2-3*_Z+1)^2*x-120*R
ootOf(9*_Z^2-3*_Z+1)*x^2-147*(-(1-x)/(1+x))^(2/3)+441*RootOf(9*_Z^2-3*_Z+1
)*(-(1-x)/(1+x))^(1/3)-1152*RootOf(9*_Z^2-3*_Z+1)^2-1884*RootOf(9*_Z^2-3*_
Z+1)*x+187*x^2+168*(-(1-x)/(1+x))^(1/3)-120*RootOf(9*_Z^2-3*_Z+1)+306*x+18
7)/x)+ln((945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1890*RootOf(9
*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-168*(-(1-x)/(1+x))^(2/3)*x^2-504*(-(1
-x)/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+72*RootOf(9*_Z^2-3*_Z+1)^2*x^2+
945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-336*(-(1-x)/(1+x))^(2/3)...
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-1\right)+\frac{1}{2}\log\left(\frac{(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+x+1}{x+1}\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="fricas")
```

output
```
sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(2/3) + 1/3*sqrt(3)) - log(((
x - 1)/(x + 1))^(2/3) - 1) + 1/2*log(((x + 1)*((x - 1)/(x + 1))^(2/3) + (x
 - 1)*((x - 1)/(x + 1))^(1/3) + x + 1)/(x + 1))
```

3.123.6 Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{1}{x\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/3)/x,x)
```

output
```
Integral(1/(x*((x - 1)/(x + 1))**(1/3)), x)
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = &-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)\\ &+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)\\ &+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\ &+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\ &-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + sqrt(3)*arc
tan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 1/2*log(((x - 1)/(x + 1
))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*log(((x - 1)/(x + 1))^(2/3)
- ((x - 1)/(x + 1))^(1/3) + 1) - log(((x - 1)/(x + 1))^(1/3) + 1) - log(((
x - 1)/(x + 1))^(1/3) - 1)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.51

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+1\right)\right) + \frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}{x+1}+1\right) - \log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-1\right|\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="giac")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(2/3) + 1)) + 1/2*log(((x
- 1)/(x + 1))^(2/3) + (x - 1)*((x - 1)/(x + 1))^(1/3)/(x + 1) + 1) - log(a
bs(((x - 1)/(x + 1))^(2/3) - 1))
```

3.123.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = -\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}-1296\right) - \ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}+648-\sqrt{3}\,648\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}+648+\sqrt{3}\,648\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

input
```
int(1/(x*((x - 1)/(x + 1))^(1/3)),x)
```

output
```
log(3^(1/2)*648i + 1296*((x - 1)/(x + 1))^(2/3) + 648)*((3^(1/2)*1i)/2 + 1
/2) - log(1296*((x - 1)/(x + 1))^(2/3) - 3^(1/2)*648i + 648)*((3^(1/2)*1i)
/2 - 1/2) - log(1296*((x - 1)/(x + 1))^(2/3) - 1296)
```

3.124 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx$

3.124.1 Optimal result

Integrand size = 12, antiderivative size = 99

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx=\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}-\frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(1+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)-\frac{1}{3}\log\left(1+\frac{1}{x}\right)$$

output
```
(1+1/x)^(1/3)*((-1+x)/x)^(2/3)-ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3))-1/3*ln
(1+1/x)+2/3*arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2)
)*3^(1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2e^{\frac{2}{3}\coth^{-1}(x)}}{1+e^{2\coth^{-1}(x)}} - \frac{2\arctan\left(\frac{-1+2e^{\frac{2}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}\right) + \frac{1}{3}\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3)/x^2,x]
```

output
```
(2*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x])) - (2*ArcTan[(-1 + 2*E^((2*
ArcCoth[x])/3))/Sqrt[3]])/Sqrt[3] - (2*Log[1 + E^((2*ArcCoth[x])/3)])/3 +
Log[1 - E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)]/3
```

3.124.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6721, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{2}{3}\int \frac{1}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \text{72}$$

$$\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)+\frac{3}{2}\log\left(\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)+\frac{1}{2}\log\left(\frac{1}{x}+1\right)\right)$$

input

```
Int[E^((2*ArcCoth[x])/3)/x^2,x]
```

output

```
(1 - x^(-1))^(2/3)*(1 + x^(-1))^(1/3) - (2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*
(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))] + (3*Log[1 + (1 - x^(-1)
)^(1/3)/(1 + x^(-1))^(1/3)])/2 + Log[1 + x^(-1)]/2))/3
```

3.124.3.1 Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 6721

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.124.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.22

method	result
trager	$\frac{(1+x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{x}-\frac{2\ln\left(\frac{9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}-3\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}\left(9\_\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/((x-1)/(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(1+x)/x*(-(1-x)/(1+x))^(2/3)-2/3*ln((9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x)
)^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2
/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(2/3
)+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-36*RootOf(9*_Z^2-3*_Z+1)^2+
6*RootOf(9*_Z^2-3*_Z+1)*x+12*RootOf(9*_Z^2-3*_Z+1)-x-1)/x)+2/3*ln(-(9*Root
Of(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(
1+x))^(2/3)+3*(-(1-x)/(1+x))^(1/3)*x-18*RootOf(9*_Z^2-3*_Z+1)^2-3*RootOf(9
*_Z^2-3*_Z+1)*x+3*(-(1-x)/(1+x))^(1/3)-3*RootOf(9*_Z^2-3*_Z+1)-x+1)/x)-2*l
n(-(9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)
*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(1/3)*x-18*RootOf(9*_Z^2-3*_Z+1)^2-
3*RootOf(9*_Z^2-3*_Z+1)*x+3*(-(1-x)/(1+x))^(1/3)-3*RootOf(9*_Z^2-3*_Z+1)-x
+1)/x)*RootOf(9*_Z^2-3*_Z+1)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-2\,x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3\,(x+1)\left(\right.}{3\,x}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3))
 + x*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 2*x*log(
((x - 1)/(x + 1))^(1/3) + 1) + 3*(x + 1)*((x - 1)/(x + 1))^(2/3))/x
```

3.124.6 Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/3)/x**2,x)
```

output
```
Integral(1/(x**2*((x - 1)/(x + 1))**(1/3)), x)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right) + \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3)
- ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) + 1)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right|\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3)
- ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}+1}-\ln\left(9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) + \ln\left(9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)-\frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}+4\right)}{3}$$

input
```
int(1/(x^2*((x - 1)/(x + 1))^(1/3)),x)
```

output
```
log(9*((3^(1/2)*1i)/3 + 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3^(1/2)*1i)/
3 + 1/3) - log(9*((3^(1/2)*1i)/3 - 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3
^(1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) + 4))/3 + (2*((x -
1)/(x + 1))^(2/3))/((x - 1)/(x + 1) + 1)
```

3.125 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx$

3.125.1 Optimal result

Integrand size = 12, antiderivative size = 130

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(1+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{9}\log\left(1+\frac{1}{x}\right)$$

output

```
1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)+1/2*(1+1/x)^(4/3)*((-1+x)/x)^(2/3)-1/3*
ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3))-1/9*ln(1+1/x)+2/9*arctan(-1/3*3^(1/2)
+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(1/2)
```

3.125.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = -\frac{2}{27}\left(\frac{27e^{\frac{2}{3}\coth^{-1}(x)}}{\left(1+e^{2\coth^{-1}(x)}\right)^2} - \frac{36e^{\frac{2}{3}\coth^{-1}(x)}}{1+e^{2\coth^{-1}(x)}} - 2\coth^{-1}(x) + 3\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}\right) - \text{RootSum}\left[1-\#1^2+\#1^4\&, \frac{\coth^{-1}(x)-3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+\coth^{-1}(x)\#1^2-3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-2+\#1^2}\&\right]\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3)/x^3,x]
```

output
```
(-2*((27*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x]))^2 - (36*E^((2*ArcCot
h[x])/3))/(1 + E^(2*ArcCoth[x])) - 2*ArcCoth[x] + 3*Log[1 + E^((2*ArcCoth[
x])/3)] - RootSum[1 - #1^2 + #1^4 & , (ArcCoth[x] - 3*Log[E^(ArcCoth[x]/3)
 - #1] + ArcCoth[x]*#1^2 - 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-2 + #1^2)
& ]))/27
```

3.125.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6721, 90, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}x}d\frac{1}{x}$$

$$\Big\downarrow 90$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}-\frac{1}{3}\int\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow 60$$

$$\frac{1}{3}\left(\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{2}{3}\int\frac{1}{\sqrt[3]{1-\frac{1}{x}}\,\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}$$

$$\Big\downarrow 72$$

$$\frac{1}{3}\left(\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)+\frac{3}{2}\log\left(\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)+\frac{1}{2}\log\left(\frac{1}{x}+1\right)\right)\right)+$$
$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}$$

input

```
Int[E^((2*ArcCoth[x])/3)/x^3,x]
```

output

```
((1 - x^(-1))^(2/3)*(1 + x^(-1))^(4/3))/2 + ((1 - x^(-1))^(2/3)*(1 + x^(-1
))^(1/3) - (2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - x^(-1))^(1/3))/(Sqrt[3]*
(1 + x^(-1))^(1/3))] + (3*Log[1 + (1 - x^(-1))^(1/3)/(1 + x^(-1))^(1/3)])/
2 + Log[1 + x^(-1)]/2))/3)/3
```

3.125.3.1 Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.125.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.89

method	result
risch	$\frac{(x-1)(5x+3)}{6x^2\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2x^2-8\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2x+27\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}}{}\right.}{}\right.}{}$
trager	$\frac{(1+x)(5x+3)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{6x^2}+\frac{2\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\ln\left(\frac{-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)}{}\right.}{}$

input
```
int(1/((x-1)/(1+x))^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(x-1)*(5*x+3)/x^2/((x-1)/(1+x))^(1/3)+(-2/9*ln((8*RootOf(_Z^2-3*_Z+9)^
2*x^2-8*RootOf(_Z^2-3*_Z+9)^2*x+27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)
-45*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x-30*RootOf(_Z^2-3*_Z+9)*x^2-1
6*RootOf(_Z^2-3*_Z+9)^2-45*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-54*Root
Of(_Z^2-3*_Z+9)*x-216*(x^3+x^2-x-1)^(2/3)-81*(x^3+x^2-x-1)^(1/3)*x-27*x^2-
24*RootOf(_Z^2-3*_Z+9)-81*(x^3+x^2-x-1)^(1/3)-36*x-9)/x/(1+x))+2/27*RootOf
(_Z^2-3*_Z+9)*ln((2*RootOf(_Z^2-3*_Z+9)^2*x^2-2*RootOf(_Z^2-3*_Z+9)^2*x-27
*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-72*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x
-1)^(1/3)*x+27*RootOf(_Z^2-3*_Z+9)*x^2-4*RootOf(_Z^2-3*_Z+9)^2-72*RootOf(_
Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-6*RootOf(_Z^2-3*_Z+9)*x-135*(x^3+x^2-x-1)^
(2/3)+81*(x^3+x^2-x-1)^(1/3)*x+36*x^2-33*RootOf(_Z^2-3*_Z+9)+81*(x^3+x^2-x
-1)^(1/3)+216*x+180)/x/(1+x)))/((x-1)/(1+x))^(1/3)*((1+x)^2*(x-1))^(1/3)/(
1+x)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{4\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\,x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-4\,x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3\,(5\,x}{18\,x^2}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="fricas")
```

output
```
1/18*(4*sqrt(3)*x^2*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(
3)) + 2*x^2*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 4
*x^2*log(((x - 1)/(x + 1))^(1/3) + 1) + 3*(5*x^2 + 8*x + 3)*((x - 1)/(x +
1))^(2/3))/x^2
```

3.125.6 Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/3)/x**3,x)
```

output
```
Integral(1/(x**3*((x - 1)/(x + 1))**(1/3)), x)
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1\right)}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*(((x
 - 1)/(x + 1))^(5/3) + 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) + (x
- 1)^2/(x + 1)^2 + 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1)
)^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) + 1)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.94

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}+1\right)^2}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right|\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*((x
- 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) + 4*((x - 1)/(x + 1))^(2/3))/((x - 1)
/(x + 1) + 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3
) + 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

3.125.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}+\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1} - \frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}+\frac{4}{9}\right)}{9}$$
$$-\ln\left(9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)+\ln\left(9\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(\frac{1}{9}+\frac{\sqrt{3}}{9}\right.$$

input
```
int(1/(x^3*((x - 1)/(x + 1))^(1/3)),x)
```

output
```
((8*((x - 1)/(x + 1))^(2/3))/3 + (2*((x - 1)/(x + 1))^(5/3))/3)/((2*(x - 1
))/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))
/9 + 4/9))/9 - log(9*((3^(1/2)*1i)/9 - 1/9)^2 + (4*((x - 1)/(x + 1))^(1/3)
)/9)*((3^(1/2)*1i)/9 - 1/9) + log(9*((3^(1/2)*1i)/9 + 1/9)^2 + (4*((x - 1)
/(x + 1))^(1/3))/9)*((3^(1/2)*1i)/9 + 1/9)
```

3.126 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

3.126.1 Optimal result

Integrand size = 14, antiderivative size = 429

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2}+\frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3-\frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+\frac{11\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+\frac{11\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}+\frac{11\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}-\frac{11\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3}+\frac{11\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128\sqrt{2}a^3}$$

output
```
37/96*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a^2+3/8*(1-1/a/x)^(7/8)*(1+1/a/x)^
(1/8)*x^2/a+1/3*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x^3+11/64*arctan((1+1/a/x)
^(1/8)/(1-1/a/x)^(1/8))/a^3+11/64*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))
/a^3-11/128*arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)+
11/128*arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)-11/25
6*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(
1/8))/a^3*2^(1/2)+11/256*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+(1+1/a/x)^(1
/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)
```

3.126.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.39

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$$
$$=\frac{-4\Bigg(-\frac{1024e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{1600e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{840e^{\frac{1}{4}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-66\arctan\left(e^{\frac{1}{4}\coth^{-1}(ax)}\right)+33\log\Big(1-e^{\frac{1}{4}\mathrm{cot}}}{1536a^3}$$

input
```
Integrate[E^(ArcCoth[a*x]/4)*x^2,x]
```

output
```
(-4*((-1024*E^(ArcCoth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^3 - (1600*E^(Arc
Coth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^2 - (840*E^(ArcCoth[a*x]/4))/(-1 +
 E^(2*ArcCoth[a*x])) - 66*ArcTan[E^(ArcCoth[a*x]/4)] + 33*Log[1 - E^(ArcCo
th[a*x]/4)] - 33*Log[1 + E^(ArcCoth[a*x]/4)]) - 33*RootSum[1 + #1^4 & , (A
rcCoth[a*x] - 4*Log[E^(ArcCoth[a*x]/4) - #1])/#1^3 & ])/(1536*a^3)
```

3.126.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.91, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}\,x^4}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 110$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{1}{3}\int \frac{\left(9a+\frac{8}{x}\right)x^3}{4a^2\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\int \frac{\left(9a+\frac{8}{x}\right)x^3}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{12a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{-\frac{1}{2}\int -\frac{\left(37a+\frac{36}{x}\right)x^2}{4a\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{\int \frac{\left(37a+\frac{36}{x}\right)x^2}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$
$$\frac{\frac{-\int -\frac{33x}{4\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \; 27$$

3.126. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{\frac{33}{4}\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \ 104$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\int \frac{1}{\frac{1}{x^8}-1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \ 758$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \ 755$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7}}{12a^2}$$

$$\Big\downarrow \ 756$$

3.126. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{66\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)-37ax\left(1-\frac{1}{ax}\right)^{7/8}}{8a}}{12a^2}$$

$\Big\downarrow$ 216

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{66\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)-37ax\left(1-\frac{1}{ax}\right)^{7/8}}{8a}}{12a^2}$$

$\Big\downarrow$ 219

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{66\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)\right)-37ax\left(1-\frac{1}{ax}\right.}{8a}}{12a^2}$$

$\Big\downarrow$ 1476

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{66\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\operatorname{arct}\right.\right.}{8a}}{12a^2}$$

$\Big\downarrow$ 1082

3.126. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\right)\right)}{8a}}{12a^2}$$

$$\Big\downarrow 217$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\arctan\right)\right)}{8a}}{12a^2}$$

$$\Big\downarrow 1479$$

3.126. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$\frac{66\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\arctan\Bigg(}{8a}$$

$$\Big\downarrow 25$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$\frac{66\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\arctan\Bigg(}{8a}$$

$$\Big\downarrow 27$$

3.126. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$66\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\Bigg(}{}$$

$$8a$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$66\Bigg(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(-$$

$$8a$$

$$12a^2$$

input
```
Int[E^(ArcCoth[a*x]/4)*x^2,x]
```

3.126. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

output
```
((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x^3)/3 - ((-9*a*(1 - 1/(a*x))^(7/
8)*(1 + 1/(a*x))^(1/8)*x^2)/2 + (-37*a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(
1/8)*x + 66*((-1/2*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] - ArcTa
nh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/2)/2 + ((ArcTan[1 - (Sqrt[2]*(
1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2] - ArcTan[1 + (Sqrt[2]*(1
+ 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 +
 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt
[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2]))/2)/2))
/(8*a))/(12*a^2)
```

3.126.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

3.126. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.126.4 Maple [F]

$$\int \frac{x^{2}}{\left(\frac{a x -1}{a x +1}\right)^{\frac{1}{8}}}d x$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

3.126.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.62

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{33\,a^3\left(-\frac{1}{a^{12}}\right)^{\frac{1}{4}}\log\left(a^9\left(-\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)-33i\,a^3\left(-\frac{1}{a^{12}}\right)^{\frac{1}{4}}\log\left(i\,a^9\left(-\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+33i\,a^3\left(-\frac{1}{a^{12}}\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="fricas")
```

output
```
-1/384*(33*a^3*(-1/a^12)^(1/4)*log(a^9*(-1/a^12)^(3/4) + ((a*x - 1)/(a*x +
 1))^(1/8)) - 33*I*a^3*(-1/a^12)^(1/4)*log(I*a^9*(-1/a^12)^(3/4) + ((a*x -
 1)/(a*x + 1))^(1/8)) + 33*I*a^3*(-1/a^12)^(1/4)*log(-I*a^9*(-1/a^12)^(3/4
) + ((a*x - 1)/(a*x + 1))^(1/8)) - 33*a^3*(-1/a^12)^(1/4)*log(-a^9*(-1/a^1
2)^(3/4) + ((a*x - 1)/(a*x + 1))^(1/8)) - 4*(32*a^3*x^3 + 68*a^2*x^2 + 73*
a*x + 37)*((a*x - 1)/(a*x + 1))^(7/8) + 66*arctan(((a*x - 1)/(a*x + 1))^(1
/8)) - 33*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) + 33*log(((a*x - 1)/(a*x +
1))^(1/8) - 1))/a^3
```

3.126.6 Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(1/8), x)
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.79

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{768}a\left(\frac{16\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{23}{8}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}+105\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="maxima")
```

output
```
-1/768*a*(16*(33*((a*x - 1)/(a*x + 1))^(23/8) - 10*((a*x - 1)/(a*x + 1))^(
15/8) + 105*((a*x - 1)/(a*x + 1))^(7/8))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a
*x - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt
(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4)
+ 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x
+ 1))^(1/4) + 1))/a^4 + 132*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^4 - 66*l
og(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*log(((a*x - 1)/(a*x + 1))^(1/
8) - 1)/a^4)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.72

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{768}a\left(\frac{66\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}+\frac{66\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}-\frac{33\sqrt{2}}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="giac")
```

output
```
-1/768*a*(66*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/8)))/a^4 + 66*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/8)))/a^4 - 33*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) +
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 33*sqrt(2)*log(-sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 132*arctan(((a*x
 - 1)/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4
+ 66*log(-((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 16*(33*((a*x - 1)/(a*x +
1))^(23/8) - 10*((a*x - 1)/(a*x + 1))^(15/8) + 105*((a*x - 1)/(a*x + 1))^(
7/8))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

3.126.9 Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.53

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{35\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{16}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{24}+\frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{23/8}}{16}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} - \frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}1\mathrm{i}\right)\,11\mathrm{i}}{64\,a^3} - \frac{11\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{64\,a^3} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{11}{128}+\frac{11}{128}\mathrm{i}\right)}{a^3} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{11}{128}-\frac{11}{128}\mathrm{i}\right)}{a^3}$$

input
```
int(x^2/((a*x - 1)/(a*x + 1))^(1/8),x)
```

output
```
((35*((a*x - 1)/(a*x + 1))^(7/8))/16 - (5*((a*x - 1)/(a*x + 1))^(15/8))/24
 + (11*((a*x - 1)/(a*x + 1))^(23/8))/16)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x +
 1)^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (at
an(((a*x - 1)/(a*x + 1))^(1/8)*1i)*11i)/(64*a^3) - (11*atan(((a*x - 1)/(a*
x + 1))^(1/8)))/(64*a^3) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/
8)*(1/2 - 1i/2))*(11/128 - 11i/128))/a^3 - (2^(1/2)*atan(2^(1/2)*((a*x - 1
)/(a*x + 1))^(1/8)*(1/2 + 1i/2))*(11/128 + 11i/128))/a^3
```

3.127 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

3.127.1 Optimal result

Integrand size = 12, antiderivative size = 392

$$\begin{aligned}\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = {} & \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} \\ & +\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2} \\ & +\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}+\frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} \\ & +\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}-\frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2} \\ & +\frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{32\sqrt{2}a^2}\end{aligned}$$

output
```
1/8*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a+1/2*(1-1/a/x)^(7/8)*(1+1/a/x)^(9/8
)*x^2+1/16*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^2+1/16*arctanh((1+1/a
/x)^(1/8)/(1-1/a/x)^(1/8))/a^2-1/32*arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/
a/x)^(1/8))/a^2*2^(1/2)+1/32*arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1
/8))/a^2*2^(1/2)-1/64*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)
*2^(1/2)/(1-1/a/x)^(1/8))/a^2*2^(1/2)+1/64*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^
(1/4)+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^2*2^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.81

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$$
$$=\frac{\frac{2}{\left(-1+e^{\frac{1}{4}\coth^{-1}(ax)}\right)^2}+\frac{6}{-1+e^{\frac{1}{4}\coth^{-1}(ax)}}-\frac{2}{\left(1+e^{\frac{1}{4}\coth^{-1}(ax)}\right)^2}+\frac{6}{1+e^{\frac{1}{4}\coth^{-1}(ax)}}+\frac{8e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)^2}-\frac{12e^{\frac{1}{4}\coth^{-1}(ax)}}{1+e^{\frac{1}{2}\coth^{-1}(ax)}}+}{}$$

input
```
Integrate[E^(ArcCoth[a*x]/4)*x,x]
```

output
```
(2/(-1 + E^(ArcCoth[a*x]/4))^2 + 6/(-1 + E^(ArcCoth[a*x]/4)) - 2/(1 + E^(A
rcCoth[a*x]/4))^2 + 6/(1 + E^(ArcCoth[a*x]/4)) + (8*E^(ArcCoth[a*x]/4))/(1
 + E^(ArcCoth[a*x]/2))^2 - (12*E^(ArcCoth[a*x]/4))/(1 + E^(ArcCoth[a*x]/2)
) + (32*E^(ArcCoth[a*x]/4))/(1 + E^ArcCoth[a*x])^2 - (40*E^(ArcCoth[a*x]/4
))/(1 + E^ArcCoth[a*x]) + 4*ArcTan[E^(ArcCoth[a*x]/4)] - 2*Sqrt[2]*ArcTan[
1 - Sqrt[2]*E^(ArcCoth[a*x]/4)] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[
a*x]/4)] + 4*ArcTanh[E^(ArcCoth[a*x]/4)] - Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcC
oth[a*x]/4) + E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x
]/4) + E^(ArcCoth[a*x]/2)])/(64*a^2)
```

3.127.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.89, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6721, 107, 105, 104, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{4}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}x^3}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 107$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{\int \frac{\sqrt[8]{1+\frac{1}{ax}}x^2}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}}{8a}$$

$$\Big\downarrow \; 105$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{\frac{\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{4a}-x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

$$\Big\downarrow \; 104$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{\frac{2\int \frac{1}{\frac{1}{x^8}-1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{a}-x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

$$\Big\downarrow \; 758$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}-x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

$$\Big\downarrow \; 755$$

3.127. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}-x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)}{a}-x\left(1-\frac{1}{ax}\right)^{7/8}}{8a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)}{a}-x\left(1-\frac{1}{ax}\right)}{8a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)\right)}{a}-x(1-}{8a}$$

$$\Big\downarrow \text{1476}$$

3.127. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\text{arcta}\right.\right.}{a}}{8a}$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\right.\right.}{a}}{8a}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctan}\right.\right.}{a}}{8a}$$

$$\Big\downarrow \; 1479$$

3.127. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.}{a}}{8a}$$

$$\Big\downarrow 25$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.}{a}}{8a}$$

$$\Big\downarrow 27$$

3.127. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\right.\right.\right.}{a}}{8a}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\log\right.\right.\right.}{a}}{8a}$$

input
```
Int[E^(ArcCoth[a*x]/4)*x,x]
```

3.127. $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

output
```
((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(9/8)*x^2)/2 - (-((1 - 1/(a*x))^(7/8)*(
1 + 1/(a*x))^(1/8)*x) + (2*((-1/2*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))
^(1/8)] - ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/2)/2 + ((ArcTan
[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2] - ArcTan[1
 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2])/2 + (Log[1
- (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2])
- Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*S
qrt[2]))/2)/2))/a)/(8*a)
```

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.127. $\int e^{\frac{1}{4}\coth^{-1}(ax)} x \, dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.127.4 Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

3.127.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{a^2\left(-\frac{1}{a^8}\right)^{\frac{1}{4}}\log\left(a^6\left(-\frac{1}{a^8}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)-i\,a^2\left(-\frac{1}{a^8}\right)^{\frac{1}{4}}\log\left(i\,a^6\left(-\frac{1}{a^8}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+i\,a^2\left(-\frac{1}{a^8}\right)^{\frac{1}{4}}\log\Bigl(-i}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="fricas")
```

output
```
-1/32*(a^2*(-1/a^8)^(1/4)*log(a^6*(-1/a^8)^(3/4) + ((a*x - 1)/(a*x + 1))^(
1/8)) - I*a^2*(-1/a^8)^(1/4)*log(I*a^6*(-1/a^8)^(3/4) + ((a*x - 1)/(a*x +
1))^(1/8)) + I*a^2*(-1/a^8)^(1/4)*log(-I*a^6*(-1/a^8)^(3/4) + ((a*x - 1)/(
a*x + 1))^(1/8)) - a^2*(-1/a^8)^(1/4)*log(-a^6*(-1/a^8)^(3/4) + ((a*x - 1)
/(a*x + 1))^(1/8)) - 4*(4*a^2*x^2 + 9*a*x + 5)*((a*x - 1)/(a*x + 1))^(7/8)
 + 2*arctan(((a*x - 1)/(a*x + 1))^(1/8)) - log(((a*x - 1)/(a*x + 1))^(1/8)
 + 1) + log(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^2
```

3.127.6 Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx=\int\frac{x}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(1/8), x)
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.78

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$$

$$=\frac{1}{64}\,a\Bigg(\frac{16\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{2\,(ax-1)a^{3}}{ax+1}-\frac{(ax-1)^{2}a^{3}}{(ax+1)^{2}}-a^{3}}-\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\Big(-\frac{1}{2}\sqrt{2}\Big(\sqrt{2}-}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="maxima")
```

output
```
1/64*a*(16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*((a*x - 1)/(a*x + 1))^(7/8))/
(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - (2*sqrt(
2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2
)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) +
1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x +
1))^(1/4) + 1))/a^3 - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^3 + 2*log(((
a*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1
)/a^3)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.73

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{1}{64}a\Bigg(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}-\sqrt{2}\log\Big(\sqrt{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="giac")
```

output
```
-1/64*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/8)))/a^3 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
1))^(1/8)))/a^3 - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^3 + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 + 4*arctan(((a*x - 1)/(a*x
+ 1))^(1/8))/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 + 2*log(-((a
*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 + 16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*(
(a*x - 1)/(a*x + 1))^(7/8))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

3.127.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.48

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx=\frac{\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{16\,a^2}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\right)}{16\,a^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}+\frac{1}{32}{}\mathrm{i}\right)}{a^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}-\frac{1}{32}{}\mathrm{i}\right)}{a^2}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(1/8),x)
```

output
```
((9*((a*x - 1)/(a*x + 1))^(7/8))/4 - ((a*x - 1)/(a*x + 1))^(15/8)/4)/(a^2
+ (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (atan(((a
*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/(16*a^2) - atan(((a*x - 1)/(a*x + 1))^(1/
8))/(16*a^2) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i
/2))*(1/32 - 1i/32))/a^2 - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/
8)*(1/2 + 1i/2))*(1/32 + 1i/32))/a^2
```

3.128 $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

3.128.1 Optimal result

Integrand size = 10, antiderivative size = 352

$$\begin{aligned}\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx &= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}\\ &\quad+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}+\frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}\\ &\quad+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}-\frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}\\ &\quad+\frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}a}\end{aligned}$$

output
```
(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x+1/2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/
8))/a+1/2*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a-1/4*arctan(1-(1+1/a/x
)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)+1/4*arctan(1+(1+1/a/x)^(1/8)*2^
(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)-1/8*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-
(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)+1/8*ln(1+(1+1/a/x)^(1/4
)/(1-1/a/x)^(1/4)+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)
```

3.128.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.16

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \frac{2e^{\frac{1}{4}\coth^{-1}(ax)}\left(1+\left(-1+e^{2\coth^{-1}(ax)}\right)\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{2\coth^{-1}(ax)}\right)\right)}{a\left(-1+e^{2\coth^{-1}(ax)}\right)}$$

input
```
Integrate[E^(ArcCoth[a*x]/4),x]
```

output
```
(2*E^(ArcCoth[a*x]/4)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[1/8
, 1, 9/8, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

3.128.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.87, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {6720, 105, 104, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}x^2}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{4a}$$

$$\Big\downarrow \text{104}$$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\int \frac{1}{\frac{1}{x^8}-1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{a}$$

$$\Big\downarrow \text{758}$$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$\Big\downarrow \text{755}$$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$\Big\downarrow \text{756}$$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)}{a}$$

$$\Big\downarrow \text{216}$$

3.128. $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\Bigg(\frac{1}{2}\Bigg(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\Bigg(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)\Bigg)+\frac{1}{2}\Bigg(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)\Bigg)}{a}$$

$$\Big\downarrow \text{219}$$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\Bigg(\frac{1}{2}\Bigg(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)+\frac{1}{2}\Bigg(-\frac{1}{2}\arctan\Bigg(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)-\frac{1}{2}\text{arctanh}\Bigg(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)}{a}$$

$$\Big\downarrow \text{1476}$$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(-\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)}{a}$$

$$\Big\downarrow \text{1082}$$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\Bigg(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\Bigg)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\Bigg(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)}{\sqrt{2}}\Bigg)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)+\frac{1}{2}\Bigg(-\frac{1}{2}\arctan\Bigg(}{a}$$

$$\Big\downarrow \text{217}$$

3.128. $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

$$\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-2\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\Bigg)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)+\frac{1}{2}\Bigg(-\frac{1}{2}\arctan\Bigg(\frac{\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}}{a}$$

$$\Big\downarrow \text{1479}$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-2\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}}{a}$$

$$\Big\downarrow \text{25}$$

3.128. $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

$$\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-2\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}}{a}$$

$$\Big\downarrow \ 27$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-2\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}}{a}$$

$$\Big\downarrow \ 1103$$

3.128. $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$\frac{2\Bigg(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\Bigg(\frac{1}{2}\Bigg(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\Bigg(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+}}{\sqrt[8]{1-\frac{1}{a}}}}{\sqrt{2}}}{a}$$

input
```
Int[E^(ArcCoth[a*x]/4),x]
```

output
```
(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x - (2*((-1/2*ArcTan[(1 + 1/(a*x))
^(1/8)/(1 - 1/(a*x))^(1/8)] - ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1
/8)]/2)/2 + ((ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)
]/Sqrt[2] - ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/
Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) +
x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^
(1/8) + x^(-2)]/(2*Sqrt[2]))/2)/2))/a
```

3.128.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

3.128. $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.128.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

3.128.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.68

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx =$$

$$-\frac{a\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)-i\,a\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(i\,a^3\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+i\,a\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-i\,a^3\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="fricas")
```

output
```
-1/4*(a*(-1/a^4)^(1/4)*log(a^3*(-1/a^4)^(3/4) + ((a*x - 1)/(a*x + 1))^(1/8
)) - I*a*(-1/a^4)^(1/4)*log(I*a^3*(-1/a^4)^(3/4) + ((a*x - 1)/(a*x + 1))^(
1/8)) + I*a*(-1/a^4)^(1/4)*log(-I*a^3*(-1/a^4)^(3/4) + ((a*x - 1)/(a*x + 1
))^(1/8)) - a*(-1/a^4)^(1/4)*log(-a^3*(-1/a^4)^(3/4) + ((a*x - 1)/(a*x + 1
))^(1/8)) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(7/8) + 2*arctan(((a*x - 1)/
(a*x + 1))^(1/8)) - log(((a*x - 1)/(a*x + 1))^(1/8) + 1) + log(((a*x - 1)/
(a*x + 1))^(1/8) - 1))/a
```

3.128.6 Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \int \frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-1/8), x)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.75

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx =$$

$$-\frac{1}{8}a\left(\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right.}{a}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="maxima")
```

output
```
-1/8*a*(16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)*a^2/(a*x + 1) - a^2) + (
2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) -
 sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(
1/4) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1))/a^2 + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^2 - 2
*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^2 + 2*log(((a*x - 1)/(a*x + 1))^(1
/8) - 1)/a^2)
```

3.128.8 Giac [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(-1/8), x)
```

3.128.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.42

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx=\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a}-\frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{2\,a}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{a}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(1/8),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(7/8))/(a - (a*(a*x - 1))/(a*x + 1)) - (atan(((a*
x - 1)/(a*x + 1))^(1/8)*1i)*1i)/(2*a) - atan(((a*x - 1)/(a*x + 1))^(1/8))/
(2*a) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1
/4 - 1i/4))/a - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 + 1
i/2))*(1/4 + 1i/4))/a
```

3.129 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

3.129.1 Optimal result

Integrand size = 14, antiderivative size = 919

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$-\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$+\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$+\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+2\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

output
```
2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))+2*arctanh((1+1/a/x)^(1/8)/(1-1/a
/x)^(1/8))-1/2*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1/2
)/(1-1/a/x)^(1/8))*2^(1/2)+1/2*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+(1+1/a
/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))*2^(1/2)-arctan(1-(1+1/a/x)^(1/8)*2^(1/2
)/(1-1/a/x)^(1/8))*2^(1/2)+arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8
))*2^(1/2)-arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(
2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+arctan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1
/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/2*ln(1+(1-1/
a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/
8))*(2-2^(1/2))^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(
1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-arctan((-2*(1-1/
a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2
))^(1/2)+arctan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-
(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/2*l
n(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1
/a/x)^(1/8))*(2+2^(1/2))^(1/2)
```

3.129.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.03

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx=\frac{16}{9}e^{\frac{9}{4}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{9}{16},1,\frac{25}{16},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/4)/x,x]
```

output
```
(16*E^((9*ArcCoth[a*x])/4)*Hypergeometric2F1[9/16, 1, 25/16, E^(4*ArcCoth[
a*x])])/9
```

3.129.3 Rubi [A] (warning: unable to verify)

Time = 1.17 (sec) , antiderivative size = 866, normalized size of antiderivative = 0.94, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.786, Rules used = {6721, 140, 73, 104, 758, 755, 756, 216, 219, 854, 828, 1442, 1476, 1082, 217, 1479, 25, 27, 1103, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}\,x}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-\frac{\int \frac{1}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}$$

$$\Big\downarrow \text{104}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-8\int \frac{1}{\frac{1}{x^8}-1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{758}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-8\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$\Big\downarrow \text{755}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$\Big\downarrow \text{756}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\right.\right.$$

$$\Big\downarrow \text{216}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\right.\right.$$

$$\Big\downarrow \text{219}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\right.\right.$$

$$\Big\downarrow \text{854}$$

$$8\int \frac{1}{\left(1+\frac{1}{x^8}\right)x^6}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\right.\right.$$

$$\Big\downarrow \text{828}$$

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\int \frac{1}{\left(1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}\right.\right.\right.$$

$$\Big\downarrow \text{1442}$$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}\right.\right.\right.$$

$$\Big\downarrow \text{1476}$$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\right.\right.$$

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$\Big\downarrow \text{1082}$$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\right.\right.$$

$$\Big\downarrow \text{217}$$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\text{arc}\right.\right.$$

$$\Big\downarrow \text{1479}$$

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{}}{\sqrt[8]{1}}\right.}{\sqrt{2}}\right.\right.\right.$$

$\downarrow$ 25

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{a}}}{\sqrt[8]{1-}}\right.}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow 27$$

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}}{\sqrt[8]{}}\right.}{\sqrt{2}}\right.\right.\right.$$

$\downarrow$ 1103

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 1483

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{\int \frac{\sqrt{2-\sqrt{2}}-\frac{\left(1-\sqrt{}\right.}{\sqrt[8]{}}}{-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1}}{\sqrt[8]{2-}}}}{}}{2\sqrt{2}}-\frac{}{}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{}\right.}{}\right.\right.\right.$$

$$\downarrow 1142$$

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int-\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int-}{}}{2\sqrt{2}}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{}\right.}{}\right.\right.\right.$$

$\downarrow$ 25

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\cdots}{\cdots}}{2\sqrt{2}}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\cdots\right.}{\cdots}\right.\right.\right.$$

↓ 1083

3.129. $\int\frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d\left(\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d\left(\right.}{}}{2\sqrt{2}}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.$$

$\downarrow$ 217

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}}{}}{2\sqrt{2}}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{}\right.}{}\right.\right.\right.$$

$\downarrow$ 1103

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

$$8\left(\frac{-\frac{-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)-}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.$$

input
```
Int[E^(ArcCoth[a*x]/4)/x,x]
```

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

output
```
8*(-1/2*((1 - 1/(a*x))^(1/8)/(2 - x^(-8))^(1/8) - (ArcTan[(-Sqrt[2 - Sqrt[
2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 + Sqrt[2]]] - ((1
 - Sqrt[2])*Log[1 - (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(
1/8) + x^(-2)])/2)/(2*Sqrt[2 - Sqrt[2]]) - (ArcTan[(Sqrt[2 - Sqrt[2]] + (2
*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[
2])*Log[1 + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x
^(-2)])/2)/(2*Sqrt[2 - Sqrt[2]]))/Sqrt[2] + ((1 - 1/(a*x))^(1/8)/(2 - x^(-
8))^(1/8) - (-ArcTan[(-Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^
(-8))^(1/8))/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 - (Sqrt[2 + Sqrt[2]
]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 + Sqrt[2
]]) - (-ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(
1/8))/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log[1 + (Sqrt[2 + Sqrt[2]]*(1 -
1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 + Sqrt[2]]))/(2
*Sqrt[2])) - 8*((-1/2*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] - Ar
cTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/2)/2 + ((ArcTan[1 - (Sqrt[2
]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2] - ArcTan[1 + (Sqrt[2]*
(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(
1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (S
qrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2]))/2)/
2)
```

3.129.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.129. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.129.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

3.129.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.45

$$\begin{aligned}\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = & -\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i+1)\sqrt{2}(-1)^{\frac{7}{8}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&+\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i-1)\sqrt{2}(-1)^{\frac{7}{8}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i-1)\sqrt{2}(-1)^{\frac{7}{8}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&+\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i+1)\sqrt{2}(-1)^{\frac{7}{8}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&+\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&+\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&+(-1)^{\frac{1}{8}}\log\left((-1)^{\frac{7}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&-i(-1)^{\frac{1}{8}}\log\left(i(-1)^{\frac{7}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&+i(-1)^{\frac{1}{8}}\log\left(-i(-1)^{\frac{7}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&-(-1)^{\frac{1}{8}}\log\left(-(-1)^{\frac{7}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\\
&+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right)\end{aligned}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="fricas")
```

output
```
-(1/2*I - 1/2)*sqrt(2)*(-1)^(1/8)*log((I + 1)*sqrt(2)*(-1)^(7/8) + 2*((a*x
 - 1)/(a*x + 1))^(1/8)) + (1/2*I + 1/2)*sqrt(2)*(-1)^(1/8)*log(-(I - 1)*sq
rt(2)*(-1)^(7/8) + 2*((a*x - 1)/(a*x + 1))^(1/8)) - (1/2*I + 1/2)*sqrt(2)*
(-1)^(1/8)*log((I - 1)*sqrt(2)*(-1)^(7/8) + 2*((a*x - 1)/(a*x + 1))^(1/8))
 + (1/2*I - 1/2)*sqrt(2)*(-1)^(1/8)*log(-(I + 1)*sqrt(2)*(-1)^(7/8) + 2*((
a*x - 1)/(a*x + 1))^(1/8)) - (1/2*I - 1/2)*sqrt(2)*log((I + 1)*sqrt(2) + 2
*((a*x - 1)/(a*x + 1))^(1/8)) + (1/2*I + 1/2)*sqrt(2)*log(-(I - 1)*sqrt(2)
 + 2*((a*x - 1)/(a*x + 1))^(1/8)) - (1/2*I + 1/2)*sqrt(2)*log((I - 1)*sqrt
(2) + 2*((a*x - 1)/(a*x + 1))^(1/8)) + (1/2*I - 1/2)*sqrt(2)*log(-(I + 1)*
sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8)) + (-1)^(1/8)*log((-1)^(7/8) + ((a
*x - 1)/(a*x + 1))^(1/8)) - I*(-1)^(1/8)*log(I*(-1)^(7/8) + ((a*x - 1)/(a*
x + 1))^(1/8)) + I*(-1)^(1/8)*log(-I*(-1)^(7/8) + ((a*x - 1)/(a*x + 1))^(1
/8)) - (-1)^(1/8)*log(-(-1)^(7/8) + ((a*x - 1)/(a*x + 1))^(1/8)) - 2*arcta
n(((a*x - 1)/(a*x + 1))^(1/8)) + log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - lo
g(((a*x - 1)/(a*x + 1))^(1/8) - 1)
```

3.129.6 Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

3.129.7 Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="maxima")
```

output
```
integrate(1/(x*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

3.129.8 Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 661, normalized size of antiderivative = 0.72

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a}-\sqrt{2}\log\left(\sqrt{2}\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="giac")
```

output
```
-1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/8)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/8)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8)
 + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/
8))/a - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a + 2*log(-((a*x - 1)/(a*x
+ 1))^(1/8) + 1)/a - 4*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))
^(1/8))/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sqrt(2) + 4)) - 4*arctan(-(sqrt(sqrt
(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sq
rt(2) + 4)) - 4*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8)
)/sqrt(sqrt(2) + 2))/(a*sqrt(-2*sqrt(2) + 4)) - 4*arctan(-(sqrt(-sqrt(2) +
 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2))/(a*sqrt(-2*sqrt(2)
 + 4)) + 2*log(sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1)/(a*sqrt(-2*sqrt(2) + 4)) - 2*log(-sqrt(sqrt(2) + 2)*
((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(-2*
sqrt(2) + 4)) + 2*log(sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(2*sqrt(2) + 4)) - 2*log(-sqrt(-sqrt(
2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*
sqrt(2*sqrt(2) + 4)))
```

3.129.9 Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 648, normalized size of antiderivative = 0.71

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8} 1\mathrm{i}\right) 2\mathrm{i} - 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)$$
$$+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(-1+1\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)(-1-\mathrm{i})+\mathrm{at}$$

input

```
int(1/(x*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

output

```
atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (((a*x - 1)/(a
*x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1
/8)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*1i)
/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i + (2^(1/2) + 2)^(1/2)*1i
) - 2*atan(((a*x - 1)/(a*x + 1))^(1/8)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/
(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1 - 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/
(a*x + 1))^(1/8)*(1/2 + 1i/2))*(1 + 1i) - atan(((a*x - 1)/(a*x + 1))^(1/8)
*1i)*2i - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (((a
*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (2^(1/2)*((a*x - 1)/(a*
x + 1))^(1/8)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))
^(1/8)*1i)/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i - (2^(1/2) + 2
)^(1/2)*1i) - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) - 2)^(1/2)
- (((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*((a*x -
1)/(a*x + 1))^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a
*x + 1))^(1/8)*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i + (2
 - 2^(1/2))^(1/2)*1i) - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) -
 2)^(1/2) + (((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2
)*((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) - (2^(1/2)*((a
*x - 1)/(a*x + 1))^(1/8)*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/
2)*1i - (2 - 2^(1/2))^(1/2)*1i)
```

3.130 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

3.130.1 Optimal result

Integrand size = 14, antiderivative size = 676

$$
\begin{aligned}
\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = {} & a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} \\
& -\frac{1}{4}\sqrt{2+\sqrt{2}}a\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) \\
& -\frac{1}{4}\sqrt{2-\sqrt{2}}a\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) \\
& +\frac{1}{4}\sqrt{2+\sqrt{2}}a\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) \\
& +\frac{1}{4}\sqrt{2-\sqrt{2}}a\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) \\
& +\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) \\
& -\frac{1}{8}\sqrt{2-\sqrt{2}}a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{}}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{}}}\right)
\end{aligned}
$$

output
```
a*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)-1/4*a*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/
x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*a*arc
tan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2
))*(2-2^(1/2))^(1/2)+1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^
(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-1/8*a*ln(1+(1-1
/a/x)^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1
/8))*(2-2^(1/2))^(1/2)-1/4*a*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2
-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/4*a*arctan((2*(1-1
/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/
2))^(1/2)+1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2+2^
(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/8*a*ln(1+(1-1/a/x)^(1/4)
/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(
1/2))^(1/2)
```

3.130.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.07

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = -2ae^{\frac{1}{4}\coth^{-1}(ax)}\left(-\frac{1}{1+e^{2\coth^{-1}(ax)}} + \text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/4)/x^2,x]
```

output
```
-2*a*E^(ArcCoth[a*x]/4)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometric2F
1[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])])
```

3.130.3 Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 624, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 60, 73, 854, 828, 1442, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{1}{4}\int \frac{1}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$2a\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}+a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{854}$$

$$2a\int \frac{1}{\left(1+\frac{1}{x^8}\right)x^6}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{828}$$

$$2a\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\int \frac{1}{\left(1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)+a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{1442}$$

3.130. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}dx$

$$2a\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)+$$

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 1483$$

$$2a\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\right.$$

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 1142$$

3.130. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

$$2\left(\frac{-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int -}{}}{2\sqrt{2}}\right.$$

$$\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}a$$

$$\Big\downarrow 25$$

3.130. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

$$2\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int}{}}{2\sqrt{2}}\right.$$

$$\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}a$$

$$\Big\downarrow \; 1083$$

3.130. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

$$2\left(\frac{-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d\left(\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d\left(\right.}{}}{2\sqrt{2}}\right.$$

$$\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}a$$

$$\Big\downarrow \text{217}$$

3.130. $\int\frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

$$2\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}}{}}{2\sqrt{2}}\right.$$

$$\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}a$$

$$\Big\downarrow \ 1103$$

3.130. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

$$2a\left(\frac{-\frac{-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}\right.$$

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

input
```
Int[E^(ArcCoth[a*x]/4)/x^2,x]
```

output
```
a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8) + 2*a*(-1/2*((1 - 1/(a*x))^(1/8)
/(2 - x^(-8))^(1/8) - (ArcTan[(-Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8)
)/(2 - x^(-8))^(1/8))/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 - (Sqrt[2
 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2
 - Sqrt[2]]) - (ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x
^(-8))^(1/8))/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 + (Sqrt[2 - Sqrt[2
]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 - Sqrt[
2]]))/Sqrt[2] + ((1 - 1/(a*x))^(1/8)/(2 - x^(-8))^(1/8) - (-ArcTan[(-Sqrt[
2 + Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 - Sqrt[2
]]] - ((1 + Sqrt[2])*Log[1 - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 -
x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 + Sqrt[2]]) - (-ArcTan[(Sqrt[2 + Sqr
t[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 - Sqrt[2]]] + (
(1 + Sqrt[2])*Log[1 + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))
^(1/8) + x^(-2)])/2)/(2*Sqrt[2 + Sqrt[2]]))/(2*Sqrt[2]))
```

3.130. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.130.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^{2}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

3.130.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$$

$$= \frac{-(i-1)\sqrt{2}(-a^8)^{\frac{1}{8}}\,x\log\left(2\,a^7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} + (i+1)\sqrt{2}(-a^8)^{\frac{7}{8}}\right) + (i+1)\sqrt{2}(-a^8)^{\frac{1}{8}}\,x\log\left(2\,a^7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} - (\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="fricas")
```

output
```
1/8*(-(I - 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*x + 1))^(1/8)
 + (I + 1)*sqrt(2)*(-a^8)^(7/8)) + (I + 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^
7*((a*x - 1)/(a*x + 1))^(1/8) - (I - 1)*sqrt(2)*(-a^8)^(7/8)) - (I + 1)*sq
rt(2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*x + 1))^(1/8) + (I - 1)*sqrt(
2)*(-a^8)^(7/8)) + (I - 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*
x + 1))^(1/8) - (I + 1)*sqrt(2)*(-a^8)^(7/8)) + 2*(-a^8)^(1/8)*x*log(a^7*(
(a*x - 1)/(a*x + 1))^(1/8) + (-a^8)^(7/8)) - 2*I*(-a^8)^(1/8)*x*log(a^7*((
a*x - 1)/(a*x + 1))^(1/8) + I*(-a^8)^(7/8)) + 2*I*(-a^8)^(1/8)*x*log(a^7*(
(a*x - 1)/(a*x + 1))^(1/8) - I*(-a^8)^(7/8)) - 2*(-a^8)^(1/8)*x*log(a^7*((
a*x - 1)/(a*x + 1))^(1/8) - (-a^8)^(7/8)) + 8*(a*x + 1)*((a*x - 1)/(a*x +
1))^(7/8))/x
```

3.130.6 Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

3.130.7 Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.64

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=\frac{1}{8}\left(2\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="giac")
```

output
```
1/8*(2*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x +
1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2
) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(sqrt(
2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(s
qrt(2) + 2)) + 2*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a*x -
 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - sqrt(sqrt(2) + 2)*log(sqrt(sqrt
(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1) - sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*
((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(-sqr
t(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)
/(a*x + 1))^(1/4) + 1) + 16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)/(a*x +
1) + 1))*a
```

3.130.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.24

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \frac{(-1)^{1/8}\,a\,\mathrm{atan}\Big((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\Big)}{2}$$

$$+\frac{(-1)^{1/8}\,a\,\mathrm{atan}\Big((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\Big)\;1\mathrm{i}}{2}+\frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{\frac{a\,x-1}{a\,x+1}+1}$$

$$+(-1)^{1/8}\sqrt{2}\,a\,\mathrm{atan}\left((-1)^{1/8}\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+(-1)^{1/8}\sqrt{2}\,a\,\mathrm{atan}\Bigg((-1)^{1/8}\sqrt{2}\Bigg($$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

output
```
((-1)^(1/8)*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/2 + ((-1)^(1/8
)*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/2 + (2*a*((a*x - 1
)/(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1) + (-1)^(1/8)*2^(1/2)*a*atan(
(-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/4 - 1i/4)
+ (-1)^(1/8)*2^(1/2)*a*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)
*(1/2 + 1i/2))*(1/4 + 1i/4)
```

3.131 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

3.131.1 Optimal result

Integrand size = 14, antiderivative size = 731

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{32}\sqrt{2+\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{32}\sqrt{2-\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

output
```
1/8*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)+1/2*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^
(9/8)-1/32*a^2*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2
))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/32*a^2*arctan((2*(1-1/a/x)^(1/8)
/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1
/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2-2^(1/2))^(
1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1
/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))
^(1/2)-1/32*a^2*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/
2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/32*a^2*arctan((2*(1-1/a/x)^(1/8
)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+
1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2+2^(1/2))^
(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+
1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2)
)^(1/2)
```

3.131.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.10

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{a^2 e^{\frac{1}{4}\coth^{-1}(ax)}\left(-1-9e^{2\coth^{-1}(ax)}+\left(1+e^{2\coth^{-1}(ax)}\right)^2 \text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\coth^{-1}(ax)}\right)\right)}{4\left(1+e^{2\coth^{-1}(ax)}\right)^2}$$

input
```
Integrate[E^(ArcCoth[a*x]/4)/x^3,x]
```

output
```
-1/4*(a^2*E^(ArcCoth[a*x]/4)*(-1 - 9*E^(2*ArcCoth[a*x]) + (1 + E^(2*ArcCot
h[a*x]))^2*Hypergeometric2F1[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])]))/(1 + E^(2
*ArcCoth[a*x]))^2
```

3.131.3 Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 665, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 90, 60, 73, 854, 828, 1442, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\int \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\left(\frac{1}{4}\int \frac{1}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\left(-2a\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\left(-2a\int \frac{1}{\left(1+\frac{1}{x^8}\right)x^6}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{828}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\left(-2a\left(\frac{\int\frac{1}{\left(1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\int\frac{1}{\left(1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\Bigg(-2a\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}$$

$$\Big\downarrow \text{1483}$$

3.131. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2a\left(\frac{-\frac{\int\frac{\sqrt{2+\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int\frac{\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\right.\right.$$

↓ 1142

3.131. $\int\frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-$$

$$\frac{1}{8}a\Bigg(-2\Bigg(\frac{-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int-\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}\quad\frac{1}{2}\big(1+}{2\sqrt{2+\sqrt{2}}}-}{2\sqrt{2}}$$

$$\Big\downarrow\ 25$$

3.131. $\int\frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}\quad\frac{1}{2}\left(1+\sqrt{2}\right)}{2\sqrt{2+\sqrt{2}}}-}{2\sqrt{2}}\right.\right.$$

↓ 1083

3.131. $\int\frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2\left(\frac{-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d\left(\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2-\sqrt{2}}\int\frac{}{\sqrt{2}-}}{}}{2\sqrt{2}}\right.\right.$$

↓ 217

3.131. $\int\frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}}{}}{2\sqrt{2}}\right.\right.$$

↓ 1103

3.131. $\int\frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2a\left(\frac{-\frac{-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\right.}{2}}{2\sqrt{2}}\right.\right.$$

input
```
Int[E^(ArcCoth[a*x]/4)/x^3,x]
```

output
```
(a^2*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(9/8))/2 - (a*(-(a*(1 - 1/(a*x))^(7
/8)*(1 + 1/(a*x))^(1/8)) - 2*a*(-1/2*((1 - 1/(a*x))^(1/8)/(2 - x^(-8))^(1/
8) - (ArcTan[(-Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1
/8))/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 - (Sqrt[2 - Sqrt[2]]*(1 - 1
/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 - Sqrt[2]]) - (A
rcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqr
t[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^
(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 - Sqrt[2]]))/Sqrt[2] + (
(1 - 1/(a*x))^(1/8)/(2 - x^(-8))^(1/8) - (-ArcTan[(-Sqrt[2 + Sqrt[2]] + (2
*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[
2])*Log[1 - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x
^(-2)])/2)/(2*Sqrt[2 + Sqrt[2]]) - (-ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1
/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log
[1 + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])
/2)/(2*Sqrt[2 + Sqrt[2]]))/(2*Sqrt[2]))))/8
```

3.131. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

3.131.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

3.131. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.131. $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

3.131.4 Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^3}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

3.131.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{-(i-1)\,\sqrt{2}(-a^{16})^{\frac{1}{8}}\,x^2\log\left(2\,a^{14}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+(i+1)\,\sqrt{2}(-a^{16})^{\frac{7}{8}}\right)+(i+1)\,\sqrt{2}(-a^{16})^{\frac{1}{8}}\,x^2\log\left(2\,a^{14}\left(\frac{ax-1}{ax+1}\right.\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="fricas")
```

output
```
1/64*(-(I - 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2*a^14*((a*x - 1)/(a*x + 1))^
(1/8) + (I + 1)*sqrt(2)*(-a^16)^(7/8)) + (I + 1)*sqrt(2)*(-a^16)^(1/8)*x^2
*log(2*a^14*((a*x - 1)/(a*x + 1))^(1/8) - (I - 1)*sqrt(2)*(-a^16)^(7/8)) -
 (I + 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2*a^14*((a*x - 1)/(a*x + 1))^(1/8)
+ (I - 1)*sqrt(2)*(-a^16)^(7/8)) + (I - 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2
*a^14*((a*x - 1)/(a*x + 1))^(1/8) - (I + 1)*sqrt(2)*(-a^16)^(7/8)) + 2*(-a
^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) + (-a^16)^(7/8)) - 2*I
*(-a^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) + I*(-a^16)^(7/8))
 + 2*I*(-a^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) - I*(-a^16)^
(7/8)) - 2*(-a^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) - (-a^16
)^(7/8)) + 8*(5*a^2*x^2 + 9*a*x + 4)*((a*x - 1)/(a*x + 1))^(7/8))/x^2
```

3.131.6 Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3 \sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

3.131.7 Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{64}\left(2\,a\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\,a\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="giac")
```

output
```
1/64*(2*a*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x
 + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(s
qrt(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqr
t(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))
/sqrt(sqrt(2) + 2)) + 2*a*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) -
2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - a*sqrt(sqrt(2) + 2)*lo
g(sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1
/4) + 1) + a*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1)
)^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - a*sqrt(-sqrt(2) + 2)*log(sqrt
(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) +
 1) + a*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(
1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 16*(a*((a*x - 1)/(a*x + 1))^(15/
8) + 9*a*((a*x - 1)/(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

3.131.9 Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.29

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{9\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4} + \frac{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}$$

$$+\frac{(-1)^{1/8}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{16} + \frac{(-1)^{1/8}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}1\mathrm{i}\right)\,1\mathrm{i}}{16}$$

$$+(-1)^{1/8}\sqrt{2}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{32}-\frac{1}{32}\mathrm{i}\right)+(-1)^{1/8}\sqrt{2}\,a^2\,\mathrm{atan}\Bigg((-1)^{1/8}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

output
```
((9*a^2*((a*x - 1)/(a*x + 1))^(7/8))/4 + (a^2*((a*x - 1)/(a*x + 1))^(15/8)
)/4)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/8)
*a^2*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/16 + ((-1)^(1/8)*a^2*at
an((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/16 + (-1)^(1/8)*2^(1/2)*
a^2*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/3
2 - 1i/32) + (-1)^(1/8)*2^(1/2)*a^2*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*
x + 1))^(1/8)*(1/2 + 1i/2))*(1/32 + 1i/32)
```

3.132 $\int e^{4\coth^{-1}(ax)}x^m\,dx$

3.132.1 Optimal result

Integrand size = 12, antiderivative size = 45

$$\int e^{4\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-ax}-4x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)$$

output
```
x^(1+m)/(1+m)+4*x^(1+m)/(-a*x+1)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)
```

3.132.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}x^m\,dx\\=\frac{x^{1+m}(-5-4m+ax-4(1+m)(-1+ax)\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{(1+m)(-1+ax)}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*x^m,x]
```

output
```
(x^(1 + m)*(-5 - 4*m + a*x - 4*(1 + m)*(-1 + a*x)*Hypergeometric2F1[1, 1 +
 m, 2 + m, a*x]))/((1 + m)*(-1 + a*x))
```

3.132.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6676, 100, 27, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int x^m e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{(ax+1)^2x^m}{(1-ax)^2}dx$$

$$\Big\downarrow \text{100}$$

$$\frac{4x^{m+1}}{1-ax}-\frac{\int \frac{a^2x^m(4m+ax+3)}{1-ax}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4x^{m+1}}{1-ax}-\int \frac{x^m(4m+ax+3)}{1-ax}dx$$

$$\Big\downarrow \text{90}$$

$$-4(m+1)\int \frac{x^m}{1-ax}dx+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

$$\Big\downarrow \text{74}$$

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

input
```
Int[E^(4*ArcCoth[a*x])*x^m,x]
```

output
```
x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, a*x]
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.132.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.66 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.47

method	result
meijerg	$-\frac{(-a)^{-m}\left(\frac{x^{m}(-a)^{m}\left(a^{2}m\,x^{2}+amx+2ax-m^{2}-3m-2\right)}{(1+m)m(-ax+1)}+x^{m}(-a)^{m}(2+m)\,\mathrm{LerchPhi}(ax,1,m)\right)}{a}+\frac{2(-a)^{-m}\left(-\frac{x^{m}(-a)^{m}(ax-m-1)}{m(-ax+1)}\right.}{}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x^m,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(-m)/a*(x^m*(-a)^m*(a^2*m*x^2+a*m*x+2*a*x-m^2-3*m-2)/(1+m)/m/(-a*x+1
)+x^m*(-a)^m*(2+m)*LerchPhi(a*x,1,m))+2*(-a)^(-m)/a*(-x^m*(-a)^m*(a*x-m-1)
/m/(-a*x+1)-x^m*(-a)^m*(1+m)*LerchPhi(a*x,1,m))-(-a)^(-m)/a*(1/(1+m)*x^m*(
-a)^m*(-m-1)/(-a*x+1)+x^m*(-a)^m*m*LerchPhi(a*x,1,m))
```

3.132.5 Fricas [F]

$$\int e^{4\coth^{-1}(ax)}x^m\,dx=\int\frac{(ax+1)^2x^m}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

3.132.6 Sympy [F]

$$\int e^{4\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m(ax+1)^2}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*x**m,x)
```

output
```
Integral(x**m*(a*x + 1)**2/(a*x - 1)**2, x)
```

3.132.7 Maxima [F]

$$\int e^{4\coth^{-1}(ax)}x^m\,dx=\int\frac{(ax+1)^2x^m}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

3.132.8 Giac [F]

$$\int e^{4\coth^{-1}(ax)}x^m\,dx=\int\frac{(ax+1)^2x^m}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="giac")
```

output
```
integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int e^{4\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m\,(a\,x+1)^2}{(a\,x-1)^2}\,dx$$

input
```
int((x^m*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
int((x^m*(a*x + 1)^2)/(a*x - 1)^2, x)
```

3.133 $\int e^{3\coth^{-1}(ax)}x^m\,dx$

3.133.1 Optimal result

Integrand size = 12, antiderivative size = 151

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} + \frac{4x^m\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

output
```
-3*x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[-1/2*m+1/2],1/a^2/x^2)/(1+m)-x^m*h
ypergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m+4*x^(1+m)*hypergeom([3/2,
-1/2-1/2*m],[-1/2*m+1/2],1/a^2/x^2)/(1+m)+4*x^m*hypergeom([3/2, -1/2*m],[1
-1/2*m],1/a^2/x^2)/a/m
```

3.133.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.35 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}x^m\,dx$$
$$= \frac{x^{1+m}\left(3(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1-ax}\sqrt{\frac{1+ax}{a^2}}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right)-2(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1}\right.}{m(1}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^m,x]
```

output
```
(x^(1 + m)*(3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a
^2]*AppellF1[m, -1/2, 1/2, 1 + m, -(a*x), a*x] - 2*(1 + m)*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, -(a
*x), a*x] + m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]*Hypergeomet
ric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]))/(m*(1 + m)*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2])
```

3.133.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6722, 27, 2355, 557, 278, 583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6722}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a+\frac{1}{x}\right)^2 \left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a+\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2355}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4a^2\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}+\int \frac{\left(-3a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{557}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4a^2\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}-3a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4a^2\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{m}\right.}{a}$$

$$\Big\downarrow \textcolor{blue}{583}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4\int \frac{\left(a+\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2}\right.}{m}\right.}{a}$$

$$\Big\downarrow \textcolor{blue}{557}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4\left(a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\frac{\left(\frac{1}{x}\right)^{-m}}{}\right.}{a}$$

$$\Big\downarrow \textcolor{blue}{278}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m}+4\left(-\frac{a\left(\frac{1}{x}\right.}{}\right.\right.}{a}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^m,x]
```

output
```
-(((x^(-1))^m*x^m*((3*a*(x^(-1))^(-1 - m)*Hypergeometric2F1[1/2, (-1 - m)/
2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m) + Hypergeometric2F1[1/2, -1/2*m, 1 - m
/2, 1/(a^2*x^2)]/(m*(x^(-1))^m) + 4*(-((a*(x^(-1))^(-1 - m)*Hypergeometric
2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m)) - Hypergeometric2F1
[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m))))/a)
```

3.133. $\int e^{3\coth^{-1}(ax)}x^m\,dx$

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 6722
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c
*x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n -
 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, c, m}, x] && Intege
rQ[(n - 1)/2] &&  !IntegerQ[m]
```

3.133.4 Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

3.133.5 Fricas [F]

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a^2*x^2 - 2*
a*x + 1), x)
```

3.133.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m,x)
```

output
```
Integral(x**m/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.133.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.133.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^m\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int(x^m/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.134 $\int e^{2\coth^{-1}(ax)}x^m\,dx$

3.134.1 Optimal result

Integrand size = 12, antiderivative size = 35

$$\int e^{2\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{1+m}$$

output
```
x^(1+m)/(1+m)-2*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)/(1+m)
```

3.134.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{2\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}(1-2\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{1+m}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^m,x]
```

output
```
(x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(1 + m)
```

3.134.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^m dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^m(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{x^{m+1}}{m+1}-2\int \frac{x^m}{1-ax}dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{m+1}$$

input

```
Int[E^(2*ArcCoth[a*x])*x^m,x]
```

output

```
x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/
(1 + m)
```

3.134.3.1 Defintions of rubi rules used

rule 74

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.134.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.03

method	result
meijerg	$-\frac{(-a)^{-m}\left(-\frac{x^{m}(-a)^{m}(amx+m+1)}{(1+m)m}+x^{m}(-a)^{m}\,\mathrm{LerchPhi}(ax,1,m)\right)}{a}+\frac{(-a)^{-m}\Big(-\frac{x^{m}(-a)^{m}(-m-1)}{(1+m)m}-x^{m}(-a)^{m}\,\mathrm{LerchPhi}(ax,1,m}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*x^m,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(-m)/a*(-x^m*(-a)^m*(a*m*x+m+1)/(1+m)/m+x^m*(-a)^m*LerchPhi(a*x,1,m)
)+(-a)^(-m)/a*(-1/(1+m)*x^m*(-a)^m*(-m-1)/m-x^m*(-a)^m*LerchPhi(a*x,1,m))
```

3.134.5 Fricas [F]

$$\int e^{2\coth^{-1}(ax)}x^m\,dx=\int\frac{(ax+1)x^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*x^m/(a*x - 1), x)
```

3.134.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(26) = 52$.

Time = 1.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

$$\int e^{2\coth^{-1}(ax)}x^m\,dx = -\frac{amx^{m+2}\Phi(ax,1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)} - \frac{2ax^{m+2}\Phi(ax,1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)} - \frac{mx^{m+1}\Phi(ax,1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)} - \frac{x^{m+1}\Phi(ax,1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**m,x)
```

output
```
-a*m*x**(m + 2)*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - 2*a*x*
*(m + 2)*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - m*x**(m + 1)*
lerchphi(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) - x**(m + 1)*lerchphi(a*
x, 1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

3.134.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax+1)x^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*x^m/(a*x - 1), x)
```

3.134.8 Giac [F]

$$\int e^{2\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax+1)x^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="giac")
```

output
```
integrate((a*x + 1)*x^m/(a*x - 1), x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m\,(a\,x+1)}{a\,x-1}\,dx$$

input

```
int((x^m*(a*x + 1))/(a*x - 1),x)
```

output

```
int((x^m*(a*x + 1))/(a*x - 1), x)
```

3.135 $\int e^{\coth^{-1}(ax)}x^m\,dx$

3.135.1 Optimal result

Integrand size = 10, antiderivative size = 74

$$\int e^{\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m}+\frac{x^m\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

output
```
x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[-1/2*m+1/2],1/a^2/x^2)/(1+m)+x^m*hype
rgeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

3.135.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.73

$$\int e^{\coth^{-1}(ax)}x^m\,dx=x^{1+m}\left(-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{-\frac{1}{a^2}+x^2}\operatorname{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right)}{m\sqrt{-1+ax}\sqrt{\frac{1+ax}{a^2}}\sqrt{1-a^2x^2}}+\frac{\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{1+m}\right)$$

input
```
Integrate[E^ArcCoth[a*x]*x^m,x]
```

output
```
x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[-a^(-2) + x^2]*AppellF1[m, -1/2,
1/2, 1 + m, -(a*x), a*x])/(m*Sqrt[-1 + a*x]*Sqrt[(1 + a*x)/a^2]*Sqrt[1 - a
^2*x^2])) + Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1
 + m))
```

3.135.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6722, 27, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6722}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a+\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a+\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{557}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(-\frac{a\left(\frac{1}{x}\right)^{-m-1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m}\right)}{a}$$

input
```
Int[E^ArcCoth[a*x]*x^m,x]
```

output

```
-(((x^(-1))^m*x^m*(-((a*(x^(-1))^(-1 - m)*Hypergeometric2F1[1/2, (-1 - m)/
2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m)) - Hypergeometric2F1[1/2, -1/2*m, 1 -
m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m)))/a)
```

3.135.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557

```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 6722

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c
*x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n -
 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, c, m}, x] && Intege
rQ[(n - 1)/2] &&  !IntegerQ[m]
```

3.135.4 Maple [F]

$$\int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input

```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

output

```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

3.135.5 Fricas [F]

$$\int e^{\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

3.135.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m,x)
```

output
```
Integral(x**m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.135.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.135.8 Giac [F]

$$\int e^{\coth^{-1}(ax)} x^m\,dx = \int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)} x^m\,dx = \int \frac{x^m}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^m/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^m/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.136 $\int e^{-\coth^{-1}(ax)}x^m\,dx$

3.136.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int e^{-\coth^{-1}(ax)}x^m\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

output
```
x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[-1/2*m+1/2],1/a^2/x^2)/(1+m)-x^m*hype
rgeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

3.136.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.53

$$\int e^{-\coth^{-1}(ax)}x^m\,dx = x^{1+m}\left(-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,ax,-ax\right)}{m\sqrt{1-ax}\sqrt{-\frac{1}{a^2}+x^2}} + \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{1+m}\right)$$

input
```
Integrate[x^m/E^ArcCoth[a*x],x]
```

output
```
x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[m, -1/2,
 1/2, 1 + m, a*x, -(a*x)])/(m*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])) + Hyperg
eometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 + m))
```

3.136.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 27, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6722}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{557}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m}-\frac{a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}\right)}{a}$$

input
```
Int[x^m/E^ArcCoth[a*x],x]
```

output
```
-(((x^(-1))^m*x^m*(-((a*(x^(-1))^(-1 - m)*Hypergeometric2F1[1/2, (-1 - m)/
2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m)) + Hypergeometric2F1[1/2, -1/2*m, 1 -
m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m)))/a)
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 6722
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c
*x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n -
 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, c, m}, x] && Intege
rQ[(n - 1)/2] &&  !IntegerQ[m]
```

3.136.4 Maple [F]

$$\int x^m \sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

3.136.5 Fricas [F]

$$\int e^{-\coth^{-1}(ax)}x^m\,dx=\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.136.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^m\,dx=\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**m*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.136.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}x^m\,dx=\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.136.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}x^m\,dx=\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^m\,dx=\int x^m\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x^m*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^m*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.137 $\int e^{-2\coth^{-1}(ax)}x^m\,dx$

3.137.1 Optimal result

Integrand size = 12, antiderivative size = 36

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{1+m}$$

output
```
x^(1+m)/(1+m)-2*x^(1+m)*hypergeom([1, 1+m],[2+m],-a*x)/(1+m)
```

3.137.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}(1-2\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax))}{1+m}$$

input
```
Integrate[x^m/E^(2*ArcCoth[a*x]),x]
```

output
```
(x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(1 + m)
```

3.137.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^m dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^m(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{x^{m+1}}{m+1}-2\int \frac{x^m}{ax+1}dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{m+1}$$

input
```
Int[x^m/E^(2*ArcCoth[a*x]),x]
```

output
```
x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)
])/(1 + m)
```

3.137.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.137.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.58

method	result
meijerg	$a^{-m-1}\left(\frac{x^m a^m (amx-m-1)}{(1+m)m}+x^m a^m \operatorname{LerchPhi}\left(-ax,1,m\right)\right)-a^{-m-1}\left(\frac{x^m a^m}{m}+\frac{x^m a^m (-m-1)\operatorname{LerchPhi}(-ax}{1+m}\right.$

input
```
int(x^m*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
a^(-m-1)*(x^m*a^m*(a*m*x-m-1)/(1+m)/m+x^m*a^m*LerchPhi(-a*x,1,m))-a^(-m-1)
*(x^m*a^m/m+1/(1+m)*x^m*a^m*(-m-1)*LerchPhi(-a*x,1,m))
```

3.137.5 Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\int \frac{(ax-1)x^m}{ax+1}\,dx$$

input
```
integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*x^m/(a*x + 1), x)
```

3.137.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.22

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx = \frac{amx^{m+2}\Phi(axe^{i\pi},1,m+2)\,\Gamma(m+2)}{\Gamma\,(m+3)} + \frac{2ax^{m+2}\Phi(axe^{i\pi},1,m+2)\,\Gamma(m+2)}{\Gamma\,(m+3)} - \frac{mx^{m+1}\Phi(axe^{i\pi},1,m+1)\,\Gamma(m+1)}{\Gamma\,(m+2)} - \frac{x^{m+1}\Phi(axe^{i\pi},1,m+1)\,\Gamma(m+1)}{\Gamma\,(m+2)}$$

input
```
integrate(x**m*(a*x-1)/(a*x+1),x)
```

output
```
a*m*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/gamma(
m + 3) + 2*a*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m +
2)/gamma(m + 3) - m*x**(m + 1)*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gam
ma(m + 1)/gamma(m + 2) - x**(m + 1)*lerchphi(a*x*exp_polar(I*pi), 1, m + 1
)*gamma(m + 1)/gamma(m + 2)
```

3.137.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx = \int \frac{(ax-1)x^m}{ax+1}\,dx$$

input
```
integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*x^m/(a*x + 1), x)
```

3.137.8 Giac [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\int \frac{(ax-1)x^m}{ax+1}\,dx$$

input
```
integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
integrate((a*x - 1)*x^m/(a*x + 1), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^m*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^m*(a*x - 1))/(a*x + 1), x)
```

3.138 $\int e^{-3\coth^{-1}(ax)}x^m\,dx$

3.138.1 Optimal result

Integrand size = 12, antiderivative size = 150

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx = -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} - \frac{4x^m\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

output

```
-3*x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[-1/2*m+1/2],1/a^2/x^2)/(1+m)+x^m*h
ypergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m+4*x^(1+m)*hypergeom([3/2,
-1/2-1/2*m],[-1/2*m+1/2],1/a^2/x^2)/(1+m)-4*x^m*hypergeom([3/2, -1/2*m],[1
-1/2*m],1/a^2/x^2)/a/m
```

3.138.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.28

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx = \frac{x^{1+m}\left(-3(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,ax,-ax\right)+2(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}}{m(1+m)}$$

input
```
Integrate[x^m/E^(3*ArcCoth[a*x]),x]
```

output
```
(x^(1 + m)*(-3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1
[m, -1/2, 1/2, 1 + m, a*x, -(a*x)] + 2*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[
(-1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, a*x, -(a*x)] + m*Sqrt[1 - a*
x]*Sqrt[-a^(-2) + x^2]*Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a
^2*x^2)]))/(m*(1 + m)*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])
```

3.138.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6722, 27, 2355, 557, 278, 583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6722}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2355}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4a^2\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}+\int \frac{\left(\frac{1}{x}-3a\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{557}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4a^2\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}-3a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4a^2 \int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{m}\right.}{a}$$

$$\Big\downarrow \; 583$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4\int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2}\right.}{m}\right.}{a}$$

$$\Big\downarrow \; 557$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(4\left(a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}-\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m}}{}\right.}{a}$$

$$\Big\downarrow \; 278$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \left(\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m}+4\left(\frac{\left(\frac{1}{x}\right)^{-m}}{}\right.\right.}{a}$$

input
```
Int[x^m/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((x^(-1))^m*x^m*((3*a*(x^(-1))^(-1 - m)*Hypergeometric2F1[1/2, (-1 - m)/
2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m) - Hypergeometric2F1[1/2, -1/2*m, 1 - m
/2, 1/(a^2*x^2)]/(m*(x^(-1))^m) + 4*(-((a*(x^(-1))^(-1 - m)*Hypergeometric
2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m)) + Hypergeometric2F1
[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m))))/a)
```

3.138. $\int e^{-3\coth^{-1}(ax)}x^m\,dx$

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 6722
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c
*x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n -
 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, c, m}, x] && Intege
rQ[(n - 1)/2] &&  !IntegerQ[m]
```

3.138.4 Maple [F]

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

3.138.5 Fricas [F]

$$\int e^{-3\coth^{-1}(ax)} x^m \, dx = \int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)} x^m \, dx = \text{Timed out}$$

input
```
integrate(x**m*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.138.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

input
```
int(x^m*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.139 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$

3.139.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,\frac{5}{4},-\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,5/4,-5/4,-m,1/a/x,-1/a/x)/(1+m)
```

3.139.2 Mathematica [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

output
```
Integrate[E^((5*ArcCoth[a*x])/2)*x^m, x]
```

3.139.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(1+\frac{1}{ax}\right)^{5/4}\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{5}{4},-\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input

```
Int[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

output

```
(x^(1 + m)*AppellF1[-1 - m, 5/4, -5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

3.139.3.1 Defintions of rubi rules used

rule 150

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.139.4 Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

3.139.5 Fricas [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a^2*x^2 -
2*a*x + 1), x)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**m,x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.139.8 Giac [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}\,dx$$

input
```
int(x^m/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
int(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.140 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

3.140.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,\frac{3}{4},-\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,3/4,-3/4,-m,1/a/x,-1/a/x)/(1+m)
```

3.140.2 Mathematica [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

output
```
Integrate[E^((3*ArcCoth[a*x])/2)*x^m, x]
```

3.140.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(1+\frac{1}{ax}\right)^{3/4}\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{3}{4},-\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[-1 - m, 3/4, -3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

3.140.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.140.4 Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
```

3.140.5 Fricas [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x - 1), x)
```

3.140.6 Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**m,x)
```

output
```
Integral(x**m/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.140.7 Maxima [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.140.8 Giac [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}\,dx$$

input
```
int(x^m/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
int(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.141 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$

3.141.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,\frac{1}{4},-\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,1/4,-1/4,-m,1/a/x,-1/a/x)/(1+m)
```

3.141.2 Mathematica [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x^m,x]
```

output
```
Integrate[E^(ArcCoth[a*x]/2)*x^m, x]
```

3.141.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\sqrt[4]{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{1}{4},-\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^(ArcCoth[a*x]/2)*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[-1 - m, 1/4, -1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

3.141.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.141.4 Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
```

3.141.5 Fricas [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a*x - 1), x)
```

3.141.6 Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**m,x)
```

output
```
Integral(x**m/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.141.7 Maxima [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.141.8 Giac [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}\,dx$$

input
```
int(x^m/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
int(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.142 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$

3.142.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,-1/4,1/4,-m,1/a/x,-1/a/x)/(1+m)
```

3.142.2 Mathematica [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

input
```
Integrate[x^m/E^(ArcCoth[a*x]/2),x]
```

output
```
Integrate[x^m/E^(ArcCoth[a*x]/2), x]
```

3.142.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\,\text{AppellF1}\left(-m-1,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input

```
Int[x^m/E^(ArcCoth[a*x]/2),x]
```

output

```
(x^(1 + m)*AppellF1[-1 - m, -1/4, 1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

3.142.3.1 Defintions of rubi rules used

rule 150

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.142.4 Maple [F]

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
```

3.142.5 Fricas [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^m \, dx = \int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
integral(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.142.6 Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^m \, dx = \int x^m \sqrt[4]{\frac{ax-1}{ax+1}} \, dx$$

input
```
integrate(x**m*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x**m*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.142.7 Maxima [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.142.8 Giac [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,dx$$

input
```
int(x^m*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
int(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.143 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

3.143.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,-\frac{3}{4},\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,-3/4,3/4,-m,1/a/x,-1/a/x)/(1+m)
```

3.143.2 Mathematica [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

input
```
Integrate[x^m/E^((3*ArcCoth[a*x])/2),x]
```

output
```
Integrate[x^m/E^((3*ArcCoth[a*x])/2), x]
```

3.143.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-\frac{3}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\,\text{AppellF1}\left(-m-1,-\frac{3}{4},\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[x^m/E^((3*ArcCoth[a*x])/2),x]
```

output
```
(x^(1 + m)*AppellF1[-1 - m, -3/4, 3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

3.143.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.143.4 Maple [F]

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
```

3.143.5 Fricas [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x^m \, dx = \int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
integral(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x^m \, dx = \text{Timed out}$$

input
```
integrate(x**m*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Timed out
```

3.143.7 Maxima [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.143.8 Giac [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}\,dx$$

input
```
int(x^m*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
int(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.144 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$

3.144.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(-1-m,-\frac{5}{4},\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,-5/4,5/4,-m,1/a/x,-1/a/x)/(1+m)
```

3.144.2 Mathematica [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$$

input
```
Integrate[x^m/E^((5*ArcCoth[a*x])/2),x]
```

output
```
Integrate[x^m/E^((5*ArcCoth[a*x])/2), x]
```

3.144.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-\frac{5}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \ 6723$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \ 150$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,-\frac{5}{4},\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[x^m/E^((5*ArcCoth[a*x])/2),x]
```

output
```
(x^(1 + m)*AppellF1[-1 - m, -5/4, 5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

3.144.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.144.4 Maple [F]

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

input
```
int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
```

3.144.5 Fricas [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1), x)
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\text{Timed out}$$

input
```
integrate(x**m*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Timed out
```

3.144.7 Maxima [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.144.8 Giac [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx=\int x^m\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}\,dx$$

input
```
int(x^m*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
int(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.145 $\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx$

3.145.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\frac{x^{1+m}\,\text{AppellF1}\left(-1-m,\frac{1}{3},-\frac{1}{3},-m,\frac{1}{x},-\frac{1}{x}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,1/3,-1/3,-m,1/x,-1/x)/(1+m)
```

3.145.2 Mathematica [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx$$

input
```
Integrate[E^((2*ArcCoth[x])/3)*x^m,x]
```

output
```
Integrate[E^((2*ArcCoth[x])/3)*x^m, x]
```

3.145.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{2}{3}\coth^{-1}(x)}dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \frac{\sqrt[3]{1+\frac{1}{x}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\,\text{AppellF1}\left(-m-1,\frac{1}{3},-\frac{1}{3},-m,\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

input
```
Int[E^((2*ArcCoth[x])/3)*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[-1 - m, 1/3, -1/3, -m, x^(-1), -x^(-1)])/(1 + m)
```

3.145.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.145.4 Maple [F]

$$\int \frac{x^m}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/((x-1)/(1+x))^(1/3)*x^m,x)
```

output
```
int(1/((x-1)/(1+x))^(1/3)*x^m,x)
```

3.145.5 Fricas [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="fricas")
```

output
```
integral((x + 1)*x^m*((x - 1)/(x + 1))^(2/3)/(x - 1), x)
```

3.145.6 Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/3)*x**m,x)
```

output
```
Integral(x**m/((x - 1)/(x + 1))**(1/3), x)
```

3.145.7 Maxima [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

3.145.8 Giac [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx=\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{1/3}}\,dx$$

input
```
int(x^m/((x - 1)/(x + 1))^(1/3),x)
```

output
```
int(x^m/((x - 1)/(x + 1))^(1/3), x)
```

3.146 $\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx$

3.146.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(-1-m, \frac{1}{6}, -\frac{1}{6}, -m, \frac{1}{x}, -\frac{1}{x}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,1/6,-1/6,-m,1/x,-1/x)/(1+m)
```

3.146.2 Mathematica [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx = \int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx$$

input
```
Integrate[E^(ArcCoth[x]/3)*x^m,x]
```

output
```
Integrate[E^(ArcCoth[x]/3)*x^m, x]
```

3.146.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{1}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m x^m\int \frac{\sqrt[6]{1+\frac{1}{x}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{1}{6},-\frac{1}{6},-m,\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

input

```
Int[E^(ArcCoth[x]/3)*x^m,x]
```

output

```
(x^(1 + m)*AppellF1[-1 - m, 1/6, -1/6, -m, x^(-1), -x^(-1)])/(1 + m)
```

3.146.3.1 Defintions of rubi rules used

rule 150

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.146.4 Maple [F]

$$\int \frac{x^m}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{6}}}dx$$

input
```
int(1/((x-1)/(1+x))^(1/6)*x^m,x)
```

output
```
int(1/((x-1)/(1+x))^(1/6)*x^m,x)
```

3.146.5 Fricas [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="fricas")
```

output
```
integral((x + 1)*x^m*((x - 1)/(x + 1))^(5/6)/(x - 1), x)
```

3.146.6 Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx = \int \frac{x^m}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/6)*x**m,x)
```

output
```
Integral(x**m/((x - 1)/(x + 1))**(1/6), x)
```

3.146.7 Maxima [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx=\int\frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

3.146.8 Giac [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx=\int\frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx=\int\frac{x^m}{\left(\frac{x-1}{x+1}\right)^{1/6}}\,dx$$

input
```
int(x^m/((x - 1)/(x + 1))^(1/6),x)
```

output
```
int(x^m/((x - 1)/(x + 1))^(1/6), x)
```

3.147 $\int e^{\frac{1}{4}\coth^{-1}(ax)} x^m \, dx$

3.147.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int e^{\frac{1}{4}\coth^{-1}(ax)} x^m \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(-1-m, \frac{1}{8}, -\frac{1}{8}, -m, \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,1/8,-1/8,-m,1/a/x,-1/a/x)/(1+m)
```

3.147.2 Mathematica [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)} x^m \, dx = \int e^{\frac{1}{4}\coth^{-1}(ax)} x^m \, dx$$

input
```
Integrate[E^(ArcCoth[a*x]/4)*x^m,x]
```

output
```
Integrate[E^(ArcCoth[a*x]/4)*x^m, x]
```

3.147.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6723}$$

$$-\left(\frac{1}{x}\right)^m x^m\int \frac{\sqrt[8]{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{1}{8},-\frac{1}{8},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^(ArcCoth[a*x]/4)*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[-1 - m, 1/8, -1/8, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

3.147.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

3.147.4 Maple [F]

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
```

3.147.5 Fricas [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(7/8)/(a*x - 1), x)
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**m,x)
```

output
```
Timed out
```

3.147.7 Maxima [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

3.147.8 Giac [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx=\int\frac{x^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}}\,dx$$

input
```
int(x^m/((a*x - 1)/(a*x + 1))^(1/8),x)
```

output
```
int(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

3.148 $\int e^{n\coth^{-1}(ax)}x^m\,dx$

3.148.1 Optimal result

Integrand size = 12, antiderivative size = 45

$$\int e^{n\coth^{-1}(ax)}x^m\,dx=\frac{x^{1+m}\,\text{AppellF1}\left(-1-m,\frac{n}{2},-\frac{n}{2},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(-1-m,1/2*n,-1/2*n,-m,1/a/x,-1/a/x)/(1+m)
```

3.148.2 Mathematica [F]

$$\int e^{n\coth^{-1}(ax)}x^m\,dx=\int e^{n\coth^{-1}(ax)}x^m\,dx$$

input
```
Integrate[E^(n*ArcCoth[a*x])*x^m,x]
```

output
```
Integrate[E^(n*ArcCoth[a*x])*x^m, x]
```

3.148.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m x^m\int\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-m-2}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(-m-1,\frac{n}{2},-\frac{n}{2},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^(n*ArcCoth[a*x])*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[-1 - m, n/2, -1/2*n, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

3.148.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

3.148.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)} x^m dx$$

input
```
int(exp(n*arccoth(a*x))*x^m,x)
```

output
```
int(exp(n*arccoth(a*x))*x^m,x)
```

3.148.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)} x^m\, dx = \int x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\, dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="fricas")
```

output
```
integral(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.148.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)} x^m\, dx = \int x^m e^{n\,\mathrm{acoth}\,(ax)}\, dx$$

input
```
integrate(exp(n*acoth(a*x))*x**m,x)
```

output
```
Integral(x**m*exp(n*acoth(a*x)), x)
```

3.148.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.148.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}x^m\,dx = \int x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}x^m\,dx = \int x^m\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

input
```
int(x^m*exp(n*acoth(a*x)),x)
```

output
```
int(x^m*exp(n*acoth(a*x)), x)
```

3.149 $\int e^{n\coth^{-1}(ax)}x^2\,dx$

3.149.1 Optimal result

Integrand size = 12, antiderivative size = 174

$$\begin{aligned}
&\int e^{n\coth^{-1}(ax)}x^2\,dx\\
&=\frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3\\
&\quad+\frac{2(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}
\end{aligned}$$

output
```
1/6*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/a+1/3*(1-1/a/x)^(1-1/2*n
)*(1+1/a/x)^(1+1/2*n)*x^3+2/3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/
2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^3/(2-n)
```

3.149.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\begin{aligned}
&\int e^{n\coth^{-1}(ax)}x^2\,dx\\
&=\frac{e^{n\coth^{-1}(ax)}\Bigl(e^{2\coth^{-1}(ax)}n(2+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\Bigl(an^2x+2}{6a^3(2+n)}
\end{aligned}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*x^2,x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometric2F1[1, 1
 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n^2*x + 2*a^3*x^3 + n*(-
1 + a^2*x^2) + (2 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a
*x])])))/(6*a^3*(2 + n))
```

3.149.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6721, 114, 25, 27, 168, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{n \coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3}\int -\frac{\left(an+\frac{1}{x}\right) \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^3}{a^2}d\frac{1}{x}+\frac{1}{3}x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{1}{3}\int \frac{\left(an+\frac{1}{x}\right) \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^3}{a^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{\int \left(an+\frac{1}{x}\right) \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^3d\frac{1}{x}}{3a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{-\frac{1}{2}\int -\left(\left(n^2+2\right) \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^2\right)d\frac{1}{x}-\frac{1}{2}anx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{\frac{1}{2}\int \left(n^2+2\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}-\frac{1}{2}anx^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{\frac{1}{2}\left(n^2+2\right)\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}-\frac{1}{2}anx^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3a^2}$$

$$\Big\downarrow \text{141}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{-\frac{2\left(n^2+2\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}-\frac{1}{2}anx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3a^2}$$

input
```
Int[E^(n*ArcCoth[a*x])*x^2,x]
```

output
```
((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3)/3 - (-1/2*(a*n*(1
- 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2) - (2*(2 + n^2)*(1 - 1/
(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2,
2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n)))/(3*a^2)
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.149.4 Maple [F]

$$\int \mathrm{e}^{n \operatorname{arccoth}(ax)} x^2 dx$$

input
```
int(exp(n*arccoth(a*x))*x^2,x)
```

output
```
int(exp(n*arccoth(a*x))*x^2,x)
```

3.149.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="fricas")
```

output
```
integral(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.149.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**2,x)
```

output
```
Integral(x**2*exp(n*acoth(a*x)), x)
```

3.149.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.149.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

input
```
int(x^2*exp(n*acoth(a*x)),x)
```

output
```
int(x^2*exp(n*acoth(a*x)), x)
```

3.150 $\int e^{n \coth^{-1}(ax)} x \, dx$

3.150.1 Optimal result

Integrand size = 10, antiderivative size = 122

$$\int e^{n \coth^{-1}(ax)} x \, dx = \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x^2 + \frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}$$

output
```
1/2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2+2*n*(1-1/a/x)^(1-1/2*n)*(1
+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^2/(
2-n)
```

3.150.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int e^{n \coth^{-1}(ax)} x \, dx = \frac{e^{n \coth^{-1}(ax)}\left(e^{2 \coth^{-1}(ax)} n^2 \text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2 \coth^{-1}(ax)}\right) + (2+n)\left(-1+anx+a^2x^2\right.\right.}{2a^2(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*x,x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2,
2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + a^2*x^2 + n*Hypergeom
etric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(2*a^2*(2 + n))
```

3.150.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6721, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{n\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{141}$$

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}+\frac{1}{2}x^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

input
```
Int[E^(n*ArcCoth[a*x])*x,x]
```

output
```
((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/2 + (2*n*(1 - 1/(a
*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2
- n/2, (a - x^(-1))/(a + x^(-1))])/(a^2*(2 - n))
```

3.150.3.1 Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.150.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)} x dx$$

input
```
int(exp(n*arccoth(a*x))*x,x)
```

output
```
int(exp(n*arccoth(a*x))*x,x)
```

3.150.5 Fricas [F]

$$\int e^{n \coth^{-1}(ax)} x \, dx = \int x \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))*x,x, algorithm="fricas")
```

output
```
integral(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.150.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int xe^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x,x)
```

output
```
Integral(x*exp(n*acoth(a*x)), x)
```

3.150.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x,x, algorithm="maxima")
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.150.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x,x, algorithm="giac")
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int x\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

input
```
int(x*exp(n*acoth(a*x)),x)
```

output
```
int(x*exp(n*acoth(a*x)), x)
```

3.151 $\int e^{n\coth^{-1}(ax)}\,dx$

3.151.1 Optimal result

Integrand size = 8, antiderivative size = 78

$$\int e^{n\coth^{-1}(ax)}\,dx=\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

output
```
4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n
],(a-1/x)/(a+1/x))/a/(2-n)
```

3.151.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int e^{n\coth^{-1}(ax)}\,dx$$
$$=\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(ax+\text{Hypergeome}\right.\right.}{a(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x]),x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
+ n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 +
n/2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

3.151.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6720, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow 6720$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}$$

$$\Big\downarrow 141$$

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

input

```
Int[E^(n*ArcCoth[a*x]),x]
```

output

```
(4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
 1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

3.151.3.1 Defintions of rubi rules used

rule 141

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6720

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.151.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}dx$$

input
```
int(exp(n*arccoth(a*x)),x)
```

output
```
int(exp(n*arccoth(a*x)),x)
```

3.151.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.151.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\,dx = \int e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x)),x)
```

output
```
Integral(exp(n*acoth(a*x)), x)
```

3.151.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.151.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

input
```
int(exp(n*acoth(a*x)),x)
```

output
```
int(exp(n*acoth(a*x)), x)
```

3.152 $\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx$

3.152.1 Optimal result

Integrand size = 12, antiderivative size = 127

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx = -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n} + \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}$$

output
```
-2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/n/((
1-1/a/x)^(1/2*n))+2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],1/2*(a-
1/x)/a)/n/((1-1/a/x)^(1/2*n))
```

3.152.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx$$

$$= e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}n\,\text{Hypergeome}\right.$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x,x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
+ n/2, -E^(2*ArcCoth[a*x])] + E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1
+ n/2, 2 + n/2, E^(2*ArcCoth[a*x])] - (2 + n)*(Hypergeometric2F1[1, n/2, 1
 + n/2, -E^(2*ArcCoth[a*x])] - Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*Arc
Coth[a*x])])))/(n*(2 + n))
```

3.152.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6721, 140, 79, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{ 140}$$

$$\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}}{a}-\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}-\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{ 141}$$

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}$$

input
```
Int[E^(n*ArcCoth[a*x])/x,x]
```

output
```
(-2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))
/(a + x^(-1))])/(n*(1 - 1/(a*x))^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-
1/2*n, -1/2*n, 1 - n/2, (a - x^(-1))/(2*a)])/(n*(1 - 1/(a*x))^(n/2))
```

3.152.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.152.4 Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{x}dx$$

input
```
int(exp(n*arccoth(a*x))/x,x)
```

output
```
int(exp(n*arccoth(a*x))/x,x)
```

3.152.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

3.152.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x,x)
```

output
```
Integral(exp(n*acoth(a*x))/x, x)
```

3.152.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

3.152.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x}\,dx$$

input
```
int(exp(n*acoth(a*x))/x,x)
```

output
```
int(exp(n*acoth(a*x))/x, x)
```

3.153 $\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx$

3.153.1 Optimal result

Integrand size = 12, antiderivative size = 70

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \frac{2^{1+\frac{n}{2}}a\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

output
```
2^(1+1/2*n)*a*(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/
2*(a-1/x)/a)/(2-n)
```

3.153.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = -\frac{4ae^{(2+n)\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(2,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)}{2+n}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x^2,x]
```

output
```
(-4*a*E^((2 + n)*ArcCoth[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(
2*ArcCoth[a*x])])/(2 + n)
```

3.153.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6721, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}$$

$$\Big\downarrow \text{79}$$

$$\frac{a2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

input
```
Int[E^(n*ArcCoth[a*x])/x^2,x]
```

output
```
(2^(1 + n/2)*a*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n,
2 - n/2, (a - x^(-1))/(2*a)])/(2 - n)
```

3.153.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.153.4 Maple [F]

$$\int \frac{\mathrm{e}^{n \,\mathrm{arccoth}(ax)}}{x^2}\,dx$$

input
```
int(exp(n*arccoth(a*x))/x^2,x)
```

output
```
int(exp(n*arccoth(a*x))/x^2,x)
```

3.153.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

3.153.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^2}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x**2,x)
```

output
```
Integral(exp(n*acoth(a*x))/x**2, x)
```

3.153.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

3.153.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^2}\,dx$$

input
```
int(exp(n*acoth(a*x))/x^2,x)
```

output
```
int(exp(n*acoth(a*x))/x^2, x)
```

3.154 $\int \frac{e^{n \coth^{-1}(ax)}}{x^3} \, dx$

3.154.1 Optimal result

Integrand size = 12, antiderivative size = 114

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} \, dx = \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{2^{n/2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

output
```
1/2*a^2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+2^(1/2*n)*a^2*n*(1-1/a/x)^
(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

3.154.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} \, dx = -\frac{a^2e^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n^2\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+\frac{1}{a^2x}\right.\right.}{2(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x^3,x]
```

output
```
-1/2*(a^2*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1
, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-1 + 1/(a^2*x^2) + n/
(a*x) + n*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 +
n)
```

3.154.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6721, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^3} \, dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{1}{2}an\int\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}$$

$$\Big\downarrow \text{79}$$

$$\frac{a^2 2^{n/2} n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}+\frac{1}{2}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/x^3,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/2 + (2^(n/2)*a^2*n
*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (a -
x^(-1))/(2*a)])/(2 - n)
```

3.154.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.154.4 Maple [F]

$$\int \frac{e^{n\ \mathrm{arccoth}(ax)}}{x^3}dx$$

input
```
int(exp(n*arccoth(a*x))/x^3,x)
```

output
```
int(exp(n*arccoth(a*x))/x^3,x)
```

3.154.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

3.154.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^3}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x**3,x)
```

output
```
Integral(exp(n*acoth(a*x))/x**3, x)
```

3.154.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

3.154.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^3}\,dx$$

input
```
int(exp(n*acoth(a*x))/x^3,x)
```

output
```
int(exp(n*acoth(a*x))/x^3, x)
```

3.155 $\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx$

3.155.1 Optimal result

Integrand size = 12, antiderivative size = 167

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx &= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x}\\&\quad+\frac{2^{n/2}a^3(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}\end{aligned}$$

output
```
1/6*a^3*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+1/3*a^2*(1-1/a/x)^(1-1/2
*n)*(1+1/a/x)^(1+1/2*n)/x+1/3*2^(1/2*n)*a^3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*hy
pergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

3.155.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = -\frac{a^3e^{n\coth^{-1}(ax)}\big(-e^{2\coth^{-1}(ax)}n(2+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+(2+n)\big(-}{6(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x^4,x]
```

output
```
-1/6*(a^3*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeomet
ric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-((1 - 1/(a^2
*x^2))*(n + 2/(a*x))) + (2 + n^2)/(a*x) + (2 + n^2)*Hypergeometric2F1[1, n
/2, 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 + n)
```

3.155.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6721, 101, 25, 27, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{3}a^2\int -\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n}{x}\right)}{a}d\frac{1}{x}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3x}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3x}-\frac{1}{3}a^2\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n}{x}\right)}{a}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3x}-\frac{1}{3}a\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{90}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3x}-$$
$$\frac{1}{3}a\left(\frac{1}{2}a\left(n^2+2\right)\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}-\frac{1}{2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\right)$$

$$\Big\downarrow 79$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3x}-$$

$$\frac{1}{3}a\left(-\frac{a^2 2^{n/2}\left(n^2+2\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}-\frac{1}{2}a^2n\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right.\right.$$

input
```
Int[E^(n*ArcCoth[a*x])/x^4,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(3*x) - (a*(-1/2*(
a^2*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)) - (2^(n/2)*a^2*(2
 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2
, (a - x^(-1))/(2*a)])/(2 - n)))/3
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.155.4 Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x^4} dx$$

input
```
int(exp(n*arccoth(a*x))/x^4,x)
```

output
```
int(exp(n*arccoth(a*x))/x^4,x)
```

3.155.5 Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^4} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^4} dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

3.155.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^4}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x**4,x)
```

output
```
Integral(exp(n*acoth(a*x))/x**4, x)
```

3.155.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^4}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

3.155.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^4}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x^4}\,dx$$

input
```
int(exp(n*acoth(a*x))/x^4,x)
```

output
```
int(exp(n*acoth(a*x))/x^4, x)
```

3.156 $\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$

3.156.1 Optimal result

Integrand size = 12, antiderivative size = 183

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a(6+n^2)+\frac{2n}{x}\right)+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2}+\frac{2^{-2+\frac{n}{2}}a^4n(8+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}$$

output
```
1/24*a^3*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(a*(n^2+6)+2*n/x)+1/4*a^2
*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)/x^2+1/3*2^(-2+1/2*n)*a^4*n*(n^2+8
)*(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)
/(2-n)
```

3.156.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$$
$$= -\frac{1}{24}a^4e^{n\coth^{-1}(ax)}\left(-6-n^2+\frac{6}{a^4x^4}+\frac{2n}{a^3x^3}+\frac{n^2}{a^2x^2}+\frac{6n}{ax}+\frac{n^3}{ax}-\frac{e^{2\coth^{-1}(ax)}n^2(8+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)}{2+n}+n(8+n^2)\,\text{Hypergeometric2F1}\left(1,\frac{n}{2},1+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x^5,x]
```

output
```
-1/24*(a^4*E^(n*ArcCoth[a*x])*(-6 - n^2 + 6/(a^4*x^4) + (2*n)/(a^3*x^3) +
n^2/(a^2*x^2) + (6*n)/(a*x) + n^3/(a*x) - (E^(2*ArcCoth[a*x])*n^2*(8 + n^2
)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])])/(2 + n) + n
*(8 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])]))
```

3.156.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6721, 111, 25, 27, 164, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{x^3}d\frac{1}{x}$$
$$\Big\downarrow \text{111}$$

$$\frac{1}{4}a^2\int -\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(2a+\frac{n}{x}\right)}{ax}d\frac{1}{x}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{4x^2}$$

$$\Big\downarrow \; 25$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{4x^2}-\frac{1}{4}a^2\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(2a+\frac{n}{x}\right)}{ax}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{4x^2}-\frac{1}{4}a\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(2a+\frac{n}{x}\right)}{x}d\frac{1}{x}$$

$$\Big\downarrow \; 164$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{4x^2}-$$
$$\frac{1}{4}a\left(\frac{1}{6}a^2n\left(n^2+8\right)\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}-\frac{1}{6}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a\left(n^2+6\right)+\frac{2n}{x}\right)\right)$$

$$\Big\downarrow \; 79$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{4x^2}-$$
$$\frac{1}{4}a\Bigg(-\frac{a^3 2^{n/2}n\left(n^2+8\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}-\frac{1}{6}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\Big(a\big(n^2+$$

input
```
Int[E^(n*ArcCoth[a*x])/x^5,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4*x^2) - (a*(-1/6
*(a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(a*(6 + n^2) + (2*
n)/x)) - (2^(n/2)*a^3*n*(8 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F
1[1 - n/2, -1/2*n, 2 - n/2, (a - x^(-1))/(2*a)])/(3*(2 - n))))/4
```

3.156. $\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

3.156. $\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$

3.156.4 Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{x^5}dx$$

input
```
int(exp(n*arccoth(a*x))/x^5,x)
```

output
```
int(exp(n*arccoth(a*x))/x^5,x)
```

3.156.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^5}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

3.156.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^5}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x**5,x)
```

output
```
Integral(exp(n*acoth(a*x))/x**5, x)
```

3.156.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^5}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

3.156.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^5}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^5}\,dx$$

input
```
int(exp(n*acoth(a*x))/x^5,x)
```

output
```
int(exp(n*acoth(a*x))/x^5, x)
```

3.157 $\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx$

3.157.1 Optimal result

Integrand size = 16, antiderivative size = 143

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx =\frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p}{1+p} +\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(1+p)\sqrt{1-\frac{1}{ax}}}$$

output
```
((a-1/x)/(a+1/x))^(1/2-p)*(-a*c*x+c)^p*hypergeom([-p, 1/2-p],[1-p],2/(a+1/
x)/x)*(1+1/a/x)^(1/2)/a/p/(p+1)/(1-1/a/x)^(1/2)+x*(-a*c*x+c)^p*(1-1/a/x)^(
1/2)*(1+1/a/x)^(1/2)/(p+1)
```

3.157.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx =\frac{\sqrt{1+\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-p}(c-acx)^p\left(p(-1+ax)\left(\frac{-1+ax}{1+ax}\right)^p+\sqrt{\frac{-1+ax}{1+ax}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\right.\right.}{ap(1+p)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

output
```
(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*(p*(-1 + a*x)*((-1 + a*x)/(1 + a*x))^p +
Sqrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1/2 - p, -p, 1 - p, 2/(1 + a*
x)]))/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^p)
```

3.157.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6727, 105, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \; 6727$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-p-2}d\frac{1}{x}$$

$$\Big\downarrow \; 105$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\left(\frac{\int\frac{\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{x}\right)^{-p-1}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a(p+1)}-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-p-1}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}}{p+1}\right)$$

$$\Big\downarrow \; 142$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-$$

$$acx)^p\left(-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(p+1)}-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)}{p}\right.$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

output
```
-(((x^(-1))^p*(c - a*c*x)^p*(-(((1 - 1/(a*x))^(1/2 + p)*Sqrt[1 + 1/(a*x)]*
(x^(-1))^(-1 - p))/(1 + p)) - (((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 -
1/(a*x))^(-1/2 + p)*Sqrt[1 + 1/(a*x)]*Hypergeometric2F1[1/2 - p, -p, 1 - p
, 2/((a + x^(-1))*x)])/(a*p*(1 + p)*(x^(-1))^p)))/(1 - 1/(a*x))^p)
```

3.157.3.1 Defintions of rubi rules used

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.157.4 Maple [F]

$$\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

3.157.5 Fricas [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

3.157.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-c(ax-1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**p,x)
```

output
```
Integral((-c*(a*x - 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.157.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.157.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(c-a\,c\,x)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.158 $\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$

3.158.1 Optimal result

Integrand size = 16, antiderivative size = 132

$$\begin{aligned}\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = &-\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\\ &-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4\\ &+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{7c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}\end{aligned}$$

output
```
17/15*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3-3/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+
1/5*a^4*c^4*(1-1/a^2/x^2)^(3/2)*x^5+7/8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a
-7/8*a*c^4*x^2*(1-1/a^2/x^2)^(1/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-136-15ax+112a^2x^2-90a^3x^3+24a^4x^4)+105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

output
```
(c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3
+ 24*a^4*x^4) + 105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

3.158.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6724, 25, 27, 540, 2338, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4 e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6724}$$

$$ac\int -c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^6d\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-ac\int c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^6d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-ac^4\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^6d\frac{1}{x}$$

$$\Big\downarrow \text{ 540}$$

$$-ac^4\left(-\frac{1}{5}\int \sqrt{1-\frac{1}{a^2x^2}}\left(15a^2-\frac{17a}{x}+\frac{5}{x^2}\right)x^5d\frac{1}{x}-\frac{1}{5}a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 2338}$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\int \sqrt{1-\frac{1}{a^2x^2}}\left(68a-\frac{35}{x}\right)x^4d\frac{1}{x}+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 534}$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-35\int\sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}a^3x^5\left(1-\frac{1}{a^2x^2}\right.\right.$$

$$\Big\downarrow \; 243$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{35}{2}\int\sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}a^3x^5\left(1-\frac{1}{a^2x}\right.\right.$$

$$\Big\downarrow \; 51$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{35}{2}\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right.\right.$$

$$\Big\downarrow \; 73$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{35}{2}\left(\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1\right.\right.\right.$$

$$\Big\downarrow \; 221$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{35}{2}\left(\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right.\right.$$

input

```
Int[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

output

```
-(a*c^4*(-1/5*(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^5) + ((15*a^2*(1 - 1/(a^2*x^2
))^(3/2)*x^4)/4 + ((-68*a*(1 - 1/(a^2*x^2))^(3/2)*x^3)/3 - (35*(-(Sqrt[1 -
 1/(a^2*x^2)]*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2))/2)/4)/5))
```

3.158. $\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.158.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\left(24 a^{4} x^{4}-90 a^{3} x^{3}+112 a^{2} x^{2}-15 a x -136\right) \left(a x -1\right) c^{4}}{120 a \sqrt{\frac{a x -1}{a x +1}}}+\frac{7 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{4} \sqrt{\left(a x -1\right) \left(a x +1\right)}}{8 \sqrt{a^{2}}\, \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$
default	$\frac{\left(a x -1\right) c^{4} \left(24 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{2} x^{2}-90 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a x +16 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}-105 \sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}\, a x +120 \left(\left(a x -1\right) \left(a x +1\right)\right)^{\frac{3}{2}} \sqrt{a^{2}}+}{120 a \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*a^4*x^4-90*a^3*x^3+112*a^2*x^2-15*a*x-136)*(a*x-1)/a*c^4/((a*x-1
)/(a*x+1))^(1/2)+7/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c
^4/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(24\,a^5c^4x^5-66\,a^4c^4x^4+22\,a^3c^4x^3+97\,a^2c^4x^2-15}{120\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (24*a^5*c^4*x^5 - 66*a^4*c^4*x^4 + 22*a^3*c^4*x^3 +
 97*a^2*c^4*x^2 - 151*a*c^4*x - 136*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.158.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = c^4\left(\int\left(-\frac{4ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right.$$
$$+\int\left(-\frac{4a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx$$
$$\left.+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**4,x)
```

output
```
c**4*(Integral(-4*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(6*a
**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**3*x**3/sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**4*x**4/sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(112) = 224$.

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{1}{120}\left(\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^4\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3}{(ax+1)^3}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^4*((a*x - 1)/(a*x + 1))^(9/2) + 79
0*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^4*((a*x - 1)/(a*x + 1))^(5/2) +
490*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^4*sqrt((a*x - 1)/(a*x + 1)))/(
5*(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^
3*a^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x +
 1)^5 - a^2))*a
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = -\frac{7\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)}$$

$$-\frac{1}{120}\sqrt{a^2x^2-1}\left(\left(\frac{15\,c^4}{\mathrm{sgn}\,(ax+1)}-2\left(\frac{56\,ac^4}{\mathrm{sgn}\,(ax+1)}+3\left(\frac{4\,a^3c^4x}{\mathrm{sgn}\,(ax+1)}-\frac{15\,a^2c^4}{\mathrm{sgn}\,(ax+1)}\right)x\right)x\right)x+\frac{1}{a\mathrm{sgn}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-7/8*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1
/120*sqrt(a^2*x^2 - 1)*((15*c^4/sgn(a*x + 1) - 2*(56*a*c^4/sgn(a*x + 1) +
3*(4*a^3*c^4*x/sgn(a*x + 1) - 15*a^2*c^4/sgn(a*x + 1))*x)*x)*x + 136*c^4/(
a*sgn(a*x + 1)))
```

3.158.9 Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.62

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$$

$$=\frac{\frac{49\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{7\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{79\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}+\frac{7\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}$$

$$+\frac{7\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((49*c^4*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^4*((a*x - 1)/(a*x + 1))^(1/
2))/4 - (224*c^4*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^4*((a*x - 1)/(a*x
 + 1))^(7/2))/6 + (7*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x -
1))/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x +
 1)^3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*
c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.158. $\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$

3.159 $\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$

3.159.1 Optimal result

Integrand size = 16, antiderivative size = 105

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{5c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
2/3*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3-1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4+5/
8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a-5/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{c^3\left(-a\sqrt{1-\frac{1}{a^2x^2}}x(16+9ax-16a^2x^2+6a^3x^3)+15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{24a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

output
```
(c^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3)) +
 15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

3.159.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 27, 540, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6724}$$

$$ac\int c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x^5d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$ac^3\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x^5d\frac{1}{x}$$

$$\Big\downarrow \text{ 540}$$

$$ac^3\left(-\frac{1}{4}\int \sqrt{1-\frac{1}{a^2x^2}}\left(8a-\frac{5}{x}\right)x^4d\frac{1}{x}-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 534}$$

$$ac^3\left(\frac{1}{4}\left(5\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 243}$$

$$ac^3\left(\frac{1}{4}\left(\frac{5}{2}\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 51}$$

$$ac^3\left(\frac{1}{4}\left(\frac{5}{2}\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$ac^3\left(\frac{1}{4}\left(\frac{5}{2}\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3}\right.$$

$$\Big\downarrow 221$$

$$ac^3\left(\frac{1}{4}\left(\frac{5}{2}\left(\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

output
```
a*c^3*(-1/4*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^4) + ((8*a*(1 - 1/(a^2*x^2))^(3
/2)*x^3)/3 + (5*(-(Sqrt[1 - 1/(a^2*x^2)]*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]]/a^2))/2)/4)
```

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.159.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

method	result	size
risch	$-\frac{\left(6 a^{3} x^{3}-16 a^{2} x^{2}+9 a x +16\right) \left(a x -1\right) c^{3}}{24 a \sqrt{\frac{a x -1}{a x +1}}}+\frac{5 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{3} \sqrt{\left(a x -1\right) \left(a x +1\right)}}{8 \sqrt{a^{2}}\, \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	120
default	$-\frac{\left(a x -1\right) c^{3} \left(6 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a x +15 \sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}\, a x -16 \left(\left(a x -1\right) \left(a x +1\right)\right)^{\frac{3}{2}} \sqrt{a^{2}}-15 \ln \left(\frac{a^{2} x +\sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}}{\sqrt{a^{2}}}\right) a \right)}{24 a \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$	141

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3-16*a^2*x^2+9*a*x+16)*(a*x-1)/a*c^3/((a*x-1)/(a*x+1))^(1/2
)+5/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x+1)/((a*
x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^3x^4-10\,a^3c^3x^3-7\,a^2c^3x^2+25\,ac^3x+16\,c^3\right)\sqrt{}}{24\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^3*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (6*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 7*a^2*c^3*x^2 + 25*a
*c^3*x + 16*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.159.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=-c^3\left(\int\frac{3ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right.$$
$$\left.+\int\frac{a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**3,x)
```

output
```
-c**3*(Integral(3*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*
a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3/sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1)), x))
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(89) = 178$.

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.10

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{1}{24}\left(\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(15\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+73\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1}{(ax+}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1)/a^2 + 2*(15*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 73*c^3
*((a*x - 1)/(a*x + 1))^(5/2) - 55*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^3
*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2
/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4
 - a^2))*a
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)}$$
$$-\frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2\left(\frac{3\,a^2c^3x}{\mathrm{sgn}\,(ax+1)}-\frac{8\,ac^3}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{9\,c^3}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{16\,c^3}{a\mathrm{sgn}\,(ax+1)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-5/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1
/24*sqrt(a^2*x^2 - 1)*((2*(3*a^2*c^3*x/sgn(a*x + 1) - 8*a*c^3/sgn(a*x + 1)
)*x + 9*c^3/sgn(a*x + 1))*x + 16*c^3/(a*sgn(a*x + 1)))
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{55\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{73\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}+\frac{5\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}$$

input

```
int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
(5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a) - ((5*c^3*((a*x - 1)/(a*x
 + 1))^(1/2))/4 - (55*c^3*((a*x - 1)/(a*x + 1))^(3/2))/12 + (73*c^3*((a*x
- 1)/(a*x + 1))^(5/2))/12 + (5*c^3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a - (4
*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2 - (4*a*(a*x - 1)^3
)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4)
```

3.160 $\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$

3.160.1 Optimal result

Integrand size = 16, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx=-\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3+1/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
-1/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-2-3ax+2a^2x^2)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - 3*a*x + 2*a^2*x^2) + 3*Log[a*(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

3.160.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 25, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6724}$$

$$ac\int -c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-ac\int c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-ac^2\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{ 534}$$

$$-ac^2\left(-\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 243}$$

$$-ac^2\left(-\frac{1}{2}\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 51}$$

$$-ac^2\left(\frac{1}{2}\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}+x\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$-ac^2\left(\frac{1}{2}\left(x\sqrt{1-\frac{1}{a^2x^2}}-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$-ac^2\left(\frac{1}{2}\left(x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}\right)-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

output
```
-(a*c^2*(-1/3*(a*(1 - 1/(a^2*x^2))^(3/2)*x^3) + (Sqrt[1 - 1/(a^2*x^2)]*x -
 ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2)/2))
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.160. $\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.160.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{\left(2 a^{2} x^{2}-3 a x -2\right) \left(a x -1\right) c^{2}}{6 a \sqrt{\frac{a x -1}{a x +1}}}+\frac{\ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{2} \sqrt{\left(a x -1\right) \left(a x +1\right)}}{2 \sqrt{a^{2}}\, \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	112
default	$-\frac{\left(a x -1\right) c^{2} \left(3 \sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}\, a x -2 \left(\left(a x -1\right) \left(a x +1\right)\right)^{\frac{3}{2}} \sqrt{a^{2}}-3 \ln \left(\frac{a^{2} x +\sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}}{\sqrt{a^{2}}}\right) a \right)}{6 \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}\, a}$	121

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-3*a*x-2)*(a*x-1)/a*c^2/((a*x-1)/(a*x+1))^(1/2)+1/2*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/(a*x+1)/((a*x-1)/(a*x+1))^
(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$$
$$=\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3c^2x^3-a^2c^2x^2-5\,ac^2x-2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) + (2*a^3*c^2*x^3 - a^2*c^2*x^2 - 5*a*c^2*x - 2*c^2)*sqrt((a
*x - 1)/(a*x + 1)))/a
```

3.160.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-\frac{2ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**2,x)
```

output
```
c**2*(Integral(-2*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**
2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x +
1) - 1/(a*x + 1)), x))
```

3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(66) = 132$.

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.32

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{1}{6}a\left(\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 8*c^2*((
a*x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*
a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^
3 - a^2))
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\begin{aligned}\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx\\ &= \frac{1}{6}\,\sqrt{a^2x^2-1}\left(\left(\frac{2\,ac^2x}{\operatorname{sgn}\left(ax+1\right)}-\frac{3\,c^2}{\operatorname{sgn}\left(ax+1\right)}\right)x-\frac{2\,c^2}{a\operatorname{sgn}\left(ax+1\right)}\right)\\ &\quad-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}\left(ax+1\right)}\end{aligned}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
1/6*sqrt(a^2*x^2 - 1)*((2*a*c^2*x/sgn(a*x + 1) - 3*c^2/sgn(a*x + 1))*x - 2
*c^2/(a*sgn(a*x + 1))) - 1/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(
abs(a)*sgn(a*x + 1))
```

3.160.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.77

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{\frac{8\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - c^2*((a*x - 1)/(a*x + 1))^(1/2) +
 c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a
*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (c^2*atanh(((a*x -
 1)/(a*x + 1))^(1/2)))/a
```

3.161 $\int e^{\coth^{-1}(ax)}(c-acx)\,dx$

3.161.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
1/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a-1/2*a*c*x^2*(1-1/a^2/x^2)^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=\frac{c\left(-a^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x),x]
```

output
```
(c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x
]))/(2*a)
```

3.161.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6724, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}ac\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}ac\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}ac\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}ac\left(\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)$$

input

```
Int[E^ArcCoth[a*x]*(c - a*c*x),x]
```

output

```
(a*c*(-(Sqrt[1 - 1/(a^2*x^2)]*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2))/2
```

3.161.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.161.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(39) = 78$.

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

method	result	size
risch	$-\frac{x(ax-1)c}{2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)c\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^{2}}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	92
default	$-\frac{(ax-1)c\left(x\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}-\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}$	93

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(a*x-1)*c/((a*x-1)/(a*x+1))^(1/2)+1/2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
-1)^(1/2))/(a^2)^(1/2)*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))
^(1/2)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2cx^2+acx)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="fricas")
```

output
```
1/2*(c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - c*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (a^2*c*x^2 + a*c*x)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.161.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=-c\left(\int\frac{ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c),x)
```

output
```
-c*(Integral(a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(39) = 78.

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.81

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx$$
$$=\frac{1}{2}a\left(\frac{2\left(c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

output
```
1/2*a*(2*(c*((a*x - 1)/(a*x + 1))^(3/2) + c*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + c*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=-\frac{\sqrt{a^2x^2-1}cx}{2\,\mathrm{sgn}\,(ax+1)}-\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="giac")
```

output
```
-1/2*sqrt(a^2*x^2 - 1)*c*x/sgn(a*x + 1) - 1/2*c*log(abs(-x*abs(a) + sqrt(a
^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

3.161.9 Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{2\,a\,(a\,x-1)}{a\,x+1}+\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}$$

input
```
int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2)
+ c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2*a*(a*x - 1))/(a*x + 1) + (a*(a*x
- 1)^2)/(a*x + 1)^2)
```

3.161. $\int e^{\coth^{-1}(ax)}(c-acx)\,dx$

3.162 $\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx$

3.162.1 Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
-arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}x+(1-ax)\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*x + (1 - a*x)*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])
*x])/(a*c*(-1 + a*x))
```

3.162.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6724, 27, 564, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{564}$$

$$\frac{a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}\right)}{c}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x),x]
```

output
```
(a*((2*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) - ArcTanh[Sqrt[1 - 1/(a^2*x
^2)]]/a^2))/c
```

3.162.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.162.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(47) = 94$.

Time = 0.42 (sec) , antiderivative size = 247, normalized size of antiderivative = 4.84

method	result
default	$-\frac{\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x^{2}-2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{2}x-((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}-2}{a\sqrt{a^{2}}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-1}{ax+1}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^3*x^2+
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2-2*ln((a^2*x+(a^2)^(1/2)*((a*x-
1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x-((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-
2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2))/(a^2)^(1/2))+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(
1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx$$
$$= -\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^{2}cx-ac}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - 2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x - a*
c)
```

3.162.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{1}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

output
```
-Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2}{a^2c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c) - 2/(a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

3.162.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{1}{(acx-c)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

output
```
undef
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
2/(a*c*((a*x - 1)/(a*x + 1))^(1/2)) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/(a*c)
```

3.163 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

3.163.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

output
```
-1/3*a^2*(1-1/a^2/x^2)^(3/2)/c^2/(a-1/x)^3
```

3.163.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(1+ax)}{3c^2(-1+ax)^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
```

output
```
-1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x))/(c^2*(-1 + a*x)^2)
```

3.163.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6724, 25, 27, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^3\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^3\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{460}$$

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
```

output
```
-1/3*(a^2*(1 - 1/(a^2*x^2))^(3/2))/(c^2*(a - x^(-1))^3)
```

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.163.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{ax+1}{3(ax-1)c^{2}\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
default	$-\frac{ax+1}{3(ax-1)c^{2}\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
trager	$-\frac{(ax+1)^{2}\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^{2}(ax-1)^{2}}$	40

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)/a
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\,\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*
c^2*x + a*c^2)
```

3.163.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

output
```
Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{3\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/3/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1
)^3*a*c^2)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{3\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

input
```
int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
-1/(3*a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

3.164 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

3.164.1 Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

output
```
1/5*a^3*(1-1/a^2/x^2)^(3/2)/c^3/(a-1/x)^4-4/15*a^2*(1-1/a^2/x^2)^(3/2)/c^3
/(a-1/x)^3
```

3.164.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-4-3ax+a^2x^2)}{15c^3(-1+ax)^3}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

output
```
-1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(-4 - 3*a*x + a^2*x^2))/(c^3*(-1 + a*x)^3)
```

3.164.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6724, 27, 571, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \textbf{6724}$$

$$ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^4x}d\frac{1}{x}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^4x}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \textbf{571}$$

$$\frac{a\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}-\frac{4}{5}\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}\right)}{c^3}$$

$$\Big\downarrow \textbf{460}$$

$$\frac{a\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}-\frac{4a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(a-\frac{1}{x}\right)^3}\right)}{c^3}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

output
```
(a*((a^2*(1 - 1/(a^2*x^2))^(3/2))/(5*(a - x^(-1))^4) - (4*a*(1 - 1/(a^2*x^
2))^(3/2))/(15*(a - x^(-1))^3)))/c^3
```

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.164.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{(ax-4)(ax+1)}{15(ax-1)^{2}c^{3}\sqrt{\frac{ax-1}{ax+1}}\,a}$	41
default	$-\frac{(ax-4)(ax+1)}{15(ax-1)^{2}c^{3}\sqrt{\frac{ax-1}{ax+1}}\,a}$	41
trager	$-\frac{(ax+1)\left(a^{2}x^{2}-3ax-4\right)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^{3}(ax-1)^{3}}$	51

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(a*x-4)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(1/2)/a
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\left(a^3x^3-2\,a^2x^2-7\,ax-4\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/15*(a^3*x^3 - 2*a^2*x^2 - 7*a*x - 4)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^3
*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

3.164.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{1}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

output
```
-Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x**2*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) -
 sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{5\,(ax-1)}{ax+1}-3}{30\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/30*(5*(a*x - 1)/(a*x + 1) - 3)/(a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{2\left(15\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
2/15*(15*(a + sqrt(a^2 - 1/x^2))^3*x^3 + 5*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 5*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^3
)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{3\,(a\,x+1)}-\frac{1}{5}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

input
```
int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
-((a*x - 1)/(3*(a*x + 1)) - 1/5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

3.165 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4} \, dx$

3.165.1 Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4} \, dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

output
```
-1/7*a^4*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^5+12/35*a^3*(1-1/a^2/x^2)^(3/2)/c
^4/(a-1/x)^4-23/105*a^2*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^3
```

3.165.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.51

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4} \, dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(23+13ax-8a^2x^2+2a^3x^3)}{105c^4(-1+ax)^4}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

output
```
-1/105*(Sqrt[1 - 1/(a^2*x^2)]*x*(23 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^
4*(-1 + a*x)^4)
```

3.165.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6724, 25, 27, 581, 25, 27, 671, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{ 6724}$$

$$ac\int -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^5x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^5x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^5x^2}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{ 581}$$

$$-\frac{a\left(-\int -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(4a-\frac{3}{x}\right)}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\left(\int \frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(4a-\frac{3}{x}\right)}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\left(a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(4a-\frac{3}{x}\right)}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow \text{ 671}$$

$$-\frac{a\left(a\left(\frac{23}{7}\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow 461$$

$$-\frac{a\left(a\left(\frac{23}{7}\left(\frac{\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{5a}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow 460$$

$$-\frac{a\left(a\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}+\frac{23}{7}\left(\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(a-\frac{1}{x}\right)^3}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}\right)\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

output
```
-((a*(a*((23*((a*(1 - 1/(a^2*x^2))^(3/2))/(5*(a - x^(-1))^4) + (1 - 1/(a^2
*x^2))^(3/2)/(15*(a - x^(-1))^3)))/7 + (a^2*(1 - 1/(a^2*x^2))^(3/2))/(7*(a
 - x^(-1))^5)) - (a^2*(1 - 1/(a^2*x^2))^(3/2))/(a - x^(-1))^4))/c^4)
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.165. $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.165.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{(2a^2x^2-10ax+23)(ax+1)}{105(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	50
default	$-\frac{(2a^2x^2-10ax+23)(ax+1)}{105(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	50
trager	$-\frac{(ax+1)(2a^3x^3-8a^2x^2+13ax+23)\sqrt{-\frac{-ax+1}{ax+1}}}{105a\,c^4(ax-1)^4}$	60

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)
/a
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+36\,ax+23)\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/105*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + 36*a*x + 23)*sqrt((a*x - 1)/(a
*x + 1))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^
4)
```

3.165.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

output
```
Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x**3*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))), x)/c**4
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{42\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}-15}{420\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/420*(42*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 - 15)/(a*c^4*((
a*x - 1)/(a*x + 1))^(7/2))
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{4\left(70\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+\right.}{105\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-4/105*(70*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 35*(a + sqrt(a^2 - 1/x^2))^3*x^
3 + 21*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 7*(a + sqrt(a^2 - 1/x^2))*x + 1)/((
(a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^4)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{\frac{(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{2\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
-((a*x - 1)^2/(3*(a*x + 1)^2) - (2*(a*x - 1))/(5*(a*x + 1)) + 1/7)/(4*a*c^
4*((a*x - 1)/(a*x + 1))^(7/2))
```

3.166 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

3.166.1 Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6}-\frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5}+\frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4}-\frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

output

```
1/9*a^5*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^6-8/21*a^4*(1-1/a^2/x^2)^(3/2)/c^5
/(a-1/x)^5+47/105*a^3*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^4-58/315*a^2*(1-1/a^
2/x^2)^(3/2)/c^5/(a-1/x)^3
```

3.166.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.44

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-58-25ax+21a^2x^2-10a^3x^3+2a^4x^4)}{315c^5(-1+ax)^5}$$

input

```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

output

```
-1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(-58 - 25*a*x + 21*a^2*x^2 - 10*a^3*x^3 +
2*a^4*x^4))/(c^5*(-1 + a*x)^5)
```

3.166.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.44, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6724, 27, 581, 25, 2170, 27, 671, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^6\left(a-\frac{1}{x}\right)^6x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^6x^3}d\frac{1}{x}}{c^5}$$

$$\Big\downarrow \text{581}$$

$$\frac{a\left(\int -\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(4a^3-\frac{7a^2}{x}+\frac{2a}{x^2}\right)}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}-\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(4a^3-\frac{7a^2}{x}+\frac{2a}{x^2}\right)}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}\right)}{c^5}$$

$$\Big\downarrow \text{2170}$$

$$\frac{a\left(-\frac{1}{2}a^2\int \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(9a-\frac{10}{x}\right)}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(-a^2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(9a-\frac{10}{x}\right)}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \text{671}$$

$$\frac{a\left(-a^2\left(\frac{29}{3}\int\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(a-\frac{1}{x}\right)^6}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \; 461$$

$$\frac{a\left(-a^2\left(\frac{29}{3}\left(\frac{2\int\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{7a}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(a-\frac{1}{x}\right)^6}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \; 461$$

$$\frac{a\left(-a^2\left(\frac{29}{3}\left(\frac{2\left(\frac{\int\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{5a}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}\right)}{7a}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(a-\frac{1}{x}\right)^6}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \; 460$$

$$\frac{a\left(-a^2\left(\frac{29}{3}\left(\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}+\frac{2\left(\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(a-\frac{1}{x}\right)^3}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}\right)}{7a}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(a-\frac{1}{x}\right)^6}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

input

```
Int[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

output

```
(a*(-(a^2*((29*((2*((a*(1 - 1/(a^2*x^2))^(3/2))/(5*(a - x^(-1))^4) + (1 -
1/(a^2*x^2))^(3/2)/(15*(a - x^(-1))^3)))/(7*a) + (a*(1 - 1/(a^2*x^2))^(3/2
))/(7*(a - x^(-1))^5)))/3 - (a^2*(1 - 1/(a^2*x^2))^(3/2))/(9*(a - x^(-1))^
6))) + (a^3*(1 - 1/(a^2*x^2))^(3/2))/(a - x^(-1))^5 + (a^2*(1 - 1/(a^2*x^2
))^(3/2))/(a - x^(-1))^4))/c^5
```

3.166. $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

3.166. $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

rule 2170

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

rule 6724

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.166.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax+1)}{315(ax-1)^4c^5\sqrt{\frac{ax-1}{ax+1}}\,a}$	58
default	$-\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax+1)}{315(ax-1)^4c^5\sqrt{\frac{ax-1}{ax+1}}\,a}$	58
trager	$-\frac{(ax+1)(2a^4x^4-10a^3x^3+21a^2x^2-25ax-58)\sqrt{-\frac{-ax+1}{ax+1}}}{315a\,c^5(ax-1)^5}$	68

input

```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output

```
-1/315*(2*a^3*x^3-12*a^2*x^2+33*a*x-58)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a*
x+1))^(1/2)/a
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{(2\,a^5x^5-8\,a^4x^4+11\,a^3x^3-4\,a^2x^2-83\,ax-58)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^6c^5x^5-5\,a^5c^5x^4+10\,a^4c^5x^3-10\,a^3c^5x^2+5\,a^2c^5x-ac^5)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 - 4*a^2*x^2 - 83*a*x - 58)*sqrt
((a*x - 1)/(a*x + 1))/(a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*a
^3*c^5*x^2 + 5*a^2*c^5*x - a*c^5)
```

3.166.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{1}{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

output
```
-Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 5*a**4*x**4*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + 10*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1)) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 5*a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**5
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{135\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{105\,(ax-1)^3}{(ax+1)^3}-35}{2520\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/2520*(135*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 105*(a*x
- 1)^3/(a*x + 1)^3 - 35)/(a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{4\left(315\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+189\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+84\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3-36\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x}{315\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^9ac^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
4/315*(315*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 189*(a + sqrt(a^2 - 1/x^2))^4*x
^4 + 84*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 36*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 9*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^9*a*c^5
)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\frac{3\,{\left(a\,x-1\right)}^2}{5\,{\left(a\,x+1\right)}^2}-\frac{{\left(a\,x-1\right)}^3}{3\,{\left(a\,x+1\right)}^3}-\frac{3\,\left(a\,x-1\right)}{7\,\left(a\,x+1\right)}+\frac{1}{9}}{8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}$$

input
```
int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
((3*(a*x - 1)^2)/(5*(a*x + 1)^2) - (a*x - 1)^3/(3*(a*x + 1)^3) - (3*(a*x -
 1))/(7*(a*x + 1)) + 1/9)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

3.167 $\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx$

3.167.1 Optimal result

Integrand size = 18, antiderivative size = 42

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}$$

output
```
2*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(p+1)/a/c/(p+1)
```

3.167.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^p(2+p+apx)}{ap(1+p)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
((c - a*c*x)^p*(2 + p + a*p*x))/(a*p*(1 + p))
```

3.167.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}(c-acx)^pdx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)(c-acx)^p}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int (ax+1)(c-acx)^{p-1}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(2(c-acx)^{p-1}-\frac{(c-acx)^p}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{(c-acx)^{p+1}}{ac^2(p+1)}-\frac{2(c-acx)^p}{acp}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
-(c*((-2*(c - a*c*x)^p)/(a*c*p) + (c - a*c*x)^(1 + p)/(a*c^2*(1 + p))))
```

3.167.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.167.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{(pax+p+2)(-acx+c)^{p}}{ap(p+1)}$	29
risch	$\frac{(pax+p+2)(-acx+c)^{p}}{ap(p+1)}$	29
norman	$\frac{x\,\mathrm{e}^{p\ln(-acx+c)}}{p+1}+\frac{(2+p)\mathrm{e}^{p\ln(-acx+c)}}{ap(p+1)}$	46
parallelrisch	$\frac{x(-acx+c)^{p}ap+(-acx+c)^{p}p+2(-acx+c)^{p}}{ap(p+1)}$	49

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x,method=_RETURNVERBOSE)
```

output
```
(a*p*x+p+2)*(-a*c*x+c)^p/a/p/(p+1)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(apx+p+2)(-acx+c)^p}{ap^2+ap}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
(a*p*x + p + 2)*(-a*c*x + c)^p/(a*p^2 + a*p)
```

3.167.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(29) = 58.

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\begin{cases}-c^px & \text{for } a=0\\ -\frac{ax\log\left(x-\frac{1}{a}\right)}{a^2cx-ac}+\frac{\log\left(x-\frac{1}{a}\right)}{a^2cx-ac}+\frac{2}{a^2cx-ac} & \text{for } p=-1\\ x+\frac{2\log\left(x-\frac{1}{a}\right)}{a} & \text{for } p=0\\ \frac{apx(-acx+c)^p}{ap^2+ap}+\frac{p(-acx+c)^p}{ap^2+ap}+\frac{2(-acx+c)^p}{ap^2+ap} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**p,x)
```

output
```
Piecewise((-c**p*x, Eq(a, 0)), (-a*x*log(x - 1/a)/(a**2*c*x - a*c) + log(x
 - 1/a)/(a**2*c*x - a*c) + 2/(a**2*c*x - a*c), Eq(p, -1)), (x + 2*log(x -
1/a)/a, Eq(p, 0)), (a*p*x*(-a*c*x + c)**p/(a*p**2 + a*p) + p*(-a*c*x + c)*
*p/(a*p**2 + a*p) + 2*(-a*c*x + c)**p/(a*p**2 + a*p), True))
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{(ac^ppx+c^p)(-ax+1)^p}{(p^2+p)a}+\frac{(-ax+1)^pc^p}{ap}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
(a*c^p*p*x + c^p)*(-a*x + 1)^p/((p^2 + p)*a) + (-a*x + 1)^p*c^p/(a*p)
```

3.167.8 Giac [F]

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(ax+1)(-acx+c)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 4.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{(c-a\,c\,x)^p\,(p+a\,p\,x+2)}{a\,p\,(p+1)}$$

input
```
int(((c - a*c*x)^p*(a*x + 1))/(a*x - 1),x)
```

output
```
((c - a*c*x)^p*(p + a*p*x + 2))/(a*p*(p + 1))
```

3.168 $\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx$

3.168.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

output
```
2/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

3.168.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(-1+ax)^5(7+5ax)}{30a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

output
```
-1/30*(c^5*(-1 + a*x)^5*(7 + 5*a*x))/a
```

3.168.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^5 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^5 e^{2\text{arctanh}(ax)}(1-ax)^5dx$$

$$\Big\downarrow \text{27}$$

$$-c^5\int e^{2\text{arctanh}(ax)}(1-ax)^5dx$$

$$\Big\downarrow \text{6679}$$

$$-c^5\int (1-ax)^4(ax+1)dx$$

$$\Big\downarrow \text{49}$$

$$-c^5\int \left(2(1-ax)^4-(1-ax)^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^5\left(\frac{(1-ax)^6}{6a}-\frac{2(1-ax)^5}{5a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

output
```
-(c^5*((-2*(1 - a*x)^5)/(5*a) + (1 - a*x)^6/(6*a)))
```

3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.168.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result
gosper	$-\frac{\left(5 a^{5} x^{5}-18 a^{4} x^{4}+15 a^{3} x^{3}+20 a^{2} x^{2}-45 a x +30\right) c^{5} x}{30}$
default	$c^{5} \left(-\frac{1}{6} a^{5} x^{6}+\frac{3}{5} a^{4} x^{5}-\frac{1}{2} a^{3} x^{4}-\frac{2}{3} a^{2} x^{3}+\frac{3}{2} a \,x^{2}-x \right)$
norman	$-c^{5} x +\frac{3}{2} a \,c^{5} x^{2}-\frac{1}{2} a^{3} c^{5} x^{4}+\frac{3}{5} a^{4} c^{5} x^{5}-\frac{1}{6} a^{5} c^{5} x^{6}-\frac{2}{3} c^{5} a^{2} x^{3}$
risch	$-c^{5} x +\frac{3}{2} a \,c^{5} x^{2}-\frac{1}{2} a^{3} c^{5} x^{4}+\frac{3}{5} a^{4} c^{5} x^{5}-\frac{1}{6} a^{5} c^{5} x^{6}-\frac{2}{3} c^{5} a^{2} x^{3}$
parallelrisch	$-c^{5} x +\frac{3}{2} a \,c^{5} x^{2}-\frac{1}{2} a^{3} c^{5} x^{4}+\frac{3}{5} a^{4} c^{5} x^{5}-\frac{1}{6} a^{5} c^{5} x^{6}-\frac{2}{3} c^{5} a^{2} x^{3}$
meijerg	$-\frac{c^{5} \left(\frac{a x \left(70 a^{5} x^{5}+84 a^{4} x^{4}+105 a^{3} x^{3}+140 a^{2} x^{2}+210 a x +420\right)}{420}+\ln \left(-a x +1\right)\right)}{a}-\frac{4 c^{5} \left(-\frac{a x \left(12 a^{4} x^{4}+15 a^{3} x^{3}+20 a^{2} x^{2}+30 a x +60\right)}{60}-\ln(\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*c^5*x
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{1}{6}\,a^5c^5x^6 + \frac{3}{5}\,a^4c^5x^5 - \frac{1}{2}\,a^3c^5x^4 - \frac{2}{3}\,a^2c^5x^3 + \frac{3}{2}\,ac^5x^2 - c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3
/2*a*c^5*x^2 - c^5*x
```

3.168.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{a^5c^5x^6}{6} + \frac{3a^4c^5x^5}{5} - \frac{a^3c^5x^4}{2} - \frac{2a^2c^5x^3}{3} + \frac{3ac^5x^2}{2} - c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**5,x)
```

output
```
-a**5*c**5*x**6/6 + 3*a**4*c**5*x**5/5 - a**3*c**5*x**4/2 - 2*a**2*c**5*x*
*3/3 + 3*a*c**5*x**2/2 - c**5*x
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{1}{6}\,a^5c^5x^6 + \frac{3}{5}\,a^4c^5x^5 - \frac{1}{2}\,a^3c^5x^4 - \frac{2}{3}\,a^2c^5x^3 + \frac{3}{2}\,ac^5x^2 - c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3
/2*a*c^5*x^2 - c^5*x
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}a^5c^5x^6+\frac{3}{5}a^4c^5x^5-\frac{1}{2}a^3c^5x^4-\frac{2}{3}a^2c^5x^3+\frac{3}{2}ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
-1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3
/2*a*c^5*x^2 - c^5*x
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{a^5\,c^5\,x^6}{6}+\frac{3\,a^4\,c^5\,x^5}{5}-\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}+\frac{3\,a\,c^5\,x^2}{2}-c^5\,x$$

input
```
int(((c - a*c*x)^5*(a*x + 1))/(a*x - 1),x)
```

output
```
(3*a*c^5*x^2)/2 - c^5*x - (2*a^2*c^5*x^3)/3 - (a^3*c^5*x^4)/2 + (3*a^4*c^5
*x^5)/5 - (a^5*c^5*x^6)/6
```

3.169 $\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx$

3.169.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

output
```
1/2*c^4*(-a*x+1)^4/a-1/5*c^4*(-a*x+1)^5/a
```

3.169.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{10}c^4x\left(-10+10ax-5a^3x^3+2a^4x^4\right)$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
(c^4*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

3.169.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = $\{6717, 27, 6679, 49, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^4 e^{2\text{arctanh}(ax)}(1-ax)^4dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^4\int e^{2\text{arctanh}(ax)}(1-ax)^4dx$$

$$\Big\downarrow \text{ 6679}$$

$$-c^4\int (1-ax)^3(ax+1)dx$$

$$\Big\downarrow \text{ 49}$$

$$-c^4\int \left(2(1-ax)^3-(1-ax)^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^4\left(\frac{(1-ax)^5}{5a}-\frac{(1-ax)^4}{2a}\right)$$

input

```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output

```
-(c^4*(-1/2*(1 - a*x)^4/a + (1 - a*x)^5/(5*a)))
```

3.169.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.169.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{x\left(2a^4x^4-5a^3x^3+10ax-10\right)c^4}{10}$
default	$c^4\left(\frac{1}{5}a^4x^5-\frac{1}{2}a^3x^4+a\,x^2-x\right)$
norman	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
risch	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
parallelrisch	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
meijerg	$-\frac{c^4\left(-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\ln(-ax+1)\right)}{a}-\frac{3c^4\left(\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln(-ax+1)\right)}{a}-2c^4$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/10*x*(2*a^4*x^4-5*a^3*x^3+10*a*x-10)*c^4
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{5}\,a^4c^4x^5 - \frac{1}{2}\,a^3c^4x^4 + ac^4x^2 - c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

3.169.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{a^4c^4x^5}{5} - \frac{a^3c^4x^4}{2} + ac^4x^2 - c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**4,x)
```

output
```
a**4*c**4*x**5/5 - a**3*c**4*x**4/2 + a*c**4*x**2 - c**4*x
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{5}\,a^4c^4x^5 - \frac{1}{2}\,a^3c^4x^4 + ac^4x^2 - c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{1}{2}\,a^3c^4x^4+ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

3.169.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{a^4\,c^4\,x^5}{5}-\frac{a^3\,c^4\,x^4}{2}+a\,c^4\,x^2-c^4\,x$$

input
```
int(((c - a*c*x)^4*(a*x + 1))/(a*x - 1),x)
```

output
```
a*c^4*x^2 - c^4*x - (a^3*c^4*x^4)/2 + (a^4*c^4*x^5)/5
```

3.170 $\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx$

3.170.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

output
```
2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a
```

3.170.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{12}c^3x\left(12-6ax-4a^2x^2+3a^3x^3\right)$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
-1/12*(c^3*x*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))
```

3.170.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^3 e^{2\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{2\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{6679}$$

$$-c^3\int (1-ax)^2(ax+1)dx$$

$$\Big\downarrow \text{49}$$

$$-c^3\int \left(2(1-ax)^2-(1-ax)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^3\left(\frac{(1-ax)^4}{4a}-\frac{2(1-ax)^3}{3a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
-(c^3*((-2*(1 - a*x)^3)/(3*a) + (1 - a*x)^4/(4*a)))
```

3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.170.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$-\frac{\left(3 a^{3} x^{3}-4 a^{2} x^{2}-6 a x +12\right) c^{3} x}{12}$
default	$c^{3} \left(-\frac{1}{4} a^{3} x^{4}+\frac{1}{3} a^{2} x^{3}+\frac{1}{2} a \,x^{2}-x \right)$
norman	$-c^{3} x +\frac{1}{2} a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}-\frac{1}{4} a^{3} c^{3} x^{4}$
risch	$-c^{3} x +\frac{1}{2} a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}-\frac{1}{4} a^{3} c^{3} x^{4}$
parallelrisch	$-c^{3} x +\frac{1}{2} a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}-\frac{1}{4} a^{3} c^{3} x^{4}$
meijerg	$-\frac{c^{3} \left(\frac{a x \left(15 a^{3} x^{3}+20 a^{2} x^{2}+30 a x +60\right)}{60}+\ln \left(-a x +1\right)\right)}{a}-\frac{2 c^{3} \left(-\frac{a x \left(4 a^{2} x^{2}+6 a x +12\right)}{12}-\ln \left(-a x +1\right)\right)}{a}+\frac{2 c^{3} \left(-a x -\ln \left(-a x +1\right)\right)}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*c^3*x
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{a^3c^3x^4}{4}+\frac{a^2c^3x^3}{3}+\frac{ac^3x^2}{2}-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**3,x)
```

output
```
-a**3*c**3*x**4/4 + a**2*c**3*x**3/3 + a*c**3*x**2/2 - c**3*x
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{a^3\,c^3\,x^4}{4}+\frac{a^2\,c^3\,x^3}{3}+\frac{a\,c^3\,x^2}{2}-c^3\,x$$

input
```
int(((c - a*c*x)^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(a*c^3*x^2)/2 - c^3*x + (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

3.171 $\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx$

3.171.1 Optimal result

Integrand size = 18, antiderivative size = 20

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=-c^2x+\frac{1}{3}a^2c^2x^3$$

output
```
-c^2*x+1/3*a^2*c^2*x^3
```

3.171.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=-c^2\left(x-\frac{a^2x^3}{3}\right)$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
-(c^2*(x - (a^2*x^3)/3))
```

3.171.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 39, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^2 e^{2\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^2\int e^{2\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{ 6679}$$

$$-c^2\int (1-ax)(ax+1)dx$$

$$\Big\downarrow \text{ 39}$$

$$-c^2\int \left(1-a^2x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^2\left(x-\frac{a^2x^3}{3}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
-(c^2*(x - (a^2*x^3)/3))
```

3.171.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.171.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	s
gosper	$\frac{x(a^2x^2-3)c^2}{3}$	1
default	$c^2\left(\frac{1}{3}a^2x^3-x\right)$	1
norman	$-c^2x+\frac{1}{3}a^2c^2x^3$	1
risch	$-c^2x+\frac{1}{3}a^2c^2x^3$	1
parallelrisch	$-c^2x+\frac{1}{3}a^2c^2x^3$	1
meijerg	$-\frac{c^2\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}-\frac{c^2\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c^2(-ax-\ln(-ax+1))}{a}+\frac{c^2\ln(-ax+1)}{a}$	9

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(a^2*x^2-3)*c^2
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*a^2*c^2*x^3 - c^2*x
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^2c^2x^3}{3}-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**2,x)
```

output
```
a**2*c**2*x**3/3 - c**2*x
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*c^2*x^3 - c^2*x
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
1/3*a^2*c^2*x^3 - c^2*x
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\,x\,(a^2\,x^2-3)}{3}$$

input
```
int(((c - a*c*x)^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^2*x*(a^2*x^2 - 3))/3
```

3.172 $\int e^{2\coth^{-1}(ax)}(c-acx)\,dx$

3.172.1 Optimal result

Integrand size = 16, antiderivative size = 14

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-cx-\frac{1}{2}acx^2$$

output
```
-c*x-1/2*a*c*x^2
```

3.172.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=\frac{ce^{2\coth^{-1}(ax)}(1-a^2x^2)}{2a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
(c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)
```

3.172.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{2726}$$

$$\frac{c(1-a^2x^2)\,e^{2\coth^{-1}(ax)}}{2a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
(c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)
```

3.172.3.1 Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

3.172.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{cx(ax+2)}{2}$	10
default	$c\left(-\frac{1}{2}a\,x^2-x\right)$	13
norman	$-cx-\frac{1}{2}ac\,x^2$	13
risch	$-cx-\frac{1}{2}ac\,x^2$	13
parallelrisch	$-cx-\frac{1}{2}ac\,x^2$	13
meijerg	$-\frac{c\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c\ln(-ax+1)}{a}$	38

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*x*(a*x+2)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="fricas")
```

output
```
-1/2*a*c*x^2 - c*x
```

3.172.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{acx^2}{2}-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x)
```

output
```
-a*c*x**2/2 - c*x
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/2*a*c*x^2 - c*x
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="giac")
```

output
```
-1/2*a*c*x^2 - c*x
```

3.172.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\,x\,(a\,x+2)}{2}$$

input
```
int(((c - a*c*x)*(a*x + 1))/(a*x - 1),x)
```

output
```
-(c*x*(a*x + 2))/2
```

3.173 $\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx$

3.173.1 Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

output
```
-2/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

3.173.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}}{c}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
-((2/(a*(1 - a*x)) + Log[1 - a*x]/a)/c)
```

3.173.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{tikzcd}
\displaystyle\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}dx \arrow[d, "6717"] \\
\displaystyle -\int \frac{e^{2\text{arctanh}(ax)}}{c(1-ax)}dx \arrow[d, "27"] \\
\displaystyle -\frac{\int \frac{e^{2\text{arctanh}(ax)}}{1-ax}dx}{c} \arrow[d, "6679"] \\
\displaystyle -\frac{\int \frac{ax+1}{(1-ax)^2}dx}{c} \arrow[d, "49"] \\
\displaystyle -\frac{\int \left(\frac{1}{ax-1}+\frac{2}{(ax-1)^2}\right)dx}{c} \arrow[d, "2009"] \\
\displaystyle -\frac{\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}}{c}
\end{tikzcd}$$

input

```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

output

```
-((2/(a*(1 - a*x)) + Log[1 - a*x]/a)/c)
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.173.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\frac{2}{a(ax-1)}-\frac{\ln(ax-1)}{a}}{c}$	29
norman	$\frac{2x}{c(ax-1)}-\frac{\ln(ax-1)}{ac}$	29
risch	$\frac{2}{ac(ax-1)}-\frac{\ln(ax-1)}{ac}$	31
parallelrisch	$\frac{-a\ln(ax-1)x+2ax+\ln(ax-1)}{c(ax-1)a}$	36

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/c*(2/a/(a*x-1)-1/a*ln(a*x-1))
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-((a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2}{a^2cx-ac} - \frac{\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x)
```

output
```
2/(a**2*c*x - a*c) - log(a*x - 1)/(a*c)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2}{a^2cx-ac} - \frac{\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
2/(a^2*c*x - a*c) - log(a*x - 1)/(a*c)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(|ax-1|)}{ac}+\frac{2}{(ax-1)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="giac")
```

output
```
-log(abs(a*x - 1))/(a*c) + 2/((a*x - 1)*a*c)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{2}{a\,(c-a\,c\,x)}-\frac{\ln(a\,x-1)}{a\,c}$$

input
```
int((a*x + 1)/((c - a*c*x)*(a*x - 1)),x)
```

output
```
- 2/(a*(c - a*c*x)) - log(a*x - 1)/(a*c)
```

3.174 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

3.174.1 Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{c^2(1-ax)^2}$$

output
```
-x/c^2/(-a*x+1)^2
```

3.174.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(1+ax)^2}{4ac^2(1-ax)^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-1/4*(1 + a*x)^2/(a*c^2*(1 - a*x)^2)
```

3.174.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6717, 27, 6679, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^2(1-ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{ax+1}{(1-ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{38}$$

$$-\frac{x}{c^2(1-ax)^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-(x/(c^2*(1 - a*x)^2))
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.174.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{x}{c^{2}\left(ax-1\right)^{2}}$	14
norman	$-\frac{x}{c^{2}\left(ax-1\right)^{2}}$	14
risch	$-\frac{x}{c^{2}\left(ax-1\right)^{2}}$	14
parallelrisch	$-\frac{x}{c^{2}\left(ax-1\right)^{2}}$	14
default	$\frac{-\frac{1}{\left(ax-1\right)^{2}a}-\frac{1}{a\left(ax-1\right)}}{c^{2}}$	30

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/c^2/(a*x-1)^2
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{x}{a^2c^2x^2-2\,ac^2x+c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

3.174.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{a^2c^2x^2-2ac^2x+c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**2,x)
```

output
```
-x/(a**2*c**2*x**2 - 2*a*c**2*x + c**2)
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{a^2c^2x^2-2\,ac^2x+c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{(acx-c)^2a}-\frac{1}{(acx-c)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-1/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{c^2\,{\left(a\,x-1\right)}^2}$$

input
```
int((a*x + 1)/((c - a*c*x)^2*(a*x - 1)),x)
```

output
```
-x/(c^2*(a*x - 1)^2)
```

3.175 $\int \frac{e^{2 \coth^{-1}(ax)}}{(c-acx)^3} \, dx$

3.175.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{e^{2 \coth^{-1}(ax)}}{(c-acx)^3} \, dx = -\frac{2}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}$$

output
```
-2/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

3.175.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{e^{2 \coth^{-1}(ax)}}{(c-acx)^3} \, dx = \frac{1+3ax}{6ac^3(-1+ax)^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(1 + 3*a*x)/(6*a*c^3*(-1 + a*x)^3)
```

3.175.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^3(1-ax)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 6679}$$

$$-\frac{\int \frac{ax+1}{(1-ax)^4}dx}{c^3}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{\int \left(\frac{1}{(ax-1)^3}+\frac{2}{(ax-1)^4}\right)dx}{c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{2}{3a(1-ax)^3}-\frac{1}{2a(1-ax)^2}}{c^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
-((2/(3*a*(1 - a*x)^3) - 1/(2*a*(1 - a*x)^2))/c^3)
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.175.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{\frac{x}{2}+\frac{1}{6a}}{(ax-1)^3 c^3}$	21
gosper	$\frac{3ax+1}{6a\,c^3(ax-1)^3}$	22
default	$\frac{\frac{1}{2(ax-1)^2 a}+\frac{2}{3a(ax-1)^3}}{c^3}$	30
parallelrisch	$\frac{a^2x^3-3a\,x^2+6x}{6c^3(ax-1)^3}$	30
norman	$\frac{\frac{x}{c}-\frac{a\,x^2}{2c}+\frac{a^2x^3}{6c}}{(ax-1)^3 c^2}$	38

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*x+1/6/a)/(a*x-1)^3/c^3
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,ax+1}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{-3ax-1}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**3,x)
```

output
```
-(-3*a*x - 1)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**2*c**3*x - 6*a
*c**3)
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,ax+1}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{3\,ax+1}{6\,(ax-1)^3ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
1/6*(3*a*x + 1)/((a*x - 1)^3*a*c^3)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx=-\frac{\frac{x}{2}+\frac{1}{6\,a}}{-a^3\,c^3\,x^3+3\,a^2\,c^3\,x^2-3\,a\,c^3\,x+c^3}$$

input
```
int((a*x + 1)/((c - a*c*x)^3*(a*x - 1)),x)
```

output
```
-(x/2 + 1/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c^3*x)
```

3.176 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.176.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{1}{2ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}$$

output
```
-1/2/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

3.176.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{1+2ax}{6ac^4(-1+ax)^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-1/6*(1 + 2*a*x)/(a*c^4*(-1 + a*x)^4)
```

3.176.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^4(1-ax)^4}dx$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \ 6679$$

$$-\frac{\int \frac{ax+1}{(1-ax)^5}dx}{c^4}$$

$$\Big\downarrow \ 53$$

$$-\frac{\int \left(-\frac{1}{(ax-1)^4}-\frac{2}{(ax-1)^5}\right)dx}{c^4}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{1}{2a(1-ax)^4}-\frac{1}{3a(1-ax)^3}}{c^4}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-((1/(2*a*(1 - a*x)^4) - 1/(3*a*(1 - a*x)^3))/c^4)
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.176.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{-\frac{x}{3}-\frac{1}{6a}}{c^{4}\left(ax-1\right)^{4}}$	21
gosper	$-\frac{2ax+1}{6a\,c^{4}\left(ax-1\right)^{4}}$	22
default	$\frac{-\frac{1}{2a\left(ax-1\right)^{4}}-\frac{1}{3a\left(ax-1\right)^{3}}}{c^{4}}$	30
parallelrisch	$\frac{a^{3}x^{4}-4a^{2}x^{3}+6a\,x^{2}-6x}{6c^{4}\left(ax-1\right)^{4}}$	38
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{x}{c}-\frac{2a^{2}x^{3}}{3c}+\frac{a^{3}x^{4}}{6c}}{\left(ax-1\right)^{4}c^{3}}$	49

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*x-1/6/a)/c^4/(a*x-1)^4
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*
x + a*c^4)
```

3.176.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2ax-1}{6a^5c^4x^4-24a^4c^4x^3+36a^3c^4x^2-24a^2c^4x+6ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**4,x)
```

output
```
(-2*a*x - 1)/(6*a**5*c**4*x**4 - 24*a**4*c**4*x**3 + 36*a**3*c**4*x**2 - 2
4*a**2*c**4*x + 6*a*c**4)
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
-1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*
x + a*c^4)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(ax-1)^4ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-1/6*(2*a*x + 1)/((a*x - 1)^4*a*c^4)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{x}{3}+\frac{1}{6\,a}}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

input
```
int((a*x + 1)/((c - a*c*x)^4*(a*x - 1)),x)
```

output
```
-(x/3 + 1/(6*a))/(c^4 + 6*a^2*c^4*x^2 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*
c^4*x)
```

3.177 $\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$

3.177.1 Optimal result

Integrand size = 18, antiderivative size = 202

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{3\sqrt{1+\frac{1}{ax}}(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a^2p\left(1-p^2\right)\left(1-\frac{1}{ax}\right)^{3/2}x}$$

output
```
(1+1/a/x)^(3/2)*x*(-a*c*x+c)^p/(p+1)/(1-1/a/x)^(1/2)-3*((a-1/x)/(a+1/x))^(
3/2-p)*(-a*c*x+c)^p*hypergeom([1-p, 3/2-p],[2-p],2/(a+1/x)/x)*(1+1/a/x)^(1
/2)/a^2/p/(-p^2+1)/(1-1/a/x)^(3/2)/x+3*(-a*c*x+c)^p*(1+1/a/x)^(1/2)/a/p/(p
+1)/(1-1/a/x)^(1/2)
```

3.177.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$$
$$=\frac{\left(\frac{-1+ax}{1+ax}\right)^{-p}(c-acx)^p\left((-1+p)\left(\frac{-1+ax}{1+ax}\right)^p(1+ax)(3+p+apx)+3\sqrt{\frac{-1+ax}{1+ax}}\,\text{Hypergeometric2F1}\left(1-p,\right.\right.}{a^2(-1+p)p(1+p)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
((c - a*c*x)^p*((-1 + p)*((-1 + a*x)/(1 + a*x))^p*(1 + a*x)*(3 + p + a*p*x
) + 3*Sqrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1 - p, 3/2 - p, 2 - p,
2/(1 + a*x)]))/(a^2*(-1 + p)*p*(1 + p)*Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)
/(1 + a*x))^p)
```

3.177.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6727, 105, 105, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$$
$$\Big\downarrow \text{6727}$$
$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{-p-2}d\frac{1}{x}$$
$$\Big\downarrow \text{105}$$
$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\left(\frac{3\int\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-p-1}d\frac{1}{x}}{a(p+1)}-\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-p-1}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}}{p+1}\right)$$
$$\Big\downarrow \text{105}$$

$$\left(\frac{1}{x}\right)^p \left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p \left(\frac{3\left(\frac{\int \frac{\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{x}\right)^{-p}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{ap}-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}}{p}\right)}{a(p+1)}-\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-p-1}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}}{p+1}\right)$$

$$\Big\downarrow \text{142}$$

$$\left(\frac{1}{x}\right)^p \left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p \left(\frac{3\left(\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{1-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a(1-p)p}-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}}{p}\right)}{a(p+1)}-\right.$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
-(((x^(-1))^p*(c - a*c*x)^p*(-(((1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*x))^(3/
2)*(x^(-1))^(-1 - p))/(1 + p)) + (3*(-(((1 - 1/(a*x))^(-1/2 + p)*Sqrt[1 +
1/(a*x)])/(p*(x^(-1))^p)) + (((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/
(a*x))^(-3/2 + p)*Sqrt[1 + 1/(a*x)]*(x^(-1))^(1 - p)*Hypergeometric2F1[1 -
 p, 3/2 - p, 2 - p, 2/((a + x^(-1))*x)])/(a*(1 - p)*p)))/(a*(1 + p))))/(1
- 1/(a*x))^p)
```

3.177.3.1 Defintions of rubi rules used

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.177.4 Maple [F]

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

3.177.5 Fricas [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a
^2*x^2 - 2*a*x + 1), x)
```

3.177.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-c(ax-1))^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**p,x)
```

output
```
Integral((-c*(a*x - 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.177.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.177.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(c-a\,c\,x)^p}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.178 $\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$

3.178.1 Optimal result

Integrand size = 18, antiderivative size = 105

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
-1/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1/5*a^4*c^4*(1-1/a^2/x^2)^(5/2)*x^5-3
/8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+3/8*a*c^4*x^2*(1-1/a^2/x^2)^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(8+25ax-16a^2x^2-10a^3x^3+8a^4x^4)-15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
(c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*
a^4*x^4) - 15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

3.178.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 25, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6724$$

$$a^3c^3\int -c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^6d\frac{1}{x}$$

$$\Big\downarrow \; 25$$

$$-a^3c^3\int c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^6d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$-a^3c^4\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^6d\frac{1}{x}$$

$$\Big\downarrow \; 534$$

$$-a^3c^4\left(-\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5d\frac{1}{x}-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \; 243$$

$$-a^3c^4\left(-\frac{1}{2}\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3d\frac{1}{x^2}-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \; 51$$

$$-a^3c^4\left(\frac{1}{2}\left(\frac{3\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \; 51$$

$$-a^3c^4\left(\frac{1}{2}\left(\frac{3\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow 73$$

$$-a^3c^4\left(\frac{1}{2}\left(\frac{3\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow 221$$

$$-a^3c^4\left(\frac{1}{2}\left(\frac{3\left(\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
-(a^3*c^4*(-1/5*(a*(1 - 1/(a^2*x^2))^(5/2)*x^5) + (((1 - 1/(a^2*x^2))^(3/2
)*x^2)/2 + (3*(-(Sqrt[1 - 1/(a^2*x^2)]*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
/a^2))/(4*a^2))/2))
```

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.178. $\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$

3.178.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8)(ax-1)c^4}{40a\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^4\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^4\Big(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+45\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-40((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-4}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/40*(8*a^4*x^4-10*a^3*x^3-16*a^2*x^2+25*a*x+8)*(a*x-1)/a*c^4/((a*x-1)/(a*
x+1))^(1/2)-3/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4/(a
*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx =$$

$$-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\Big(8\,a^5c^4x^5-2\,a^4c^4x^4-26\,a^3c^4x^3+9\,a^2c^4x^2+33\,ac^4}{40\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^4*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (8*a^5*c^4*x^5 - 2*a^4*c^4*x^4 - 26*a^3*c^4*x^3 + 9*a
^2*c^4*x^2 + 33*a*c^4*x + 8*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.178.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = c^4\left(\int\left(-\frac{4ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{6a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{4a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**4,x)
```

output
```
c**4*(Integral(-4*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2*x**2/(a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1)), x) + Integral(-4*a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
 Integral(a**4*x**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1)), x))
```

3.178.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(89) = 178$.

Time = 0.21 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.47

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx =$$

$$-\frac{1}{40}\left(\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^4\left(\frac{ax-}{ax+}\right.\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
-1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^4*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^
4*((a*x - 1)/(a*x + 1))^(7/2) - 128*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 70*c
^4*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x
 - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/
(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5
- a^2))*a
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)}$$
$$+\frac{1}{40}\sqrt{a^2x^2-1}\left(\left(\left(\frac{25\,c^4}{\mathrm{sgn}\,(ax+1)}-2\left(\frac{8\,ac^4}{\mathrm{sgn}\,(ax+1)}-\left(\frac{4\,a^3c^4x}{\mathrm{sgn}\,(ax+1)}-\frac{5\,a^2c^4}{\mathrm{sgn}\,(ax+1)}\right)x\right)x\right)x+\frac{8\,c}{a\mathrm{sgn}\,(a}\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
3/8*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + 1/
40*sqrt(a^2*x^2 - 1)*((25*c^4/sgn(a*x + 1) - 2*(8*a*c^4/sgn(a*x + 1) - (4*
a^3*c^4*x/sgn(a*x + 1) - 5*a^2*c^4/sgn(a*x + 1))*x)*x)*x + 8*c^4/(a*sgn(a*
x + 1)))
```

3.178.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.04

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{\frac{3c^4\sqrt{\frac{ax-1}{ax+1}}}{4}-\frac{7c^4\left(\frac{ax-1}{ax+1}\right)^{3/2}}{2}+\frac{32c^4\left(\frac{ax-1}{ax+1}\right)^{5/2}}{5}+\frac{7c^4\left(\frac{ax-1}{ax+1}\right)^{7/2}}{2}-\frac{3c^4\left(\frac{ax-1}{ax+1}\right)^{9/2}}{4}}{a-\frac{5a(ax-1)}{ax+1}+\frac{10a(ax-1)^2}{(ax+1)^2}-\frac{10a(ax-1)^3}{(ax+1)^3}+\frac{5a(ax-1)^4}{(ax+1)^4}-\frac{a(ax-1)^5}{(ax+1)^5}}$$
$$-\frac{3c^4\operatorname{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{4a}$$

input
```
int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((3*c^4*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^4*((a*x - 1)/(a*x + 1))^(3/2
))/2 + (32*c^4*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^4*((a*x - 1)/(a*x + 1
))^(7/2))/2 - (3*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/
(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^4*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.179 $\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx$

3.179.1 Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output

```
-1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4-3/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/
a+3/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)
```

3.179.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{c^3\left(a^2\sqrt{1-\frac{1}{a^2x^2}}x^2(5-2a^2x^2)-3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{8a}$$

input

```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output

```
(c^3*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*(5 - 2*a^2*x^2) - 3*Log[a*(1 + Sqrt[1
- 1/(a^2*x^2)])*x]))/(8*a)
```

3.179.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6724, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5d\frac{1}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a^3c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a^3c^3\left(-\frac{3\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}}{4a^2}-\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a^3c^3\left(-\frac{3\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)}{4a^2}-\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a^3c^3\left(-\frac{3\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)}{4a^2}-\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}a^3c^3\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)}{4a^2}-\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
(a^3*c^3*(-1/2*((1 - 1/(a^2*x^2))^(3/2)*x^2) - (3*(-(Sqrt[1 - 1/(a^2*x^2)]
*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2))/(4*a^2)))/2
```

3.179.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.179.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

method	result	size
risch	$-\frac{x(2a^2x^2-5)(ax-1)c^3}{8\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	106
default	$-\frac{(ax-1)^2c^3\left(2x(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3x\sqrt{a^2x^2-1}\,\sqrt{a^2}+3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	124

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(2*a^2*x^2-5)*(a*x-1)*c^3/((a*x-1)/(a*x+1))^(1/2)-3/8*ln(a^2*x/(a^2
)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
*((a*x-1)*(a*x+1))^(1/2)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx =$$
$$-\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2\,a^4c^3x^4+2\,a^3c^3x^3-5\,a^2c^3x^2-5\,ac^3x)\sqrt{\frac{ax-1}{ax+1}}}{8\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^3*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) + (2*a^4*c^3*x^4 + 2*a^3*c^3*x^3 - 5*a^2*c^3*x^2 - 5*a*c^3
*x)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.179.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=-c^3\left(\int\frac{3ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**3,x)
```

output
```
-c**3*(Integral(3*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2*x**2/(a
*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a
*x + 1))/(a*x + 1)), x) + Integral(a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + I
ntegral(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

3.179.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(66) = 132$.

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.83

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{8}\left(\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4}{(ax+1)}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^2 + 2*(3*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 11*c^3*((
a*x - 1)/(a*x + 1))^(5/2) - 11*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqr
t((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*
x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a
^2))*a
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{8}\left(\frac{2\,a^2c^3x^2}{\operatorname{sgn}(ax+1)}-\frac{5\,c^3}{\operatorname{sgn}(ax+1)}\right)\sqrt{a^2x^2-1}x + \frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(2*a^2*c^3*x^2/sgn(a*x + 1) - 5*c^3/sgn(a*x + 1))*sqrt(a^2*x^2 - 1)*x
 + 3/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

3.179.9 Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.26

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{\frac{3\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{11\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4}-\frac{11\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{4}+\frac{3\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a-\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{6\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}} - \frac{3\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((3*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(3/
2))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 + (3*c^3*((a*x - 1)/(a*x +
1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)
^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) - (3*c^3
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.179. $\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx$

3.180 $\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx$

3.180.1 Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3-1/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
+1/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-2+3ax+2a^2x^2)-3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*a*x + 2*a^2*x^2) - 3*Log[a*(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

3.180.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 25, 27, 566, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int -\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{c\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{c\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^3c^2\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a-\frac{1}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{566}$$

$$-a^3c^2\int \sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{a}+\frac{1}{xa^2}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{534}$$

$$-a^3c^2\left(\frac{\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}}{a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

$$\Big\downarrow \text{243}$$

$$-a^3c^2\left(\frac{\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}}{2a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

$$\Big\downarrow \text{51}$$

$$-a^3c^2\left(\frac{x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}}{2a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

$$\Big\downarrow \ 73$$

$$-a^3c^2\left(\frac{\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

$$\Big\downarrow \ 221$$

$$-a^3c^2\left(\frac{\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
-(a^3*c^2*(-1/3*((1 - 1/(a^2*x^2))^(3/2)*x^3)/a + (-(Sqrt[1 - 1/(a^2*x^2)]
*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2)/(2*a^2)))
```

3.180.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.180. $\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx$

3.180.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{(2a^2x^2+3ax-2)(ax-1)c^2}{6a\sqrt{\frac{ax-1}{ax+1}}}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	112
default	$\frac{(ax-1)^2c^2\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}$	130

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2+3*a*x-2)*(a*x-1)/a*c^2/((a*x-1)/(a*x+1))^(1/2)-1/2*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/(a*x+1)/((a*x-1)/(a*x+1))^
(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3+5\,a^2c^2x^2+ac^2x-2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) - (2*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + a*c^2*x - 2*c^2)*sqrt((
a*x - 1)/(a*x + 1)))/a
```

3.180.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-\frac{2ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**2,x)
```

output
```
c**2*(Integral(-2*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2*x**2/(a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

3.180.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(66) = 132$.

Time = 0.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{1}{6}\,a\left(\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 8*c^2*(
(a*x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)
*a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)
^3 - a^2))
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\begin{aligned}&\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx\\&=\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,ac^2x}{\mathrm{sgn}\,(ax+1)}+\frac{3\,c^2}{\mathrm{sgn}\,(ax+1)}\right)x-\frac{2\,c^2}{a\mathrm{sgn}\,(ax+1)}\right)\\&\quad+\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}\end{aligned}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
1/6*sqrt(a^2*x^2 - 1)*((2*a*c^2*x/sgn(a*x + 1) + 3*c^2/sgn(a*x + 1))*x - 2
*c^2/(a*sgn(a*x + 1))) + 1/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(
abs(a)*sgn(a*x + 1))
```

3.180.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(c^2*((a*x - 1)/(a*x + 1))^(1/2) + (8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 -
 c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a
*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^2*atanh(((a*x -
 1)/(a*x + 1))^(1/2)))/a
```

3.181 $\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$

3.181.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-2c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
-3/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a-2*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^
2*(1-1/a^2/x^2)^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(4+ax)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
-1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^
2*x^2)])*x]))/a
```

3.181.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6724, 27, 570, 540, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{c^2\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$a^3c\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{570}$$

$$\frac{c\int \frac{\left(a+\frac{1}{x}\right)^2x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{540}$$

$$\frac{c\left(-\frac{1}{2}\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\left(\frac{1}{2}\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{534}$$

$$\frac{c\left(\frac{1}{2}\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{243}$$

$$\frac{c\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow 73$$

$$\frac{c\left(\frac{1}{2}\left(-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow 221$$

$$\frac{c\left(\frac{1}{2}\left(-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
(c*(-1/2*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x -
 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/2))/a
```

3.181.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.181.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

method	result	size
risch	$-\frac{(ax+4)(ax-1)c}{2a\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)c\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^{2}}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	99
default	$-\frac{(ax-1)^{2}c\left(\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax-\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a+4\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}+4a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^{2}}}$	162

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+4)*(a*x-1)/a*c/((a*x-1)/(a*x+1))^(1/2)-3/2*ln(a^2*x/(a^2)^(1/2)+
(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*
(a*x+1))^(1/2)
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2+5\,acx+4\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="fricas")
```

output
```
-1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) + (a^2*c*x^2 + 5*a*c*x + 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.181.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int \frac{ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c),x)
```

output
```
-c*(Integral(a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x))
```

3.181.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.08

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$$
$$=-\frac{1}{2}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/2*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(3/2) - 5*c*sqrt((a*x - 1)/(a*x + 1))
)/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 3*c*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/a^2)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{cx}{\operatorname{sgn}(ax+1)}+\frac{4\,c}{a\operatorname{sgn}(ax+1)}\right)+\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="giac")
```

output
```
-1/2*sqrt(a^2*x^2 - 1)*(c*x/sgn(a*x + 1) + 4*c/(a*sgn(a*x + 1))) + 3/2*c*l
og(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

3.181.9 Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-\frac{5\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (5*c*((a*x - 1)/(a*x + 1))^(1/2) - 3*c*((a*x - 1)/(a*x + 1))^(3/2))/(a -
 (2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a
*x - 1)/(a*x + 1))^(1/2)))/a
```

3.182 $\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$

3.182.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
8/3*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(3/2)-arctanh((1-1/a^2/x^2)^(1/2))/a/c+4/3
/a^2/c/x/(1-1/a^2/x^2)^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{\frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+2ax)}{(-1+ax)^2}-\frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
((4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x))/(-1 + a*x)^2 - (3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

3.182.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6724, 27, 570, 532, 25, 2336, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$$

$$\Big\downarrow 6724$$

$$a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{c^4\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}$$

$$\Big\downarrow 27$$

$$\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c}$$

$$\Big\downarrow 570$$

$$\frac{\int \frac{\left(a+\frac{1}{x}\right)^4x}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^5c}$$

$$\Big\downarrow 532$$

$$\frac{\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(3a^4+\frac{4a^3}{x}-\frac{3a^2}{x^2}\right)x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^5c}$$

$$\Big\downarrow 25$$

$$\frac{\frac{1}{3}\int \frac{\left(3a^4+\frac{4a^3}{x}-\frac{3a^2}{x^2}\right)x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\Big\downarrow 2336$$

$$\frac{\frac{1}{3}\left(\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{3a^4x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{3}\left(3a^4\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{3}\left(\frac{3}{2}a^4\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{3}\left(\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}-3a^6\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}-3a^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5c}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
((8*a^3*(a + x^(-1)))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((4*a^3)/(Sqrt[1 - 1/(
a^2*x^2)]*x) - 3*a^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/3)/(a^5*c)
```

3.182.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.182. $\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.182. $\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$

3.182.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(70) = 140$.

Time = 0.39 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.31

method	result
default	$-\frac{3\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-3\sqrt{a^2}\,((ax-1)(ax+1)}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*(3*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^4*
x^3+3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^3*x^3-9*ln((a^2*x+(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^3*x^2-3*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*a*x-9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2+9*ln((a^2*x+(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x+((a*x-1)*(a*x+1))^(3
/2)*(a^2)^(1/2)+9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x-3*a*ln((a^2*x+(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-3*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/(a^2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1
)/(a*x+1))^(3/2)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.50

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx =$$

$$-\frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-4\left(2\,a^2x^2+ax-1\right)}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*
x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - 4*(2*a^2*x^2 + a*x -
 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

3.182.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{1}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

output
```
-Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1)), x)/c
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{1}{3}a\left(\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\left(\frac{3(ax-1)}{ax+1}+1\right)}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/3*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 3*log(sqrt((a*x - 1
)/(a*x + 1)) - 1)/(a^2*c) - 2*(3*(a*x - 1)/(a*x + 1) + 1)/(a^2*c*((a*x - 1
)/(a*x + 1))^(3/2)))
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

output
```
log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c*abs(a)*sgn(a*x + 1))
```

3.182.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\frac{2\,(a\,x-1)}{a\,x+1}+\frac{2}{3}}{a\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
((2*(a*x - 1))/(a*x + 1) + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2)) - (2*ata
nh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.183 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

3.183.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

output
```
-1/5*a^4*(1-1/a^2/x^2)^(5/2)/c^2/(a-1/x)^5
```

3.183.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(1+ax)^2}{5c^2(-1+ax)^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-1/5*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2)/(c^2*(-1 + a*x)^3)
```

3.183.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6724, 25, 27, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int -\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{460}$$

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-1/5*(a^4*(1 - 1/(a^2*x^2))^(5/2))/(c^2*(a - x^(-1))^5)
```

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.183.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{ax+1}{5(ax-1)c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	36
default	$-\frac{ax+1}{5(ax-1)c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	36
trager	$-\frac{(ax+1)\left(a^{2}x^{2}+2ax+1\right)\sqrt{-\frac{-ax+1}{ax+1}}}{5a\,c^{2}(ax-1)^{3}}$	51

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

3.183.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(a^3x^3+3\,a^2x^2+3\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/5*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*
x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

3.183.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

output
```
Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**2
*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(a*x +
1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1))
, x)/c**2
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{5\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/5/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.183.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2\left(5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{5\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-2/5*(5*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 10*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^2)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{5\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-1/(5*a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.184 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

3.184.1 Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6} - \frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

output

```
1/7*a^5*(1-1/a^2/x^2)^(5/2)/c^3/(a-1/x)^6-6/35*a^4*(1-1/a^2/x^2)^(5/2)/c^3
/(a-1/x)^5
```

3.184.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-6+ax)(1+ax)^2}{35c^3(-1+ax)^4}$$

input

```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output

```
-1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-6 + a*x)*(1 + a*x)^2)/(c^3*(-1 + a*x)^4)
```

3.184.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6724, 27, 571, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \textcolor{blue}{6724}$$

$$a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^6\left(a-\frac{1}{x}\right)^6x}d\frac{1}{x}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^6x}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \textcolor{blue}{571}$$

$$\frac{a^3\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(a-\frac{1}{x}\right)^6}-\frac{6}{7}\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}\right)}{c^3}$$

$$\Big\downarrow \textcolor{blue}{460}$$

$$\frac{a^3\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(a-\frac{1}{x}\right)^6}-\frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35\left(a-\frac{1}{x}\right)^5}\right)}{c^3}$$

input

```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output

```
(a^3*((a^2*(1 - 1/(a^2*x^2))^(5/2))/(7*(a - x^(-1))^6) - (6*a*(1 - 1/(a^2*
x^2))^(5/2))/(35*(a - x^(-1))^5)))/c^3
```

3.184.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.184.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{(ax-6)(ax+1)}{35(ax-1)^{2}c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	41
default	$-\frac{(ax-6)(ax+1)}{35(ax-1)^{2}c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	41
trager	$-\frac{(ax+1)\left(a^{3}x^{3}-4a^{2}x^{2}-11ax-6\right)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^{3}(ax-1)^{4}}$	59

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(a*x-6)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(3/2)/a
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\left(a^4x^4-3\,a^3x^3-15\,a^2x^2-17\,ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output

```
-1/35*(a^4*x^4 - 3*a^3*x^3 - 15*a^2*x^2 - 17*a*x - 6)*sqrt((a*x - 1)/(a*x
+ 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

3.184.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$$
$$= -\frac{\int \frac{1}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

output

```
-Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**
3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{7\,(ax-1)}{ax+1}-5}{70\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output

```
-1/70*(7*(a*x - 1)/(a*x + 1) - 5)/(a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

3.184.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(59) = 118$.

Time $= 0.34$ (sec) , antiderivative size $= 125$, normalized size of antiderivative $= 1.87$

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$$
$$= \frac{2\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+70\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+14\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+\right.}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
2/35*(35*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 35*(a + sqrt(a^2 - 1/x^2))^4*x^4
+ 70*(a + sqrt(a^2 - 1/x^2))^3*x^3 + 14*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 7*
(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^3)
```

3.184.9 Mupad [B] (verification not implemented)

Time $= 0.04$ (sec) , antiderivative size $= 39$, normalized size of antiderivative $= 0.58$

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{5\,(a\,x+1)}-\frac{1}{7}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-((a*x - 1)/(5*(a*x + 1)) - 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

3.185 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.185.1 Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

output
```
-47/315*(a+1/x)^5/a^6/c^4/(1-1/a^2/x^2)^(5/2)+16/63*(a+1/x)^6/a^7/c^4/(1-1
/a^2/x^2)^(7/2)-1/9*(a+1/x)^7/a^8/c^4/(1-1/a^2/x^2)^(9/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(1+ax)^2\left(47-14ax+2a^2x^2\right)}{315c^4(-1+ax)^5}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(47 - 14*a*x + 2*a^2*x^2))/(c^
4*(-1 + a*x)^5)
```

3.185.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6724, 25, 27, 570, 529, 27, 669, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int -\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^7\left(a-\frac{1}{x}\right)^7x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^7\left(a-\frac{1}{x}\right)^7x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^7x^2}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^7}{\left(1-\frac{1}{a^2x^2}\right)^{11/2}x^2}d\frac{1}{x}}{a^{11}c^4}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\frac{a^3\left(a+\frac{1}{x}\right)^7}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a\int \frac{a\left(a+\frac{1}{x}\right)^6\left(7a+\frac{9}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^{11}c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{a^3\left(a+\frac{1}{x}\right)^7}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a^2\int \frac{\left(a+\frac{1}{x}\right)^6\left(7a+\frac{9}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^{11}c^4}$$

$$\Big\downarrow \text{669}$$

$$-\frac{\frac{a^3\left(a+\frac{1}{x}\right)^7}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a^2\left(\frac{16a^2\left(a+\frac{1}{x}\right)^6}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47}{7}a^2\int\frac{\left(a+\frac{1}{x}\right)^5}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}\right)}{a^{11}c^4}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\frac{a^3\left(a+\frac{1}{x}\right)^7}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a^2\left(\frac{16a^2\left(a+\frac{1}{x}\right)^6}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47a^3\left(a+\frac{1}{x}\right)^5}{35\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)}{a^{11}c^4}$$

input

```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output

```
-((-1/9*(a^2*((-47*a^3*(a + x^(-1))^5)/(35*(1 - 1/(a^2*x^2))^(5/2)) + (16*
a^2*(a + x^(-1))^6)/(7*(1 - 1/(a^2*x^2))^(7/2)))) + (a^3*(a + x^(-1))^7)/(
9*(1 - 1/(a^2*x^2))^(9/2)))/(a^11*c^4))
```

3.185.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 529

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.185.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{(2a^2x^2-14ax+47)(ax+1)}{315(ax-1)^3c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
default	$-\frac{(2a^2x^2-14ax+47)(ax+1)}{315(ax-1)^3c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
trager	$-\frac{(ax+1)(2a^4x^4-10a^3x^3+21a^2x^2+80ax+47)\sqrt{-\frac{-ax+1}{ax+1}}}{315a\,c^4(ax-1)^5}$	68

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(2*a^2*x^2-14*a*x+47)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)
/a
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{(2\,a^5x^5-8\,a^4x^4+11\,a^3x^3+101\,a^2x^2+127\,ax+47)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output

```
-1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 + 101*a^2*x^2 + 127*a*x + 47)*s
qrt((a*x - 1)/(a*x + 1))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 1
0*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

3.185.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{\int \frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

output

```
Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 5*a**4
*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{90\,(ax-1)}{ax+1} - \frac{63\,(ax-1)^2}{(ax+1)^2} - 35}{1260\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/1260*(90*(a*x - 1)/(a*x + 1) - 63*(a*x - 1)^2/(a*x + 1)^2 - 35)/(a*c^4*(
(a*x - 1)/(a*x + 1))^(9/2))
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.54

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx =$$

$$-\frac{4\left(210\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^6 x^6 + 315\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5 x^5 + 441\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4 x^4 + 126\left(a+\sqrt{a^2-\frac{1}{x}}\right.\right.}{315\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^9 ac^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-4/315*(210*(a + sqrt(a^2 - 1/x^2))^6*x^6 + 315*(a + sqrt(a^2 - 1/x^2))^5*
x^5 + 441*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 126*(a + sqrt(a^2 - 1/x^2))^3*x^
3 + 36*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 9*(a + sqrt(a^2 - 1/x^2))*x + 1)/((
(a + sqrt(a^2 - 1/x^2))*x - 1)^9*a*c^4)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{(a\,x-1)^2}{5\,(a\,x+1)^2}-\frac{2\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{9}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

input
```
int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-((a*x - 1)^2/(5*(a*x + 1)^2) - (2*(a*x - 1))/(7*(a*x + 1)) + 1/9)/(4*a*c^
4*((a*x - 1)/(a*x + 1))^(9/2))
```

3.186 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

3.186.1 Optimal result

Integrand size = 18, antiderivative size = 125

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}$$

output
```
-152/1155*(a+1/x)^5/a^6/c^5/(1-1/a^2/x^2)^(5/2)+79/231*(a+1/x)^6/a^7/c^5/(
1-1/a^2/x^2)^(7/2)-10/33*(a+1/x)^7/a^8/c^5/(1-1/a^2/x^2)^(9/2)+1/11*(a+1/x
)^8/a^9/c^5/(1-1/a^2/x^2)^(11/2)
```

3.186.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2\left(-152+61ax-16a^2x^2+2a^3x^3\right)}{1155c^5(-1+ax)^6}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

output
```
-1/1155*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(-152 + 61*a*x - 16*a^2*x^2 +
 2*a^3*x^3))/(c^5*(-1 + a*x)^6)
```

3.186.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6724, 27, 570, 529, 2166, 27, 669, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^8\left(a-\frac{1}{x}\right)^8x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^8x^3}d\frac{1}{x}}{c^5}$$

$$\Big\downarrow \text{570}$$

$$\frac{\int \frac{\left(a+\frac{1}{x}\right)^8}{\left(1-\frac{1}{a^2x^2}\right)^{13/2}x^3}d\frac{1}{x}}{a^{13}c^5}$$

$$\Big\downarrow \text{529}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\int \frac{\left(a+\frac{1}{x}\right)^7\left(8a^3+\frac{11a^2}{x}+\frac{11a}{x^2}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{11/2}}d\frac{1}{x}}{a^{13}c^5}$$

$$\Big\downarrow \text{2166}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\left(\frac{10a^4\left(a+\frac{1}{x}\right)^7}{3\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a\int \frac{3a^2\left(a+\frac{1}{x}\right)^6\left(46a+\frac{33}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}\right)}{a^{13}c^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\left(\frac{10a^4\left(a+\frac{1}{x}\right)^7}{3\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{3}a^3\int \frac{\left(a+\frac{1}{x}\right)^6\left(46a+\frac{33}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}\right)}{a^{13}c^5}$$

$$\Big\downarrow \text{669}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\left(\frac{10a^4\left(a+\frac{1}{x}\right)^7}{3\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{3}a^3\left(\frac{79a^2\left(a+\frac{1}{x}\right)^6}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152}{7}a^2\int\frac{\left(a+\frac{1}{x}\right)^5}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}\right)\right)}{a^{13}c^5}$$

$$\Big\downarrow \text{460}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\left(\frac{10a^4\left(a+\frac{1}{x}\right)^7}{3\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{3}a^3\left(\frac{79a^2\left(a+\frac{1}{x}\right)^6}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152a^3\left(a+\frac{1}{x}\right)^5}{35\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)\right)}{a^{13}c^5}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

output
```
(-1/11*(a*(-1/3*(a^3*((-152*a^3*(a + x^(-1))^5)/(35*(1 - 1/(a^2*x^2))^(5/2
)) + (79*a^2*(a + x^(-1))^6)/(7*(1 - 1/(a^2*x^2))^(7/2)))) + (10*a^4*(a +
x^(-1))^7)/(3*(1 - 1/(a^2*x^2))^(9/2)))) + (a^4*(a + x^(-1))^8)/(11*(1 - 1
/(a^2*x^2))^(11/2)))/(a^13*c^5)
```

3.186.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.186.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax+1)}{1155(ax-1)^4c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
default	$-\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax+1)}{1155(ax-1)^4c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
trager	$-\frac{(ax+1)(2a^5x^5-12a^4x^4+31a^3x^3-46a^2x^2-243ax-152)\sqrt{-\frac{-ax+1}{ax+1}}}{1155a\,c^5(ax-1)^6}$	76

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(2*a^3*x^3-16*a^2*x^2+61*a*x-152)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(
a*x+1))^(3/2)/a
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{(2\,a^6x^6-10\,a^5x^5+19\,a^4x^4-15\,a^3x^3-289\,a^2x^2-395\,ax-152)\sqrt{\frac{ax-1}{ax+1}}}{1155\,(a^7c^5x^6-6\,a^6c^5x^5+15\,a^5c^5x^4-20\,a^4c^5x^3+15\,a^3c^5x^2-6\,a^2c^5x+ac^5)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/1155*(2*a^6*x^6 - 10*a^5*x^5 + 19*a^4*x^4 - 15*a^3*x^3 - 289*a^2*x^2 -
395*a*x - 152)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 15
*a^5*c^5*x^4 - 20*a^4*c^5*x^3 + 15*a^3*c^5*x^2 - 6*a^2*c^5*x + a*c^5)
```

3.186.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{1}{\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{20a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+}}{c^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

output
```
-Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 6*a**
5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 15*a**4*x**4*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 20*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1) + 15*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1) - 6*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**5
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{385\,(ax-1)}{ax+1} - \frac{495\,(ax-1)^2}{(ax+1)^2} + \frac{231\,(ax-1)^3}{(ax+1)^3} - 105}{9240\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/9240*(385*(a*x - 1)/(a*x + 1) - 495*(a*x - 1)^2/(a*x + 1)^2 + 231*(a*x
- 1)^3/(a*x + 1)^3 - 105)/(a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$

$$= \frac{4\Bigg(1155\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^7 x^7 + 2079\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^6 x^6 + 2541\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5 x^5 + 825\Bigg(a+\sqrt{a^2-}}{1155\Bigg(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
4/1155*(1155*(a + sqrt(a^2 - 1/x^2))^7*x^7 + 2079*(a + sqrt(a^2 - 1/x^2))^
6*x^6 + 2541*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 825*(a + sqrt(a^2 - 1/x^2))^4
*x^4 + 165*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 55*(a + sqrt(a^2 - 1/x^2))^2*x^
2 + 11*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^11*
a*c^5)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\frac{3\,(a\,x-1)^2}{7\,(a\,x+1)^2}-\frac{(a\,x-1)^3}{5\,(a\,x+1)^3}-\frac{a\,x-1}{3\,(a\,x+1)}+\frac{1}{11}}{8\,a\,c^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}$$

input
```
int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
((3*(a*x - 1)^2)/(7*(a*x + 1)^2) - (a*x - 1)^3/(5*(a*x + 1)^3) - (a*x - 1)
/(3*(a*x + 1)) + 1/11)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

3.187 $\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx$

3.187.1 Optimal result

Integrand size = 18, antiderivative size = 66

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{4c(c-acx)^{-1+p}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}$$

output

```
4*c*(-a*c*x+c)^(-1+p)/a/(1-p)+4*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(p+1)/a/c/(p+1
)
```

3.187.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^p\left(\frac{4+3p}{p(1+p)}+\frac{ax}{1+p}+\frac{4}{(-1+p)(-1+ax)}\right)}{a}$$

input

```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output

```
((c - a*c*x)^p*((4 + 3*p)/(p*(1 + p)) + (a*x)/(1 + p) + 4/((-1 + p)*(-1 +
a*x))))/a
```

3.187.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int e^{4\text{arctanh}(ax)}(c-acx)^pdx$$

$$\Big\downarrow \text{6680}$$

$$\int \frac{(ax+1)^2(c-acx)^p}{(1-ax)^2}dx$$

$$\Big\downarrow \text{35}$$

$$c^2\int (ax+1)^2(c-acx)^{p-2}dx$$

$$\Big\downarrow \text{53}$$

$$c^2\int \left(4(c-acx)^{p-2}-\frac{4(c-acx)^{p-1}}{c}+\frac{(c-acx)^p}{c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^2\left(-\frac{(c-acx)^{p+1}}{ac^3(p+1)}+\frac{4(c-acx)^p}{ac^2p}+\frac{4(c-acx)^{p-1}}{ac(1-p)}\right)$$

input

```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output

```
c^2*((4*(c - a*c*x)^(-1 + p))/(a*c*(1 - p)) + (4*(c - a*c*x)^p)/(a*c^2*p)
- (c - a*c*x)^(1 + p)/(a*c^3*(1 + p)))
```

3.187.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.187.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

method	result
gosper	$\frac{\left(a^{2} p^{2} x^{2}-a^{2} x^{2} p+2 a\,p^{2} x+2 p a x-4 a x+p^{2}+3 p+4\right) \left(-a c x+c\right)^{p}}{\left(a x-1\right) a p \left(p^{2}-1\right)}$
risch	$\frac{\left(a^{2} p^{2} x^{2}-a^{2} x^{2} p+2 a\,p^{2} x+2 p a x-4 a x+p^{2}+3 p+4\right) \left(-a c x+c\right)^{p}}{a p \left(p+1\right) \left(-1+p\right) \left(a x-1\right)}$
norman	$\frac{\frac{a\,x^{2} {\mathrm e}^{p \ln \left(-a c x+c\right)}}{p+1}+\frac{\left(p^{2}+3 p+4\right) {\mathrm e}^{p \ln \left(-a c x+c\right)}}{a p \left(p^{2}-1\right)}+\frac{2 \left(2+p\right) x\,{\mathrm e}^{p \ln \left(-a c x+c\right)}}{p \left(p+1\right)}}{a x-1}$
parallelrisch	$\frac{x^{2} \left(-a c x+c\right)^{p} a^{2} p^{2}-x^{2} \left(-a c x+c\right)^{p} a^{2} p+2 x \left(-a c x+c\right)^{p} a\,p^{2}+2 x \left(-a c x+c\right)^{p} a p-4 \left(-a c x+c\right)^{p} x a+\left(-a c x+c\right)^{p} p^{2}+3 \left(-a c x+c\right)^{p} p+}{\left(a x-1\right) a p \left(p^{2}-1\right)}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x,method=_RETURNVERBOSE)
```

output
```
(a^2*p^2*x^2-a^2*p*x^2+2*a*p^2*x+2*a*p*x-4*a*x+p^2+3*p+4)*(-a*c*x+c)^p/(a*
x-1)/a/p/(p^2-1)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = -\frac{((a^2p^2-a^2p)x^2+p^2+2\,(ap^2+ap-2\,a)x+3\,p+4)(-acx+c)^p}{ap^3-ap-(a^2p^3-a^2p)x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
-((a^2*p^2 - a^2*p)*x^2 + p^2 + 2*(a*p^2 + a*p - 2*a)*x + 3*p + 4)*(-a*c*x
 + c)^p/(a*p^3 - a*p - (a^2*p^3 - a^2*p)*x)
```

3.187.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(48) = 96.

Time = 0.58 (sec) , antiderivative size = 530, normalized size of antiderivative = 8.03

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx$$
$$= \begin{cases} c^p x \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{2ax\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{4ax}{a^3cx^2-2a^2cx+ac}-\frac{\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}-\frac{2}{a^3cx^2-2a^2cx+ac} \\ \frac{a^2x^2}{a^2x-a}+\frac{4ax\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{4\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{5}{a^2x-a} \\ -\frac{acx^2}{2}-3cx-\frac{4c\log\left(x-\frac{1}{a}\right)}{a} \\ \frac{a^2p^2x^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{a^2px^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2ap^2x(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2apx(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{4ax(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+ \end{cases}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**p,x)
```

output
```
Piecewise((c**p*x, Eq(a, 0)), (-a**2*x**2*log(x - 1/a)/(a**3*c*x**2 - 2*a*
*2*c*x + a*c) + 2*a*x*log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 4*a*
x/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*
x + a*c) - 2/(a**3*c*x**2 - 2*a**2*c*x + a*c), Eq(p, -1)), (a**2*x**2/(a**
2*x - a) + 4*a*x*log(x - 1/a)/(a**2*x - a) - 4*log(x - 1/a)/(a**2*x - a) -
 5/(a**2*x - a), Eq(p, 0)), (-a*c*x**2/2 - 3*c*x - 4*c*log(x - 1/a)/a, Eq(
p, 1)), (a**2*p**2*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 +
 a*p) - a**2*p*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p
) + 2*a*p**2*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2
*a*p*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) - 4*a*x*(-a
*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + p**2*(-a*c*x + c)**
p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 3*p*(-a*c*x + c)**p/(a**2*p**3
*x - a**2*p*x - a*p**3 + a*p) + 4*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x
- a*p**3 + a*p), True))
```

3.187.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.32

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{((p^2-p)a^2c^px^2+2\,ac^p(p-1)x+2\,c^p)(-ax+1)^pa^2}{(p^3-p)a^4x-(p^3-p)a^3} + \frac{2\,(ac^p(p-1)x+c^p)(-ax+1)^pa}{(p^2-p)a^3x-(p^2-p)a^2} + \frac{(-ax+1)^pc^p}{a^2(p-1)x-a(p-1)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
((p^2 - p)*a^2*c^p*x^2 + 2*a*c^p*(p - 1)*x + 2*c^p)*(-a*x + 1)^p*a^2/((p^3
 - p)*a^4*x - (p^3 - p)*a^3) + 2*(a*c^p*(p - 1)*x + c^p)*(-a*x + 1)^p*a/((
p^2 - p)*a^3*x - (p^2 - p)*a^2) + (-a*x + 1)^p*c^p/(a^2*(p - 1)*x - a*(p -
 1))
```

3.187.8 Giac [F]

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(ax+1)^2(-acx+c)^p}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)^2*(-a*c*x + c)^p/(a*x - 1)^2, x)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{4\,(c-a\,c\,x)^p}{a\,(a\,x-1)\,(p-1)}+\frac{(c-a\,c\,x)^p\,(3\,p+a\,p\,x+4)}{a\,p\,(p+1)}$$

input
```
int(((c - a*c*x)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(4*(c - a*c*x)^p)/(a*(a*x - 1)*(p - 1)) + ((c - a*c*x)^p*(3*p + a*p*x + 4)
)/(a*p*(p + 1))
```

3.188 $\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx$

3.188.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

output
```
-c^5*(-a*x+1)^4/a+4/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

3.188.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(-1+ax)^4\,(11+14ax+5a^2x^2)}{30a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

output
```
-1/30*(c^5*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2))/a
```

3.188.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^5 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^5(1-ax)^5 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^5\int e^{4\text{arctanh}(ax)}(1-ax)^5dx$$

$$\Big\downarrow \text{6679}$$

$$c^5\int (1-ax)^3(ax+1)^2dx$$

$$\Big\downarrow \text{49}$$

$$c^5\int \left((1-ax)^5-4(1-ax)^4+4(1-ax)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^5\left(-\frac{(1-ax)^6}{6a}+\frac{4(1-ax)^5}{5a}-\frac{(1-ax)^4}{a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

output
```
c^5*(-((1 - a*x)^4/a) + (4*(1 - a*x)^5)/(5*a) - (1 - a*x)^6/(6*a))
```

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.188.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{x\left(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30\right)c^5}{30}$
default	$c^5\left(-\frac{1}{6}a^5x^6+\frac{1}{5}a^4x^5+\frac{1}{2}a^3x^4-\frac{2}{3}a^2x^3-\frac{1}{2}a\,x^2+x\right)$
risch	$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}c^5a^2x^3-\frac{1}{2}a\,c^5x^2+c^5x$
parallelrisch	$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}c^5a^2x^3-\frac{1}{2}a\,c^5x^2+c^5x$
norman	$\frac{-c^5x+\frac{3}{2}a\,c^5x^2-\frac{7}{6}a^3c^5x^4+\frac{3}{10}a^4c^5x^5+\frac{11}{30}a^5c^5x^6-\frac{1}{6}a^6c^5x^7+\frac{1}{6}c^5a^2x^3}{ax-1}$
meijerg	$-\frac{c^5\left(\frac{ax\left(-20a^6x^6-28a^5x^5-42a^4x^4-70a^3x^3-140a^2x^2-420ax+840\right)}{-120ax+120}+7\ln(-ax+1)\right)}{a}-\frac{3c^5\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3-70\right.}{70(-ax+1)}\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/30*x*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*c^5
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1
/2*a*c^5*x^2 + c^5*x
```

3.188.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{a^5c^5x^6}{6}+\frac{a^4c^5x^5}{5}+\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}-\frac{ac^5x^2}{2}+c^5x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**5,x)
```

output
```
-a**5*c**5*x**6/6 + a**4*c**5*x**5/5 + a**3*c**5*x**4/2 - 2*a**2*c**5*x**3
/3 - a*c**5*x**2/2 + c**5*x
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1
/2*a*c^5*x^2 + c^5*x
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{\left(5\,c^5+\frac{24\,c^5}{ax-1}+\frac{30\,c^5}{(ax-1)^2}\right)(ax-1)^6}{30\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
-1/30*(5*c^5 + 24*c^5/(a*x - 1) + 30*c^5/(a*x - 1)^2)*(a*x - 1)^6/a
```

3.188.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{a^5\,c^5\,x^6}{6}+\frac{a^4\,c^5\,x^5}{5}+\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}-\frac{a\,c^5\,x^2}{2}+c^5\,x$$

input
```
int(((c - a*c*x)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^5*x - (a*c^5*x^2)/2 - (2*a^2*c^5*x^3)/3 + (a^3*c^5*x^4)/2 + (a^4*c^5*x^5
)/5 - (a^5*c^5*x^6)/6
```

3.189 $\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx$

3.189.1 Optimal result

Integrand size = 18, antiderivative size = 32

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5$$

output
```
c^4*x-2/3*a^2*c^4*x^3+1/5*a^4*c^4*x^5
```

3.189.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=c^4\left(x-\frac{2a^2x^3}{3}+\frac{a^4x^5}{5}\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

3.189.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 27, 6679, 39, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^4(1-ax)^4 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^4\int e^{4\text{arctanh}(ax)}(1-ax)^4dx$$

$$\Big\downarrow \text{6679}$$

$$c^4\int (1-ax)^2(ax+1)^2dx$$

$$\Big\downarrow \text{39}$$

$$c^4\int \left(1-a^2x^2\right)^2dx$$

$$\Big\downarrow \text{210}$$

$$c^4\int \left(a^4x^4-2a^2x^2+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^4\left(\frac{a^4x^5}{5}-\frac{2a^2x^3}{3}+x\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

3.189.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.189.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

method	result
default	$c^{4} \left(\frac{1}{5} a^{4} x^{5}-\frac{2}{3} a^{2} x^{3}+x \right)$
gosper	$\frac{x \left(3 a^{4} x^{4}-10 a^{2} x^{2}+15\right) c^{4}}{15}$
risch	$c^{4} x -\frac{2}{3} a^{2} c^{4} x^{3}+\frac{1}{5} a^{4} c^{4} x^{5}$
parallelrisch	$c^{4} x -\frac{2}{3} a^{2} c^{4} x^{3}+\frac{1}{5} a^{4} c^{4} x^{5}$
norman	$\frac{-c^{4} x +a \,c^{4} x^{2}+\frac{2}{3} a^{2} c^{4} x^{3}-\frac{2}{3} a^{3} c^{4} x^{4}-\frac{1}{5} a^{4} c^{4} x^{5}+\frac{1}{5} a^{5} c^{4} x^{6}}{a x -1}$
meijerg	$-\frac{c^{4} \left(-\frac{a x \left(-14 a^{5} x^{5}-21 a^{4} x^{4}-35 a^{3} x^{3}-70 a^{2} x^{2}-210 a x +420\right)}{70 \left(-a x +1\right)}-6 \ln \left(-a x +1\right)\right)}{a}-\frac{2 c^{4} \Bigg(\frac{a x \left(-3 a^{4} x^{4}-5 a^{3} x^{3}-10 a^{2} x^{2}-30 a x +60\right)}{-12 a x +12}+5}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/5*a^4*x^5-2/3*a^2*x^3+x)
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

3.189.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{a^4c^4x^5}{5}-\frac{2a^2c^4x^3}{3}+c^4x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**4,x)
```

output
```
a**4*c**4*x**5/5 - 2*a**2*c**4*x**3/3 + c**4*x
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{\left(3\,c^4+\frac{15\,c^4}{ax-1}+\frac{20\,c^4}{(ax-1)^2}\right)(ax-1)^5}{15\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
1/15*(3*c^4 + 15*c^4/(a*x - 1) + 20*c^4/(a*x - 1)^2)*(a*x - 1)^5/a
```

3.189.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4\,x\,(3\,a^4\,x^4-10\,a^2\,x^2+15)}{15}$$

input
```
int(((c - a*c*x)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^4*x*(3*a^4*x^4 - 10*a^2*x^2 + 15))/15
```

3.190 $\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx$

3.190.1 Optimal result

Integrand size = 18, antiderivative size = 35

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}$$

output
```
2/3*c^3*(a*x+1)^3/a-1/4*c^3*(a*x+1)^4/a
```

3.190.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{12}c^3x\left(-12-6ax+4a^2x^2+3a^3x^3\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
-1/12*(c^3*x*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))
```

3.190.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^3(1-ax)^3e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^3\int e^{4\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{6679}$$

$$c^3\int (1-ax)(ax+1)^2dx$$

$$\Big\downarrow \text{49}$$

$$c^3\int \left(2(ax+1)^2-(ax+1)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{2(ax+1)^3}{3a}-\frac{(ax+1)^4}{4a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
c^3*((2*(1 + a*x)^3)/(3*a) - (1 + a*x)^4/(4*a))
```

3.190.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.190.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result
gosper	$-\frac{x\left(3a^{3}x^{3}+4a^{2}x^{2}-6ax-12\right)c^{3}}{12}$
default	$c^{3}\left(-\frac{1}{4}a^{3}x^{4}-\frac{1}{3}a^{2}x^{3}+\frac{1}{2}a\,x^{2}+x\right)$
risch	$-\frac{1}{4}a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+\frac{1}{2}a\,c^{3}x^{2}+c^{3}x$
parallelrisch	$-\frac{1}{4}a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+\frac{1}{2}a\,c^{3}x^{2}+c^{3}x$
norman	$\frac{-c^{3}x+\frac{1}{2}a\,c^{3}x^{2}+\frac{5}{6}a^{2}c^{3}x^{3}-\frac{1}{12}a^{3}c^{3}x^{4}-\frac{1}{4}a^{4}c^{3}x^{5}}{ax-1}$
meijerg	$-\frac{c^{3}\left(\frac{ax\left(-3a^{4}x^{4}-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{-12ax+12}+5\ln\left(-ax+1\right)\right)}{a}-\frac{c^{3}\left(-\frac{ax\left(-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{15\left(-ax+1\right)}-4\ln\left(-ax+1\right)\right)}{a}+2c^{3}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/12*x*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*c^3
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

3.190.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{a^3c^3x^4}{4}-\frac{a^2c^3x^3}{3}+\frac{ac^3x^2}{2}+c^3x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**3,x)
```

output
```
-a**3*c**3*x**4/4 - a**2*c**3*x**3/3 + a*c**3*x**2/2 + c**3*x
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{\left(3\,c^3+\frac{16\,c^3}{ax-1}+\frac{24\,c^3}{(ax-1)^2}\right)(ax-1)^4}{12\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-1/12*(3*c^3 + 16*c^3/(a*x - 1) + 24*c^3/(a*x - 1)^2)*(a*x - 1)^4/a
```

3.190.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{a^3\,c^3\,x^4}{4}-\frac{a^2\,c^3\,x^3}{3}+\frac{a\,c^3\,x^2}{2}+c^3\,x$$

input
```
int(((c - a*c*x)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^3*x + (a*c^3*x^2)/2 - (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

3.191 $\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx$

3.191.1 Optimal result

Integrand size = 18, antiderivative size = 17

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2(1+ax)^3}{3a}$$

output
```
1/3*c^2*(a*x+1)^3/a
```

3.191.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=c^2\left(x+ax^2+\frac{a^2x^3}{3}\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
c^2*(x + a*x^2 + (a^2*x^3)/3)
```

3.191.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6717, 27, 6679, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^2(1-ax)^2e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{4\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{6679}$$

$$c^2\int (ax+1)^2dx$$

$$\Big\downarrow \text{17}$$

$$\frac{c^2(ax+1)^3}{3a}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
(c^2*(1 + a*x)^3)/(3*a)
```

3.191.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.191.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{c^2(ax+1)^3}{3a}$	16
gosper	$\frac{x(a^2x^2+3ax+3)c^2}{3}$	20
parallelrisch	$\frac{1}{3}a^2c^2x^3+a\,c^2x^2+c^2x$	26
risch	$\frac{a^2c^2x^3}{3}+a\,c^2x^2+c^2x+\frac{c^2}{3a}$	34
norman	$\frac{-\frac{c^2}{a}+\frac{2a^2c^2x^3}{3}+\frac{a^3c^2x^4}{3}}{ax-1}$	40
meijerg	$-\frac{c^2\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a}+\frac{2c^2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c^2x}{-ax+1}$	103

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^2*(a*x+1)^3/a
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3+ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

3.191.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^2c^2x^3}{3}+ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**2,x)
```

output
```
a**2*c**2*x**3/3 + a*c**2*x**2 + c**2*x
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3+ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

3.191.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.35

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{\left(c^2+\frac{6\,c^2}{ax-1}+\frac{12\,c^2}{(ax-1)^2}\right)(ax-1)^3}{3\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
1/3*(c^2 + 6*c^2/(a*x - 1) + 12*c^2/(a*x - 1)^2)*(a*x - 1)^3/a
```

3.191.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

input
```
int(((c - a*c*x)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^2*x*(3*a*x + a^2*x^2 + 3))/3
```

3.192 $\int e^{4\coth^{-1}(ax)}(c-acx)\,dx$

3.192.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}$$

output
```
-3*c*x-1/2*a*c*x^2-4*c*ln(-a*x+1)/a
```

3.192.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=c\left(-3x-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

3.192.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = $\{6717, 27, 6679, 49, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int c(1-ax)e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c\int e^{4\text{arctanh}(ax)}(1-ax)dx$$

$$\Big\downarrow \text{ 6679}$$

$$c\int \frac{(ax+1)^2}{1-ax}dx$$

$$\Big\downarrow \text{ 49}$$

$$c\int \left(-ax+\frac{4}{1-ax}-3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)$$

input

```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

output

```
c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

3.192.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.192.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	s
default	$c\left(-\frac{a\,x^{2}}{2}-3x-\frac{4\ln\left(ax-1\right)}{a}\right)$	2
risch	$-\frac{ac\,x^{2}}{2}-3cx-\frac{4c\ln\left(ax-1\right)}{a}$	2
parallelrisch	$-\frac{a^{2}c\,x^{2}+6acx+8c\ln\left(ax-1\right)}{2a}$	2
norman	$\frac{3cx-\frac{5}{2}ac\,x^{2}-\frac{1}{2}a^{2}c\,x^{3}}{ax-1}-\frac{4c\ln\left(ax-1\right)}{a}$	4
meijerg	$-\frac{c\left(\frac{ax\left(-2a^{2}x^{2}-6ax+12\right)}{-4ax+4}+3\ln\left(-ax+1\right)\right)}{a}+\frac{c\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}+\frac{c\left(\frac{ax}{-ax+1}+\ln\left(-ax+1\right)\right)}{a}+\frac{cx}{-ax+1}$	1

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2*a*x^2-3*x-4/a*ln(a*x-1))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{a^2cx^2+6\,acx+8\,c\log{(ax-1)}}{2\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="fricas")
```

output
```
-1/2*(a^2*c*x^2 + 6*a*c*x + 8*c*log(a*x - 1))/a
```

3.192.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{acx^2}{2}-3cx-\frac{4c\log{(ax-1)}}{a}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c),x)
```

output
```
-a*c*x**2/2 - 3*c*x - 4*c*log(a*x - 1)/a
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-3\,cx-\frac{4\,c\log{(ax-1)}}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/2*a*c*x^2 - 3*c*x - 4*c*log(a*x - 1)/a
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{(ax-1)^2\left(c+\frac{8\,c}{ax-1}\right)}{2\,a}+\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="giac")
```

output
```
-1/2*(a*x - 1)^2*(c + 8*c/(a*x - 1))/a + 4*c*log(abs(a*x - 1)/((a*x - 1)^2
*abs(a)))/a
```

3.192.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\,(8\,\ln(a\,x-1)+6\,a\,x+a^2\,x^2)}{2\,a}$$

input
```
int(((c - a*c*x)*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
-(c*(8*log(a*x - 1) + 6*a*x + a^2*x^2))/(2*a)
```

3.193 $\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx$

3.193.1 Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2}{ac(1-ax)^2} - \frac{4}{ac(1-ax)} - \frac{\log(1-ax)}{ac}$$

output
```
2/a/c/(-a*x+1)^2-4/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

3.193.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{-2+4ax-(-1+ax)^2\log(1-ax)}{ac(-1+ax)^2}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
(-2 + 4*a*x - (-1 + a*x)^2*Log[1 - a*x])/(a*c*(-1 + a*x)^2)
```

3.193.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{tikzcd}
\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}dx \arrow[d, "6717"] \\
\int \frac{e^{4\text{arctanh}(ax)}}{c(1-ax)}dx \arrow[d, "27"] \\
\frac{\int \frac{e^{4\text{arctanh}(ax)}}{1-ax}dx}{c} \arrow[d, "6679"] \\
\frac{\int \frac{(ax+1)^2}{(1-ax)^3}dx}{c} \arrow[d, "49"] \\
\frac{\int \left(-\frac{4}{(ax-1)^2}-\frac{4}{(ax-1)^3}+\frac{1}{1-ax}\right)dx}{c} \arrow[d, "2009"] \\
\frac{-\frac{4}{a(1-ax)}+\frac{2}{a(1-ax)^2}-\frac{\log(1-ax)}{a}}{c}
\end{tikzcd}$$

input

```
Int[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

output

```
(2/(a*(1 - a*x)^2) - 4/(a*(1 - a*x)) - Log[1 - a*x]/a)/c
```

3.193.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.193.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

method	result	size
norman	$\frac{2a\,x^2}{c(ax-1)^2}-\frac{\ln(ax-1)}{ac}$	32
risch	$\frac{4x-\frac{2}{a}}{c(ax-1)^2}-\frac{\ln(ax-1)}{ac}$	36
default	$\frac{\frac{2}{(ax-1)^2a}+\frac{4}{a(ax-1)}-\frac{\ln(ax-1)}{a}}{c}$	41
parallelrisch	$\frac{-a^2\ln(ax-1)x^2+2a^2x^2+2a\ln(ax-1)x-\ln(ax-1)}{(ax-1)^2ca}$	56

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*a/c*x^2/(a*x-1)^2-1/a/c*ln(a*x-1)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{4\,ax-(a^2x^2-2\,ax+1)\log(ax-1)-2}{a^3cx^2-2\,a^2cx+ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="fricas")
```

output
```
(4*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x - 1) - 2)/(a^3*c*x^2 - 2*a^2*c*x +
a*c)
```

3.193.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{-4ax+2}{a^3cx^2-2a^2cx+ac} - \frac{\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c),x)
```

output
```
-(-4*a*x + 2)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(a*x - 1)/(a*c)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2\,(2\,ax-1)}{a^3cx^2-2\,a^2cx+ac} - \frac{\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="maxima")
```

output
```
2*(2*a*x - 1)/(a^3*c*x^2 - 2*a^2*c*x + a*c) - log(a*x - 1)/(a*c)
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac} + \frac{2\left(\frac{2\,ac}{ax-1}+\frac{ac}{(ax-1)^2}\right)}{a^2c^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="giac")
```

output
```
log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) + 2*(2*a*c/(a*x - 1) + a*c/(a
*x - 1)^2)/(a^2*c^2)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{4\,x-\frac{2}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c} - \frac{\ln\left(a\,x-1\right)}{a\,c}$$

input
```
int((a*x + 1)^2/((c - a*c*x)*(a*x - 1)^2),x)
```

output
```
(4*x - 2/a)/(c + a^2*c*x^2 - 2*a*c*x) - log(a*x - 1)/(a*c)
```

3.194 $\int \frac{e^{4 \coth^{-1}(ax)}}{(c-acx)^2} \, dx$

3.194.1 Optimal result

Integrand size = 18, antiderivative size = 25

$$\int \frac{e^{4 \coth^{-1}(ax)}}{(c-acx)^2} \, dx = \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

output
```
1/6*(a*x+1)^3/a/c^2/(-a*x+1)^3
```

3.194.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{4 \coth^{-1}(ax)}}{(c-acx)^2} \, dx = \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
(1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

3.194.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6717, 27, 6679, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^2(1-ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{6679}$$

$$\frac{\int \frac{(ax+1)^2}{(1-ax)^4}dx}{c^2}$$

$$\Big\downarrow \text{48}$$

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
(1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.194.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{-a\,x^{2}-\frac{1}{3a}}{\left(ax-1\right)^{3}c^{2}}$	24
parallelrisch	$\frac{-a^{2}x^{3}-3x}{3\left(ax-1\right)^{3}c^{2}}$	25
gosper	$-\frac{3a^{2}x^{2}+1}{3\left(ax-1\right)^{3}a\,c^{2}}$	26
norman	$\frac{-\frac{x}{c}-\frac{a^{2}x^{3}}{3c}}{\left(ax-1\right)^{3}c}$	30
default	$\frac{-\frac{2}{\left(ax-1\right)^{2}a}-\frac{1}{a\left(ax-1\right)}-\frac{4}{3a\left(ax-1\right)^{3}}}{c^{2}}$	42

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(-a*x^2-1/3/a)/(a*x-1)^3/c^2
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3\,a^2x^2+1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

3.194.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{-3a^2x^2-1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**2,x)
```

output
```
(-3*a**2*x**2 - 1)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x -
3*a*c**2)
```

3.194.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3\,a^2x^2+1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

3.194.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2}{(acx-c)^2a} - \frac{1}{(acx-c)ac} - \frac{4c}{3(acx-c)^3a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-2/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c) - 4/3*c/((a*c*x - c)^3*a)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3\,a^2\,x^2+1}{3\,a\,c^2\,(a\,x-1)^3}$$

input
```
int((a*x + 1)^2/((c - a*c*x)^2*(a*x - 1)^2),x)
```

output
```
-(3*a^2*x^2 + 1)/(3*a*c^2*(a*x - 1)^3)
```

3.195 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

3.195.1 Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{ac^3(1-ax)^4} - \frac{4}{3ac^3(1-ax)^3} + \frac{1}{2ac^3(1-ax)^2}$$

output
```
1/a/c^3/(-a*x+1)^4-4/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

3.195.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1+2ax+3a^2x^2}{6ac^3(-1+ax)^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(1 + 2*a*x + 3*a^2*x^2)/(6*a*c^3*(-1 + a*x)^4)
```

3.195.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^3(1-ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{6679}$$

$$\frac{\int \frac{(ax+1)^2}{(1-ax)^5}dx}{c^3}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(-\frac{1}{(ax-1)^3}-\frac{4}{(ax-1)^4}-\frac{4}{(ax-1)^5}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2a(1-ax)^2}-\frac{4}{3a(1-ax)^3}+\frac{1}{a(1-ax)^4}}{c^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(1/(a*(1 - a*x)^4) - 4/(3*a*(1 - a*x)^3) + 1/(2*a*(1 - a*x)^2))/c^3
```

3.195. $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}dx$

3.195.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.195.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{\frac{a\,x^{2}}{2}+\frac{x}{3}+\frac{1}{6a}}{\left(ax-1\right)^{4}c^{3}}$	27
gosper	$\frac{3a^{2}x^{2}+2ax+1}{6\left(ax-1\right)^{4}a\,c^{3}}$	30
parallelrisch	$\frac{-a^{3}x^{4}+4a^{2}x^{3}-3a\,x^{2}+6x}{6\left(ax-1\right)^{4}c^{3}}$	39
default	$\frac{\frac{1}{a\left(ax-1\right)^{4}}+\frac{1}{2\left(ax-1\right)^{2}a}+\frac{4}{3a\left(ax-1\right)^{3}}}{c^{3}}$	41
norman	$\frac{\frac{x}{c}-\frac{a\,x^{2}}{2c}+\frac{2a^{2}x^{3}}{3c}-\frac{a^{3}x^{4}}{6c}}{\left(ax-1\right)^{4}c^{2}}$	49

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*a*x^2+1/3*x+1/6/a)/(a*x-1)^4/c^3
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2x^2+2\,ax+1}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
 4*a^2*c^3*x + a*c^3)
```

3.195.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{-3a^2x^2-2ax-1}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**3,x)
```

output
```
-(-3*a**2*x**2 - 2*a*x - 1)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**
3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2x^2+2\,ax+1}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
 4*a^2*c^3*x + a*c^3)
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\frac{3}{(ax-1)^2a} + \frac{8}{(ax-1)^3a} + \frac{6}{(ax-1)^4a}}{6\,c^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
1/6*(3/((a*x - 1)^2*a) + 8/((a*x - 1)^3*a) + 6/((a*x - 1)^4*a))/c^3
```

3.195.9 Mupad [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2\,x^2+2\,a\,x+1}{6\,a\,c^3\,{\left(a\,x-1\right)}^4}$$

input
```
int((a*x + 1)^2/((c - a*c*x)^3*(a*x - 1)^2),x)
```

output
```
(2*a*x + 3*a^2*x^2 + 1)/(6*a*c^3*(a*x - 1)^4)
```

3.196 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.196.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx=\frac{4}{5ac^4(1-ax)^5}-\frac{1}{ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}$$

output
```
4/5/a/c^4/(-a*x+1)^5-1/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

3.196.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{2+5ax+5a^2x^2}{15ac^4(-1+ax)^5}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-1/15*(2 + 5*a*x + 5*a^2*x^2)/(a*c^4*(-1 + a*x)^5)
```

3.196.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^4(1-ax)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{\int \frac{(ax+1)^2}{(1-ax)^6}dx}{c^4}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{1}{(ax-1)^4}+\frac{4}{(ax-1)^5}+\frac{4}{(ax-1)^6}\right)dx}{c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{3a(1-ax)^3}-\frac{1}{a(1-ax)^4}+\frac{4}{5a(1-ax)^5}}{c^4}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
(4/(5*a*(1 - a*x)^5) - 1/(a*(1 - a*x)^4) + 1/(3*a*(1 - a*x)^3))/c^4
```

3.196.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.196.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{-\frac{a x^2}{3}-\frac{x}{3}-\frac{2}{15a}}{(ax-1)^5 c^4}$	27
gosper	$-\frac{5a^2x^2+5ax+2}{15(ax-1)^5 a c^4}$	30
default	$\frac{-\frac{1}{a(ax-1)^4}-\frac{1}{3a(ax-1)^3}-\frac{4}{5a(ax-1)^5}}{c^4}$	42
parallelrisch	$\frac{-2a^4x^5+10a^3x^4-20a^2x^3+15a x^2-15x}{15(ax-1)^5 c^4}$	47
norman	$\frac{\frac{a x^2}{c}-\frac{x}{c}-\frac{4a^2x^3}{3c}+\frac{2a^3x^4}{3c}-\frac{2a^4x^5}{15c}}{(ax-1)^5 c^3}$	60

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*a*x^2-1/3*x-2/15/a)/(a*x-1)^5/c^4
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{5\,a^2x^2+5\,ax+2}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^
3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

3.196.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-5a^2x^2-5ax-2}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**4,x)
```

output
```
(-5*a**2*x**2 - 5*a*x - 2)/(15*a**6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a*
*4*c**4*x**3 - 150*a**3*c**4*x**2 + 75*a**2*c**4*x - 15*a*c**4)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{5\,a^2x^2+5\,ax+2}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
-1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^
3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{5}{(ax-1)^3a}+\frac{15}{(ax-1)^4a}+\frac{12}{(ax-1)^5a}}{15\,c^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-1/15*(5/((a*x - 1)^3*a) + 15/((a*x - 1)^4*a) + 12/((a*x - 1)^5*a))/c^4
```

3.196.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{5\,a^2\,x^2+5\,a\,x+2}{15\,a\,c^4\,\left(a\,x-1\right)^5}$$

input
```
int((a*x + 1)^2/((c - a*c*x)^4*(a*x - 1)^2),x)
```

output
```
-(5*a*x + 5*a^2*x^2 + 2)/(15*a*c^4*(a*x - 1)^5)
```

3.197 $\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$

3.197.1 Optimal result

Integrand size = 18, antiderivative size = 94

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,-\frac{1}{2}-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

output
```
((a-1/x)/(a+1/x))^(-1/2-p)*x*(-a*c*x+c)^p*hypergeom([-1-p, -1/2-p],[-p],2/
(a+1/x)/x)*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/(p+1)
```

3.197.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,-\frac{1}{2}-p,-p,\frac{2}{1+ax}\right)}{1+p}$$

input
```
Integrate[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(c - a*c*x)^p*H
ypergeometric2F1[-1 - p, -1/2 - p, -p, 2/(1 + a*x)])/(1 + p)
```

3.197.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \; 6727$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int\frac{\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{x}\right)^{-p-2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 142$$

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}(c-acx)^p\,\text{Hypergeometric2F1}\left(-p-1,-p-\frac{1}{2},-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

input
```
Int[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)
]*x*(c - a*c*x)^p*Hypergeometric2F1[-1 - p, -1/2 - p, -p, 2/((a + x^(-1))*
x)])/(1 + p)
```

3.197.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.197.4 Maple [F]

$$\int (-acx+c)^p \sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

3.197.5 Fricas [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \int (-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
integral((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.197.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \int \sqrt{\frac{ax-1}{ax+1}}(-c(ax-1))^p\,dx$$

input
```
integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**p, x)
```

3.197.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.197.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int(c-a\,c\,x)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.198 $\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$

3.198.1 Optimal result

Integrand size = 18, antiderivative size = 127

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x-\frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4-\frac{35c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
-35/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+20/3*c^3*x*(1-1/a^2/x^2)^(1/2)-27
/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)+4/3*a^2*c^3*x^3*(1-1/a^2/x^2)^(1/2)-1/4*a
^3*c^3*x^4*(1-1/a^2/x^2)^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx =\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(160-81ax+32a^2x^2-6a^3x^3)-105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{24a}$$

input
```
Integrate[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

output
```
(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(160 - 81*a*x + 32*a^2*x^2 - 6*a^3*x^3) -
105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

3.198.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6724, 27, 540, 2338, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{c^4\left(a-\frac{1}{x}\right)^4 x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\int \frac{\left(a-\frac{1}{x}\right)^4 x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{540}$$

$$\frac{c^3\left(-\frac{1}{4}\int \frac{\left(16a^3-\frac{27a^2}{x}+\frac{16a}{x^2}-\frac{4}{x^3}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{2338}$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\int \frac{\left(81a^2-\frac{80a}{x}+\frac{12}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{2338}$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{5\left(32a-\frac{21}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{5}{2}\int \frac{\left(32a-\frac{21}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{534}$$

$$\frac{c^3\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(-\frac{5}{2}\Bigg(-21\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-32ax\sqrt{1-\frac{1}{a^2x^2}}\Bigg)-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\Bigg)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\Bigg)-\frac{1}{4}a^4x^4\sqrt{1-}}{a}$$

$$\Big\downarrow 243$$

$$\frac{c^3\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(-\frac{5}{2}\Bigg(-\frac{21}{2}\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-32ax\sqrt{1-\frac{1}{a^2x^2}}\Bigg)-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\Bigg)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\Bigg)-\frac{1}{4}a^4x^4\sqrt{1-}}{a}$$

$$\Big\downarrow 73$$

$$\frac{c^3\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(-\frac{5}{2}\Bigg(21a^2\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-32ax\sqrt{1-\frac{1}{a^2x^2}}\Bigg)-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\Bigg)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\Bigg)}{a}$$

$$\Big\downarrow 221$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{5}{2}\left(21\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-32ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1}\right.\right.}{a}$$

input
```
Int[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

output
```
(c^3*(-1/4*(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4) + ((16*a^3*Sqrt[1 - 1/(a^2*x^2)
]*x^3)/3 + ((-81*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (5*(-32*a*Sqrt[1 - 1/(
a^2*x^2)]*x + 21*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/2)/3)/4))/a
```

3.198.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.198. $\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$

3.198.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(6a^3x^3-32a^2x^2+81ax-160)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{24a}-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\Bigg(6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+87\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-32((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-87\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-192\sqrt{a^2}}{24a\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

input

```
int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/24*(6*a^3*x^3-32*a^2*x^2+81*a*x-160)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1
/2)-35/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x-1)*(
(a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx =$$

$$-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^3x^4-26\,a^3c^3x^3+49\,a^2c^3x^2-79\,ac^3x-16}{24\,a}$$

input

```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output

```
-1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (6*a^4*c^3*x^4 - 26*a^3*c^3*x^3 + 49*a^2*c^3*x^2 -
79*a*c^3*x - 160*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.198.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\left(\int 3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

input
```
integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
-c**3*(Integral(3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*
a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x +
 1)), x))
```

3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(107) = 214$.

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{24}\Bigg(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} - \frac{2\left(279\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} - 511\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 385\,c^3\right.}{\frac{4\,(ax-1)a^2}{ax+1} - \frac{6\,(ax-1)^2a^2}{(ax+1)^2} + \frac{4\,(ax-1)^3a^2}{(ax+1)^3}}$$

input
```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(279*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 51
1*c^3*((a*x - 1)/(a*x + 1))^(5/2) + 385*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
105*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1
)^2*a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
 + 1)^4 - a^2))*a
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{35\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{8\,|a|}$$
$$+\frac{1}{24}\sqrt{a^2x^2-1}\Bigg(\frac{160\,c^3\operatorname{sgn}(ax+1)}{a}-\big(81\,c^3\operatorname{sgn}(ax+1)+2\,\big(3\,a^2c^3x\operatorname{sgn}(ax+1)-16\,ac^3\operatorname{sgn}(ax+1)\big)x$$

input
```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
35/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/2
4*sqrt(a^2*x^2 - 1)*(160*c^3*sgn(a*x + 1)/a - (81*c^3*sgn(a*x + 1) + 2*(3*
a^2*c^3*x*sgn(a*x + 1) - 16*a*c^3*sgn(a*x + 1))*x)*x)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{\frac{35\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{385\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{511\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}$$
$$-\frac{35\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((35*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (385*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/12 + (511*c^3*((a*x - 1)/(a*x + 1))^(5/2))/12 - (93*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*
x + 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) -
(35*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.199 $\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx$

3.199.1 Optimal result

Integrand size = 18, antiderivative size = 100

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{5c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output

```
-5/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+11/3*c^2*x*(1-1/a^2/x^2)^(1/2)-3/2
*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)+1/3*a^2*c^2*x^3*(1-1/a^2/x^2)^(1/2)
```

3.199.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(22-9ax+2a^2x^2)-15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input

```
Integrate[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

output

```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 - 9*a*x + 2*a^2*x^2) - 15*Log[a*(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

3.199.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 25, 27, 540, 2338, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int -\frac{c^3\left(a-\frac{1}{x}\right)^3x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{c^3\left(a-\frac{1}{x}\right)^3x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{\left(a-\frac{1}{x}\right)^3x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^2\left(-\frac{1}{3}\int \frac{\left(9a^2-\frac{11a}{x}+\frac{3}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(22a-\frac{15}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{534}$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\left(-15\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-22ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{15}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-22ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\left(15a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-22ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\left(15\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-22ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

input
```
Int[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

output
```
-((c^2*(-1/3*(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3) + ((9*a^2*Sqrt[1 - 1/(a^2*x^2
)]*x^2)/2 + (-22*a*Sqrt[1 - 1/(a^2*x^2)]*x + 15*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/2)/3))/a)
```

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.199. $\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx$

3.199.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(2a^2x^2-9ax+22)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{6a}-\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-24\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+24a\ln\left(\right.\right.}{6\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$

input
```
int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-9*a*x+22)*(a*x+1)/a*c^2*((a*x-1)/(a*x+1))^(1/2)-5/2*ln(a^2*
x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2*((a*x-1)/(a*x+1))^(1/2)/(
a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx =$$
$$-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^2x^3-7\,a^2c^2x^2+13\,ac^2x+22\,c^2)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 7*a^2*c^2*x^2 + 13*a*c^2*x + 22*c^2)*
sqrt((a*x - 1)/(a*x + 1)))/a
```

3.199.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

input
```
integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**2*(Integral(-2*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**
2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1)), x))
```

3.199.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(84) = 168$.

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.81

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{1}{6}a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} + \frac{2\left(33\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 40\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 15\,c^2\sqrt{\frac{ax-}{ax+}}\right.}{\frac{3\,(ax-1)a^2}{ax+1} - \frac{3\,(ax-1)^2a^2}{(ax+1)^2} + \frac{(ax-1)^3a^2}{(ax+1)^3} - a^2}\right.$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 + 2*(33*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 40*c
^2*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x
 - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x
 + 1)^3 - a^2))
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx &= \frac{5\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}\\ &\quad+\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(2\,ac^2x\operatorname{sgn}(ax+1)-9\,c^2\operatorname{sgn}(ax+1)\right)x+\frac{22\,c^2\operatorname{sgn}(ax+1)}{a}\right)\end{aligned}$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
5/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/6*
sqrt(a^2*x^2 - 1)*((2*a*c^2*x*sgn(a*x + 1) - 9*c^2*sgn(a*x + 1))*x + 22*c^
2*sgn(a*x + 1)/a)
```

3.199.9 Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.40

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{5\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+11\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}} - \frac{5\,c^2\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (40*c^2*((a*x - 1)/(a*x + 1))^(3/2))/
3 + 11*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (
3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c^2*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.200 $\int e^{-\coth^{-1}(ax)}(c-acx)\,dx$

3.200.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=2c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
-3/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+2*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^
2*(1-1/a^2/x^2)^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-4+ax)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

input
```
Integrate[(c - a*c*x)/E^ArcCoth[a*x],x]
```

output
```
-1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a
^2*x^2)])*x]))/a
```

3.200.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6724, 27, 540, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6724$$

$$\frac{\int \frac{c^2\left(a-\frac{1}{x}\right)^2x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{\left(a-\frac{1}{x}\right)^2x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \; 540$$

$$\frac{c\left(-\frac{1}{2}\int \frac{\left(4a-\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \; 534$$

$$\frac{c\left(\frac{1}{2}\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \; 243$$

$$\frac{c\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \; 73$$

$$\frac{c\left(\frac{1}{2}\left(4ax\sqrt{1-\frac{1}{a^2x^2}}-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$\frac{c\left(\frac{1}{2}\left(4ax\sqrt{1-\frac{1}{a^2x^2}}-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

input
```
Int[(c - a*c*x)/E^ArcCoth[a*x],x]
```

output
```
(c*(-1/2*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[1 - 1/(a^2*x^2)]*x -
3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/2))/a
```

3.200.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6724

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.200.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{(ax-4)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{2a}-\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^{2}}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax-\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a-4\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}+4a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)\right)}{2\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^{2}}}$

input

```
int((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(a*x-4)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)-3/2*ln(a^2*x/(a^2)^(1/2)+
(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))
^(1/2)/(a*x-1)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx$$
$$=-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^{2}cx^{2}-3\,acx-4\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) + (a^2*c*x^2 - 3*a*c*x - 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.200.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=-c\left(\int ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
-c*(Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.08

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=\frac{1}{2}a\left(\frac{2\left(5c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
1/2*a*(2*(5*c*((a*x - 1)/(a*x + 1))^(3/2) - 3*c*sqrt((a*x - 1)/(a*x + 1)))
/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 3*c*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) -
1)/a^2)
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = \frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|} - \frac{1}{2}\,\sqrt{a^2x^2-1}\left(cx\operatorname{sgn}(ax+1)-\frac{4\,c\operatorname{sgn}(ax+1)}{a}\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
3/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/2*sq
rt(a^2*x^2 - 1)*(c*x*sgn(a*x + 1) - 4*c*sgn(a*x + 1)/a)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = \frac{3\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-5\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}} - \frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(3*c*((a*x - 1)/(a*x + 1))^(1/2) - 5*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (
2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a*x
 - 1)/(a*x + 1))^(1/2)))/a
```

3.201 $\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx$

3.201.1 Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
-arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

3.201.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log\left(ax\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)}{ac}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
```

output
```
-(Log[a*x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]/(a*c))
```

3.201.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6724, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} \, dx$$

$$\Big\downarrow \text{ 6724}$$

$$\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2ac}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
```

output
```
-(ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c))
```

3.201.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.201.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(21) = 42$.

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)}{\sqrt{(ax-1)(ax+1)}\,c\sqrt{a^{2}}}$	76

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2))/(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c/(a^2)^(1/2)
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-(log(sqrt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))
/(a*c)
```

3.201.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

output
```
-Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c))
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx=\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

output
```
log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a))
```

3.201.9 Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x),x)
```

output
```
-(2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.202 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

3.202.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

output
```
-(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)
```

3.202.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2(-1+ax)}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

output
```
-((Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(-1 + a*x)))
```

3.202.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6724, 25, 27, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int -\frac{1}{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{ac^2}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

output
```
-(Sqrt[1 - 1/(a^2*x^2)]/(c^2*(a - x^(-1))))
```

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.202.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)a\,c^{2}}$	36
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)a\,c^{2}}$	36
trager	$-\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^{2}(ax-1)}$	38

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/(a*x-1)/a/c^2
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

3.202.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

output
```
Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1), x)/c**
2
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/(a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

3.202.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(acx-c)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
undef
```

3.202.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{1}{a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^2,x)
```

output
```
-1/(a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

3.203 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

3.203.1 Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

output
```
1/3*a*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)^2-2/3*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x
)
```

3.203.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-2+ax)}{3c^3(-1+ax)^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

output
```
-1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(c^3*(-1 + a*x)^2)
```

3.203.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6724, 27, 571, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx$$

$$\Big\downarrow \; 6724$$

$$\frac{\int \frac{1}{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x}d\frac{1}{x}}{ac^3}$$

$$\Big\downarrow \; 571$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}-\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{ac^3}$$

$$\Big\downarrow \; 460$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)}}{ac^3}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

output
```
((a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*(a - x^(-1))^2) - (2*a*Sqrt[1 - 1/(a^2*x^2
)])/(3*(a - x^(-1))))/(a*c^3)
```

3.203.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.203.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax-2)(ax+1)}{3(ax-1)^{2}c^{3}a}$	41
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax-2)(ax+1)}{3(ax-1)^{2}c^{3}a}$	41
trager	$-\frac{(ax-2)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^{3}(ax-1)^{2}}$	43

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((a*x-1)/(a*x+1))^(1/2)*(a*x-2)*(a*x+1)/(a*x-1)^2/c^3/a
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\left(a^2x^2-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*(a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^3*x^2 - 2*a^2*c^
3*x + a*c^3)
```

3.203.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-3a^2x^2+3ax-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

output
```
-Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x**2 + 3*a
*x - 1), x)/c**3
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{3\,(ax-1)}{ax+1}-1}{6\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/6*(3*(a*x - 1)/(a*x + 1) - 1)/(a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
2/3*(3*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^3*a
*c^3)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{a\,x+1}-\frac{1}{3}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^3,x)
```

output
```
-((a*x - 1)/(a*x + 1) - 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

3.204 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.204.1 Optimal result

Integrand size = 18, antiderivative size = 95

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

output
```
-1/5*a^2*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)^3+8/15*a*(1-1/a^2/x^2)^(1/2)/c^4/
(a-1/x)^2-7/15*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)
```

3.204.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(7-6ax+2a^2x^2)}{15c^4(-1+ax)^3}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

output
```
-1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^4*(-1 + a*x)^3)
```

3.204.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 25, 27, 581, 25, 27, 671, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int -\frac{1}{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2}d\frac{1}{x}}{ac^4}$$

$$\Big\downarrow \text{581}$$

$$-\frac{-\int -\frac{a\left(2a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a\left(2a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{2a-\frac{1}{x}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{671}$$

$$-\frac{a\left(\frac{7}{5}\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{461}$$

$$-\frac{a\left(\frac{7}{5}\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{3a}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{460}$$

$$-\frac{a\left(\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}+\frac{7}{5}\left(\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

output
```
-((a*((7*((a*Sqrt[1 - 1/(a^2*x^2)])/(3*(a - x^(-1))^2) + Sqrt[1 - 1/(a^2*x
^2)]/(3*(a - x^(-1)))))/5 + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(5*(a - x^(-1))^3)
) - (a^2*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1))^2)/(a*c^4))
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.204.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2-6ax+7)(ax+1)}{15(ax-1)^3c^4a}$	50
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2-6ax+7)(ax+1)}{15(ax-1)^3c^4a}$	50
trager	$-\frac{(2a^2x^2-6ax+7)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^4(ax-1)^3}$	52

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/15*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2-6*a*x+7)*(a*x+1)/(a*x-1)^3/c^4/a
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\left(2\,a^3x^3-4\,a^2x^2+ax+7\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^4
*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

3.204.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

output
```
Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x**3 + 6*a*
*2*x**2 - 4*a*x + 1), x)/c**4
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-3}{60\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^4*((a*
x - 1)/(a*x + 1))^(5/2))
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^4}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)
/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^4)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{3\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^4,x)
```

output
```
-((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^4*((
a*x - 1)/(a*x + 1))^(5/2))
```

3.205 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5} \, dx$

3.205.1 Optimal result

Integrand size = 18, antiderivative size = 128

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5} \, dx = \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

output
```
1/7*a^3*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^4-18/35*a^2*(1-1/a^2/x^2)^(1/2)/c^
5/(a-1/x)^3+23/35*a*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^2-12/35*(1-1/a^2/x^2)^
(1/2)/c^5/(a-1/x)
```

3.205.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5} \, dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-12+13ax-8a^2x^2+2a^3x^3)}{35c^5(-1+ax)^4}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

output
```
-1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-12 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^
5*(-1 + a*x)^4)
```

3.205.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 27, 581, 25, 27, 671, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$

$$\Big\downarrow \text{ 6724}$$

$$\frac{\int \frac{1}{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4x^3}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4x^3}d\frac{1}{x}}{ac^5}$$

$$\Big\downarrow \text{ 581}$$

$$\frac{\int -\frac{a^2\left(2a-\frac{3}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^5}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-\int \frac{a^2\left(2a-\frac{3}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{ac^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\int \frac{2a-\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{ac^5}$$

$$\Big\downarrow \text{ 671}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\left(\frac{18}{7}\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{7\left(a-\frac{1}{x}\right)^4}\right)}{ac^5}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\left(\frac{18}{7}\left(\frac{2\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{5a}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{7\left(a-\frac{1}{x}\right)^4}\right)}{ac^5}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\left(\frac{18}{7}\left(\frac{2\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{3a}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}\right)}{5a}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{7\left(a-\frac{1}{x}\right)^4}\right)}{ac^5}$$

$$\Big\downarrow \text{460}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\left(\frac{18}{7}\left(\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}+\frac{2\left(\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)}\right)}{5a}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{7\left(a-\frac{1}{x}\right)^4}\right)}{ac^5}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

output
```
(-(a^2*((18*((2*((a*Sqrt[1 - 1/(a^2*x^2)])/(3*(a - x^(-1))^2) + Sqrt[1 - 1
/(a^2*x^2)]/(3*(a - x^(-1)))))/(5*a) + (a*Sqrt[1 - 1/(a^2*x^2)])/(5*(a - x
^(-1))^3)))/7 - (a^2*Sqrt[1 - 1/(a^2*x^2)])/(7*(a - x^(-1))^4))) + (a^2*Sq
rt[1 - 1/(a^2*x^2)])/(a - x^(-1))^2)/(a*c^5)
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.205. $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

3.205.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^{3}x^{3}-8a^{2}x^{2}+13ax-12)\,(ax+1)}{35(ax-1)^{4}c^{5}a}$	58
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^{3}x^{3}-8a^{2}x^{2}+13ax-12)\,(ax+1)}{35(ax-1)^{4}c^{5}a}$	58
trager	$-\frac{(2a^{3}x^{3}-8a^{2}x^{2}+13ax-12)\,(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^{5}(ax-1)^{4}}$	60

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/35*((a*x-1)/(a*x+1))^(1/2)*(2*a^3*x^3-8*a^2*x^2+13*a*x-12)*(a*x+1)/(a*x
-1)^4/c^5/a
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5}}\,dx=-\frac{(2\,a^{4}x^{4}-6\,a^{3}x^{3}+5\,a^{2}x^{2}+ax-12)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^{5}c^{5}x^{4}-4\,a^{4}c^{5}x^{3}+6\,a^{3}c^{5}x^{2}-4\,a^{2}c^{5}x+ac^{5})}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/35*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + a*x - 12)*sqrt((a*x - 1)/(a*x +
 1))/(a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 6*a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5)
```

3.205.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5}}\,dx=-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{5}x^{5}-5a^{4}x^{4}+10a^{3}x^{3}-10a^{2}x^{2}+5ax-1}\,dx}{c^{5}}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

output
```
-Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 5*a**4*x**4 + 10*
a**3*x**3 - 10*a**2*x**2 + 5*a*x - 1), x)/c**5
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{21\,(ax-1)}{ax+1} - \frac{35\,(ax-1)^2}{(ax+1)^2} + \frac{35\,(ax-1)^3}{(ax+1)^3} - 5}{280\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/280*(21*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 + 35*(a*x - 1)
^3/(a*x + 1)^3 - 5)/(a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{4\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3 - 21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2 + 7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x - 1\right)}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^5}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
4/35*(35*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 21*(a + sqrt(a^2 - 1/x^2))^2*x^2
+ 7*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^
5)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{8\,a\,c^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^5,x)
```

output
```
((a*x - 1)^2/(a*x + 1)^2 - (a*x - 1)^3/(a*x + 1)^3 - (3*(a*x - 1))/(5*(a*x
 + 1)) + 1/7)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

3.206 $\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$

3.206.1 Optimal result

Integrand size = 18, antiderivative size = 44

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^{2+p}\,\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}$$

output
```
1/2*(-a*c*x+c)^(2+p)*hypergeom([1, 2+p],[3+p],-1/2*a*x+1/2)/a/c^2/(2+p)
```

3.206.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$$
$$=-\frac{(-1+ax)(c-acx)^p\left(-1+\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{1}{2}(1-ax)\right)\right)}{a(1+p)}$$

input
```
Integrate[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((-1 + a*x)*(c - a*c*x)^p*(-1 + Hypergeometric2F1[1, 1 + p, 2 + p, (1 -
a*x)/2]))/(a*(1 + p)))
```

3.206.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = $\{6717, 6680, 35, 78\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(c-acx)^pdx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)(c-acx)^p}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{p+1}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{78}$$

$$\frac{(c-acx)^{p+2}\,\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

input
```
Int[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

output
```
((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a
*c^2*(2 + p))
```

3.206.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.206.4 Maple [F]

$$\int \frac{(-acx+c)^p\,(ax-1)}{ax+1}dx$$

input
```
int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
```

output
```
int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
```

3.206.5 Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

input
```
integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

3.206.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-c(ax-1))^p\,(ax-1)}{ax+1}\,dx$$

input
```
integrate((-a*c*x+c)**p*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(a*x - 1))**p*(a*x - 1)/(a*x + 1), x)
```

3.206.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

input
```
integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

3.206.8 Giac [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

input
```
integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(c-a\,c\,x)^p\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1), x)
```

3.207 $\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$

3.207.1 Optimal result

Integrand size = 18, antiderivative size = 91

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=16c^4x-\frac{4c^4(1-ax)^2}{a}-\frac{4c^4(1-ax)^3}{3a}-\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}-\frac{32c^4\log(1+ax)}{a}$$

output
```
16*c^4*x-4*c^4*(-a*x+1)^2/a-4/3*c^4*(-a*x+1)^3/a-1/2*c^4*(-a*x+1)^4/a-1/5*
c^4*(-a*x+1)^5/a-32*c^4*ln(a*x+1)/a
```

3.207.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4(-181+930ax-390a^2x^2+160a^3x^3-45a^4x^4+6a^5x^5-960\log(1+ax))}{30a}$$

input
```
Integrate[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^4*(-181 + 930*a*x - 390*a^2*x^2 + 160*a^3*x^3 - 45*a^4*x^4 + 6*a^5*x^5
- 960*Log[1 + a*x]))/(30*a)
```

3.207.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^4e^{-2\text{arctanh}(ax)}(1-ax)^4dx$$

$$\Big\downarrow \text{27}$$

$$-c^4\int e^{-2\text{arctanh}(ax)}(1-ax)^4dx$$

$$\Big\downarrow \text{6679}$$

$$-c^4\int \frac{(1-ax)^5}{ax+1}dx$$

$$\Big\downarrow \text{49}$$

$$-c^4\int \left(-(1-ax)^4-2(1-ax)^3-4(1-ax)^2-8(1-ax)+\frac{32}{ax+1}-16\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^4\left(\frac{(1-ax)^5}{5a}+\frac{(1-ax)^4}{2a}+\frac{4(1-ax)^3}{3a}+\frac{4(1-ax)^2}{a}+\frac{32\log(ax+1)}{a}-16x\right)$$

input
```
Int[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^4*(-16*x + (4*(1 - a*x)^2)/a + (4*(1 - a*x)^3)/(3*a) + (1 - a*x)^4/(2*
a) + (1 - a*x)^5/(5*a) + (32*Log[1 + a*x])/a))
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.207.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

method	result
default	$c^4\left(\frac{a^4x^5}{5}-\frac{3a^3x^4}{2}+\frac{16a^2x^3}{3}-13a\,x^2+31x-\frac{32\ln(ax+1)}{a}\right)$
norman	$31c^4x-13a\,c^4x^2+\frac{16a^2c^4x^3}{3}-\frac{3a^3c^4x^4}{2}+\frac{a^4c^4x^5}{5}-\frac{32c^4\ln(ax+1)}{a}$
risch	$31c^4x-13a\,c^4x^2+\frac{16a^2c^4x^3}{3}-\frac{3a^3c^4x^4}{2}+\frac{a^4c^4x^5}{5}-\frac{32c^4\ln(ax+1)}{a}$
parallelrisch	$-\frac{-6a^5c^4x^5+45a^4c^4x^4-160a^3c^4x^3+390a^2c^4x^2-930a\,c^4x+960c^4\ln(ax+1)}{30a}$
meijerg	$\frac{c^4\left(\frac{ax\left(12a^4x^4-15a^3x^3+20a^2x^2-30ax+60\right)}{60}-\ln(ax+1)\right)}{a}-\frac{5c^4\left(-\frac{ax\left(-15a^3x^3+20a^2x^2-30ax+60\right)}{60}+\ln(ax+1)\right)}{a}+$ $10c^4$

input
```
int((-a*c*x+c)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/5*a^4*x^5-3/2*a^3*x^4+16/3*a^2*x^3-13*a*x^2+31*x-32*ln(a*x+1)/a)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{6\,a^5c^4x^5-45\,a^4c^4x^4+160\,a^3c^4x^3-390\,a^2c^4x^2+930\,ac^4x-960\,c^4\log\left(ax+1\right)}{30\,a}$$

input
```
integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/30*(6*a^5*c^4*x^5 - 45*a^4*c^4*x^4 + 160*a^3*c^4*x^3 - 390*a^2*c^4*x^2 +
 930*a*c^4*x - 960*c^4*log(a*x + 1))/a
```

3.207.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{a^4c^4x^5}{5}-\frac{3a^3c^4x^4}{2}+\frac{16a^2c^4x^3}{3} - 13ac^4x^2+31c^4x-\frac{32c^4\log\left(ax+1\right)}{a}$$

input
```
integrate((-a*c*x+c)**4*(a*x-1)/(a*x+1),x)
```

output
```
a**4*c**4*x**5/5 - 3*a**3*c**4*x**4/2 + 16*a**2*c**4*x**3/3 - 13*a*c**4*x*
*2 + 31*c**4*x - 32*c**4*log(a*x + 1)/a
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{3}{2}\,a^3c^4x^4+\frac{16}{3}\,a^2c^4x^3-13\,ac^4x^2+31\,c^4x-\frac{32\,c^4\log\left(ax+1\right)}{a}$$

input
```
integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/5*a^4*c^4*x^5 - 3/2*a^3*c^4*x^4 + 16/3*a^2*c^4*x^3 - 13*a*c^4*x^2 + 31*c
^4*x - 32*c^4*log(a*x + 1)/a
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=-\frac{32\,c^4\log\left(|ax+1|\right)}{a}+\frac{6\,a^9c^4x^5-45\,a^8c^4x^4+160\,a^7c^4x^3-390\,a^6c^4x^2+930\,a^5c^4x}{30\,a^5}$$

input
```
integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-32*c^4*log(abs(a*x + 1))/a + 1/30*(6*a^9*c^4*x^5 - 45*a^8*c^4*x^4 + 160*a
^7*c^4*x^3 - 390*a^6*c^4*x^2 + 930*a^5*c^4*x)/a^5
```

3.207.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=31\,c^4\,x-13\,a\,c^4\,x^2+\frac{16\,a^2\,c^4\,x^3}{3}-\frac{3\,a^3\,c^4\,x^4}{2}+\frac{a^4\,c^4\,x^5}{5}-\frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - a*c*x)^4*(a*x - 1))/(a*x + 1),x)
```

output
```
31*c^4*x - 13*a*c^4*x^2 + (16*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + (a^4*c^
4*x^5)/5 - (32*c^4*log(a*x + 1))/a
```

3.207. $\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$

3.208 $\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx$

3.208.1 Optimal result

Integrand size = 18, antiderivative size = 73

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx=8c^3x-\frac{2c^3(1-ax)^2}{a}-\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}-\frac{16c^3\log(1+ax)}{a}$$

output
```
8*c^3*x-2*c^3*(-a*x+1)^2/a-2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a-16*c^
3*ln(a*x+1)/a
```

3.208.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{c^3(35-180ax+66a^2x^2-20a^3x^3+3a^4x^4+192\log(1+ax))}{12a}$$

input
```
Integrate[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-1/12*(c^3*(35 - 180*a*x + 66*a^2*x^2 - 20*a^3*x^3 + 3*a^4*x^4 + 192*Log[1
 + a*x]))/a
```

3.208.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^3 e^{-2\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{-2\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{6679}$$

$$-c^3\int \frac{(1-ax)^4}{ax+1}dx$$

$$\Big\downarrow \text{49}$$

$$-c^3\int \left(-(1-ax)^3-2(1-ax)^2-4(1-ax)+\frac{16}{ax+1}-8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^3\left(\frac{(1-ax)^4}{4a}+\frac{2(1-ax)^3}{3a}+\frac{2(1-ax)^2}{a}+\frac{16\log(ax+1)}{a}-8x\right)$$

input
```
Int[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^3*(-8*x + (2*(1 - a*x)^2)/a + (2*(1 - a*x)^3)/(3*a) + (1 - a*x)^4/(4*a
) + (16*Log[1 + a*x])/a))
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.208.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

method	result
default	$c^{3} \left(-\frac{a^{3} x^{4}}{4}+\frac{5 a^{2} x^{3}}{3}-\frac{11 a \,x^{2}}{2}+15 x -\frac{16 \ln \left(a x +1\right)}{a}\right)$
norman	$15 c^{3} x -\frac{11 a \,c^{3} x^{2}}{2}+\frac{5 a^{2} c^{3} x^{3}}{3}-\frac{a^{3} c^{3} x^{4}}{4}-\frac{16 c^{3} \ln \left(a x +1\right)}{a}$
risch	$15 c^{3} x -\frac{11 a \,c^{3} x^{2}}{2}+\frac{5 a^{2} c^{3} x^{3}}{3}-\frac{a^{3} c^{3} x^{4}}{4}-\frac{16 c^{3} \ln \left(a x +1\right)}{a}$
parallelrisch	$-\frac{3 a^{4} c^{3} x^{4}-20 a^{3} c^{3} x^{3}+66 a^{2} c^{3} x^{2}-180 a \,c^{3} x +192 c^{3} \ln \left(a x +1\right)}{12 a}$
meijerg	$-\frac{c^{3} \left(-\frac{a x \left(-15 a^{3} x^{3}+20 a^{2} x^{2}-30 a x +60\right)}{60}+\ln \left(a x +1\right)\right)}{a}+\frac{4 c^{3} \left(\frac{a x \left(4 a^{2} x^{2}-6 a x +12\right)}{12}-\ln \left(a x +1\right)\right)}{a}-\frac{6 c^{3} \left(-\frac{a x \left(-3 a x +6\right)}{6}+\ln (ax}{a}$

input
```
int((-a*c*x+c)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/4*a^3*x^4+5/3*a^2*x^3-11/2*a*x^2+15*x-16*ln(a*x+1)/a)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{3\,a^4c^3x^4-20\,a^3c^3x^3+66\,a^2c^3x^2-180\,ac^3x+192\,c^3\log(ax+1)}{12\,a}$$

input
```
integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
-1/12*(3*a^4*c^3*x^4 - 20*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 180*a*c^3*x + 192
*c^3*log(a*x + 1))/a
```

3.208.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{a^3c^3x^4}{4}+\frac{5a^2c^3x^3}{3}-\frac{11ac^3x^2}{2}+15c^3x-\frac{16c^3\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)**3*(a*x-1)/(a*x+1),x)
```

output
```
-a**3*c**3*x**4/4 + 5*a**2*c**3*x**3/3 - 11*a*c**3*x**2/2 + 15*c**3*x - 16
*c**3*log(a*x + 1)/a
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}\,a^3c^3x^4+\frac{5}{3}\,a^2c^3x^3-\frac{11}{2}\,ac^3x^2+15\,c^3x-\frac{16\,c^3\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/4*a^3*c^3*x^4 + 5/3*a^2*c^3*x^3 - 11/2*a*c^3*x^2 + 15*c^3*x - 16*c^3*lo
g(a*x + 1)/a
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{16\,c^3\log\left(|ax+1|\right)}{a} - \frac{3\,a^7c^3x^4 - 20\,a^6c^3x^3 + 66\,a^5c^3x^2 - 180\,a^4c^3x}{12\,a^4}$$

input
```
integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-16*c^3*log(abs(a*x + 1))/a - 1/12*(3*a^7*c^3*x^4 - 20*a^6*c^3*x^3 + 66*a^
5*c^3*x^2 - 180*a^4*c^3*x)/a^4
```

3.208.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = 15\,c^3\,x - \frac{11\,a\,c^3\,x^2}{2} + \frac{5\,a^2\,c^3\,x^3}{3} - \frac{a^3\,c^3\,x^4}{4} - \frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - a*c*x)^3*(a*x - 1))/(a*x + 1),x)
```

output
```
15*c^3*x - (11*a*c^3*x^2)/2 + (5*a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4 - (16*c^
3*log(a*x + 1))/a
```

3.209 $\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx$

3.209.1 Optimal result

Integrand size = 18, antiderivative size = 55

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=4c^2x-\frac{c^2(1-ax)^2}{a}-\frac{c^2(1-ax)^3}{3a}-\frac{8c^2\log(1+ax)}{a}$$

output
```
4*c^2*x-c^2*(-a*x+1)^2/a-1/3*c^2*(-a*x+1)^3/a-8*c^2*ln(a*x+1)/a
```

3.209.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2(-4+21ax-6a^2x^2+a^3x^3-24\log(1+ax))}{3a}$$

input
```
Integrate[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*(-4 + 21*a*x - 6*a^2*x^2 + a^3*x^3 - 24*Log[1 + a*x]))/(3*a)
```

3.209.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^2 e^{-2\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{27}$$

$$-c^2\int e^{-2\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{6679}$$

$$-c^2\int \frac{(1-ax)^3}{ax+1}dx$$

$$\Big\downarrow \text{49}$$

$$-c^2\int \left(-(1-ax)^2-2(1-ax)+\frac{8}{ax+1}-4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^2\left(\frac{(1-ax)^3}{3a}+\frac{(1-ax)^2}{a}+\frac{8\log(ax+1)}{a}-4x\right)$$

input
```
Int[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^2*(-4*x + (1 - a*x)^2/a + (1 - a*x)^3/(3*a) + (8*Log[1 + a*x])/a))
```

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.209.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

method	result	size
default	$c^{2} \left(\frac{a^{2} x^{3}}{3}-2 a \,x^{2}+7 x -\frac{8 \ln \left(a x +1\right)}{a}\right)$	34
norman	$7 c^{2} x -2 a \,c^{2} x^{2}+\frac{a^{2} c^{2} x^{3}}{3}-\frac{8 c^{2} \ln \left(a x +1\right)}{a}$	42
risch	$7 c^{2} x -2 a \,c^{2} x^{2}+\frac{a^{2} c^{2} x^{3}}{3}-\frac{8 c^{2} \ln \left(a x +1\right)}{a}$	42
parallelrisch	$-\frac{-a^{3} c^{2} x^{3}+6 a^{2} c^{2} x^{2}-21 a \,c^{2} x +24 c^{2} \ln \left(a x +1\right)}{3 a}$	47
meijerg	$\frac{c^{2} \left(\frac{a x \left(4 a^{2} x^{2}-6 a x +12\right)}{12}-\ln \left(a x +1\right)\right)}{a}-\frac{3 c^{2} \left(-\frac{a x \left(-3 a x +6\right)}{6}+\ln \left(a x +1\right)\right)}{a}+\frac{3 c^{2} \left(a x -\ln \left(a x +1\right)\right)}{a}-\frac{c^{2} \ln \left(a x +1\right)}{a}$	95

input
```
int((-a*c*x+c)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/3*a^2*x^3-2*a*x^2+7*x-8*ln(a*x+1)/a)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^3c^2x^3-6\,a^2c^2x^2+21\,ac^2x-24\,c^2\log\left(ax+1\right)}{3\,a}$$

input
```
integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/3*(a^3*c^2*x^3 - 6*a^2*c^2*x^2 + 21*a*c^2*x - 24*c^2*log(a*x + 1))/a
```

3.209.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^2c^2x^3}{3}-2ac^2x^2+7c^2x-\frac{8c^2\log\left(ax+1\right)}{a}$$

input
```
integrate((-a*c*x+c)**2*(a*x-1)/(a*x+1),x)
```

output
```
a**2*c**2*x**3/3 - 2*a*c**2*x**2 + 7*c**2*x - 8*c**2*log(a*x + 1)/a
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-2\,ac^2x^2+7\,c^2x-\frac{8\,c^2\log\left(ax+1\right)}{a}$$

input
```
integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/3*a^2*c^2*x^3 - 2*a*c^2*x^2 + 7*c^2*x - 8*c^2*log(a*x + 1)/a
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=-\frac{8\,c^2\log\left(|ax+1|\right)}{a}+\frac{a^5c^2x^3-6\,a^4c^2x^2+21\,a^3c^2x}{3\,a^3}$$

input
```
integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-8*c^2*log(abs(a*x + 1))/a + 1/3*(a^5*c^2*x^3 - 6*a^4*c^2*x^2 + 21*a^3*c^2
*x)/a^3
```

3.209.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=7\,c^2\,x-2\,a\,c^2\,x^2+\frac{a^2\,c^2\,x^3}{3}-\frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - a*c*x)^2*(a*x - 1))/(a*x + 1),x)
```

output
```
7*c^2*x - 2*a*c^2*x^2 + (a^2*c^2*x^3)/3 - (8*c^2*log(a*x + 1))/a
```

3.210 $\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx$

3.210.1 Optimal result

Integrand size = 16, antiderivative size = 26

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

output
```
3*c*x-1/2*a*c*x^2-4*c*ln(a*x+1)/a
```

3.210.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

input
```
Integrate[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

3.210.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int ce^{-2\text{arctanh}(ax)}(1-ax)dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-2\text{arctanh}(ax)}(1-ax)dx$$

$$\Big\downarrow \text{6679}$$

$$-c\int \frac{(1-ax)^2}{ax+1}dx$$

$$\Big\downarrow \text{49}$$

$$-c\int \left(ax+\frac{4}{ax+1}-3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)$$

input
```
Int[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))
```

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.210.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
default	$c\left(-\frac{a\,x^{2}}{2}+3x-\frac{4\ln(ax+1)}{a}\right)$	24
norman	$3cx-\frac{ac\,x^{2}}{2}-\frac{4c\ln(ax+1)}{a}$	25
risch	$3cx-\frac{ac\,x^{2}}{2}-\frac{4c\ln(ax+1)}{a}$	25
parallelrisch	$-\frac{a^{2}c\,x^{2}-6acx+8c\ln(ax+1)}{2a}$	29
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a}+\frac{2c(ax-\ln(ax+1))}{a}-\frac{c\ln(ax+1)}{a}$	55

input
```
int((-a*c*x+c)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2*a*x^2+3*x-4*ln(a*x+1)/a)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{a^2cx^2-6\,acx+8\,c\log(ax+1)}{2\,a}$$

input
```
integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
-1/2*(a^2*c*x^2 - 6*a*c*x + 8*c*log(a*x + 1))/a
```

3.210.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{acx^2}{2}+3cx-\frac{4c\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x)
```

output
```
-a*c*x**2/2 + 3*c*x - 4*c*log(a*x + 1)/a
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2+3\,cx-\frac{4\,c\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/2*a*c*x^2 + 3*c*x - 4*c*log(a*x + 1)/a
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{4\,c\log\left(|ax+1|\right)}{a}-\frac{a^3cx^2-6\,a^2cx}{2\,a^2}$$

input
```
integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-4*c*log(abs(a*x + 1))/a - 1/2*(a^3*c*x^2 - 6*a^2*c*x)/a^2
```

3.210.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\,\left(8\,\ln\left(a\,x+1\right)-6\,a\,x+a^2\,x^2\right)}{2\,a}$$

input
```
int(((c - a*c*x)*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c*(8*log(a*x + 1) - 6*a*x + a^2*x^2))/(2*a)
```

3.211 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx$

3.211.1 Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\log(1+ax)}{ac}$$

output
```
-ln(a*x+1)/a/c
```

3.211.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\log(1+ax)}{ac}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

output
```
-(Log[1 + a*x]/(a*c))
```

3.211.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6717, 27, 6679, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c(1-ax)}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{1-ax}dx}{c}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{1}{ax+1}dx}{c}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\log(ax+1)}{ac}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

output
```
-(Log[1 + a*x]/(a*c))
```

3.211.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.211.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{\ln(ax+1)}{ac}$	15
norman	$-\frac{\ln(ax+1)}{ac}$	15
risch	$-\frac{\ln(ax+1)}{ac}$	15
parallelrisch	$-\frac{\ln(ax+1)}{ac}$	15

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-ln(a*x+1)/a/c
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(ax+1)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-log(a*x + 1)/(a*c)
```

3.211.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log{(acx+c)}}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x)
```

output
```
-log(a*c*x + c)/(a*c)
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log{(ax+1)}}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-log(a*x + 1)/(a*c)
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log{(|ax+1|)}}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="giac")
```

output
```
-log(abs(a*x + 1))/(a*c)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\ln\left(a\,x+1\right)}{a\,c}$$

input
```
int((a*x - 1)/((c - a*c*x)*(a*x + 1)),x)
```

output
```
-log(a*x + 1)/(a*c)
```

3.212 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

3.212.1 Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\operatorname{arctanh}(ax)}{ac^2}$$

output
```
-arctanh(a*x)/a/c^2
```

3.212.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\operatorname{arctanh}(ax)}{ac^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

output
```
-(ArcTanh[a*x]/(a*c^2))
```

3.212.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 39, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}dx$$
$$\Big\downarrow \color{blue}{6717}$$
$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^2(1-ax)^2}dx$$
$$\Big\downarrow \color{blue}{27}$$
$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-ax)^2}dx}{c^2}$$
$$\Big\downarrow \color{blue}{6679}$$
$$-\frac{\int \frac{1}{(1-ax)(ax+1)}dx}{c^2}$$
$$\Big\downarrow \color{blue}{39}$$
$$-\frac{\int \frac{1}{1-a^2x^2}dx}{c^2}$$
$$\Big\downarrow \color{blue}{219}$$
$$-\frac{\text{arctanh}(ax)}{ac^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

output
```
-(ArcTanh[a*x]/(a*c^2))
```

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.212.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

method	result	size
parallelrisch	$\frac{-\ln(ax+1)+\ln(ax-1)}{2a\,c^2}$	24
default	$\frac{-\frac{\ln(ax+1)}{2a}+\frac{\ln(ax-1)}{2a}}{c^2}$	28
norman	$\frac{\ln(ax-1)}{2a\,c^2}-\frac{\ln(ax+1)}{2a\,c^2}$	30
risch	$-\frac{\ln(ax+1)}{2a\,c^2}+\frac{\ln(-ax+1)}{2a\,c^2}$	31

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-ln(a*x+1)+ln(a*x-1))/a/c^2
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\log(ax+1)-\log(ax-1)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(log(a*x + 1) - log(a*x - 1))/(a*c^2)
```

3.212.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**2,x)
```

output
```
(log(x - 1/a)/2 - log(x + 1/a)/2)/(a*c**2)
```

3.212.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\log(ax+1)}{2\,ac^2}+\frac{\log(ax-1)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

3.212.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\log\left(\left|-\frac{2c}{acx-c}-1\right|\right)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^2)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\operatorname{atanh}(a\,x)}{a\,c^2}$$

input
```
int((a*x - 1)/((c - a*c*x)^2*(a*x + 1)),x)
```

output
```
-atanh(a*x)/(a*c^2)
```

3.213 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

3.213.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{1}{2ac^3(1-ax)} - \frac{\operatorname{arctanh}(ax)}{2ac^3}$$

output
```
-1/2/a/c^3/(-a*x+1)-1/2*arctanh(a*x)/a/c^3
```

3.213.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{1}{2a(1-ax)} + \frac{\operatorname{arctanh}(ax)}{2a}}{c^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

output
```
-((1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3)
```

3.213.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^3(1-ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{1}{(1-ax)^2(ax+1)}dx}{c^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{2(ax-1)^2}-\frac{1}{2(a^2x^2-1)}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{2a}+\frac{1}{2a(1-ax)}}{c^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

output
```
-((1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3)
```

3.213. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}dx$

3.213.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.213.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{-\frac{\ln(ax+1)}{4a}+\frac{1}{2a(ax-1)}+\frac{\ln(ax-1)}{4a}}{c^3}$	40
risch	$\frac{1}{2a(ax-1)c^3}+\frac{\ln(-ax+1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$	46
parallelrisch	$\frac{a\ln(ax-1)x-a\ln(ax+1)x+2ax-\ln(ax-1)+\ln(ax+1)}{4c^3(ax-1)a}$	54
norman	$\frac{-\frac{x}{2c}+\frac{a\,x^2}{2c}}{c^2(ax-1)^2}+\frac{\ln(ax-1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$	57

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-1/4*ln(a*x+1)/a+1/2/a/(a*x-1)+1/4/a*ln(a*x-1))
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(ax-1)\log{(ax+1)}-(ax-1)\log{(ax-1)}-2}{4\,(a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*c^3*x - a*
c^3)
```

3.213.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2a^2c^3x-2ac^3} - \frac{-\frac{\log{\left(x-\frac{1}{a}\right)}}{4}+\frac{\log{\left(x+\frac{1}{a}\right)}}{4}}{ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**3,x)
```

output
```
1/(2*a**2*c**3*x - 2*a*c**3) - (-log(x - 1/a)/4 + log(x + 1/a)/4)/(a*c**3)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2\,(a^2c^3x-ac^3)} - \frac{\log{(ax+1)}}{4\,ac^3} + \frac{\log{(ax-1)}}{4\,ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/2/(a^2*c^3*x - a*c^3) - 1/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)/(a*c
^3)
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\log\left(|ax+1|\right)}{4\,ac^3}+\frac{\log\left(|ax-1|\right)}{4\,ac^3}+\frac{1}{2\,(ax-1)ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) + 1/2/((a*x
 - 1)*a*c^3)
```

3.213.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}-\frac{\mathrm{atanh}(a\,x)}{2\,a\,c^3}$$

input
```
int((a*x - 1)/((c - a*c*x)^3*(a*x + 1)),x)
```

output
```
- 1/(2*a*(c^3 - a*c^3*x)) - atanh(a*x)/(2*a*c^3)
```

3.214 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.214.1 Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^4}$$

output
```
-1/4/a/c^4/(-a*x+1)^2-1/4/a/c^4/(-a*x+1)-1/4*arctanh(a*x)/a/c^4
```

3.214.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2+ax-(-1+ax)^2\operatorname{arctanh}(ax)}{4ac^4(-1+ax)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

output
```
(-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^4*(-1 + a*x)^2)
```

3.214.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^4(1-ax)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{1}{(1-ax)^3(ax+1)}dx}{c^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{4(ax-1)^2}-\frac{1}{2(ax-1)^3}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}+\frac{1}{4a(1-ax)}+\frac{1}{4a(1-ax)^2}}{c^4}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

output
```
-((1/(4*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)) + ArcTanh[a*x]/(4*a))/c^4)
```

3.214.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.214.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\frac{x}{4}-\frac{1}{2a}}{\left(ax-1\right)^{2} c^{4}}+\frac{\ln\left(-ax+1\right)}{8a\,c^{4}}-\frac{\ln\left(ax+1\right)}{8a\,c^{4}}$	51
default	$\frac{-\frac{\ln\left(ax+1\right)}{8a}-\frac{1}{4\left(ax-1\right)^{2} a}+\frac{1}{4a\left(ax-1\right)}+\frac{\ln\left(ax-1\right)}{8a}}{c^{4}}$	52
norman	$\frac{\frac{3x}{4c}-\frac{5a\,x^{2}}{4c}+\frac{a^{2} x^{3}}{2c}}{c^{3} \left(ax-1\right)^{3}}+\frac{\ln\left(ax-1\right)}{8a\,c^{4}}-\frac{\ln\left(ax+1\right)}{8a\,c^{4}}$	68
parallelrisch	$\frac{a^{2} \ln\left(ax-1\right) x^{2}-a^{2} \ln\left(ax+1\right) x^{2}+4a^{2} x^{2}-2a\ln\left(ax-1\right) x+2a\ln\left(ax+1\right) x-6ax+\ln\left(ax-1\right)-\ln\left(ax+1\right)}{8c^{4} \left(ax-1\right)^{2} a}$	90

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x-1/2/a)/(a*x-1)^2/c^4+1/8*ln(-a*x+1)/a/c^4-1/8*ln(a*x+1)/a/c^4
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{2\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+(a^2x^2-2\,ax+1)\log(ax-1)-4}{8\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 4)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

3.214.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{ax-2}{4a^3c^4x^2-8a^2c^4x+4ac^4} + \frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**4,x)
```

output
```
(a*x - 2)/(4*a**3*c**4*x**2 - 8*a**2*c**4*x + 4*a*c**4) + (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**4)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{ax-2}{4\,(a^3c^4x^2-2\,a^2c^4x+ac^4)} - \frac{\log(ax+1)}{8\,ac^4} + \frac{\log(ax-1)}{8\,ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/4*(a*x - 2)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) - 1/8*log(a*x + 1)/(a*c^
4) + 1/8*log(a*x - 1)/(a*c^4)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\log\left(|ax+1|\right)}{8\,ac^4}+\frac{\log\left(|ax-1|\right)}{8\,ac^4}+\frac{ax-2}{4\,(ax-1)^2ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^4) + 1/8*log(abs(a*x - 1))/(a*c^4) + 1/4*(a*x
- 2)/((a*x - 1)^2*a*c^4)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}-\frac{\operatorname{atanh}(a\,x)}{4\,a\,c^4}$$

input
```
int((a*x - 1)/((c - a*c*x)^4*(a*x + 1)),x)
```

output
```
(x/4 - 1/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) - atanh(a*x)/(4*a*c^4)
```

3.215 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

3.215.1 Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx=-\frac{1}{6ac^5(1-ax)^3}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{8ac^5(1-ax)}-\frac{\text{arctanh}(ax)}{8ac^5}$$

output
```
-1/6/a/c^5/(-a*x+1)^3-1/8/a/c^5/(-a*x+1)^2-1/8/a/c^5/(-a*x+1)-1/8*arctanh(
a*x)/a/c^5
```

3.215.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx=\frac{10-9ax+3a^2x^2-3(-1+ax)^3\text{arctanh}(ax)}{24ac^5(-1+ax)^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

output
```
(10 - 9*a*x + 3*a^2*x^2 - 3*(-1 + a*x)^3*ArcTanh[a*x])/(24*a*c^5*(-1 + a*x
)^3)
```

3.215.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^5(1-ax)^5}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-ax)^5}dx}{c^5}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{1}{(1-ax)^4(ax+1)}dx}{c^5}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{8(ax-1)^2}-\frac{1}{4(ax-1)^3}+\frac{1}{2(ax-1)^4}-\frac{1}{8(a^2x^2-1)}\right)dx}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{8a}+\frac{1}{8a(1-ax)}+\frac{1}{8a(1-ax)^2}+\frac{1}{6a(1-ax)^3}}{c^5}$$

input

```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

output

```
-((1/(6*a*(1 - a*x)^3) + 1/(8*a*(1 - a*x)^2) + 1/(8*a*(1 - a*x)) + ArcTanh
[a*x]/(8*a))/c^5)
```

3.215.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.215.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\frac{a\,x^{2}}{8}-\frac{3x}{8}+\frac{5}{12a}}{\left(ax-1\right)^{3}c^{5}}-\frac{\ln\left(ax+1\right)}{16c^{5}a}+\frac{\ln\left(-ax+1\right)}{16c^{5}a}$
default	$\frac{-\frac{\ln\left(ax+1\right)}{16a}+\frac{1}{6a\left(ax-1\right)^{3}}-\frac{1}{8\left(ax-1\right)^{2}a}+\frac{1}{8a\left(ax-1\right)}+\frac{\ln\left(ax-1\right)}{16a}}{c^{5}}$
norman	$\frac{-\frac{7x}{8c}+\frac{2a\,x^{2}}{c}-\frac{37a^{2}x^{3}}{24c}+\frac{5a^{3}x^{4}}{12c}}{c^{4}\left(ax-1\right)^{4}}+\frac{\ln\left(ax-1\right)}{16a\,c^{5}}-\frac{\ln\left(ax+1\right)}{16c^{5}a}$
parallelrisch	$\frac{3a^{3}\ln\left(ax-1\right)x^{3}-3a^{3}\ln\left(ax+1\right)x^{3}+20a^{3}x^{3}-9a^{2}\ln\left(ax-1\right)x^{2}+9a^{2}\ln\left(ax+1\right)x^{2}-54a^{2}x^{2}+9a\ln\left(ax-1\right)x-9a\ln\left(ax+1\right)x+42ax-}{48c^{5}\left(ax-1\right)^{3}a}$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/8*a*x^2-3/8*x+5/12/a)/(a*x-1)^3/c^5-1/16/c^5/a*ln(a*x+1)+1/16/c^5/a*ln(
-a*x+1)
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= \frac{6\,a^2x^2 - 18\,ax - 3\,(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\log(ax+1) + 3\,(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\log(ax-1) + 2}{48\,(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
1/48*(6*a^2*x^2 - 18*a*x - 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x + 1
) + 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x - 1) + 20)/(a^4*c^5*x^3 -
3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

3.215.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{-3a^2x^2 + 9ax - 10}{24a^4c^5x^3 - 72a^3c^5x^2 + 72a^2c^5x - 24ac^5} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16} + \frac{\log\left(x+\frac{1}{a}\right)}{16}}{ac^5}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**5,x)
```

output
```
-(-3*a**2*x**2 + 9*a*x - 10)/(24*a**4*c**5*x**3 - 72*a**3*c**5*x**2 + 72*a
**2*c**5*x - 24*a*c**5) - (-log(x - 1/a)/16 + log(x + 1/a)/16)/(a*c**5)
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{3\,a^2x^2-9\,ax+10}{24\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}-\frac{\log{(ax+1)}}{16\,ac^5}+\frac{\log{(ax-1)}}{16\,ac^5}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
1/24*(3*a^2*x^2 - 9*a*x + 10)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x -
 a*c^5) - 1/16*log(a*x + 1)/(a*c^5) + 1/16*log(a*x - 1)/(a*c^5)
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\log{\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}}{16\,ac^5}+\frac{\frac{3\,a^2c^2}{acx-c}-\frac{3\,a^2c^3}{(acx-c)^2}+\frac{4\,a^2c^4}{(acx-c)^3}}{24\,a^3c^6}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
-1/16*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^5) + 1/24*(3*a^2*c^2/(a*c*x - c)
 - 3*a^2*c^3/(a*c*x - c)^2 + 4*a^2*c^4/(a*c*x - c)^3)/(a^3*c^6)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{a\,x^2}{8}-\frac{3\,x}{8}+\frac{5}{12\,a}}{-a^3\,c^5\,x^3+3\,a^2\,c^5\,x^2-3\,a\,c^5\,x+c^5}-\frac{\mathrm{atanh}(a\,x)}{8\,a\,c^5}$$

input
```
int((a*x - 1)/((c - a*c*x)^5*(a*x + 1)),x)
```

output
```
- ((a*x^2)/8 - (3*x)/8 + 5/(12*a))/(c^5 + 3*a^2*c^5*x^2 - a^3*c^5*x^3 - 3*
a*c^5*x) - atanh(a*x)/(8*a*c^5)
```

3.216 $\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx$

3.216.1 Optimal result

Integrand size = 18, antiderivative size = 94

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{3/2}x(c-acx)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,-1-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1+\frac{1}{ax}}}$$

output
```
((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2)*x*(-a*c*x+c)^p*hypergeom([-1-p,
 -3/2-p],[-p],2/(a+1/x)/x)/(p+1)/(1+1/a/x)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(1+ax)(c-acx)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,-1-p,-p,\frac{2}{1+ax}\right)}{a(1+p)\sqrt{1+\frac{1}{ax}}}$$

input
```
Integrate[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(1 + a*x)*(c - a*c*x)
^p*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*Sqrt[1
 + 1/(a*x)])
```

3.216.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int\frac{\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{x}\right)^{-p-2}}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}(c-acx)^p\,\text{Hypergeometric2F1}\left(-p-\frac{3}{2},-p-1,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(p+1)\sqrt{\frac{1}{ax}+1}}$$

input
```
Int[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2)*x*(c - a*c*x)^
p*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/((a + x^(-1))*x)])/((1 + p)*Sq
rt[1 + 1/(a*x)])
```

3.216.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[p]
```

3.216.4 Maple [F]

$$\int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

3.216.5 Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.216.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.216.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\int (c-a\,c\,x)^p\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.217 $\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$

3.217.1 Optimal result

Integrand size = 18, antiderivative size = 152

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+30c^3\sqrt{1-\frac{1}{a^2x^2}}x-\frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2+2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4-\frac{315c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
-315/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+32*c^3*(a-1/x)/a^2/(1-1/a^2/x^2)
^(1/2)+30*c^3*x*(1-1/a^2/x^2)^(1/2)-67/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)+2*a
^2*c^3*x^3*(1-1/a^2/x^2)^(1/2)-1/4*a^3*c^3*x^4*(1-1/a^2/x^2)^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.57

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{1}{8}c^3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}x(496+173ax-51a^2x^2+14a^3x^3-2a^4x^4)}{1+ax}-\frac{315\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

input
```
Integrate[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^3*((Sqrt[1 - 1/(a^2*x^2)]*x*(496 + 173*a*x - 51*a^2*x^2 + 14*a^3*x^3 -
2*a^4*x^4))/(1 + a*x) - (315*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/8
```

3.217.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6724, 27, 528, 2338, 2338, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{-3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6724}$$

$$\frac{\int \frac{c^6\left(a-\frac{1}{x}\right)^6 x^5}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^3\int \frac{\left(a-\frac{1}{x}\right)^6 x^5}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \text{ 528}$$

$$\frac{c^3\left(a^2\int \frac{\left(a^4-\frac{6a^3}{x}+\frac{16a^2}{x^2}-\frac{26a}{x^3}+\frac{31}{x^4}\right)x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{32a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{c^3\left(a^2\left(-\frac{1}{4}\int \frac{\left(24a^3-\frac{67a^2}{x}+\frac{104a}{x^2}-\frac{124}{x^3}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{32a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{c^3\left(a^2\left(\frac{1}{4}\left(\frac{1}{3}\int \frac{3\left(67a^2-\frac{120a}{x}+\frac{124}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{32a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\left(a^2\left(\frac{1}{4}\left(\int \frac{\left(67a^2-\frac{120a}{x}+\frac{124}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{32a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{2338}$$

$$\frac{c^3\left(a^2\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{15\left(16a-\frac{21}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{67}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}+8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{32a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\left(a^2\left(\frac{1}{4}\left(-\frac{15}{2}\int \frac{\left(16a-\frac{21}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{67}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}+8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{32a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{534}$$

$$\frac{c^3\Bigg(a^2\Bigg(\frac{1}{4}\Bigg(-\frac{15}{2}\left(-21\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{67}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}+8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\Bigg)-\frac{1}{4}a^4x^4\sqrt{1-}}{a^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{c^3\Bigg(a^2\Bigg(\frac{1}{4}\Bigg(-\frac{15}{2}\left(-\frac{21}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-16ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{67}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}+8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\Bigg)-\frac{1}{4}a^4x^4\sqrt{1-}}{a^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{c^3\Bigg(a^2\Bigg(\frac{1}{4}\Bigg(-\frac{15}{2}\left(21a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-16ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{67}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}+8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\Bigg)}{a^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{c^3\Bigg(\frac{32a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+a^2\Bigg(\frac{1}{4}\Bigg(-\frac{15}{2}\left(21\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-16ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{67}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}+8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\Bigg)}{a^3}$$

3.217. $\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$

input
```
Int[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^3*((32*a*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-1/4*(a^4*Sqrt[1 -
1/(a^2*x^2)]*x^4) + ((-67*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + 8*a^3*Sqrt[1
- 1/(a^2*x^2)]*x^3 - (15*(-16*a*Sqrt[1 - 1/(a^2*x^2)]*x + 21*ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]]))/2)/4)))/a^3
```

3.217.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.217.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(2 a^{3} x^{3}-16 a^{2} x^{2}+67 a x -240\right) \left(a x +1\right) c^{3} \sqrt{\frac{a x -1}{a x +1}}}{8 a}-\frac{\left(\frac{315 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right)}{8 \sqrt{a^{2}}}-\frac{32 \sqrt{a^{2} \left(x +\frac{1}{a}\right)^{2}-2 a \left(x +\frac{1}{a}\right)}}{a^{2} \left(x +\frac{1}{a}\right)}\right) c^{3} \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{(a x -1)(a}}{a x -1}$
default	$-\frac{\Big(2 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{3} x^{3}+4 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{2} x^{2}+69 \sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}\, a^{3} x^{3}-16 \sqrt{a^{2}}\, \left(\left(a x -1\right) \left(a x +1\right)\right)^{\frac{3}{2}} a^{2} x^{2}+2 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a x +}{}$

input
```
int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*a^3*x^3-16*a^2*x^2+67*a*x-240)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1/
2)-(315/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-32/a^2/(x+1/
a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*c^3/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*
((a*x-1)*(a*x+1))^(1/2)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx =$$
$$-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4-14\,a^3c^3x^3+51\,a^2c^3x^2-173\,ac^3x-49\right.}{8\,a}$$

input
```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (2*a^4*c^3*x^4 - 14*a^3*c^3*x^3 + 51*a^2*c^3*x^2 - 1
73*a*c^3*x - 496*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.217.6 Sympy [F]

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\Bigg(&\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx\\&+\int\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\\&+\int\left(-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx\\&+\int\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\Bigg)\end{aligned}$$

input
```
integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
-c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral
(-4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(6*a**2*
x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**3*x*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x**4*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.61

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx =$$

$$-\frac{1}{8}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{256\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}-\frac{2\left(325\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-765\,c^3\left(\right.\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2}{(ax+1)}}\right.$$

input
```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 256*c^3*sqrt((a*x - 1)/(a*x + 1))/a^2 - 2*(3
25*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 765*c^3*((a*x - 1)/(a*x + 1))^(5/2) +
 643*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 187*c^3*sqrt((a*x - 1)/(a*x + 1)))/
(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x + 1)^2 + 4*(a*x - 1)^3
*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*a
```

3.217.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx=\int -(acx-c)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.217.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.31

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$$

$$=\frac{\frac{187\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{643\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4}+\frac{765\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{4}-\frac{325\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a-\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{6\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}$$

$$+\frac{32\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{315\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((187*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (643*c^3*((a*x - 1)/(a*x + 1))^
(3/2))/4 + (765*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 - (325*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*
x + 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) +
(32*c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (315*c^3*atanh(((a*x - 1)/(a*x +
1))^(1/2)))/(4*a)
```

3.217. $\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$

3.218 $\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx$

3.218.1 Optimal result

Integrand size = 18, antiderivative size = 129

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{35c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
-35/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+16*c^2*(a-1/x)/a^2/(1-1/a^2/x^2)^
(1/2)+35/3*c^2*x*(1-1/a^2/x^2)^(1/2)-5/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)+1/3
*a^2*c^2*x^3*(1-1/a^2/x^2)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{6}c^2\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(166+55ax-13a^2x^2+2a^3x^3)}{1+ax}-\frac{105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

input
```
Integrate[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*((Sqrt[1 - 1/(a^2*x^2)]*x*(166 + 55*a*x - 13*a^2*x^2 + 2*a^3*x^3))/(1
 + a*x) - (105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/6
```

3.218.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6724, 25, 27, 528, 2338, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{-3\coth^{-1}(ax)}\, dx$$

$$\Big\downarrow \ 6724$$

$$\frac{\int -\frac{c^5\left(a-\frac{1}{x}\right)^5 x^4}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{c^5\left(a-\frac{1}{x}\right)^5 x^4}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^2\int \frac{\left(a-\frac{1}{x}\right)^5 x^4}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \ 528$$

$$-\frac{c^2\left(a^2\int \frac{\left(a^3-\frac{5a^2}{x}+\frac{11a}{x^2}-\frac{15}{x^3}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \ 2338$$

$$-\frac{c^2\left(a^2\left(-\frac{1}{3}\int \frac{5\left(3a^2-\frac{7a}{x}+\frac{9}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\int\frac{\left(3a^2-\frac{7a}{x}+\frac{9}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \textbf{2338}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{1}{2}\int\frac{7\left(2a-\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\int\frac{\left(2a-\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \textbf{534}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\left(-3\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \textbf{243}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\left(-\frac{3}{2}\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-2ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right.}{a^3}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\left(3a^2\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-2ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)\right.}{a^3}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\left(3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right.}{a^3}$$

input
```
Int[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

3.218. $\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx$

output
```
-((c^2*((-16*a*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-1/3*(a^3*Sqrt[1
 - 1/(a^2*x^2)]*x^3) - (5*((-3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (7*(-2*a
*Sqrt[1 - 1/(a^2*x^2)]*x + 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/2))/3)))/a^3
)
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.218.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(2a^2x^2-15ax+70)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{6a}+\frac{\left(-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2\sqrt{a^2}}+\frac{16\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{(15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-2\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+30\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-4\sqrt{a^2}\,((ax-1)}{}$

input
```
int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-15*a*x+70)*(a*x+1)/a*c^2*((a*x-1)/(a*x+1))^(1/2)+(-35/2*ln(
a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+16/a^2/(x+1/a)*(a^2*(x+1/
a)^2-2*a*(x+1/a))^(1/2))*c^2/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x
+1))^(1/2)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3-13\,a^2c^2x^2+55\,ac^2x+166\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 13*a^2*c^2*x^2 + 55*a*c^2*x + 166*c
^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.218.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

input
```
integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral
(3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**2*
x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.58

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx =$$
$$-\frac{1}{6}\,a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{96\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{2\left(87\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-136\,c^2\left(\right.\right.}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}}\right.$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/6*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 96*c^2*sqrt((a*x - 1)/(a*x + 1))/a^2 + 2*(
87*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 136*c^2*((a*x - 1)/(a*x + 1))^(3/2) +
 57*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1
)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2))
```

3.218.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\int (acx-c)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.218.9 Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.26

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{19\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{136\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{35\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(19*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (136*c^2*((a*x - 1)/(a*x + 1))^(3/2)
)/3 + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) +
 (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (16*c^2*((
a*x - 1)/(a*x + 1))^(1/2))/a - (35*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))
/a
```

3.218. $\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx$

3.219 $\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx$

3.219.1 Optimal result

Integrand size = 16, antiderivative size = 92

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{15c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
-15/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+8*c*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)
+4*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2*(1-1/a^2/x^2)^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\frac{1}{2}c\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}x(24+7ax-a^2x^2)}{1+ax}-\frac{15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

input
```
Integrate[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*((Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 7*a*x - a^2*x^2))/(1 + a*x) - (15*Log[a
*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/2
```

3.219.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 27, 528, 2338, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6724}$$

$$\frac{\int \frac{c^4\left(a-\frac{1}{x}\right)^4x^3}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\int \frac{\left(a-\frac{1}{x}\right)^4x^3}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \text{ 528}$$

$$\frac{c\left(a^2\int \frac{\left(a^2-\frac{4a}{x}+\frac{7}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{c\left(a^2\left(-\frac{1}{2}\int \frac{\left(8a-\frac{15}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{c\left(a^2\left(\frac{1}{2}\left(15\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow 243$$

$$\frac{c\left(a^2\left(\frac{1}{2}\left(\frac{15}{2}\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+8ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow 73$$

$$\frac{c\left(a^2\left(\frac{1}{2}\left(8ax\sqrt{1-\frac{1}{a^2x^2}}-15a^2\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow 221$$

$$\frac{c\left(a^2\left(\frac{1}{2}\left(8ax\sqrt{1-\frac{1}{a^2x^2}}-15\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

input
```
Int[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*((8*a*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-1/2*(a^2*Sqrt[1 - 1/(
a^2*x^2)]*x^2) + (8*a*Sqrt[1 - 1/(a^2*x^2)]*x - 15*ArcTanh[Sqrt[1 - 1/(a^2
*x^2)]])/2)))/a^3
```

3.219.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.219.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{(ax-8)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{2a}-\frac{\left(\frac{15\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2\sqrt{a^2}}-\frac{8\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-16\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+16\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(a}{\sqrt{a}}\right.\right.}{}$

input
```
int((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-8)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)-(15/2*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-8/a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))
^(1/2))*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2-7\,acx-24\,c)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) + (a^2*c*x^2 - 7*a*c*x - 24*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.219.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
-c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2
*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.70

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = \frac{1}{2}\,a\left(\frac{2\left(9\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-7\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{16\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
1/2*a*(2*(9*c*((a*x - 1)/(a*x + 1))^(3/2) - 7*c*sqrt((a*x - 1)/(a*x + 1)))
/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 15*c*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1))
- 1)/a^2 + 16*c*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

3.219.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = \int -(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.219.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = \frac{7\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-9\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,(a\,x-1)}{a\,x+1}+\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}}-\frac{15\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}$$

input
```
int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(7*c*((a*x - 1)/(a*x + 1))^(1/2) - 9*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (
2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (15*c*atanh(((a*
x - 1)/(a*x + 1))^(1/2)))/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

3.219. $\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx$

3.220 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx$

3.220.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
-arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{1+ax}-\frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

output
```
((2*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])
*x]/a)/c
```

3.220.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6724, 27, 528, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{c^2\left(a-\frac{1}{x}\right)^2x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(a-\frac{1}{x}\right)^2x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c}$$

$$\Big\downarrow \text{528}$$

$$\frac{a^2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^3c}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}a^2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+\frac{2a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^3c}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-a^4\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}}{a^3c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-a^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^3c}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

3.220. $\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}dx$

output
```
((2*a*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] - a^2*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(a^3*c)
```

3.220.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.220. $\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx$

3.220.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(49) = 98$.

Time = 0.41 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.68

method	result
default	$-\frac{\left(\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2-\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^2x+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-}{a\sqrt{a^2}\,c(ax-1)\sqrt{(ax-1)(ax+1)}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2+2*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-2*(a
^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(1/2))/(a^2)^(1/2))-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/a*((a*x-1)/
(a*x+1))^(3/2)/(a^2)^(1/2)/c/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
(2*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) + log(sq
rt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

3.220.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

output
```
-(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Integr
al(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c) - 2*sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

3.220.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{acx-c}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

output
```
undef
```

3.220.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/
2)))/(a*c)
```

3.221 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

3.221.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(a-1/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2(1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(1 + a*x))
```

3.221.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6724, 25, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$$

$$\Big\downarrow \; 6724$$

$$\frac{\int -\frac{c\left(a-\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{c\left(a-\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{a-\frac{1}{x}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^2}$$

$$\Big\downarrow \; 453$$

$$\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

output
```
(a - x^(-1))/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)])
```

3.221.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.221.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
trager	$\frac{\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^{2}}$	25
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)a\,c^{2}}$	35
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)a\,c^{2}}$	35

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2*(-(-a*x+1)/(a*x+1))^(1/2)
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

3.221.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

output
```
(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x
+ 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**
2*x**2 - a*x + 1), x))/c**2
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

3.221.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
undef
```

3.221.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^2,x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(a*c^2)
```

3.222 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

3.222.1 Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/a/c^3/(1-1/a^2/x^2)^(1/2)
```

3.222.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^3\left(-1+a^2x^2\right)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(c^3*(-1 + a^2*x^2))
```

3.222.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6724, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \displaystyle\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}dx \\ \Big\downarrow \color{blue}{6724} \\ \dfrac{\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}d\frac{1}{x}}{a^3c^3} \\ \Big\downarrow \color{blue}{241} \\ \dfrac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}} \end{array}$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

output

```
1/(a*c^3*Sqrt[1 - 1/(a^2*x^2)])
```

3.222.3.1 Defintions of rubi rules used

rule 241

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6724

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.222.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

method	result	size
trager	$\frac{x\sqrt{-\frac{-ax+1}{ax+1}}}{c^{3}(ax-1)}$	30
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x}{(ax-1)^{2}c^{3}}$	33
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x}{(ax-1)^{2}c^{3}}$	33

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*x/(a*x-1)*(-(-a*x+1)/(a*x+1))^(1/2)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{x\sqrt{\frac{ax-1}{ax+1}}}{ac^3x-c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
x*sqrt((a*x - 1)/(a*x + 1))/(a*c^3*x - c^3)
```

3.222.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

output
```
-(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*x**3 + 2
*a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4
- 2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2}\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3} + \frac{1}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*a*(sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + 1/(a^2*c^3*sqrt((a*x - 1)/(a*
x + 1))))
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{x\mathrm{sgn}(ax+1)}{\sqrt{a^2x^2-1}c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
x*sgn(a*x + 1)/(sqrt(a^2*x^2 - 1)*c^3)
```

3.222.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\frac{a\,x-1}{a\,x+1}+1}{2\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^3,x)
```

output
```
((a*x - 1)/(a*x + 1) + 1)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(1/2))
```

3.223 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.223.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=\frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}$$

output
```
2/3/a/c^4/(1-1/a^2/x^2)^(1/2)-1/3/a^2/c^4/(a-1/x)/x^2/(1-1/a^2/x^2)^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-1-2ax+2a^2x^2)}{3c^4(-1+ax)^2(1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^4*(-1 + a*x)^2*(1
+ a*x))
```

3.223.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6724, 25, 27, 567, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$

$$\Big\downarrow \; 6724$$

$$\frac{\int -\frac{1}{c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1}{c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2}d\frac{1}{x}}{a^3c^4}$$

$$\Big\downarrow \; 567$$

$$-\frac{\frac{a}{3x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{2}{3}\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}d\frac{1}{x}}{a^3c^4}$$

$$\Big\downarrow \; 241$$

$$-\frac{\frac{a}{3x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{2a^2}{3\sqrt{1-\frac{1}{a^2x^2}}}}{a^3c^4}$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

output

```
-(((-2*a^2)/(3*Sqrt[1 - 1/(a^2*x^2)]) + a/(3*Sqrt[1 - 1/(a^2*x^2)]*(a - x^
(-1))*x^2))/(a^3*c^4))
```

3.223. $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.223.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 567

```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[c*x^m*((a + b*x^2)^(p + 1)/(2*a*d*p*(c + d*x))), x] - Simp[m/(2*d*p)
   Int[x^(m - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[m + 2*p + 1, 0]
```

rule 6724

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.223.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(2a^{3}x^{3}-3ax-1\right)}{3\left(ax-1\right)^{3}c^{4}a}$	45
trager	$\frac{\left(2a^{2}x^{2}-2ax-1\right)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^{4}\left(ax-1\right)^{2}}$	47
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(2a^{2}x^{2}-2ax-1\right)\left(ax+1\right)}{3\left(ax-1\right)^{3}c^{4}a}$	50

input

```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output

```
1/3*((a*x-1)/(a*x+1))^(3/2)*(2*a^3*x^3-3*a*x-1)/(a*x-1)^3/c^4/a
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{(2\,a^2x^2-2\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^4*x^2 - 2*a^2
*c^4*x + a*c^4)
```

3.223.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

output
```
(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*
a**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a
*x + 1), x))/c**4
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{1}{12}\,a\left(\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4} + \frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4) + (6*(a*x - 1)/(a*x + 1) - 1
)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

3.223.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^4, x)
```

3.223.9 Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2\,a^2\,x^2+2\,a\,x+1}{\left(3\,a\,c^4-3\,a^3\,c^4\,x^2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^4,x)
```

output
```
(2*a*x - 2*a^2*x^2 + 1)/((3*a*c^4 - 3*a^3*c^4*x^2)*((a*x - 1)/(a*x + 1))^(
1/2))
```

3.224 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

3.224.1 Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-4/5*(a+1/x)/a^2/c^5/(1-1/a^2/x^2)^(3/2)+1/5*(a+1/x)^2/a^3/c^5/(1-1/a^2/x^
2)^(5/2)+1/5*(5*a+2/x)/a^2/c^5/(1-1/a^2/x^2)^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(2+ax-4a^2x^2+2a^3x^3)}{5c^5(-1+ax)^3(1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3))/(5*c^5*(-1 + a
*x)^3*(1 + a*x))
```

3.224.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6724, 27, 570, 529, 2166, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{1}{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^2x^3}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^2x^3}d\frac{1}{x}}{a^3c^5}$$

$$\Big\downarrow \text{570}$$

$$\frac{\int \frac{\left(a+\frac{1}{x}\right)^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^3}d\frac{1}{x}}{a^7c^5}$$

$$\Big\downarrow \text{529}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\int \frac{\left(a+\frac{1}{x}\right)\left(2a^3+\frac{5a^2}{x}+\frac{5a}{x^2}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^7c^5}$$

$$\Big\downarrow \text{2166}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{4a^4\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}a\int \frac{3a^2\left(2a+\frac{5}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)}{a^7c^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{4a^4\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-a^3\int \frac{2a+\frac{5}{x}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)}{a^7c^5}$$

$$\Big\downarrow \text{453}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{4a^4\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{a^4\left(5a+\frac{2}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7c^5}$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

output

```
(-1/5*(a*((4*a^4*(a + x^(-1)))/(1 - 1/(a^2*x^2))^(3/2) - (a^4*(5*a + 2/x))
/Sqrt[1 - 1/(a^2*x^2)])) + (a^4*(a + x^(-1))^2)/(5*(1 - 1/(a^2*x^2))^(5/2)
))/(a^7*c^5)
```

3.224.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453

```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 529

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570

```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.224.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

method	result	size
trager	$\frac{(2a^3x^3-4a^2x^2+ax+2)\sqrt{-\frac{-ax+1}{ax+1}}}{5a\,c^5(ax-1)^3}$	54
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^3x^3-4a^2x^2+ax+2)(ax+1)}{5(ax-1)^4c^5a}$	57
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^4x^4-2a^3x^3-3a^2x^2+3ax+2)}{5(ax-1)^4c^5a}$	61

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5/a/c^5*(2*a^3*x^3-4*a^2*x^2+a*x+2)/(a*x-1)^3*(-(-a*x+1)/(a*x+1))^(1/2)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\left(2\,a^3x^3-4\,a^2x^2+ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^5*x
^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

3.224.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx}{c^5}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

output
```
-(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5
*a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*
a*x - 1), x))/c**5
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{1}{40}\,a\left(\frac{5\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5}-\frac{\frac{5\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-1}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
1/40*a*(5*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^5) - (5*(a*x - 1)/(a*x + 1) - 1
5*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(5/2)))
```

3.224.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^5}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^5, x)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=\frac{2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2}{5\,a\,c^5\,(a\,x+1)^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^5,x)
```

output
```
(a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2)/(5*a*c^5*(a*x + 1)^3*((a*x - 1)/(a*x + 1
))^(5/2))
```

3.225 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$

3.225.1 Optimal result

Integrand size = 18, antiderivative size = 125

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-46/35*(a+1/x)/a^2/c^6/(1-1/a^2/x^2)^(3/2)+24/35*(a+1/x)^2/a^3/c^6/(1-1/a^
2/x^2)^(5/2)-1/7*(a+1/x)^3/a^4/c^6/(1-1/a^2/x^2)^(7/2)+1/35*(35*a+13/x)/a^
2/c^6/(1-1/a^2/x^2)^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(-13+4ax+20a^2x^2-24a^3x^3+8a^4x^4)}{35c^6(-1+ax)^4(1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^
4))/(35*c^6*(-1 + a*x)^4*(1 + a*x))
```

3.225.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 25, 27, 570, 529, 2166, 2166, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int -\frac{1}{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^3x^4}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^3x^4}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^3x^4}d\frac{1}{x}}{a^3c^6}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^3}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^4}d\frac{1}{x}}{a^9c^6}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\int \frac{\left(a+\frac{1}{x}\right)^2\left(3a^4+\frac{7a^3}{x}+\frac{7a^2}{x^2}+\frac{7a}{x^3}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^9c^6}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\left(\frac{24a^5\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\int \frac{\left(a+\frac{1}{x}\right)\left(33a^4+\frac{70a^3}{x}+\frac{35a^2}{x^2}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}\right)}{a^9c^6}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\left(\frac{24a^5\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{46a^5\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}a\int\frac{3a^3\left(13a+\frac{35}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)\right)}{a^9c^6}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\left(\frac{24a^5\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{46a^5\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-a^4\int\frac{13a+\frac{35}{x}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)\right)}{a^9c^6}$$

$$\Big\downarrow 453$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\left(\frac{24a^5\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{46a^5\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{a^5\left(35a+\frac{13}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)\right)}{a^9c^6}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

output
```
-((-1/7*(a*(-1/5*(a*((46*a^5*(a + x^(-1)))/(1 - 1/(a^2*x^2))^(3/2) - (a^5*
(35*a + 13/x))/Sqrt[1 - 1/(a^2*x^2)])) + (24*a^5*(a + x^(-1))^2)/(5*(1 - 1
/(a^2*x^2))^(5/2)))) + (a^5*(a + x^(-1))^3)/(7*(1 - 1/(a^2*x^2))^(7/2)))/(
a^9*c^6))
```

3.225.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

3.225. $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$

rule 529

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570

```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6724

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.225.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

method	result	size
trager	$\frac{\left(8 a^{4} x^{4}-24 a^{3} x^{3}+20 a^{2} x^{2}+4 a x -13\right) \sqrt{-\frac{-a x +1}{a x +1}}}{35 a \,c^{6} \left(a x -1\right)^{4}}$	63
gosper	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(8 a^{4} x^{4}-24 a^{3} x^{3}+20 a^{2} x^{2}+4 a x -13\right) \left(a x +1\right)}{35 \left(a x -1\right)^{5} c^{6} a}$	66
default	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(8 a^{5} x^{5}-16 a^{4} x^{4}-4 a^{3} x^{3}+24 a^{2} x^{2}-9 a x -13\right)}{35 \left(a x -1\right)^{5} c^{6} a}$	69

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/35/a/c^6*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a*x-1)^4*(-(-a*x+1)
/(a*x+1))^(1/2)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx=\frac{(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^6x^4-4\,a^4c^6x^3+6\,a^3c^6x^2-4\,a^2c^6x+ac^6)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="fricas")
```

output
```
1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^6*x^4 - 4*a^4*c^6*x^3 + 6*a^3*c^6*x^2 - 4*a^2*c^6*x + a*c^6
)
```

3.225.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$$
$$=\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\,dx}{c^6}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**6,x)
```

output
```
(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*
a**5*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x) + Int
egral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*a
**5*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x))/c**6
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{1}{560}a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^6}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="maxima")
```

output
```
1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^6) + (28*(a*x - 1)/(a*x + 1)
- 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^6*(
(a*x - 1)/(a*x + 1))^(7/2)))
```

3.225.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^6}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^6, x)
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{8\,a^4\,x^4-24\,a^3\,x^3+20\,a^2\,x^2+4\,a\,x-13}{35\,a\,c^6\,(a\,x+1)^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^6,x)
```

output
```
(4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^6*(a*x + 1)^4*(
(a*x - 1)/(a*x + 1))^(7/2))
```

3.226 $\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

3.226.1 Optimal result

Integrand size = 18, antiderivative size = 254

$$\begin{aligned}\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx &= -\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}\\ &\quad+\frac{9088\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{3465a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3}-\frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}\\ &\quad+\frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}\end{aligned}$$

output
```
-32/99*(a-1/x)^3*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)+9088
/3465*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)/x^3-768/385*(1+
1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+128/231*(1+1/a/x)^(3
/2)*(-a*c*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^4*(1+1/a/x)^(3/2)*
x*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2c^4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(5419-977ax-1866a^2x^2+2710a^3x^3-1505a^4x^4+315a^5x^5)}{3465a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

output
```
(2*c^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(5419 - 977*a*x - 1866*a^2*x^2 +
2710*a^3*x^3 - 1505*a^4*x^4 + 315*a^5*x^5))/(3465*a*Sqrt[1 - 1/(a*x)])
```

3.226.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6727, 27, 105, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{9/2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}}{a^4\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{16}{11}\int\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^4}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{16}{11}\left(-\frac{4}{3}\int\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^4}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 100$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{16}{11}\Bigg(-\frac{4}{3}\Bigg(\frac{2}{7}\int-\frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\Bigg)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)}{11\left(\frac{1}{x}\right)^{11/2}}}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{16}{11}\Bigg(-\frac{4}{3}\Bigg(-\frac{1}{7}\int\frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\Bigg)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)}{11\left(\frac{1}{x}\right)^{11/2}}}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 87$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{16}{11}\Bigg(-\frac{4}{3}\Bigg(\frac{1}{7}\Bigg(\frac{71}{5}\int\frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\Bigg)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\Bigg)-2\Big(}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}\Bigg(-\frac{16}{11}\Bigg(-\frac{4}{3}\Bigg(\frac{1}{7}\Bigg(\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}-\frac{142\left(\frac{1}{ax}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}\Bigg)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\Bigg)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^4}{11\left(\frac{1}{x}\right)^{11/2}}\Bigg)\Big(}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

3.226. $\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

output

```
-((((-16*((-4*(((36*a*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) - (142*(1 +
1/(a*x))^(3/2))/(15*(x^(-1))^(3/2)))/7 - (2*a^2*(1 + 1/(a*x))^(3/2))/(7*(x
^(-1))^(7/2))))/3 - (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2))/(9*(x^(-1))^(9/
2))))/11 - (2*(a - x^(-1))^4*(1 + 1/(a*x))^(3/2))/(11*(x^(-1))^(11/2)))*(x
^(-1))^(9/2)*(c - a*c*x)^(9/2))/(a^4*(1 - 1/(a*x))^(9/2)))
```

3.226.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.226.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.27

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\,c^{4}(ax+1)\left(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419\right)}{3465\sqrt{\frac{ax-1}{ax+1}}\,a}$	69
gosper	$\frac{2(ax+1)\left(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419\right)(-acx+c)^{\frac{9}{2}}}{3465a(ax-1)^{4}\sqrt{\frac{ax-1}{ax+1}}}$	72
risch	$-\frac{2c^{5}(ax-1)\left(315a^{5}x^{5}-1505a^{4}x^{4}+2710a^{3}x^{3}-1866a^{2}x^{2}-977ax+5419\right)}{3465\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	77

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^4*(a*x+1)*(315*a^4*x^4
-1820*a^3*x^3+4530*a^2*x^2-6396*a*x+5419)/a
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\left(315\,a^{6}c^{4}x^{6}-1190\,a^{5}c^{4}x^{5}+1205\,a^{4}c^{4}x^{4}+844\,a^{3}c^{4}x^{3}-2843\,a^{2}c^{4}x^{2}+4442\,ac^{4}x+5419\,c^{4}\right.}{3465\left(a^{2}x-a\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas"
)
```

output
```
2/3465*(315*a^6*c^4*x^6 - 1190*a^5*c^4*x^5 + 1205*a^4*c^4*x^4 + 844*a^3*c^
4*x^3 - 2843*a^2*c^4*x^2 + 4442*a*c^4*x + 5419*c^4)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1))/(a^2*x - a)
```

3.226.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(9/2),x)
```

output
```
Timed out
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\,(315\,a^5\sqrt{-c}c^4x^5-1505\,a^4\sqrt{-c}c^4x^4+2710\,a^3\sqrt{-c}c^4x^3-1866\,a^2\sqrt{-c}c^4x^2-977\,a\sqrt{-c}c^4x}{3465\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima"
)
```

output
```
2/3465*(315*a^5*sqrt(-c)*c^4*x^5 - 1505*a^4*sqrt(-c)*c^4*x^4 + 2710*a^3*sq
rt(-c)*c^4*x^3 - 1866*a^2*sqrt(-c)*c^4*x^2 - 977*a*sqrt(-c)*c^4*x + 5419*s
qrt(-c)*c^4)*sqrt(a*x + 1)/a
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\,\Big(4096\,\sqrt{2}\sqrt{-c}c^3-\frac{315\,(acx+c)^5\sqrt{-acx-c}-3080\,(acx+c)^4\sqrt{-acx-c}c+11880\,(acx+c)^3\sqrt{-acx-c}c^2-22176\,(acx+c)}{c^2}}{3465\,a|c|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

output
```
2/3465*(4096*sqrt(2)*sqrt(-c)*c^3 - (315*(a*c*x + c)^5*sqrt(-a*c*x - c) -
3080*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 11880*(a*c*x + c)^3*sqrt(-a*c*x -
c)*c^2 - 22176*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3 - 18480*(-a*c*x - c)^(3/
2)*c^4)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))
```

3.226.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\,c^4\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(315\,a^4\,x^4-1820\,a^3\,x^3+4530\,a^2\,x^2-6396\,a\,x+5419\right)}{3465\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^4*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(4530*a^2
*x^2 - 6396*a*x - 1820*a^3*x^3 + 315*a^4*x^4 + 5419))/(3465*a*(a*x - 1))
```

3.227 $\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

3.227.1 Optimal result

Integrand size = 18, antiderivative size = 197

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{568\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{315a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2}+\frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

output
```
-8/21*(1+1/a/x)^(3/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)-568/315*(1+1/a/x)
^(3/2)*(-a*c*x+c)^(7/2)/a^3/(1-1/a/x)^(7/2)/x^2+48/35*(1+1/a/x)^(3/2)*(-a*
c*x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(a-1/x)^3*(1+1/a/x)^(3/2)*x*(-a*c*x
+c)^(7/2)/a^3/(1-1/a/x)^(7/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{2c^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}\left(-319+2ax+156a^2x^2-130a^3x^3+35a^4x^4\right)}{315a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

output
```
(-2*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-319 + 2*a*x + 156*a^2*x^2 - 13
0*a^3*x^3 + 35*a^4*x^4))/(315*a*Sqrt[1 - 1/(a*x)])
```

3.227.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6727, 27, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}{a^3\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{4}{3}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{4}{3}\left(\frac{2}{7}\int -\frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{4}{3}\left(-\frac{1}{7}\int \frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow 87$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{4}{3}\left(\frac{1}{7}\left(\frac{71}{5}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}\left(-\frac{4}{3}\left(\frac{1}{7}\left(\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}-\frac{142\left(\frac{1}{ax}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)(c-acx)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

output
```
-((((-4*(((36*a*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) - (142*(1 + 1/(a*x
))^(3/2))/(15*(x^(-1))^(3/2)))/7 - (2*a^2*(1 + 1/(a*x))^(3/2))/(7*(x^(-1))
^(7/2))))/3 - (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2))/(9*(x^(-1))^(9/2)))*(
x^(-1))^(7/2)*(c - a*c*x)^(7/2))/(a^3*(1 - 1/(a*x))^(7/2)))
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.227.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.31

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\,c^{3}(ax+1)\left(35a^{3}x^{3}-165a^{2}x^{2}+321ax-319\right)}{315\sqrt{\frac{ax-1}{ax+1}}\,a}$	61
gosper	$\frac{2(ax+1)\left(35a^{3}x^{3}-165a^{2}x^{2}+321ax-319\right)(-acx+c)^{\frac{7}{2}}}{315a(ax-1)^{3}\sqrt{\frac{ax-1}{ax+1}}}$	64
risch	$\frac{2c^{4}(ax-1)\left(35a^{4}x^{4}-130a^{3}x^{3}+156a^{2}x^{2}+2ax-319\right)}{315\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	69

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^3*(a*x+1)*(35*a^3*x^3-
165*a^2*x^2+321*a*x-319)/a
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(35\,a^5c^3x^5-95\,a^4c^3x^4+26\,a^3c^3x^3+158\,a^2c^3x^2-317\,ac^3x-319\,c^3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^2x-a)}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

output

```
-2/315*(35*a^5*c^3*x^5 - 95*a^4*c^3*x^4 + 26*a^3*c^3*x^3 + 158*a^2*c^3*x^2
 - 317*a*c^3*x - 319*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

3.227.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Timed out}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(7/2),x)
```

output

```
Timed out
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\left(35\,a^4\sqrt{-c}c^3x^4-130\,a^3\sqrt{-c}c^3x^3+156\,a^2\sqrt{-c}c^3x^2+2\,a\sqrt{-c}c^3x-319\,\sqrt{-c}c^3\right)\sqrt{ax+1}}{315\,a}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

output
```
-2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 130*a^3*sqrt(-c)*c^3*x^3 + 156*a^2*sqrt(
-c)*c^3*x^2 + 2*a*sqrt(-c)*c^3*x - 319*sqrt(-c)*c^3)*sqrt(a*x + 1)/a
```

3.227.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.227.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-35\,a^4\,x^4+60\,a^3\,x^3+34\,a^2\,x^2-124\,a\,x+193\right)}{315\,a}+\frac{1024\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(34*a^2*x^2 - 124*a*x
 + 60*a^3*x^3 - 35*a^4*x^4 + 193))/(315*a) + (1024*c^3*(c - a*c*x)^(1/2)*(
(a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

3.228 $\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.228.1 Optimal result

Integrand size = 18, antiderivative size = 115

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{64a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{105(c-acx)^{3/2}}+\frac{16a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{35\sqrt{c-acx}}+\frac{2}{7}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\sqrt{c-acx}$$

output
```
64/105*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+16/35*a^2*c^3*(1-1
/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)+2/7*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3*(
-a*c*x+c)^(1/2)
```

3.228.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(71+17ax-39a^2x^2+15a^3x^3)}{105a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

output
```
(2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(71 + 17*a*x - 39*a^2*x^2 + 15*a^
3*x^3))/(105*a*Sqrt[1 - 1/(a*x)])
```

3.228.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6727$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}{a^2\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \; 100$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(\frac{2}{7}\int -\frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{1}{7}\int \frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \; 87$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(\frac{1}{7}\left(\frac{71}{5}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{7}\left(\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}-\frac{142\left(\frac{1}{ax}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)(c-acx)^{5/2}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

output
```
-(((((36*a*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) - (142*(1 + 1/(a*x))^(3
/2))/(15*(x^(-1))^(3/2)))/7 - (2*a^2*(1 + 1/(a*x))^(3/2))/(7*(x^(-1))^(7/2
)))*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))/(a^2*(1 - 1/(a*x))^(5/2)))
```

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.228.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\, c^{2}(ax+1)\left(15a^{2}x^{2}-54ax+71\right)}{105\sqrt{\frac{ax-1}{ax+1}}\, a}$	53
gosper	$\frac{2(ax+1)\left(15a^{2}x^{2}-54ax+71\right)\left(-acx+c\right)^{\frac{5}{2}}}{105a\left(ax-1\right)^{2}\sqrt{\frac{ax-1}{ax+1}}}$	56
risch	$-\frac{2c^{3}(ax-1)\left(15a^{3}x^{3}-39a^{2}x^{2}+17ax+71\right)}{105\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\, a}$	61

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^2*(a*x+1)*(15*a^2*x^2-5
4*a*x+71)/a
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(15\,a^{4}c^{2}x^{4}-24\,a^{3}c^{2}x^{3}-22\,a^{2}c^{2}x^{2}+88\,ac^{2}x+71\,c^{2}\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^{2}x-a\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

output
```
2/105*(15*a^4*c^2*x^4 - 24*a^3*c^2*x^3 - 22*a^2*c^2*x^2 + 88*a*c^2*x + 71*
c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(15\,a^{3}\sqrt{-c}c^{2}x^{3}-39\,a^{2}\sqrt{-c}c^{2}x^{2}+17\,a\sqrt{-c}c^{2}x+71\,\sqrt{-c}c^{2}\right)\sqrt{ax+1}}{105\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

output
```
2/105*(15*a^3*sqrt(-c)*c^2*x^3 - 39*a^2*sqrt(-c)*c^2*x^2 + 17*a*sqrt(-c)*c
^2*x + 71*sqrt(-c)*c^2)*sqrt(a*x + 1)/a
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(64\,\sqrt{2}\sqrt{-c}c-\frac{15\,(acx+c)^{3}\sqrt{-acx-c}-84\,(acx+c)^{2}\sqrt{-acx-c}c-140\,(-acx-c)^{\frac{3}{2}}c^{2}}{c^{2}}\right)c^{2}}{105\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
2/105*(64*sqrt(2)*sqrt(-c)*c - (15*(a*c*x + c)^3*sqrt(-a*c*x - c) - 84*(a*
c*x + c)^2*sqrt(-a*c*x - c)*c - 140*(-a*c*x - c)^(3/2)*c^2)/c^2)*c^2/(a*ab
s(c)*sgn(a*x + 1))
```

3.228.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^2\,x^2-54\,a\,x+71\right)}{105\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x
^2 - 54*a*x + 71))/(105*a*(a*x - 1))
```

3.229 $\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

3.229.1 Optimal result

Integrand size = 18, antiderivative size = 77

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{8a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{15(c-acx)^{3/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{5\sqrt{c-acx}}$$

output
```
8/15*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+2/5*a^2*c^2*(1-1/a^2
/x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)
```

3.229.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\sqrt{1+\frac{1}{ax}}(1+ax)(-7+3ax)\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

output
```
(-2*c*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a*Sqrt
[1 - 1/(a*x)])
```

3.229.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}{a\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{7}{5}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{14\left(\frac{1}{ax}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}-\frac{2a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)(c-acx)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

output
```
-((((-2*a*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) + (14*(1 + 1/(a*x))^(3/2
))/(15*(x^(-1))^(3/2)))*(x^(-1))^(3/2)*(c - a*c*x)^(3/2))/(a*(1 - 1/(a*x))
^(3/2)))
```

3.229.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.229.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\, c(ax+1)(3ax-7)}{15\sqrt{\frac{ax-1}{ax+1}}\, a}$	43
gosper	$\frac{2(ax+1)(3ax-7)(-acx+c)^{\frac{3}{2}}}{15a(ax-1)\sqrt{\frac{ax-1}{ax+1}}}$	48
risch	$\frac{2c^2(ax-1)(3a^2x^2-4ax-7)}{15\sqrt{\frac{ax-1}{ax+1}}\, \sqrt{-c(ax-1)}\, a}$	53

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c*(a*x+1)*(3*a*x-7)/a
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\,(3\,a^3cx^3-a^2cx^2-11\,acx-7\,c)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

output
```
-2/15*(3*a^3*c*x^3 - a^2*c*x^2 - 11*a*c*x - 7*c)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

3.229.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int\frac{(-c(ax-1))^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1))**(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\,\left(3\,a^2\sqrt{-c}cx^2-4\,a\sqrt{-c}cx-7\,\sqrt{-c}c\right)\sqrt{ax+1}}{15\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

output
```
-2/15*(3*a^2*sqrt(-c)*c*x^2 - 4*a*sqrt(-c)*c*x - 7*sqrt(-c)*c)*sqrt(a*x +
1)/a
```

3.229.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.229.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,c\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\left(3\,a\,x-7\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
-(2*c*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 7)*((a*x - 1)/(a*x + 1))^(1/2
))/(15*a*(a*x - 1))
```

3.230 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.230.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

3.230.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

output
```
(2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

3.230.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6726$$

$$\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

output
```
(2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

3.230.3.1 Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

3.230.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	35
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
risch	$-\frac{2c(ax+1)(ax-1)}{3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	42

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

3.230.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)\sqrt{ax+1}}{3\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,c^2\left(\frac{2\,\sqrt{2}\sqrt{-c}}{c}+\frac{(-acx-c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*c^2*(2*sqrt(2)*sqrt(-c)/c + (-a*c*x - c)^(3/2)/c^2)/(a*abs(c)*sgn(a*x
+ 1))
```

3.230.9 Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,{(a\,x+1)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x -
1))
```

3.231 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.231.1 Optimal result

Integrand size = 18, antiderivative size = 118

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-acx}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

output
```
2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)-2*arctanh(2^(1/2)*(1/
x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a^(1/2)/(1/x)^(1
/2)/(-a*c*x+c)^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}\,x\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}-\sqrt{2}\sqrt{\frac{1}{x}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\sqrt{c-acx}}$$

input
```
Integrate[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[1 - 1/(a*x)]*x*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[2]*Sqrt[x^(-1)]*A
rcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt
[c - a*c*x])
```

3.231.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6727, 27, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \textcolor{blue}{6727}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \textcolor{blue}{105}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \textcolor{blue}{104}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

3.231. $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$

input
```
Int[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[1 - 1/(a*x)]*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[
2]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/
(Sqrt[x^(-1)]*Sqrt[c - a*c*x]))
```

3.231.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.231.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\left(\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-\sqrt{-c(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}\,ca}$	83
risch	$\frac{2ax-2}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a\sqrt{c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	115

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*2^(1/2)*arctan(1/2*(
-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/
c/a
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.03

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$

$$= \left[\frac{\sqrt{2}(acx-c)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2ax-3}{a^2x^2-2ax+1}\right)-2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac},\right.$$

$$\left.-\frac{2\left(\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}-\frac{\sqrt{2}(acx-c)\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{\sqrt{c}}\right)}{a^2cx-ac}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt(2)*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2
*a*x + 1)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*
c*x - a*c), -2*(sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - sqr
t(2)*(a*c*x - c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))
/((a*x - 1)*sqrt(c)))/sqrt(c))/(a^2*c*x - a*c)]
```

3.231.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))), x)
```

3.231.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\,c\left(\frac{\sqrt{2}\left(\sqrt{c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{-c}\right)}{c}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-\sqrt{-acx-c}}{c}\right)}{a|c|\mathrm{sgn}\left(ax+1\right)}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output

```
2*c*(sqrt(2)*(sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - sqrt(-c))/c - (sqrt(2)*sq
rt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - sqrt(-a*c*x - c))/c)/
(a*abs(c)*sgn(a*x + 1))
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input

```
int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output

```
int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.232 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

3.232.1 Optimal result

Integrand size = 18, antiderivative size = 128

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

output
```
-1/2*(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*
a^(1/2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)-a*(1-1/a/x)^(3/2)*x*(1+1/a/x)
^(1/2)/(a-1/x)/(-a*c*x+c)^(3/2)
```

3.232.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{2\sqrt{a}c(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(
-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2
*Sqrt[a]*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

3.232.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6727, 27, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \; 6727$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{a^2\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \; 104$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

output
```
-((a^2*(1 - 1/(a*x))^(3/2)*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1
))) + ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]
*a^(3/2))))/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

3.232.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.232.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\sqrt{-c(ax-1)}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c+2\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)\sqrt{-c(ax+1)}\,c^{\frac{5}{2}}\,a}$	118

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a*x-1))^(1/2)*(2^(1/2)*arctan(1/
2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-2^(1/2)*arctan(1/2*(-c*(a*x+1)
)^(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2
)/c^(5/2)/a
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.20

$$\int\frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\left[-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-}}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}\right.$$

$$\left.-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

output
```
[-1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x -
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/
(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/2*(sqrt(2)*(a^2*x^2 -
2*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/
(a*x + 1))/(a*c*x - c)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

3.232.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**(3/2)), x)
```

3.232.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\int \frac{1}{(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.47

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{-acx-c}}{acx-c}}{2\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + 2*sqrt
(-a*c*x - c)/(a*c*x - c))/(a*abs(c))
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.233 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.233.1 Optimal result

Integrand size = 18, antiderivative size = 193

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

output
```
-1/4*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)+1/
16*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(
1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+1/8*a^2*(1-1/a/x)^(5/2)*x^2*(1+
1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\,x\left(-2\sqrt{a}\sqrt{1+\frac{1}{ax}}(3+ax)+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{16\sqrt{a}c^2(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(3 + a*x) + Sqrt[2]*Sqr
t[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/
(a*x)])]))/(16*Sqrt[a]*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

3.233.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \ 6727$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{a^3\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}-\frac{1}{8}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{8}\left(-\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \ 104$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{8}\left(-\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow 219$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{8}\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

output
```
-((a^3*(1 - 1/(a*x))^(5/2)*(((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(4*(a - x^(
-1))^2) + (-((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1)))) - ArcTanh[
(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2)))/8))
/((x^(-1))^(5/2)*(c - a*c*x)^(5/2)))
```

3.233.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.233.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

method	result
default	$\frac{\sqrt{-c(ax-1)}\,\left(-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}+2\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+2ax\sqrt{c}\,\sqrt{-c(ax+1)}-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+}}{2\sqrt{c}}\right.\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)^{2}c^{\frac{7}{2}}\,\sqrt{-c(ax+1)}\,a}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-c*(a*x-1))^(1/2)*(-2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^
(1/2))*a^2*c*x^2+2*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*
a*c*x+2*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)-2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/
2)*2^(1/2)/c^(1/2))*c+6*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(1/2
)/(a*x-1)^2/c^(7/2)/(-c*(a*x+1))^(1/2)/a
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.75

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)}{32\,(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3})}\right.$$

$$\left.-\frac{\sqrt{2}(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(a^{2}x^{2}+4\,ax+3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{16\,(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3})}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

output
```
[-1/32*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2
 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 + 4*a*x + 3)*sqrt(-a
*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*
c^3*x - a*c^3), -1/16*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*a
rctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x -
c)) - 2*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

3.233.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left((-acx-c)^{\frac{3}{2}}-2\sqrt{-acx-c}c\right)}{(acx-c)^2c}}{16\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
1/16*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) + 2*((-
a*c*x - c)^(3/2) - 2*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c))/(a*abs(c))
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.234 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

3.234.1 Optimal result

Integrand size = 18, antiderivative size = 250

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

output
```
-1/6*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(7/2)+1/
16*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)-1/64
*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/
2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/32*a^3*(1-1/a/x)^(7/2)*x^3*(1+1
/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{1+\frac{1}{ax}}(25+10ax-3a^2x^2)}{\sqrt{\frac{1}{x}}} + 3\sqrt{2}(-1+ax)^3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{192\sqrt{a}c^3\sqrt{\frac{1}{x}}(-1+ax)^3\sqrt{c-acx}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(25 + 10*a*x - 3*a^2*x^2)
)/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])]))/(192*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 + a*x)^3*Sqr
t[c - a*c*x])
```

3.234.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6727, 27, 105, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{a^4\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}-\frac{1}{4}\int \frac{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{4}\left(\frac{1}{8}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)+\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow 105$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)+\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow 104$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)+\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow 219$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)+\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

output
```
-((a^4*(1 - 1/(a*x))^(7/2)*(((1 + 1/(a*x))^(3/2)*(x^(-1))^(3/2))/(6*(a - x
^(-1))^3) + (-1/4*((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(a - x^(-1))^2 + ((Sq
rt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1))) + ArcTanh[(Sqrt[2]*Sqrt[x^(
-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2)))/8)/4))/((x^(-1))^(7/
2)*(c - a*c*x)^(7/2)))
```

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.234.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.88

method	result
default	$\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{3}c\,x^{3}+9\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}+6a^{2}x^{2}\sqrt{-c(ax+1)}\,\sqrt{c}-9\sqrt{2}\arctan\left(\sqrt{\right.\right.}{192\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)^{3}c^{\frac{9}{2}}\sqrt{-c(ax+1)}\,a}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)
/c^(1/2))*a^3*c*x^3+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2
))*a^2*c*x^2+6*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)-9*2^(1/2)*arctan(1/2*(-c
*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-20*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+3
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-50*(-c*(a*x+1))^
(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^3/c^(9/2)/(-c*(a*x+1))^(1/2
)/a
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.57

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-}}{a^2x^2-2\,ax+1}\right.}{384\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2}\right.$$

$$-\frac{3\sqrt{2}(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(3\,a^3x^3-7\,a^2x^2-35\,ax-2}{192\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

output
```
[-1/384*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*
log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*
sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 - 7
*a^2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c
^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/192*(3*s
qrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2
)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a
^3*x^3 - 7*a^2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1
)))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

3.234.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.234.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}} - \frac{2\left(3(acx+c)^2\sqrt{-acx-c}+16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}c^2\right)}{(acx-c)^3c^2}}{192a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
1/192*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*
(3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c
*x - c)*c^2)/((a*c*x - c)^3*c^2))/(a*abs(c))
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.235 $\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

3.235.1 Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

output
```
4/7*(-a*c*x+c)^(7/2)/a-2/9*(-a*c*x+c)^(9/2)/a/c
```

3.235.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{2c^3(-1+ax)^3(11+7ax)\sqrt{c-acx}}{63a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

output
```
(-2*c^3*(-1 + a*x)^3*(11 + 7*a*x)*Sqrt[c - a*c*x])/(63*a)
```

3.235.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}(c-acx)^{7/2}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(ax+1)(c-acx)^{7/2}}{1-ax}dx$$

$$\Big\downarrow \text{ 35}$$

$$-c\int (ax+1)(c-acx)^{5/2}dx$$

$$\Big\downarrow \text{ 53}$$

$$-c\int \left(2(c-acx)^{5/2}-\frac{(c-acx)^{7/2}}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c\left(\frac{2(c-acx)^{9/2}}{9ac^2}-\frac{4(c-acx)^{7/2}}{7ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

output
```
-(c*((-4*(c - a*c*x)^(7/2))/(7*a*c) + (2*(c - a*c*x)^(9/2))/(9*a*c^2)))
```

3.235.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.235.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{7}{2}}(7ax+11)}{63a}$	21
pseudoelliptic	$-\frac{2\sqrt{-c(ax-1)}\,(ax-1)^{3}(ax+\frac{11}{7})c^{3}}{9a}$	31
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{9}{2}}}{9}+\frac{4c(-acx+c)^{\frac{7}{2}}}{7}}{ac}$	33
trager	$-\frac{2c^{3}(7a^{4}x^{4}-10a^{3}x^{3}-12a^{2}x^{2}+26ax-11)\sqrt{-acx+c}}{63a}$	48
risch	$\frac{2c^{4}(7a^{4}x^{4}-10a^{3}x^{3}-12a^{2}x^{2}+26ax-11)(ax-1)}{63a\sqrt{-c(ax-1)}}$	54

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/63*(-a*c*x+c)^(7/2)*(7*a*x+11)/a
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(7\,a^4c^3x^4-10\,a^3c^3x^3-12\,a^2c^3x^2+26\,ac^3x-11\,c^3)\sqrt{-acx+c}}{63\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/63*(7*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 12*a^2*c^3*x^2 + 26*a*c^3*x - 11*c
^3)*sqrt(-a*c*x + c)/a
```

3.235.6 Sympy [A] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \begin{cases} -\frac{2\left(-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{ac} & \text{for } ac\neq 0 \\ c^{\frac{7}{2}}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(7/2),x)
```

output
```
Piecewise((-2*(-2*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a*c),
Ne(a*c, 0)), (c**(7/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) -
1)/a, True)), True))
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{2\left(7\left(-acx+c\right)^{\frac{9}{2}}-18\left(-acx+c\right)^{\frac{7}{2}}c\right)}{63\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-2/63*(7*(-a*c*x + c)^(9/2) - 18*(-a*c*x + c)^(7/2)*c)/(a*c)
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.12

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{2\left(90\left(acx-c\right)^{3}\sqrt{-acx+c}+378\left(acx-c\right)^{2}\sqrt{-acx+c}c-630\left(-acx+c\right)^{\frac{3}{2}}c^{2}+945\sqrt{-ac}\right.}{}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
2/315*(90*(a*c*x - c)^3*sqrt(-a*c*x + c) + 378*(a*c*x - c)^2*sqrt(-a*c*x +
 c)*c - 630*(-a*c*x + c)^(3/2)*c^2 + 945*sqrt(-a*c*x + c)*c^3 + 210*((-a*c
*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c^2 - (35*(a*c*x - c)^4*sqrt(-a*c*x
+ c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 378*(a*c*x - c)^2*sqrt(-a*c*
x + c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sqrt(-a*c*x + c)*c^4)/c)/a
```

3.235.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{4\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a\,c}$$

input
```
int(((c - a*c*x)^(7/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

3.236 $\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.236.1 Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

output
```
4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c
```

3.236.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2(-1+ax)^2(9+5ax)\sqrt{c-acx}}{35a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
(2*c^2*(-1 + a*x)^2*(9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a)
```

3.236.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}(c-acx)^{5/2}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)(c-acx)^{5/2}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int (ax+1)(c-acx)^{3/2}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(2(c-acx)^{3/2}-\frac{(c-acx)^{5/2}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2(c-acx)^{7/2}}{7ac^2}-\frac{4(c-acx)^{5/2}}{5ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
-(c*((-4*(c - a*c*x)^(5/2))/(5*a*c) + (2*(c - a*c*x)^(7/2))/(7*a*c^2)))
```

3.236.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.236.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{5}{2}}(5ax+9)}{35a}$	21
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(ax+\frac{9}{5})(ax-1)^{2}c^{2}}{7a}$	31
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{4c(-acx+c)^{\frac{5}{2}}}{5}}{ac}$	33
trager	$\frac{2c^{2}(5a^{3}x^{3}-a^{2}x^{2}-13ax+9)\sqrt{-acx+c}}{35a}$	40
risch	$-\frac{2c^{3}(5a^{3}x^{3}-a^{2}x^{2}-13ax+9)(ax-1)}{35a\sqrt{-c(ax-1)}}$	46

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(-a*c*x+c)^(5/2)*(5*a*x+9)/a
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,(5\,a^3c^2x^3-a^2c^2x^2-13\,ac^2x+9\,c^2)\sqrt{-acx+c}}{35\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^3*c^2*x^3 - a^2*c^2*x^2 - 13*a*c^2*x + 9*c^2)*sqrt(-a*c*x + c)/a
```

3.236.6 Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\begin{cases}-\frac{2\left(-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{ac} & \text{for } ac\neq 0\\ c^{\frac{5}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log{(ax-1)}-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(5/2),x)
```

output
```
Piecewise((-2*(-2*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a*c),
Ne(a*c, 0)), (c**(5/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) -
1)/a, True)), True))
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{2\left(5\,(-acx+c)^{\frac{7}{2}} - 14\,(-acx+c)^{\frac{5}{2}}c\right)}{35\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-2/35*(5*(-a*c*x + c)^(7/2) - 14*(-a*c*x + c)^(5/2)*c)/(a*c)
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.52

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx =$$

$$-\frac{2\left(21\,(acx-c)^{2}\sqrt{-acx+c} - 70\,(-acx+c)^{\frac{3}{2}}c - 35\left((-acx+c)^{\frac{3}{2}} - 3\,\sqrt{-acx+c}c\right)c - \frac{3\left(5\,(acx-c)^{3}\sqrt{-acx-}\right.}{}\right.}{105\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2/105*(21*(a*c*x - c)^2*sqrt(-a*c*x + c) - 70*(-a*c*x + c)^(3/2)*c - 35*(
(-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c - 3*(5*(a*c*x - c)^3*sqrt(-a*
c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2
 + 35*sqrt(-a*c*x + c)*c^3)/c)/a
```

3.236.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{4\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a} - \frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a\,c}$$

input
```
int(((c - a*c*x)^(5/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

3.237 $\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

3.237.1 Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

output
```
4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c
```

3.237.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c(-1+ax)(7+3ax)\sqrt{c-acx}}{15a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(-2*c*(-1 + a*x)*(7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a)
```

3.237.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}(c-acx)^{3/2}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(ax+1)(c-acx)^{3/2}}{1-ax}dx$$

$$\Big\downarrow \text{ 35}$$

$$-c\int (ax+1)\sqrt{c-acx}dx$$

$$\Big\downarrow \text{ 53}$$

$$-c\int \left(2\sqrt{c-acx}-\frac{(c-acx)^{3/2}}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c\left(\frac{2(c-acx)^{5/2}}{5ac^2}-\frac{4(c-acx)^{3/2}}{3ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
-(c*((-4*(c - a*c*x)^(3/2))/(3*a*c) + (2*(c - a*c*x)^(5/2))/(5*a*c^2)))
```

3.237.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.237.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{3}{2}}(3ax+7)}{15a}$	21
pseudoelliptic	$-\frac{2\sqrt{-c(ax-1)}\,(ax+\frac{7}{3})(ax-1)c}{5a}$	27
trager	$-\frac{2c(3a^2x^2+4ax-7)\sqrt{-acx+c}}{15a}$	30
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}-\frac{2c(-acx+c)^{\frac{3}{2}}}{3}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{4c(-acx+c)^{\frac{3}{2}}}{3}}{ac}$	33
risch	$\frac{2c^2(3a^2x^2+4ax-7)(ax-1)}{15a\sqrt{-c(ax-1)}}$	38

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(-a*c*x+c)^(3/2)*(3*a*x+7)/a
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\,(3\,a^2cx^2+4\,acx-7\,c)\sqrt{-acx+c}}{15\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^2*c*x^2 + 4*a*c*x - 7*c)*sqrt(-a*c*x + c)/a
```

3.237.6 Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\begin{cases}-\frac{2\left(-\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{ac} & \text{for } ac\neq 0\\ c^{\frac{3}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log{(ax-1)}-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(3/2),x)
```

output
```
Piecewise((-2*(-2*c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/(a*c),
Ne(a*c, 0)), (c**(3/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) -
1)/a, True)), True))
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(3\left(-acx+c\right)^{\frac{5}{2}}-10\left(-acx+c\right)^{\frac{3}{2}}c\right)}{15\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/15*(3*(-a*c*x + c)^(5/2) - 10*(-a*c*x + c)^(3/2)*c)/(a*c)
```

3.237.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{2\left(15\sqrt{-acx+c}c-\frac{3(acx-c)^2\sqrt{-acx+c}-10(-acx+c)^{\frac{3}{2}}c+15\sqrt{-acx+c}c^2}{c}\right)}{15\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(-a*c*x + c)*c - (3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c
*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/c)/a
```

3.237.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{4\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a\,c}$$

input
```
int(((c - a*c*x)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

3.238 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.238.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

output
```
-2/3*(-a*c*x+c)^(3/2)/a/c+4*(-a*c*x+c)^(1/2)/a
```

3.238.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(5+ax)\sqrt{c-acx}}{3a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

3.238.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2(c-acx)^{3/2}}{3ac^2}-\frac{4\sqrt{c-acx}}{ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a*c) + (2*(c - a*c*x)^(3/2))/(3*a*c^2)))
```

3.238.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.238.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
trager	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(ax+5)}{3a}$	21
risch	$-\frac{2c(ax+5)(ax-1)}{3a\sqrt{-c(ax-1)}}$	27
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2c\sqrt{-acx+c}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}+4c\sqrt{-acx+c}}{ac}$	33

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{-acx+c}(ax+5)}{3\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

3.238.6 Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\begin{cases}-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0\\ \sqrt{c}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*
c, 0)), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a,
True)), True))
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c}{c}\right)}{3\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

3.238.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

3.239 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.239.1 Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{4}{a\sqrt{c-acx}} - \frac{2\sqrt{c-acx}}{ac}$$

output
```
-4/a/(-a*c*x+c)^(1/2)-2*(-a*c*x+c)^(1/2)/a/c
```

3.239.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{-6+2ax}{a\sqrt{c-acx}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
(-6 + 2*a*x)/(a*Sqrt[c - a*c*x])
```

3.239.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{ax+1}{(1-ax)\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(\frac{2}{(c-acx)^{3/2}}-\frac{1}{c\sqrt{c-acx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2\sqrt{c-acx}}{ac^2}+\frac{4}{ac\sqrt{c-acx}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
-(c*(4/(a*c*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x])/(a*c^2)))
```

3.239. $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$

3.239.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.239.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{2ax-6}{a\sqrt{-acx+c}}$	20
pseudoelliptic	$\frac{2ax-6}{a\sqrt{-c(ax-1)}}$	21
trager	$-\frac{2(ax-3)\sqrt{-acx+c}}{ca(ax-1)}$	30
derivativedivides	$-\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ca}$	31
default	$\frac{-2\sqrt{-acx+c}-\frac{4c}{\sqrt{-acx+c}}}{ac}$	33
risch	$\frac{2ax-2}{a\sqrt{-c(ax-1)}}-\frac{4}{a\sqrt{-c(ax-1)}}$	37

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x-3)/a/(-a*c*x+c)^(1/2)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{-acx+c}(ax-3)}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*c*x + c)*(a*x - 3)/(a^2*c*x - a*c)
```

3.239.6 Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{\sqrt{-acx+c}}+\sqrt{-acx+c}\right)}{ac} & \text{for}\ ac\neq 0 \\ \frac{\begin{cases} -x & \text{for}\ a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(2*c/sqrt(-a*c*x + c) + sqrt(-a*c*x + c))/(a*c), Ne(a*c, 0))
, (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a, True))/sqrt(c)
, True))
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2*(sqrt(-a*c*x + c) + 2*c/sqrt(-a*c*x + c))/(a*c)
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{4}{\sqrt{-acx+c}a} - \frac{2\sqrt{-acx+c}}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-4/(sqrt(-a*c*x + c)*a) - 2*sqrt(-a*c*x + c)/(a*c)
```

3.239.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\,a\,x-6}{a\,\sqrt{c-a\,c\,x}}$$

input
```
int((a*x + 1)/((c - a*c*x)^(1/2)*(a*x - 1)),x)
```

output
```
(2*a*x - 6)/(a*(c - a*c*x)^(1/2))
```

3.240 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

3.240.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{4}{3a(c-acx)^{3/2}}+\frac{2}{ac\sqrt{c-acx}}$$

output
```
-4/3/a/(-a*c*x+c)^(3/2)+2/a/c/(-a*c*x+c)^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{-2+6ax}{3ac(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
(-2 + 6*a*x)/(3*a*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

3.240.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{ax+1}{(1-ax)(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(\frac{2}{(c-acx)^{5/2}}-\frac{1}{c(c-acx)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{4}{3ac(c-acx)^{3/2}}-\frac{2}{ac^2\sqrt{c-acx}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
-(c*(4/(3*a*c*(c - a*c*x)^(3/2)) - 2/(a*c^2*Sqrt[c - a*c*x])))
```

3.240.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.240.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(3ax-1)}{3a(-acx+c)^{\frac{3}{2}}}$	21
default	$\frac{\frac{2}{\sqrt{-acx+c}}-\frac{4c}{3(-acx+c)^{\frac{3}{2}}}}{ac}$	31
trager	$-\frac{2(3ax-1)\sqrt{-acx+c}}{3c^{2}(ax-1)^{2}a}$	31
pseudoelliptic	$\frac{6ax-2}{3ac(ax-1)\sqrt{-c(ax-1)}}$	32
derivativedivides	$-\frac{2\left(-\frac{1}{\sqrt{-acx+c}}+\frac{2c}{3(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	33

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*a*x-1)/a/(-a*c*x+c)^(3/2)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\sqrt{-acx+c}(3\,ax-1)}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-a*c*x + c)*(3*a*x - 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

3.240.6 Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{3(-acx+c)^{\frac{3}{2}}}-\frac{1}{\sqrt{-acx+c}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log{(ax-1)}-1}{a} & \text{otherwise} \end{cases}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(3/2),x)
```

output
```
Piecewise((-2*(2*c/(3*(-a*c*x + c)**(3/2)) - 1/sqrt(-a*c*x + c))/(a*c), Ne
(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a, True)
)/c**(3/2), True))
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\,(3\,acx-c)}{3\,(-acx+c)^{\frac{3}{2}}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/3*(3*a*c*x - c)/((-a*c*x + c)^(3/2)*a*c)
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\,(3\,acx-c)}{3\,(acx-c)\sqrt{-acx+c}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
2/3*(3*a*c*x - c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c)
```

3.240.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{6\,a\,x-2}{3\,a\,(c-a\,c\,x)^{3/2}}$$

input
```
int((a*x + 1)/((c - a*c*x)^(3/2)*(a*x - 1)),x)
```

output
```
-(6*a*x - 2)/(3*a*(c - a*c*x)^(3/2))
```

3.241 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.241.1 Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=-\frac{4}{5a(c-acx)^{5/2}}+\frac{2}{3ac(c-acx)^{3/2}}$$

output
```
-4/5/a/(-a*c*x+c)^(5/2)+2/3/a/c/(-a*c*x+c)^(3/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=-\frac{2(1+5ax)}{15ac^2(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
(-2*(1 + 5*a*x))/(15*a*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

3.241.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{ax+1}{(1-ax)(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{ 35}$$

$$-c\int \frac{ax+1}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{ 53}$$

$$-c\int \left(\frac{2}{(c-acx)^{7/2}}-\frac{1}{c(c-acx)^{5/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c\left(\frac{4}{5ac(c-acx)^{5/2}}-\frac{2}{3ac^2(c-acx)^{3/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
-(c*(4/(5*a*c*(c - a*c*x)^(5/2)) - 2/(3*a*c^2*(c - a*c*x)^(3/2))))
```

3.241.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.241.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2(5ax+1)}{15(-acx+c)^{\frac{5}{2}} a}$	21
trager	$\frac{2(5ax+1)\sqrt{-acx+c}}{15c^{3}(ax-1)^{3} a}$	31
pseudoelliptic	$\frac{-\frac{2ax}{3}-\frac{2}{15}}{c^{2}(ax-1)^{2} \sqrt{-c(ax-1)}\, a}$	32
derivativedivides	$-\frac{2\left(\frac{2c}{5(-acx+c)^{\frac{5}{2}}}-\frac{1}{3(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	33
default	$\frac{\frac{2}{3(-acx+c)^{\frac{3}{2}}}-\frac{4c}{5(-acx+c)^{\frac{5}{2}}}}{ac}$	33

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(5*a*x+1)/(-a*c*x+c)^(5/2)/a
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{2\sqrt{-acx+c}(5\,ax+1)}{15\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/15*sqrt(-a*c*x + c)*(5*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)
```

3.241.6 Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{5(-acx+c)^{\frac{5}{2}}}-\frac{1}{3(-acx+c)^{\frac{3}{2}}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(5/2),x)
```

output
```
Piecewise((-2*(2*c/(5*(-a*c*x + c)**(5/2)) - 1/(3*(-a*c*x + c)**(3/2)))/(a
*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a
, True))/c**(5/2), True))
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2\,(5\,acx+c)}{15\,(-acx+c)^{\frac{5}{2}}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-2/15*(5*a*c*x + c)/((-a*c*x + c)^(5/2)*a*c)
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2\,(5\,acx+c)}{15\,(acx-c)^{2}\sqrt{-acx+c}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2/15*(5*a*c*x + c)/((a*c*x - c)^2*sqrt(-a*c*x + c)*a*c)
```

3.241.9 Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{10\,a\,x+2}{15\,a\,(c-a\,c\,x)^{5/2}}$$

input
```
int((a*x + 1)/((c - a*c*x)^(5/2)*(a*x - 1)),x)
```

output
```
-(10*a*x + 2)/(15*a*(c - a*c*x)^(5/2))
```

3.242 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

3.242.1 Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{4}{7a(c-acx)^{7/2}}+\frac{2}{5ac(c-acx)^{5/2}}$$

output
```
-4/7/a/(-a*c*x+c)^(7/2)+2/5/a/c/(-a*c*x+c)^(5/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{6+14ax}{35ac^3(-1+ax)^3\sqrt{c-acx}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
(6 + 14*a*x)/(35*a*c^3*(-1 + a*x)^3*Sqrt[c - a*c*x])
```

3.242.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{ax+1}{(1-ax)(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{(c-acx)^{9/2}}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(\frac{2}{(c-acx)^{9/2}}-\frac{1}{c(c-acx)^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{4}{7ac(c-acx)^{7/2}}-\frac{2}{5ac^2(c-acx)^{5/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
-(c*(4/(7*a*c*(c - a*c*x)^(7/2)) - 2/(5*a*c^2*(c - a*c*x)^(5/2))))
```

3.242.3.1 Defintions of rubi rules used

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.242.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2(7ax+3)}{35a(-acx+c)^{\frac{7}{2}}}$	21
trager	$-\frac{2(7ax+3)\sqrt{-acx+c}}{35c^{4}(ax-1)^{4}a}$	31
pseudoelliptic	$\frac{\frac{2ax}{5}+\frac{6}{35}}{a\,c^{3}(ax-1)^{3}\sqrt{-c(ax-1)}}$	32
derivativedivides	$-\frac{2\left(-\frac{1}{5(-acx+c)^{\frac{5}{2}}}+\frac{2c}{7(-acx+c)^{\frac{7}{2}}}\right)}{ca}$	33
default	$\frac{\frac{2}{5(-acx+c)^{\frac{5}{2}}}-\frac{4c}{7(-acx+c)^{\frac{7}{2}}}}{ac}$	33

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(7*a*x+3)/a/(-a*c*x+c)^(7/2)
```

3.242.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{2\sqrt{-acx+c}(7\,ax+3)}{35\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/35*sqrt(-a*c*x + c)*(7*a*x + 3)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^
4*x^2 - 4*a^2*c^4*x + a*c^4)
```

3.242.6 Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{7(-acx+c)^{\frac{7}{2}}}-\frac{1}{5(-acx+c)^{\frac{5}{2}}}\right)}{ac} & \text{for}\ ac\neq 0 \\ \frac{\begin{cases} -x & \text{for}\ a=0 \\ \frac{ax+2\log{(ax-1)}-1}{a} & \text{otherwise} \end{cases}}{c^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(7/2),x)
```

output
```
Piecewise((-2*(2*c/(7*(-a*c*x + c)**(7/2)) - 1/(5*(-a*c*x + c)**(5/2)))/(a
*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a
, True))/c**(7/2), True))
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{2\,(7\,acx+3\,c)}{35\,(-acx+c)^{\frac{7}{2}}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-2/35*(7*a*c*x + 3*c)/((-a*c*x + c)^(7/2)*a*c)
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{2\,(7\,acx+3\,c)}{35\,(acx-c)^{3}\sqrt{-acx+c}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
2/35*(7*a*c*x + 3*c)/((a*c*x - c)^3*sqrt(-a*c*x + c)*a*c)
```

3.242.9 Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{14\,a\,x+6}{35\,a\,(c-a\,c\,x)^{7/2}}$$

input
```
int((a*x + 1)/((c - a*c*x)^(7/2)*(a*x - 1)),x)
```

output
```
-(14*a*x + 6)/(35*a*(c - a*c*x)^(7/2))
```

3.243 $\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

3.243.1 Optimal result

Integrand size = 20, antiderivative size = 197

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=-\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{856\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{1155a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

output
```
-8/33*(1+1/a/x)^(5/2)*(-a*c*x+c)^(9/2)/a/(1-1/a/x)^(9/2)-856/1155*(1+1/a/x
)^(5/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+16/21*(1+1/a/x)^(5/2)*(-a
*c*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^3*(1+1/a/x)^(5/2)*x*(-a*c
*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.39

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2c^4\sqrt{1+\frac{1}{ax}}(1+ax)^2\sqrt{c-acx}\left(-533+755ax-455a^2x^2+105a^3x^3\right)}{1155a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

output
```
(2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(-533 + 755*a*x - 455
*a^2*x^2 + 105*a^3*x^3))/(1155*a*Sqrt[1 - 1/(a*x)])
```

3.243.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6727, 27, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{9/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}}{a^3\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{12}{11}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{12}{11}\left(\frac{2}{9}\int -\frac{\left(22a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{2\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{12}{11}\left(-\frac{1}{9}\int\frac{\left(22a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow 87$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{12}{11}\left(\frac{1}{9}\left(\frac{107}{7}\int\frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}+\frac{44a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}\left(-\frac{12}{11}\left(\frac{1}{9}\left(\frac{44a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{214\left(\frac{1}{ax}+1\right)^{5/2}}{35\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)(c-acx)^{9/2}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

output
```
-((((-12*(((44*a*(1 + 1/(a*x))^(5/2))/(7*(x^(-1))^(7/2)) - (214*(1 + 1/(a*
x))^(5/2))/(35*(x^(-1))^(5/2)))/9 - (2*a^2*(1 + 1/(a*x))^(5/2))/(9*(x^(-1)
)^(9/2))))/11 - (2*(a - x^(-1))^3*(1 + 1/(a*x))^(5/2))/(11*(x^(-1))^(11/2)
))*(x^(-1))^(9/2)*(c - a*c*x)^(9/2))/(a^3*(1 - 1/(a*x))^(9/2)))
```

3.243.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.243.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

method	result	size
gosper	$\frac{2(ax+1)\left(105a^{3}x^{3}-455a^{2}x^{2}+755ax-533\right)\left(-acx+c\right)^{\frac{9}{2}}}{1155a\left(ax-1\right)^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64
default	$\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^{4}\left(105a^{3}x^{3}-455a^{2}x^{2}+755ax-533\right)}{1155\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	66
risch	$-\frac{2c^{5}(ax-1)\left(105a^{5}x^{5}-245a^{4}x^{4}-50a^{3}x^{3}+522a^{2}x^{2}-311ax-533\right)}{1155\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	77

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(a*x+1)*(105*a^3*x^3-455*a^2*x^2+755*a*x-533)*(-a*c*x+c)^(9/2)/a/(a
*x-1)^3/((a*x-1)/(a*x+1))^(3/2)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\,(105\,a^6c^4x^6-140\,a^5c^4x^5-295\,a^4c^4x^4+472\,a^3c^4x^3+211\,a^2c^4x^2-844\,ac^4x-533\,c^4)\sqrt{-a}}{1155\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas"
)
```

output
```
2/1155*(105*a^6*c^4*x^6 - 140*a^5*c^4*x^5 - 295*a^4*c^4*x^4 + 472*a^3*c^4*
x^3 + 211*a^2*c^4*x^2 - 844*a*c^4*x - 533*c^4)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1))/(a^2*x - a)
```

3.243.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(9/2),x)
```

output
```
Timed out
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\,\left(105\,a^5\sqrt{-c}c^4x^5-455\,a^4\sqrt{-c}c^4x^4+650\,a^3\sqrt{-c}c^4x^3-78\,a^2\sqrt{-c}c^4x^2-755\,a\sqrt{-c}c^4x+5\right.}{1155\,(ax-1)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima"
)
```

output
```
2/1155*(105*a^5*sqrt(-c)*c^4*x^5 - 455*a^4*sqrt(-c)*c^4*x^4 + 650*a^3*sqrt
(-c)*c^4*x^3 - 78*a^2*sqrt(-c)*c^4*x^2 - 755*a*sqrt(-c)*c^4*x + 533*sqrt(-
c)*c^4)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = -\frac{2\left(512\sqrt{2}\sqrt{-c}c^3+\frac{105\,(acx+c)^5\sqrt{-acx-c}-770\,(acx+c)^4\sqrt{-acx-c}c+1980\,(acx+c)^3\sqrt{-acx-c}c^2-1848\,(acx+c)^2\sqrt{-acx-c}c^3}{c^2}\right)c^2}{1155\,a|c|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

output
```
-2/1155*(512*sqrt(2)*sqrt(-c)*c^3 + (105*(a*c*x + c)^5*sqrt(-a*c*x - c) -
770*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 1980*(a*c*x + c)^3*sqrt(-a*c*x - c)
*c^2 - 1848*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3)/c^2)*c^2/(a*abs(c)*sgn(a*x
 + 1))
```

3.243.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(105\,a^5\,x^5-35\,a^4\,x^4-330\,a^3\,x^3+142\,a^2\,x^2+353\,a\,x-491\right)}{1155\,a} - \frac{2048\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{1155\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(353*a*x + 142*a^2*x^
2 - 330*a^3*x^3 - 35*a^4*x^4 + 105*a^5*x^5 - 491))/(1155*a) - (2048*c^4*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(1155*a*(a*x - 1))
```

3.243. $\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

3.244 $\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

3.244.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{214\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{315a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

output
```
-44/63*(1+1/a/x)^(5/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)+214/315*(1+1/a/x
)^(5/2)*(-a*c*x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(1+1/a/x)^(5/2)*x*(-a*c
*x+c)^(7/2)/(1-1/a/x)^(7/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.50

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{2c^3\sqrt{1+\frac{1}{ax}}(1+ax)^2\sqrt{c-acx}(107-110ax+35a^2x^2)}{315a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

output
```
(-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(107 - 110*a*x + 35*
a^2*x^2))/(315*a*Sqrt[1 - 1/(a*x)])
```

3.244.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}{a^2\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(\frac{2}{9}\int -\frac{\left(22a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{2\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{1}{9}\int \frac{\left(22a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(\frac{1}{9}\left(\frac{107}{7}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}+\frac{44a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}\left(\frac{1}{9}\left(\frac{44a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{214\left(\frac{1}{ax}+1\right)^{5/2}}{35\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)(c-acx)^{7/2}}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

output
```
-(((((44*a*(1 + 1/(a*x))^(5/2))/(7*(x^(-1))^(7/2)) - (214*(1 + 1/(a*x))^(5
/2))/(35*(x^(-1))^(5/2)))/9 - (2*a^2*(1 + 1/(a*x))^(5/2))/(9*(x^(-1))^(9/2
)))*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))/(a^2*(1 - 1/(a*x))^(7/2)))
```

3.244.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.244.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{2(ax+1)(35a^2x^2-110ax+107)(-acx+c)^{\frac{7}{2}}}{315a(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	56
default	$-\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^3(35a^2x^2-110ax+107)}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
risch	$\frac{2c^4(ax-1)(35a^4x^4-40a^3x^3-78a^2x^2+104ax+107)}{315\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	69

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(a*x+1)*(35*a^2*x^2-110*a*x+107)*(-a*c*x+c)^(7/2)/a/(a*x-1)^2/((a*x-
1)/(a*x+1))^(3/2)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx =$$
$$-\frac{2\,(35\,a^5c^3x^5-5\,a^4c^3x^4-118\,a^3c^3x^3+26\,a^2c^3x^2+211\,ac^3x+107\,c^3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

output
```
-2/315*(35*a^5*c^3*x^5 - 5*a^4*c^3*x^4 - 118*a^3*c^3*x^3 + 26*a^2*c^3*x^2
+ 211*a*c^3*x + 107*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=$$
$$-\frac{2\left(35\,a^4\sqrt{-c}c^3x^4-110\,a^3\sqrt{-c}c^3x^3+72\,a^2\sqrt{-c}c^3x^2+110\,a\sqrt{-c}c^3x-107\,\sqrt{-c}c^3\right)(ax+1)^{\frac{3}{2}}}{315\,(ax-1)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

output
```
-2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 110*a^3*sqrt(-c)*c^3*x^3 + 72*a^2*sqrt(-
c)*c^3*x^2 + 110*a*sqrt(-c)*c^3*x - 107*sqrt(-c)*c^3)*(a*x + 1)^(3/2)/((a*
x - 1)*a)
```

3.244.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.244.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(35\,a^4\,x^4+30\,a^3\,x^3-88\,a^2\,x^2-62\,a\,x+149\right)}{315\,a} - \frac{512\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(30*a^3*x^3 - 88*a^
2*x^2 - 62*a*x + 35*a^4*x^4 + 149))/(315*a) - (512*c^3*(c - a*c*x)^(1/2)*(
(a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

3.245 $\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.245.1 Optimal result

Integrand size = 20, antiderivative size = 89

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{18\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

output
```
-18/35*(1+1/a/x)^(5/2)*(-a*c*x+c)^(5/2)/a/(1-1/a/x)^(5/2)+2/7*(1+1/a/x)^(5
/2)*x*(-a*c*x+c)^(5/2)/(1-1/a/x)^(5/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(-9+5ax)\sqrt{c-acx}(c+acx)^2}{35a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*(-9 + 5*a*x)*Sqrt[c - a*c*x]*(c + a*c*x)^2)/(35*a*Sqr
t[1 - 1/(a*x)])
```

3.245.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow 6727$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{a\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 27$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 87$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{9}{7}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}\left(\frac{18\left(\frac{1}{ax}+1\right)^{5/2}}{35\left(\frac{1}{x}\right)^{5/2}}-\frac{2a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)(c-acx)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
-((((-2*a*(1 + 1/(a*x))^(5/2))/(7*(x^(-1))^(7/2)) + (18*(1 + 1/(a*x))^(5/2
))/(35*(x^(-1))^(5/2)))*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))/(a*(1 - 1/(a*x))
^(5/2)))
```

3.245.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.245.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{2(ax+1)(5ax-9)(-acx+c)^{\frac{5}{2}}}{35a(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	48
default	$\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^{2}(5ax-9)}{35\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
risch	$-\frac{2c^{3}(ax-1)\left(5a^{3}x^{3}+a^{2}x^{2}-13ax-9\right)}{35\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	60

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(a*x+1)*(5*a*x-9)*(-a*c*x+c)^(5/2)/a/(a*x-1)/((a*x-1)/(a*x+1))^(3/2)
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,(5\,a^4c^2x^4+6\,a^3c^2x^3-12\,a^2c^2x^2-22\,ac^2x-9\,c^2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

output
```
2/35*(5*a^4*c^2*x^4 + 6*a^3*c^2*x^3 - 12*a^2*c^2*x^2 - 22*a*c^2*x - 9*c^2)
*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^3\sqrt{-c}c^2x^3-9\,a^2\sqrt{-c}c^2x^2-5\,a\sqrt{-c}c^2x+9\,\sqrt{-c}c^2\right)(ax+1)^{\frac{3}{2}}}{35\,(ax-1)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

output
```
2/35*(5*a^3*sqrt(-c)*c^2*x^3 - 9*a^2*sqrt(-c)*c^2*x^2 - 5*a*sqrt(-c)*c^2*x
 + 9*sqrt(-c)*c^2)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{2\left(16\sqrt{2}\sqrt{-c}c+\frac{5(acx+c)^3\sqrt{-acx-c}-14(acx+c)^2\sqrt{-acx-c}c}{c^2}\right)c^2}{35\,a|c|\text{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2/35*(16*sqrt(2)*sqrt(-c)*c + (5*(a*c*x + c)^3*sqrt(-a*c*x - c) - 14*(a*c
*x + c)^2*sqrt(-a*c*x - c)*c)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))
```

3.245.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=$$
$$-\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-5\,a^3\,x^3-11\,a^2\,x^2+a\,x+23\right)}{35\,a}-\frac{64\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a*x - 11*a^2*x^2 -
 5*a^3*x^3 + 23))/(35*a) - (64*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))
^(1/2))/(35*a*(a*x - 1))
```

3.246 $\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

3.246.1 Optimal result

Integrand size = 20, antiderivative size = 31

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2e^{3\coth^{-1}(ax)}(1+ax)(c-acx)^{3/2}}{5a}$$

output
```
2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

3.246.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

3.246.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6726}$$

$$\frac{2(ax+1)(c-acx)^{3/2}e^{3\coth^{-1}(ax)}}{5a}$$

input

```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output

```
(2*E^(3*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(3/2))/(5*a)
```

3.246.3.1 Defintions of rubi rules used

rule 6726

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

3.246.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{2(ax+1)(-acx+c)^{\frac{3}{2}}}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	35
default	$-\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	42
risch	$\frac{2c^{2}(ax-1)\left(a^{2}x^{2}+2ax+1\right)}{5\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	52

input

```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(a^3cx^3+3\,a^2cx^2+3\,acx+c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^2x-a\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

output
```
-2/5*(a^3*c*x^3 + 3*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a*c*x + c)*sqrt((a*x -
1)/(a*x + 1))/(a^2*x - a)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(3/2),x)
```

output
```
Timed out
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(a^2\sqrt{-c}cx^2-\sqrt{-c}c\right)(ax+1)^{\frac{3}{2}}}{5\,(ax-1)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

output
```
-2/5*(a^2*sqrt(-c)*c*x^2 - sqrt(-c)*c)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

3.246.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.246.9 Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.61

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2+4\,a\,x+7\right)}{5\,a}-\frac{16\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + a^2*x^2 + 7)
)/(5*a) - (16*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x -
 1))
```

3.247 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.247.1 Optimal result

Integrand size = 20, antiderivative size = 163

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

output
```
2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(
-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1
+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(3/2)/(1-1/a/x)^(1/2
)
```

3.247.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(7+ax)-6\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^
(3/2)*Sqrt[1 - 1/(a*x)])
```

3.247.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 104$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{2\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output

```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*(1 + 1/(a*x))^(3/2))/(3*a*(x^(-1))^
(3/2)) + (2*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[2]*ArcTanh[
(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/a))/Sqrt[1
- 1/(a*x)])
```

3.247. $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.247.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.247.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(6\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-ax\sqrt{-c(ax+1)}-7\sqrt{-c(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a}$	107
risch	$-\frac{2(ax+7)c(ax-1)}{3a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	121

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-a*x*(-c*(a*x+1))^(
1/2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.53

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$

$$= \left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(a^2x^2+8\,ax+7)\sqrt{-acx+c}\right.}{3\,(a^2x-a)}\right.$$

$$\left.-\frac{2\left(6\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*s
qrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(
a*x + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*s
qrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2
+ 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

3.247.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.247.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.247.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \frac{\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.248 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.248.1 Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

output
```
2*a*(1+1/a/x)^(3/2)*x*(1-1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(1/2)-6*(1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(1/2)-3*arctanh(2^(1/2)*(1/x)^(1
/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a^(1/2)/(1/x)^(1/2)/(
-a*c*x+c)^(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.66

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(-2+ax)-3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}(-1+ax)\sqrt{c-acx}}$$

input

```
Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output

```
(Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 3*Sqrt[2]*S
qrt[x^(-1)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/
(a*x)])]))/(Sqrt[a]*(-1 + a*x)*Sqrt[c - a*c*x])
```

3.248.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}}{a\sqrt{\frac{1}{x}}\left(a-\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{2a}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}}{a\sqrt{\frac{1}{x}}\left(a-\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow 104$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{2a}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}}{a\sqrt{\frac{1}{x}}\left(a-\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow 219$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{2a}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}}{a\sqrt{\frac{1}{x}}\left(a-\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

input

```
Int[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output

```
-((a^2*Sqrt[1 - 1/(a*x)]*((1 + 1/(a*x))^(3/2)/(a*(a - x^(-1))*Sqrt[x^(-1)]
) + (3*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[2]*ArcTanh[(Sqrt
[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/(2*a)))/(Sqrt[x^
(-1)]*Sqrt[c - a*c*x]))
```

3.248. $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.248.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.248. $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.248.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result	s
default	$-\frac{\sqrt{-c(ax-1)}\,\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+2ax\sqrt{c}\,\sqrt{-c(ax+1)}+3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c-4\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)c^{\frac{3}{2}}\sqrt{-c(ax+1)}\,a}$	1
risch	$\frac{2ax-2}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}+\frac{\left(-\frac{2\sqrt{-acx-c}}{a(-acx+c)}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	1

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1
/2))*a*c*x+2*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+
1))^(1/2)*2^(1/2)/c^(1/2))*c-4*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1
))^(3/2)/(a*x+1)/c^(3/2)/(-c*(a*x+1))^(1/2)/a
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.63

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$

$$=\left[\frac{3\sqrt{2}(a^2cx^2-2\,acx+c)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)-4\,(a^2x^2-ax-2)\sqrt{-ac}}{2\,(a^3cx^2-2\,a^2cx+ac)}\right.$$

$$\left.-\frac{2\,(a^2x^2-ax-2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}-\frac{3\sqrt{2}(a^2cx^2-2\,acx+c)\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{\sqrt{c}}}{a^3cx^2-2\,a^2cx+ac}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/2*(3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqr
t(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a
*x - 3)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sq
rt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c), -(2*(a^2*x^2 - a*x
 - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)) - 3*sqrt(2)*(a^2*c*x^2 -
2*a*c*x + c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a
*x - 1)*sqrt(c)))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

3.248.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
```

output
```
Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1))), x)
```

3.248.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{1}{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{3\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-2\sqrt{-acx-c}+\frac{2\sqrt{-acx-c}c}{acx-c}}{a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-(3*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - 2*sqrt(
-a*c*x - c) + 2*sqrt(-a*c*x - c)*c/(a*c*x - c))/(a*abs(c))
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.249 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

3.249.1 Optimal result

Integrand size = 20, antiderivative size = 187

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

output

```
-1/2*a^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x/(a-1/x)^2/(-a*c*x+c)^(3/2)-3/8*
(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a^(1/
2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)-3/4*a*(1-1/a/x)^(3/2)*x*(1+1/a/x)^
(1/2)/(a-1/x)/(-a*c*x+c)^(3/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.67

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(-1+5ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{8\sqrt{a}c(-1+ax)^2\sqrt{c-acx}}$$

input

```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1 + 5*a*x) + 3*Sqrt[2]
*Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1
+ 1/(a*x)])]))/(8*Sqrt[a]*c*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

3.249.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{a^3\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{4a}+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{3\left(\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{3\left(\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

input

```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output

```
-((a^3*(1 - 1/(a*x))^(3/2)*(((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(2*a*(a - x
^(-1))^2) + (3*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1))) + ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2))))/
(4*a)))/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

3.249.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.249.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\sqrt{-c(ax-1)}\,\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}-6\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+10ax\sqrt{c}\,\sqrt{-c(ax+1)}+3\sqrt{2}\,\arctan\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)c^{\frac{5}{2}}\sqrt{-c(ax+1)}\,a}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(5/2)*(3
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-6*2^(1/2
)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+10*a*x*c^(1/2)*(-c*
(a*x+1))^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-
2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.82

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[-\frac{3\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3}{a^2x^2-2\,ax+1}\right.}{16\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}\right.$$

$$-\frac{3\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+2\,(5\,a^2x^2+4\,ax-1)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{8\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

output
```
[-1/16*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x
^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1
)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(5*a^2*x^2 + 4*a*x - 1)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*
a^2*c^2*x - a*c^2), -1/8*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt
(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c
*x - c)) + 2*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
```

output
```
Timed out
```

3.249.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}} - \frac{2\left(5(-acx-c)^{\frac{3}{2}}+6\sqrt{-acx-c}\,c\right)}{(acx-c)^2}}{8a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - 2*(5
*(-a*c*x - c)^(3/2) + 6*sqrt(-a*c*x - c)*c)/(a*c*x - c)^2)/(a*abs(c))
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.250 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.250.1 Optimal result

Integrand size = 20, antiderivative size = 250

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

output
```
1/24*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)-1/
6*a^4*(1-1/a/x)^(5/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(5/2)+1/32*
a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2
))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+1/16*a^2*(1-1/a/x)^(5/2)*x^2*(1+1/
a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

3.250.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}\left(7+22ax+3a^2x^2\right)-\frac{3\sqrt{2}(-1+ax)^3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{x}\right)}{96\sqrt{a}c^2\left(\frac{1}{x}\right)^{3/2}(-1+ax)^3\sqrt{c-acx}}$$

input

```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output

```
-1/96*(Sqrt[1 - 1/(a*x)]*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(7 + 22
*a*x + 3*a^2*x^2) - (3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])
/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/x))/(Sqrt[a]*c^2*(x^(-1))^(3/2)*(-1 + a*x)^
3*Sqrt[c - a*c*x])
```

3.250.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6727, 27, 105, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}-\frac{1}{12}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{12}\left(-\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input

```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output

```
-((a^4*(1 - 1/(a*x))^(5/2)*(((1 + 1/(a*x))^(5/2)*Sqrt[x^(-1)])/(6*(a - x^(
-1))^3) + (-1/2*((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(a*(a - x^(-1))^2) - (3
*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1))) + ArcTanh[(Sqrt[2]*Sqr
t[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2))))/(4*a))/12))/((
x^(-1))^(5/2)*(c - a*c*x)^(5/2)))
```

3.250. $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.250.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90

method	result
default	$\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^{3}c\,x^{3}+9\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}+6a^{2}x^{2}\sqrt{-c(ax+1)}\sqrt{c}-9\sqrt{2}\arctan\left(\right.\right.}{96\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)^{2}(ax+1)c^{\frac{7}{2}}\sqrt{-c(ax+}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/96*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*a^3*c*x^3+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2)
)*a^2*c*x^2+6*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)-9*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+44*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+3*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+14*(-c*(a*x+1))^(
1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)/c^(7/2)/(-c*(a*x+1
))^(1/2)/a
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \Bigg[-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{-c}\log\Bigg(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-}}{a^2x^2-2ax+1}}{192\,(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2}$$

$$-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(3a^3x^3+25a^2x^2+29ax+}{96\,(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

output
```
[-1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*
log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*
sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 + 2
5*a^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c
^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), -1/96*(3*sq
rt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)
*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^
3*x^3 + 25*a^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)
))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

3.250.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{2\left(3(acx+c)^2\sqrt{-acx-c}-16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}c^2\right)}{(acx-c)^3c}}{96\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
1/96*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - 2*(
3*(a*c*x + c)^2*sqrt(-a*c*x - c) - 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*
x - c)*c^2)/((a*c*x - c)^3*c))/(a*abs(c))
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\int \frac{1}{{\left(c-a\,c\,x\right)}^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

input
```
int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.251 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

3.251.1 Optimal result

Integrand size = 20, antiderivative size = 307

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx =& -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} \\
& -\frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} \\
& +\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{256\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}
\end{aligned}
$$

output
```
-1/8*a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^4/(-a*c*x+c)^(7/2)-1/
128*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)+1/3
2*a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^3/(a-1/x)^3/(-a*c*x+c)^(7/2)-3/512
*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/
2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-3/256*a^3*(1-1/a/x)^(7/2)*x^3*(1+
1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

3.251.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{1+\frac{1}{ax}}(39+79ax+13a^2x^2-3a^3x^3)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}(-1+ax)^4\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{512\sqrt{a}c^3\sqrt{\frac{1}{x}}(-1+ax)^4\sqrt{c-acx}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(39 + 79*a*x + 13*a^2*x^2
 - 3*a^3*x^3))/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^4*ArcTanh[(Sqrt[2]*Sqrt
[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(512*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1
 + a*x)^4*Sqrt[c - a*c*x])
```

3.251.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6727, 27, 105, 105, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \ 6727$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{a^5\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\int\frac{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}-\frac{1}{12}\int\frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{1}{12}\left(-\frac{3\int\frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right.\right.}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{ 219}$$

3.251. $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right.\right.}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
-((a^5*(1 - 1/(a*x))^(7/2)*(((1 + 1/(a*x))^(5/2)*(x^(-1))^(3/2))/(8*(a - x
^(-1))^4) - (3*(((1 + 1/(a*x))^(5/2)*Sqrt[x^(-1)])/(6*(a - x^(-1))^3) + (-
1/2*((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(a*(a - x^(-1))^2) - (3*((Sqrt[1 +
1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1))) + ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2))))/(4*a))/12))/16))/((x^(-1))
^(7/2)*(c - a*c*x)^(7/2)))
```

3.251.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.251.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.91

method	result
default	$\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{4}c\,x^{4}+12\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{3}c\,x^{3}+6a^{3}x^{3}\sqrt{-c(ax+1)}\,\sqrt{c}-18\sqrt{2}\,\arctan\left(\right.\right.}{512\left(\frac{ax-}{ax-}\right.}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/512*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)
/c^(1/2))*a^4*c*x^4+12*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/
2))*a^3*c*x^3+6*a^3*x^3*(-c*(a*x+1))^(1/2)*c^(1/2)-18*2^(1/2)*arctan(1/2*(
-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-26*a^2*x^2*(-c*(a*x+1))^(1/2)
*c^(1/2)+12*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-1
58*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*
2^(1/2)/c^(1/2))*c-78*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/
(a*x-1)^3/(a*x+1)/c^(9/2)/(-c*(a*x+1))^(1/2)/a
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.46

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-ac}}{a^2x^2}\right.}{1024\,(a^6c^4x^5-5a^5c^4x^4+10}\right.$$

$$-\frac{3\sqrt{2}(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(3a^4x^4-10a^3x^3}{512\,(a^6c^4x^5-5a^5c^4x^4+10a^4c^4x^3-10a^3c^4x^2+5a^2c^4x-ac^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

output
```
[-1/1024*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x
 - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x
 + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4
*(3*a^4*x^4 - 10*a^3*x^3 - 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqr
t((a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10
*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4), -1/512*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^
4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*
x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^4*x^4 - 10*
a^3*x^3 - 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*
a^2*c^4*x - a*c^4)]
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

3.251.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}} - \frac{2\left(3(acx+c)^3\sqrt{-acx-c}-22(acx+c)^2\sqrt{-acx-c}c+44(-acx-c)^{\frac{3}{2}}c^2+24\sqrt{-acx-c}c^3\right)}{(acx-c)^4c^2}}{512\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
1/512*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*
(3*(a*c*x + c)^3*sqrt(-a*c*x - c) - 22*(a*c*x + c)^2*sqrt(-a*c*x - c)*c +
44*(-a*c*x - c)^(3/2)*c^2 + 24*sqrt(-a*c*x - c)*c^3)/((a*c*x - c)^4*c^2))/
(a*abs(c))
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.252 $\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

3.252.1 Optimal result

Integrand size = 20, antiderivative size = 194

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{16384c^5\sqrt{1-\frac{1}{a^2x^2}}\,x}{693\sqrt{c-acx}}+\frac{4096}{693}c^4\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}+\frac{512}{231}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}+\frac{640}{693}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{5/2}+\frac{40}{99}c\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{7/2}+\frac{2}{11}\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{9/2}$$

output

```
512/231*c^3*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+640/693*c^2*x*(-a*c*x+c
)^(5/2)*(1-1/a^2/x^2)^(1/2)+40/99*c*x*(-a*c*x+c)^(7/2)*(1-1/a^2/x^2)^(1/2)
+2/11*x*(-a*c*x+c)^(9/2)*(1-1/a^2/x^2)^(1/2)+16384/693*c^5*x*(1-1/a^2/x^2)
^(1/2)/(-a*c*x+c)^(1/2)+4096/693*c^4*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2
)
```

3.252.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.44

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2c^4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}\left(-11531+5419ax-3198a^2x^2+1510a^3x^3-455a^4x^4+63a^5x^5\right)}{693a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

output
```
(2*c^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-11531 + 5419*a*x - 3198*a^2*x^2
 + 1510*a^3*x^3 - 455*a^4*x^4 + 63*a^5*x^5))/(693*a*Sqrt[1 - 1/(a*x)])
```

3.252.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6727, 27, 105, 105, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{9/2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6727$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^5}{a^5\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^5}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{20}{11}\int \frac{\left(a-\frac{1}{x}\right)^4}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^5}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{20}{11}\left(-\frac{16}{9}\int \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^5}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{20}{11}\Bigg(-\frac{16}{9}\Bigg(-\frac{12}{7}\int\frac{\left(a-\frac{1}{x}\right)^2}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^5}{11\left(\frac{1}{x}\right)^{11/2}}\Bigg)}{a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \textcolor{blue}{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{20}{11}\Bigg(-\frac{16}{9}\Bigg(-\frac{12}{7}\Bigg(\frac{2}{5}\int-\frac{14a-\frac{5}{x}}{2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{20}{11}\Bigg(-\frac{16}{9}\Bigg(-\frac{12}{7}\Bigg(-\frac{1}{5}\int\frac{14a-\frac{5}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \textcolor{blue}{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{20}{11}\Bigg(-\frac{16}{9}\Bigg(-\frac{12}{7}\Bigg(\frac{1}{5}\Bigg(\frac{43}{3}\int\frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\Bigg)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)}{a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \textcolor{blue}{48}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}\Bigg(-\frac{20}{11}\Bigg(-\frac{16}{9}\Bigg(-\frac{12}{7}\Bigg(\frac{1}{5}\Bigg(\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{86\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}\Bigg)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\Bigg)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}\Bigg)-2\sqrt{\ }}{a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

input
```
Int[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

output
```
-((((-20*((-16*((-12*(((28*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) - (86*S
qrt[1 + 1/(a*x)])/(3*Sqrt[x^(-1)]))/5 - (2*a^2*Sqrt[1 + 1/(a*x)])/(5*(x^(-
1))^(5/2))))/7 - (2*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)])/(7*(x^(-1))^(7/2))))
/9 - (2*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)])/(9*(x^(-1))^(9/2))))/11 - (2*(a
- x^(-1))^5*Sqrt[1 + 1/(a*x)])/(11*(x^(-1))^(11/2)))*(x^(-1))^(9/2)*(c - a
*c*x)^(9/2))/(a^5*(1 - 1/(a*x))^(9/2)))
```

3.252. $\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.252.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.40

method	result	size
risch	$-\frac{2c^{5} \sqrt{\frac{ax-1}{ax+1}}\, \left(63a^{5}x^{5}-455a^{4}x^{4}+1510a^{3}x^{3}-3198a^{2}x^{2}+5419ax-11531\right) \left(ax+1\right)}{693 \sqrt{-c \left(ax-1\right)}\, a}$	77
gosper	$\frac{2 \left(ax+1\right) \left(63a^{5}x^{5}-455a^{4}x^{4}+1510a^{3}x^{3}-3198a^{2}x^{2}+5419ax-11531\right) \left(-acx+c\right)^{\frac{9}{2}} \sqrt{\frac{ax-1}{ax+1}}}{693a \left(ax-1\right)^{5}}$	80
default	$\frac{2 \sqrt{\frac{ax-1}{ax+1}}\, \left(ax+1\right) \sqrt{-c \left(ax-1\right)}\, c^{4} \left(63a^{5}x^{5}-455a^{4}x^{4}+1510a^{3}x^{3}-3198a^{2}x^{2}+5419ax-11531\right)}{693 \left(ax-1\right) a}$	84

input
```
int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/693*c^5*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(63*a^5*x^5-455*a^4*
x^4+1510*a^3*x^3-3198*a^2*x^2+5419*a*x-11531)/a*(a*x+1)
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(63\,a^{6}c^{4}x^{6} - 392\,a^{5}c^{4}x^{5} + 1055\,a^{4}c^{4}x^{4} - 1688\,a^{3}c^{4}x^{3} + 2221\,a^{2}c^{4}x^{2} - 6112\,ac^{4}x - 11531\,c^{4}}{693\left(a^{2}x - a\right)}$$

input
```
integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
2/693*(63*a^6*c^4*x^6 - 392*a^5*c^4*x^5 + 1055*a^4*c^4*x^4 - 1688*a^3*c^4*
x^3 + 2221*a^2*c^4*x^2 - 6112*a*c^4*x - 11531*c^4)*sqrt(-a*c*x + c)*sqrt((
a*x - 1)/(a*x + 1))/(a^2*x - a)
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(63\,a^{6}\sqrt{-c}c^{4}x^{6}-392\,a^{5}\sqrt{-c}c^{4}x^{5}+1055\,a^{4}\sqrt{-c}c^{4}x^{4}-1688\,a^{3}\sqrt{-c}c^{4}x^{3}+2221\,a^{2}\sqrt{-c}c^{4}x}{693\,(a^{2}x-a)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
2/693*(63*a^6*sqrt(-c)*c^4*x^6 - 392*a^5*sqrt(-c)*c^4*x^5 + 1055*a^4*sqrt(
-c)*c^4*x^4 - 1688*a^3*sqrt(-c)*c^4*x^3 + 2221*a^2*sqrt(-c)*c^4*x^2 - 6112
*a*sqrt(-c)*c^4*x - 11531*sqrt(-c)*c^4)*(a*x - 1)/((a^2*x - a)*sqrt(a*x +
1))
```

3.252.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.252.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(63\,a^5\,x^5-329\,a^4\,x^4+726\,a^3\,x^3-962\,a^2\,x^2+1259\,a\,x-4853\right)}{693\,a}-\frac{32768\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{693\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(1259*a*x - 962*a^2*x
^2 + 726*a^3*x^3 - 329*a^4*x^4 + 63*a^5*x^5 - 4853))/(693*a) - (32768*c^4*
(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(693*a*(a*x - 1))
```

3.253 $\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

3.253.1 Optimal result

Integrand size = 20, antiderivative size = 161

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx &= \frac{4096c^4\sqrt{1-\frac{1}{a^2x^2}}\,x}{315\sqrt{c-acx}}\\ &+\frac{1024}{315}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}+\frac{128}{105}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}\\ &+\frac{32}{63}c\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{5/2}+\frac{2}{9}\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{7/2}\end{aligned}$$

output

```
128/105*c^2*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+32/63*c*x*(-a*c*x+c)^(5
/2)*(1-1/a^2/x^2)^(1/2)+2/9*x*(-a*c*x+c)^(7/2)*(1-1/a^2/x^2)^(1/2)+4096/31
5*c^4*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+1024/315*c^3*x*(1-1/a^2/x^2)^
(1/2)*(-a*c*x+c)^(1/2)
```

3.253.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2c^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}\,(2867-1276ax+642a^2x^2-220a^3x^3+35a^4x^4)}{315a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

output
```
(-2*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(2867 - 1276*a*x + 642*a^2*x^2 -
 220*a^3*x^3 + 35*a^4*x^4))/(315*a*Sqrt[1 - 1/(a*x)])
```

3.253.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6727, 27, 105, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^4}{a^4\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^4}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{16}{9}\int \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{16}{9}\left(-\frac{12}{7}\int \frac{\left(a-\frac{1}{x}\right)^2}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{16}{9}\left(-\frac{12}{7}\left(\frac{2}{5}\int -\frac{14a-\frac{5}{x}}{2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{16}{9}\left(-\frac{12}{7}\left(-\frac{1}{5}\int \frac{14a-\frac{5}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 87$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{16}{9}\left(-\frac{12}{7}\left(\frac{1}{5}\left(\frac{43}{3}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)-}{a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}\left(-\frac{16}{9}\left(-\frac{12}{7}\left(\frac{1}{5}\left(\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{86\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4}{9\left(\frac{1}{x}\right)^{9/2}}\right)(c-acx)}{a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

input

```
Int[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

output

```
-((((-16*((-12*(((28*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) - (86*Sqrt[1
+ 1/(a*x)])/(3*Sqrt[x^(-1)]))/5 - (2*a^2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5
/2))))/7 - (2*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)])/(7*(x^(-1))^(7/2))))/9 - (
2*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)])/(9*(x^(-1))^(9/2)))*(x^(-1))^(7/2)*(c
- a*c*x)^(7/2))/(a^4*(1 - 1/(a*x))^(7/2)))
```

3.253. $\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

3.253.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.253. $\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

3.253.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{2c^{4}\sqrt{\frac{ax-1}{ax+1}}\,(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)(ax+1)}{315\sqrt{-c(ax-1)}\,a}$	69
gosper	$\frac{2(ax+1)(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}{315a(ax-1)^{4}}$	72
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,c^{3}(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)}{315(ax-1)a}$	76

input
```
int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*c^4*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(35*a^4*x^4-220*a^3*x
^3+642*a^2*x^2-1276*a*x+2867)/a*(a*x+1)
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx =$$
$$-\frac{2\left(35\,a^{5}c^{3}x^{5}-185\,a^{4}c^{3}x^{4}+422\,a^{3}c^{3}x^{3}-634\,a^{2}c^{3}x^{2}+1591\,ac^{3}x+2867\,c^{3}\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^{2}x-a)}$$

input
```
integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(35*a^5*c^3*x^5 - 185*a^4*c^3*x^4 + 422*a^3*c^3*x^3 - 634*a^2*c^3*x
^2 + 1591*a*c^3*x + 2867*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(
a^2*x - a)
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=$$
$$-\frac{2\left(35\,a^5\sqrt{-c}c^3x^5-185\,a^4\sqrt{-c}c^3x^4+422\,a^3\sqrt{-c}c^3x^3-634\,a^2\sqrt{-c}c^3x^2+1591\,a\sqrt{-c}c^3x+2867\,\sqrt{-c}c^3\right)(}{315\,(a^2x-a)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-2/315*(35*a^5*sqrt(-c)*c^3*x^5 - 185*a^4*sqrt(-c)*c^3*x^4 + 422*a^3*sqrt(
-c)*c^3*x^3 - 634*a^2*sqrt(-c)*c^3*x^2 + 1591*a*sqrt(-c)*c^3*x + 2867*sqrt
(-c)*c^3)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

3.253.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.253.9 Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(35\,a^4\,x^4-150\,a^3\,x^3+272\,a^2\,x^2-362\,a\,x+1229\right)}{315\,a} - \frac{8192\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

input

```
int((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
- (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(272*a^2*x^2 - 362*
a*x - 150*a^3*x^3 + 35*a^4*x^4 + 1229))/(315*a) - (8192*c^3*(c - a*c*x)^(1
/2)*((a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

3.254 $\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.254.1 Optimal result

Integrand size = 20, antiderivative size = 128

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{256c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}{35\sqrt{c-acx}}+\frac{64}{35}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}+\frac{24}{35}c\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}+\frac{2}{7}\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{5/2}$$

output
```
24/35*c*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+2/7*x*(-a*c*x+c)^(5/2)*(1-1
/a^2/x^2)^(1/2)+256/35*c^3*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+64/35*c^
2*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

3.254.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2c^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}\,(-177+71ax-27a^2x^2+5a^3x^3)}{35a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

output
```
(2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-177 + 71*a*x - 27*a^2*x^2 + 5*a
^3*x^3))/(35*a*Sqrt[1 - 1/(a*x)])
```

3.254.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6727, 27, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \textcolor{blue}{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^3}{a^3\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{12}{7}\int \frac{\left(a-\frac{1}{x}\right)^2}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{12}{7}\left(\frac{2}{5}\int -\frac{14a-\frac{5}{x}}{2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{12}{7}\left(-\frac{1}{5}\int \frac{14a-\frac{5}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{12}{7}\left(\frac{1}{5}\left(\frac{43}{3}\int\frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}\left(-\frac{12}{7}\left(\frac{1}{5}\left(\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{86\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)(c-acx)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

input
```
Int[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

output
```
-((((-12*(((28*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) - (86*Sqrt[1 + 1/(a
*x)])/(3*Sqrt[x^(-1)]))/5 - (2*a^2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5/2))))
/7 - (2*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)])/(7*(x^(-1))^(7/2)))*(x^(-1))^(5/
2)*(c - a*c*x)^(5/2))/(a^3*(1 - 1/(a*x))^(5/2)))
```

3.254.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.254.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{2 c^{3} \sqrt{\frac{a x -1}{a x +1}}\, \left(5 a^{3} x^{3}-27 a^{2} x^{2}+71 a x -177\right) \left(a x +1\right)}{35 \sqrt{-c \left(a x -1\right)}\, a}$	61
gosper	$\frac{2 \left(a x +1\right) \left(5 a^{3} x^{3}-27 a^{2} x^{2}+71 a x -177\right) \left(-a c x +c \right)^{\frac{5}{2}} \sqrt{\frac{a x -1}{a x +1}}}{35 a \left(a x -1\right)^{3}}$	64
default	$\frac{2 \sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) \sqrt{-c \left(a x -1\right)}\, c^{2} \left(5 a^{3} x^{3}-27 a^{2} x^{2}+71 a x -177\right)}{35 \left(a x -1\right) a}$	68

input
```
int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*c^3*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(5*a^3*x^3-27*a^2*x^2
+71*a*x-177)/a*(a*x+1)
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^4c^2x^4-22\,a^3c^2x^3+44\,a^2c^2x^2-106\,ac^2x-177\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^4*c^2*x^4 - 22*a^3*c^2*x^3 + 44*a^2*c^2*x^2 - 106*a*c^2*x - 177*
c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

3.254.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.75

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^4\sqrt{-c}c^2x^4-22\,a^3\sqrt{-c}c^2x^3+44\,a^2\sqrt{-c}c^2x^2-106\,a\sqrt{-c}c^2x-177\,\sqrt{-c}c^2\right)(ax-1)}{35\,(a^2x-a)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*a^4*sqrt(-c)*c^2*x^4 - 22*a^3*sqrt(-c)*c^2*x^3 + 44*a^2*sqrt(-c)*c
^2*x^2 - 106*a*sqrt(-c)*c^2*x - 177*sqrt(-c)*c^2)*(a*x - 1)/((a^2*x - a)*s
qrt(a*x + 1))
```

3.254.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.254.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-17\,a^2\,x^2+27\,a\,x-79\right)}{35\,a} - \frac{512\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(27*a*x - 17*a^2*x^2
+ 5*a^3*x^3 - 79))/(35*a) - (512*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1
))^(1/2))/(35*a*(a*x - 1))
```

3.255 $\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

3.255.1 Optimal result

Integrand size = 20, antiderivative size = 95

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{64c^2\sqrt{1-\frac{1}{a^2x^2}}x}{15\sqrt{c-acx}}+\frac{16}{15}c\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}+\frac{2}{5}\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}$$

output
```
2/5*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+64/15*c^2*x*(1-1/a^2/x^2)^(1/2)
/(-a*c*x+c)^(1/2)+16/15*c*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(43-14ax+3a^2x^2)}{15a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

output
```
(-2*c*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(43 - 14*a*x + 3*a^2*x^2))/(15*a*S
qrt[1 - 1/(a*x)])
```

3.255.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)^2}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(\frac{2}{5}\int -\frac{14a-\frac{5}{x}}{2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{1}{5}\int \frac{14a-\frac{5}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(\frac{1}{5}\left(\frac{43}{3}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{5}\left(\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{86\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

output
```
-(((((28*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) - (86*Sqrt[1 + 1/(a*x)])/
(3*Sqrt[x^(-1)]))/5 - (2*a^2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5/2)))*(x^(-1
))^(3/2)*(c - a*c*x)^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)))
```

3.255.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.255.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{2 c^{2} \sqrt{\frac{a x -1}{a x +1}}\, \left(3 a^{2} x^{2}-14 a x +43\right) \left(a x +1\right)}{15 \sqrt{-c \left(a x -1\right)}\, a}$	53
gosper	$\frac{2 \left(a x +1\right) \left(3 a^{2} x^{2}-14 a x +43\right) \left(-a c x +c \right)^{\frac{3}{2}} \sqrt{\frac{a x -1}{a x +1}}}{15 a \left(a x -1\right)^{2}}$	56
default	$-\frac{2 \sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) \sqrt{-c \left(a x -1\right)}\, c \left(3 a^{2} x^{2}-14 a x +43\right)}{15 \left(a x -1\right) a}$	58

input
```
int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*c^2*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(3*a^2*x^2-14*a*x+43)/
a*(a*x+1)
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,(3\,a^3cx^3-11\,a^2cx^2+29\,acx+43\,c)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^3*c*x^3 - 11*a^2*c*x^2 + 29*a*c*x + 43*c)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

3.255.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(3\,a^3\sqrt{-c}cx^3-11\,a^2\sqrt{-c}cx^2+29\,a\sqrt{-c}cx+43\,\sqrt{-c}c\right)(ax-1)}{15\left(a^2x-a\right)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(3*a^3*sqrt(-c)*c*x^3 - 11*a^2*sqrt(-c)*c*x^2 + 29*a*sqrt(-c)*c*x +
43*sqrt(-c)*c)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

3.255.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.255.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(3\,a^2\,x^2-8\,a\,x+21\right)}{15\,a}-\frac{128\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
- (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 8*a*x +
21))/(15*a) - (128*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(15*a*
(a*x - 1))
```

3.255. $\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

3.256 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.256.1 Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{8c\sqrt{1-\frac{1}{a^2x^2}}x}{3\sqrt{c-acx}}+\frac{2}{3}\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}$$

output
```
8/3*c*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+2/3*x*(1-1/a^2/x^2)^(1/2)*(-a
*c*x+c)^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(-5+ax)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

3.256.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{5}{3}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{10\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}-\frac{2a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

output
```
-((((-2*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (10*Sqrt[1 + 1/(a*x)])/(
3*Sqrt[x^(-1)]))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]))
```

3.256.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.256.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(ax-5)(ax+1)}{3\sqrt{-c(ax-1)}\,a}$	42
gosper	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(ax-5)}{3(ax-1)a}$	48

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-5)/a*(a*x+1)
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(a^2x^2-4\,ax-5\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

3.256.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \int \sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(ax-1\right)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(a^2\sqrt{-c}x^2-4\,a\sqrt{-c}x-5\,\sqrt{-c}\right)(ax-1)}{3\left(a^2x-a\right)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a
)*sqrt(a*x + 1))
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,(-acx-c)^{\frac{3}{2}}|c|}{3\,ac^2}+\frac{4\,\sqrt{-acx-c}|c|}{ac}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

3.256.9 Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,(a\,x-3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.257 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.257.1 Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2e^{-\coth^{-1}(ax)}(1+ax)}{a\sqrt{c-acx}}$$

output
```
2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

3.257.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-acx}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
```

output
```
(2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - a*c*x]
```

3.257.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$

$$\Big\downarrow \, 6726$$

$$\frac{2(ax+1)e^{-\coth^{-1}(ax)}}{a\sqrt{c-acx}}$$

input
```
Int[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
```

output
```
(2*(1 + a*x))/(a*E^ArcCoth[a*x]*Sqrt[c - a*c*x])
```

3.257.3.1 Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

3.257.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-acx+c}}$	35
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{\sqrt{-c(ax-1)}\,a}$	36
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}{(ax-1)ca}$	46

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

3.257.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\,(ax-1)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)), x)
```

3.257.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)}{\sqrt{ax+1}ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2*(a*sqrt(-c)*x + sqrt(-c))/(sqrt(a*x + 1)*a*c)
```

3.257.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.257.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(2\,x+\frac{2}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(1/2),x)
```

output
```
((2*x + 2/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

3.258 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

3.258.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

output
```
-(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a^(1
/2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)
```

3.258.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

output
```
-((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqr
t[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

3.258.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6727, 27, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{a}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

output
```
-((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqr
t[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

3.258.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.258.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{(ax-1)\sqrt{-c(ax+1)}\,c^{\frac{3}{2}}\,a}$	78

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*2^(1/2)*arctan(1/2*(-c
*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)/c^(3/2)/a
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.86

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)}{2\,ac}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{ac^{\frac{3}{2}}}\right]$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/c)*log(-(a^2*x^2 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x +
1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a*x + 1)
)/(a*c), -sqrt(2)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)
)/((a*x - 1)*sqrt(c)))/(a*c^(3/2))]
```

3.258.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1))**(3/2), x)
```

3.258.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(3/2), x)
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{a\sqrt{c}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)|c|\mathrm{sgn}(ax+1)}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*sqrt(c)) - sqrt(2
)*arctan(sqrt(-c)/sqrt(c))/(a*sqrt(c)))*abs(c)*sgn(a*x + 1)/c^2
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a\,c\,x\right)}^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2), x)
```

3.259 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.259.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

output

```
1/4*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^
(1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)-1/2*a^2*(1-1/a/x)^(5/2)*x^2*(1
+1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\,x\left(-2\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{4\sqrt{a}c^2(-1+ax)\sqrt{c-acx}}$$

input

```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

output

```
(Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*
(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(
4*Sqrt[a]*c^2*(-1 + a*x)*Sqrt[c - a*c*x])
```

3.259.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6727, 27, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$$

$$\Big\downarrow \textcolor{blue}{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{a^2\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{105}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{1}{4}\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{104}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{1}{2}\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

output

```
-((a^2*(1 - 1/(a*x))^(5/2)*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(2*(a - x^(-1
))) - ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(2*Sqrt[
2]*Sqrt[a])))/((x^(-1))^(5/2)*(c - a*c*x)^(5/2)))
```

3.259.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.259. $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.259.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c+2\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{4c^{\frac{7}{2}}\,(ax-1)^{2}\sqrt{-c(ax+1)}\,a}$	123

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-2^(1/2)*arctan(1/
2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+2^(1/2)*arctan(1/2*(-c*(a*x+1)
)^(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(7/2)/(a*x-1)^2
/(-c*(a*x+1))^(1/2)/a
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.07

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\,\sqrt{-}}{8\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}\right.$$

$$\left.-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{4\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}\right]$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x -
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/
(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), -1/4*(sqrt(2)*(a^2*x^2 -
2*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/
(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

3.259.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(5/2), x)
```

3.259.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a\,c\,x\right)}^{5/2}}\,dx$$

input

```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2),x)
```

output

```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2), x)
```

3.260 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

3.260.1 Optimal result

Integrand size = 20, antiderivative size = 193

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}+\frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

output
```
-3/32*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x
)^(1/2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/4*a^3*(1-1/a/x)^(7/2)*x^2*
(1+1/a/x)^(1/2)/(a-1/x)^2/(-a*c*x+c)^(7/2)+3/16*a^3*(1-1/a/x)^(7/2)*x^3*(1
+1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.65

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(7-3ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{32\sqrt{a}c^3(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 - 3*a*x) + 3*Sqrt[2]*
Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 +
 1/(a*x)])]))/(32*Sqrt[a]*c^3*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

3.260.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{a^3\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{3}{8}\int \frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{3}{8}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{1}{4}\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{3}{8}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{1}{2}\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{3}{8}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\sqrt{a}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

output
```
-((a^3*(1 - 1/(a*x))^(7/2)*((Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))/(4*(a - x^(
-1))^2) - (3*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(2*(a - x^(-1))) - ArcTanh[
(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(2*Sqrt[2]*Sqrt[a])))/
8))/((x^(-1))^(7/2)*(c - a*c*x)^(7/2)))
```

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.260.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}+6\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+6ax\sqrt{c}\,\sqrt{-c(ax+1)}-3\sqrt{2}\right.}{32c^{\frac{9}{2}}\,(ax-1)^{3}\sqrt{-c(ax+1)}\,a}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan
(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2+6*2^(1/2)*arctan(1/2*(-
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+6*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)-3
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-14*(-c*(a*x+1))^
(1/2)*c^(1/2))/c^(9/2)/(a*x-1)^3/(-c*(a*x+1))^(1/2)/a
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.77

$$\int\frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\left[-\frac{3\,\sqrt{2}(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\right.}{a^{2}x^{2}-2\,ax+1}\right.}{64\,(a^{4}c^{4}x^{3}-3\,a^{3}c^{4}x^{2}+3\,a^{2}c^{4}x-ac^{4})}\right.$$

$$-\frac{3\,\sqrt{2}(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(3\,a^{2}x^{2}-4\,ax-7)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{32\,(a^{4}c^{4}x^{3}-3\,a^{3}c^{4}x^{2}+3\,a^{2}c^{4}x-ac^{4})}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x
^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1
)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^2*x^2 - 4*a*x - 7)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*
a^2*c^4*x - a*c^4), -1/32*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqr
t(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*
c*x - c)) - 2*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

3.260.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(7/2), x)
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3(-acx-c)^{\frac{3}{2}}+10\sqrt{-acx-c}c\right)}{(acx-c)^2c^2}\right)|c|\mathrm{sgn}(ax+1)}{32\,ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
1/32*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) + 2*(
3*(-a*c*x - c)^(3/2) + 10*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c^2))*abs(c)*
sgn(a*x + 1)/(a*c^2)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a\,c\,x)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2), x)
```

3.261 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

3.261.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{32c^3\sqrt{c-acx}}{a}-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{8c(c-acx)^{5/2}}{5a}-\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}+\frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-16/3*c^2*(-a*c*x+c)^(3/2)/a-8/5*c*(-a*c*x+c)^(5/2)/a-4/7*(-a*c*x+c)^(7/2)
/a-2/9*(-a*c*x+c)^(9/2)/a/c+32*c^(7/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2
)/c^(1/2))*2^(1/2)/a-32*c^3*(-a*c*x+c)^(1/2)/a
```

3.261.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.64

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{2c^3\left(\sqrt{c-acx}(-6257+1754ax-732a^2x^2+230a^3x^3-35a^4x^4)+5040\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a}}{\sqrt{2}\sqrt{}}\right.\right.}{315a}$$

input
```
Integrate[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*c^3*(Sqrt[c - a*c*x]*(-6257 + 1754*a*x - 732*a^2*x^2 + 230*a^3*x^3 - 35
*a^4*x^4) + 5040*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])
]))/(315*a)
```

3.261.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6680, 35, 60, 60, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(c-acx)^{7/2}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(1-ax)(c-acx)^{7/2}}{ax+1}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{(c-acx)^{9/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\int \frac{(c-acx)^{7/2}}{ax+1}dx+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\left(2c\int \frac{(c-acx)^{5/2}}{ax+1}dx+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\left(2c\left(2c\int \frac{(c-acx)^{3/2}}{ax+1}dx+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\left(2c\left(2c\left(2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow \; 60$$

$$-\frac{2c\left(2c\left(2c\left(2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{2c\left(2c\left(2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow \; 219$$

$$-\frac{2c\left(2c\left(2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

input

```
Int[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output

```
-(((2*(c - a*c*x)^(9/2))/(9*a) + 2*c*((2*(c - a*c*x)^(7/2))/(7*a) + 2*c*((
2*(c - a*c*x)^(5/2))/(5*a) + 2*c*((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((2*Sq
rt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqr
t[c])])/a)))))/c)
```

3.261.3.1 Defintions of rubi rules used

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

3.261. $\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.261.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

method	result
pseudoelliptic	$\frac{32\left(\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-\frac{\left(35a^{4}x^{4}-230a^{3}x^{3}+732a^{2}x^{2}-1754ax+6257\right)\sqrt{-c(ax-1)}}{5040}\right)c^{3}}{a}$
risch	$\frac{2(35a^{4}x^{4}-230a^{3}x^{3}+732a^{2}x^{2}-1754ax+6257)(ax-1)c^{4}}{315a\sqrt{-c(ax-1)}}+\frac{32c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}+\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{4c^{2}(-acx+c)^{\frac{5}{2}}}{5}+\frac{8c^{3}(-acx+c)^{\frac{3}{2}}}{3}+16c^{4}\sqrt{-acx+c}-16c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$
default	$\frac{-\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{4c(-acx+c)^{\frac{7}{2}}}{7}-\frac{8c^{2}(-acx+c)^{\frac{5}{2}}}{5}-\frac{16c^{3}(-acx+c)^{\frac{3}{2}}}{3}-32c^{4}\sqrt{-acx+c}+32c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$

input
```
int((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
32*(c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))-1/5040
*(35*a^4*x^4-230*a^3*x^3+732*a^2*x^2-1754*a*x+6257)*(-c*(a*x-1))^(1/2))*c^
3/a
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\left[\frac{2\left(2520\,\sqrt{2}c^{\frac{7}{2}}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)-(35\,a^{4}c^{3}x^{4}-230\,a^{3}c^{3}x^{3}+732\,a^{2}c^{3}x^{2}-1754\,ac}{315\,a}\right.$$

$$-\frac{2\left(5040\,\sqrt{2}\sqrt{-c}c^{3}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)+(35\,a^{4}c^{3}x^{4}-230\,a^{3}c^{3}x^{3}+732\,a^{2}c^{3}x^{2}-1754\,ac^{3}x+6257\,c^{3})\right.}{315\,a}$$

input
```
integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/315*(2520*sqrt(2)*c^(7/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(
c) - 3*c)/(a*x + 1)) - (35*a^4*c^3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2
 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c))/a, -2/315*(5040*sqrt(2)*sqrt
(-c)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*c^3*x^4
 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*
x + c))/a]
```

3.261.6 Sympy [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\begin{cases}-\frac{2\cdot\left(\frac{16\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+16c^4\sqrt{-acx+c}+\frac{8c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{4c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{ac} & \text{for } ac \\ c^{\frac{7}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise}\end{cases}\right) & \text{other}\end{cases}$$

input
```
integrate((-a*c*x+c)**(7/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(16*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))
/sqrt(-c) + 16*c**4*sqrt(-a*c*x + c) + 8*c**3*(-a*c*x + c)**(3/2)/3 + 4*c*
*2*(-a*c*x + c)**(5/2)/5 + 2*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)
/9)/(a*c), Ne(a*c, 0)), (c**(7/2)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(
a*x + 1) + 1)/a, True)), True))
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=$$

$$-\frac{2\left(2520\sqrt{2}c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35(-acx+c)^{\frac{9}{2}}+90(-acx+c)^{\frac{7}{2}}c+252(-acx+c)^{\frac{5}{2}}c^2+840(-a\right.}{315\,ac}$$

input
```
integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/315*(2520*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sq
rt(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) + 90*(-a*c*x +
c)^(7/2)*c + 252*(-a*c*x + c)^(5/2)*c^2 + 840*(-a*c*x + c)^(3/2)*c^3 + 504
0*sqrt(-a*c*x + c)*c^4)/(a*c)
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{32\sqrt{2}c^4\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}$$

$$-\frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}a^8c^8-90\,(acx-c)^3\sqrt{-acx+c}a^8c^9+252\,(acx-c)^2\sqrt{-acx+c}a^8c^{10}+840\,(\right.}{315\,a^9c^9}$$

input
```
integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-32*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
 - 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^8*c^8 - 90*(a*c*x - c)^3*sqr
t(-a*c*x + c)*a^8*c^9 + 252*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^10 + 840*
(-a*c*x + c)^(3/2)*a^8*c^11 + 5040*sqrt(-a*c*x + c)*a^8*c^12)/(a^9*c^9)
```

3.261.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{4\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a}-\frac{8\,c\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a}-\frac{32\,c^3\,\sqrt{c-a\,c\,x}}{a}$$

$$-\frac{16\,c^2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a\,c}-\frac{\sqrt{2}\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,32{}\mathrm{i}}{a}$$

input
```
int(((c - a*c*x)^(7/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
- (4*(c - a*c*x)^(7/2))/(7*a) - (8*c*(c - a*c*x)^(5/2))/(5*a) - (32*c^3*(c
 - a*c*x)^(1/2))/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(9/
2))/(9*a*c) - (2^(1/2)*c^(7/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1
/2)))*32i)/a
```

3.262 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.262.1 Optimal result

Integrand size = 20, antiderivative size = 116

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{16c^2\sqrt{c-acx}}{a}-\frac{8c(c-acx)^{3/2}}{3a}-\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}+\frac{16\sqrt{2}c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-8/3*c*(-a*c*x+c)^(3/2)/a-4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c+
16*c^(5/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-16*c^2*
(-a*c*x+c)^(1/2)/a
```

3.262.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\left(\sqrt{c-acx}(-1037+269ax-87a^2x^2+15a^3x^3)+840\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{105a}$$

input
```
Integrate[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*c^2*(Sqrt[c - a*c*x]*(-1037 + 269*a*x - 87*a^2*x^2 + 15*a^3*x^3) + 840*
Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(105*a)
```

3.262.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6717, 6680, 35, 60, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(c-acx)^{5/2}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(1-ax)(c-acx)^{5/2}}{ax+1}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{(c-acx)^{7/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\int \frac{(c-acx)^{5/2}}{ax+1}dx+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\left(2c\int \frac{(c-acx)^{3/2}}{ax+1}dx+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\left(2c\left(2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\left(2c\left(2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2c\left(2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow 219$$

$$-\frac{2c\left(2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

input
```
Int[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((2*(c - a*c*x)^(7/2))/(7*a) + 2*c*((2*(c - a*c*x)^(5/2))/(5*a) + 2*c*((
2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[
c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a))))/c)
```

3.262.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.262.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{2\left(56\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{\left(15a^{3}x^{3}-87a^{2}x^{2}+269ax-1037\right)\sqrt{-c(ax-1)}}{15}\right)c^{2}}{7a}$	71
risch	$-\frac{2(15a^{3}x^{3}-87a^{2}x^{2}+269ax-1037)(ax-1)c^{3}}{105a\sqrt{-c(ax-1)}}+\frac{16c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$	76
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{4c^{2}(-acx+c)^{\frac{3}{2}}}{3}+8c^{3}\sqrt{-acx+c}-8c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	87
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}-\frac{8c^{2}(-acx+c)^{\frac{3}{2}}}{3}-16c^{3}\sqrt{-acx+c}+16c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	87

input
```
int((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
2/7*(56*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+1/
15*(15*a^3*x^3-87*a^2*x^2+269*a*x-1037)*(-c*(a*x-1))^(1/2))*c^2/a
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.57

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \left[\frac{2\left(420\,\sqrt{2}c^{\frac{5}{2}}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+(15\,a^3c^2x^3-87\,a^2c^2x^2+269\,ac^2x-1037\,c^2)\sqrt{-}\right.}{105\,a}\right.$$

$$\left.-\frac{2\left(840\,\sqrt{2}\sqrt{-c}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-(15\,a^3c^2x^3-87\,a^2c^2x^2+269\,ac^2x-1037\,c^2)\sqrt{-acx+c}\right)}{105\,a}\right]$$

input
```
integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/105*(420*sqrt(2)*c^(5/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) + (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c^2*x - 103
7*c^2)*sqrt(-a*c*x + c))/a, -2/105*(840*sqrt(2)*sqrt(-c)*c^2*arctan(1/2*sq
rt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 26
9*a*c^2*x - 1037*c^2)*sqrt(-a*c*x + c))/a]
```

3.262.6 Sympy [A] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{8\sqrt{2}c^4\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+8c^3\sqrt{-acx+c}+\frac{4c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{ac} & \text{for } ac\neq 0 \\ c^{\frac{5}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((-a*c*x+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(8*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 8*c**3*sqrt(-a*c*x + c) + 4*c**2*(-a*c*x + c)**(3/2)/3 + 2*c*(-
a*c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a*c), Ne(a*c, 0)), (c**(5/2)
*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)), True))
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx =$$
$$-\frac{2\left(420\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+15\,(-acx+c)^{\frac{7}{2}}+42\,(-acx+c)^{\frac{5}{2}}c+140\,(-acx+c)^{\frac{3}{2}}c^{2}+840\,\sqrt{-a}}{105\,ac}$$

input
```
integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/105*(420*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqr
t(2)*sqrt(c) + sqrt(-a*c*x + c))) + 15*(-a*c*x + c)^(7/2) + 42*(-a*c*x + c
)^(5/2)*c + 140*(-a*c*x + c)^(3/2)*c^2 + 840*sqrt(-a*c*x + c)*c^3)/(a*c)
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{16\sqrt{2}c^{3}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}$$
$$+\frac{2\left(15\,(acx-c)^{3}\sqrt{-acx+c}a^{6}c^{6}-42\,(acx-c)^{2}\sqrt{-acx+c}a^{6}c^{7}-140\,(-acx+c)^{\frac{3}{2}}a^{6}c^{8}-840\,\sqrt{-acx+c}a}{105\,a^{7}c^{7}}$$

input
```
integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-16*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
 + 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^6*c^6 - 42*(a*c*x - c)^2*sqr
t(-a*c*x + c)*a^6*c^7 - 140*(-a*c*x + c)^(3/2)*a^6*c^8 - 840*sqrt(-a*c*x +
 c)*a^6*c^9)/(a^7*c^7)
```

3.262.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{4\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a}-\frac{8\,c\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a}-\frac{16\,c^2\,\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a\,c}-\frac{\sqrt{2}\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,16{}\mathrm{i}}{a}$$

input
```
int(((c - a*c*x)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
- (4*(c - a*c*x)^(5/2))/(5*a) - (8*c*(c - a*c*x)^(3/2))/(3*a) - (16*c^2*(c
 - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(7/2))/(7*a*c) - (2^(1/2)*c^(5/2)*atan
((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*16i)/a
```

3.263 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

3.263.1 Optimal result

Integrand size = 20, antiderivative size = 95

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} + \frac{8\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c+8*c^(3/2)*arctanh(1/2*(-a
*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-8*c*(-a*c*x+c)^(1/2)/a
```

3.263.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{-2c\sqrt{c-acx}(73-16ax+3a^2x^2)+120\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a}$$

input
```
Integrate[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*c*Sqrt[c - a*c*x]*(73 - 16*a*x + 3*a^2*x^2) + 120*Sqrt[2]*c^(3/2)*ArcT
anh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a)
```

3.263.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6680, 35, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(c-acx)^{3/2}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)(c-acx)^{3/2}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{5/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{(c-acx)^{3/2}}{ax+1}dx+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

input
```
Int[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((2*(c - a*c*x)^(5/2))/(5*a) + 2*c*((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((
2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]
*Sqrt[c])])/a)))/c)
```

3.263.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.263. $\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.263.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{2\left(-20\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{\left(3a^{2}x^{2}-16ax+73\right)\sqrt{-c(ax-1)}}{3}\right)c}{5a}$	61
risch	$\frac{2(3a^{2}x^{2}-16ax+73)(ax-1)c^{2}}{15a\sqrt{-c(ax-1)}}+\frac{8c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$	68
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^{2}\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	73
default	$\frac{-\frac{2(-acx+c)^{\frac{5}{2}}}{5}-\frac{4c(-acx+c)^{\frac{3}{2}}}{3}-8c^{2}\sqrt{-acx+c}+8c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	73

input
```
int((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-20*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+
1/3*(3*a^2*x^2-16*a*x+73)*(-c*(a*x-1))^(1/2))*c/a
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.54

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\left[\frac{2\left(30\sqrt{2}c^{\frac{3}{2}}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-(3a^{2}cx^{2}-16acx+73c)\sqrt{-acx+c}\right)}{15a},\right.$$
$$\left.-\frac{2\left(60\sqrt{2}\sqrt{-c}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)+(3a^{2}cx^{2}-16acx+73c)\sqrt{-acx+c}\right)}{15a}\right]$$

input
```
integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/15*(30*sqrt(2)*c^(3/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
- 3*c)/(a*x + 1)) - (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a, -
2/15*(60*sqrt(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c
) + (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a]
```

3.263.6 Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int e^{-2\coth^{-1}(ax)}(c - acx)^{3/2}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{4\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+4c^2\sqrt{-acx+c}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{ac} & \text{for } ac \neq 0 \\ c^{\frac{3}{2}}\left(\begin{cases} -x & \text{for } a = 0 \\ \frac{ax-2\log{(ax+1)}+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-a*c*x+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(4*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 4*c**2*sqrt(-a*c*x + c) + 2*c*(-a*c*x + c)**(3/2)/3 + (-a*c*x +
 c)**(5/2)/5)/(a*c), Ne(a*c, 0)), (c**(3/2)*Piecewise((-x, Eq(a, 0)), ((a*
x - 2*log(a*x + 1) + 1)/a, True)), True))
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c - acx)^{3/2}\,dx = -\frac{2\left(30\sqrt{2}c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3(-acx+c)^{\frac{5}{2}}+10(-acx+c)^{\frac{3}{2}}c+60\sqrt{-acx+c}c^2\right)}{15\,ac}$$

input
```
integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/15*(30*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(
2)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 10*(-a*c*x + c)^(
3/2)*c + 60*sqrt(-a*c*x + c)*c^2)/(a*c)
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{8\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}a^4c^4+10\,(-acx+c)^{\frac{3}{2}}a^4c^5+60\,\sqrt{-acx+c}a^4c^6\right)}{15\,a^5c^5}$$

input
```
integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-8*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
- 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^4 + 10*(-a*c*x + c)^(3/2)*a
^4*c^5 + 60*sqrt(-a*c*x + c)*a^4*c^6)/(a^5*c^5)
```

3.263.9 Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{4\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a} - \frac{8\,c\,\sqrt{c-a\,c\,x}}{a} - \frac{2\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a\,c} - \frac{\sqrt{2}\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,8{}\mathrm{i}}{a}$$

input
```
int(((c - a*c*x)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
- (4*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)
^(5/2))/(5*a*c) - (2^(1/2)*c^(3/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*
c^(1/2)))*8i)/a
```

3.264 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.264.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-2/3*(-a*c*x+c)^(3/2)/a/c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*
2^(1/2)*c^(1/2)/a-4*(-a*c*x+c)^(1/2)/a
```

3.264.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(-7+ax)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]
/(Sqrt[2]*Sqrt[c])])/(3*a)
```

3.264.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6717, 6680, 35, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

input

```
Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

output

```
-(((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*S
qrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a))/c)
```

3.264.3.1 Defintions of rubi rules used

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.264.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{2(ax-7)(ax-1)c}{3a\sqrt{-c(ax-1)}}+\frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a}$	57
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}-4c\sqrt{-acx+c}+4c^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	59
pseudoelliptic	$\frac{4\sqrt{c}\,\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{2ax\sqrt{-c(ax-1)}}{3}-\frac{14\sqrt{-c(ax-1)}}{3}}{a}$	59

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a*x-7)*(a*x-1)/a/(-c*(a*x-1))^(1/2)*c+4*arctanh(1/2*(-a*c*x+c)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$

$$= \left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a},\right.$$
$$\left.-\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
3*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*
arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x -
7))/a]
```

3.264.6 Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax-2\log{(ax+1)}+1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*c, 0)
), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)
), True))
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$= -\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}c\right)}{3ac}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)
/(a*c)
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3\,a^3c^3}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
2/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

3.264.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{2}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a} - \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c} - \frac{4\sqrt{c-a\,c\,x}}{a}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
(4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*
(c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

3.265 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.265.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{c-acx}}{ac} + \frac{2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

output
```
2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)-2*(-a*c*
x+c)^(1/2)/a/c
```

3.265.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{c-acx}}{ac} + \frac{2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

output
```
(-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/(a*Sqrt[c])
```

3.265.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 6680, 35, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{1-ax}{(ax+1)\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{\sqrt{c-acx}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

output
```
-(((2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqr
t[2]*Sqrt[c])])/a)/c)
```

3.265.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, x}, x] && IntegerQ[n/2]
```

3.265. $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.265.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2\left(\sqrt{-acx+c}-\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}\right)}{ca}$	45
default	$\frac{-2\sqrt{-acx+c}+2\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{ac}$	46
pseudoelliptic	$\frac{-2\sqrt{-c(ax-1)}+2\sqrt{c}\,\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	48
risch	$\frac{2ax-2}{a\sqrt{-c(ax-1)}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\sqrt{c}}$	51

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/a*((-a*c*x+c)^(1/2)-arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(
1/2)*c^(1/2))
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)-2\sqrt{-acx+c}}{ac}, \frac{2\left(\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)-\sqrt{-acx+c}\right)}{ac}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*sqrt(c)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x
+ 1)) - 2*sqrt(-a*c*x + c))/(a*c), 2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(2)*
sqrt(-a*c*x + c)*sqrt(-1/c)/(a*x - 1)) - sqrt(-a*c*x + c))/(a*c)]
```

3.265.6 Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \begin{cases} -\frac{2\left(\frac{\sqrt{2}c\,\text{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\sqrt{-acx+c}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

input

```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(1/2),x)
```

output

```
Piecewise((-2*(sqrt(2)*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(
-c) + sqrt(-a*c*x + c))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a
*x - 2*log(a*x + 1) + 1)/a, True))/sqrt(c), True))
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+2\sqrt{-acx+c}}{ac}$$

input

```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output

```
-(sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c))) + 2*sqrt(-a*c*x + c))/(a*c)
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\sqrt{-acx+c}}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2*
sqrt(-a*c*x + c)/(a*c)
```

3.265.9 Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a\,\sqrt{c}}-\frac{2\,\sqrt{c-a\,c\,x}}{a\,c}$$

input
```
int((a*x - 1)/((c - a*c*x)^(1/2)*(a*x + 1)),x)
```

output
```
(2*2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(1/2)) - (
2*(c - a*c*x)^(1/2))/(a*c)
```

3.266 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

3.266.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

output
```
arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

3.266.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{1-ax}{(ax+1)(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{1}{(ax+1)\sqrt{c-acx}}dx}{c}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{ac^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

input

```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

output

```
(Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

3.266.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.266.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\,c^{\frac{3}{2}}}$	29
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\,c^{\frac{3}{2}}}$	29
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{c^{\frac{3}{2}}\,a}$	30

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.38

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)}{2\,ac^{\frac{3}{2}}}, \frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)}{ac}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x + 1))
/(a*c^(3/2)), sqrt(2)*sqrt(-1/c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-1/c
)/(a*x - 1))/(a*c)]
```

3.266.6 Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \begin{cases} -\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{ac\sqrt{-c}} & \text{for}\ ac \neq 0 \\ \frac{\begin{cases} -x & \text{for}\ a = 0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(3/2),x)
```

output
```
Piecewise((-sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*c*sqrt(
-c)), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/
a, True))/c**(3/2), True))
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{2\,ac^{\frac{3}{2}}}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) +
sqrt(-a*c*x + c)))/(a*c^(3/2))
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}c}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a\,c^{3/2}}$$

input
```
int((a*x - 1)/((c - a*c*x)^(3/2)*(a*x + 1)),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(3/2))
```

3.267 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.267.1 Optimal result

Integrand size = 20, antiderivative size = 57

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=-\frac{1}{ac^2\sqrt{c-acx}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

output
```
1/2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)-1/a/c^
2/(-a*c*x+c)^(1/2)
```

3.267.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)}{ac^2\sqrt{c-acx}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]/(a*c^2*Sqrt[c - a*c*x]))
```

3.267.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 6680, 35, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{1-ax}{(ax+1)(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{1}{(ax+1)(c-acx)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{1}{(ax+1)\sqrt{c-acx}}dx}{2c}+\frac{1}{ac\sqrt{c-acx}}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{ac^2}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{3/2}}}{c}$$

input

```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

output

```
-((1/(a*c*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(S
qrt[2]*a*c^(3/2)))/c)
```

3.267.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.267. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.267.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}+\frac{1}{2c\sqrt{-acx+c}}\right)}{ca}$	50
default	$\frac{-\frac{1}{c\sqrt{-acx+c}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{2c^{\frac{3}{2}}}}{ac}$	50
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax-1)}-2\sqrt{c}}{2c^{\frac{5}{2}}\sqrt{-c(ax-1)}\,a}$	58

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/a*(-1/4/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
+1/2/c/(-a*c*x+c)^(1/2))
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+4\sqrt{-acx+c}}{4\left(a^2c^3x-ac^3\right)},\right.$$
$$\left.-\frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-2\sqrt{-acx+c}}{2\left(a^2c^3x-ac^3\right)}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3), -1/2*(s
qrt(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c)
- 2*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3)]
```

3.267.6 Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\begin{cases}-\frac{2\cdot\left(\frac{1}{2c\sqrt{-acx+c}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4c\sqrt{-c}}\right)}{ac} & \text{for } ac\neq 0\\ \frac{\begin{cases}-x & \text{for } a=0\\ \frac{ax-2\log\left(ax+1\right)+1}{a} & \text{otherwise}\end{cases}}{c^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(5/2),x)
```

output
```
Piecewise((-2*(1/(2*c*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x
 + c)/(2*sqrt(-c)))/(4*c*sqrt(-c)))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq
(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True))/c**(5/2), True))
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=-\frac{\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}+\frac{4}{\sqrt{-acx+c}c}}{4\,ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) +
 sqrt(-a*c*x + c)))/c^(3/2) + 4/(sqrt(-a*c*x + c)*c))/(a*c)
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2\,a\sqrt{-c}c^2} - \frac{1}{\sqrt{-acx+c}ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^2
) - 1/(sqrt(-a*c*x + c)*a*c^2)
```

3.267.9 Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{2\,a\,c^{5/2}} - \frac{1}{a\,c^2\,\sqrt{c-a\,c\,x}}$$

input
```
int((a*x - 1)/((c - a*c*x)^(5/2)*(a*x + 1)),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(2*a*c^(5/2)) - 1
/(a*c^2*(c - a*c*x)^(1/2))
```

3.268 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

3.268.1 Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

output
```
-1/3/a/c^2/(-a*c*x+c)^(3/2)+1/4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/
2))/a/c^(7/2)*2^(1/2)-1/2/a/c^3/(-a*c*x+c)^(1/2)
```

3.268.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.47

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-ax)\right)}{3ac^2(c-acx)^{3/2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(3/2
))
```

3.268.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6717, 6680, 35, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{1-ax}{(ax+1)(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{1}{(ax+1)(c-acx)^{5/2}}dx}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{1}{(ax+1)(c-acx)^{3/2}}dx}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\frac{\int \frac{1}{(ax+1)\sqrt{c-acx}}dx}{2c}+\frac{1}{ac\sqrt{c-acx}}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{ac^2}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{3/2}}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{c}$$

input

```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

output

```
-((1/(3*a*c*(c - a*c*x)^(3/2)) + (1/(a*c*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c
 - a*c*x]/(Sqrt[2]*Sqrt[c])]/(Sqrt[2]*a*c^(3/2)))/(2*c))/c)
```

3.268.3.1 Defintions of rubi rules used

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.268.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}+\frac{1}{4c^{2}\sqrt{-acx+c}}+\frac{1}{6c(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	64
default	$\frac{-\frac{1}{2c^{2}\sqrt{-acx+c}}-\frac{1}{3c(-acx+c)^{\frac{3}{2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{5}{2}}}}{ac}$	64
pseudoelliptic	$\frac{\sqrt{2}\,\sqrt{-c(ax-1)}\,(ax-1)\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\sqrt{c}\,\left(-2ax+\frac{10}{3}\right)}{4c^{\frac{7}{2}}\sqrt{-c(ax-1)}\,(ax-1)a}$	75

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/a*(-1/8/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
+1/4/c^2/(-a*c*x+c)^(1/2)+1/6/c/(-a*c*x+c)^(3/2))
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.36

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+4\sqrt{-acx+c}(3\,ax-5)}{24\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)},\right.$$
$$\left.-\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-2\sqrt{-acx+c}(3\,ax-5)}{12\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt
(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 5))/(
a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)
*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x
+ c)*(3*a*x - 5))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

3.268.6 Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{1}{6c(-acx+c)^{\frac{3}{2}}}+\frac{1}{4c^2\sqrt{-acx+c}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8c^2\sqrt{-c}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(7/2),x)
```

output
```
Piecewise((-2*(1/(6*c*(-a*c*x + c)**(3/2)) + 1/(4*c**2*sqrt(-a*c*x + c)) +
 sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(8*c**2*sqrt(-c)))/(a
*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a
, True))/c**(7/2), True))
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}-\frac{4(3acx-5c)}{(-acx+c)^{\frac{3}{2}}c^2}}{24ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c)))/c^(5/2) - 4*(3*a*c*x - 5*c)/((-a*c*x + c)^(3/2)*c^2
))/(a*c)
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4\,a\sqrt{-c}c^3} - \frac{3\,acx-5\,c}{6\,(acx-c)\sqrt{-acx+c}ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^3
) - 1/6*(3*a*c*x - 5*c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c^3)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{4\,a\,c^{7/2}} - \frac{\frac{c-a\,c\,x}{2\,c^2}+\frac{1}{3\,c}}{a\,c\,{\left(c-a\,c\,x\right)}^{3/2}}$$

input
```
int((a*x - 1)/((c - a*c*x)^(7/2)*(a*x + 1)),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(4*a*c^(7/2)) - (
(c - a*c*x)/(2*c^2) + 1/(3*c))/(a*c*(c - a*c*x)^(3/2))
```

3.269 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

3.269.1 Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx=-\frac{1}{5ac^2(c-acx)^{5/2}}-\frac{1}{6ac^3(c-acx)^{3/2}}-\frac{1}{4ac^4\sqrt{c-acx}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

output
```
-1/5/a/c^2/(-a*c*x+c)^(5/2)-1/6/a/c^3/(-a*c*x+c)^(3/2)+1/8*arctanh(1/2*(-a
*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)-1/4/a/c^4/(-a*c*x+c)^(1/2
)
```

3.269.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-ax)\right)}{5ac^2(c-acx)^{5/2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

output
```
-1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(5/2
))
```

3.269.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6680, 35, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{9/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{1-ax}{(ax+1)(c-acx)^{9/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{1}{(ax+1)(c-acx)^{7/2}}dx}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{1}{(ax+1)(c-acx)^{5/2}}dx}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\frac{\int \frac{1}{(ax+1)(c-acx)^{3/2}}dx}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\frac{\frac{\int \frac{1}{(ax+1)\sqrt{c-acx}}dx}{2c}+\frac{1}{ac\sqrt{c-acx}}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{ac^2}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$-\frac{\frac{\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{3/2}}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

input

```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

output

```
-((1/(5*a*c*(c - a*c*x)^(5/2)) + (1/(3*a*c*(c - a*c*x)^(3/2)) + (1/(a*c*Sq
rt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(Sqrt[2]*a*c^(
3/2)))/(2*c))/(2*c))/c)
```

3.269.3.1 Defintions of rubi rules used

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.269. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.269.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}+\frac{1}{8c^{3}\sqrt{-acx+c}}+\frac{1}{12c^{2}(-acx+c)^{\frac{3}{2}}}+\frac{1}{10c(-acx+c)^{\frac{5}{2}}}\right)}{ca}$	78
default	$\frac{-\frac{1}{4c^{3}\sqrt{-acx+c}}-\frac{1}{6c^{2}(-acx+c)^{\frac{3}{2}}}-\frac{1}{5c(-acx+c)^{\frac{5}{2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{7}{2}}}}{ac}$	78
pseudoelliptic	$\frac{\sqrt{2}\,\sqrt{-c(ax-1)}\,(ax-1)^{2}\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-2\left(a^{2}x^{2}-\frac{8}{3}ax+\frac{37}{15}\right)\sqrt{c}}{8c^{\frac{9}{2}}\sqrt{-c(ax-1)}\,(ax-1)^{2}a}$	85

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/a*(-1/16/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)+1/8/c^3/(-a*c*x+c)^(1/2)+1/12/c^2/(-a*c*x+c)^(3/2)+1/10/c/(-a*c*x+c)^(5/
2))
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.42

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \left[\frac{15\,\sqrt{2}(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\sqrt{c}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right) + 4\,(15\,a^2x^2 - 40\,ax}{240\,(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5)}\right.$$

$$\left.-\frac{15\,\sqrt{2}(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right) - 2\,(15\,a^2x^2 - 40\,ax + 37)\sqrt{-acx+c}}{120\,(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5)}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((a*c*x -
2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*(15*a^2*x^2 - 40*
a*x + 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a
*c^5), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arcta
n(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*(15*a^2*x^2 - 40*a*x + 37)*
sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

3.269.6 Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx=\begin{cases} -\frac{2\cdot\left(\frac{1}{10c(-acx+c)^{\frac{5}{2}}}+\frac{1}{12c^{2}(-acx+c)^{\frac{3}{2}}}+\frac{1}{8c^{3}\sqrt{-acx+c}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{16c^{3}\sqrt{-c}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log{(ax+1)}+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(9/2),x)
```

output
```
Piecewise((-2*(1/(10*c*(-a*c*x + c)**(5/2)) + 1/(12*c**2*(-a*c*x + c)**(3/
2)) + 1/(8*c**3*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/
(2*sqrt(-c)))/(16*c**3*sqrt(-c)))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a
, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True))/c**(9/2), True))
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx=-\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{7}{2}}}+\frac{4\left(15\,(acx-c)^{2}-10\,(acx-c)c+12\,c^{2}\right)}{(-acx+c)^{\frac{5}{2}}\,c^{3}}}{240\,ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

output
```
-1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt
(c) + sqrt(-a*c*x + c)))/c^(7/2) + 4*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c
+ 12*c^2)/((-a*c*x + c)^(5/2)*c^3))/(a*c)
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8\,a\sqrt{-c}c^{4}} - \frac{15\,(acx-c)^{2}-10\,(acx-c)c+12\,c^{2}}{60\,(acx-c)^{2}\sqrt{-acx+c}ac^{4}}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^4
) - 1/60*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 12*c^2)/((a*c*x - c)^2*sqr
t(-a*c*x + c)*a*c^4)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{8\,a\,c^{9/2}} - \frac{\frac{c-a\,c\,x}{6\,c^{2}}+\frac{1}{5\,c}+\frac{(c-a\,c\,x)^{2}}{4\,c^{3}}}{a\,c\,(c-a\,c\,x)^{5/2}}$$

input
```
int((a*x - 1)/((c - a*c*x)^(9/2)*(a*x + 1)),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(8*a*c^(9/2)) - (
(c - a*c*x)/(6*c^2) + 1/(5*c) + (c - a*c*x)^2/(4*c^3))/(a*c*(c - a*c*x)^(5
/2))
```

3.270 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

3.270.1 Optimal result

Integrand size = 20, antiderivative size = 368

$$
\begin{aligned}
\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = & -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} \\
& -\frac{94208(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^5}-\frac{40960(c-acx)^{9/2}}{231a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^4} \\
& +\frac{4096(c-acx)^{9/2}}{231a^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^3}-\frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^2} \\
& +\frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2\left(a-\frac{1}{x}\right)^6x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}
\end{aligned}
$$

output
```
-16/33*(a-1/x)^5*(-a*c*x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)-9420
8/231*(-a*c*x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/x^5/(1+1/a/x)^(1/2)-40960/231*(
-a*c*x+c)^(9/2)/a^5/(1-1/a/x)^(9/2)/x^4/(1+1/a/x)^(1/2)+4096/231*(-a*c*x+c
)^(9/2)/a^4/(1-1/a/x)^(9/2)/x^3/(1+1/a/x)^(1/2)-1024/231*(a-1/x)^3*(-a*c*x
+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/x^2/(1+1/a/x)^(1/2)+320/231*(a-1/x)^4*(-a*c*
x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/x/(1+1/a/x)^(1/2)+2/11*(a-1/x)^6*x*(-a*c*x+
c)^(9/2)/a^6/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2c^4\sqrt{c-acx}(-46355-23062ax+5419a^2x^2-2132a^3x^3+755a^4x^4-182a^5x^5+21a^6x^6)}{231a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*c^4*Sqrt[c - a*c*x]*(-46355 - 23062*a*x + 5419*a^2*x^2 - 2132*a^3*x^3 +
 755*a^4*x^4 - 182*a^5*x^5 + 21*a^6*x^6))/(231*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

3.270.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.73, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6727, 27, 105, 105, 105, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{9/2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^6}{a^6\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^6}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{24}{11}\int\frac{\left(a-\frac{1}{x}\right)^5}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^6}{11\left(\frac{1}{x}\right)^{11/2}\sqrt{\frac{1}{ax}+1}}\Bigg)}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{24}{11}\Bigg(-\frac{20}{9}\int\frac{\left(a-\frac{1}{x}\right)^4}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^5}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right)^6}{11\left(\frac{1}{x}\right)^{11/2}\sqrt{\frac{1}{ax}+1}}\Bigg)}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{24}{11}\Bigg(-\frac{20}{9}\Bigg(-\frac{16}{7}\int\frac{\left(a-\frac{1}{x}\right)^3}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right)^5}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right)^6}{11\left(\frac{1}{x}\right)^{11/2}\sqrt{\frac{1}{ax}+1}}}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{24}{11}\Bigg(-\frac{20}{9}\Bigg(-\frac{16}{7}\Bigg(-\frac{12}{5}\int\frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right.}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{}}}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{24}{11}\Bigg(-\frac{20}{9}\Bigg(-\frac{16}{7}\Bigg(-\frac{12}{5}\Bigg(\frac{2}{3}\int-\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{}{7(}}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{24}{11}\Bigg(-\frac{20}{9}\Bigg(-\frac{16}{7}\Bigg(-\frac{12}{5}\Bigg(-\frac{1}{3}\int\frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2}{7\left(\frac{1}{x}\right.}}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\Bigg(-\frac{24}{11}\Bigg(-\frac{20}{9}\Bigg(-\frac{16}{7}\Bigg(-\frac{12}{5}\Bigg(\frac{1}{3}\Bigg(23\int\frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2(a-}{5\left(\frac{1}{x}\right)^{5/2}}}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{ 48}$$

3.270. $\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}\Bigg(-\frac{24}{11}\Bigg(-\frac{20}{9}\Bigg(-\frac{16}{7}\Bigg(-\frac{12}{5}\Bigg(\frac{1}{3}\Bigg(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\Bigg)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^6\left(1-\frac{1}{ax}\right)^{9/2}}$$

input
```
Int[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((((-24*((-20*((-16*((-12*(((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46
*Sqrt[x^(-1)])/Sqrt[1 + 1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1)
)^(3/2))))/5 - (2*(a - x^(-1))^3)/(5*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2))))/7
 - (2*(a - x^(-1))^4)/(7*Sqrt[1 + 1/(a*x)]*(x^(-1))^(7/2))))/9 - (2*(a - x
^(-1))^5)/(9*Sqrt[1 + 1/(a*x)]*(x^(-1))^(9/2))))/11 - (2*(a - x^(-1))^6)/(
11*Sqrt[1 + 1/(a*x)]*(x^(-1))^(11/2)))*(x^(-1))^(9/2)*(c - a*c*x)^(9/2))/(
a^6*(1 - 1/(a*x))^(9/2)))
```

3.270.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.270.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.24

method	result	size
gosper	$\frac{2(ax+1)\left(21a^{6}x^{6}-182a^{5}x^{5}+755a^{4}x^{4}-2132a^{3}x^{3}+5419a^{2}x^{2}-23062ax-46355\right)(-acx+c)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{231a(ax-1)^{6}}$	88
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^{4}\left(21a^{6}x^{6}-182a^{5}x^{5}+755a^{4}x^{4}-2132a^{3}x^{3}+5419a^{2}x^{2}-23062ax-46355\right)}{231(ax-1)^{2}a}$	92
risch	$-\frac{2\left(21a^{5}x^{5}-203a^{4}x^{4}+958a^{3}x^{3}-3090a^{2}x^{2}+8509ax-31571\right)(ax+1)c^{5}\sqrt{\frac{ax-1}{ax+1}}}{231a\sqrt{-c(ax-1)}}+\frac{128c^{5}\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	111

input
```
int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/231*(a*x+1)*(21*a^6*x^6-182*a^5*x^5+755*a^4*x^4-2132*a^3*x^3+5419*a^2*x^
2-23062*a*x-46355)*(-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^6
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}(c$$

$$-acx)^{9/2}\,dx = \frac{2\,(21\,a^6c^4x^6 - 182\,a^5c^4x^5 + 755\,a^4c^4x^4 - 2132\,a^3c^4x^3 + 5419\,a^2c^4x^2 - 23062\,ac^4x - 46355\,c^4}{231\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
2/231*(21*a^6*c^4*x^6 - 182*a^5*c^4*x^5 + 755*a^4*c^4*x^4 - 2132*a^3*c^4*x
^3 + 5419*a^2*c^4*x^2 - 23062*a*c^4*x - 46355*c^4)*sqrt(-a*c*x + c)*sqrt((
a*x - 1)/(a*x + 1))/(a^2*x - a)
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.41

$$\int e^{-3\coth^{-1}(ax)}(c$$

$$-acx)^{9/2}\,dx = \frac{2\,(21\,a^7\sqrt{-c}c^4x^7 - 161\,a^6\sqrt{-c}c^4x^6 + 573\,a^5\sqrt{-c}c^4x^5 - 1377\,a^4\sqrt{-c}c^4x^4 + 3287\,a^3\sqrt{-c}c^4x^3}{231\,(a^3x^2 - 2\,a^2x + a)(ax + 1}$$

input
```
integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
2/231*(21*a^7*sqrt(-c)*c^4*x^7 - 161*a^6*sqrt(-c)*c^4*x^6 + 573*a^5*sqrt(-
c)*c^4*x^5 - 1377*a^4*sqrt(-c)*c^4*x^4 + 3287*a^3*sqrt(-c)*c^4*x^3 - 17643
*a^2*sqrt(-c)*c^4*x^2 - 69417*a*sqrt(-c)*c^4*x - 46355*sqrt(-c)*c^4)*(a*x
- 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

3.270.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.270.9 Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(21\,a^5\,x^5-161\,a^4\,x^4+594\,a^3\,x^3-1538\,a^2\,x^2+3881\,a\,x-19181\right)}{231\,a} - \frac{131072\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{231\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3881*a*x - 1538*a^2*
x^2 + 594*a^3*x^3 - 161*a^4*x^4 + 21*a^5*x^5 - 19181))/(231*a) - (131072*c
^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(231*a*(a*x - 1))
```

3.271 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

3.271.1 Optimal result

Integrand size = 20, antiderivative size = 311

$$\begin{aligned}
&\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \\
&-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{11776(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^4} \\
&+\frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}-\frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2} \\
&+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}
\end{aligned}$$

output

```
-40/63*(a-1/x)^4*(-a*c*x+c)^(7/2)/a^5/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+1177
6/63*(-a*c*x+c)^(7/2)/a^5/(1-1/a/x)^(7/2)/x^4/(1+1/a/x)^(1/2)+5120/63*(-a*
c*x+c)^(7/2)/a^4/(1-1/a/x)^(7/2)/x^3/(1+1/a/x)^(1/2)-512/63*(-a*c*x+c)^(7/
2)/a^3/(1-1/a/x)^(7/2)/x^2/(1+1/a/x)^(1/2)+128/63*(a-1/x)^3*(-a*c*x+c)^(7/
2)/a^5/(1-1/a/x)^(7/2)/x/(1+1/a/x)^(1/2)+2/9*(a-1/x)^5*x*(-a*c*x+c)^(7/2)/
a^5/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.24

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2c^3\sqrt{c-acx}(5797+2867ax-638a^2x^2+214a^3x^3-55a^4x^4+7a^5x^5)}{63a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(-2*c^3*Sqrt[c - a*c*x]*(5797 + 2867*a*x - 638*a^2*x^2 + 214*a^3*x^3 - 55*
a^4*x^4 + 7*a^5*x^5))/(63*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.271.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6727, 27, 105, 105, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^5}{a^5\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^5}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{20}{9}\int \frac{\left(a-\frac{1}{x}\right)^4}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^5}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{20}{9}\left(-\frac{16}{7}\int \frac{\left(a-\frac{1}{x}\right)^3}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^5}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{20}{9}\left(-\frac{16}{7}\left(-\frac{12}{5}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^5}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right.}{a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{20}{9}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{2}{3}\int -\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right.}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{}}\right.\right.}{a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{20}{9}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(-\frac{1}{3}\int \frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{a}}\right.\right.}{a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{20}{9}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{1}{3}\left(23\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right.\right.\right.}{a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}\left(-\frac{20}{9}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{}{9(}\right.}{a^5\left(1-\frac{1}{ax}\right)^{7/2}}$$

input
```
Int[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

3.271. $\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

output
```
-((((-20*((-16*((-12*(((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[
x^(-1)])/Sqrt[1 + 1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2
))))/5 - (2*(a - x^(-1))^3)/(5*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2))))/7 - (2*
(a - x^(-1))^4)/(7*Sqrt[1 + 1/(a*x)]*(x^(-1))^(7/2))))/9 - (2*(a - x^(-1))
^5)/(9*Sqrt[1 + 1/(a*x)]*(x^(-1))^(9/2)))*(x^(-1))^(7/2)*(c - a*c*x)^(7/2)
)/(a^5*(1 - 1/(a*x))^(7/2)))
```

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.271.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

method	result	size
gosper	$\frac{2(ax+1)\left(7a^{5}x^{5}-55a^{4}x^{4}+214a^{3}x^{3}-638a^{2}x^{2}+2867ax+5797\right)\left(-acx+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{63a\left(ax-1\right)^{5}}$	80
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^{3}\left(7a^{5}x^{5}-55a^{4}x^{4}+214a^{3}x^{3}-638a^{2}x^{2}+2867ax+5797\right)}{63\left(ax-1\right)^{2}a}$	84
risch	$\frac{2\left(7a^{4}x^{4}-62a^{3}x^{3}+276a^{2}x^{2}-914ax+3781\right)(ax+1)c^{4}\sqrt{\frac{ax-1}{ax+1}}}{63a\sqrt{-c(ax-1)}}+\frac{64c^{4}\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	103

input
```
int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/63*(a*x+1)*(7*a^5*x^5-55*a^4*x^4+214*a^3*x^3-638*a^2*x^2+2867*a*x+5797)*
(-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^5
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx =$$
$$-\frac{2\left(7\,a^{5}c^{3}x^{5}-55\,a^{4}c^{3}x^{4}+214\,a^{3}c^{3}x^{3}-638\,a^{2}c^{3}x^{2}+2867\,ac^{3}x+5797\,c^{3}\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^{2}x-a\right)}$$

input
```
integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-2/63*(7*a^5*c^3*x^5 - 55*a^4*c^3*x^4 + 214*a^3*c^3*x^3 - 638*a^2*c^3*x^2
+ 2867*a*c^3*x + 5797*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

3.271.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=$$
$$-\frac{2\left(7\,a^6\sqrt{-c}c^3x^6-48\,a^5\sqrt{-c}c^3x^5+159\,a^4\sqrt{-c}c^3x^4-424\,a^3\sqrt{-c}c^3x^3+2229\,a^2\sqrt{-c}c^3x^2+8664\,a\sqrt{-c}c^3\right.}{63\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-2/63*(7*a^6*sqrt(-c)*c^3*x^6 - 48*a^5*sqrt(-c)*c^3*x^5 + 159*a^4*sqrt(-c)
*c^3*x^4 - 424*a^3*sqrt(-c)*c^3*x^3 + 2229*a^2*sqrt(-c)*c^3*x^2 + 8664*a*s
qrt(-c)*c^3*x + 5797*sqrt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a
*x + 1)^(3/2))
```

3.271.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.271.9 Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.33

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx =$$
$$-\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(7\,a^4\,x^4-48\,a^3\,x^3+166\,a^2\,x^2-472\,a\,x+2395\right)}{63\,a}$$
$$-\frac{16384\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{63\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(166*a^2*x^2 - 472*
a*x - 48*a^3*x^3 + 7*a^4*x^4 + 2395))/(63*a) - (16384*c^3*(c - a*c*x)^(1/2
)*((a*x - 1)/(a*x + 1))^(1/2))/(63*a*(a*x - 1))
```

3.272 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.272.1 Optimal result

Integrand size = 20, antiderivative size = 254

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx &= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} \\ &\quad -\frac{2944(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3}-\frac{256(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2} \\ &\quad +\frac{128(c-acx)^{5/2}}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}\end{aligned}$$

output

```
-32/35*(a-1/x)^3*(-a*c*x+c)^(5/2)/a^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)-2944
/35*(-a*c*x+c)^(5/2)/a^4/(1-1/a/x)^(5/2)/x^3/(1+1/a/x)^(1/2)-256/7*(-a*c*x
+c)^(5/2)/a^3/(1-1/a/x)^(5/2)/x^2/(1+1/a/x)^(1/2)+128/35*(-a*c*x+c)^(5/2)/
a^2/(1-1/a/x)^(5/2)/x/(1+1/a/x)^(1/2)+2/7*(a-1/x)^4*x*(-a*c*x+c)^(5/2)/a^4
/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\sqrt{c-acx}(-1451-708ax+142a^2x^2-36a^3x^3+5a^4x^4)}{35a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*c^2*Sqrt[c - a*c*x]*(-1451 - 708*a*x + 142*a^2*x^2 - 36*a^3*x^3 + 5*a^4
*x^4))/(35*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.272.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6727, 27, 105, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^4}{a^4\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^4}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\int \frac{\left(a-\frac{1}{x}\right)^3}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\int\frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 100$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{2}{3}\int-\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 27$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(-\frac{1}{3}\int\frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 87$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{1}{3}\left(23\int\frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)(c-acx)^{5}}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

input

```
Int[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output

```
-((((-16*((-12*(((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[x^(-1)
])/Sqrt[1 + 1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))))/5
 - (2*(a - x^(-1))^3)/(5*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2))))/7 - (2*(a - x
^(-1))^4)/(7*Sqrt[1 + 1/(a*x)]*(x^(-1))^(7/2)))*(x^(-1))^(5/2)*(c - a*c*x)
^(5/2))/(a^4*(1 - 1/(a*x))^(5/2)))
```

3.272. $\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.272.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.272. $\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.272.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2(ax+1)(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451)(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{35a(ax-1)^4}$	72
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^2(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451)}{35(ax-1)^2a}$	76
risch	$-\frac{2(5a^3x^3-41a^2x^2+183ax-891)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{35a\sqrt{-c(ax-1)}}+\frac{32c^3\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	95

input
```
int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(a*x+1)*(5*a^4*x^4-36*a^3*x^3+142*a^2*x^2-708*a*x-1451)*(-a*c*x+c)^(5
/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^4
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^4c^2x^4-36\,a^3c^2x^3+142\,a^2c^2x^2-708\,ac^2x-1451\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^2x-a\right)}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^4*c^2*x^4 - 36*a^3*c^2*x^3 + 142*a^2*c^2*x^2 - 708*a*c^2*x - 145
1*c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.47

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^5\sqrt{-c}c^2x^5-31\,a^4\sqrt{-c}c^2x^4+106\,a^3\sqrt{-c}c^2x^3-566\,a^2\sqrt{-c}c^2x^2-2159\,a\sqrt{-c}c^2x-145\right.}{35\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
2/35*(5*a^5*sqrt(-c)*c^2*x^5 - 31*a^4*sqrt(-c)*c^2*x^4 + 106*a^3*sqrt(-c)*
c^2*x^3 - 566*a^2*sqrt(-c)*c^2*x^2 - 2159*a*sqrt(-c)*c^2*x - 1451*sqrt(-c)
*c^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

3.272.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.272.9 Mupad [B] (verification not implemented)

Time = 4.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.37

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-31\,a^2\,x^2+111\,a\,x-597\right)}{35\,a} - \frac{4096\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(111*a*x - 31*a^2*x^2
 + 5*a^3*x^3 - 597))/(35*a) - (4096*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x
+ 1))^(1/2))/(35*a*(a*x - 1))
```

3.273 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

3.273.1 Optimal result

Integrand size = 20, antiderivative size = 195

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{184(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2}+\frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}+\frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

output
```
-8/5*(-a*c*x+c)^(3/2)/a/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+184/5*(-a*c*x+c)^(
3/2)/a^3/(1-1/a/x)^(3/2)/x^2/(1+1/a/x)^(1/2)+16*(-a*c*x+c)^(3/2)/a^2/(1-1/
a/x)^(3/2)/x/(1+1/a/x)^(1/2)+2/5*(a-1/x)^3*x*(-a*c*x+c)^(3/2)/a^3/(1-1/a/x
)^(3/2)/(1+1/a/x)^(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\sqrt{c-acx}(91+43ax-7a^2x^2+a^3x^3)}{5a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(-2*c*Sqrt[c - a*c*x]*(91 + 43*a*x - 7*a^2*x^2 + a^3*x^3))/(5*a^2*Sqrt[1 -
 1/(a^2*x^2)]*x)
```

3.273.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6727, 27, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)^3}{a^3\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)^3}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{12}{5}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{12}{5}\left(\frac{2}{3}\int -\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{12}{5}\left(-\frac{1}{3}\int \frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{12}{5}\left(\frac{1}{3}\left(23\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}\left(-\frac{12}{5}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)(c-acx)^{3/2}}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((((-12*(((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[x^(-1)])/Sqr
t[1 + 1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))))/5 - (2*
(a - x^(-1))^3)/(5*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2)))*(x^(-1))^(3/2)*(c -
a*c*x)^(3/2))/(a^3*(1 - 1/(a*x))^(3/2)))
```

3.273.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.273.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.32

method	result	size
gosper	$\frac{2(ax+1)\left(a^{3}x^{3}-7a^{2}x^{2}+43ax+91\right)(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{5a(ax-1)^{3}}$	63
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c\left(a^{3}x^{3}-7a^{2}x^{2}+43ax+91\right)}{5(ax-1)^{2}a}$	65
risch	$\frac{2\left(a^{2}x^{2}-8ax+51\right)(ax+1)c^{2}\sqrt{\frac{ax-1}{ax+1}}}{5a\sqrt{-c(ax-1)}}+\frac{16c^{2}\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	86

input
```
int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a*x+1)*(a^3*x^3-7*a^2*x^2+43*a*x+91)*(-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1
))^(3/2)/a/(a*x-1)^3
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.32

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,(a^3cx^3-7\,a^2cx^2+43\,acx+91\,c)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(a^3*c*x^3 - 7*a^2*c*x^2 + 43*a*c*x + 91*c)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

3.273.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(a^4\sqrt{-c}cx^4-6\,a^3\sqrt{-c}cx^3+36\,a^2\sqrt{-c}cx^2+134\,a\sqrt{-c}cx+91\,\sqrt{-c}c\right)(ax-1)^2}{5\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-2/5*(a^4*sqrt(-c)*c*x^4 - 6*a^3*sqrt(-c)*c*x^3 + 36*a^2*sqrt(-c)*c*x^2 +
134*a*sqrt(-c)*c*x + 91*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(
a*x + 1)^(3/2))
```

3.273.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.273.9 Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2-6\,a\,x+37\right)}{5\,a}-\frac{256\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a^2*x^2 - 6*a*x + 37
))/(5*a) - (256*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x
 - 1))
```

3.274 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.274.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

output
```
-20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(
1/2)/a^2/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2)
```

3.274.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-23-10ax+a^2x^2)}{3a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*
x)
```

3.274.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{3}\int -\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{3}\int \frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{3}\left(23\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[x^(-1)])/Sqrt[1 +
1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2)))*Sqrt[x^(-1)]*S
qrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.274.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.274.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.40

method	result	size
gosper	$\frac{2(ax+1)(a^{2}x^{2}-10ax-23)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a(ax-1)^{2}}$	55
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(a^{2}x^{2}-10ax-23)}{3(ax-1)^{2}a}$	56
risch	$-\frac{2(ax-11)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{3a\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	74

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a
/(a*x-1)^2
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^{2}x^{2}-10\,ax-23\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{2}x-a\right)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x - a)
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^3\sqrt{-c}x^3-9\,a^2\sqrt{-c}x^2-33\,a\sqrt{-c}x-23\,\sqrt{-c}\right)(ax-1)^2}{3\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c)
)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

3.274.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.274.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-a\,c\,x}\,(a\,x-9)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{64\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.275 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.275.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}+\frac{2\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}$$

output
```
6*(1-1/a/x)^(1/2)/a/(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)+2*x*(1-1/a/x)^(1/2)/(
1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}(3+ax)}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

output
```
(2*Sqrt[1 - 1/(a*x)]*(3 + a*x))/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

3.275.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a-\frac{1}{x}}{a\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a-\frac{1}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\left(-3\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{2a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{6\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{1-\frac{1}{ax}}}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

output
```
-((((-2*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) - (6*Sqrt[x^(-1)])/Sqrt[1 + 1/
(a*x)])*Sqrt[1 - 1/(a*x)])/(a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]))
```

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.275.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{2(ax+1)(ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(ax-1)\sqrt{-acx+c}}$	47
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(ax+3)}{(ax-1)^{2}ca}$	51
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{\sqrt{-c(ax-1)}\,a}+\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	67

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x+1)*(a*x+3)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)/(-a*c*x+c)^(1/2)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{-acx+c}(ax+3)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*c*x + c)*(a*x + 3)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
```

output
```
Timed out
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\,(a^2x^2+4\,ax+3)(ax-1)}{\left(a^2\sqrt{-c}x-a\sqrt{-c}\right)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2*(a^2*x^2 + 4*a*x + 3)*(a*x - 1)/((a^2*sqrt(-c)*x - a*sqrt(-c))*(a*x + 1)
^(3/2))
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = 2\left(\frac{\sqrt{-acx-c}}{ac^2}-\frac{2}{\sqrt{-acx-cac}}\right)|c|$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2*(sqrt(-a*c*x - c)/(a*c^2) - 2/(sqrt(-a*c*x - c)*a*c))*abs(c)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(2\,x+\frac{6}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(1/2),x)
```

output
```
((2*x + 6/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

3.276 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

3.276.1 Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2e^{-3\coth^{-1}(ax)}(1+ax)}{a(c-acx)^{3/2}}$$

output
```
-2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

3.276.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}x}{\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

output
```
(-2*(1 - 1/(a*x))^(3/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))
```

3.276.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{6726\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \text{6726}$$

$$-\frac{2(ax+1)e^{-3\coth^{-1}(ax)}}{a(c-acx)^{3/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

output
```
(-2*(1 + a*x))/(a*E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2))
```

3.276.3.1 Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

3.276.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(-acx+c)^{\frac{3}{2}}}$	35
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}}{(ax-1)^{2}c^{2}a}$	46

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
```

output
```
Timed out
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)(ax-1)}{(a^2c^2x-ac^2)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-2*(a*sqrt(-c)*x + sqrt(-c))*(a*x - 1)/((a^2*c^2*x - a*c^2)*(a*x + 1)^(3/2
))
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\frac{\sqrt{2}}{a\sqrt{-c}}-\frac{2}{\sqrt{-acx-ca}}\right)|c|\mathrm{sgn}(ax+1)}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(2)/(a*sqrt(-c)) - 2/(sqrt(-a*c*x - c)*a))*abs(c)*sgn(a*x + 1)/c^2
```

3.276.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c\,\sqrt{c-a\,c\,x}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(3/2),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c*(c - a*c*x)^(1/2))
```

3.277 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.277.1 Optimal result

Integrand size = 20, antiderivative size = 120

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}} - \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

output

```
-1/2*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)
^(1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+a*(1-1/a/x)^(5/2)*x^2/(-a*c*x
+c)^(5/2)/(1+1/a/x)^(1/2)
```

3.277.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}-\sqrt{2}\sqrt{a}\sqrt{1+\frac{1}{ax}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{2ac^2\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

input

```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

output

```
(Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)] - Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*Arc
Tanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*a*c^2*Sqrt[1
 + 1/(a*x)]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

3.277.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6727, 27, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{a\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{2}a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\left(a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

output
```
-((a*(1 - 1/(a*x))^(5/2)*(-(Sqrt[x^(-1)]/Sqrt[1 + 1/(a*x)]) + (Sqrt[a]*Arc
Tanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[2]))/((x^(-
1))^(5/2)*(c - a*c*x)^(5/2)))
```

3.277.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.277.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1) \sqrt{-c(ax-1)}\, \left(\arctan\left(\frac{\sqrt{-c(ax+1)}\, \sqrt{2}}{2\sqrt{c}}\right) \sqrt{2}\, \sqrt{-c(ax+1)}+2\sqrt{c}\right)}{2(ax-1)^{2} c^{\frac{7}{2}} a}$	85

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)/c^(7/2)*
(arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^(1/2)
+2*c^(1/2))/a
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.96

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-acx+c}}{4\left(a^2c^3x-ac^3\right)}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^
2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3*x
 - a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*sqrt((
a*x - 1)/(a*x + 1)))/(a^2*c^3*x - a*c^3)]
```

3.277.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

3.277.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(5/2), x)
```

3.277.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{{\left(c-a\,c\,x\right)}^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2), x)
```

3.278 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

3.278.1 Optimal result

Integrand size = 20, antiderivative size = 184

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}-\frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}+\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

output
```
3/8*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^
(1/2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/2*a^2*(1-1/a/x)^(7/2)*x^2/(a
-1/x)/(-a*c*x+c)^(7/2)/(1+1/a/x)^(1/2)-3/4*a^2*(1-1/a/x)^(7/2)*x^3/(-a*c*x
+c)^(7/2)/(1+1/a/x)^(1/2)
```

3.278.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.71

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(-2+6ax-\frac{3\sqrt{2}\sqrt{a}\sqrt{1+\frac{1}{ax}}(-1+ax)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{\frac{1}{x}}}\right)}{8ac^3\sqrt{1+\frac{1}{ax}}(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(-2 + 6*a*x - (3*Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1
+ a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[x
^(-1)]))/(8*a*c^3*Sqrt[1 + 1/(a*x)]*(-1 + a*x)*Sqrt[c - a*c*x])
```

3.278.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{a^2\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\int \frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(\frac{1}{2}a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(a\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

output
```
-((a^2*(1 - 1/(a*x))^(7/2)*((x^(-1))^(3/2)/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x
)]) - (3*(-(Sqrt[x^(-1)]/Sqrt[1 + 1/(a*x)]) + (Sqrt[a]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[2]))/4))/((x^(-1))^(7/2)*(c
 - a*c*x)^(7/2)))
```

3.278.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.278.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)ax\sqrt{-c(ax+1)}-3\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c(ax+1)}+6\sqrt{c}\,a\right.}{8(ax-1)^{3}c^{\frac{9}{2}}a}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^3*(-c*(a*x-1))^(1/2)/c^(9/2)*
(3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*x*(-c*(a*x+1))
^(1/2)-3*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1
))^(1/2)+6*c^(1/2)*a*x-2*c^(1/2))/a
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4}{16\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
 - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x
- 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/8*(3*sqrt(2)*(a^2*
x^2 - 2*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x
 - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x -
 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

3.278.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

3.278.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(7/2), x)
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{5}{2}}} - \frac{2\,(3\,acx-c)}{\left((-acx-c)^{\frac{3}{2}}+2\sqrt{-acx-c}c\right)ac^{2}}\right)|c|\text{sgn}(ax+1)}{8\,c^{2}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
-1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(5/2)) -
 2*(3*a*c*x - c)/(((-a*c*x - c)^(3/2) + 2*sqrt(-a*c*x - c)*c)*a*c^2))*abs(
c)*sgn(a*x + 1)/c^2
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-a\,c\,x\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2), x)
```

3.279 $\int e^{\coth^{-1}(x)}x(1+x)\,dx$

3.279.1 Optimal result

Integrand size = 9, antiderivative size = 99

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

output
```
arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+1/3*(1+1/x)^(3/2)*x^2*((-1+x)/x)^(
1/2)+1/3*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+x*(1+1/x)^(1/2)*((-1+x)/x)^(1/
2)
```

3.279.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x(5+3x+x^2)+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*x*(1 + x),x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(5 + 3*x + x^2))/3 + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

3.279.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6729, 107, 105, 105, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{ 6729}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^{3/2}x^4}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 107}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{2}{3}\int \frac{\left(1+\frac{1}{x}\right)^{3/2}x^3}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{2}{3}\left(\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{x}}x^2}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{ 105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-$$
$$\frac{2}{3}\left(\frac{3}{2}\left(\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{ 103}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-$$
$$\frac{2}{3}\left(\frac{3}{2}\left(-\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-$$
$$\frac{2}{3}\left(\frac{3}{2}\left(-\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1x}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

input
```
Int[E^ArcCoth[x]*x*(1 + x),x]
```

output
```
(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(5/2)*x^3)/3 - (2*(-1/2*(Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2) + (3*(-(Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x) - ArcT
anh[Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]]))/2))/3
```

3.279.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

3.279.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

method	result	size
trager	$\frac{(1+x)(x^2+3x+5)\sqrt{-\frac{1-x}{1+x}}}{3}+\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)$	62
risch	$\frac{(x^2+3x+5)(x-1)}{3\sqrt{\frac{x-1}{1+x}}}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	62
default	$\frac{(x-1)\left(((x-1)(1+x))^{\frac{3}{2}}+3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)+6\sqrt{x^2-1}\right)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	67

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1+x)*(x^2+3*x+5)*(-(1-x)/(1+x))^(1/2)+ln((-(1-x)/(1+x))^(1/2)*x+(-(1-
x)/(1+x))^(1/2)+x)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\frac{1}{3}\left(x^3+4\,x^2+8\,x+5\right)\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 4*x^2 + 8*x + 5)*sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x +
1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

3.279.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\int\frac{x(x+1)}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x),x)
```

output
```
Integral(x*(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=-\frac{2\left(3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{x-1}{x+1}}\right)}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="maxima")
```

output
```
-2/3*(3*((x - 1)/(x + 1))^(5/2) - 8*((x - 1)/(x + 1))^(3/2) + 9*sqrt((x -
1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1
)^3 - 1) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\frac{1}{3}\sqrt{x^2-1}\left(x\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{3}{\operatorname{sgn}(x+1)}\right)+\frac{5}{\operatorname{sgn}(x+1)}\right)-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="giac")
```

output
```
1/3*sqrt(x^2 - 1)*(x*(x/sgn(x + 1) + 3/sgn(x + 1)) + 5/sgn(x + 1)) - log(a
bs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{6\,\sqrt{\frac{x-1}{x+1}}-\frac{16\,{\left(\frac{x-1}{x+1}\right)}^{3/2}}{3}+2\,{\left(\frac{x-1}{x+1}\right)}^{5/2}}{\frac{3\,\left(x-1\right)}{x+1}-\frac{3\,{\left(x-1\right)}^2}{{\left(x+1\right)}^2}+\frac{{\left(x-1\right)}^3}{{\left(x+1\right)}^3}-1}$$

input
```
int((x*(x + 1))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
2*atanh(((x - 1)/(x + 1))^(1/2)) - (6*((x - 1)/(x + 1))^(1/2) - (16*((x -
1)/(x + 1))^(3/2))/3 + 2*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (
3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.280 $\int e^{\coth^{-1}(x)}(1+x)\,dx$

3.280.1 Optimal result

Integrand size = 8, antiderivative size = 79

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=\frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2+\frac{3}{2}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

output
```
3/2*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+1/2*(1+1/x)^(3/2)*x^2*((-1+x)/
x)^(1/2)+3/2*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

3.280.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x(4+x)+\frac{3}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 + x),x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(4 + x))/2 + (3*Log[(1 + Sqrt[1 - x^(-2)])*x])/2
```

3.280.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6725, 105, 105, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6725}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^{3/2}x^3}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2-\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{x}}x^2}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2-\frac{3}{2}\left(\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2-\frac{3}{2}\left(-\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2-\frac{3}{2}\left(-\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)$$

input
```
Int[E^ArcCoth[x]*(1 + x),x]
```

output
```
(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(3/2)*x^2)/2 - (3*(-(Sqrt[1 - x^(-1)]*Sqrt[
1 + x^(-1)]*x) - ArcTanh[Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]]))/2
```

3.280.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.280.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{(x-1)\left(x\sqrt{x^{2}-1}+4\sqrt{x^{2}-1}+3\ln\left(x+\sqrt{x^{2}-1}\right)\right)}{2\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	57
risch	$\frac{(x+4)(x-1)}{2\sqrt{\frac{x-1}{1+x}}}+\frac{3\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	58
trager	$\frac{(1+x)(x+4)\sqrt{-\frac{1-x}{1+x}}}{2}-\frac{3\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	62

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x-1)*(x*(x^2-1)^(1/2)+4*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/((x-1)/(
1+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \frac{1}{2}\left(x^2+5x+4\right)\sqrt{\frac{x-1}{x+1}} + \frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="fricas")
```

output
```
1/2*(x^2 + 5*x + 4)*sqrt((x - 1)/(x + 1)) + 3/2*log(sqrt((x - 1)/(x + 1))
+ 1) - 3/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.280.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \int \frac{x+1}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1+x),x)
```

output
```
Integral((x + 1)/sqrt((x - 1)/(x + 1)), x)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \frac{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} - 5\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1} - \frac{(x-1)^2}{(x+1)^2} - 1} + \frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="maxima")
```

output
```
(3*((x - 1)/(x + 1))^(3/2) - 5*sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) -
 (x - 1)^2/(x + 1)^2 - 1) + 3/2*log(sqrt((x - 1)/(x + 1)) + 1) - 3/2*log(s
qrt((x - 1)/(x + 1)) - 1)
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \frac{1}{2}\sqrt{x^2-1}\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{4}{\operatorname{sgn}(x+1)}\right)-\frac{3\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 1)*(x/sgn(x + 1) + 4/sgn(x + 1)) - 3/2*log(abs(-x + sqrt(x^
2 - 1)))/sgn(x + 1)
```

3.280.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = 3\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)+\frac{5\sqrt{\frac{x-1}{x+1}}-3\left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2\,(x-1)}{x+1}+1}$$

input
```
int((x + 1)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
3*atanh(((x - 1)/(x + 1))^(1/2)) + (5*((x - 1)/(x + 1))^(1/2) - 3*((x - 1)
/(x + 1))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

3.281 $\int e^{\coth^{-1}(x)}(1-x)x\,dx$

3.281.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx=-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

output
```
-1/3*(1-1/x^2)^(3/2)*x^3
```

3.281.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx=-\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\left(-1+x^2\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)*x,x]
```

output
```
-1/3*(Sqrt[1 - x^(-2)]*x*(-1 + x^2))
```

3.281.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6728, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)xe^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6728}$$

$$\int \sqrt{1-\frac{1}{x^2}}x^4d\frac{1}{x}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

input
```
Int[E^ArcCoth[x]*(1 - x)*x,x]
```

output
```
-1/3*((1 - x^(-2))^(3/2)*x^3)
```

3.281.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6728
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_))^(p_.), x_S
ymbol] :> Simp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + p + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && In
tegerQ[p] && IntegerQ[n] && IntegerQ[m]
```

3.281.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
gosper	$-\frac{(x-1)^{2}(1+x)}{3\sqrt{\frac{x-1}{1+x}}}$	22
default	$-\frac{(x-1)^{2}(1+x)}{3\sqrt{\frac{x-1}{1+x}}}$	22
risch	$-\frac{(x^{2}-1)(x-1)}{3\sqrt{\frac{x-1}{1+x}}}$	22
trager	$-\frac{(1+x)(x^{2}-1)\sqrt{-\frac{1-x}{1+x}}}{3}$	25

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x-1)^2*(1+x)/((x-1)/(1+x))^(1/2)
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{1}{3}\left(x^{3}+x^{2}-x-1\right)\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="fricas")
```

output
```
-1/3*(x^3 + x^2 - x - 1)*sqrt((x - 1)/(x + 1))
```

3.281.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\int\left(-\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx - \int\frac{x^{2}}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)*x,x)
```

output
```
-Integral(-x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(x**2/sqrt(x/(x + 1
) - 1/(x + 1)), x)
```

3.281.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = \frac{8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}{3\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="maxima")
```

output
```
8/3*((x - 1)/(x + 1))^(3/2)/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (
x - 1)^3/(x + 1)^3 - 1)
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{(x^2-1)^{\frac{3}{2}}}{3\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="giac")
```

output
```
-1/3*(x^2 - 1)^(3/2)/sgn(x + 1)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{\left(\frac{x-1}{x+1}\right)^{3/2}(x+1)^3}{3}$$

input
```
int(-(x*(x - 1))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
-(((x - 1)/(x + 1))^(3/2)*(x + 1)^3)/3
```

3.282 $\int e^{\coth^{-1}(x)}(1-x)\,dx$

3.282.1 Optimal result

Integrand size = 10, antiderivative size = 35

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
1/2*arctanh((1-1/x^2)^(1/2))-1/2*x^2*(1-1/x^2)^(1/2)
```

3.282.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 - x),x]
```

output
```
-1/2*(Sqrt[1 - x^(-2)]*x^2) + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

3.282.3 Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 243, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6724}$$

$$\int \sqrt{1-\frac{1}{x^2}}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \sqrt{1-\frac{1}{x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}-\sqrt{1-\frac{1}{x^2}}x\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}-\sqrt{1-\frac{1}{x^2}}x\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\sqrt{1-\frac{1}{x^2}}x\right)$$

input
```
Int[E^ArcCoth[x]*(1 - x),x]
```

output
```
(-(Sqrt[1 - x^(-2)]*x) + ArcTanh[Sqrt[1 - x^(-2)]])/2
```

3.282.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.282.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result	size
default	$-\frac{\left(x-1\right) \left(x \sqrt{x^{2}-1}-\ln \left(x+\sqrt{x^{2}-1}\right)\right)}{2 \sqrt{\frac{x-1}{1+x}}\, \sqrt{\left(x-1\right) \left(1+x\right)}}$	48
risch	$-\frac{x \left(x-1\right)}{2 \sqrt{\frac{x-1}{1+x}}}+\frac{\ln \left(x+\sqrt{x^{2}-1}\right) \sqrt{\left(x-1\right) \left(1+x\right)}}{2 \sqrt{\frac{x-1}{1+x}}\, \left(1+x\right)}$	56
trager	$-\frac{\left(1+x\right) \sqrt{-\frac{1-x}{1+x}}\, x}{2}+\frac{\ln \left(\sqrt{-\frac{1-x}{1+x}}\, x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	57

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x-1)*(x*(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/((x-1)/(1+x))^(1/2)/((x-1
)*(1+x))^(1/2)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{1}{2}\left(x^2+x\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="fricas")
```

output
```
-1/2*(x^2 + x)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1)
- 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.282.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\int\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx-\int\left(-\frac{1}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1-x),x)
```

output
```
-Integral(x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(-1/sqrt(x/(x + 1) -
 1/(x + 1)), x)
```

3.282.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(27) = 54$.

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = \frac{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} + \sqrt{\frac{x-1}{x+1}}}{\frac{2\,(x-1)}{x+1} - \frac{(x-1)^2}{(x+1)^2} - 1} + \frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) - \frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="maxima")
```

output
```
(((x - 1)/(x + 1))^(3/2) + sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) - (x
- 1)^2/(x + 1)^2 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt(
(x - 1)/(x + 1)) - 1)
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = -\frac{\sqrt{x^2-1}x}{2\,\mathrm{sgn}\,(x+1)} - \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="giac")
```

output
```
-1/2*sqrt(x^2 - 1)*x/sgn(x + 1) - 1/2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x +
 1)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = \mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{\sqrt{\frac{x-1}{x+1}} + \left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2} - \frac{2\,(x-1)}{x+1} + 1}$$

input
```
int(-(x - 1)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
atanh(((x - 1)/(x + 1))^(1/2)) - (((x - 1)/(x + 1))^(1/2) + ((x - 1)/(x +
1))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

3.283 $\int e^{\coth^{-1}(x)}x(1+x)^2\,dx$

3.283.1 Optimal result

Integrand size = 11, antiderivative size = 133

$$\begin{aligned}\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=&\frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2\\&+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3\\&+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}x^4+\frac{15}{8}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)\end{aligned}$$

output
```
15/8*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+5/8*(1+1/x)^(3/2)*x^2*((-1+x)
/x)^(1/2)+1/4*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+1/4*(1+1/x)^(7/2)*x^4*((-
1+x)/x)^(1/2)+15/8*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

3.283.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{1}{8}\sqrt{1-\frac{1}{x^2}}x\left(24+15x+8x^2+2x^3\right)+\frac{15}{8}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*x*(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(24 + 15*x + 8*x^2 + 2*x^3))/8 + (15*Log[(1 + Sqrt[1 -
 x^(-2)])*x])/8
```

3.283.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {6729, 107, 105, 105, 105, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)^2 e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6729}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^{5/2} x^5}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2} x^4-\frac{3}{4}\int \frac{\left(1+\frac{1}{x}\right)^{5/2} x^4}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2} x^4-\frac{3}{4}\left(\frac{5}{3}\int \frac{\left(1+\frac{1}{x}\right)^{3/2} x^3}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}-\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2} x^3\right)$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2} x^4-$$
$$\frac{3}{4}\left(\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{x}}x^2}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2} x^2\right)-\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2} x^3\right)$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-$$
$$\frac{3}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(\int\frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1x}\Bigg)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\Bigg)-\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3$$

$$\Big\downarrow 103$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-$$
$$\frac{3}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-\int\frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1x}\Bigg)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\Bigg)-\frac{1}{3}\sqrt{1-\frac{1}{x}}\Bigg(\frac{1}{x}$$

$$\Big\downarrow 219$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-$$
$$\frac{3}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1x}\Bigg)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\Bigg)-\frac{1}{3}\sqrt{1-\frac{1}{x}}\Bigg(\frac{1}{x}+1$$

input
```
Int[E^ArcCoth[x]*x*(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(7/2)*x^4)/4 - (3*(-1/3*(Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(5/2)*x^3) + (5*(-1/2*(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(3/2)*x^2
) + (3*(-(Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x) - ArcTanh[Sqrt[1 - x^(-1)]*
Sqrt[1 + x^(-1)]]))/2))/3))/4
```

3.283.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

3.283.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

method	result	size
risch	$\frac{\left(2 x^{3}+8 x^{2}+15 x +24\right) \left(x -1\right)}{8 \sqrt{\frac{x -1}{1+x}}}+\frac{15 \ln \left(x +\sqrt{x^{2}-1}\right) \sqrt{\left(x -1\right) \left(1+x \right)}}{8 \sqrt{\frac{x -1}{1+x}}\, \left(1+x \right)}$	70
trager	$\frac{\left(1+x \right) \left(2 x^{3}+8 x^{2}+15 x +24\right) \sqrt{-\frac{1-x}{1+x}}}{8}-\frac{15 \ln \left(-\sqrt{-\frac{1-x}{1+x}}\, x -\sqrt{-\frac{1-x}{1+x}}+x \right)}{8}$	74
default	$\frac{\left(x -1\right) \left(2 x \left(x^{2}-1\right)^{\frac{3}{2}}+8 \left(\left(x -1\right) \left(1+x \right)\right)^{\frac{3}{2}}+17 x \sqrt{x^{2}-1}+32 \sqrt{x^{2}-1}+15 \ln \left(x +\sqrt{x^{2}-1}\right)\right)}{8 \sqrt{\frac{x -1}{1+x}}\, \sqrt{\left(x -1\right) \left(1+x \right)}}$	79

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*x^3+8*x^2+15*x+24)*(x-1)/((x-1)/(1+x))^(1/2)+15/8*ln(x+(x^2-1)^(1/2
))/((x-1)/(1+x))^(1/2)/(1+x)*((x-1)*(1+x))^(1/2)
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.50

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \frac{1}{8}\left(2\,x^4+10\,x^3+23\,x^2+39\,x+24\right)\sqrt{\frac{x-1}{x+1}} + \frac{15}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{15}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="fricas")
```

output
```
1/8*(2*x^4 + 10*x^3 + 23*x^2 + 39*x + 24)*sqrt((x - 1)/(x + 1)) + 15/8*log
(sqrt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.283.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \int \frac{x(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**2,x)
```

output
```
Integral(x*(x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \frac{15\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}} - 55\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} + 73\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} - 49\sqrt{\frac{x-1}{x+1}}}{4\left(\frac{4\,(x-1)}{x+1} - \frac{6\,(x-1)^2}{(x+1)^2} + \frac{4\,(x-1)^3}{(x+1)^3} - \frac{(x-1)^4}{(x+1)^4} - 1\right)} + \frac{15}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{15}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="maxima")
```

output
```
1/4*(15*((x - 1)/(x + 1))^(7/2) - 55*((x - 1)/(x + 1))^(5/2) + 73*((x - 1)
/(x + 1))^(3/2) - 49*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)
^2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 15/8*log
(sqrt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx$$
$$=\frac{1}{8}\left(\left(2\,x\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{4}{\operatorname{sgn}(x+1)}\right)+\frac{15}{\operatorname{sgn}(x+1)}\right)x+\frac{24}{\operatorname{sgn}(x+1)}\right)\sqrt{x^2-1}$$
$$-\frac{15\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{8\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="giac")
```

output
```
1/8*((2*x*(x/sgn(x + 1) + 4/sgn(x + 1)) + 15/sgn(x + 1))*x + 24/sgn(x + 1)
)*sqrt(x^2 - 1) - 15/8*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\frac{15\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}+\frac{\frac{49\sqrt{\frac{x-1}{x+1}}}{4}-\frac{73\left(\frac{x-1}{x+1}\right)^{3/2}}{4}+\frac{55\left(\frac{x-1}{x+1}\right)^{5/2}}{4}-\frac{15\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6\,(x-1)^2}{(x+1)^2}-\frac{4\,(x-1)}{x+1}-\frac{4\,(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}$$

input
```
int((x*(x + 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(15*atanh(((x - 1)/(x + 1))^(1/2)))/4 + ((49*((x - 1)/(x + 1))^(1/2))/4 -
(73*((x - 1)/(x + 1))^(3/2))/4 + (55*((x - 1)/(x + 1))^(5/2))/4 - (15*((x
- 1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (
4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

3.284 $\int e^{\coth^{-1}(x)}(1+x)^2\,dx$

3.284.1 Optimal result

Integrand size = 10, antiderivative size = 106

$$\begin{aligned}\int e^{\coth^{-1}(x)}(1+x)^2\,dx &= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x+\frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}x^2\\&\quad+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}x^3+\frac{5}{2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)\end{aligned}$$

output
```
5/2*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+5/6*(1+1/x)^(3/2)*x^2*((-1+x)/
x)^(1/2)+1/3*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+5/2*x*(1+1/x)^(1/2)*((-1+x
)/x)^(1/2)
```

3.284.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x\left(22+9x+2x^2\right)+\frac{5}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(22 + 9*x + 2*x^2))/6 + (5*Log[(1 + Sqrt[1 - x^(-2)])*
x])/2
```

3.284.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6725, 105, 105, 105, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^2 e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6725}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^{5/2} x^4}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2} x^3-\frac{5}{3}\int \frac{\left(1+\frac{1}{x}\right)^{3/2} x^3}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2} x^3-\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{x}}x^2}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2} x^2\right)$$

$$\Big\downarrow \text{105}$$

$$\begin{gathered}\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2} x^3-\\ \frac{5}{3}\left(\frac{3}{2}\left(\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2} x^2\right)\end{gathered}$$

$$\Big\downarrow \text{103}$$

$$\begin{gathered}\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2} x^3-\\ \frac{5}{3}\left(\frac{3}{2}\left(-\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2} x^2\right)\end{gathered}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{5}{3}\left(\frac{3}{2}\left(-\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1x}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

input
```
Int[E^ArcCoth[x]*(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(5/2)*x^3)/3 - (5*(-1/2*(Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2) + (3*(-(Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x) - ArcT
anh[Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]]))/2))/3
```

3.284.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.284.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{(2x^2+9x+22)(x-1)}{6\sqrt{\frac{x-1}{1+x}}}+\frac{5\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	65
trager	$\frac{(1+x)(2x^2+9x+22)\sqrt{-\frac{1-x}{1+x}}}{6}+\frac{5\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	66
default	$\frac{(x-1)\left(2((x-1)(1+x))^{\frac{3}{2}}+9x\sqrt{x^2-1}+24\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)\right)}{6\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	69

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^2+9*x+22)*(x-1)/((x-1)/(1+x))^(1/2)+5/2*ln(x+(x^2-1)^(1/2))/((x-1
)/(1+x))^(1/2)/(1+x)*((x-1)*(1+x))^(1/2)
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=\frac{1}{6}\left(2\,x^3+11\,x^2+31\,x+22\right)\sqrt{\frac{x-1}{x+1}}+\frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="fricas")
```

output
```
1/6*(2*x^3 + 11*x^2 + 31*x + 22)*sqrt((x - 1)/(x + 1)) + 5/2*log(sqrt((x -
 1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.284.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \int \frac{(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**2,x)
```

output
```
Integral((x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\begin{aligned}\int e^{\coth^{-1}(x)}(1+x)^2\,dx = &-\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} - 40\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} + 33\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)} \\ &+ \frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="maxima")
```

output
```
-1/3*(15*((x - 1)/(x + 1))^(5/2) - 40*((x - 1)/(x + 1))^(3/2) + 33*sqrt((x
 - 1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x
+ 1)^3 - 1) + 5/2*log(sqrt((x - 1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x
 + 1)) - 1)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$\begin{aligned}\int e^{\coth^{-1}(x)}(1+x)^2\,dx = &\frac{1}{6}\sqrt{x^2-1}\left(x\left(\frac{2x}{\operatorname{sgn}(x+1)} + \frac{9}{\operatorname{sgn}(x+1)}\right) + \frac{22}{\operatorname{sgn}(x+1)}\right) \\ &- \frac{5\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}\end{aligned}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="giac")
```

output
```
1/6*sqrt(x^2 - 1)*(x*(2*x/sgn(x + 1) + 9/sgn(x + 1)) + 22/sgn(x + 1)) - 5/
2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

3.284.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = 5\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{11\,\sqrt{\frac{x-1}{x+1}} - \frac{40\,\left(\frac{x-1}{x+1}\right)^{3/2}}{3} + 5\,\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3\,(x-1)}{x+1} - \frac{3\,(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1}$$

input
```
int((x + 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

output
```
5*atanh(((x - 1)/(x + 1))^(1/2)) - (11*((x - 1)/(x + 1))^(1/2) - (40*((x -
 1)/(x + 1))^(3/2))/3 + 5*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) -
(3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.285 $\int e^{\coth^{-1}(x)}(1-x)^2x\,dx$

3.285.1 Optimal result

Integrand size = 13, antiderivative size = 71

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{8}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-1/3*(1-1/x^2)^(3/2)*x^3+1/4*(1-1/x^2)^(3/2)*x^4-1/8*arctanh((1-1/x^2)^(1/
2))+1/8*x^2*(1-1/x^2)^(1/2)
```

3.285.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{1}{24}\sqrt{1-\frac{1}{x^2}}x(8-3x-8x^2+6x^3)-\frac{1}{8}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)^2*x,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(8 - 3*x - 8*x^2 + 6*x^3))/24 - Log[(1 + Sqrt[1 - x^(-
2)])*x]/8
```

3.285.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6728, 25, 539, 534, 243, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^2 x e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6728}$$

$$\int -\sqrt{1-\frac{1}{x^2}}\left(1-\frac{1}{x}\right)x^5d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{1-\frac{1}{x^2}}\left(1-\frac{1}{x}\right)x^5d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}\int \sqrt{1-\frac{1}{x^2}}\left(4-\frac{1}{x}\right)x^4d\frac{1}{x}+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{4}\left(-\int \sqrt{1-\frac{1}{x^2}}x^3d\frac{1}{x}-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \sqrt{1-\frac{1}{x^2}}x^2d\frac{1}{x^2}-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}+\sqrt{1-\frac{1}{x^2}}x\right)-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{1-\frac{1}{x^2}}x-\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}\right)-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{1-\frac{1}{x^2}}x-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)\right)-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

input
```
Int[E^ArcCoth[x]*(1 - x)^2*x,x]
```

output
```
((1 - x^(-2))^(3/2)*x^4)/4 + ((-4*(1 - x^(-2))^(3/2)*x^3)/3 + (Sqrt[1 - x^
(-2)]*x - ArcTanh[Sqrt[1 - x^(-2)]])/2)/4
```

3.285.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6728
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_))^(p_.), x_S
ymbol] :> Simp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + p + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && In
tegerQ[p] && IntegerQ[n] && IntegerQ[m]
```

3.285.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{(x-1)\left(6x\left(x^{2}-1\right)^{\frac{3}{2}}-8((x-1)(1+x))^{\frac{3}{2}}+3x\sqrt{x^{2}-1}-3\ln\left(x+\sqrt{x^{2}-1}\right)\right)}{24\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	70
risch	$\frac{(6x^{3}-8x^{2}-3x+8)(x-1)}{24\sqrt{\frac{x-1}{1+x}}}-\frac{\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{8\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	70
trager	$\frac{(1+x)(6x^{3}-8x^{2}-3x+8)\sqrt{-\frac{1-x}{1+x}}}{24}-\frac{\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{8}$	71

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/24*(x-1)*(6*x*(x^2-1)^(3/2)-8*((x-1)*(1+x))^(3/2)+3*x*(x^2-1)^(1/2)-3*ln
(x+(x^2-1)^(1/2)))/((x-1)/(1+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{1}{24}\left(6\,x^{4}-2\,x^{3}-11\,x^{2}+5\,x+8\right)\sqrt{\frac{x-1}{x+1}}-\frac{1}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="fricas")
```

output
```
1/24*(6*x^4 - 2*x^3 - 11*x^2 + 5*x + 8)*sqrt((x - 1)/(x + 1)) - 1/8*log(sq
rt((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.285.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\int\frac{x(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2*x,x)
```

output
```
Integral(x*(x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

3.285.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(55)=110$.

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}+53\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-11\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+3\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}-\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="maxima")
```

output
```
-1/12*(3*((x - 1)/(x + 1))^(7/2) + 53*((x - 1)/(x + 1))^(5/2) - 11*((x - 1
)/(x + 1))^(3/2) + 3*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)
^2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) - 1/8*log(
sqrt((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx = \frac{1}{24}\left(\left(2\,x\left(\frac{3\,x}{\operatorname{sgn}(x+1)}-\frac{4}{\operatorname{sgn}(x+1)}\right)-\frac{3}{\operatorname{sgn}(x+1)}\right)x+\frac{8}{\operatorname{sgn}(x+1)}\right)\sqrt{x^2-1} + \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{8\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="giac")
```

output
```
1/24*((2*x*(3*x/sgn(x + 1) - 4/sgn(x + 1)) - 3/sgn(x + 1))*x + 8/sgn(x + 1
))*sqrt(x^2 - 1) + 1/8*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

3.285.9 Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx = \frac{\frac{\sqrt{\frac{x-1}{x+1}}}{4}-\frac{11\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{53\left(\frac{x-1}{x+1}\right)^{5/2}}{12}+\frac{\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6\,(x-1)^2}{(x+1)^2}-\frac{4\,(x-1)}{x+1}-\frac{4\,(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}-\frac{\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}$$

input
```
int((x*(x - 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(((x - 1)/(x + 1))^(1/2)/4 - (11*((x - 1)/(x + 1))^(3/2))/12 + (53*((x - 1
)/(x + 1))^(5/2))/12 + ((x - 1)/(x + 1))^(7/2)/4)/((6*(x - 1)^2)/(x + 1)^2
 - (4*(x - 1))/(x + 1) - (4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1
) - atanh(((x - 1)/(x + 1))^(1/2))/4
```

3.286 $\int e^{\coth^{-1}(x)}(1-x)^2\,dx$

3.286.1 Optimal result

Integrand size = 12, antiderivative size = 53

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
1/3*(1-1/x^2)^(3/2)*x^3+1/2*arctanh((1-1/x^2)^(1/2))-1/2*x^2*(1-1/x^2)^(1/
2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x\left(-2-3x+2x^2\right)+\frac{1}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(-2 - 3*x + 2*x^2))/6 + Log[(1 + Sqrt[1 - x^(-2)])*x]/
2
```

3.286.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6724, 25, 534, 243, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^2 e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{ 6724}$$

$$\int -\sqrt{1-\frac{1}{x^2}}\left(1-\frac{1}{x}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \sqrt{1-\frac{1}{x^2}}\left(1-\frac{1}{x}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{ 534}$$

$$\int \sqrt{1-\frac{1}{x^2}}x^3d\frac{1}{x}+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \sqrt{1-\frac{1}{x^2}}x^2d\frac{1}{x^2}+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}-\sqrt{1-\frac{1}{x^2}}x\right)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}-\sqrt{1-\frac{1}{x^2}}x\right)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\sqrt{1-\frac{1}{x^2}}x\right)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

input
```
Int[E^ArcCoth[x]*(1 - x)^2,x]
```

output
```
((1 - x^(-2))^(3/2)*x^3)/3 + (-(Sqrt[1 - x^(-2)]*x) + ArcTanh[Sqrt[1 - x^(
-2)]])/2
```

3.286.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.286.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{(x-1)\left(2((x-1)(1+x))^{\frac{3}{2}}-3x\sqrt{x^{2}-1}+3\ln\left(x+\sqrt{x^{2}-1}\right)\right)}{6\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	60
risch	$\frac{(2x^{2}-3x-2)(x-1)}{6\sqrt{\frac{x-1}{1+x}}}+\frac{\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	65
trager	$\frac{(1+x)(2x^{2}-3x-2)\sqrt{-\frac{1-x}{1+x}}}{6}-\frac{\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	69

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(x-1)*(2*((x-1)*(1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/
((x-1)/(1+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\frac{1}{6}\left(2\,x^{3}-x^{2}-5\,x-2\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="fricas")
```

output
```
1/6*(2*x^3 - x^2 - 5*x - 2)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(
x + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.286.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\int\frac{(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2,x)
```

output
```
Integral((x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

3.286.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(41) = 82.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3\,(x-1)}{x+1}-\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*((x - 1)/(x + 1))^(5/2) + 8*((x - 1)/(x + 1))^(3/2) - 3*sqrt((x -
1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1
)^3 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x +
1)) - 1)
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\frac{1}{6}\sqrt{x^2-1}\left(x\left(\frac{2\,x}{\operatorname{sgn}(x+1)}-\frac{3}{\operatorname{sgn}(x+1)}\right)-\frac{2}{\operatorname{sgn}(x+1)}\right)-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="giac")
```

output
```
1/6*sqrt(x^2 - 1)*(x*(2*x/sgn(x + 1) - 3/sgn(x + 1)) - 2/sgn(x + 1)) - 1/2
*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{3/2}}{3}-\sqrt{\frac{x-1}{x+1}}+\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3\,(x-1)}{x+1}-\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

input
```
int((x - 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

output
```
atanh(((x - 1)/(x + 1))^(1/2)) - ((8*((x - 1)/(x + 1))^(3/2))/3 - ((x - 1)
/(x + 1))^(1/2) + ((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3*(x -
1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.287 $\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx$

3.287.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx=\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}x$$

output
```
x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx=x\sqrt{\frac{-1+x^2}{x^2}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 + x),x]
```

output
```
x*Sqrt[(-1 + x^2)/x^2]
```

3.287.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6729, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{x+1}dx$$

$$\Big\downarrow \text{6729}$$

$$-\int \frac{x^2}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{106}$$

$$\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x$$

input
```
Int[(E^ArcCoth[x]*x)/(1 + x),x]
```

output
```
Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x
```

3.287.3.1 Defintions of rubi rules used

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

3.287.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}}$	16
risch	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}}$	16
trager	$(1+x)\sqrt{-\frac{1-x}{1+x}}$	19
default	$\frac{(x-1)\sqrt{x^2-1}}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	32

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/((x-1)/(1+x))^(1/2)
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt((x - 1)/(x + 1))
```

3.287.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x),x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)), x)
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="maxima")
```

output
```
-2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \frac{\sqrt{x^2-1}}{\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="giac")
```

output
```
sqrt(x^2 - 1)/sgn(x + 1)
```

3.287.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
```

output
```
-(2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x + 1) - 1)
```

3.288 $\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx$

3.288.1 Optimal result

Integrand size = 10, antiderivative size = 22

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

output
```
arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))
```

3.288.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \log\left(x\left(1+\sqrt{\frac{-1+x^2}{x^2}}\right)\right)$$

input
```
Integrate[E^ArcCoth[x]/(1 + x),x]
```

output
```
Log[x*(1 + Sqrt[(-1 + x^2)/x^2])]
```

3.288.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6725, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{x+1}\,dx$$

$$\Big\downarrow \text{6725}$$

$$-\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{103}$$

$$\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)$$

$$\Big\downarrow \text{219}$$

$$\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)$$

input
```
Int[E^ArcCoth[x]/(1 + x),x]
```

output
```
ArcTanh[Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]]
```

3.288.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.288.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

method	result	size
default	$\frac{(x-1)\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	35
trager	$-\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	39

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x),x,method=_RETURNVERBOSE)
```

output
```
1/((x-1)/(1+x))^(1/2)*(x-1)/((x-1)*(1+x))^(1/2)*ln(x+(x^2-1)^(1/2))
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx=\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

3.288.6 Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = -\log\left(\sqrt{1-\frac{2}{x+1}}-1\right)+\log\left(\sqrt{1-\frac{2}{x+1}}+1\right)$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)/(1+x),x)
```

output
```
-log(sqrt(1 - 2/(x + 1)) - 1) + log(sqrt(1 - 2/(x + 1)) + 1)
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

3.288.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

input

```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
```

output

```
2*atanh(((x - 1)/(x + 1))^(1/2))
```

3.289 $\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx$

3.289.1 Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx=\frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}-\sqrt{1-\frac{1}{x^2}}x-2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-2*arctanh((1-1/x^2)^(1/2))+2*(1+1/x)/(1-1/x^2)^(1/2)-x*(1-1/x^2)^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx=-\frac{\sqrt{1-\frac{1}{x^2}}(-3+x)x}{-1+x}-2\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 - x),x]
```

output
```
-((Sqrt[1 - x^(-2)]*(-3 + x)*x)/(-1 + x)) - 2*Log[(1 + Sqrt[1 - x^(-2)])*x
]
```

3.289.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6728, 564, 534, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{1-x}dx$$

$$\Big\downarrow \text{6728}$$

$$\int \frac{\sqrt{1-\frac{1}{x^2}}x^2}{\left(1-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{564}$$

$$\int \frac{\left(1+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{534}$$

$$2\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x^2}}x+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{243}$$

$$\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}-\sqrt{1-\frac{1}{x^2}}x+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{73}$$

$$-2\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}-\sqrt{1-\frac{1}{x^2}}x+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{219}$$

$$-2\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\sqrt{1-\frac{1}{x^2}}x+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

input
```
Int[(E^ArcCoth[x]*x)/(1 - x),x]
```

3.289. $\int \frac{e^{\coth^{-1}(x)}x}{1-x}dx$

output
```
(2*Sqrt[1 - x^(-2)])/(1 - x^(-1)) - Sqrt[1 - x^(-2)]*x - 2*ArcTanh[Sqrt[1
- x^(-2)]]
```

3.289.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6728
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_))^(p_.), x_S
ymbol] :> Simp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + p + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && In
tegerQ[p] && IntegerQ[n] && IntegerQ[m]
```

3.289.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

method	result	size
trager	$-\frac{(1+x)(-3+x)\sqrt{-\frac{1-x}{1+x}}}{x-1}-2\ln\left(\sqrt{-\frac{1-x}{1+x}}\, x+\sqrt{-\frac{1-x}{1+x}}+x\right)$	64
risch	$-\frac{x^{2}-2x-3}{\sqrt{\frac{x-1}{1+x}}\, (1+x)}-\frac{2\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\, (1+x)}$	65
default	$\frac{\left(x^{2}-1\right)^{\frac{3}{2}}-2x^{2}\sqrt{x^{2}-1}-2\ln\left(x+\sqrt{x^{2}-1}\right)x^{2}+4x\sqrt{x^{2}-1}+4\ln\left(x+\sqrt{x^{2}-1}\right)x-2\sqrt{x^{2}-1}-2\ln\left(x+\sqrt{x^{2}-1}\right)}{(x-1)\sqrt{(x-1)(1+x)}\, \sqrt{\frac{x-1}{1+x}}}$	106

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x),x,method=_RETURNVERBOSE)
```

output
```
-(1+x)*(-3+x)/(x-1)*(-(1-x)/(1+x))^(1/2)-2*ln((-(1-x)/(1+x))^(1/2)*x+(-(1-
x)/(1+x))^(1/2)+x)
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\frac{2\,(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-2\,(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)+(x^{2}-2\,x-3)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="fricas")
```

output
```
-(2*(x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 2*(x - 1)*log(sqrt((x - 1)/(x
 + 1)) - 1) + (x^2 - 2*x - 3)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

3.289.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\int \frac{x}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x),x)
```

output
```
-Integral(x/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
 x)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = \frac{2\left(\frac{2(x-1)}{x+1}-1\right)}{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-\sqrt{\frac{x-1}{x+1}}} - 2\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) + 2\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="maxima")
```

output
```
2*(2*(x - 1)/(x + 1) - 1)/(((x - 1)/(x + 1))^(3/2) - sqrt((x - 1)/(x + 1))
) - 2*log(sqrt((x - 1)/(x + 1)) + 1) + 2*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = \frac{2\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} - \frac{\sqrt{x^2-1}}{\operatorname{sgn}(x+1)} - \frac{4}{\left(x-\sqrt{x^2-1}-1\right)\operatorname{sgn}(x+1)} - 2\operatorname{sgn}(x+1)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="giac")
```

output
```
2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - sqrt(x^2 - 1)/sgn(x + 1) - 4/(
(x - sqrt(x^2 - 1) - 1)*sgn(x + 1)) - 2*sgn(x + 1)
```

3.289.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\frac{2\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)\,\sqrt{\frac{x-1}{x+1}}-6}{2\,\sqrt{\frac{x-1}{x+1}}}$$

input
```
int(-x/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
```

output
```
-(2*x + 8*atanh(((x - 1)/(x + 1))^(1/2))*((x - 1)/(x + 1))^(1/2) - 6)/(2*(
(x - 1)/(x + 1))^(1/2))
```

3.290 $\int \frac{e^{\coth^{-1}(x)}}{1-x} \, dx$

3.290.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{e^{\coth^{-1}(x)}}{1-x} \, dx = \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-arctanh((1-1/x^2)^(1/2))+2*(1+1/x)/(1-1/x^2)^(1/2)
```

3.290.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(x)}}{1-x} \, dx = \frac{2\sqrt{1-\frac{1}{x^2}}x}{-1+x} - \log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]/(1 - x),x]
```

output
```
(2*Sqrt[1 - x^(-2)]*x)/(-1 + x) - Log[(1 + Sqrt[1 - x^(-2)])*x]
```

3.290.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6724, 564, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx$$

$$\Big\downarrow \text{ 6724}$$

$$\int \frac{\sqrt{1-\frac{1}{x^2}}x}{\left(1-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 564}$$

$$\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}-\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

input

```
Int[E^ArcCoth[x]/(1 - x),x]
```

output

```
(2*Sqrt[1 - x^(-2)])/(1 - x^(-1)) - ArcTanh[Sqrt[1 - x^(-2)]]
```

3.290.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6724

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.290. $\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx$

3.290.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

method	result	size
risch	$\frac{2}{\sqrt{\frac{x-1}{1+x}}}-\frac{\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	52
trager	$\frac{2(1+x)\sqrt{-\frac{1-x}{1+x}}}{x-1}+\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	62
default	$\frac{\left(x^{2}-1\right)^{\frac{3}{2}}-x^{2}\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)x^{2}+2x\sqrt{x^{2}-1}+2\ln\left(x+\sqrt{x^{2}-1}\right)x-\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)}{(x-1)\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$	106

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x),x,method=_RETURNVERBOSE)
```

output
```
2/((x-1)/(1+x))^(1/2)-ln(x+(x^2-1)^(1/2))/((x-1)/(1+x))^(1/2)/(1+x)*((x-1)
*(1+x))^(1/2)
```

3.290.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(29) = 58.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx=-\frac{(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-2\,(x+1)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="fricas")
```

output
```
-((x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - (x - 1)*log(sqrt((x - 1)/(x + 1
)) - 1) - 2*(x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

3.290.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = -\int \frac{1}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)/(1-x),x)
```

output
```
-Integral(1/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
 x)
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{2}{\sqrt{\frac{x-1}{x+1}}} - \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) + \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="maxima")
```

output
```
2/sqrt((x - 1)/(x + 1)) - log(sqrt((x - 1)/(x + 1)) + 1) + log(sqrt((x - 1
)/(x + 1)) - 1)
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\mathrm{sgn}\,(x+1)} - \frac{4}{\left(x-\sqrt{x^2-1}-1\right)\mathrm{sgn}\,(x+1)} - 2\,\mathrm{sgn}(x+1)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="giac")
```

output
```
log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - 4/((x - sqrt(x^2 - 1) - 1)*sgn(x
 + 1)) - 2*sgn(x + 1)
```

3.290.9 Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{2}{\sqrt{\frac{x-1}{x+1}}} - 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

input
```
int(-1/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
```

output
```
2/((x - 1)/(x + 1))^(1/2) - 2*atanh(((x - 1)/(x + 1))^(1/2))
```

3.291 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx$

3.291.1 Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}+\operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

output
```
arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))-((-1+x)/x)^(1/2)/(1+1/x)^(1/2)
```

3.291.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}x}{1+x}+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

output
```
-((Sqrt[1 - x^(-2)]*x)/(1 + x)) + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

3.291.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6729, 107, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{(x+1)^2} \, dx$$

$$\Big\downarrow \ 6729$$

$$-\int \frac{x}{\sqrt{1-\frac{1}{x}} \left(1+\frac{1}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \ 107$$

$$-\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \ 103$$

$$\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \ 219$$

$$\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

input
```
Int[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

output
```
-(Sqrt[1 - x^(-1)]/Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[1 - x^(-1)]*Sqrt[1 + x
^(-1)]]
```

3.291.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

3.291.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

method	result	size
trager	$-\sqrt{-\frac{1-x}{1+x}}-\ln\left(-\sqrt{-\frac{1-x}{1+x}}\, x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	56
risch	$-\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\, \left(1+x\right)}+\frac{\ln\left(x+\sqrt{x^{2}-1}\right) \sqrt{\left(x-1\right) \left(1+x\right)}}{\sqrt{\frac{x-1}{1+x}}\, \left(1+x\right)}$	59
default	$\frac{\left(x-1\right) \left(\left(x^{2}-1\right)^{\frac{3}{2}}-x^{2} \sqrt{x^{2}-1}+2 \ln\left(x+\sqrt{x^{2}-1}\right) x^{2}-2 x \sqrt{x^{2}-1}+4 \ln\left(x+\sqrt{x^{2}-1}\right) x-\sqrt{x^{2}-1}+2 \ln\left(x+\sqrt{x^{2}-1}\right)\right)}{2 \sqrt{\frac{x-1}{1+x}}\, \sqrt{\left(x-1\right) \left(1+x\right)}\, \left(1+x\right)^{2}}$	110

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-x)/(1+x))^(1/2)-ln(-(-(1-x)/(1+x))^(1/2)*x-(-(1-x)/(1+x))^(1/2)+x)
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="fricas")
```

output
```
-sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)
/(x + 1)) - 1)
```

3.291.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**2,x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**2), x)
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="maxima")
```

output
```
-sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)
/(x + 1)) - 1)
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} - \frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - 2/((x - sqrt(x^2 - 1) + 1)*sgn(
x + 1))
```

3.291.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \sqrt{\frac{x-1}{x+1}}$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
```

output
```
2*atanh(((x - 1)/(x + 1))^(1/2)) - ((x - 1)/(x + 1))^(1/2)
```

3.292 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx$

3.292.1 Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx=\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}$$

output
```
((-1+x)/x)^(1/2)/(1+1/x)^(1/2)
```

3.292.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx=\frac{\sqrt{1-\frac{1}{x^2}}x}{1+x}$$

input
```
Integrate[E^ArcCoth[x]/(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*x)/(1 + x)
```

3.292.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6725, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{(x+1)^2}dx$$

$$\Big\downarrow \text{6725}$$

$$-\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

input
```
Int[E^ArcCoth[x]/(1 + x)^2,x]
```

output
```
Sqrt[1 - x^(-1)]/Sqrt[1 + x^(-1)]
```

3.292.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.292.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\sqrt{\frac{x-1}{1+x}}$	12
trager	$\sqrt{-\frac{1-x}{1+x}}$	15
gosper	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	21
risch	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	21
default	$\frac{\sqrt{x^{2}-1}\,(x-1)}{(1+x)\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$	37

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
((x-1)/(1+x))^(1/2)
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="fricas")
```

output
```
sqrt((x - 1)/(x + 1))
```

3.292.6 Sympy [A] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**2,x)
```

output
```
sqrt((x - 1)/(x + 1))
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="maxima")
```

output
```
sqrt((x - 1)/(x + 1))
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}\left(x+1\right)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="giac")
```

output
```
2/((x - sqrt(x^2 - 1) + 1)*sgn(x + 1))
```

3.292.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{1-\frac{2}{x+1}}$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
```

output
```
(1 - 2/(x + 1))^(1/2)
```

3.293 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx$

3.293.1 Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-4/3*(1+1/x)/(1-1/x^2)^(3/2)+arctanh((1-1/x^2)^(1/2))+1/3*(-3-5/x)/(1-1/x^
2)^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \frac{\sqrt{1-\frac{1}{x^2}}(5-7x)x}{3(-1+x)^2} + \log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*(5 - 7*x)*x)/(3*(-1 + x)^2) + Log[(1 + Sqrt[1 - x^(-2)])
*x]
```

3.293.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {6728, 25, 570, 532, 25, 532, 27, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x e^{\coth^{-1}(x)}}{(1-x)^2}dx$$

$$\Big\downarrow \text{ 6728}$$

$$\int -\frac{\sqrt{1-\frac{1}{x^2}}x}{\left(1-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sqrt{1-\frac{1}{x^2}}x}{\left(1-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 570}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^3 x}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 532}$$

$$\frac{1}{3}\int -\frac{\left(3+\frac{5}{x}\right)x}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{3}\int \frac{\left(3+\frac{5}{x}\right)x}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 532}$$

$$\frac{1}{3}\left(\int -\frac{3x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(-3\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{243}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$\frac{1}{3}\left(3\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{1}{3}\left(3\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

input
```
Int[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

output
```
(-4*(1 + x^(-1)))/(3*(1 - x^(-2))^(3/2)) + (-((3 + 5/x)/Sqrt[1 - x^(-2)])
+ 3*ArcTanh[Sqrt[1 - x^(-2)]])/3
```

3.293.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.293. $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx$

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 570

```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6728

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_))^(p_.), x_S
ymbol] :> Simp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + p + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && In
tegerQ[p] && IntegerQ[n] && IntegerQ[m]
```

3.293.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

method	result
trager	$-\frac{(1+x)(7x-5)\sqrt{-\frac{1-x}{1+x}}}{3(x-1)^2}+\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)$
risch	$-\frac{7x^2+2x-5}{3(x-1)\sqrt{\frac{x-1}{1+x}}\,(1+x)}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$
default	$-\frac{3x\left(x^2-1\right)^{\frac{3}{2}}-3\sqrt{x^2-1}\,x^3-3\ln\left(x+\sqrt{x^2-1}\right)x^3-2\left(x^2-1\right)^{\frac{3}{2}}+9x^2\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)x^2-9x\sqrt{x^2-1}-9\ln\left(x+\sqrt{x^2-1}\right)x+3}{3(x-1)^2\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$

input

```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(1+x)*(7*x-5)/(x-1)^2*(-(1-x)/(1+x))^(1/2)+ln((-(1-x)/(1+x))^(1/2)*x+
(-(1-x)/(1+x))^(1/2)+x)
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \frac{3\,(x^2-2\,x+1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-3\,(x^2-2\,x+1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-(7\,x^2+2\,x-5)\sqrt{\frac{x-1}{x+1}}}{3\,(x^2-2\,x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(x^2 - 2*x + 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 3*(x^2 - 2*x + 1)*
log(sqrt((x - 1)/(x + 1)) - 1) - (7*x^2 + 2*x - 5)*sqrt((x - 1)/(x + 1)))/
(x^2 - 2*x + 1)
```

3.293.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**2,x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = -\frac{\frac{6\,(x-1)}{x+1}+1}{3\,\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="maxima")
```

output
```
-1/3*(6*(x - 1)/(x + 1) + 1)/((x - 1)/(x + 1))^(3/2) + log(sqrt((x - 1)/(x
 + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} + \frac{2\left(9\left(x-\sqrt{x^2-1}\right)^2-12\,x+12\,\sqrt{x^2-1}+7\right)}{3\left(x-\sqrt{x^2-1}-1\right)^3\operatorname{sgn}(x+1)}+\frac{7}{3}\operatorname{sgn}(x+1)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) + 2/3*(9*(x - sqrt(x^2 - 1))^2 -
12*x + 12*sqrt(x^2 - 1) + 7)/((x - sqrt(x^2 - 1) - 1)^3*sgn(x + 1)) + 7/3*
sgn(x + 1)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\frac{2\,(x-1)}{x+1}+\frac{1}{3}}{\left(\frac{x-1}{x+1}\right)^{3/2}}$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
```

output
```
2*atanh(((x - 1)/(x + 1))^(1/2)) - ((2*(x - 1))/(x + 1) + 1/3)/((x - 1)/(x
 + 1))^(3/2)
```

3.293. $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx$

3.294 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx$

3.294.1 Optimal result

Integrand size = 12, antiderivative size = 24

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

output
```
-1/3*(1-1/x^2)^(3/2)/(1-1/x)^3
```

3.294.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}x(1+x)}{3(-1+x)^2}$$

input
```
Integrate[E^ArcCoth[x]/(1 - x)^2,x]
```

output
```
-1/3*(Sqrt[1 - x^(-2)]*x*(1 + x))/(-1 + x)^2
```

3.294.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6724, 25, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2} dx$$

$$\Big\downarrow \text{6724}$$

$$\int -\frac{\sqrt{1-\frac{1}{x^2}}}{\left(1-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{1-\frac{1}{x^2}}}{\left(1-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

input
```
Int[E^ArcCoth[x]/(1 - x)^2,x]
```

output
```
-1/3*(1 - x^(-2))^(3/2)/(1 - x^(-1))^3
```

3.294.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

3.294.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{1+x}{3(x-1)\sqrt{\frac{x-1}{1+x}}}$	22
trager	$-\frac{(1+x)^{2}\sqrt{-\frac{1-x}{1+x}}}{3(x-1)^{2}}$	27
risch	$-\frac{x^{2}+2x+1}{3\sqrt{\frac{x-1}{1+x}}\,(1+x)(x-1)}$	32
default	$-\frac{(x^{2}-1)^{\frac{3}{2}}}{3\sqrt{\frac{x-1}{1+x}}\,(x-1)^{2}\sqrt{(x-1)(1+x)}}$	35

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(1+x)/(x-1)/((x-1)/(1+x))^(1/2)
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{(x^2+2\,x+1)\sqrt{\frac{x-1}{x+1}}}{3\,(x^2-2\,x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="fricas")
```

output
```
-1/3*(x^2 + 2*x + 1)*sqrt((x - 1)/(x + 1))/(x^2 - 2*x + 1)
```

3.294.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**2,x)
```

output
```
Integral(1/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{1}{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="maxima")
```

output
```
-1/3/((x - 1)/(x + 1))^(3/2)
```

3.294.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \frac{2\left(3\left(x-\sqrt{x^2-1}\right)^2+1\right)}{3\left(x-\sqrt{x^2-1}-1\right)^3\operatorname{sgn}(x+1)} + \frac{1}{3}\operatorname{sgn}(x+1)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="giac")
```

output
```
2/3*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2 - 1) - 1)^3*sgn(x + 1)) +
 1/3*sgn(x + 1)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{1}{3\left(\frac{x-1}{x+1}\right)^{3/2}}$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
```

output
```
-1/(3*((x - 1)/(x + 1))^(3/2))
```

3.295 $\int e^{\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx$

3.295.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int e^{\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx = \frac{2x^{1+m}\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{ax}\right)}{(3+2m)\sqrt{1-\frac{1}{ax}}}$$

output
```
2*x^(1+m)*hypergeom([-1/2, -3/2-m],[-1/2-m],-1/a/x)*(-a*c*x+c)^(1/2)/(3+2*
m)/(1-1/a/x)^(1/2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int e^{\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx = -\frac{x^{1+m}\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{ax}\right)}{\left(-\frac{3}{2}-m\right)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
```

output
```
-((x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -
(1/(a*x))])/((-3/2 - m)*Sqrt[1 - 1/(a*x)]))
```

3.295.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6730, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-acx}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}}x^m\sqrt{c-acx}\int\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{74}$$

$$\frac{2x^{m+1}\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-\frac{3}{2},-m-\frac{1}{2},-\frac{1}{ax}\right)}{(2m+3)\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
```

output
```
(2*x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -
(1/(a*x))])/((3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

3.295.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.295.4 Maple [F]

$$\int \frac{x^m\sqrt{-acx+c}}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

3.295.5 Fricas [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \int \frac{\sqrt{-acx+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-a*c*x + c)*(a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1
), x)
```

3.295.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***m*(-a*c*x+c)**(1/2),x)
```

output
```
Timed out
```

3.295.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int \frac{\sqrt{-acx+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.295.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int \frac{\sqrt{-acx+c}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int \frac{x^m\sqrt{c-a\,c\,x}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.296 $\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.296.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{16\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}$$

output
```
16/105*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)-8/35*(1+1/a/
x)^(3/2)*x^2*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*(1+1/a/x)^(3/2)*x^3*(-
a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

3.296.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(8-4ax+3a^2x^2+15a^3x^3)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(8 - 4*a*x + 3*a^2*x^2 + 15*a^3*x^3))
/(105*a^3*Sqrt[1 - 1/(a*x)])
```

3.296.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6730, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{4\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{4\left(-\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{4\left(\frac{4\left(\frac{1}{ax}+1\right)^{3/2}}{15a\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

output
```
-((((-4*((-2*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) + (4*(1 + 1/(a*x))^(3
/2))/(15*a*(x^(-1))^(3/2))))/(7*a) - (2*(1 + 1/(a*x))^(3/2))/(7*(x^(-1))^(
7/2)))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)])
```

3.296.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.296.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{2(ax+1)(15a^{2}x^{2}-12ax+8)\sqrt{-acx+c}}{105a^{3}\sqrt{\frac{ax-1}{ax+1}}}$	49
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)(15a^{2}x^{2}-12ax+8)}{105\sqrt{\frac{ax-1}{ax+1}}\,a^{3}}$	50
risch	$-\frac{2c(ax-1)(15a^{3}x^{3}+3a^{2}x^{2}-4ax+8)}{105\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a^{3}}$	59

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
2/105*(a*x+1)*(15*a^2*x^2-12*a*x+8)*(-a*c*x+c)^(1/2)/a^3/((a*x-1)/(a*x+1))
^(1/2)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\,(15\,a^4x^4+18\,a^3x^3-a^2x^2+4\,ax+8)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
2/105*(15*a^4*x^4 + 18*a^3*x^3 - a^2*x^2 + 4*a*x + 8)*sqrt(-a*c*x + c)*sqr
t((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

3.296.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\left(15\,a^3\sqrt{-c}x^3+3\,a^2\sqrt{-c}x^2-4\,a\sqrt{-c}x+8\,\sqrt{-c}\right)\sqrt{ax+1}}{105\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
2/105*(15*a^3*sqrt(-c)*x^3 + 3*a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x + 8*sqrt(
-c))*sqrt(a*x + 1)/a^3
```

3.296.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.296.9 Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^2\,x^2-12\,a\,x+8\right)}{105\,a^3\,\left(a\,x-1\right)}$$

input
```
int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x^2 -
 12*a*x + 8))/(105*a^3*(a*x - 1))
```

3.297 $\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

3.297.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=-\frac{4\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}$$

output
```
-4/15*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(
3/2)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

3.297.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(1+ax)(-2+3ax)\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-2 + 3*a*x)*Sqrt[c - a*c*x])/(15*a^2*Sqrt[
1 - 1/(a*x)])
```

3.297.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6730, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{4\left(\frac{1}{ax}+1\right)^{3/2}}{15a\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

output

```
-((((-2*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) + (4*(1 + 1/(a*x))^(3/2))/
(15*a*(x^(-1))^(3/2)))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)])
```

3.297.3.1 Defintions of rubi rules used

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6730

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.297.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{2(ax+1)(3ax-2)\sqrt{-acx+c}}{15a^{2}\sqrt{\frac{ax-1}{ax+1}}}$	41
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)(3ax-2)}{15\sqrt{\frac{ax-1}{ax+1}}\,a^{2}}$	42
risch	$-\frac{2c(ax-1)\left(3a^{2}x^{2}+ax-2\right)}{15\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a^{2}}$	50

input

```
int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/15*(a*x+1)*(3*a*x-2)*(-a*c*x+c)^(1/2)/a^2/((a*x-1)/(a*x+1))^(1/2)
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,(3\,a^3x^3+4\,a^2x^2-ax-2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3x-a^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

output
```
2/15*(3*a^3*x^3 + 4*a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*
x + 1))/(a^3*x - a^2)
```

3.297.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{x\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\left(3\,a^2\sqrt{-c}x^2+a\sqrt{-c}x-2\,\sqrt{-c}\right)\sqrt{ax+1}}{15\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

output
```
2/15*(3*a^2*sqrt(-c)*x^2 + a*sqrt(-c)*x - 2*sqrt(-c))*sqrt(a*x + 1)/a^2
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,c^2\left(\frac{2\,\sqrt{2}\sqrt{-c}}{ac} - \frac{3\,(acx+c)^2\sqrt{-acx-c}+5\,(-acx-c)^{\frac{3}{2}}c}{ac^3}\right)}{15\,a|c|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

output
```
2/15*c^2*(2*sqrt(2)*sqrt(-c)/(a*c) - (3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 5
*(-a*c*x - c)^(3/2)*c)/(a*c^3))/(a*abs(c)*sgn(a*x + 1))
```

3.297.9 Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,(a\,x+1)^2\,(3\,a\,x-2)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a^2\,(a\,x-1)}$$

input
```
int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/
(15*a^2*(a*x - 1))
```

3.298 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.298.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

3.298.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

output
```
(2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

3.298.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6726}$$

$$\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

output
```
(2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

3.298.3.1 Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

3.298.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	35
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
risch	$-\frac{2c(ax+1)(ax-1)}{3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	42

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,(a^2x^2+2\,ax+1)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

3.298.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\left(a\sqrt{-c}x+\sqrt{-c}\right)\sqrt{ax+1}}{3\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,c^2\left(\frac{2\,\sqrt{2}\sqrt{-c}}{c}+\frac{(-acx-c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*c^2*(2*sqrt(2)*sqrt(-c)/c + (-a*c*x - c)^(3/2)/c^2)/(a*abs(c)*sgn(a*x
+ 1))
```

3.298.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x -
1))
```

3.299 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.299.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-2*arcsinh((1/x)^(1/2)/a
^(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}} - \sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[x^(-1)]*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

3.299.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6730, 57, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \; 6730$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 57$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 63$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 222$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*Sqrt[1 + 1/(a*x)])/Sqrt[x^(-1)] + (2*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[a]))/Sqrt[1 - 1/(a*x)])
```

3.299.3.1 Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.299.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\,\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)-\sqrt{-c(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}$	70

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a*x+1))
^(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.20

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1},\ -\frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="frica
s")
```

output
```
[((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*
arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - s
qrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

3.299.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.299.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{c\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{2}\sqrt{-c}\sqrt{c}}{c^{\frac{5}{2}}} - \frac{\sqrt{-acx-c}}{c^2}\right)}{|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac"
)
```

output
```
2*c^3*(arctan(sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - (c*arctan(sqrt(2)*sqrt(-
c)/sqrt(c)) - sqrt(2)*sqrt(-c)*sqrt(c))/c^(5/2) - sqrt(-a*c*x - c)/c^2)/(a
bs(c)*sgn(a*x + 1))
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.300 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

3.300.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\mathrm{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
-(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)-arcsinh((1/x)^(1/2)/a^
(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}+\sqrt{a}\,\mathrm{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + Sqrt[a]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

3.300.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6730, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \; 6730$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 63$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}+\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 222$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)+\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + Sqrt[a]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

3.300.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.300.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\left(\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right) acx+\sqrt{-c(ax+1)}\,\sqrt{c}\right)\sqrt{-c(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}\,x\sqrt{c}}$	78
risch	$\frac{c(ax-1)}{x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{a\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	106

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE
)
```

output
```
-(arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))*(-c
*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2)/x/c^(1/2)
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.36

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \left[\frac{(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)}, -\frac{(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fri
cas")
```

output
```
[1/2*((a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -((a^2*x^2
 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/
(a*c*x - c)) + sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^
2 - x)]
```

3.300.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.300.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)+\sqrt{2}a^2\sqrt{-c}\sqrt{c}}{c^{\frac{3}{2}}} + \frac{\sqrt{-acx-c}a}{cx}\right)c^2}{a|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="gia
c")
```

output
```
(a^2*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - (a^2*c*arctan(sqrt(2)*sqrt
(-c)/sqrt(c)) + sqrt(2)*a^2*sqrt(-c)*sqrt(c))/c^(3/2) + sqrt(-a*c*x - c)*a
/(c*x))*c^2/(a*abs(c)*sgn(a*x + 1))
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.301 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

3.301.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^4}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}$$

output
```
-14/3*(-a*c*x+c)^(3/2)/a^4/c+18/5*(-a*c*x+c)^(5/2)/a^4/c^2-10/7*(-a*c*x+c)
^(7/2)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4+4*(-a*c*x+c)^(1/2)/a^4
```

3.301.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(272+136ax+102a^2x^2+85a^3x^3+35a^4x^4)}{315a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(272 + 136*a*x + 102*a^2*x^2 + 85*a^3*x^3 + 35*a^4*x^4)
)/(315*a^4)
```

3.301.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6717, 6680, 35, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-acx}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^3\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x^3(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{x^3(ax+1)}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{86}$$

$$-c\int \left(\frac{(c-acx)^{7/2}}{a^3c^4}-\frac{5(c-acx)^{5/2}}{a^3c^3}+\frac{9(c-acx)^{3/2}}{a^3c^2}-\frac{7\sqrt{c-acx}}{a^3c}+\frac{2}{a^3\sqrt{c-acx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(-\frac{2(c-acx)^{9/2}}{9a^4c^5}+\frac{10(c-acx)^{7/2}}{7a^4c^4}-\frac{18(c-acx)^{5/2}}{5a^4c^3}+\frac{14(c-acx)^{3/2}}{3a^4c^2}-\frac{4\sqrt{c-acx}}{a^4c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a^4*c) + (14*(c - a*c*x)^(3/2))/(3*a^4*c^2) - (
18*(c - a*c*x)^(5/2))/(5*a^4*c^3) + (10*(c - a*c*x)^(7/2))/(7*a^4*c^4) - (
2*(c - a*c*x)^(9/2))/(9*a^4*c^5)))
```

3.301.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.301.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
trager	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	46
risch	$-\frac{2c(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)(ax-1)}{315a^4\sqrt{-c(ax-1)}}$	52
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{10c(-acx+c)^{\frac{7}{2}}}{7}+\frac{18c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{14c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}}{a^4c^4}$	75
default	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{10c(-acx+c)^{\frac{7}{2}}}{7}+\frac{18c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{14c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}}{a^4c^4}$	75

input
```
int(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(-a*c*x+c)^(1/2)*(35*a^4*x^4+85*a^3*x^3+102*a^2*x^2+136*a*x+272)/a^4
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{2\,(35\,a^4x^4+85\,a^3x^3+102\,a^2x^2+136\,ax+272)\sqrt{-acx+c}}{315\,a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^4*x^4 + 85*a^3*x^3 + 102*a^2*x^2 + 136*a*x + 272)*sqrt(-a*c*x
+ c)/a^4
```

3.301.6 Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\begin{aligned}&\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx\\&=\begin{cases}\frac{2\cdot\left(2c^4\sqrt{-acx+c}-\frac{7c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{9c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{5c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4} & \text{for}\ ac\neq 0\\ \sqrt{c}\left(\frac{x^4}{4}+\frac{2x^3}{3a}+\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\left(\begin{cases}-x & \text{for}\ a=0\\ \frac{\log(ax-1)}{a} & \text{otherwise}\end{cases}\right)}{a^3}\right) & \text{otherwise}\end{cases}\end{aligned}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**3*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((2*(2*c**4*sqrt(-a*c*x + c) - 7*c**3*(-a*c*x + c)**(3/2)/3 + 9*c
**2*(-a*c*x + c)**(5/2)/5 - 5*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2
)/9)/(a**4*c**4), Ne(a*c, 0)), (sqrt(c)*(x**4/4 + 2*x**3/(3*a) + x**2/a**2
 + 2*x/a**3 + 2*Piecewise((-x, Eq(a, 0)), (log(a*x - 1)/a, True))/a**3), T
rue))
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(35\left(-acx+c\right)^{\frac{9}{2}}-225\left(-acx+c\right)^{\frac{7}{2}}c+567\left(-acx+c\right)^{\frac{5}{2}}c^2-735\left(-acx+c\right)^{\frac{3}{2}}c^3+630\sqrt{-acx+c}c^4\right)}{315\,a^4c^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(-a*c*x + c)^(9/2) - 225*(-a*c*x + c)^(7/2)*c + 567*(-a*c*x + c)
^(5/2)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c
^4)
```

3.301.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.87

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(\frac{9\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\sqrt{-acx+c}c^3\right)}{a^3c^3}+\frac{35\,(acx-c)^4\sqrt{-acx+c}+180\,(acx-c)^3\sqrt{-acx+c}c}{}\right.}{315\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/315*(9*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x
+ c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*x + c)*c^3)/(a^3*c^3) +
(35*(a*c*x - c)^4*sqrt(-a*c*x + c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c
+ 378*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 31
5*sqrt(-a*c*x + c)*c^4)/(a^3*c^4))/a
```

3.301.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^4}-\frac{14\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^4\,c}+\frac{18\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^4\,c^2}-\frac{10\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^4\,c^3}+\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a^4\,c^4}$$

input

```
int((x^3*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output

```
(4*(c - a*c*x)^(1/2))/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*
c*x)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c
*x)^(9/2))/(9*a^4*c^4)
```

3.302 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.302.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^3}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{2(c-acx)^{7/2}}{7a^3c^3}$$

output
```
-10/3*(-a*c*x+c)^(3/2)/a^3/c+8/5*(-a*c*x+c)^(5/2)/a^3/c^2-2/7*(-a*c*x+c)^(
7/2)/a^3/c^3+4*(-a*c*x+c)^(1/2)/a^3
```

3.302.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(104+52ax+39a^2x^2+15a^3x^3)}{105a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(104 + 52*a*x + 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3)
```

3.302.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6717, 6680, 35, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^2\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x^2(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{x^2(ax+1)}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{86}$$

$$-c\int \left(-\frac{(c-acx)^{5/2}}{a^2c^3}+\frac{4(c-acx)^{3/2}}{a^2c^2}-\frac{5\sqrt{c-acx}}{a^2c}+\frac{2}{a^2\sqrt{c-acx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2(c-acx)^{7/2}}{7a^3c^4}-\frac{8(c-acx)^{5/2}}{5a^3c^3}+\frac{10(c-acx)^{3/2}}{3a^3c^2}-\frac{4\sqrt{c-acx}}{a^3c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a^3*c) + (10*(c - a*c*x)^(3/2))/(3*a^3*c^2) - (
8*(c - a*c*x)^(5/2))/(5*a^3*c^3) + (2*(c - a*c*x)^(7/2))/(7*a^3*c^4)))
```

3.302.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.302.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
trager	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	38
risch	$-\frac{2c(15a^3x^3+39a^2x^2+52ax+104)(ax-1)}{105a^3\sqrt{-c(ax-1)}}$	44
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-2c^3\sqrt{-acx+c}\right)}{c^3a^3}$	61
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{8c(-acx+c)^{\frac{5}{2}}}{5}-\frac{10c^2(-acx+c)^{\frac{3}{2}}}{3}+4c^3\sqrt{-acx+c}}{a^3c^3}$	61

input
```
int(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(-a*c*x+c)^(1/2)*(15*a^3*x^3+39*a^2*x^2+52*a*x+104)/a^3
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.45

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\left(15\,a^3x^3+39\,a^2x^2+52\,ax+104\right)\sqrt{-acx+c}}{105\,a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*a^3*x^3 + 39*a^2*x^2 + 52*a*x + 104)*sqrt(-a*c*x + c)/a^3
```

3.302.6 Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\left(-2c^3\sqrt{-acx+c}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^3}{3}+\frac{x^2}{a}+\frac{2x}{a^2}+\frac{2\left(\begin{cases} -x & \text{for } a=0 \\ \frac{\log(ax-1)}{a} & \text{otherwise} \end{cases}\right)}{a^2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**2*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(-2*c**3*sqrt(-a*c*x + c) + 5*c**2*(-a*c*x + c)**(3/2)/3 - 4
*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a**3*c**3), Ne(a*c, 0))
, (sqrt(c)*(x**3/3 + x**2/a + 2*x/a**2 + 2*Piecewise((-x, Eq(a, 0)), (log(
a*x - 1)/a, True))/a**2), True))
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2\left(15(-acx+c)^{\frac{7}{2}}-84(-acx+c)^{\frac{5}{2}}c+175(-acx+c)^{\frac{3}{2}}c^2-210\sqrt{-acx+c}c^3\right)}{105a^3c^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/105*(15*(-a*c*x + c)^(7/2) - 84*(-a*c*x + c)^(5/2)*c + 175*(-a*c*x + c)
^(3/2)*c^2 - 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

3.302.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(66) = 132$.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$= \frac{2\left(\frac{7\left(3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}c^2\right)}{a^2c^2}+\frac{3\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{}\right.}{a^2c^3}}{105\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/105*(7*(3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*
sqrt(-a*c*x + c)*c^2)/(a^2*c^2) + 3*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21
*(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a
*c*x + c)*c^3)/(a^2*c^3))/a
```

3.302.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{4\,\sqrt{c-a\,c\,x}}{a^3}-\frac{10\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^3\,c}+\frac{8\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^3\,c^2}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^3\,c^3}$$

input
```
int((x^2*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c
*x)^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

3.303 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

3.303.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}$$

output
```
-2*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c^2+4*(-a*c*x+c)^(1/2)/
a^2
```

3.303.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(6+3ax+a^2x^2)}{5a^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(6 + 3*a*x + a^2*x^2))/(5*a^2)
```

3.303.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6717, 6680, 35, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{x(ax+1)}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{86}$$

$$-c\int \left(\frac{(c-acx)^{3/2}}{ac^2}-\frac{3\sqrt{c-acx}}{ac}+\frac{2}{a\sqrt{c-acx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(-\frac{2(c-acx)^{5/2}}{5a^2c^3}+\frac{2(c-acx)^{3/2}}{a^2c^2}-\frac{4\sqrt{c-acx}}{a^2c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a^2*c) + (2*(c - a*c*x)^(3/2))/(a^2*c^2) - (2*(
c - a*c*x)^(5/2))/(5*a^2*c^3)))
```

3.303.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.303.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(a^2x^2+3ax+6)}{5a^2}$	28
trager	$\frac{2\sqrt{-acx+c}\,(a^2x^2+3ax+6)}{5a^2}$	28
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(a^2x^2+3ax+6)}{5a^2}$	29
risch	$-\frac{2c(a^2x^2+3ax+6)(ax-1)}{5a^2\sqrt{-c(ax-1)}}$	35
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}-2c(-acx+c)^{\frac{3}{2}}+4c^2\sqrt{-acx+c}}{a^2c^2}$	47
default	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}-2c(-acx+c)^{\frac{3}{2}}+4c^2\sqrt{-acx+c}}{a^2c^2}$	47

input
```
int(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(-a*c*x+c)^(1/2)*(a^2*x^2+3*a*x+6)/a^2
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int e^{2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{2\left(a^2x^2+3\,ax+6\right)\sqrt{-acx+c}}{5\,a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)/a^2
```

3.303.6 Sympy [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \begin{cases} \frac{2\cdot\left(2c^2\sqrt{-acx+c}-c(-acx+c)^{\frac{3}{2}}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^2}{2}+\frac{2x}{a}+\frac{2\left(\begin{cases} -x & \text{for } a=0 \\ \frac{\log(ax-1)}{a} & \text{otherwise} \end{cases}\right)}{a}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((2*(2*c**2*sqrt(-a*c*x + c) - c*(-a*c*x + c)**(3/2) + (-a*c*x +
c)**(5/2)/5)/(a**2*c**2), Ne(a*c, 0)), (sqrt(c)*(x**2/2 + 2*x/a + 2*Piecew
ise((-x, Eq(a, 0)), (log(a*x - 1)/a, True))/a), True))
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\left((-acx+c)^{\frac{5}{2}}-5\,(-acx+c)^{\frac{3}{2}}c+10\,\sqrt{-acx+c}c^{2}\right)}{5\,a^{2}c^{2}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2/5*((-a*c*x + c)^(5/2) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c*x + c)*c^2
)/(a^2*c^2)
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$= -\frac{2\left(\frac{5\left((-acx+c)^{\frac{3}{2}}-3\,\sqrt{-acx+c}c\right)}{ac}-\frac{3\,(acx-c)^{2}\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}c^{2}}{ac^{2}}\right)}{15\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(5*((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/(a*c) - (3*(a*c*x - c
)^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/
(a*c^2))/a
```

3.303.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,(c-a\,c\,x)^{5/2}-10\,c\,(c-a\,c\,x)^{3/2}+20\,c^{2}\,\sqrt{c-a\,c\,x}}{5\,a^{2}\,c^{2}}$$

input
```
int((x*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(2*(c - a*c*x)^(5/2) - 10*c*(c - a*c*x)^(3/2) + 20*c^2*(c - a*c*x)^(1/2))/
(5*a^2*c^2)
```

3.304 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.304.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

output
```
-2/3*(-a*c*x+c)^(3/2)/a/c+4*(-a*c*x+c)^(1/2)/a
```

3.304.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(5+ax)\sqrt{c-acx}}{3a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

3.304.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2(c-acx)^{3/2}}{3ac^2}-\frac{4\sqrt{c-acx}}{ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a*c) + (2*(c - a*c*x)^(3/2))/(3*a*c^2)))
```

3.304.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.304.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
trager	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(ax+5)}{3a}$	21
risch	$-\frac{2c(ax+5)(ax-1)}{3a\sqrt{-c(ax-1)}}$	27
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2c\sqrt{-acx+c}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}+4c\sqrt{-acx+c}}{ac}$	33

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{-acx+c}(ax+5)}{3a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

3.304.6 Sympy [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\begin{cases}-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0\\ \sqrt{c}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*
c, 0)), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a,
True)), True))
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c}{c}\right)}{3\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

3.304.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

3.305 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.305.1 Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

3.305.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

output
```
2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

3.305.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6717, 6680, 35, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x(1-ax)}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{x\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{90}$$

$$-c\left(\int \frac{1}{x\sqrt{c-acx}}dx-\frac{2\sqrt{c-acx}}{c}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(-\frac{2\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}-\frac{2\sqrt{c-acx}}{c}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\sqrt{c-acx}}{c}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

output
```
-(c*((-2*Sqrt[c - a*c*x])/c - (2*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c]
))
```

3.305.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.305.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}+2\sqrt{-acx+c}$	32
default	$2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}+2\sqrt{-acx+c}$	32
pseudoelliptic	$2\,\mathrm{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\sqrt{c}+2\sqrt{-c\,(ax-1)}$	34

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c},\right.$$
$$\left.-2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)+2\sqrt{-acx+c}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x
 + c), -2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x + c
)]
```

3.305.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(34) = 68$.

Time = 3.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.05

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \begin{cases} -\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c} & \text{for}\ ac \neq 0 \\ \sqrt{c}\left(-\frac{3a\left(\frac{\log\left(\frac{2}{x}\right)}{a} - \frac{\log\left(2a-\frac{2}{x}\right)}{a}\right)}{2} + \frac{\log\left(\frac{a}{x}-\frac{1}{x^2}\right)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x,x)
```

output
```
Piecewise((-2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x +
 c), Ne(a*c, 0)), (sqrt(c)*(-3*a*(log(2/x)/a - log(2*a - 2/x)/a)/2 + log(a
/x - 1/x**2)/2), True))
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -\sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right) + 2\sqrt{-acx+c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
-sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c))) +
2*sqrt(-a*c*x + c)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -2\,c\left(\frac{\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{-acx+c}}{c}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

output
```
-2*c*(arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/c)
```

3.305.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+2\sqrt{c-a\,c\,x}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

output
```
2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2)
```

3.306 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

3.306.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} + 3a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
3*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+(-a*c*x+c)^(1/2)/x
```

3.306.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} + 3a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

output
```
Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

3.306.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6717, 6680, 35, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^2}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x^2(1-ax)}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{x^2\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{87}$$

$$-c\left(\frac{3}{2}a\int \frac{1}{x\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{cx}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(-\frac{3\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{c}-\frac{\sqrt{c-acx}}{cx}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(-\frac{3a\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-acx}}{cx}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

output
```
-(c*(-(Sqrt[c - a*c*x]/(c*x)) - (3*a*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqr
t[c]))
```

3.306.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.306.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
risch	$-\frac{(ax-1)c}{x\sqrt{-c(ax-1)}}+3a\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}$	43
pseudoelliptic	$\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax-1)}\,\sqrt{c}}{x\sqrt{c}}$	43
derivativedivides	$-2ca\left(-\frac{\sqrt{-acx+c}}{2acx}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	45
default	$2ca\left(\frac{\sqrt{-acx+c}}{2acx}+\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	45

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)/x/(-c*(a*x-1))^(1/2)*c+3*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1
/2)
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.31

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \left[\frac{3\,a\sqrt{c}x\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}}{2\,x},\right.$$
$$\left.-\frac{3\,a\sqrt{-c}x\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(3*a*sqrt(c)*x*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*
sqrt(-a*c*x + c))/x, -(3*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c)
- sqrt(-a*c*x + c))/x]
```

3.306.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^2\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{1}{2}\,ac\left(\frac{3\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{2\,\sqrt{-acx+c}}{acx}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/2*a*c*(3*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))
/sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{\frac{3\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{-acx+c}a}{x}}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-(3*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)*a/
x)/a
```

3.306.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-a\,c\,x}}{x}+3\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

output
```
(c - a*c*x)^(1/2)/x + 3*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2))
```

3.307 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

3.307.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}+\frac{7}{4}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
7/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a*c*x+c)^(1/2)/x^2
+7/4*a*(-a*c*x+c)^(1/2)/x
```

3.307.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\frac{(2+7ax)\sqrt{c-acx}}{4x^2}+\frac{7}{4}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

output
```
((2 + 7*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (7*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/Sqrt[c]])/4
```

3.307.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6717, 6680, 35, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^3}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x^3(1-ax)}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{x^3\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{87}$$

$$-c\left(\frac{7}{4}a\int \frac{1}{x^2\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{2cx^2}\right)$$

$$\Big\downarrow \text{52}$$

$$-c\left(\frac{7}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(\frac{7}{4}a\left(-\frac{\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{c}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(\frac{7}{4}a\left(-\frac{a\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)$$

input

```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

output
```
-(c*(-1/2*Sqrt[c - a*c*x]/(c*x^2) + (7*a*(-(Sqrt[c - a*c*x]/(c*x)) - (a*Ar
cTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c]))/4))
```

3.307.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.307. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.307.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right) a^{2} c\, x^{2}}{4}+\frac{7 \sqrt{c}\, \sqrt{-c(ax-1)}\, \left(ax+\frac{2}{7}\right)}{4}}{\sqrt{c}\, x^{2}}$	52
risch	$-\frac{(7a^{2}x^{2}-5ax-2)c}{4x^{2}\sqrt{-c(ax-1)}}+\frac{7a^{2} \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}}{4}$	54
derivativedivides	$2a^{2}c^{2}\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{a^{2}c^{2}x^{2}}+\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$	65
default	$2a^{2}c^{2}\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{a^{2}c^{2}x^{2}}+\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$	65

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
7/4/c^(1/2)*(arctanh((-c*(a*x-1))^(1/2)/c^(1/2))*a^2*c*x^2+c^(1/2)*(-c*(a*
x-1))^(1/2)*(a*x+2/7))/x^2
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.72

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^{3}}\, dx = \left[\frac{7\, a^{2}\sqrt{c}x^{2} \log\left(\frac{acx-2\, \sqrt{-acx+c}\sqrt{c}-2\, c}{x}\right)+2\, \sqrt{-acx+c}(7\, ax+2)}{8\, x^{2}}, -\frac{7\, a^{2}\sqrt{-c}x^{2} \arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}(7\, ax+2)}{4\, x^{2}}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(7*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x)
+ 2*sqrt(-a*c*x + c)*(7*a*x + 2))/x^2, -1/4*(7*a^2*sqrt(-c)*x^2*arctan(sqr
t(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(7*a*x + 2))/x^2]
```

3.307.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^3\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= -\frac{1}{8}\,a^2c^2\left(\frac{2\left(7\,(-acx+c)^{\frac{3}{2}}-9\,\sqrt{-acx+c}c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{7\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/8*a^2*c^2*(2*(7*(-a*c*x + c)^(3/2) - 9*sqrt(-a*c*x + c)*c)/((a*c*x - c)
^2*c + 2*(a*c*x - c)*c^2 + c^3) + 7*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt
(-a*c*x + c) + sqrt(c)))/c^(3/2))
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{\frac{7\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{7\,(-acx+c)^{\frac{3}{2}}a^3c-9\,\sqrt{-acx+c}a^3c^2}{a^2c^2x^2}}{4\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*(7*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + (7*(-a*c*x + c)
^(3/2)*a^3*c - 9*sqrt(-a*c*x + c)*a^3*c^2)/(a^2*c^2*x^2))/a
```

3.307.9 Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{9\,\sqrt{c-a\,c\,x}}{4\,x^2} + \frac{7\,a^2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)}{4} - \frac{7\,(c-a\,c\,x)^{3/2}}{4\,c\,x^2}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

output
```
(9*(c - a*c*x)^(1/2))/(4*x^2) + (7*a^2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(
1/2)))/4 - (7*(c - a*c*x)^(3/2))/(4*c*x^2)
```

3.308 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

3.308.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{11}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
11/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a*c*x+c)^(1/2)/x^
3+11/12*a*(-a*c*x+c)^(1/2)/x^2+11/8*a^2*(-a*c*x+c)^(1/2)/x
```

3.308.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}(8+22ax+33a^2x^2)}{24x^3}+\frac{11}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

output
```
(Sqrt[c - a*c*x]*(8 + 22*a*x + 33*a^2*x^2))/(24*x^3) + (11*a^3*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

3.308.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6717, 6680, 35, 87, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^4}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x^4(1-ax)}dx$$

$$\Big\downarrow \text{ 35}$$

$$-c\int \frac{ax+1}{x^4\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{ 87}$$

$$-c\left(\frac{11}{6}a\int \frac{1}{x^3\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

$$\Big\downarrow \text{ 52}$$

$$-c\left(\frac{11}{6}a\left(\frac{3}{4}a\int \frac{1}{x^2\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

$$\Big\downarrow \text{ 52}$$

$$-c\left(\frac{11}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

$$\Big\downarrow \text{ 73}$$

$$-c\left(\frac{11}{6}a\left(\frac{3}{4}a\left(-\frac{\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{c}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

$$\Big\downarrow \text{ 221}$$

$$-c\left(\frac{11}{6}a\left(\frac{3}{4}a\left(-\frac{a\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

output
```
-(c*(-1/3*Sqrt[c - a*c*x]/(c*x^3) + (11*a*(-1/2*Sqrt[c - a*c*x]/(c*x^2) +
(3*a*(-(Sqrt[c - a*c*x]/(c*x)) - (a*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt
[c]))/4))/6))
```

3.308.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.308.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\left(33 a^{3} x^{3}-11 a^{2} x^{2}-14 a x -8\right) c}{24 x^{3} \sqrt{-c \left(a x -1\right)}}+\frac{11 a^{3} \operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right) \sqrt{c}}{8}$	62
pseudoelliptic	$\frac{\frac{\sqrt{-c \left(a x -1\right)}\, \left(33 a^{2} x^{2}+22 a x +8\right) \sqrt{c}}{24}+\frac{11 \,\operatorname{arctanh}\left(\frac{\sqrt{-c \left(a x -1\right)}}{\sqrt{c}}\right) a^{3} c \,x^{3}}{8}}{\sqrt{c}\, x^{3}}$	62
default	$2 c^{3} a^{3} \left(\frac{\frac{11 \left(-a c x +c \right)^{\frac{5}{2}}}{16 c^{2}}-\frac{11 \left(-a c x +c \right)^{\frac{3}{2}}}{6 c}+\frac{21 \sqrt{-a c x +c}}{16}}{a^{3} c^{3} x^{3}}+\frac{11 \,\operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right)}{16 c^{\frac{5}{2}}}\right)$	79
derivativedivides	$-2 c^{3} a^{3} \left(-\frac{\frac{11 \left(-a c x +c \right)^{\frac{5}{2}}}{16 c^{2}}-\frac{11 \left(-a c x +c \right)^{\frac{3}{2}}}{6 c}+\frac{21 \sqrt{-a c x +c}}{16}}{a^{3} c^{3} x^{3}}-\frac{11 \,\operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right)}{16 c^{\frac{5}{2}}}\right)$	80

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(33*a^3*x^3-11*a^2*x^2-14*a*x-8)/x^3/(-c*(a*x-1))^(1/2)*c+11/8*a^3*a
rctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.49

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \left[\frac{33\,a^3\sqrt{c}x^3\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,(33\,a^2x^2+22\,ax+8)\sqrt{-acx+c}}{48\,x^3}, -\frac{33\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-(33\,a^2x^2+22\,ax+8)\sqrt{-acx+c}}{24\,x^3}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(33*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x
) + 2*(33*a^2*x^2 + 22*a*x + 8)*sqrt(-a*c*x + c))/x^3, -1/24*(33*a^3*sqrt(
-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (33*a^2*x^2 + 22*a*x + 8)*sq
rt(-a*c*x + c))/x^3]
```

3.308.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^4\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$=\frac{1}{48}\,a^3c^3\left(\frac{2\left(33\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+63\,\sqrt{-acx+c}c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{33\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/48*a^3*c^3*(2*(33*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 63*sqrt
(-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)
*c^4 + c^5) - 33*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt
(c)))/c^(5/2))
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$=-\frac{\frac{33\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{33\,(acx-c)^2\sqrt{-acx+c}a^4c-88\,(-acx+c)^{\frac{3}{2}}\,a^4c^2+63\,\sqrt{-acx+c}a^4c^3}{a^3c^3x^3}}{24\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/24*(33*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (33*(a*c*x -
c)^2*sqrt(-a*c*x + c)*a^4*c - 88*(-a*c*x + c)^(3/2)*a^4*c^2 + 63*sqrt(-a*c
*x + c)*a^4*c^3)/(a^3*c^3*x^3))/a
```

3.308.9 Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{21\sqrt{c-a\,c\,x}}{8\,x^3} - \frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3} + \frac{11\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3} - \frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,11\mathrm{i}}{8}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

output
```
(21*(c - a*c*x)^(1/2))/(8*x^3) - (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/
c^(1/2))*11i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (11*(c - a*c*x)^(5/2)
)/(8*c^2*x^3)
```

3.309 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

3.309.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
75/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a*c*x+c)^(1/2)/x
^4+5/8*a*(-a*c*x+c)^(1/2)/x^3+25/32*a^2*(-a*c*x+c)^(1/2)/x^2+75/64*a^3*(-a
*c*x+c)^(1/2)/x
```

3.309.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}(16+40ax+50a^2x^2+75a^3x^3)}{64x^4}+\frac{75}{64}a^4\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

output
```
(Sqrt[c - a*c*x]*(16 + 40*a*x + 50*a^2*x^2 + 75*a^3*x^3))/(64*x^4) + (75*a
^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

3.309.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6717, 6680, 35, 87, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \textcolor{blue}{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^5}dx$$

$$\Big\downarrow \textcolor{blue}{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x^5(1-ax)}dx$$

$$\Big\downarrow \textcolor{blue}{35}$$

$$-c\int \frac{ax+1}{x^5\sqrt{c-acx}}dx$$

$$\Big\downarrow \textcolor{blue}{87}$$

$$-c\left(\frac{15}{8}a\int \frac{1}{x^4\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \textcolor{blue}{52}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\int \frac{1}{x^3\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \textcolor{blue}{52}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\int \frac{1}{x^2\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \textcolor{blue}{52}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(-\frac{\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{c}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow 221$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

input

```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

output

```
-(c*(-1/4*Sqrt[c - a*c*x]/(c*x^4) + (15*a*(-1/3*Sqrt[c - a*c*x]/(c*x^3) +
(5*a*(-1/2*Sqrt[c - a*c*x]/(c*x^2) + (3*a*(-(Sqrt[c - a*c*x]/(c*x)) - (a*A
rcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c]))/4))/6))/8))
```

3.309.3.1 Defintions of rubi rules used

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.309.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{\left(75 a^{4} x^{4}-25 a^{3} x^{3}-10 a^{2} x^{2}-24 a x -16\right) c}{64 x^{4} \sqrt{-c \left(a x -1\right)}}+\frac{75 a^{4} \operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right) \sqrt{c}}{64}$	70
pseudoelliptic	$\frac{\frac{\sqrt{-c \left(a x -1\right)}\, \left(75 a^{3} x^{3}+50 a^{2} x^{2}+40 a x +16\right) \sqrt{c}}{64}+\frac{75 \,\operatorname{arctanh}\left(\frac{\sqrt{-c \left(a x -1\right)}}{\sqrt{c}}\right) c \,a^{4} x^{4}}{64}}{\sqrt{c}\, x^{4}}$	70
derivativedivides	$2 c^{4} a^{4} \left(\frac{-\frac{75 \left(-a c x +c \right)^{\frac{7}{2}}}{128 c^{3}}+\frac{275 \left(-a c x +c \right)^{\frac{5}{2}}}{128 c^{2}}-\frac{365 \left(-a c x +c \right)^{\frac{3}{2}}}{128 c}+\frac{181 \sqrt{-a c x +c}}{128}}{a^{4} c^{4} x^{4}}+\frac{75 \,\operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right)}{128 c^{\frac{7}{2}}}\right)$	93
default	$2 c^{4} a^{4} \left(\frac{-\frac{75 \left(-a c x +c \right)^{\frac{7}{2}}}{128 c^{3}}+\frac{275 \left(-a c x +c \right)^{\frac{5}{2}}}{128 c^{2}}-\frac{365 \left(-a c x +c \right)^{\frac{3}{2}}}{128 c}+\frac{181 \sqrt{-a c x +c}}{128}}{a^{4} c^{4} x^{4}}+\frac{75 \,\operatorname{arctanh}\left(\frac{\sqrt{-a c x +c}}{\sqrt{c}}\right)}{128 c^{\frac{7}{2}}}\right)$	93

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/64*(75*a^4*x^4-25*a^3*x^3-10*a^2*x^2-24*a*x-16)/x^4/(-c*(a*x-1))^(1/2)*
c+75/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.35

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \left[\frac{75\,a^4\sqrt{c}x^4\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,(75\,a^3x^3+50\,a^2x^2+40\,ax+16)\sqrt{-acx+c}}{128\,x^4},\right.$$
$$\left.-\frac{75\,a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-(75\,a^3x^3+50\,a^2x^2+40\,ax+16)\sqrt{-acx+c}}{64\,x^4}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/128*(75*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/
x) + 2*(75*a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4, -1/6
4*(75*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (75*a^3*x^3 +
 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4]
```

3.309.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^5\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx =$$
$$-\frac{1}{128}\,a^4c^4\left(\frac{2\left(75\,(-acx+c)^{\frac{7}{2}}-275\,(-acx+c)^{\frac{5}{2}}c+365\,(-acx+c)^{\frac{3}{2}}c^2-181\,\sqrt{-acx+c}c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{75\log}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/128*a^4*c^4*(2*(75*(-a*c*x + c)^(7/2) - 275*(-a*c*x + c)^(5/2)*c + 365*
(-a*c*x + c)^(3/2)*c^2 - 181*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*
(a*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 75*lo
g((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(7/2))
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx =$$
$$-\frac{\frac{75\,a^5c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{75\,(acx-c)^3\sqrt{-acx+c}a^5c+275\,(acx-c)^2\sqrt{-acx+c}a^5c^2-365\,(-acx+c)^{\frac{3}{2}}a^5c^3+181\,\sqrt{-acx+c}a^5c^4}{a^4c^4x^4}}{64\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/64*(75*a^5*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (75*(a*c*x -
c)^3*sqrt(-a*c*x + c)*a^5*c + 275*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^5*c^2 -
 365*(-a*c*x + c)^(3/2)*a^5*c^3 + 181*sqrt(-a*c*x + c)*a^5*c^4)/(a^4*c^4*x
^4))/a
```

3.309.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{181\sqrt{c-a\,c\,x}}{64\,x^4} - \frac{365\,(c-a\,c\,x)^{3/2}}{64\,c\,x^4} + \frac{275\,(c-a\,c\,x)^{5/2}}{64\,c^2\,x^4} - \frac{75\,(c-a\,c\,x)^{7/2}}{64\,c^3\,x^4} - \frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,75\mathrm{i}}{64}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

output
```
(181*(c - a*c*x)^(1/2))/(64*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i
)/c^(1/2))*75i)/64 - (365*(c - a*c*x)^(3/2))/(64*c*x^4) + (275*(c - a*c*x)
^(5/2))/(64*c^2*x^4) - (75*(c - a*c*x)^(7/2))/(64*c^3*x^4)
```

3.310 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

3.310.1 Optimal result

Integrand size = 23, antiderivative size = 309

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}\\ &\quad+\frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}\\ &\quad+\frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}\\ &\quad-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{9/2}\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output

```
1576/315*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(1-1/a/x)^(1/2)+472/315*x*(1
+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+92/105*x^2*(1+1/a/x)^(1
/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+38/63*x^3*(1+1/a/x)^(1/2)*(-a*c*x
+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/9*x^4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/
a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*
(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(9/2)/(1-1/a/x)^(1/2)
```

3.310.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.42

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(788+236ax+138a^2x^2+95a^3x^3+35a^4x^4)-630\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{315a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(788 + 236*a*x + 138*a^2*x^2
 + 95*a^3*x^3 + 35*a^4*x^4) - 630*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(315*a^(9/2)*Sqrt[1 - 1/(a*x)])
```

3.310.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.75, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {6730, 27, 109, 27, 169, 27, 169, 25, 169, 27, 169, 27, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-acx}e^{3\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6730}$$
$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$
$$\Big\downarrow \text{27}$$
$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$
$$\Big\downarrow \text{109}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{2\int -\frac{19a+\frac{17}{x}}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{9a}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{19a+\frac{17}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 169}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{-\frac{2\int -\frac{3\left(23a+\frac{19}{x}\right)}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\int \frac{23a+\frac{19}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 169}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(-\frac{2\int -\frac{59a+\frac{46}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 25}$$

3.310. $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(\frac{2\int \frac{59a+\frac{46}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(\frac{2\left(-\frac{2\int -\frac{197a+\frac{118}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

3.310. $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(\frac{2\left(\frac{\int \frac{197a+\frac{118}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 169$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(\frac{2\left(\frac{-\frac{2\int -\frac{315a}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{394\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

3.310. $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(\frac{2\left(\frac{315\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{394\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 104$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(\frac{2\left(\frac{630\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{394\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 219$$

3.310. $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{\frac{6\left(\frac{2\left(\frac{\frac{315\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}}-\frac{394\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*Sqrt[1 + 1/(a*x)])/(9*a*(x^(-1))^(9
/2)) + ((-38*Sqrt[1 + 1/(a*x)])/(7*(x^(-1))^(7/2)) + (6*((-46*Sqrt[1 + 1/(
a*x)])/(5*(x^(-1))^(5/2)) + (2*((-118*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)
) + ((-394*Sqrt[1 + 1/(a*x)])/Sqrt[x^(-1)] + (315*Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[a])/(3*a)))/(5*a)))/(7*a
))/(9*a^2)))/Sqrt[1 - 1/(a*x)])
```

3.310. $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

3.310.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.310. $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.310.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.47

method	result
risch	$-\frac{2(35a^4x^4+95a^3x^3+138a^2x^2+236ax+788)c(ax-1)}{315a^4\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$
default	$\frac{2(ax-1)\sqrt{-c(ax-1)}\Bigg(35a^4x^4\sqrt{-c(ax+1)}+95a^3x^3\sqrt{-c(ax+1)}+138a^2x^2\sqrt{-c(ax+1)}+236ax\sqrt{-c(ax+1)}-630\sqrt{c}\sqrt{2}\arctan\Bigg(}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}a^4}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/315*(35*a^4*x^4+95*a^3*x^3+138*a^2*x^2+236*a*x+788)/a^4*c/((a*x-1)/(a*x
+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-1)-4/a^4*2^(1/2)*c^(1/2)*arctan(1/2*(-a
*c*x-c)^(1/2)*2^(1/2)/c^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x+1)
)^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-1)
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.98

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$

$$=\Bigg[\frac{2\left(315\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(35\,a^5x^5+130\,a^4x^4+233\,a\right.}{315\,(a^5x-a^4)}$$

$$-\frac{2\left(630\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(35\,a^5x^5+130\,a^4x^4+233\,a^3x^3+374\,a^2x^2+102\right.}{315\,(a^5x-a^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
[2/315*(315*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(
2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) + (35*a^5*x^5 + 130*a^4*x^4 + 233*a^3*x^3 + 374*a^2*x
^2 + 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*x -
a^4), -2/315*(630*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c
)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (35*a^5*x^5 + 130*a^4*x
^4 + 233*a^3*x^3 + 374*a^2*x^2 + 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1)))/(a^5*x - a^4)]
```

3.310.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int\frac{x^3\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.310.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.310.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int \frac{x^3\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.311 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.311.1 Optimal result

Integrand size = 23, antiderivative size = 261

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

output
```
104/21*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+32/21*x*(1+1/a
/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+6/7*x^2*(1+1/a/x)^(1/2)*(-a
*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*x^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(
1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1
/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(7/2)/(1-1/a/x)^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(52+16ax+9a^2x^2+3a^3x^3)-42\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{21a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

input

```
Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

output

```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(52 + 16*a*x + 9*a^2*x^2 + 3
*a^3*x^3) - 42*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a
]*Sqrt[1 + 1/(a*x)])]))/(21*a^(7/2)*Sqrt[1 - 1/(a*x)])
```

3.311.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.75, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6730, 27, 109, 27, 169, 27, 169, 27, 169, 27, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 109}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{2\int -\frac{15a+\frac{13}{x}}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{15a+\frac{13}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 169$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{-\frac{2\int -\frac{10\left(4a+\frac{3}{x}\right)}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\int \frac{4a+\frac{3}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 169$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(-\frac{2\int -\frac{13a+\frac{8}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

3.311. $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(\frac{\int \frac{13a+\frac{8}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(\frac{-\frac{2\int -\frac{21a}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{26\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(\frac{21\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{26\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{104}$$

3.311. $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(\frac{42\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{26\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{\frac{4\left(\frac{\frac{21\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}}-\frac{26\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*Sqrt[1 + 1/(a*x)])/(7*a*(x^(-1))^(7
/2)) + ((-6*Sqrt[1 + 1/(a*x)])/(x^(-1))^(5/2) + (4*((-8*Sqrt[1 + 1/(a*x)])
/(3*(x^(-1))^(3/2)) + ((-26*Sqrt[1 + 1/(a*x)])/Sqrt[x^(-1)] + (21*Sqrt[2]*
ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[a])/(3*a
)))/a)/(7*a^2)))/Sqrt[1 - 1/(a*x)])
```

3.311. $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.311.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.311.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{2(3a^3x^3+9a^2x^2+16ax+52)c(ax-1)}{21a^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a^3(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-3a^3x^3\sqrt{-c(ax+1)}-9a^2x^2\sqrt{-c(ax+1)}+42\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-16ax\sqrt{-c(ax+1)}-52\sqrt{-c}\right.}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}a^3}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/21*(3*a^3*x^3+9*a^2*x^2+16*a*x+52)/a^3*c/((a*x-1)/(a*x+1))^(1/2)/(-c*(a
*x-1))^(1/2)*(a*x-1)-4/a^3*2^(1/2)*c^(1/2)*arctan(1/2*(-a*c*x-c)^(1/2)*2^(
1/2)/c^(1/2))/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x+1))^(1/2)/(-c*(a*x-
1))^(1/2)*(a*x-1)
```

3.311.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$

$$=\left[\frac{2\left(21\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+(3a^4x^4+12a^3x^3+25a^2x^2-\right.}{21\left(a^4x-a^3\right)}\right.$$

$$-\frac{2\left(42\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(3a^4x^4+12a^3x^3+25a^2x^2+68ax+52)\sqrt{-acx}\right.}{21\left(a^4x-a^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
[2/21*(21*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)
*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) + (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*s
qrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3), -2/21*(42*sqrt(2
)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))/(a*c*x - c)) - (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x +
 52)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)]
```

3.311.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.311.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.311.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx =$$
$$-\frac{2\,c^2\left(\frac{2\sqrt{2}\left(21\sqrt{c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-40\sqrt{-c}\right)}{a^2c}-\frac{42\sqrt{2}c^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-3\,(acx+c)^3\sqrt{-acx-c}+7\,(-acx-c)^{\frac{3}{2}}c^2-42\sqrt{-acx-c}\,c^3}{a^2c^4}\right)}{21\,a|c|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
-2/21*c^2*(2*sqrt(2)*(21*sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - 40*sqrt(-c))/(
a^2*c) - (42*sqrt(2)*c^(7/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))
- 3*(a*c*x + c)^3*sqrt(-a*c*x - c) + 7*(-a*c*x - c)^(3/2)*c^2 - 42*sqrt(-a
*c*x - c)*c^3)/(a^2*c^4))/(a*abs(c)*sgn(a*x + 1))
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int \frac{x^2\sqrt{c-a\,c\,x}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.312 $\int e^{3\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

3.312.1 Optimal result

Integrand size = 21, antiderivative size = 211

$$\int e^{3\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2} x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

output
```
2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(5/
2)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)
/a^2/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2)
)*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(5/2)/(1-1/a/x)^(1/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(38+11ax+3a^2x^2)-30\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{15a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(38 + 11*a*x + 3*a^2*x^2) -
30*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1
/(a*x)])]))/(15*a^(5/2)*Sqrt[1 - 1/(a*x)])
```

3.312.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6730, 27, 107, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 107}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{2\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 104$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{2\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 219$$

3.312. $\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{\frac{2\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*(1 + 1/(a*x))^(5/2))/(5*a*(x^(-1))^
(5/2)) + ((-2*(1 + 1/(a*x))^(3/2))/(3*a*(x^(-1))^(3/2)) + (2*((-2*Sqrt[1 +
 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(S
qrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/a)/a))/Sqrt[1 - 1/(a*x)])
```

3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.312.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-3a^{2}x^{2}\sqrt{-c(ax+1)}+30\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-11ax\sqrt{-c(ax+1)}-38\sqrt{-c(ax+1)}\right)}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^{2}}$	125
risch	$-\frac{2(3a^{2}x^{2}+11ax+38)c(ax-1)}{15a^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a^{2}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	130

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*a^2*x
^2*(-c*(a*x+1))^(1/2)+30*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(
1/2)/c^(1/2))-11*a*x*(-c*(a*x+1))^(1/2)-38*(-c*(a*x+1))^(1/2))/(-c*(a*x+1)
)^(1/2)/a^2
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.29

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$

$$= \Bigg[\frac{2\left(15\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(3\,a^3x^3+14\,a^2x^2+49\,ax+}{15\,(a^3x-a^2)}$$

$$-\frac{2\left(30\,\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(3\,a^3x^3+14\,a^2x^2+49\,ax+38)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3x-a^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

output
```
[2/15*(15*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)
*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) + (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x +
c)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x - a^2), -2/15*(30*sqrt(2)*(a*x - 1)*s
qrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(
a*c*x - c)) - (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1)))/(a^3*x - a^2)]
```

3.312.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{x\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.312.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+cx}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.312.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{x\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.313 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.313.1 Optimal result

Integrand size = 20, antiderivative size = 163

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

output
```
2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(
-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1
+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(3/2)/(1-1/a/x)^(1/2
)
```

3.313.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(7+ax)-6\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^
(3/2)*Sqrt[1 - 1/(a*x)])
```

3.313.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 104$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{2\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output

```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*(1 + 1/(a*x))^(3/2))/(3*a*(x^(-1))^
(3/2)) + (2*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[2]*ArcTanh[
(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/a))/Sqrt[1
- 1/(a*x)])
```

3.313. $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.313.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.313.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(6\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-ax\sqrt{-c(ax+1)}-7\sqrt{-c(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a}$	107
risch	$-\frac{2(ax+7)c(ax-1)}{3a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	121

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-a*x*(-c*(a*x+1))^(
1/2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.53

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$= \left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{}\right.}{3\,(a^2x-a)}\right.$$
$$\left.-\frac{2\left(6\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*s
qrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(
a*x + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*s
qrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2
+ 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

3.313.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.313.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.313.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \frac{\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.314 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.314.1 Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*arcsinh((1/x)^(1/2)/a
^(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(2^(
1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(
1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

3.314.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-2\sqrt{2}\sqrt{\frac{1}{x}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]] - 2*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

3.314.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6730, 27, 109, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \color{blue}{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{109}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{2\int -\frac{3a+\frac{1}{x}}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{3a+\frac{1}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 175$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{4a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 63$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{4a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 104$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 219$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{4\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 222$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{4\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)])
 + (-2*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] + 4*Sqrt[2]*Sqrt[a]*ArcTanh[(
Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^2))/Sqrt[1 - 1/(a*x)
])
```

3.314. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.314.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.314. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.314.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-2\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c(ax+1)}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}}$	107

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(a*x-1)*(-c*(a*x-1))^(1/2)*(-2*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(
1/2)*2^(1/2)/c^(1/2))+c^(1/2)*arctan((-c*(a*x+1))^(1/2)/c^(1/2))+(-c*(a*x+
1))^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.07

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$$

$$= \left[\frac{2\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{}}{}\right.}{ax-1}\right.$$

$$-\frac{2\left(2\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+}\right.}{ax-1}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="frica
s")
```

output
```
[(2*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 -
 2*a*x + 1)) + (a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x
+ c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*(2*sqr
t(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x -
 1)/(a*x + 1))/(a*c*x - c)) - (a*x - 1)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*
sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

3.314.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\int \frac{\sqrt{-c\,(ax-1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.314.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\int \frac{\sqrt{-acx+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.314.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.315 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

3.315.1 Optimal result

Integrand size = 23, antiderivative size = 172

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{5\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+5*arcsinh((1/x)^(1/2)/a
^(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(2^(
1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*a^(1/2)*(1/x)^(1/2)*(-a*
c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

3.315.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}+5\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-4\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

output
```
(Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + 5*Sqrt[a]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] - 4*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/Sqrt[1 - 1/(a*x)]
```

3.315.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6730, 27, 113, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{113}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\int -\frac{3a+\frac{5}{x}}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{3a+\frac{5}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 175}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-5\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 63}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-10\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{16a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-10\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-10\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 222}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-10\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/a) +
 (-10*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] + 8*Sqrt[2]*Sqrt[a]*ArcTanh[(S
qrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(2*a)))/Sqrt[1 - 1/(a*x
)])
```

3.315. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

3.315.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.315. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.315.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-4\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+5\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,\sqrt{c}\,x}$	117
risch	$-\frac{c(ax-1)}{x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{5a\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c\sqrt{-c(ax+1)}\,(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$	140

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE
)
```

output
```
1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-4*2^(1/2)*a
rctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+5*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/c^(1/2
)/x
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.27

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$= \Bigg[\frac{4\sqrt{2}(a^2x^2-ax)\sqrt{-c}\log\Bigg(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\Bigg)+5\,(a^2x^2-ax)\sqrt{-c}\log\Bigg(-\frac{a^2cx^2}{}}{2\,(ax^2-x)}$$
$$-\frac{4\sqrt{2}(a^2x^2-ax)\sqrt{c}\arctan\Bigg(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\Bigg)-5\,(a^2x^2-ax)\sqrt{c}\arctan\Bigg(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\Bigg)-\sqrt{-c}}{ax^2-x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fri
cas")
```

output
```
[1/2*(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqr
t(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/
(a^2*x^2 - 2*a*x + 1)) + 5*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*
x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)
/(a*x^2 - x)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a
*x^2 - x), -(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x
+ c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 5*(a^2*x^2 - a*x)*sq
rt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)]
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

output
```
Timed out
```

3.315.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\int \frac{\sqrt{-acx+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.315.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\int \frac{\sqrt{c-a\,c\,x}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.316 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

3.316.1 Optimal result

Integrand size = 23, antiderivative size = 224

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x} + \frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
1/2*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+7/4*a*(1+1/a/x)^(
1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+23/4*a^(3/2)*arcsinh((1/x)^(1/2)/a
^(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(3/2)*arctanh(2^(
1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(
1/2)/(1-1/a/x)^(1/2)
```

3.316.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}(2+9ax)+\frac{23a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}-\frac{16\sqrt{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}}\right)}{4\sqrt{1-\frac{1}{ax}}x^2}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

output
```
(Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(2 + 9*a*x) + (23*a^(3/2)*ArcSinh[Sqrt
[x^(-1)]/Sqrt[a]])/(x^(-1))^(3/2) - (16*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*S
qrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(3/2)))/(4*Sqrt[1 - 1/
(a*x)]*x^2)
```

3.316.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.70, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6730, 27, 112, 27, 171, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{112}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\int\frac{\left(a+\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\int\frac{\left(a+\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(-\int-\frac{9a+\frac{23}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{9a+\frac{23}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{175}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(32a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-23\int\frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(32a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-46\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(64a\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-46\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

3.316. $\int\frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

$$\Big\downarrow 219$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(32\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-46\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3}\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 222$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(32\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-46\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-1/2*((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])
 + (-7*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + (-46*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/
Sqrt[a]] + 32*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt
[1 + 1/(a*x)])])/2)/4))/Sqrt[1 - 1/(a*x)])
```

3.316.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.316. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

3.316.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.64

method	result
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-16\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^2c\,x^2+23c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^2x^2+9ax\sqrt{c}\,\sqrt{-c(ax+1)}+2\sqrt{-c(ax+1)}\right.}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\,\sqrt{-c(ax+1)}\,x^2}$
risch	$-\frac{(9a^2x^2+11ax+2)c(ax-1)}{4x^2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^2\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{23a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)c\sqrt{-c(ax+1)}\,(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE
)
```

output
```
1/4*(a*x-1)*(-c*(a*x-1))^(1/2)*(-16*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*
2^(1/2)/c^(1/2))*a^2*c*x^2+23*c*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^2*x^2
+9*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+2*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(
a*x+1))^(3/2)/(a*x+1)/c^(1/2)/(-c*(a*x+1))^(1/2)/x^2
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.91

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= \left[\frac{16\sqrt{2}(a^3x^3-a^2x^2)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+23\,(a^3x^3-a^2x^2)\sqrt{-c}\log\left(\right.}{8\,(ax^3-x^2)}\right.$$
$$-\frac{16\sqrt{2}(a^3x^3-a^2x^2)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-23\,(a^3x^3-a^2x^2)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)}{4\,(ax^3-x^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fri
cas")
```

output
```
[1/8*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x +
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
3*c)/(a^2*x^2 - 2*a*x + 1)) + 23*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*
x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x +
1)) - 2*c)/(a*x^2 - x)) + 2*(9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2), -1/4*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)
*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))
/(a*c*x - c)) - 23*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqr
t(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (9*a^2*x^2 + 11*a*x + 2)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

3.316.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**3,x)
```

output
```
Timed out
```

3.316.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-acx+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.316.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.317 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

3.317.1 Optimal result

Integrand size = 23, antiderivative size = 274

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx &= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x} \\ &\quad + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} \\ &\quad + \frac{45a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}} \\ &\quad - \frac{4\sqrt{2}a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output
```
1/3*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+3/4*a^2*(1+1/a/
x)^(3/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+13/8*a^2*(1+1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+45/8*a^(5/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1
/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(5/2)*arctanh(2^(1/2)*(1/x)
^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/
a/x)^(1/2)
```

3.317.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.51

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}(8+26ax+57a^2x^2)+\frac{135a^{5/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}-\frac{96\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{24\sqrt{1-\frac{1}{ax}}x^3}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

output
```
(Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(8 + 26*a*x + 57*a^2*x^2) + (135*a^(5/
2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2) - (96*Sqrt[2]*a^(5/2)*Arc
Tanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(5/2)))
/(24*Sqrt[1 - 1/(a*x)]*x^3)
```

3.317.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6730, 27, 112, 27, 171, 27, 171, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{112}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{3}\int \frac{3\left(a+\frac{3}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{2\left(a-\frac{1}{x}\right)}d\frac{1}{x}-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\int \frac{\left(a+\frac{3}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{a-\frac{1}{x}}d\frac{1}{x}-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(-\frac{1}{2}a\int -\frac{\left(3a+\frac{13}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\int \frac{\left(3a+\frac{13}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{2}\Bigg(\frac{1}{4}a\Bigg(-\int -\frac{19a+\frac{45}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\Big(\frac{1}{ax}+1}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{2}\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\int \frac{19a+\frac{45}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{175}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{2}\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(64a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-45\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\Bigg)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{3}{2}a\sqrt{\frac{1}{x}}\Big(\frac{1}{ax}+1}{\sqrt{1-\frac{1}{ax}}}$$

3.317. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

$$\Big\downarrow \text{63}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{2}\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(64a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-90\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\Bigg)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{2}\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(128a\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-90\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\Bigg)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{2}\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(64\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-90\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\Bigg)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{3}{2}a\sqrt{\frac{1}{x}}\Big(}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{2}\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(64\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-90\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\Bigg)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{3}{2}a}{\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

output

```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-1/3*((1 + 1/(a*x))^(3/2)*(x^(-1))^(3/2
)) + ((-3*a*(1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/2 + (a*(-13*Sqrt[1 + 1/(a*x)
]*Sqrt[x^(-1)] + (-90*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] + 64*Sqrt[2]*S
qrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/2))/4)
/2))/Sqrt[1 - 1/(a*x)])
```

3.317. $\int\frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

3.317.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.317. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.317.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

method	result
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-96\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^3c\,x^3+135c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^3x^3+57a^2x^2\sqrt{-c(ax+1)}\,\sqrt{c}+26ax\sqrt{c}\right.}{24\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\,\sqrt{-c(ax+1)}\,x^3}$
risch	$-\frac{(57a^3x^3+83a^2x^2+34ax+8)c(ax-1)}{24x^3(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^3\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{45a^3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{8\sqrt{c}}\right)c\sqrt{-c(ax+1)}\,(ax-1)}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE
)
```

output
```
1/24/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-96*2^(1/
2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^3*c*x^3+135*c*arctan((
-c*(a*x+1))^(1/2)/c^(1/2))*a^3*x^3+57*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)+2
6*a*x*c^(1/2)*(-c*(a*x+1))^(1/2)+8*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(-c
*(a*x+1))^(1/2)/x^3
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.62

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$

$$= \left[\frac{96\sqrt{2}(a^4x^4-a^3x^3)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+135\,(a^4x^4-a^3x^3)\sqrt{-c}\log}{48\,(ax^4-} \right.$$

$$-\frac{96\sqrt{2}(a^4x^4-a^3x^3)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-135\,(a^4x^4-a^3x^3)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)}{24\,(ax^4-x^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fri
cas")
```

output
```
[1/48*(96*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x +
 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 135*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*
c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x
+ 1)) - 2*c)/(a*x^2 - x)) + 2*(57*a^3*x^3 + 83*a^2*x^2 + 34*a*x + 8)*sqrt(
-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^3), -1/24*(96*sqrt(2)*(a
^4*x^4 - a^3*x^3)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))/(a*c*x - c)) - 135*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(sq
rt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (57*a^3*x^
3 + 83*a^2*x^2 + 34*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(
a*x^4 - x^3)]
```

3.317.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**4,x)
```

output
```
Timed out
```

3.317.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-acx+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.317.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.318 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

3.318.1 Optimal result

Integrand size = 23, antiderivative size = 322

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx &= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3}+\frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2}\\ &\quad+\frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}\,x}+\frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}\,x}\\ &\quad+\frac{363a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}\\ &\quad-\frac{4\sqrt{2}a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output

```
1/4*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)+11/24*a^2*(1+1/
a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+21/32*a^3*(1+1/a/x)^(3/2)*
(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+107/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/x/(1-1/a/x)^(1/2)+363/64*a^(7/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(
1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2
)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)
```

3.318.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}(48+136ax+214a^2x^2+447a^3x^3)+\frac{1089a^{7/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}-\frac{768\sqrt{2}a^{7/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{a}\sqrt{1+}}\right.}{\left(\frac{1}{x}\right)^{7/2}}\right.}{192\sqrt{1-\frac{1}{ax}}x^4}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

output
```
(Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(48 + 136*a*x + 214*a^2*x^2 + 447*a^3*
x^3) + (1089*a^(7/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2) - (768*
Sqrt[2]*a^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])
])/(x^(-1))^(7/2)))/(192*Sqrt[1 - 1/(a*x)]*x^4)
```

3.318.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.68, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6730, 27, 112, 27, 171, 27, 171, 27, 171, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x^5}\,dx$$
$$\Big\downarrow \text{6730}$$
$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$
$$\Big\downarrow \text{27}$$
$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 112}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\int \frac{\left(5a+\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}d\frac{1}{x}-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\int \frac{\left(5a+\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}{a-\frac{1}{x}}d\frac{1}{x}-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 171}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(-\frac{1}{3}a\int -\frac{3\left(11a+\frac{21}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{2\left(a-\frac{1}{x}\right)}d\frac{1}{x}-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\int \frac{\left(11a+\frac{21}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{a-\frac{1}{x}}d\frac{1}{x}-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 171}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(-\frac{1}{2}a\int -\frac{\left(21a+\frac{107}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{21}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{4}\left(\frac{1}{x}\right)^{5/}\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{\left(21a+\frac{107}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{21}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 171}$$

3.318. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{8}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(-\int -\frac{149a+\frac{363}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{21}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{8}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\int \frac{149a+\frac{363}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{21}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{175}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{8}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(512a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-363\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\Bigg)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{21}{2}a\sqrt{}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{8}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(512a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-726\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\Bigg)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{21}{2}a\sqrt{}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{8}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(1024a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-726\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\Bigg)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{21}{2}a\sqrt{\frac{1}{x}}\Bigg(}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{1}{8}\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{1}{2}\Bigg(512\sqrt{2}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-726\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\Bigg)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\Bigg)-}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

3.318. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{1}{2}\left(512\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-726\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\right.\right.\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-1/4*((1 + 1/(a*x))^(3/2)*(x^(-1))^(5/2
)) + ((-11*a*(1 + 1/(a*x))^(3/2)*(x^(-1))^(3/2))/3 + (a*((-21*a*(1 + 1/(a*
x))^(3/2)*Sqrt[x^(-1)])/2 + (a*(-107*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + (-72
6*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] + 512*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqr
t[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/2))/4))/2)/8))/Sqrt[1 - 1
/(a*x)])
```

3.318.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

3.318. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.318. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

3.318.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{(447a^4x^4+661a^3x^3+350a^2x^2+184ax+48)c(ax-1)}{192x^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^4\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{363a^4\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{64\sqrt{c}}\right)c\sqrt{-c(ax+1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}$
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-768\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^4cx^4+1089c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^4x^4+447a^3x^3\sqrt{-c(ax+1)}\sqrt{c}+214a^2x}{192\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\sqrt{-c(ax+1)}x^4}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE
)
```

output
```
-1/192*(447*a^4*x^4+661*a^3*x^3+350*a^2*x^2+184*a*x+48)/x^4*c/(a*x+1)/((a*
x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-1)-(4*a^4*2^(1/2)/c^(1/2)*arct
an(1/2*(-a*c*x-c)^(1/2)*2^(1/2)/c^(1/2))-363/64*a^4/c^(1/2)*arctan((-a*c*x
-c)^(1/2)/c^(1/2)))*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x+1))^(1/2)/(
-c*(a*x-1))^(1/2)*(a*x-1)
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.43

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$

$$= \left[\frac{768\sqrt{2}(a^5x^5-a^4x^4)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+1089\,(a^5x^5-a^4x^4)\sqrt{-c}\,\mathrm{lo}}{38}\right.$$

$$-\frac{768\sqrt{2}(a^5x^5-a^4x^4)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-1089\,(a^5x^5-a^4x^4)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right.}{192\,(ax^5-x^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fri
cas")
```

---

output
```
[1/384*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
 - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 1089*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a
^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1)) - 2*c)/(a*x^2 - x)) + 2*(447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2
+ 184*a*x + 48)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4),
 -1/192*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*
x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 1089*(a^5*x^5 - a^
4*x^4)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(
a*c*x - c)) - (447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184*a*x + 48)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

3.318.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**5,x)
```

output
```
Timed out
```

3.318.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-acx+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.318.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.319 $\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx$

3.319.1 Optimal result

Integrand size = 13, antiderivative size = 144

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\frac{46\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}}+\frac{92\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}x}+\frac{8\sqrt{-\frac{1-x}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}+\frac{2\sqrt{-\frac{1-x}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

output
```
46/21*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)+92/21*(1+x)^(3/2)*((-1+x)
/x)^(1/2)/(1+1/x)^(3/2)/x+8/7*x*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)
+2/7*x^2*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)
```

3.319.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(46+23x+12x^2+3x^3)}{21\sqrt{1+\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

output
```
(2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(46 + 23*x + 12*x^2 + 3*x^3))/(21*Sqrt[1 +
 x^(-1)])
```

3.319.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6730, 100, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)^{3/2}e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\int \frac{\left(1+\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\left(\frac{2}{7}\int \frac{20+\frac{7}{x}}{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\left(\frac{1}{7}\int \frac{20+\frac{7}{x}}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\left(\frac{1}{7}\left(23\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{8\sqrt{1-\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\left(\frac{1}{7}\left(23\left(\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{8\sqrt{1-\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{7}\left(23\left(-\frac{4\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{8\sqrt{1-\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}{\left(\frac{1}{x}+1\right)^{3/2}}$$

input

```
Int[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

output

```
-((((23*((-2*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (4*Sqrt[1 - x^(-1)])/(
3*Sqrt[x^(-1)])) - (8*Sqrt[1 - x^(-1)])/(x^(-1))^(5/2))/7 - (2*Sqrt[1 - x^
(-1)])/(7*(x^(-1))^(7/2)))*(x^(-1))^(3/2)*(1 + x)^(3/2))/(1 + x^(-1))^(3/2
))
```

3.319.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.319.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.26

method	result	size
gosper	$\frac{2(x-1)\left(3x^{3}+12x^{2}+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37
default	$\frac{2(x-1)\left(3x^{3}+12x^{2}+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37
risch	$\frac{2(x-1)\left(3x^{3}+12x^{2}+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*(x-1)*(3*x^3+12*x^2+23*x+46)/(1+x)^(1/2)/((x-1)/(1+x))^(1/2)
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.23

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\frac{2}{21}\left(3\,x^3+12\,x^2+23\,x+46\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(3*x^3 + 12*x^2 + 23*x + 46)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

3.319.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\int\frac{x(x+1)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(3/2),x)
```

output
```
Integral(x*(x + 1)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.19

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\frac{2\left(3\,x^4+9\,x^3+11\,x^2+23\,x-46\right)}{21\,\sqrt{x-1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
2/21*(3*x^4 + 9*x^3 + 11*x^2 + 23*x - 46)/sqrt(x - 1)
```

3.319.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.319.9 Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.33

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \sqrt{\frac{x-1}{x+1}}\left(\frac{46\,x\,\sqrt{x+1}}{21}+\frac{92\,\sqrt{x+1}}{21}+\frac{8\,x^2\,\sqrt{x+1}}{7}+\frac{2\,x^3\,\sqrt{x+1}}{7}\right)$$

input
```
int((x*(x + 1)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
((x - 1)/(x + 1))^(1/2)*((46*x*(x + 1)^(1/2))/21 + (92*(x + 1)^(1/2))/21 +
 (8*x^2*(x + 1)^(1/2))/7 + (2*x^3*(x + 1)^(1/2))/7)
```

3.320 $\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx$

3.320.1 Optimal result

Integrand size = 12, antiderivative size = 107

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{28\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}x}+\frac{2\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

output
```
28/15*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)+86/15*(1+x)^(3/2)*((-1+x)
/x)^(1/2)/(1+1/x)^(3/2)/x+2/5*x*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)
```

3.320.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}\left(43+14x+3x^2\right)}{15\sqrt{1+\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

output
```
(2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(43 + 14*x + 3*x^2))/(15*Sqrt[1 + x^(-1)])
```

3.320.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6727, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^{3/2} e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2} (x+1)^{3/2} \int \frac{\left(1+\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2} (x+1)^{3/2} \left(\frac{2}{5}\int \frac{14+\frac{5}{x}}{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2} (x+1)^{3/2} \left(\frac{1}{5}\int \frac{14+\frac{5}{x}}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2} (x+1)^{3/2} \left(\frac{1}{5}\left(\frac{43}{3}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{28\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{5}\left(-\frac{86\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{28\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}{\left(\frac{1}{x}+1\right)^{3/2}}$$

input
```
Int[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

output
```
-(((((-28*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (86*Sqrt[1 - x^(-1)])/(3*
Sqrt[x^(-1)]))/5 - (2*Sqrt[1 - x^(-1)])/(5*(x^(-1))^(5/2)))*(x^(-1))^(3/2)
*(1 + x)^(3/2))/(1 + x^(-1))^(3/2))
```

3.320.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.320.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

method	result	size
gosper	$\frac{2(x-1)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32
default	$\frac{2(x-1)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32
risch	$\frac{2(x-1)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(x-1)*(3*x^2+14*x+43)/(1+x)^(1/2)/((x-1)/(1+x))^(1/2)
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.26

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \frac{2}{15}\left(3\,x^2+14\,x+43\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*x^2 + 14*x + 43)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

3.320.6 Sympy [A] (verification not implemented)

Time = 42.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = 2\left(\begin{cases} 4\sqrt{2}\left(\frac{\sqrt{2}(x-1)^{\frac{5}{2}}}{40}+\frac{\sqrt{2}(x-1)^{\frac{3}{2}}}{6}+\frac{\sqrt{2}\sqrt{x-1}}{2}\right) & \text{for } \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2} \end{cases}\right)$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(3/2),x)
```

output
```
2*Piecewise((4*sqrt(2)*(sqrt(2)*(x - 1)**(5/2)/40 + sqrt(2)*(x - 1)**(3/2)
/6 + sqrt(2)*sqrt(x - 1)/2), (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqr
t(2))))
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{2\,(3\,x^{3}+11\,x^{2}+29\,x-43)}{15\,\sqrt{x-1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(3*x^3 + 11*x^2 + 29*x - 43)/sqrt(x - 1)
```

3.320.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.320.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.36

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\sqrt{\frac{x-1}{x+1}}\left(\frac{28\,x\,\sqrt{x+1}}{15}+\frac{86\,\sqrt{x+1}}{15}+\frac{2\,x^{2}\,\sqrt{x+1}}{5}\right)$$

input
```
int((x + 1)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
((x - 1)/(x + 1))^(1/2)*((28*x*(x + 1)^(1/2))/15 + (86*(x + 1)^(1/2))/15 +
 (2*x^2*(x + 1)^(1/2))/5)
```

3.320. $\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx$

3.321 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx$

3.321.1 Optimal result

Integrand size = 15, antiderivative size = 104

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx=\frac{44\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

output

```
44/105*(1+1/x)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)-22/35*(1+1/x)^(3/2)*(1-x)^(
3/2)*x/(1-1/x)^(3/2)+2/7*(1+1/x)^(3/2)*(1-x)^(3/2)*x^2/(1-1/x)^(3/2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx=-\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(22-11x-18x^2+15x^3)}{105\sqrt{\frac{-1+x}{x}}}$$

input

```
Integrate[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

output

```
(-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(22 - 11*x - 18*x^2 + 15*x^3))/(105*Sqrt[
(-1 + x)/x])
```

3.321.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6730, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{3/2} x e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\int \frac{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\left(-\frac{11}{7}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\left(-\frac{11}{7}\left(-\frac{2}{5}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{11}{7}\left(\frac{4\left(\frac{1}{x}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\right)(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{\left(1-\frac{1}{x}\right)^{3/2}}$$

input

```
Int[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

output

```
-((((-11*((-2*(1 + x^(-1))^(3/2))/(5*(x^(-1))^(5/2)) + (4*(1 + x^(-1))^(3/
2))/(15*(x^(-1))^(3/2))))/7 - (2*(1 + x^(-1))^(3/2))/(7*(x^(-1))^(7/2)))*(
1 - x)^(3/2)*(x^(-1))^(3/2))/(1 - x^(-1))^(3/2))
```

3.321.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.321.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.33

method	result	size
gosper	$-\frac{2(1+x)\sqrt{1-x}\,(15x^{2}-33x+22)}{105\sqrt{\frac{x-1}{1+x}}}$	34
default	$-\frac{2(1+x)\sqrt{1-x}\,(15x^{2}-33x+22)}{105\sqrt{\frac{x-1}{1+x}}}$	34
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)(15x^{3}-18x^{2}-11x+22)}{105\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	62

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
-2/105*(1+x)*(1-x)^(1/2)*(15*x^2-33*x+22)/((x-1)/(1+x))^(1/2)
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.43

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = -\frac{2\,(15\,x^4-3\,x^3-29\,x^2+11\,x+22)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{105\,(x-1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="fricas")
```

output
```
-2/105*(15*x^4 - 3*x^3 - 29*x^2 + 11*x + 22)*sqrt(-x + 1)*sqrt((x - 1)/(x
+ 1))/(x - 1)
```

3.321.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \text{Timed out}$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2)*x,x)
```

output
```
Timed out
```

3.321.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{2}{105}\left(-15i\,x^3+18i\,x^2+11i\,x-22i\right)\sqrt{x+1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="maxima")
```

output
```
2/105*(-15*I*x^3 + 18*I*x^2 + 11*I*x - 22*I)*sqrt(x + 1)
```

3.321.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{16}{105}i\sqrt{2}\mathrm{sgn}(x+1) - \frac{2\left(15\,(x+1)^3\sqrt{-x-1}-63\,(x+1)^2\sqrt{-x-1}-70\,(-x-1)^{\frac{3}{2}}-8i\sqrt{2}\right)\mathrm{sgn}(x)}{105\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="giac")
```

output
```
16/105*I*sqrt(2)*sgn(x + 1) - 2/105*(15*(x + 1)^3*sqrt(-x - 1) - 63*(x + 1
)^2*sqrt(-x - 1) - 70*(-x - 1)^(3/2) - 8*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

3.321.9 Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2\,(15\,x^2-33\,x+22)}{105\,\sqrt{1-x}}$$

input
```
int((x*(1 - x)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2*(15*x^2 - 33*x + 22))/(105*(1 - x)^(1
/2))
```

3.322 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx$

3.322.1 Optimal result

Integrand size = 14, antiderivative size = 68

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=-\frac{14\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}$$

output
```
-14/15*(1+1/x)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)+2/5*(1+1/x)^(3/2)*(1-x)^(3/
2)*x/(1-1/x)^(3/2)
```

3.322.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=-\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(-7-4x+3x^2)}{15\sqrt{\frac{-1+x}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

output
```
(-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(-7 - 4*x + 3*x^2))/(15*Sqrt[(-1 + x)/x])
```

3.322.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6727, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{3/2}e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\int \frac{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\left(-\frac{7}{5}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{14\left(\frac{1}{x}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{\left(1-\frac{1}{x}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

output
```
-((((-2*(1 + x^(-1))^(3/2))/(5*(x^(-1))^(5/2)) + (14*(1 + x^(-1))^(3/2))/(
15*(x^(-1))^(3/2)))*(1 - x)^(3/2)*(x^(-1))^(3/2))/(1 - x^(-1))^(3/2))
```

3.322.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.322.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2(1+x)(3x-7)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
default	$-\frac{2(1+x)(3x-7)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)\left(3x^{2}-4x-7\right)}{15\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	57

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(1+x)*(3*x-7)*(1-x)^(1/2)/((x-1)/(1+x))^(1/2)
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=-\frac{2\,(3\,x^3-x^2-11\,x-7)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*x^3 - x^2 - 11*x - 7)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

3.322.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=\int\frac{(1-x)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2),x)
```

output
```
Integral((1 - x)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

3.322.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=\frac{2}{15}\left(-3i\,x^2+4i\,x+7i\right)\sqrt{x+1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(-3*I*x^2 + 4*I*x + 7*I)*sqrt(x + 1)
```

3.322.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = -\frac{16}{15}i\sqrt{2}\mathrm{sgn}(x+1) - \frac{2\left(3\,(x+1)^2\sqrt{-x-1}+10\,(-x-1)^{\frac{3}{2}}+8i\sqrt{2}\right)\mathrm{sgn}(x)}{15\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="giac")
```

output
```
-16/15*I*sqrt(2)*sgn(x + 1) - 2/15*(3*(x + 1)^2*sqrt(-x - 1) + 10*(-x - 1)
^(3/2) + 8*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

3.322.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = \frac{2\,(3\,x-7)\,\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{15\,\sqrt{1-x}}$$

input
```
int((1 - x)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(2*(3*x - 7)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

3.323 $\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx$

3.323.1 Optimal result

Integrand size = 13, antiderivative size = 107

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{12\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{6\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}$$

output
```
12/5*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)+6/5*x*((-1+x)/x)^(1/2)*(1+
x)^(1/2)/(1+1/x)^(1/2)+2/5*x^2*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)
```

3.323.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.36

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(6+3x+x^2)}{5\sqrt{1+\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

output
```
(2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(6 + 3*x + x^2))/(5*Sqrt[1 + x^(-1)])
```

3.323.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6730, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x+1}e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\int \frac{1+\frac{1}{x}}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\left(\frac{9}{5}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\left(\frac{9}{5}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(\frac{9}{5}\left(-\frac{4\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\sqrt{\frac{1}{x}}\sqrt{x+1}}{\sqrt{\frac{1}{x}+1}}$$

input

```
Int[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

output

```
-((((9*((-2*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (4*Sqrt[1 - x^(-1)])/(3
*Sqrt[x^(-1)])))/5 - (2*Sqrt[1 - x^(-1)])/(5*(x^(-1))^(5/2)))*Sqrt[x^(-1)]
*Sqrt[1 + x])/Sqrt[1 + x^(-1)])
```

3.323.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.323.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2(x-1)\left(x^{2}+3x+6\right)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30
default	$\frac{2(x-1)\left(x^{2}+3x+6\right)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30
risch	$\frac{2(x-1)\left(x^{2}+3x+6\right)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(x-1)*(x^2+3*x+6)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

3.323.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.24

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{2}{5}\left(x^2+3\,x+6\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(x^2 + 3*x + 6)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

3.323.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\int\frac{x\sqrt{x+1}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(1/2),x)
```

output
```
Integral(x*sqrt(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.19

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{2\left(x^3+2\,x^2+3\,x-6\right)}{5\,\sqrt{x-1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(x^3 + 2*x^2 + 3*x - 6)/sqrt(x - 1)
```

3.323.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.323.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.36

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \sqrt{\frac{x-1}{x+1}}\left(\frac{6\,x\,\sqrt{x+1}}{5}+\frac{12\,\sqrt{x+1}}{5}+\frac{2\,x^2\,\sqrt{x+1}}{5}\right)$$

input
```
int((x*(x + 1)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
((x - 1)/(x + 1))^(1/2)*((6*x*(x + 1)^(1/2))/5 + (12*(x + 1)^(1/2))/5 + (2
*x^2*(x + 1)^(1/2))/5)
```

3.324 $\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx$

3.324.1 Optimal result

Integrand size = 12, antiderivative size = 70

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}$$

output
```
10/3*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)+2/3*x*((-1+x)/x)^(1/2)*(1+
x)^(1/2)/(1+1/x)^(1/2)
```

3.324.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(5+x)}{3\sqrt{1+\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*Sqrt[1 + x],x]
```

output
```
(2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(5 + x))/(3*Sqrt[1 + x^(-1)])
```

3.324.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6727, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+1}e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\int \frac{1+\frac{1}{x}}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\left(\frac{5}{3}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{10\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{\frac{1}{x}}\sqrt{x+1}}{\sqrt{\frac{1}{x}+1}}$$

input

```
Int[E^ArcCoth[x]*Sqrt[1 + x],x]
```

output

```
-((((-2*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (10*Sqrt[1 - x^(-1)])/(3*Sq
rt[x^(-1)]))*Sqrt[x^(-1)]*Sqrt[1 + x])/Sqrt[1 + x^(-1)])
```

3.324.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.324.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

method	result	size
gosper	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
default	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
risch	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x-1)*(x+5)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{2}{3}\,(x+5)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x + 5)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

3.324.6 Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx$$
$$= 2\left(\begin{cases} 2\sqrt{2}\left(\frac{\sqrt{2}(x-1)^{\frac{3}{2}}}{12} + \frac{\sqrt{2}\sqrt{x-1}}{2}\right) & \text{for } \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right)$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(1/2),x)
```

output
```
2*Piecewise((2*sqrt(2)*(sqrt(2)*(x - 1)**(3/2)/12 + sqrt(2)*sqrt(x - 1)/2)
, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2))))
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{2\,(x^2+4\,x-5)}{3\,\sqrt{x-1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x^2 + 4*x - 5)/sqrt(x - 1)
```

3.324.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.324.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}\,(x+5)}{3}$$

input
```
int((x + 1)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 5))/3
```

3.325 $\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx$

3.325.1 Optimal result

Integrand size = 15, antiderivative size = 71

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=-\frac{4\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x}{15\sqrt{1-\frac{1}{x}}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}$$

output
```
-4/15*(1+1/x)^(3/2)*x*(1-x)^(1/2)/(1-1/x)^(1/2)+2/5*(1+1/x)^(3/2)*x^2*(1-x
)^(1/2)/(1-1/x)^(1/2)
```

3.325.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(-2+x+3x^2)}{15\sqrt{\frac{-1+x}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

output
```
(2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(-2 + x + 3*x^2))/(15*Sqrt[(-1 + x)/x])
```

3.325.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6730, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}\,x e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{1-x}\sqrt{\frac{1}{x}}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{x}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{1-x}\sqrt{\frac{1}{x}}\left(-\frac{2}{5}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{x}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{4\left(\frac{1}{x}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\sqrt{1-x}\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}$$

input
```
Int[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

output
```
-((((-2*(1 + x^(-1))^(3/2))/(5*(x^(-1))^(5/2)) + (4*(1 + x^(-1))^(3/2))/(1
5*(x^(-1))^(3/2)))*Sqrt[1 - x]*Sqrt[x^(-1)])/Sqrt[1 - x^(-1)])
```

3.325.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.325.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{2(1+x)(3x-2)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
default	$\frac{2(1+x)(3x-2)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
risch	$-\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)\left(3x^{2}+x-2\right)}{15\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	55

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+x)*(3*x-2)*(1-x)^(1/2)/((x-1)/(1+x))^(1/2)
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = \frac{2\,(3\,x^3+4\,x^2-x-2)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*x^3 + 4*x^2 - x - 2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

3.325.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = \int \frac{x\sqrt{1-x}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x*(1-x)**(1/2),x)
```

output
```
Integral(x*sqrt(1 - x)/sqrt((x - 1)/(x + 1)), x)
```

3.325.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.24

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = -\frac{2}{15}\left(-3i\,x^2-i\,x+2i\right)\sqrt{x+1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(-3*I*x^2 - I*x + 2*I)*sqrt(x + 1)
```

3.325.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = -\frac{4}{15}i\,\sqrt{2}\mathrm{sgn}(x+1) + \frac{2\left(3\,(x+1)^2\sqrt{-x-1}+5\,(-x-1)^{\frac{3}{2}}-2i\,\sqrt{2}\right)\mathrm{sgn}(x)}{15\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="giac")
```

output
```
-4/15*I*sqrt(2)*sgn(x + 1) + 2/15*(3*(x + 1)^2*sqrt(-x - 1) + 5*(-x - 1)^(
3/2) - 2*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

3.325.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = -\frac{2\,(3\,x-2)\,\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{15\,\sqrt{1-x}}$$

input
```
int((x*(1 - x)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
-(2*(3*x - 2)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

3.326 $\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx$

3.326.1 Optimal result

Integrand size = 14, antiderivative size = 20

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\frac{2}{3}e^{\coth^{-1}(x)}\sqrt{1-x}(1+x)$$

output
```
2/3/((-1+x)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

3.326.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x}{3\sqrt{1-\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*Sqrt[1 - x],x]
```

output
```
(2*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(3*Sqrt[1 - x^(-1)])
```

3.326.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow 6726$$

$$\frac{2}{3}\sqrt{1-x}(x+1)e^{\coth^{-1}(x)}$$

input
```
Int[E^ArcCoth[x]*Sqrt[1 - x],x]
```

output
```
(2*E^ArcCoth[x]*Sqrt[1 - x]*(1 + x))/3
```

3.326.3.1 Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

3.326.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

method	result	size
gosper	$\frac{2(1+x)\sqrt{1-x}}{3\sqrt{\frac{x-1}{1+x}}}$	24
default	$\frac{2(1+x)\sqrt{1-x}}{3\sqrt{\frac{x-1}{1+x}}}$	24
risch	$-\frac{2(1+x)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	50

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((x-1)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = \frac{2\,(x^2+2\,x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{3\,(x-1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^2 + 2*x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

3.326.6 Sympy [F]

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(1/2),x)
```

output
```
Integral(sqrt(1 - x)/sqrt((x - 1)/(x + 1)), x)
```

3.326.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = -\frac{2}{3}\,\sqrt{x+1}(-i\,x-i)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(x + 1)*(-I*x - I)
```

3.326.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = -\frac{4}{3}i\sqrt{2}\text{sgn}(x+1) - \frac{2\left((-x-1)^{\frac{3}{2}} + 2i\sqrt{2}\right)\text{sgn}(x)}{3\,\text{sgn}\,(x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="giac")
```

output
```
-4/3*I*sqrt(2)*sgn(x + 1) - 2/3*((-x - 1)^(3/2) + 2*I*sqrt(2))*sgn(x)/sgn(
x + 1)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{3\sqrt{1-x}}$$

input
```
int((1 - x)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
-(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(3*(1 - x)^(1/2))
```

3.327 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx$

3.327.1 Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx=\frac{4\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x}{3\sqrt{1+x}}+\frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x^2}{3\sqrt{1+x}}$$

output
```
4/3*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)/(1+x)^(1/2)+2/3*x^2*(1+1/x)^(1/2)*((-
1+x)/x)^(1/2)/(1+x)^(1/2)
```

3.327.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx=\frac{2\sqrt{1-\frac{1}{x^2}}x(2+x)}{3\sqrt{1+x}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

output
```
(2*Sqrt[1 - x^(-2)]*x*(2 + x))/(3*Sqrt[1 + x])
```

3.327.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6730, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{x+1}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{\frac{1}{x}}\sqrt{x+1}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(-\frac{4\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}\sqrt{x+1}}$$

input
```
Int[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

output
```
-((((-2*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (4*Sqrt[1 - x^(-1)])/(3*Sqr
t[x^(-1)]))*Sqrt[1 + x^(-1)])/(Sqrt[x^(-1)]*Sqrt[1 + x]))
```

3.327.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.327.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

method	result	size
gosper	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
default	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
risch	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x-1)*(x+2)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

3.327. $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx$

3.327.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.29

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx=\frac{2}{3}\,(x+2)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x + 2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

3.327.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx=\begin{cases}\frac{2x\sqrt{x-1}}{3}+\frac{4\sqrt{x-1}}{3} & \text{for } |x|>1\\ \frac{2ix\sqrt{1-x}}{3}+\frac{4i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(1/2),x)
```

output
```
Piecewise((2*x*sqrt(x - 1)/3 + 4*sqrt(x - 1)/3, Abs(x) > 1), (2*I*x*sqrt(1
 - x)/3 + 4*I*sqrt(1 - x)/3, True))
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx=\frac{2\,(x^2+x-2)}{3\,\sqrt{x-1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x^2 + x - 2)/sqrt(x - 1)
```

3.327.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.327.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.29

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}\,(x+2)}{3}$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
```

output
```
(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 2))/3
```

3.328 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} dx$

3.328.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} dx = \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}x}{\sqrt{1+x}}$$

output
```
2*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)/(1+x)^(1/2)
```

3.328.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} dx = \frac{2\sqrt{1-\frac{1}{x^2}}x}{\sqrt{1+x}}$$

input
```
Integrate[E^ArcCoth[x]/Sqrt[1 + x],x]
```

output
```
(2*Sqrt[1 - x^(-2)]*x)/Sqrt[1 + x]
```

3.328.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6727, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{x+1}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x}{\sqrt{x+1}}$$

input
```
Int[E^ArcCoth[x]/Sqrt[1 + x],x]
```

output
```
(2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 + x]
```

3.328.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.328.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22
default	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22
risch	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x-1)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

3.328.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = \begin{cases} 2\sqrt{x-1} & \text{for } |x| > 1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x), True))
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.21

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{x-1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x - 1)
```

3.328.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = -\frac{2\left(i\sqrt{2}-\sqrt{x-1}\right)\text{sgn}(x)}{\text{sgn}\left(x+1\right)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2*(I*sqrt(2) - sqrt(x - 1))*sgn(x)/sgn(x + 1)
```

3.328.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
```

output
```
2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)
```

3.329 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx$

3.329.1 Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

output
```
2/3*(1+1/x)^(3/2)*x^2*(1-1/x)^(1/2)/(1-x)^(1/2)+2*x*(1-1/x)^(1/2)*(1+1/x)^
(1/2)/(1-x)^(1/2)-2*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))*2^(1/2)*(1-
1/x)^(1/2)/(1-x)^(1/2)/(1/x)^(1/2)
```

3.329.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \frac{2\sqrt{\frac{-1+x}{x}}\,x\left(\sqrt{1+\frac{1}{x}}(4+x)-3\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{3\sqrt{1-x}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

output
```
(2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)]*(4 + x) - 3*Sqrt[2]*Sqrt[x^(-1)]*A
rcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(3*Sqrt[1 - x])
```

3.329.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6730, 107, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x e^{\coth^{-1}(x)}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(2\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(4\int \frac{1}{1-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

input

```
Int[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

output
```
-((Sqrt[1 - x^(-1)]*((-2*(1 + x^(-1))^(3/2))/(3*(x^(-1))^(3/2)) - (2*Sqrt[
1 + x^(-1)])/Sqrt[x^(-1)] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[
1 + x^(-1)]]))/(Sqrt[1 - x]*Sqrt[x^(-1)]))
```

3.329.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.329. $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx$

3.329.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{2\sqrt{1-x}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-\sqrt{-1-x}\,x-4\sqrt{-1-x}\right)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{-1-x}}$	66
risch	$\frac{2(x+4)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{3\sqrt{-1-x}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	111

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*(3*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(
1/2))-(-1-x)^(1/2)*x-4*(-1-x)^(1/2))/(-1-x)^(1/2)
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx=\frac{2\left(3\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-(x^2+5x+4)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{3(x-1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(
x - 1)) - (x^2 + 5*x + 4)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

3.329.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx=\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\sqrt{1-x}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*sqrt(1 - x)), x)
```

3.329.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \int \frac{x}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

3.329.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: (-12
*atan(i)+20*i)*1/3/sqrt(2)*sign(sageVARx+1)-(-2/3*sqrt(-sageVARx-1)*(-sage
VARx-1)+2*sqrt(-sageVARx-1)+1/3*(12*atan(i)-20*i)/sqrt(2)-4*atan(sqrt(-sag
eVARx-1)/sqrt(2))/s
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

output
```
int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

3.330 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx$

3.330.1 Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

output
```
2*x*(1-1/x)^(1/2)*(1+1/x)^(1/2)/(1-x)^(1/2)-2*arctanh(2^(1/2)*(1/x)^(1/2)/
(1+1/x)^(1/2))*2^(1/2)*(1-1/x)^(1/2)/(1-x)^(1/2)/(1/x)^(1/2)
```

3.330.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{2\sqrt{\frac{-1+x}{x}}x\left(\sqrt{1+\frac{1}{x}}-\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{\sqrt{1-x}}$$

input
```
Integrate[E^ArcCoth[x]/Sqrt[1 - x],x]
```

output
```
(2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)] - Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[Sqr
t[2]*Sqrt[(1 + x)^(-1)]]))/Sqrt[1 - x]
```

3.330.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6727, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(2\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(4\int \frac{1}{1-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

input

```
Int[E^ArcCoth[x]/Sqrt[1 - x],x]
```

output

```
-((Sqrt[1 - x^(-1)]*((-2*Sqrt[1 + x^(-1)])/Sqrt[x^(-1)] + 2*Sqrt[2]*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]]))/(Sqrt[1 - x]*Sqrt[x^(-1)]))
```

3.330.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.330.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{2\sqrt{1-x}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-\sqrt{-1-x}\right)}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{-1-x}}$	55
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{-1-x}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	108

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2)
)-(-1-x)^(1/2))/(-1-x)^(1/2)
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{2\left(\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{x-1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x -
1)) - (x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

3.330.6 Sympy [A] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = -2\left(\begin{cases}\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-x-1}}{2}-\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)\right) & \text{for } \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(1/2),x)
```

output
```
-2*Piecewise((sqrt(2)*(sqrt(2)*sqrt(-x - 1)/2 - acos(sqrt(2)/sqrt(1 - x)))
, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2))))
```

3.330.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \int \frac{1}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

3.330.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: (-4*
atan(i)+4*i)/sqrt(2)*sign(sageVARx+1)-(2*sqrt(-sageVARx-1)+(4*atan(i)-4*i)
/sqrt(2)-4*atan(sqrt(-sageVARx-1)/sqrt(2))/sqrt(2))*sign(sageVARx)/sign(sa
geVARx+1)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

output
```
int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

3.331 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx$

3.331.1 Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}} + \frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

output
```
(1+1/x)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/((-1+x)/x)^(1/2))*2^(1/2)/(1/x)^(
3/2)/(1+x)^(3/2)+2*(1+1/x)^(3/2)*x^2*((-1+x)/x)^(1/2)/(1+x)^(3/2)
```

3.331.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \frac{\sqrt{1+\frac{1}{x}}\,x\left(2\sqrt{\frac{-1+x}{x}}-\sqrt{2}\sqrt{\frac{1}{x}}\arctan\left(\frac{\sqrt{\frac{-1+x}{x^2}}\,x}{\sqrt{2}}\right)\right)}{\sqrt{1+x}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

output
```
(Sqrt[1 + x^(-1)]*x*(2*Sqrt[(-1 + x)/x] - Sqrt[2]*Sqrt[x^(-1)]*ArcTan[(Sqr
t[(-1 + x)/x^2]*x)/Sqrt[2]]))/Sqrt[1 + x]
```

3.331.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6730, 107, 104, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x e^{\coth^{-1}(x)}}{(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\left(-\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\left(-2\int \frac{1}{1+\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\left(-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

input

```
Int[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

output

```
-(((1 + x^(-1))^(3/2)*((-2*Sqrt[1 - x^(-1)])/Sqrt[x^(-1)] - Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 - x^(-1)]]))/((x^(-1))^(3/2)*(1 + x)^(3/2))
)
```

3.331.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.331.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\sqrt{x-1}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)-2\sqrt{x-1}\right)}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	47
risch	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)\sqrt{x-1}}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	60

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(x-1)^(1/2)*(2^(1/2)*arctan(1/2*(x-1)^(1/2)*2^(1/2))-2*(x-1)^(1/2))/((x-1
)/(1+x))^(1/2)/(1+x)^(1/2)
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = -\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)+2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))) + 2*sqrt(x
+ 1)*sqrt((x - 1)/(x + 1))
```

3.331.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(3/2),x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**(3/2)), x)
```

3.331.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

3.331.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*(s
qrt(sageVARx-1)+(atan(i)-2*i)/sqrt(2)-atan(sqrt(sageVARx-1)/sqrt(2))/sqrt(
2))*sign(sageVARx)/sign(sageVARx+1)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
```

output
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

3.332 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx$

3.332.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = -\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

output

```
-(1+1/x)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/((-1+x)/x)^(1/2))*2^(1/2)/(1/x)^
(3/2)/(1+x)^(3/2)
```

3.332.2 Mathematica [A] (warning: unable to verify)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \sqrt{2}\sqrt{\frac{1}{1+x}}\sqrt{1+x}\arctan\left(\frac{\sqrt{\frac{-1+x}{x^2}}\,x}{\sqrt{2}}\right)$$

input

```
Integrate[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

output

```
Sqrt[2]*Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcTan[(Sqrt[(-1 + x)/x^2]*x)/Sqrt[
2]]
```

3.332.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6727, 104, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{(x+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{2\left(\frac{1}{x}+1\right)^{3/2}\int \frac{1}{1+\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

input

```
Int[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

output

```
-((Sqrt[2]*(1 + x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 - x^(-1
)]])/((x^(-1))^(3/2)*(1 + x)^(3/2)))
```

3.332.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.332.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)\sqrt{x-1}}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	37

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctan(1/2*(x-1)^(1/2)*2^(1/2))/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)*(x
-1)^(1/2)
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1)))
```

3.332.6 Sympy [A] (verification not implemented)

Time = 40.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = 2\left(\begin{cases} \frac{\sqrt{2}\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} & \text{for}\ \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right)$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(3/2),x)
```

output
```
2*Piecewise((sqrt(2)*acos(sqrt(2)/sqrt(x + 1))/2, (sqrt(x + 1) < sqrt(2))
& (sqrt(x + 1) > -sqrt(2))))
```

3.332.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \int \frac{1}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

3.332.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*(-
1/2*sqrt(2)*atan(i)+1/2*sqrt(2)*atan(sqrt(sageVARx-1)/sqrt(2)))*sign(sageV
ARx)/sign(sageVARx+1)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
```

output
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

3.333 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx$

3.333.1 Optimal result

Integrand size = 15, antiderivative size = 130

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{5\left(1-\frac{1}{x}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

output
```
-5/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))/(1-x)^(3/2)/
(1/x)^(3/2)*2^(1/2)-1/2*(1+1/x)^(3/2)*x^2*(1-1/x)^(1/2)/(1-x)^(3/2)+5/2*(1
-1/x)^(3/2)*x^2*(1+1/x)^(1/2)/(1-x)^(3/2)
```

3.333.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{\frac{-1+x}{x}}x\left(2\sqrt{1+\frac{1}{x}}(3-2x)+5\sqrt{2}(-1+x)\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{2(1-x)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

output
```
-1/2*(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)]*(3 - 2*x) + 5*Sqrt[2]*(-1 + x
)*Sqrt[x^(-1)]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(1 - x)^(3/2)
```

3.333.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6730, 107, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{(1-x)^{3/2}}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{5}{4}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{\left(\frac{1}{x}+1\right)^{3/2}}{2\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{5}{4}\left(2\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)+\frac{\left(\frac{1}{x}+1\right)^{3/2}}{2\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{5}{4}\left(4\int \frac{1}{1-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)+\frac{\left(\frac{1}{x}+1\right)^{3/2}}{2\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{5}{4}\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)+\frac{\left(\frac{1}{x}+1\right)^{3/2}}{2\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

input

```
Int[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

output

```
-(((1 - x^(-1))^(3/2)*((1 + x^(-1))^(3/2)/(2*(1 - x^(-1))*Sqrt[x^(-1)]) +
(5*((-2*Sqrt[1 + x^(-1)])/Sqrt[x^(-1)] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/Sqrt[1 + x^(-1)]]))/4))/((1 - x)^(3/2)*(x^(-1))^(3/2)))
```

3.333.3.1 Defintions of rubi rules used

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.333.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{\sqrt{1-x}\,\left(5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)x-5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-4\sqrt{-1-x}\,x+6\sqrt{-1-x}\right)}{2\sqrt{\frac{x-1}{1+x}}\,(x-1)\sqrt{-1-x}}$	90
risch	$-\frac{(2x^2-x-3)\sqrt{\frac{(1+x)(1-x)}{x-1}}}{\sqrt{-1-x}\,\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	120

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((x-1)/(1+x))^(1/2)/(x-1)*(1-x)^(1/2)*(5*2^(1/2)*arctan(1/2*(-1-x)^(1
/2)*2^(1/2))*x-5*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))-4*(-1-x)^(1/2)*x
+6*(-1-x)^(1/2))/(-1-x)^(1/2)
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.65

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx =$$
$$-\frac{5\,\sqrt{2}(x^2-2\,x+1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-2\,(2\,x^2-x-3)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x^2-2\,x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(5*sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(
x + 1))/(x - 1)) - 2*(2*x^2 - x - 3)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(
x^2 - 2*x + 1)
```

3.333.6 Sympy [A] (verification not implemented)

Time = 101.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = 2\left(\begin{cases}\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-x-1}}{2}-\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)\right) & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right) - 2\left(\begin{cases}\frac{\sqrt{2}\left(\frac{\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)}{2}-\frac{\sqrt{2}\sqrt{1-\frac{2}{1-x}}}{2\sqrt{1-x}}\right)}{2} & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(3/2),x)
```

output
```
2*Piecewise((sqrt(2)*(sqrt(2)*sqrt(-x - 1)/2 - acos(sqrt(2)/sqrt(1 - x))),
 (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2)))) - 2*Piecewise((sqrt(
2)*(acos(sqrt(2)/sqrt(1 - x))/2 - sqrt(2)*sqrt(1 - 2/(1 - x))/(2*sqrt(1 -
x)))/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2))))
```

3.333.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \int \frac{x}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right) - 2\sqrt{-x-1} + \frac{\sqrt{-x-1}}{x-1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="giac")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) - 2*sqrt(-x - 1) + sqrt(-x -
1)/(x - 1)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

output
```
int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

3.334 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx$

3.334.1 Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

output

```
-1/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))/(1-x)^(3/2)/
(1/x)^(3/2)*2^(1/2)-x*(1-1/x)^(1/2)*(1+1/x)^(1/2)/(1-x)^(3/2)
```

3.334.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{\frac{-1+x}{x}}\,x\left(2\sqrt{1+\frac{1}{x}}+\sqrt{2}(-1+x)\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{2(1-x)^{3/2}}$$

input

```
Integrate[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

output

```
-1/2*(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)] + Sqrt[2]*(-1 + x)*Sqrt[x^(-1
)]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(1 - x)^(3/2)
```

3.334.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6727, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{\sqrt{\frac{1}{x}+1}\sqrt{\frac{1}{x}}}{1-\frac{1}{x}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\int \frac{1}{1-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}+\frac{\sqrt{\frac{1}{x}+1}\sqrt{\frac{1}{x}}}{1-\frac{1}{x}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}}+\frac{\sqrt{\frac{1}{x}+1}\sqrt{\frac{1}{x}}}{1-\frac{1}{x}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

output
```
-(((1 - x^(-1))^(3/2)*((Sqrt[1 + x^(-1)]*Sqrt[x^(-1)])/(1 - x^(-1)) + ArcT
anh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]]/Sqrt[2]))/((1 - x)^(3/2)*(x^(
-1))^(3/2)))
```

3.334.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.334.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{1-x}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right) x-\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)+2\sqrt{-1-x}\right)}{2\sqrt{\frac{x-1}{1+x}}\,\left(x-1\right)\sqrt{-1-x}}$	79
risch	$\frac{\sqrt{\frac{\left(1+x\right)\left(1-x\right)}{x-1}}}{\sqrt{-1-x}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{\left(1+x\right)\left(1-x\right)}{x-1}}\,\left(x-1\right)}{2\sqrt{\frac{x-1}{1+x}}\,\left(1+x\right)\sqrt{1-x}}$	104

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((x-1)/(1+x))^(1/2)/(x-1)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/2
)*2^(1/2))*x-2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))+2*(-1-x)^(1/2))/(-1-
x)^(1/2)
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{2}(x^2-2\,x+1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)+2\,(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x^2-2\,x+1)}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x
+ 1))/(x - 1)) + 2*(x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^2 - 2*x
+ 1)
```

3.334.6 Sympy [A] (verification not implemented)

Time = 97.85 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -2\left(\begin{cases}\frac{\sqrt{2}\left(\frac{\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)}{2}-\frac{\sqrt{2}\sqrt{1-\frac{2}{1-x}}}{2\sqrt{1-x}}\right)}{2} & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

input
```
integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(3/2),x)
```

output
```
-2*Piecewise((sqrt(2)*(acos(sqrt(2)/sqrt(1 - x))/2 - sqrt(2)*sqrt(1 - 2/(1
 - x))/(2*sqrt(1 - x)))/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(
2))))
```

3.334.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \int \frac{1}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right) + \frac{\sqrt{-x-1}}{x-1}$$

input
```
integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) + sqrt(-x - 1)/(x - 1)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

output
```
int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

3.335 $\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx$

3.335.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx &= \frac{2\sqrt{1+\frac{1}{ax}}x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} \\ &\quad - \frac{2(5+4m)x^m\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2}-m,-\frac{1}{ax}\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output
```
-2*(5+4*m)*x^m*hypergeom([1/2, -1/2-m],[1/2-m],-1/a/x)*(-a*c*x+c)^(1/2)/a/
(4*m^2+8*m+3)/(1-1/a/x)^(1/2)+2*x^(1+m)*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(
3+2*m)/(1-1/a/x)^(1/2)
```

3.335.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.78

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx \\ = \frac{2x^m\sqrt{c-acx}\left(a(1+2m)\sqrt{1+\frac{1}{ax}}x-(5+4m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2}-m,-\frac{1}{ax}\right)\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

input
```
Integrate[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
(2*x^m*Sqrt[c - a*c*x]*(a*(1 + 2*m)*Sqrt[1 + 1/(a*x)]*x - (5 + 4*m)*Hyperg
eometric2F1[1/2, -1/2 - m, 1/2 - m, -(1/(a*x))]))/(a*(1 + 2*m)*(3 + 2*m)*S
qrt[1 - 1/(a*x)])
```

3.335.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6730, 27, 88, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-acx}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}} x^m \sqrt{c-acx} \int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}} x^m \sqrt{c-acx} \int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{88}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}} x^m \sqrt{c-acx} \left(-\frac{(4m+5)\int \frac{\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2m+3}-\frac{2a\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}}{2m+3}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}} x^m \sqrt{c-acx} \left(\frac{2(4m+5)\left(\frac{1}{x}\right)^{-m-\frac{1}{2}} \text{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{1}{2}-m,-\frac{1}{ax}\right)}{(2m+1)(2m+3)}-\frac{2a\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}}{2m+3}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
-(((x^(-1))^(1/2 + m)*x^m*Sqrt[c - a*c*x]*((-2*a*Sqrt[1 + 1/(a*x)]*(x^(-1)
)^(-3/2 - m))/(3 + 2*m) + (2*(5 + 4*m)*(x^(-1))^(-1/2 - m)*Hypergeometric2
F1[1/2, -1/2 - m, 1/2 - m, -(1/(a*x))])/((1 + 2*m)*(3 + 2*m))))/(a*Sqrt[1
 - 1/(a*x)]))
```

3.335.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.335. $\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-acx}\,dx$

3.335.4 Maple [F]

$$\int x^m\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

3.335.5 Fricas [F]

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int\sqrt{-acx+c}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.335.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.335.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int \sqrt{-acx+c}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.335.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int \sqrt{-acx+c}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\int x^m\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.336 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.336.1 Optimal result

Integrand size = 23, antiderivative size = 142

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{152c\sqrt{1-\frac{1}{a^2x^2}}x}{105a^2\sqrt{c-acx}}+\frac{38\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}}{105a^2}+\frac{6\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}}{35a^2c}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}x^2(c-acx)^{3/2}}{7ac}$$

output
```
6/35*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)/a^2/c-2/7*x^2*(-a*c*x+c)^(3/2)
*(1-1/a^2/x^2)^(1/2)/a/c+152/105*c*x*(1-1/a^2/x^2)^(1/2)/a^2/(-a*c*x+c)^(1
/2)+38/105*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)/a^2
```

3.336.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(-104+52ax-39a^2x^2+15a^3x^3)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-104 + 52*a*x - 39*a^2*x^2 + 15*a^3*
x^3))/(105*a^3*Sqrt[1 - 1/(a*x)])
```

3.336.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6730, 27, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{13}{7}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{13}{7}\left(-\frac{4\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{13}{7}\left(-\frac{4\left(-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(-\frac{2a\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{13}{7}\left(-\frac{4\left(\frac{4\sqrt{\frac{1}{ax}+1}}{3a\sqrt{\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
-((((-13*((-4*((-2*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (4*Sqrt[1 + 1/(
a*x)])/(3*a*Sqrt[x^(-1)])))/(5*a) - (2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5/2
))))/7 - (2*a*Sqrt[1 + 1/(a*x)])/(7*(x^(-1))^(7/2)))*Sqrt[x^(-1)]*Sqrt[c -
 a*c*x])/(a*Sqrt[1 - 1/(a*x)]))
```

3.336.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

3.336. $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6730

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.336.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(15a^3x^3-39a^2x^2+52ax-104)(ax+1)}{105\sqrt{-c(ax-1)}\,a^3}$	59
gosper	$\frac{2(ax+1)(15a^3x^3-39a^2x^2+52ax-104)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105a^3(ax-1)}$	64
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(15a^3x^3-39a^2x^2+52ax-104)}{105(ax-1)a^3}$	65

input

```
int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/105*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(15*a^3*x^3-39*a^2*x^2
+52*a*x-104)/a^3*(a*x+1)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\left(15\,a^4x^4-24\,a^3x^3+13\,a^2x^2-52\,ax-104\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^4x-a^3\right)}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
2/105*(15*a^4*x^4 - 24*a^3*x^3 + 13*a^2*x^2 - 52*a*x - 104)*sqrt(-a*c*x +
c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

3.336.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int x^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(ax-1\right)}\,dx$$

input
```
integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\left(15\,a^4\sqrt{-c}x^4-24\,a^3\sqrt{-c}x^3+13\,a^2\sqrt{-c}x^2-52\,a\sqrt{-c}x-104\,\sqrt{-c}\right)\left(ax-1\right)}{105\left(a^4x-a^3\right)\sqrt{ax+1}}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
2/105*(15*a^4*sqrt(-c)*x^4 - 24*a^3*sqrt(-c)*x^3 + 13*a^2*sqrt(-c)*x^2 - 5
2*a*sqrt(-c)*x - 104*sqrt(-c))*(a*x - 1)/((a^4*x - a^3)*sqrt(a*x + 1))
```

3.336.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.336.9 Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^3\,x^3-9\,a^2\,x^2+4\,a\,x-48\right)}{105\,a^3} - \frac{304\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,\left(a\,x-1\right)}$$

input
```
int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x - 9*a^2*x^2 + 15*a
^3*x^3 - 48))/(105*a^3) - (304*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/
2))/(105*a^3*(a*x - 1))
```

3.337 $\int e^{-\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

3.337.1 Optimal result

Integrand size = 21, antiderivative size = 104

$$\int e^{-\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = -\frac{8c\sqrt{1-\frac{1}{a^2x^2}}x}{5a\sqrt{c-acx}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}}{5a} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}}{5ac}$$

output
```
-2/5*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)/a/c-8/5*c*x*(1-1/a^2/x^2)^(1/2
)/a/(-a*c*x+c)^(1/2)-2/5*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)/a
```

3.337.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.56

$$\int e^{-\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(6-3ax+a^2x^2)}{5a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(6 - 3*a*x + a^2*x^2))/(5*a^2*Sqrt[1
- 1/(a*x)])
```

3.337.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6730, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{9}{5}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{9}{5}\left(-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(-\frac{2a\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}-\frac{9}{5}\left(\frac{4\sqrt{\frac{1}{ax}+1}}{3a\sqrt{\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
-((((-9*((-2*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (4*Sqrt[1 + 1/(a*x)])
/(3*a*Sqrt[x^(-1)])))/5 - (2*a*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5/2)))*Sqrt
[x^(-1)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]))
```

3.337.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.337. $\int e^{-\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

3.337.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-3ax+6)\,(ax+1)}{5\sqrt{-c(ax-1)}\,a^2}$	50
gosper	$\frac{2(ax+1)\,(a^2x^2-3ax+6)\,\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5a^2(ax-1)}$	55
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,\sqrt{-c(ax-1)}\,(a^2x^2-3ax+6)}{5(ax-1)a^2}$	56

input
```
int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a^2*x^2-3*a*x+6)/a^2*(a
*x+1)
```

3.337.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,(a^3x^3-2\,a^2x^2+3\,ax+6)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^3x-a^2)}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

output
```
2/5*(a^3*x^3 - 2*a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1))/(a^3*x - a^2)
```

3.337.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

input
```
integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\left(a^3\sqrt{-c}x^3-2\,a^2\sqrt{-c}x^2+3\,a\sqrt{-c}x+6\,\sqrt{-c}\right)(ax-1)}{5\,(a^3x-a^2)\sqrt{ax+1}}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

output
```
2/5*(a^3*sqrt(-c)*x^3 - 2*a^2*sqrt(-c)*x^2 + 3*a*sqrt(-c)*x + 6*sqrt(-c))*
(a*x - 1)/((a^3*x - a^2)*sqrt(a*x + 1))
```

3.337.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=-\frac{4\sqrt{-acx-c}|c|}{a^2c}-\frac{2\left((acx+c)^2\sqrt{-acx-c}|c|+5\,(-acx-c)^{\frac{3}{2}}c|c|\right)}{5\,a^2c^3}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-4*sqrt(-a*c*x - c)*abs(c)/(a^2*c) - 2/5*((a*c*x + c)^2*sqrt(-a*c*x - c)*a
bs(c) + 5*(-a*c*x - c)^(3/2)*c*abs(c))/(a^2*c^3)
```

3.337.9 Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^3\,x^3-2\,a^2\,x^2+3\,a\,x+6\right)}{5\,a^2\,\left(a\,x-1\right)}$$

input
```
int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a*x - 2*a^2*x^2 + a^3*
x^3 + 6))/(5*a^2*(a*x - 1))
```

3.338 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.338.1 Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{8c\sqrt{1-\frac{1}{a^2x^2}}x}{3\sqrt{c-acx}}+\frac{2}{3}\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}$$

output
```
8/3*c*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+2/3*x*(1-1/a^2/x^2)^(1/2)*(-a
*c*x+c)^(1/2)
```

3.338.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(-5+ax)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

3.338.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \ 6727$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 87$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{5}{3}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{10\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}-\frac{2a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

output
```
-((((-2*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (10*Sqrt[1 + 1/(a*x)])/(
3*Sqrt[x^(-1)]))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]))
```

3.338.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.338.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(ax-5)(ax+1)}{3\sqrt{-c(ax-1)}\,a}$	42
gosper	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(ax-5)}{3(ax-1)a}$	48

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-5)/a*(a*x+1)
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^2x^2-4\,ax-5\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

3.338.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(ax-1\right)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^2\sqrt{-c}x^2-4\,a\sqrt{-c}x-5\sqrt{-c}\right)\left(ax-1\right)}{3\left(a^2x-a\right)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a
)*sqrt(a*x + 1))
```

3.338.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,(-acx-c)^{\frac{3}{2}}|c|}{3\,ac^2}+\frac{4\,\sqrt{-acx-c}|c|}{ac}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

3.338.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,(a\,x-3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.339 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.339.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*arcsinh((1/x)^(1/2)/a
^(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

3.339.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

3.339.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6730, 27, 87, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}-\frac{2a\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(-2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-\frac{2a\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

3.339. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*a*Sqrt[1 + 1/(a*x)])/Sqrt[x^(-1)] - 2
*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a*x)]))
```

3.339.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.339. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.339.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c(ax+1)}\right)}{(ax-1)\sqrt{-c(ax+1)}}$	80

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(
a*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1))^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.19

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1},\ 2\Big((ax-1\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas"
)
```

output
```
[((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), 2*((a*x - 1)*sqrt(c)*a
rctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) + sq
rt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

3.339.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}{x}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x, x)
```

3.339.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.339.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -2\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{-acx-c}}{c}\right)|c|$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

output
```
-2*(arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + sqrt(-a*c*x - c)/c)*abs(c)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.340 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

3.340.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output

```
(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)-3*arcsinh((1/x)^(1/2)/a
^(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}-3\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

input

```
Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

output

```
(Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] - 3*Sqrt[a]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)]
```

3.340.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6730, 27, 90, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{3}{2}a\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(3a\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

output

```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-(a*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + 3*a
^(3/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a*x)]))
```

3.340.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.340.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(-3\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{(ax-1)\sqrt{-c(ax+1)}\,x\sqrt{c}}$	90
risch	$-\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\sqrt{-c(ax-1)}}-\frac{3a\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	95

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*arctan((-c*(a*x+1))
^(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/
2)/x/c^(1/2)
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.42

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$=\left[\frac{3\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)},\right.$$
$$\left.-\frac{3\left(a^2x^2-ax\right)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="frica
s")
```

output
```
[1/2*(3*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*s
qrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -(3*(a^2
*x^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(
a*x^2 - x)]
```

3.340.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}{x^2}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x**2, x)
```

3.340.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\int \frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = a\left(\frac{3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{\sqrt{-acx-c}}{acx}\right)|c|$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac"
)
```

output
```
a*(3*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - sqrt(-a*c*x - c)/(a*c*x))*
abs(c)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

3.341 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

3.341.1 Optimal result

Integrand size = 23, antiderivative size = 139

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^4}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{2(c-acx)^{5/2}}{5a^4c^2}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

output
```
2/3*(-a*c*x+c)^(3/2)/a^4/c+2/5*(-a*c*x+c)^(5/2)/a^4/c^2-2/7*(-a*c*x+c)^(7/
2)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4-4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(
1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4+4*(-a*c*x+c)^(1/2)/a^4
```

3.341.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{2\left(\sqrt{c-acx}(788-236ax+138a^2x^2-95a^3x^3+35a^4x^4)-630\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a^4}$$

input
```
Integrate[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*(Sqrt[c - a*c*x]*(788 - 236*a*x + 138*a^2*x^2 - 95*a^3*x^3 + 35*a^4*x^4
) - 630*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(315*
a^4)
```

3.341.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6717, 6680, 35, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-acx}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^3\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x^3(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{x^3(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{(c-acx)^{7/2}}{a^3c^2}-\frac{(c-acx)^{5/2}}{a^3c}-\frac{(c-acx)^{3/2}}{a^3(ax+1)}+\frac{(c-acx)^{3/2}}{a^3}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{4\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{2(c-acx)^{9/2}}{9a^4c^3}+\frac{2(c-acx)^{7/2}}{7a^4c^2}-\frac{2(c-acx)^{5/2}}{5a^4c}-\frac{2(c-acx)^{3/2}}{3a^4}-\frac{4c\sqrt{c-acx}}{a^4}}{c}$$

input
```
Int[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((-4*c*Sqrt[c - a*c*x])/a^4 - (2*(c - a*c*x)^(3/2))/(3*a^4) - (2*(c - a*
c*x)^(5/2))/(5*a^4*c) + (2*(c - a*c*x)^(7/2))/(7*a^4*c^2) - (2*(c - a*c*x)
^(9/2))/(9*a^4*c^3) + (4*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*
Sqrt[c])])/a^4)/c)
```

3.341.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.341. $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

3.341.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$\frac{\frac{2\left(35a^{4}x^{4}-95a^{3}x^{3}+138a^{2}x^{2}-236ax+788\right)\sqrt{-c(ax-1)}}{315}-4\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{4}}$	75
risch	$-\frac{2(35a^{4}x^{4}-95a^{3}x^{3}+138a^{2}x^{2}-236ax+788)(ax-1)c}{315a^{4}\sqrt{-c(ax-1)}}-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^{4}}$	82
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c^{2}(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c^{3}(-acx+c)^{\frac{3}{2}}}{3}+4c^{4}\sqrt{-acx+c}-4c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{4}c^{4}}$	101
default	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c^{2}(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c^{3}(-acx+c)^{\frac{3}{2}}}{3}+4c^{4}\sqrt{-acx+c}-4c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{4}c^{4}}$	101

input
```
int(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
2/315*((35*a^4*x^4-95*a^3*x^3+138*a^2*x^2-236*a*x+788)*(-c*(a*x-1))^(1/2)-
630*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2)))/a^4
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.21

$$\int e^{-2\coth^{-1}(ax)}x^{3}\sqrt{c-acx}\,dx$$

$$=\left[\frac{2\left(315\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+(35a^{4}x^{4}-95a^{3}x^{3}+138a^{2}x^{2}-236ax+788)\sqrt{-acx+c}\right)}{315a^{4}}\right.$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/315*(315*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*x + 7
88)*sqrt(-a*c*x + c))/a^4, 2/315*(630*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*
sqrt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 23
6*a*x + 788)*sqrt(-a*c*x + c))/a^4]
```

3.341.6 Sympy [A] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.26

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= \begin{cases} \frac{2\cdot\left(\frac{2\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^4\sqrt{-acx+c}+\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^4}{4}-\frac{2x^3}{3a}+\frac{x^2}{a^2}-\frac{2x}{a^3}+\frac{2\left(\begin{cases} x & \text{for } a=0 \\ \frac{\log(ax+1)}{a} & \text{otherwise} \end{cases}\right)}{a^3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((2*(2*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 2*c**4*sqrt(-a*c*x + c) + c**3*(-a*c*x + c)**(3/2)/3 + c**2*(-a*
c*x + c)**(5/2)/5 - c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4
*c**4), Ne(a*c, 0)), (sqrt(c)*(x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**
3 + 2*Piecewise((x, Eq(a, 0)), (log(a*x + 1)/a, True))/a**3), True))
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= \frac{2\left(315\sqrt{2}c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35(-acx+c)^{\frac{9}{2}}-45(-acx+c)^{\frac{7}{2}}c+63(-acx+c)^{\frac{5}{2}}c^2+105(-acx}{315a^4c^4}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
2/315*(315*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt
(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) - 45*(-a*c*x + c)
^(7/2)*c + 63*(-a*c*x + c)^(5/2)*c^2 + 105*(-a*c*x + c)^(3/2)*c^3 + 630*sq
rt(-a*c*x + c)*c^4)/(a^4*c^4)
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^4\sqrt{-c}} + \frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}a^{32}c^{32}+45\,(acx-c)^3\sqrt{-acx+c}a^{32}c^{33}+63\,(acx-c)^2\sqrt{-acx+c}a^{32}c^{34}+\right.}{315\,a^{36}c^{36}}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^4*sqrt(-c)) +
 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^32*c^32 + 45*(a*c*x - c)^3*sqr
t(-a*c*x + c)*a^32*c^33 + 63*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^32*c^34 + 10
5*(-a*c*x + c)^(3/2)*a^32*c^35 + 630*sqrt(-a*c*x + c)*a^32*c^36)/(a^36*c^3
6)
```

3.341.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= \frac{4\sqrt{c-a\,c\,x}}{a^4}+\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^4\,c}+\frac{2\,(c-a\,c\,x)^{5/2}}{5\,a^4\,c^2}\\ &\quad-\frac{2\,(c-a\,c\,x)^{7/2}}{7\,a^4\,c^3}+\frac{2\,(c-a\,c\,x)^{9/2}}{9\,a^4\,c^4}\\ &\quad+\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^4}\end{aligned}$$

input
```
int((x^3*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*
x)^(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)
^(9/2))/(9*a^4*c^4) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)
/(2*c^(1/2)))*4i)/a^4
```

3.342 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.342.1 Optimal result

Integrand size = 23, antiderivative size = 97

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c} - \frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

output
```
-2/3*(-a*c*x+c)^(3/2)/a^3/c-2/7*(-a*c*x+c)^(7/2)/a^3/c^3+4*arctanh(1/2*(-a
*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3-4*(-a*c*x+c)^(1/2)/a^3
```

3.342.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-52+16ax-9a^2x^2+3a^3x^3)+84\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{21a^3}$$

input
```
Integrate[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(-52 + 16*a*x - 9*a^2*x^2 + 3*a^3*x^3) + 84*Sqrt[2]*Sqr
t[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(21*a^3)
```

3.342.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6717, 6680, 35, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^2\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x^2(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{x^2(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{(c-acx)^{3/2}}{a^2(ax+1)}-\frac{(c-acx)^{5/2}}{a^2c}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{4\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}+\frac{2(c-acx)^{7/2}}{7a^3c^2}+\frac{2(c-acx)^{3/2}}{3a^3}+\frac{4c\sqrt{c-acx}}{a^3}}{c}$$

input
```
Int[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((4*c*Sqrt[c - a*c*x])/a^3 + (2*(c - a*c*x)^(3/2))/(3*a^3) + (2*(c - a*c
*x)^(7/2))/(7*a^3*c^2) - (4*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[
2]*Sqrt[c])])/a^3)/c)
```

3.342.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.342.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{84\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+2\left(3a^{3}x^{3}-9a^{2}x^{2}+16ax-52\right)\sqrt{-c(ax-1)}}{21a^{3}}$	68
risch	$-\frac{2\left(3a^{3}x^{3}-9a^{2}x^{2}+16ax-52\right)(ax-1)c}{21a^{3}\sqrt{-c(ax-1)}}+\frac{4\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^{3}}$	74
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{c^{2}(-acx+c)^{\frac{3}{2}}}{3}+2c^{3}\sqrt{-acx+c}-2c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{c^{3}a^{3}}$	75
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}-\frac{2c^{2}(-acx+c)^{\frac{3}{2}}}{3}-4c^{3}\sqrt{-acx+c}+4c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{3}c^{3}}$	75

input
```
int(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/21*(84*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+2
*(3*a^3*x^3-9*a^2*x^2+16*a*x-52)*(-c*(a*x-1))^(1/2))/a^3
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \left[\frac{2\left(21\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+(3a^3x^3-9a^2x^2+16ax-52)\sqrt{-acx+c}\right)}{21a^3}, -\frac{2\left(42\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-(3a^3x^3-9a^2x^2+16ax-52)\sqrt{-acx+c}\right)}{21a^3}\right]$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/21*(21*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
- 3*c)/(a*x + 1)) + (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c)
)/a^3, -2/21*(42*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt
(-c)/c) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/a^3]
```

3.342.6 Sympy [A] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^4\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^3\sqrt{-acx+c}+\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^3}{3}-\frac{x^2}{a}+\frac{2x}{a^2}-\frac{2\left(\begin{cases} x & \text{for } a=0 \\ \frac{\log(ax+1)}{a} & \text{otherwise} \end{cases}\right)}{a^2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(2*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 2*c**3*sqrt(-a*c*x + c) + c**2*(-a*c*x + c)**(3/2)/3 + (-a*c*x
+ c)**(7/2)/7)/(a**3*c**3), Ne(a*c, 0)), (sqrt(c)*(x**3/3 - x**2/a + 2*x/a
**2 - 2*Piecewise((x, Eq(a, 0)), (log(a*x + 1)/a, True))/a**2), True))
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2\left(21\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3\left(-acx+c\right)^{\frac{7}{2}}+7\left(-acx+c\right)^{\frac{3}{2}}c^2+42\sqrt{-acx+c}c^3\right)}{21\,a^3c^3}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/21*(21*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(
2)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(7/2) + 7*(-a*c*x + c)^(3
/2)*c^2 + 42*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

3.342.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^3\sqrt{-c}}\\ &\quad+\frac{2\left(3\left(acx-c\right)^3\sqrt{-acx+c}a^{18}c^{18}-7\left(-acx+c\right)^{\frac{3}{2}}a^{18}c^{20}-42\sqrt{-acx+c}a^{18}c^{21}\right)}{21\,a^{21}c^{21}}\end{aligned}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^3*sqrt(-c))
+ 2/21*(3*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^18*c^18 - 7*(-a*c*x + c)^(3/2)*
a^18*c^20 - 42*sqrt(-a*c*x + c)*a^18*c^21)/(a^21*c^21)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{4\sqrt{c-a\,c\,x}}{a^3}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^3\,c}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^3\,c^3}-\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,4{}\mathrm{i}}{a^3}$$

input
```
int((x^2*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
- (4*(c - a*c*x)^(1/2))/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*
c*x)^(7/2))/(7*a^3*c^3) - (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)
*1i)/(2*c^(1/2)))*4i)/a^3
```

3.343 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

3.343.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

output
```
2/3*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c^2-4*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2+4*(-a*c*x+c)^(1/2)/a^2
```

3.343.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(38-11ax+3a^2x^2)-60\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a^2}$$

input
```
Integrate[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(38 - 11*a*x + 3*a^2*x^2) - 60*Sqrt[2]*Sqrt[c]*ArcTanh[
Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a^2)
```

3.343.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6717, 6680, 35, 90, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x\sqrt{c-acx}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{x(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{x(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{ 90}$$

$$-\frac{-\frac{\int \frac{(c-acx)^{3/2}}{ax+1}dx}{a}-\frac{2(c-acx)^{5/2}}{5a^2c}}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{-\frac{2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}}{a}-\frac{2(c-acx)^{5/2}}{5a^2c}}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{-\frac{2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{a}-\frac{2(c-acx)^{5/2}}{5a^2c}}{c}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{a}-\frac{2(c-acx)^{5/2}}{5a^2c}}{c}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{-\frac{2(c-acx)^{5/2}}{5a^2c}-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{a}}{c}$$

input

```
Int[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output

```
-(((-2*(c - a*c*x)^(5/2))/(5*a^2*c) - ((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*(
(2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2
]*Sqrt[c])])/a))/a)/c)
```

3.343.3.1 Defintions of rubi rules used

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.343.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{-60\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\left(6a^{2}x^{2}-22ax+76\right)\sqrt{-c(ax-1)}}{15a^{2}}$	59
risch	$-\frac{2\left(3a^{2}x^{2}-11ax+38\right)(ax-1)c}{15a^{2}\sqrt{-c(ax-1)}}-\frac{4\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^{2}}$	66
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^{2}\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{2}c^{2}}$	73
default	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^{2}\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{2}c^{2}}$	73

input
```
int(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-60*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+
(6*a^2*x^2-22*a*x+76)*(-c*(a*x-1))^(1/2))/a^2
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=\left[\frac{2\left(15\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+(3a^2x^2-11ax+38)\sqrt{-acx+c}\right)}{15a^2},\ \frac{2\left(30\sqrt{2}\sqrt{-c}\arctan\left(\sqrt{2}\right.\right.}{}\right.$$

input

```
integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output

```
[2/15*(15*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
- 3*c)/(a*x + 1)) + (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2, 2/15*
(30*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (3*
a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2]
```

3.343.6 Sympy [A] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=\begin{cases}\frac{2\cdot\left(\frac{2\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^2\sqrt{-acx+c}+\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2} & \text{for}\ ac\neq 0\\ \sqrt{c}\left(\frac{x^2}{2}-\frac{2x}{a}+\frac{2\left(\begin{cases}x & \text{for}\ a=0\\ \frac{\log(ax+1)}{a} & \text{otherwise}\end{cases}\right)}{a}\right) & \text{otherwise}\end{cases}$$

input

```
integrate(x*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output

```
Piecewise((2*(2*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 2*c**2*sqrt(-a*c*x + c) + c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)
**(5/2)/5)/(a**2*c**2), Ne(a*c, 0)), (sqrt(c)*(x**2/2 - 2*x/a + 2*Piecewis
e((x, Eq(a, 0)), (log(a*x + 1)/a, True))/a), True))
```

3.343.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\left(15\sqrt{2}c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3\left(-acx+c\right)^{\frac{5}{2}}+5\left(-acx+c\right)^{\frac{3}{2}}c+30\sqrt{-acx+c}c^{2}\right)}{15\,a^{2}c^{2}}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 5*(-a*c*x + c)^(3/
2)*c + 30*sqrt(-a*c*x + c)*c^2)/(a^2*c^2)
```

3.343.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^{2}\sqrt{-c}} + \frac{2\left(3\left(acx-c\right)^{2}\sqrt{-acx+c}a^{8}c^{8}+5\left(-acx+c\right)^{\frac{3}{2}}a^{8}c^{9}+30\sqrt{-acx+c}a^{8}c^{10}\right)}{15\,a^{10}c^{10}}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^2*sqrt(-c)) +
 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^8 + 5*(-a*c*x + c)^(3/2)*a^8
*c^9 + 30*sqrt(-a*c*x + c)*a^8*c^10)/(a^10*c^10)
```

3.343.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^2}+\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^2\,c}+\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^2\,c^2} + \frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^2}$$

input
```
int((x*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*
x)^(5/2))/(5*a^2*c^2) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1
i)/(2*c^(1/2)))*4i)/a^2
```

3.344 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.344.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-2/3*(-a*c*x+c)^(3/2)/a/c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*
2^(1/2)*c^(1/2)/a-4*(-a*c*x+c)^(1/2)/a
```

3.344.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(-7+ax)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]
/(Sqrt[2]*Sqrt[c])])/(3*a)
```

3.344.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6717, 6680, 35, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*S
qrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a))/c)
```

3.344.3.1 Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.344.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{2(ax-7)(ax-1)c}{3a\sqrt{-c(ax-1)}}+\frac{4\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a}$	57
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}-4c\sqrt{-acx+c}+4c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	59
pseudoelliptic	$\frac{4\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{2ax\sqrt{-c(ax-1)}}{3}-\frac{14\sqrt{-c(ax-1)}}{3}}{a}$	59

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a*x-7)*(a*x-1)/a/(-c*(a*x-1))^(1/2)*c+4*arctanh(1/2*(-a*c*x+c)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$

$$=\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a},\right.$$
$$\left.-\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}(ax-7)\right)}{3a}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
3*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*
arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x -
7))/a]
```

3.344.6 Sympy [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log{(ax+1)}+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input

```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output

```
Piecewise((-2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*c, 0)
), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)
), True))
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx \\ = -\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}c\right)}{3\,ac}$$

input

```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output

```
-2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)
/(a*c)
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3\,a^3c^3}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
2/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

3.344.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{2}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a} - \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c} - \frac{4\sqrt{c-a\,c\,x}}{a}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
(4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*
(c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

3.345 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.345.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

3.345.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]
*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

3.345.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = $\{6717, 6680, 35, 95, 27, 174, 73, 219, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x} \, dx$$

$$\Big\downarrow \textbf{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x}dx$$

$$\Big\downarrow \textbf{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x(ax+1)}dx$$

$$\Big\downarrow \textbf{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x(ax+1)}dx}{c}$$

$$\Big\downarrow \textbf{95}$$

$$-\frac{\frac{\int \frac{ac^2(1-3ax)}{x(ax+1)\sqrt{c-acx}}dx}{a}-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{c^2\int \frac{1-3ax}{x(ax+1)\sqrt{c-acx}}dx-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \textbf{174}$$

$$-\frac{c^2\left(\int \frac{1}{x\sqrt{c-acx}}dx-4a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{c^2\left(\frac{8\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{2\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{c^2\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$-\frac{c^2\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-2c\sqrt{c-acx}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-((-2*c*Sqrt[c - a*c*x] + c^2*((-2*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[
c] + (4*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c]))/c)
```

3.345.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

3.345. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.345.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$2\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}-4\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}+2\sqrt{-acx+c}$	58
default	$2\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}-4\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}+2\sqrt{-acx+c}$	58
pseudoelliptic	$2\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\sqrt{c}-4\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+2\sqrt{-c\,(ax-1)}$	61

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

3.345. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.345.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[2\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)\right. \\ +\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right) \\ +2\sqrt{-acx+c},4\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right) \\ \left.-2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)+2\sqrt{-acx+c}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
[2*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/
(a*x + 1)) + sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2
*sqrt(-a*c*x + c), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*
sqrt(-c)/c) - 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c
*x + c)]
```

3.345.6 Sympy [A] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.65

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx \\ = \begin{cases} -\frac{2c\,\text{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}c\,\text{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c} & \text{for } ac\neq 0 \\ \sqrt{c}\left(-\frac{3a\left(\frac{\log\left(-\frac{2}{x}\right)}{a}-\frac{\log\left(2a+\frac{2}{x}\right)}{a}\right)}{2}+\frac{\log\left(\frac{a}{x}+\frac{1}{x^2}\right)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

---

output
```
Piecewise((-2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*ata
n(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(-a*c*x + c), Ne
(a*c, 0)), (sqrt(c)*(-3*a*(log(-2/x)/a - log(2*a + 2/x)/a)/2 + log(a/x + x
**(-2))/2), True))
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right) - \sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right) + 2\sqrt{-acx+c}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c))) - sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-
a*c*x + c) + sqrt(c))) + 2*sqrt(-a*c*x + c)
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*c*a
rctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

3.345.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+2\sqrt{c-a\,c\,x} - 4\sqrt{2}\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

output
```
2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2) - 4*2^(1/
2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

3.346 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

3.346.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
-5*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*arctanh(1/2*(-a*c*x+c)^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+(-a*c*x+c)^(1/2)/x
```

3.346.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[
2]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

3.346.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6717, 6680, 35, 109, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^2}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x^2(ax+1)}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x^2(ax+1)}dx}{c}$$

$$\Big\downarrow \text{ 109}$$

$$-\frac{-\int \frac{ac^2(5-3ax)}{2x(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{1}{2}ac^2\int \frac{5-3ax}{x(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \text{ 174}$$

$$-\frac{-\frac{1}{2}ac^2\left(5\int \frac{1}{x\sqrt{c-acx}}dx-8a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{-\frac{1}{2}ac^2\left(\frac{16\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{10\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \text{ 219}$$

3.346. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}dx$

$$-\frac{-\frac{1}{2}ac^2\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \color{blue}{221}$$

$$-\frac{-\frac{1}{2}ac^2\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{c\sqrt{c-acx}}{x}}{c}$$

input

```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output

```
-((-((c*Sqrt[c - a*c*x])/x) - (a*c^2*((-10*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]
])/Sqrt[c] + (8*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c
]))/2)/c)
```

3.346.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.346. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.346. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

3.346.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-5\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax-1)}\,\sqrt{c}}{x\sqrt{c}}$	70
derivativedivides	$-2ca\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-acx+c}}{2acx}+\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	71
default	$2ca\left(\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{-acx+c}}{2acx}-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	71
risch	$-\frac{(ax-1)c}{x\sqrt{-c(ax-1)}}+\frac{a\left(-\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{8\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)c}{2}$	73

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(4*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-5*arctanh
((-c*(a*x-1))^(1/2)/c^(1/2))*a*c*x+(-c*(a*x-1))^(1/2)*c^(1/2))/x/c^(1/2)
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.26

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$=\left[\frac{4\,\sqrt{2}a\sqrt{c}x\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+5\,a\sqrt{c}x\log\left(\frac{acx+2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}}{2\,x},\right.$$
$$\left.-\frac{4\,\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-5\,a\sqrt{-c}x\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(4*sqrt(2)*a*sqrt(c)*x*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) + 5*a*sqrt(c)*x*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c
) - 2*c)/x) + 2*sqrt(-a*c*x + c))/x, -(4*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*s
qrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 5*a*sqrt(-c)*x*arctan(sqrt(-a*c*x +
c)*sqrt(-c)/c) - sqrt(-a*c*x + c))/x]
```

3.346.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^2\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{1}{2}\,ac\left(\frac{4\,\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{\sqrt{c}} - \frac{5\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{2\,\sqrt{-acx+c}}{acx}\right)$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
-1/2*a*c*(4*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqr
t(c) + sqrt(-a*c*x + c)))/sqrt(c) - 5*log((sqrt(-a*c*x + c) - sqrt(c))/(sq
rt(-a*c*x + c) + sqrt(c)))/sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{4\sqrt{2}ac\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{5\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\sqrt{-acx+c}}{x}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
-4*sqrt(2)*a*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 5*
a*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

3.346.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-a\,c\,x}}{x} - 5\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right) + 4\,\sqrt{2}\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

output
```
(c - a*c*x)^(1/2)/x - 5*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 4*2^(
1/2)*a*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

3.347 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

3.347.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}+\frac{23}{4}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

output

```
23/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^2*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/2*(-a*c*x+c)^(1/2)/x^2-9/4*
a*(-a*c*x+c)^(1/2)/x
```

3.347.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\frac{(2-9ax)\sqrt{c-acx}}{4x^2}+\frac{23}{4}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

input

```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
((2 - 9*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (23*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]
*Sqrt[c])]
```

3.347.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {6717, 6680, 35, 109, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^3}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x^3(ax+1)}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x^3(ax+1)}dx}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{-\frac{1}{2}\int \frac{ac^2(9-7ax)}{2x^2(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{1}{4}ac^2\int \frac{9-7ax}{x^2(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{\int \frac{ac(23-9ax)}{2x(ax+1)\sqrt{c-acx}}dx}{c}-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\int \frac{23-9ax}{x(ax+1)\sqrt{c-acx}}dx-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{174}$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\left(23\int \frac{1}{x\sqrt{c-acx}}dx-32a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\left(\frac{64\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{46\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\left(\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{46\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\left(\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{46\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
-((-1/2*(c*Sqrt[c - a*c*x])/x^2 - (a*c^2*((-9*Sqrt[c - a*c*x])/(c*x) - (a*
((-46*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c] + (32*Sqrt[2]*ArcTanh[Sqrt
[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c]))/2))/4)/c)
```

3.347.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.347. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

rule 6680

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.347.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{\sqrt{-c(ax-1)}\,(9ax-2)\sqrt{c}+a^2c\,x^2\left(16\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-23\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\right)}{4\sqrt{c}\,x^2}$	80
risch	$\frac{(9a^2x^2-11ax+2)c}{4x^2\sqrt{-c(ax-1)}}-\frac{a^2\left(-\frac{46\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{32\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)c}{8}$	84
derivativedivides	$2c^2a^2\left(\frac{\frac{\frac{9(-acx+c)^{\frac{3}{2}}}{8}-\frac{7c\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$	94
default	$2c^2a^2\left(\frac{\frac{\frac{9(-acx+c)^{\frac{3}{2}}}{8}-\frac{7c\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$	94

input

```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output

```
-1/4/c^(1/2)*((-c*(a*x-1))^(1/2)*(9*a*x-2)*c^(1/2)+a^2*c*x^2*(16*2^(1/2)*a
rctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))-23*arctanh((-c*(a*x-1))^(1/
2)/c^(1/2))))/x^2
```

3.347. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

3.347.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.92

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= \left[\frac{16\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+23\,a^2\sqrt{c}x^2\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\sqrt{-acx+c}(9\,ax-}{8\,x^2}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(16*sqrt(2)*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c) - 3*c)/(a*x + 1)) + 23*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x +
 c)*sqrt(c) - 2*c)/x) - 2*sqrt(-a*c*x + c)*(9*a*x - 2))/x^2, 1/4*(16*sqrt(
2)*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 23*a
^2*sqrt(-c)*x^2*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(9*
a*x - 2))/x^2]
```

3.347.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^3\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= \frac{1}{8}\,a^2c^2\left(\frac{2\left(9\,(-acx+c)^{\frac{3}{2}}-7\sqrt{-acx+c}c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{16\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}-\frac{23\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
1/8*a^2*c^2*(2*(9*(-a*c*x + c)^(3/2) - 7*sqrt(-a*c*x + c)*c)/((a*c*x - c)^
2*c + 2*(a*c*x - c)*c^2 + c^3) + 16*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-
a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*c*x + c)))/c^(3/2) - 23*log((sqrt(-
a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(3/2))
```

3.347.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{4\sqrt{2}a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{23\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{9\,(-acx+c)^{\frac{3}{2}}a^2c - 7\sqrt{-acx+c}a^2c^2}{4\,a^2c^2x^2}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
4*sqrt(2)*a^2*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2
3/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/4*(9*(-a*c*x + c)
^(3/2)*a^2*c - 7*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

3.347.9 Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{9\,(c-a\,c\,x)^{3/2}}{4\,c\,x^2} - \frac{a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,23\mathrm{i}}{4} - \frac{7\,\sqrt{c-a\,c\,x}}{4\,x^2} + \sqrt{2}\,a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

output
```
(9*(c - a*c*x)^(3/2))/(4*c*x^2) - (a^2*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)
/c^(1/2))*23i)/4 - (7*(c - a*c*x)^(1/2))/(4*x^2) + 2^(1/2)*a^2*c^(1/2)*ata
n((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

3.348 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

3.348.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = {} & \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x} \\ & -\frac{45}{8}a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) \\ & +4\sqrt{2}a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\end{aligned}$$

output
```
-45/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a^3*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/3*(-a*c*x+c)^(1/2)/x^3-13/
12*a*(-a*c*x+c)^(1/2)/x^2+19/8*a^2*(-a*c*x+c)^(1/2)/x
```

3.348.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = {} & \frac{\sqrt{c-acx}(8-26ax+57a^2x^2)}{24x^3} \\ & -\frac{45}{8}a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) \\ & +4\sqrt{2}a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\end{aligned}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - a*c*x]*(8 - 26*a*x + 57*a^2*x^2))/(24*x^3) - (45*a^3*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c -
a*c*x]/(Sqrt[2]*Sqrt[c])]
```

3.348.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6717, 6680, 35, 109, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^4}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x^4(ax+1)}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x^4(ax+1)}dx}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{-\frac{1}{3}\int \frac{ac^2(13-11ax)}{2x^3(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{1}{6}ac^2\int \frac{13-11ax}{x^3(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{\int \frac{3ac(19-13ax)}{2x^2(ax+1)\sqrt{c-acx}}dx}{2c}-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\int \frac{19-13ax}{x^2(ax+1)\sqrt{c-acx}}dx-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{\int \frac{ac(45-19ax)}{2x(ax+1)\sqrt{c-acx}}dx}{c}-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\int \frac{45-19ax}{x(ax+1)\sqrt{c-acx}}dx-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \ 174$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(45\int \frac{1}{x\sqrt{c-acx}}dx-64a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \ 73$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{128\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{90\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{64\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{90\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \ 221$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{64\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{90\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

input

```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

3.348. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

output
```
-((-1/3*(c*Sqrt[c - a*c*x])/x^3 - (a*c^2*((-13*Sqrt[c - a*c*x])/(2*c*x^2)
- (3*a*((-19*Sqrt[c - a*c*x])/(c*x) - (a*((-90*ArcTanh[Sqrt[c - a*c*x]/Sqr
t[c]])/Sqrt[c] + (64*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/S
qrt[c]))/2))/4))/6)/c)
```

3.348.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.348. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.348.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\frac{\sqrt{-c(ax-1)}\left(57a^{2}x^{2}-26ax+8\right)\sqrt{c}}{3}+a^{3}c\,x^{3}\left(32\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-45\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\right)}{8\sqrt{c}\,x^{3}}$
risch	$-\frac{\left(57a^{3}x^{3}-83a^{2}x^{2}+34ax-8\right)c}{24x^{3}\sqrt{-c(ax-1)}}+\frac{a^{3}\left(-\frac{90\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{64\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)c}{16}$
derivativedivides	$-2c^{3}a^{3}\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11c(-acx+c)^{\frac{3}{2}}}{6}-\frac{13c^{2}\sqrt{-acx+c}}{16}}{a^{3}c^{3}x^{3}}+\frac{45\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16\sqrt{c}}}{c^{2}}\right.$
default	$2c^{3}a^{3}\left(\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11c(-acx+c)^{\frac{3}{2}}}{6}-\frac{13c^{2}\sqrt{-acx+c}}{16}}{a^{3}c^{3}x^{3}}+\frac{45\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16\sqrt{c}}}{c^{2}}\right)$

3.348. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^{4}}\,dx$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/c^(1/2)*(1/3*(-c*(a*x-1))^(1/2)*(57*a^2*x^2-26*a*x+8)*c^(1/2)+a^3*c*x^
3*(32*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))-45*arctanh((
-c*(a*x-1))^(1/2)/c^(1/2))))/x^3
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.73

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$

$$= \left[\frac{96\,\sqrt{2}a^3\sqrt{c}x^3\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+135\,a^3\sqrt{c}x^3\log\left(\frac{acx+2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,(57\,a^2x^2-26\,ax+}{48\,x^3}\right.$$

$$-\frac{96\,\sqrt{2}a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-135\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-(57\,a^2x^2-26\,ax+8)}{24\,x^3}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(96*sqrt(2)*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c) - 3*c)/(a*x + 1)) + 135*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(-a*c*x
 + c)*sqrt(c) - 2*c)/x) + 2*(57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^
3, -1/24*(96*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*
sqrt(-c)/c) - 135*a^3*sqrt(-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (
57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3]
```

3.348.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^4\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.44

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$=\frac{1}{48}\,a^3c^3\left(\frac{2\left(57\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+39\,\sqrt{-acx+c}c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{96\,\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}+13$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
1/48*a^3*c^3*(2*(57*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 39*sqrt
(-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)
*c^4 + c^5) - 96*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c)))/c^(5/2) + 135*log((sqrt(-a*c*x + c) - sqrt(
c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(5/2))
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$=-\frac{4\,\sqrt{2}a^3c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{45\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}}$$
$$+\frac{57\,(acx-c)^2\sqrt{-acx+c}a^3c-88\,(-acx+c)^{\frac{3}{2}}a^3c^2+39\,\sqrt{-acx+c}a^3c^3}{24\,a^3c^3x^3}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
-4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) +
45/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/24*(57*(a*c*x -
c)^2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 39*sqrt(-a*c
*x + c)*a^3*c^3)/(a^3*c^3*x^3)
```

3.348.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{13\sqrt{c-a\,c\,x}}{8\,x^3} + \frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,45\mathrm{i}}{8} - \frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3} + \frac{19\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3} - \sqrt{2}\,a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

input

```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

output

```
(13*(c - a*c*x)^(1/2))/(8*x^3) + (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/
c^(1/2))*45i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (19*(c - a*c*x)^(5/2)
)/(8*c^2*x^3) - 2^(1/2)*a^3*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2
*c^(1/2)))*4i
```

3.349 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

3.349.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = {}&\frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{107a^2\sqrt{c-acx}}{96x^2}\\&-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{363}{64}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)\\&-4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\end{aligned}$$

output
```
363/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^4*arctanh(1/2*(-a
*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/4*(-a*c*x+c)^(1/2)/x^4-17
/24*a*(-a*c*x+c)^(1/2)/x^3+107/96*a^2*(-a*c*x+c)^(1/2)/x^2-149/64*a^3*(-a*
c*x+c)^(1/2)/x
```

3.349.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = {}&\frac{\sqrt{c-acx}(48-136ax+214a^2x^2-447a^3x^3)}{192x^4}\\&+\frac{363}{64}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)\\&-4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\end{aligned}$$

input

```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output

```
(Sqrt[c - a*c*x]*(48 - 136*a*x + 214*a^2*x^2 - 447*a^3*x^3))/(192*x^4) + (
363*a^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[
c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

3.349.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {6717, 6680, 35, 109, 27, 168, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^5}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x^5(ax+1)}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x^5(ax+1)}dx}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{-\frac{1}{4}\int \frac{ac^2(17-15ax)}{2x^4(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{1}{8}ac^2\int \frac{17-15ax}{x^4(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{\int \frac{ac(107-85ax)}{2x^3(ax+1)\sqrt{c-acx}}dx}{3c}-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$$\downarrow 27$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\int \frac{107-85ax}{x^3(ax+1)\sqrt{c-acx}}dx-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$$\downarrow 168$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{\int \frac{3ac(149-107ax)}{2x^2(ax+1)\sqrt{c-acx}}dx}{2c}-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$$\downarrow 27$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\int \frac{149-107ax}{x^2(ax+1)\sqrt{c-acx}}dx-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$$\downarrow 168$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{\int \frac{ac(363-149ax)}{2x(ax+1)\sqrt{c-acx}}dx}{c}-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$$\downarrow 27$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\int \frac{363-149ax}{x(ax+1)\sqrt{c-acx}}dx-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$$\downarrow 174$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(363\int \frac{1}{x\sqrt{c-acx}}dx-512a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-}{c}$$

$$\downarrow 73$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{1024\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{726\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-}}{3cx^3}}{c}$$

$$\downarrow 219$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{512\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{726\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c}}{3c}}{c}$$

3.349. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{512\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{726\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-}}{3cx^3}\right.}{c}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
-((-1/4*(c*Sqrt[c - a*c*x])/x^4 - (a*c^2*((-17*Sqrt[c - a*c*x])/(3*c*x^3)
 - (a*((-107*Sqrt[c - a*c*x])/(2*c*x^2) - (3*a*((-149*Sqrt[c - a*c*x])/(c*x
) - (a*((-726*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c] + (512*Sqrt[2]*Arc
Tanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c]))/2))/4))/6))/8)/c)
```

3.349.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.349. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.349. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

3.349.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{-c(ax-1)}\left(447a^3x^3-214a^2x^2+136ax-48\right)\sqrt{c}}{3}+a^4cx^4\left(256\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-363\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\right.}{64\sqrt{c}\,x^4}$
risch	$\frac{(447a^4x^4-661a^3x^3+350a^2x^2-184ax+48)c}{192x^4\sqrt{-c(ax-1)}}-\frac{a^4\left(-\frac{726\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{512\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)c}{128}$
derivativedivides	$2c^4a^4\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{\frac{\frac{149(-acx+c)^{\frac{7}{2}}}{128}-\frac{1127c(-acx+c)^{\frac{5}{2}}}{384}+\frac{1049c^2(-acx+c)^{\frac{3}{2}}}{384}-\frac{107c^3\sqrt{-acx+c}}{128}}{a^4c^4x^4}+}{c^3}\right.$
default	$2c^4a^4\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{\frac{\frac{149(-acx+c)^{\frac{7}{2}}}{128}-\frac{1127c(-acx+c)^{\frac{5}{2}}}{384}+\frac{1049c^2(-acx+c)^{\frac{3}{2}}}{384}-\frac{107c^3\sqrt{-acx+c}}{128}}{a^4c^4x^4}+}{c^3}\right.$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/64/c^(1/2)*(1/3*(-c*(a*x-1))^(1/2)*(447*a^3*x^3-214*a^2*x^2+136*a*x-48)
*c^(1/2)+a^4*c*x^4*(256*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(
1/2))-363*arctanh((-c*(a*x-1))^(1/2)/c^(1/2))))/x^4
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.59

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$=\Bigg[\frac{768\,\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{acx+2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+1089\,a^4\sqrt{c}x^4\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)-2\,(447\,a^3x^3-214}{384\,x^4}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

---

output
```
[1/384*(768*sqrt(2)*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c
)*sqrt(c) - 3*c)/(a*x + 1)) + 1089*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*
c*x + c)*sqrt(c) - 2*c)/x) - 2*(447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*
sqrt(-a*c*x + c))/x^4, 1/192*(768*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(1/2*sqrt
(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 1089*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*
x + c)*sqrt(-c)/c) - (447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqrt(-a*c*
x + c))/x^4]
```

3.349.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^5\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= \frac{1}{384}\,a^4c^4\left(\frac{2\left(447\,(-acx+c)^{\frac{7}{2}} - 1127\,(-acx+c)^{\frac{5}{2}}c + 1049\,(-acx+c)^{\frac{3}{2}}c^2 - 321\,\sqrt{-acx+c}c^3\right)}{(acx-c)^4c^3 + 4\,(acx-c)^3c^4 + 6\,(acx-c)^2c^5 + 4\,(acx-c)c^6 + c^7} + 768\sqrt{}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

output
```
1/384*a^4*c^4*(2*(447*(-a*c*x + c)^(7/2) - 1127*(-a*c*x + c)^(5/2)*c + 104
9*(-a*c*x + c)^(3/2)*c^2 - 321*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 +
4*(a*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 768
*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt
(-a*c*x + c)))/c^(7/2) - 1089*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*
x + c) + sqrt(c)))/c^(7/2))
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{363\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\sqrt{-c}}$$
$$-\frac{447\,(acx-c)^3\sqrt{-acx+c}a^4c + 1127\,(acx-c)^2\sqrt{-acx+c}a^4c^2 - 1049\,(-acx+c)^{\frac{3}{2}}a^4c^3 + 321\sqrt{-acx}}{192\,a^4c^4x^4}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

output
```
4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 3
63/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/192*(447*(a*c*x
 - c)^3*sqrt(-a*c*x + c)*a^4*c + 1127*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c
^2 - 1049*(-a*c*x + c)^(3/2)*a^4*c^3 + 321*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*
c^4*x^4)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{1049\,(c-a\,c\,x)^{3/2}}{192\,c\,x^4} - \frac{a^4\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,363\mathrm{i}}{64}$$
$$-\frac{107\sqrt{c-a\,c\,x}}{64\,x^4} - \frac{1127\,(c-a\,c\,x)^{5/2}}{192\,c^2\,x^4} + \frac{149\,(c-a\,c\,x)^{7/2}}{64\,c^3\,x^4}$$
$$+\sqrt{2}\,a^4\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\sqrt{c}}\right)\,4\mathrm{i}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

output
```
(1049*(c - a*c*x)^(3/2))/(192*c*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2
)*1i)/c^(1/2))*363i)/64 - (107*(c - a*c*x)^(1/2))/(64*x^4) - (1127*(c - a*
c*x)^(5/2))/(192*c^2*x^4) + (149*(c - a*c*x)^(7/2))/(64*c^3*x^4) + 2^(1/2)
*a^4*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

3.350 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

3.350.1 Optimal result

Integrand size $=23$, antiderivative size $=281$

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= \frac{1312\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}-\frac{656\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}\\ &\quad-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}\\ &\quad-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}\end{aligned}$$

output

```
-82/9*x^2*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-8/9*x^3*(-a
*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/9*x^4*(-a*c*x+c)^(1/2)/(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1312/45*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^
4/(1-1/a/x)^(1/2)-656/45*x*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^
(1/2)+164/15*x^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)
```

3.350.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(656+328ax-82a^2x^2+41a^3x^3-20a^4x^4+5a^5x^5)}{45a^5\sqrt{1-\frac{1}{a^2x^2}}x}$$

input

```
Integrate[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output

```
(2*Sqrt[c - a*c*x]*(656 + 328*a*x - 82*a^2*x^2 + 41*a^3*x^3 - 20*a^4*x^4 +
 5*a^5*x^5))/(45*a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.350.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6730, 27, 100, 27, 87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-acx}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{9}\int -\frac{28a-\frac{9}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{9}\int \frac{28a-\frac{9}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 87$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{9}\left(41\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}+\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{9}\left(41\left(6\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}+\frac{2}{\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{9}\left(41\left(6\left(-\frac{4\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{9}\left(41\left(6\left(-\frac{4\left(-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 48$$

3.350. $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{9}\left(\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+41\left(6\left(-\frac{4\left(\frac{4\sqrt{\frac{1}{ax}+1}}{3a\sqrt{\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)\right)-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output

```
-((((41*(6*((-4*((-2*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (4*Sqrt[1 + 1
/(a*x)])/(3*a*Sqrt[x^(-1)])))/(5*a) - (2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5
/2))) + 2/(Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2))) + (8*a)/(Sqrt[1 + 1/(a*x)]*(
x^(-1))^(7/2)))/9 - (2*a^2)/(9*Sqrt[1 + 1/(a*x)]*(x^(-1))^(9/2)))*Sqrt[x^(
-1)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.350.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.350.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2(ax+1)\left(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45a^4(ax-1)^2}$	80
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,\left(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656\right)}{45(ax-1)^2a^4}$	81
risch	$-\frac{2\left(5a^4x^4-25a^3x^3+66a^2x^2-148ax+476\right)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{45a^4\sqrt{-c(ax-1)}}-\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^4\sqrt{-c(ax-1)}}$	99

input
```
int(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(a*x+1)*(5*a^5*x^5-20*a^4*x^4+41*a^3*x^3-82*a^2*x^2+328*a*x+656)*(-a*
c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\,(5\,a^5x^5-20\,a^4x^4+41\,a^3x^3-82\,a^2x^2+328\,ax+656)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{45\,(a^5x-a^4)}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
2/45*(5*a^5*x^5 - 20*a^4*x^4 + 41*a^3*x^3 - 82*a^2*x^2 + 328*a*x + 656)*sq
rt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^5*x - a^4)
```

3.350.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(5\,a^6\sqrt{-c}x^6-15\,a^5\sqrt{-c}x^5+21\,a^4\sqrt{-c}x^4-41\,a^3\sqrt{-c}x^3+246\,a^2\sqrt{-c}x^2+984\,a\sqrt{-c}x+656\,\sqrt{-c}\right)(a}{45\,(a^6x^2-2\,a^5x+a^4)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
2/45*(5*a^6*sqrt(-c)*x^6 - 15*a^5*sqrt(-c)*x^5 + 21*a^4*sqrt(-c)*x^4 - 41*
a^3*sqrt(-c)*x^3 + 246*a^2*sqrt(-c)*x^2 + 984*a*sqrt(-c)*x + 656*sqrt(-c))
*(a*x - 1)^2/((a^6*x^2 - 2*a^5*x + a^4)*(a*x + 1)^(3/2))
```

3.350.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.350.9 Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^5\,x^5-20\,a^4\,x^4+41\,a^3\,x^3-82\,a^2\,x^2+328\,a\,x+656\right)}{45\,a^4\,\left(a\,x-1\right)}$$

input
```
int(x^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(328*a*x - 82*a^2*x^2 + 4
1*a^3*x^3 - 20*a^4*x^4 + 5*a^5*x^5 + 656))/(45*a^4*(a*x - 1))
```

3.351 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.351.1 Optimal result

Integrand size = 23, antiderivative size = 231

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = &-\frac{2672\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}\\&+\frac{1336\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}\\&-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}\end{aligned}$$

output
```
-334/35*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-44/35*x^2*(
-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/7*x^3*(-a*c*x+c)^(1/2)
/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2672/105*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)
/a^3/(1-1/a/x)^(1/2)+1336/105*x*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/
a/x)^(1/2)
```

3.351.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(-1336-668ax+167a^2x^2-66a^3x^3+15a^4x^4)}{105a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(-1336 - 668*a*x + 167*a^2*x^2 - 66*a^3*x^3 + 15*a^4*x^
4))/(105*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.351.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6730, 27, 100, 27, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{7}\int -\frac{22a-\frac{7}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{7}\int \frac{22a-\frac{7}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 87$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{7}\left(\frac{167}{5}\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{44a}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{7}\left(\frac{167}{5}\left(4\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{2}{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{44a}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{7}\left(\frac{167}{5}\left(4\left(-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{44a}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 48$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{7}\left(\frac{44a}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{167}{5}\left(4\left(\frac{4\sqrt{\frac{1}{ax}+1}}{3a\sqrt{\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)\right)-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((((167*(4*((-2*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (4*Sqrt[1 + 1/(a
*x)])/(3*a*Sqrt[x^(-1)])) + 2/(Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))))/5 + (44
*a)/(5*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2)))/7 - (2*a^2)/(7*Sqrt[1 + 1/(a*x)]
*(x^(-1))^(7/2)))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.351. $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.351.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.351. $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

3.351.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.31

method	result	size
gosper	$\frac{2(ax+1)(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105a^3(ax-1)^2}$	72
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)}{105(ax-1)^2a^3}$	73
risch	$-\frac{2(15a^3x^3-81a^2x^2+248ax-916)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{105a^3\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^3\sqrt{-c(ax-1)}}$	91

input

```
int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/105*(a*x+1)*(15*a^4*x^4-66*a^3*x^3+167*a^2*x^2-668*a*x-1336)*(-a*c*x+c)^
(1/2)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$=\frac{2\,(15\,a^4x^4-66\,a^3x^3+167\,a^2x^2-668\,ax-1336)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^4x-a^3)}$$

input

```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output

```
2/105*(15*a^4*x^4 - 66*a^3*x^3 + 167*a^2*x^2 - 668*a*x - 1336)*sqrt(-a*c*x
 + c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

3.351.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.45

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$=\frac{2\left(15\,a^5\sqrt{-c}x^5-51\,a^4\sqrt{-c}x^4+101\,a^3\sqrt{-c}x^3-501\,a^2\sqrt{-c}x^2-2004\,a\sqrt{-c}x-1336\,\sqrt{-c}\right)(ax-1)^2}{105\,(a^5x^2-2\,a^4x+a^3)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
2/105*(15*a^5*sqrt(-c)*x^5 - 51*a^4*sqrt(-c)*x^4 + 101*a^3*sqrt(-c)*x^3 -
501*a^2*sqrt(-c)*x^2 - 2004*a*sqrt(-c)*x - 1336*sqrt(-c))*(a*x - 1)^2/((a^
5*x^2 - 2*a^4*x + a^3)*(a*x + 1)^(3/2))
```

3.351.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.351.9 Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^3\,x^3-51\,a^2\,x^2+116\,a\,x-552\right)}{105\,a^3} - \frac{3776\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,\left(a\,x-1\right)}$$

input
```
int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(116*a*x - 51*a^2*x^2 + 1
5*a^3*x^3 - 552))/(105*a^3) - (3776*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1)
)^(1/2))/(105*a^3*(a*x - 1))
```

3.352 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

3.352.1 Optimal result

Integrand size = 21, antiderivative size = 182

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = -\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{316\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

output
```
-158/15*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-32/15*x*(-a*c
*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/5*x^2*(-a*c*x+c)^(1/2)/(1-
1/a/x)^(1/2)/(1+1/a/x)^(1/2)+316/15*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(
1-1/a/x)^(1/2)
```

3.352.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.31

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(158+79ax-16a^2x^2+3a^3x^3)}{15a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(158 + 79*a*x - 16*a^2*x^2 + 3*a^3*x^3))/(15*a^3*Sqrt[1
 - 1/(a^2*x^2)]*x)
```

3.352.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6730, 27, 100, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{5}\int -\frac{16a-\frac{5}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{5}\int \frac{16a-\frac{5}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{5}\left(\frac{79}{3}\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{32a}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{5}\left(\frac{79}{3}\left(2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{32a}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 48$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{5}\left(\frac{32a}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{79}{3}\left(\frac{2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{4\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)\right)-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((((79*(2/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) - (4*Sqrt[1 + 1/(a*x)])/Sqrt[
x^(-1)]))/3 + (32*a)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2)))/5 - (2*a^2)/(5*
Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2)))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/(a^2*Sqrt
[1 - 1/(a*x)]))
```

3.352.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6730

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.352. $\int e^{-3\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

3.352.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{2(ax+1)(3a^3x^3-16a^2x^2+79ax+158)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15a^2(ax-1)^2}$	64
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(3a^3x^3-16a^2x^2+79ax+158)}{15(ax-1)^2a^2}$	65
risch	$-\frac{2(3a^2x^2-19ax+98)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{15a^2\sqrt{-c(ax-1)}}-\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^2\sqrt{-c(ax-1)}}$	83

input
```
int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(a*x+1)*(3*a^3*x^3-16*a^2*x^2+79*a*x+158)*(-a*c*x+c)^(1/2)*((a*x-1)/(
a*x+1))^(3/2)/a^2/(a*x-1)^2
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.34

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,(3\,a^3x^3-16\,a^2x^2+79\,ax+158)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3x-a^2)}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

output
```
2/15*(3*a^3*x^3 - 16*a^2*x^2 + 79*a*x + 158)*sqrt(-a*c*x + c)*sqrt((a*x -
1)/(a*x + 1))/(a^3*x - a^2)
```

3.352.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\text{Timed out}$$

input
```
integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx =\frac{2\left(3\,a^4\sqrt{-c}x^4-13\,a^3\sqrt{-c}x^3+63\,a^2\sqrt{-c}x^2+237\,a\sqrt{-c}x+158\,\sqrt{-c}\right)(ax-1)^2}{15\,(a^4x^2-2\,a^3x+a^2)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

output
```
2/15*(3*a^4*sqrt(-c)*x^4 - 13*a^3*sqrt(-c)*x^3 + 63*a^2*sqrt(-c)*x^2 + 237
*a*sqrt(-c)*x + 158*sqrt(-c))*(a*x - 1)^2/((a^4*x^2 - 2*a^3*x + a^2)*(a*x
+ 1)^(3/2))
```

3.352.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.352.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.32

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(3\,a^3\,x^3-16\,a^2\,x^2+79\,a\,x+158\right)}{15\,a^2\,\left(a\,x-1\right)}$$

input
```
int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a*x - 16*a^2*x^2 + 3*
a^3*x^3 + 158))/(15*a^2*(a*x - 1))
```

3.353 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.353.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

output
```
-20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(
1/2)/a^2/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2)
```

3.353.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-23-10ax+a^2x^2)}{3a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*
x)
```

3.353.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{3}\int -\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{3}\int \frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{3}\left(23\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[x^(-1)])/Sqrt[1 +
1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2)))*Sqrt[x^(-1)]*S
qrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.353.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

3.353. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.353.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.40

method	result	size
gosper	$\frac{2(ax+1)(a^{2}x^{2}-10ax-23)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a(ax-1)^{2}}$	55
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(a^{2}x^{2}-10ax-23)}{3(ax-1)^{2}a}$	56
risch	$-\frac{2(ax-11)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{3a\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	74

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a
/(a*x-1)^2
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a^{2}x^{2}-10\,ax-23\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{2}x-a\right)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x - a)
```

3.353.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\left(a^{3}\sqrt{-c}x^{3}-9\,a^{2}\sqrt{-c}x^{2}-33\,a\sqrt{-c}x-23\,\sqrt{-c}\right)(ax-1)^{2}}{3\,(a^{3}x^{2}-2\,a^{2}x+a)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c)
)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

3.353.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.353.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-a\,c\,x}\,(a\,x-9)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{64\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.354 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

3.354.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,x\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+10*(-a*c*x+c)^(1/2)/a/x
/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2
)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

3.354.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}\left(a+\frac{5}{x}-\sqrt{a}\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(2*Sqrt[c - a*c*x]*(a + 5/x - Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*ArcSi
nh[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.354.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6730, 27, 100, 27, 87, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(2\int -\frac{4a-\frac{1}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\int \frac{4a-\frac{1}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(a\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{10a\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 63$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(2a\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{10a\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 222$$

$$-\frac{\sqrt{\frac{1}{x}}\left(2a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{10a\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

output

```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*a^2)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])
 - (10*a*Sqrt[x^(-1)])/Sqrt[1 + 1/(a*x)] + 2*a^(3/2)*ArcSinh[Sqrt[x^(-1)]/
Sqrt[a]]))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.354.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.354.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)\sqrt{-c(ax+1)}+acx+5c\right)}{(ax-1)^{2}c}$	80

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(c^(1/2)*ar
ctan((-c*(a*x+1))^(1/2)/c^(1/2))*(-c*(a*x+1))^(1/2)+a*c*x+5*c)/c
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+5)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, -\frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+5)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas"
)
```

output
```
[((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x +
 c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*
arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - s
qrt(-a*c*x + c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

3.354.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Timed out
```

3.354.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.354.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.355 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

3.355.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x}+\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
-8*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-(1+1/a/x)^(1/2)*(-a*
c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+7*arcsinh((1/x)^(1/2)/a^(1/2))*a^(1/2)*(1/x
)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

3.355.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\frac{\sqrt{c-acx}\left(-1-9ax+\frac{7a^{3/2}\sqrt{1+\frac{1}{ax}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - a*c*x]*(-1 - 9*a*x + (7*a^(3/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]])/(x^(-1))^(3/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

3.355.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6730, 27, 100, 27, 90, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-2a^2\int \frac{3a-\frac{1}{x}}{2a\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-a\int \frac{3a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{7}{2}a\int\frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 63$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-a\left(7a\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 222$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-a\left(7a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((8*a^2*Sqrt[x^(-1)])/Sqrt[1 + 1/(a*x)] -
a*(-(a*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + 7*a^(3/2)*ArcSinh[Sqrt[x^(-1)]/Sq
rt[a]])))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.355.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

3.355. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.355.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(7\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)ax\sqrt{-c(ax+1)}+9\sqrt{c}\,ax+\sqrt{c}\right)\sqrt{-c(ax-1)}}{(ax-1)^{2}\sqrt{c}\,x}$	86
risch	$\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\sqrt{-c(ax-1)}}-\frac{\left(-\frac{8a}{\sqrt{-acx-c}}-\frac{7a\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	112

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(7*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*x
*(-c*(a*x+1))^(1/2)+9*c^(1/2)*a*x+c^(1/2))*(-c*(a*x-1))^(1/2)/(a*x-1)^2/c^
(1/2)/x
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.66

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$= \left[\frac{7\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\sqrt{-acx+c}(9\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)}, \; 7\,(a^2\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="frica
s")
```

output
```
[1/2*(7*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*s
qrt(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), (7*(a^
2*x^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x +
 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a*x^2 - x)]
```

3.355.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Timed out
```

3.355.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.355.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.356 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

3.356.1 Optimal result

Integrand size = 23, antiderivative size = 190

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2}+\frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x}-\frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

output
```
-8*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/2*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+47/4*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/x/(1-1/a/x)^(1/2)-47/4*a^(3/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/
2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

3.356.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{\sqrt{c-acx}\left(2-13ax-47a^2x^2+\frac{47a^{5/2}\sqrt{1+\frac{1}{ax}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
-1/4*(Sqrt[c - a*c*x]*(2 - 13*a*x - 47*a^2*x^2 + (47*a^(5/2)*Sqrt[1 + 1/(a
*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2)))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x^3)
```

3.356.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6730, 27, 100, 27, 90, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-2a^2\int \frac{\left(11a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\int \frac{\left(11a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 90$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{47}{4}a\int\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{47}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-\frac{1}{2}a\int\frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)-\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 63$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{47}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 222$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{47}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)-\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output

```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((8*a^2*(x^(-1))^(3/2))/Sqrt[1 + 1/(a*x)]
- a*(-1/2*(a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2)) + (47*a*(a*Sqrt[1 + 1/(a*x)
]*Sqrt[x^(-1)] - a^(3/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/4)))/(a^2*Sqrt[1
- 1/(a*x)]))
```

3.356. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

3.356.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.356. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.356.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1)\sqrt{-c(ax-1)}\left(47\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^{2}x^{2}\sqrt{-c(ax+1)}+47\sqrt{c}\,a^{2}x^{2}+13\sqrt{c}\,ax-2\sqrt{c}\right)}{4(ax-1)^{2}\sqrt{c}\,x^{2}}$	103
risch	$-\frac{(15a^{2}x^{2}+13ax-2)c\sqrt{\frac{ax-1}{ax+1}}}{4x^{2}\sqrt{-c(ax-1)}}-\frac{\left(\frac{8a^{2}}{\sqrt{-acx-c}}+\frac{47a^{2}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	126

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(47*arcta
n((-c*(a*x+1))^(1/2)/c^(1/2))*a^2*x^2*(-c*(a*x+1))^(1/2)+47*c^(1/2)*a^2*x^
2+13*c^(1/2)*a*x-2*c^(1/2))/c^(1/2)/x^2
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.38

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^{3}}\,dx$$

$$= \left[\frac{47\,(a^{3}x^{3}-a^{2}x^{2})\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^{2}-x}\right)+2\,(47\,a^{2}x^{2}+13\,ax-2)\sqrt{-acx+}}{8\,(ax^{3}-x^{2})}\right.$$

$$\left.-\frac{47\,(a^{3}x^{3}-a^{2}x^{2})\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(47\,a^{2}x^{2}+13\,ax-2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{4\,(ax^{3}-x^{2})}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="frica
s")
```

output
```
[1/8*(47*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*
c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x))
+ 2*(47*a^2*x^2 + 13*a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a*x^3 - x^2), -1/4*(47*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c
)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (47*a^2*x^2 + 13*a*x -
2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

3.356.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Timed out
```

3.356.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.356.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^3}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.357 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

3.357.1 Optimal result

Integrand size = 23, antiderivative size = 238

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2} - \frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x} + \frac{119a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\mathrm{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

output

```
-8*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/3*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)+119/12*a*(1+1/a/x)^(1/2)*(-a*c*x+c)
^(1/2)/x^2/(1-1/a/x)^(1/2)-119/8*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1
-1/a/x)^(1/2)+119/8*a^(5/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*(-a*c
*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

3.357.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\left(-8+38ax-119a^2x^2-357a^3x^3+\frac{357a^{7/2}\sqrt{1+\frac{1}{ax}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}\right)}{24a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - a*c*x]*(-8 + 38*a*x - 119*a^2*x^2 - 357*a^3*x^3 + (357*a^(7/2)*S
qrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2)))/(24*a*Sqr
t[1 - 1/(a^2*x^2)]*x^4)
```

3.357.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6730, 27, 100, 27, 90, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}\,e^{-3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 100$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-2a^2\int\frac{\left(19a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\int\frac{\left(19a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 90$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{119}{6}a\int\frac{\left(\frac{1}{x}\right)^{3/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{119}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\int\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\Biggl(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\Biggl(\frac{119}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-\frac{1}{2}a\int\frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)\right)-\frac{1}{3}a\left(\frac{1}{x}\right)^{5/}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 63$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\Biggl(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\Biggl(\frac{119}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)\right)-\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 222$$

3.357. $\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}dx$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{119}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)\right)-\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+}\right.\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((8*a^2*(x^(-1))^(5/2))/Sqrt[1 + 1/(a*x)]
- a*(-1/3*(a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2)) + (119*a*((a*Sqrt[1 + 1/(a*
x)]*(x^(-1))^(3/2))/2 - (3*a*(a*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] - a^(3/2)*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/4))/6)))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.357.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

3.357. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.357.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.48

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(357\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^{3}x^{3}\sqrt{-c(ax+1)}+357a^{3}x^{3}\sqrt{c}+119\sqrt{c}\,a^{2}x^{2}-38\sqrt{c}\,ax+8\sqrt{c}\right)}{24(ax-1)^{2}\sqrt{c}\,x^{3}}$	114
risch	$\frac{\left(165a^{3}x^{3}+119a^{2}x^{2}-38ax+8\right)c\sqrt{\frac{ax-1}{ax+1}}}{24x^{3}\sqrt{-c(ax-1)}}-\frac{\left(-\frac{8a^{3}}{\sqrt{-acx-c}}-\frac{119a^{3}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{8\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	134

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(357*ar
ctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^3*x^3*(-c*(a*x+1))^(1/2)+357*a^3*x^3*c^
(1/2)+119*c^(1/2)*a^2*x^2-38*c^(1/2)*a*x+8*c^(1/2))/c^(1/2)/x^3
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.17

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$= \Bigg[\frac{357\,(a^4x^4-a^3x^3)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)-2\,(357\,a^3x^3+119\,a^2x^2-38\,ax+}{48\,(ax^4-x^3)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="frica
s")
```

output
```
[1/48*(357*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)
) - 2*(357*a^3*x^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1)))/(a*x^4 - x^3), 1/24*(357*(a^4*x^4 - a^3*x^3)*sqrt(c)*arct
an(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (357*
a^3*x^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x +
 1)))/(a*x^4 - x^3)]
```

3.357.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Timed out
```

3.357.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx=\int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.357.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx=\int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.358 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

3.358.1 Optimal result

Integrand size = 23, antiderivative size = 286

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = & -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4}\\ & +\frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3}-\frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{96\sqrt{1-\frac{1}{ax}}x^2}\\ & +\frac{1115a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x}\\ & -\frac{1115a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output
```
-8*(-a*c*x+c)^(1/2)/x^4/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/4*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/x^4/(1-1/a/x)^(1/2)+223/24*a*(1+1/a/x)^(1/2)*(-a*c*x+c)
^(1/2)/x^3/(1-1/a/x)^(1/2)-1115/96*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x^
2/(1-1/a/x)^(1/2)+1115/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)
^(1/2)-1115/64*a^(7/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*(-a*c*x+c)
^(1/2)/(1-1/a/x)^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.37

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{\sqrt{c-acx}\left(48-200ax+446a^2x^2-1115a^3x^3-3345a^4x^4+\frac{3345a^{9/2}\sqrt{1+\frac{1}{ax}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{9/2}}\right)}{192a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
-1/192*(Sqrt[c - a*c*x]*(48 - 200*a*x + 446*a^2*x^2 - 1115*a^3*x^3 - 3345*
a^4*x^4 + (3345*a^(9/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(
x^(-1))^(9/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5)
```

3.358.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6730, 27, 100, 27, 90, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-3\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{5/2}}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{5/2}}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 100$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-2a^2\int\frac{\left(27a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\int\frac{\left(27a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 90$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{223}{8}a\int\frac{\left(\frac{1}{x}\right)^{5/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{4}a\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{223}{8}a\left(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\int\frac{\left(\frac{1}{x}\right)^{3/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{1}{4}a\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\Bigg(\frac{223}{8}a\Bigg(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\int\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)\Bigg)-\frac{1}{4}a\left(\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\Bigg(\frac{223}{8}a\Bigg(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\Bigg(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\Bigg(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-\frac{1}{2}a\int -}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 63$$

3.358. $\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}dx$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\Bigg(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\Bigg(\frac{223}{8}a\Bigg(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\Bigg(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\Bigg(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a\int}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{222}$$

$$-\frac{\sqrt{\frac{1}{x}}\Bigg(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\Bigg(\frac{223}{8}a\Bigg(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\Bigg(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\Bigg(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a^{3/2}\text{arcsinh}\Bigg(}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((8*a^2*(x^(-1))^(7/2))/Sqrt[1 + 1/(a*x)]
- a*(-1/4*(a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(7/2)) + (223*a*((a*Sqrt[1 + 1/(a*
x)]*(x^(-1))^(5/2))/3 - (5*a*((a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))/2 - (3*
a*(a*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] - a^(3/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]
]))/4))/6))/8)))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.358.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

3.358. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

3.358.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.44

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,\Bigg(3345\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^4x^4\sqrt{-c(ax+1)}+3345a^4x^4\sqrt{c}+1115a^3x^3\sqrt{c}-446\sqrt{c}\,a^2x^2+200\sqrt{c}\,ax-}{192(ax-1)^2\sqrt{c}\,x^4}$
risch	$-\frac{(1809a^4x^4+1115a^3x^3-446a^2x^2+200ax-48)c\sqrt{\frac{ax-1}{ax+1}}}{192x^4\sqrt{-c(ax-1)}}-\frac{\left(\frac{8a^4}{\sqrt{-acx-c}}+\frac{1115a^4\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{64\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

3.358. $\int\frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

output
```
1/192*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(3345*a
rctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^4*x^4*(-c*(a*x+1))^(1/2)+3345*a^4*x^4*
c^(1/2)+1115*a^3*x^3*c^(1/2)-446*c^(1/2)*a^2*x^2+200*c^(1/2)*a*x-48*c^(1/2
))/c^(1/2)/x^4
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.03

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$

$$= \left[\frac{3345\,(a^5x^5-a^4x^4)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\,(3345\,a^4x^4+1115\,a^3x^3-446}{384\,(ax^5-x^4)}\right.$$

$$-\frac{3345\,(a^5x^5-a^4x^4)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(3345\,a^4x^4+1115\,a^3x^3-446\,a^2x^2+200\,ax-48)}{192\,(ax^5-x^4)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="frica
s")
```

output
```
[1/384*(3345*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt
(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 -
x)) + 2*(3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a
*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/192*(3345*(a^5*x^5
- a^4*x^4)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1
))/(a*c*x - c)) - (3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 4
8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

3.358.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Timed out
```

3.358.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.358.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.359 $\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$

3.359.1 Optimal result

Integrand size = 24, antiderivative size = 278

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx &= -\frac{(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)}\\ &\quad+\frac{2(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a^2(6+n)\left(8+6n+n^2\right)x}\\ &\quad+\frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{6+n}\\ &\quad-\frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a}\end{aligned}$$

output

```
-(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(2+1/2*
n)/a/(4+n)/(6+n)+2*(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*
(-a*c*x+c)^(2+1/2*n)/a^2/(6+n)/(n^2+6*n+8)/x+(8+n)*(1-1/a/x)^(-2-1/2*n)*(1
+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/(6+n)-(a-1/x)*(1-1/a/x)^(-2-1/2*n
)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/a
```

3.359.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.42

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$$
$$=\frac{2c^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(1+ax)(c-acx)^{n/2}\,(n^2(-1+ax)^2+8(7-4ax+a^2x^2)+2n(7-10ax+3}{a(2+n)(4+n)(6+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

output
```
(2*c^2*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(n^2*(-1 + a*x)^2 +
 8*(7 - 4*a*x + a^2*x^2) + 2*n*(7 - 10*a*x + 3*a^2*x^2)))/(a*(2 + n)*(4 +
n)*(6 + n)*(1 - 1/(a*x))^(n/2))
```

3.359.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6727, 27, 101, 27, 88, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{\frac{n}{2}+2}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \textcolor{blue}{6727}$$

$$\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(-\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\right)(c-acx)^{\frac{n+4}{2}}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-4}}{a^2}d\frac{1}{x}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\int \left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-4}d\frac{1}{x}}{a^2}$$

$$\Big\downarrow \textcolor{blue}{101}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\Big(a\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-a\int -\frac{1}{2}\left(1+\frac{1}{ax}\right)^{n/2}\left(a(n+8)-\frac{n+4}{x}\right)\left(\frac{1}{x}\right)}{a^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\left(\frac{1}{2}a\int\left(1+\frac{1}{ax}\right)^{n/2}\left(a(n+8)-\frac{n+4}{x}\right)\left(\frac{1}{x}\right)^{-\frac{n}{2}-4}d\frac{1}{x}+a\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+\right.\right.}{a^2}$$

$$\Big\downarrow 88$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\left(\frac{1}{2}a\left(-\frac{(n^2+14n+56)\int\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}d\frac{1}{x}}{n+6}-\frac{2a(n+8)\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+6}\right)+a\left(\right.\right.}{a^2}$$

$$\Big\downarrow 55$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\left(\frac{1}{2}a\left(-\frac{(n^2+14n+56)\left(-\frac{2\int\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}d\frac{1}{x}}{a(n+4)}-\frac{2\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}\right)}{n+6}-\frac{2a(n+8}{}\right.\right.}{a^2}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(\frac{1}{2}a\left(-\frac{(n^2+14n+56)\left(\frac{4\left(\frac{1}{x}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{a(n+2)(n+4)}-\frac{2\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}\right)}{n+6}-\frac{2a(n+8)\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+6}\right)+a\left(a-\frac{1}{x}\right.\right.}{a^2}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

output
```
-((((a*(-(((56 + 14*n + n^2)*((-2*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^(-2 -
 n/2))/(4 + n) + (4*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^(-1 - n/2))/(a*(2 +
 n)*(4 + n))))/(6 + n)) - (2*a*(8 + n)*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^
(-3 - n/2))/(6 + n)))/2 + a*(a - x^(-1))*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1)
)^(-3 - n/2))*(1 - 1/(a*x))^(-2 - n/2)*(x^(-1))^((4 + n)/2)*(c - a*c*x)^((
4 + n)/2))/a^2)
```

3.359. $\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$

3.359.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.359. $\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$

3.359.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.37

method	result	size
gosper	$\frac{2(ax+1)\left(a^{2}n^{2}x^{2}+6n\,x^{2}a^{2}+8a^{2}x^{2}-2n^{2}xa-20anx-32ax+n^{2}+14n+56\right)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}\left(-acx+c\right)^{2+\frac{n}{2}}}{\left(ax-1\right)^{2}a\left(n^{3}+12n^{2}+44n+48\right)}$	104

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x+1)*(a^2*n^2*x^2+6*a^2*n*x^2+8*a^2*x^2-2*a*n^2*x-20*a*n*x-32*a*x+n^2
+14*n+56)*exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n)/(a*x-1)^2/a/(n^3+12*n^2
+44*n+48)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.67

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$$

$$=\frac{2\left((a^3n^2+6\,a^3n+8\,a^3)x^3-(a^2n^2+14\,a^2n+24\,a^2)x^2+n^2-(an^2+6\,an-24\,a)x+14\,n+56)(-acx+\right.}{an^3+12\,an^2+(a^3n^3+12\,a^3n^2+44\,a^3n+48\,a^3)x^2+44\,an-2\left(a^2n^3+12\,a^2n^2+44\,a^2n+48\,a\right.}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="fricas")
```

output
```
2*((a^3*n^2 + 6*a^3*n + 8*a^3)*x^3 - (a^2*n^2 + 14*a^2*n + 24*a^2)*x^2 + n
^2 - (a*n^2 + 6*a*n - 24*a)*x + 14*n + 56)*(-a*c*x + c)^(1/2*n + 2)*((a*x
+ 1)/(a*x - 1))^(1/2*n)/(a*n^3 + 12*a*n^2 + (a^3*n^3 + 12*a^3*n^2 + 44*a^3
*n + 48*a^3)*x^2 + 44*a*n - 2*(a^2*n^3 + 12*a^2*n^2 + 44*a^2*n + 48*a^2)*x
 + 48*a)
```

3.359.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx=\int\left(-c(ax-1)\right)^{\frac{n}{2}+2}e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(2+1/2*n),x)
```

output
```
Integral((-c*(a*x - 1))**(n/2 + 2)*exp(n*acoth(a*x)), x)
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.44

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$$
$$=\frac{2\,((n^2+6\,n+8)a^3(-c)^{\frac{1}{2}\,n}\,c^2x^3-(n^2+14\,n+24)a^2(-c)^{\frac{1}{2}\,n}\,c^2x^2-(n^2+6\,n-24)a(-c)^{\frac{1}{2}\,n}\,c^2x+(n^2+}{(n^3+12\,n^2+44\,n+48)a}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="maxima")
```

output
```
2*((n^2 + 6*n + 8)*a^3*(-c)^(1/2*n)*c^2*x^3 - (n^2 + 14*n + 24)*a^2*(-c)^(
1/2*n)*c^2*x^2 - (n^2 + 6*n - 24)*a*(-c)^(1/2*n)*c^2*x + (n^2 + 14*n + 56)
*(-c)^(1/2*n)*c^2)*(a*x + 1)^(1/2*n)/((n^3 + 12*n^2 + 44*n + 48)*a)
```

3.359.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n+2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n + 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.80

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$$

$$= \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{x^3\,{\left(c-a\,c\,x\right)}^{\frac{n}{2}+2}\,\left(2\,n^2+12\,n+16\right)}{n^3+12\,n^2+44\,n+48}+\frac{{\left(c-a\,c\,x\right)}^{\frac{n}{2}+2}\,\left(2\,n^2+28\,n+112\right)}{a^3\,\left(n^3+12\,n^2+44\,n+48\right)}-\frac{2\,x\,{\left(c-a\,c\,x\right)}^{\frac{n}{2}+2}\,\left(n^2+6\,n-24\right)}{a^2\,\left(n^3+12\,n^2+44\,n+48\right)}-\frac{x^2\,{\left(c-a\,c\,x\right)}^{\frac{n}{2}+2}}{a\,(n^3+12\,n^2}\right.}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 2),x)
```

output
```
(((a*x + 1)/(a*x))^(n/2)*((x^3*(c - a*c*x)^(n/2 + 2)*(12*n + 2*n^2 + 16))/
(44*n + 12*n^2 + n^3 + 48) + ((c - a*c*x)^(n/2 + 2)*(28*n + 2*n^2 + 112))/
(a^3*(44*n + 12*n^2 + n^3 + 48)) - (2*x*(c - a*c*x)^(n/2 + 2)*(6*n + n^2 -
 24))/(a^2*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(c - a*c*x)^(n/2 + 2)*(28*n
+ 2*n^2 + 48))/(a*(44*n + 12*n^2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*(
1/a^2 - (2*x)/a + x^2))
```

3.360 $\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$

3.360.1 Optimal result

Integrand size = 24, antiderivative size = 127

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{2(6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{2+n}{2}}}{a(2+n)(4+n)} + \frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n}$$

output
```
-2*(6+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(1+1/2*n)/a/(
n^2+6*n+8)+2*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(1+1/2*
n)/(4+n)
```

3.360.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{2c\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(1+ax)(c-acx)^{n/2}(-6+2ax+n(-1+ax))}{a(2+n)(4+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

output
```
(-2*c*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(-6 + 2*a*x + n*(-1
+ a*x)))/(a*(2 + n)*(4 + n)*(1 - 1/(a*x))^(n/2))
```

3.360.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6727, 27, 88, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{\frac{n}{2}+1}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6727}$$

$$\left(\frac{1}{x}\right)^{\frac{n+2}{2}}\left(-\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\right)(c-acx)^{\frac{n+2}{2}}\int \frac{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}}{a}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}(c-acx)^{\frac{n+2}{2}}\int \left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 88}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}(c-acx)^{\frac{n+2}{2}}\left(-\frac{(n+6)\int \left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}d\frac{1}{x}}{n+4}-\frac{2a\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}\right)}{a}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+2}{2}}\left(\frac{2(n+6)\left(\frac{1}{x}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+2)(n+4)}-\frac{2a\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}(c-acx)^{\frac{n+2}{2}}}{a}$$

input

```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

output

```
-((((-2*a*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^(-2 - n/2))/(4 + n) + (2*(6 +
 n)*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^(-1 - n/2))/((2 + n)*(4 + n)))*(1 -
 1/(a*x))^(-1 - n/2)*(x^(-1))^((2 + n)/2)*(c - a*c*x)^((2 + n)/2))/a)
```

3.360.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.360.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

method	result
gosper	$\frac{2(-acx+c)^{1+\frac{n}{2}} \mathrm{e}^{n \ \mathrm{arccoth}(ax)}(anx+2ax-n-6)(ax+1)}{(ax-1)a(n^{2}+6n+8)}$
parallelrisch	$-\frac{-2x^{2}(-acx+c)^{1+\frac{n}{2}} \mathrm{e}^{n \ \mathrm{arccoth}(ax)} a^{2} n-4x^{2}(-acx+c)^{1+\frac{n}{2}} \mathrm{e}^{n \ \mathrm{arccoth}(ax)} a^{2}+8 \mathrm{e}^{n \ \mathrm{arccoth}(ax)} x(-acx+c)^{1+\frac{n}{2}} a+2(-acx+c)^{1}}{(ax-1)a(n^{2}+6n+8)}$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x,method=_RETURNVERBOSE)
```

output
```
2*(-a*c*x+c)^(1+1/2*n)*exp(n*arccoth(a*x))*(a*n*x+2*a*x-n-6)*(a*x+1)/(a*x-
1)/a/(n^2+6*n+8)
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx=-\frac{2\,((a^2n+2\,a^2)x^2-4\,ax-n-6)(-acx+c)^{\frac{1}{2}\,n+1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{an^2+6\,an-(a^2n^2+6\,a^2n+8\,a^2)x+8\,a}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="fricas")
```

output
```
-2*((a^2*n + 2*a^2)*x^2 - 4*a*x - n - 6)*(-a*c*x + c)^(1/2*n + 1)*((a*x +
1)/(a*x - 1))^(1/2*n)/(a*n^2 + 6*a*n - (a^2*n^2 + 6*a^2*n + 8*a^2)*x + 8*a
)
```

3.360.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$$

$$=\begin{cases} c^{\frac{n}{2}+1}xe^{\frac{i\pi n}{2}} \\ 0^{\frac{n}{2}+1}xe^{\infty n} \\ -\frac{\int\frac{1}{axe^{4\operatorname{acoth}(ax)}-e^{4\operatorname{acoth}(ax)}}\,dx}{c} \\ \int e^{-2\operatorname{acoth}(ax)}\,dx \\ \frac{2a^2nx^2(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a}+\frac{4a^2x^2(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a}-\frac{8ax(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a}-\frac{2n(-acx+c)}{a^2n^2x+6a^2nx+8} \end{cases}$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1+1/2*n),x)
```

output
```
Piecewise((c**(n/2 + 1)*x*exp(I*pi*n/2), Eq(a, 0)), (0**(n/2 + 1)*x*exp(oo
*n), Eq(a, 1/x)), (-Integral(1/(a*x*exp(4*acoth(a*x)) - exp(4*acoth(a*x)))
, x)/c, Eq(n, -4)), (Integral(exp(-2*acoth(a*x)), x), Eq(n, -2)), (2*a**2*
n*x**2*(-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x
 + 8*a**2*x - a*n**2 - 6*a*n - 8*a) + 4*a**2*x**2*(-a*c*x + c)**(n/2 + 1)*
exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x + 8*a**2*x - a*n**2 - 6*a*n -
8*a) - 8*a*x*(-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a*
*2*n*x + 8*a**2*x - a*n**2 - 6*a*n - 8*a) - 2*n*(-a*c*x + c)**(n/2 + 1)*ex
p(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x + 8*a**2*x - a*n**2 - 6*a*n - 8*
a) - 12*(-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*
x + 8*a**2*x - a*n**2 - 6*a*n - 8*a), True))
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{2\left(a^2(-c)^{\frac{1}{2}n}c(n+2)x^2-4\,a(-c)^{\frac{1}{2}n}cx-(-c)^{\frac{1}{2}n}c(n+6)\right)(ax+1)^{\frac{1}{2}n}}{(n^2+6\,n+8)a}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="maxima")
```

output
```
-2*(a^2*(-c)^(1/2*n)*c*(n + 2)*x^2 - 4*a*(-c)^(1/2*n)*c*x - (-c)^(1/2*n)*c
*(n + 6))*(a*x + 1)^(1/2*n)/((n^2 + 6*n + 8)*a)
```

3.360.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = \int(-acx+c)^{\frac{1}{2}n+1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n + 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{\left(\frac{(2\,n+12)\,(c-a\,c\,x)^{\frac{n}{2}+1}}{a^2\,(n^2+6\,n+8)}-\frac{x^2\,(2\,n+4)\,(c-a\,c\,x)^{\frac{n}{2}+1}}{n^2+6\,n+8}+\frac{8\,x\,(c-a\,c\,x)^{\frac{n}{2}+1}}{a\,(n^2+6\,n+8)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\left(x-\frac{1}{a}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 1),x)
```

output
```
-((((2*n + 12)*(c - a*c*x)^(n/2 + 1))/(a^2*(6*n + n^2 + 8)) - (x^2*(2*n +
4)*(c - a*c*x)^(n/2 + 1))/(6*n + n^2 + 8) + (8*x*(c - a*c*x)^(n/2 + 1))/(a
*(6*n + n^2 + 8)))*((a*x + 1)/(a*x))^(n/2))/((x - 1/a)*((a*x - 1)/(a*x))^(
n/2))
```

3.360. $\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$

3.361 $\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx$

3.361.1 Optimal result

Integrand size = 22, antiderivative size = 36

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2e^{n \coth^{-1}(ax)}(1+ax)(c-acx)^{n/2}}{a(2+n)}$$

output
```
2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

3.361.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx=-\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}}x(c-acx)^{n/2}}{-1-\frac{n}{2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

output
```
-(((1 + 1/(a*x))^(1 + n/2)*x*(c - a*c*x)^(n/2))/((-1 - n/2)*(1 - 1/(a*x))^
(n/2)))
```

3.361.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{n/2}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6726}$$

$$\frac{2(ax+1)(c-acx)^{n/2}e^{n\coth^{-1}(ax)}}{a(n+2)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

output
```
(2*E^(n*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(n/2))/(a*(2 + n))
```

3.361.3.1 Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

3.361.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{2\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}(ax+1)(-acx+c)^{\frac{n}{2}}}{a(2+n)}$
parallelrisch	$-\frac{-2\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}x(-acx+c)^{\frac{n}{2}}a-2\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}(-acx+c)^{\frac{n}{2}}}{a(2+n)}$
risch	$\frac{2(ax+1)(ax+1)^{\frac{n}{2}}(ax-1)^{-\frac{n}{2}}(ax-1)^{\frac{n}{2}}c^{\frac{n}{2}}\,\mathrm{e}^{-\frac{i\pi n(-\operatorname{csgn}(i(ax-1))\operatorname{csgn}(ic(ax-1))^{2}+\operatorname{csgn}(i(ax-1))\operatorname{csgn}(ic(ax-1))\operatorname{csgn}(ic)-\operatorname{csgn}(ic(ax}{4}}}{a(2+n)}$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x,method=_RETURNVERBOSE)
```

output
```
2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2\,(ax+1)(-acx+c)^{\frac{1}{2}\,n}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{an+2\,a}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="fricas")
```

output
```
2*(a*x + 1)*(-a*c*x + c)^(1/2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*n + 2*a)
```

3.361.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\begin{cases}-\frac{x}{c} & \text{for}\ a=0\wedge n=-2\\ c^{\frac{n}{2}}xe^{\frac{i\pi n}{2}} & \text{for}\ a=0\\ -\frac{\int\frac{1}{axe^{2\,\mathrm{acoth}\,(ax)}-e^{2\,\mathrm{acoth}\,(ax)}}\,dx}{c} & \text{for}\ n=-2\\ \frac{2ax(-acx+c)^{\frac{n}{2}}e^{n\,\mathrm{acoth}\,(ax)}}{an+2a}+\frac{2(-acx+c)^{\frac{n}{2}}e^{n\,\mathrm{acoth}\,(ax)}}{an+2a} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2*n),x)
```

output
```
Piecewise((-x/c, Eq(a, 0) & Eq(n, -2)), (c**(n/2)*x*exp(I*pi*n/2), Eq(a, 0
)), (-Integral(1/(a*x*exp(2*acoth(a*x)) - exp(2*acoth(a*x))), x)/c, Eq(n,
-2)), (2*a*x*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a) + 2*(-a*c*x
 + c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a), True))
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2\left(a(-c)^{\frac{1}{2}\,n}\,x+(-c)^{\frac{1}{2}\,n}\right)(ax+1)^{\frac{1}{2}\,n}}{a(n+2)}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="maxima")
```

output
```
2*(a*(-c)^(1/2*n)*x + (-c)^(1/2*n))*(a*x + 1)^(1/2*n)/(a*(n + 2))
```

3.361.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\int(-acx+c)^{\frac{1}{2}\,n}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.361.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2\left(\frac{1}{a\,x}+1\right)^{n/2}(c-a\,c\,x)^{n/2}\,(a\,x+1)}{a\left(1-\frac{1}{a\,x}\right)^{n/2}\,(n+2)}$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2),x)
```

output
```
(2*(1/(a*x) + 1)^(n/2)*(c - a*c*x)^(n/2)*(a*x + 1))/(a*(1 - 1/(a*x))^(n/2)
*(n + 2))
```

3.362 $\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx$

3.362.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x(c-acx)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

output
```
2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x*(-a*c*x+c)^(-1+1/2*n)*hypergeom(
[1, -1/2*n],[1-1/2*n],2/(a+1/x)/x)/n
```

3.362.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(c-acx)^{n/2}\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{1+ax}\right)}{acn}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

output
```
(-2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 -
 n/2, 2/(1 + a*x)])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

3.362.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6727, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{\frac{n}{2}-1}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^{\frac{n-2}{2}}\left(-\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\right)(c-acx)^{\frac{n-2}{2}}\int \frac{a\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-1}}{a-\frac{1}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a\left(\frac{1}{x}\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}(c-acx)^{\frac{n-2}{2}}\int \frac{\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-1}}{a-\frac{1}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$\frac{2\left(\frac{1}{x}\right)^{\frac{n-2}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

output
```
(2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*(x^(-1))^((-2 + n)/2 - n/2)
*(c - a*c*x)^((-2 + n)/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, 2/((a + x^
(-1))*x)])/n
```

3.362.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.362.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)} (-acx+c)^{-1+\frac{n}{2}}\, dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

3.362.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\, dx = \int (-acx+c)^{\frac{1}{2}n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="fricas")
```

output
```
integral((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.362.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-c(ax-1))^{\frac{n}{2}-1}e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-1+1/2*n),x)
```

output
```
Integral((-c*(a*x - 1))**(n/2 - 1)*exp(n*acoth(a*x)), x)
```

3.362.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.362.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{\frac{n}{2}-1}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1), x)
```

3.363 $\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx$

3.363.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx =$$

$$-\frac{2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x(c-acx)^{\frac{1}{2}(-4+n)}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

output

```
-2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-1+1/2*n)*x*(-a*c*x+c)^(-2+1/2*n)*hyperg
eom([2, 1-1/2*n],[2-1/2*n],2/(a+1/x)/x)/(2-n)
```

3.363.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx$$

$$=\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(c-acx)^{n/2}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{1+ax}\right)}{ac^2(-2+n)(1+ax)}$$

input

```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

output

```
(2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
 n/2, 2/(1 + a*x)])/(a*c^2*(-2 + n)*(1 - 1/(a*x))^(n/2)*(1 + a*x))
```

3.363.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6727, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{\frac{n}{2}-2}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^{\frac{n-4}{2}}\left(-\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\right)(c-acx)^{\frac{n-4}{2}}\int \frac{a^2\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-n/2}}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^2\left(\frac{1}{x}\right)^{\frac{n-4}{2}}\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}(c-acx)^{\frac{n-4}{2}}\int \frac{\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-n/2}}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$-\frac{2\left(\frac{1}{x}\right)^{\frac{n-4}{2}-\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}(c-acx)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

output
```
(-2*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*(x^(-1))^(1 + (-4 +
 n)/2 - n/2)*(c - a*c*x)^((-4 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/
2, 2/((a + x^(-1))*x)])/(2 - n)
```

3.363.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.363.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}(-acx+c)^{-2+\frac{n}{2}}\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

3.363.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int (-acx+c)^{\frac{1}{2}\,n-2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="fricas")
```

output
```
integral((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.363.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int(-c(ax-1))^{\frac{n}{2}-2}\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-2+1/2*n),x)
```

output
```
Integral((-c*(a*x - 1))**(n/2 - 2)*exp(n*acoth(a*x)), x)
```

3.363.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n-2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.363.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n-2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{\frac{n}{2}-2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2), x)
```

3.364 $\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx$

3.364.1 Optimal result

Integrand size = 18, antiderivative size = 104

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$=\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2}(n-2p),-1-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

output
```
((a-1/x)/(a+1/x))^(1/2*n-p)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^p*hypergeom([
-1-p, 1/2*n-p],[-p],2/(a+1/x)/x)/(p+1)/((1-1/a/x)^(1/2*n))
```

3.364.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$=\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(n-2p)}(1+ax)(c-acx)^p\,\text{Hypergeometric2F1}\big(-1-p,\frac{n}{2}-p,-p,\frac{2}{1+ax}}{a(1+p)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
((1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((n - 2*p)/2)*(1 + a*x)*(c - a
*c*x)^p*Hypergeometric2F1[-1 - p, n/2 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*(1
 - 1/(a*x))^(n/2))
```

3.364.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^p e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int \left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-p-2}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\text{Hypergeometric2F1}\left(\frac{1}{2}(n-2p),-p-1,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^p*Hypergeometric2F1[(n - 2*p)/2, -1 - p, -p, 2/((a + x^(-1))*x)])
/((1 + p)*(1 - 1/(a*x))^(n/2))
```

3.364.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.364.4 Maple [F]

$$\int \mathrm{e}^{n \operatorname{arccoth}(ax)}(-acx+c)^p\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

3.364.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
integral((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.364.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-c(ax-1))^p\,e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**p,x)
```

output
```
Integral((-c*(a*x - 1))**p*exp(n*acoth(a*x)), x)
```

3.364.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.364.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^p,x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^p, x)
```

3.365 $\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx$

3.365.1 Optimal result

Integrand size = 18, antiderivative size = 81

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left(5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

output
```
-32*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([5, 4-1/2*n],[5
-1/2*n],(a-1/x)/(a+1/x))/a/(8-n)
```

3.365.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(81) = 162.

Time = 1.92 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.35

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{c^3e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(-48+44n-12n^2+n^3)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
-1/24*(c^3*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(-48 + 44*n - 12*n^2 +
 n^3)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)
*(a*n^3*x + n^2*(-1 - 12*a*x + a^2*x^2) + 2*n*(6 + 21*a*x - 6*a^2*x^2 + a^
3*x^3) + 6*(-7 - 4*a*x + 6*a^2*x^2 - 4*a^3*x^3 + a^4*x^4) + (-48 + 44*n -
12*n^2 + n^3)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a
*(2 + n))
```

3.365.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 81$, normalized size of antiderivative $= 1.00$, number of steps used $= 3$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used $= \{6725, 141\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6725}$$

$$a^3c^3\int \left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^5d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$-\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\text{Hypergeometric2F1}\left(5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
(-32*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric
2F1[5, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

3.365.3.1 Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.365.4 Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} (-acx+c)^3 \, dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

3.365.5 Fricas [F]

$$\int e^{n \coth^{-1}(ax)} (c-acx)^3 \, dx = \int -(acx-c)^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*((a*x + 1)/(a*x
- 1))^(1/2*n), x)
```

3.365.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=-c^3\left(\int 3axe^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-3a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right.$$
$$\left.+\int a^3x^3e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**3,x)
```

output
```
-c**3*(Integral(3*a*x*exp(n*acoth(a*x)), x) + Integral(-3*a**2*x**2*exp(n*
acoth(a*x)), x) + Integral(a**3*x**3*exp(n*acoth(a*x)), x) + Integral(-exp
(n*acoth(a*x)), x))
```

3.365.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int-(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-integrate((a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.365.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int-(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
integrate(-(a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^3\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^3,x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^3, x)
```

3.366 $\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx$

3.366.1 Optimal result

Integrand size = 18, antiderivative size = 81

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\text{Hypergeometric2F1}\left(4,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

output

```
16*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([4, 3-1/2*n],[4-
1/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

3.366.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(8-6n+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(6\right.\right.}{}$$

input

```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
(c^2*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(8 - 6*n + n^2)*Hypergeometr
ic2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(6 + 6*a*x + a*n^
2*x - 6*a^2*x^2 + 2*a^3*x^3 + n*(-1 - 6*a*x + a^2*x^2) + (8 - 6*n + n^2)*H
ypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(6*a*(2 + n))
```

3.366.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6725, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6725}$$

$$-a^2c^2\int \left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^4d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$\frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\text{Hypergeometric2F1}\left(4,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
(16*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2
F1[4, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

3.366.3.1 Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.366.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}(-acx+c)^2\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

3.366.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.366.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=c^2\left(\int \left(-2axe^{n\,\mathrm{acoth}\,(ax)}\right)\,dx+\int a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**2,x)
```

output
```
c**2*(Integral(-2*a*x*exp(n*acoth(a*x)), x) + Integral(a**2*x**2*exp(n*aco
th(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

3.366.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.366.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^2\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^2,x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^2, x)
```

3.367 $\int e^{n\coth^{-1}(ax)}(c-acx)\,dx$

3.367.1 Optimal result

Integrand size = 16, antiderivative size = 79

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx = -\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(3,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

output
```
-8*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([3, 2-1/2*n],[3-1/
2*n],(a-1/x)/(a+1/x))/a/(4-n)
```

3.367.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx = -\frac{ce^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}(-2+n)n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+\right.\right.}{2a(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
-1/2*(c*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F
1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*(-2 + n)*x +
a^2*x^2 + (-2 + n)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])
))/(a*(2 + n))
```

3.367.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6725, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6725}$$

$$ac\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$-\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(3,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
(-8*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1
[3, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

3.367.3.1 Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.367.4 Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}(-acx+c)\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

3.367.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int -(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.367.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=-c\left(\int axe^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c),x)
```

output
```
-c*(Integral(a*x*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)), x))
```

3.367.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int -(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="maxima")
```

output
```
-integrate((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.367.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int -(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x), x)
```

3.368 $\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx$

3.368.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

output
```
2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n
/((1-1/a/x)^(1/2*n))
```

3.368.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx=$$
$$-\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+\text{Hypergeo}\right.\right.}{acn(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2,
2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + Hypergeometric2F1[1, n/2, 1 +
 n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n)))
```

3.368.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6725, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx$$

$$\Big\downarrow \text{ 6725}$$

$$\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{ 141}$$

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

input

```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

output

```
(2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/
(a + x^(-1))])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

3.368.3.1 Defintions of rubi rules used

rule 141

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6725

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.368.4 Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{-acx+c}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

3.368.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{acx-c}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

3.368.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c),x)
```

output
```
-Integral(exp(n*acoth(a*x))/(a*x - 1), x)/c
```

3.368.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

3.368.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{c-a\,c\,x}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x), x)
```

3.369 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

3.369.1 Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}$$

output

```
-(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)
```

3.369.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{e^{n\coth^{-1}(ax)}(1+ax)}{ac^2(2+n)(-1+ax)}$$

input

```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output

```
-((E^(n*ArcCoth[a*x])*(1 + a*x))/(a*c^2*(2 + n)*(-1 + a*x)))
```

3.369.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6725, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx$$

$$\Big\downarrow \text{ 6725}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}}{a^2c^2}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-(((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 + n)))
```

3.369.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.369.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{\mathrm{e}^{n\ \operatorname{arccoth}(ax)}(ax+1)}{(ax-1)c^{2}(2+n)a}$	33
parallelrisch	$\frac{-x\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}a-\mathrm{e}^{n\ \operatorname{arccoth}(ax)}}{c^{2}(ax-1)(2+n)a}$	41

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(n*arccoth(a*x))*(a*x+1)/(a*x-1)/c^2/(2+n)/a
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{(ax+1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n+2\,ac^2-(a^2c^2n+2\,a^2c^2)x}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
(a*x + 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n + 2*a*c^2 - (a^2*c^2*n +
2*a^2*c^2)*x)
```

3.369.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.90

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx$$
$$=\begin{cases}-\frac{x}{c^2} & \text{for}\ a=0\wedge n=-2\\ \frac{xe^{\frac{i\pi n}{2}}}{c^2} & \text{for}\ a=0\\ -\frac{ax\operatorname{acoth}(ax)}{a^2c^2xe^{2\operatorname{acoth}(ax)}-ac^2e^{2\operatorname{acoth}(ax)}}-\frac{\operatorname{acoth}(ax)}{a^2c^2xe^{2\operatorname{acoth}(ax)}-ac^2e^{2\operatorname{acoth}(ax)}} & \text{for}\ n=-2\\ -\frac{axe^{n\operatorname{acoth}(ax)}}{a^2c^2nx+2a^2c^2x-ac^2n-2ac^2}-\frac{e^{n\operatorname{acoth}(ax)}}{a^2c^2nx+2a^2c^2x-ac^2n-2ac^2} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**2,x)
```

output
```
Piecewise((-x/c**2, Eq(a, 0) & Eq(n, -2)), (x*exp(I*pi*n/2)/c**2, Eq(a, 0)
), (-a*x*acoth(a*x)/(a**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*
x))) - acoth(a*x)/(a**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*x)
)), Eq(n, -2)), (-a*x*exp(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a
*c**2*n - 2*a*c**2) - exp(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a
*c**2*n - 2*a*c**2), True))
```

3.369.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(acx-c)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

3.369.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(acx-c)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(a\,x+1)}{a\,c^2\,(a\,x-1)\,(n+2)}$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^2,x)
```

output
```
-(exp(n*acoth(a*x))*(a*x + 1))/(a*c^2*(a*x - 1)*(n + 2))
```

3.370 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

3.370.1 Optimal result

Integrand size = 18, antiderivative size = 104

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}$$

output
```
-(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(n^2+6*n+8)+(1-1/a/x
)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(4+n)
```

3.370.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{e^{n\coth^{-1}(ax)}(3+n-ax)\left(\cosh\left(3\coth^{-1}(ax)\right)+\sinh\left(3\coth^{-1}(ax)\right)\right)}{a^2c^3(2+n)(4+n)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(E^(n*ArcCoth[a*x])*(3 + n - a*x)*(Cosh[3*ArcCoth[a*x]] + Sinh[3*ArcCoth[a
*x]]))/(a^2*c^3*(2 + n)*(4 + n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.370.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6725, 88, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6725}$$

$$\frac{\int \frac{(1-\frac{1}{ax})^{-\frac{n}{2}-3}(1+\frac{1}{ax})^{n/2}}{x}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{88}$$

$$\frac{\frac{a^2(1-\frac{1}{ax})^{-\frac{n}{2}-2}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{n+4}-\frac{a(n+3)\int (1-\frac{1}{ax})^{-\frac{n}{2}-2}(1+\frac{1}{ax})^{n/2}d\frac{1}{x}}{n+4}}{a^3c^3}$$

$$\Big\downarrow \text{48}$$

$$\frac{\frac{a^2(1-\frac{1}{ax})^{-\frac{n}{2}-2}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{n+4}-\frac{a^2(n+3)(1-\frac{1}{ax})^{-\frac{n}{2}-1}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{(n+2)(n+4)}}{a^3c^3}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
((a^2*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4 + n) - (a^2*(
3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/((2 + n)*(4 + n
)))/(a^3*c^3)
```

3.370.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 88

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 6725

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.370.4 Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{\mathrm{e}^{n \,\operatorname{arccoth}\left(a x \right)} \left(a x -n -3\right) \left(a x +1\right)}{\left(a x -1\right)^{2} c^{3} \left(n^{2}+6 n +8\right) a}$	46
parallelrisch	$\frac{2 x \,\mathrm{e}^{n \,\operatorname{arccoth}\left(a x \right)} a +x \,\mathrm{e}^{n \,\operatorname{arccoth}\left(a x \right)} a n -x^{2} \mathrm{e}^{n \,\operatorname{arccoth}\left(a x \right)} a^{2}+\mathrm{e}^{n \,\operatorname{arccoth}\left(a x \right)} n +3 \,\mathrm{e}^{n \,\operatorname{arccoth}\left(a x \right)}}{c^{3} \left(a x -1\right)^{2} \left(n^{2}+6 n +8\right) a}$	81

input

```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output

```
-exp(n*arccoth(a*x))*(a*x-n-3)*(a*x+1)/(a*x-1)^2/c^3/(n^2+6*n+8)/a
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\left(a^{2} x^{2} - {\left(a n + 2 \, a\right)} x - n - 3\right)} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n}}{a c^{3} n^{2} + 6 \, a c^{3} n + 8 \, a c^{3} + {\left(a^{3} c^{3} n^{2} + 6 \, a^{3} c^{3} n + 8 \, a^{3} c^{3}\right)} x^{2} - 2 \, {\left(a^{2} c^{3} n^{2} + 6 \, a^{2} c^{3} n + 8 \, a^{2} c^{3}\right)} x}$$

input

```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-(a^2*x^2 - (a*n + 2*a)*x - n - 3)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^
2 + 6*a*c^3*n + 8*a*c^3 + (a^3*c^3*n^2 + 6*a^3*c^3*n + 8*a^3*c^3)*x^2 - 2*
(a^2*c^3*n^2 + 6*a^2*c^3*n + 8*a^2*c^3)*x)
```

3.370.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 59.89 (sec) , antiderivative size = 1112, normalized size of antiderivative = 10.69

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**3,x)
```

output
```
Piecewise((x*exp(I*pi*n/2)/c**3, Eq(a, 0)), (a**2*x**2*acoth(a*x)/(2*a**3*
c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*e
xp(4*acoth(a*x))) + 2*a*x*acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) -
 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - a*x/(2*a*
*3*c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**
3*exp(4*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*
a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - 1/(2*a**3*c*
*3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp
(4*acoth(a*x))), Eq(n, -4)), (-a**2*x**2*acoth(a*x)/(2*a**3*c**3*x**2*exp(
2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x
))) + a*x/(2*a**3*c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(
a*x)) + 2*a*c**3*exp(2*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(2*a
coth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x)))
 + 1/(2*a**3*c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x))
 + 2*a*c**3*exp(2*acoth(a*x))), Eq(n, -2)), (-a**2*x**2*exp(n*acoth(a*x))/
(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3
*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8
*a*c**3) + a*n*x*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x*
*2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c*
*3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + 2*a*x*exp(n*acoth(a*x))/(...
```

3.370.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

3.370.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

3.370.9 Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n+3}{a^3\,c^3\,(n^2+6\,n+8)}-\frac{x^2}{a\,c^3\,(n^2+6\,n+8)}+\frac{x\,(n+2)}{a^2\,c^3\,(n^2+6\,n+8)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^3,x)
```

output
```
(((a*x + 1)/(a*x))^(n/2)*((n + 3)/(a^3*c^3*(6*n + n^2 + 8)) - x^2/(a*c^3*(
6*n + n^2 + 8)) + (x*(n + 2))/(a^2*c^3*(6*n + n^2 + 8))))/(((a*x - 1)/(a*x
))^(n/2)*(1/a^2 - (2*x)/a + x^2))
```

3.371 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.371.1 Optimal result

Integrand size = 18, antiderivative size = 224

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)}\\ &\quad-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)}\\ &\quad-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)\left(8+6n+n^2\right)}\\ &\quad-\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x}\end{aligned}$$

output
```
-(n^2+8*n+14)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(n^2+10*n+24)
-(n^2+8*n+14)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(n^3+12*n^2+4
4*n+48)+(5+n)*(1-1/a/x)^(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(6+n)-(1-1/a/
x)^(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a^2/c^4/x
```

3.371.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.37

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{e^{n\coth^{-1}(ax)}(-12-8n-n^2+(4+n)^2\cosh(2\coth^{-1}(ax))-2(4+n)\sinh(2\coth^{-1}(ax)))\,(\cosh(4\,\mathrm{co}}{2ac^4(2+n)(4+n)(6+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-1/2*(E^(n*ArcCoth[a*x])*(-12 - 8*n - n^2 + (4 + n)^2*Cosh[2*ArcCoth[a*x]]
 - 2*(4 + n)*Sinh[2*ArcCoth[a*x]])*(Cosh[4*ArcCoth[a*x]] + Sinh[4*ArcCoth[
a*x]]))/(a*c^4*(2 + n)*(4 + n)*(6 + n))
```

3.371.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6725, 101, 25, 27, 88, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{6725}$$

$$-\frac{\int \frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-4}\left(1+\frac{1}{ax}\right)^{n/2}}{x^2}d\frac{1}{x}}{a^4c^4}$$

$$\Big\downarrow \text{101}$$

$$-\frac{a^2\int -\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-4}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n+4}{x}\right)}{a}d\frac{1}{x}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{x}}{a^4c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{a^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{x}-a^2\int \frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-4}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n+4}{x}\right)}{a}d\frac{1}{x}}{a^4c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{a^2(1-\frac{1}{ax})^{-\frac{n}{2}-3}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{x}-a\int\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-4}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n+4}{x}\right)d\frac{1}{x}}{a^4c^4}$$

$$\Big\downarrow 88$$

$$-\frac{\frac{a^2(1-\frac{1}{ax})^{-\frac{n}{2}-3}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{x}-a\left(\frac{a^2(n+5)(1-\frac{1}{ax})^{-\frac{n}{2}-3}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{n+6}-\frac{a(n^2+8n+14)\int(1-\frac{1}{ax})^{-\frac{n}{2}-3}(1+\frac{1}{ax})^{n/2}d\frac{1}{x}}{n+6}\right)}{a^4c^4}$$

$$\Big\downarrow 55$$

$$-\frac{\frac{a^2(1-\frac{1}{ax})^{-\frac{n}{2}-3}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{x}-a\left(\frac{a^2(n+5)(1-\frac{1}{ax})^{-\frac{n}{2}-3}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{n+6}-\frac{a(n^2+8n+14)\left(\frac{\int\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}}{n+4}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(1}{n+4}\right.}{n+6}\right.}{a^4c^4}$$

$$\Big\downarrow 48$$

$$-\frac{\frac{a^2(1-\frac{1}{ax})^{-\frac{n}{2}-3}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{x}-a\left(\frac{a^2(n+5)(1-\frac{1}{ax})^{-\frac{n}{2}-3}(\frac{1}{ax}+1)^{\frac{n+2}{2}}}{n+6}-\frac{a(n^2+8n+14)\left(\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}}{n+4}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(1-}{(n+2)(n+4}\right.}{n+6}\right.}{a^4c^4}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-((-(a*(-((a*(14 + 8*n + n^2)*((a*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^(
(2 + n)/2))/(4 + n) + (a*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2
))/((2 + n)*(4 + n))))/(6 + n)) + (a^2*(5 + n)*(1 - 1/(a*x))^(-3 - n/2)*(1
 + 1/(a*x))^((2 + n)/2))/(6 + n))) + (a^2*(1 - 1/(a*x))^(-3 - n/2)*(1 + 1/
(a*x))^((2 + n)/2))/x)/(a^4*c^4))
```

3.371. $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.371.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

3.371. $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

3.371.4 Maple [A] (verified)

Time = 12.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.30

method	result
gosper	$-\frac{(ax+1)\left(2a^{2}x^{2}-2anx-8ax+n^{2}+8n+14\right)\mathrm{e}^{n\ \operatorname{arccoth}(ax)}}{(ax-1)^{3}c^{4}a(n^{2}+8n+12)(4+n)}$
parallelrisch	$\frac{2x^{2}\mathrm{e}^{n\ \operatorname{arccoth}(ax)}a^{2}n-x\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}a\,n^{2}-6x\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}a-6x\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}an+6x^{2}\mathrm{e}^{n\ \operatorname{arccoth}(ax)}a^{2}-8\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}n-}{c^{4}(ax-1)^{3}a(n^{2}+8n+12)(4+n)}$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-(a*x+1)*(2*a^2*x^2-2*a*n*x-8*a*x+n^2+8*n+14)*exp(n*arccoth(a*x))/(a*x-1)^
3/c^4/a/(n^2+8*n+12)/(4+n)
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{4}}\,dx$$

$$=\frac{(2\,a^{3}x^{3}-2\,(a^{2}n+3\,a^{2})x^{2}+n^{2}+(an^{2}+6\,an+6\,a)}{ac^{4}n^{3}+12\,ac^{4}n^{2}+44\,ac^{4}n+48\,ac^{4}-(a^{4}c^{4}n^{3}+12\,a^{4}c^{4}n^{2}+44\,a^{4}c^{4}n+48\,a^{4}c^{4})x^{3}+3\,(a^{3}c^{4}n^{3}+12\,a^{3}c^{4}}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
(2*a^3*x^3 - 2*(a^2*n + 3*a^2)*x^2 + n^2 + (a*n^2 + 6*a*n + 6*a)*x + 8*n +
 14)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^3 + 12*a*c^4*n^2 + 44*a*c^4*n
+ 48*a*c^4 - (a^4*c^4*n^3 + 12*a^4*c^4*n^2 + 44*a^4*c^4*n + 48*a^4*c^4)*x^
3 + 3*(a^3*c^4*n^3 + 12*a^3*c^4*n^2 + 44*a^3*c^4*n + 48*a^3*c^4)*x^2 - 3*(
a^2*c^4*n^3 + 12*a^2*c^4*n^2 + 44*a^2*c^4*n + 48*a^2*c^4)*x)
```

3.371.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**4,x)
```

output
```
Timed out
```

3.371.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^4}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

3.371.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^4}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

3.371.9 Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,x^3}{a\,c^4\,\left(n^3+12\,n^2+44\,n+48\right)}+\frac{n^2+8\,n+14}{a^4\,c^4\,\left(n^3+12\,n^2+44\,n+48\right)}-\frac{x^2\,\left(2\,n+6\right)}{a^2\,c^4\,\left(n^3+12\,n^2+44\,n+48\right)}+\frac{x\,\left(n^2+6\,n+6\right)}{a^3\,c^4\,\left(n^3+12\,n^2+44\,n+48\right)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{3\,x}{a^2}-\frac{1}{a^3}+x^3-\frac{3\,x^2}{a}\right)}$$

input

```
int(exp(n*acoth(a*x))/(c - a*c*x)^4,x)
```

output

```
-(((a*x + 1)/(a*x))^(n/2)*((2*x^3)/(a*c^4*(44*n + 12*n^2 + n^3 + 48)) + (8
*n + n^2 + 14)/(a^4*c^4*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(2*n + 6))/(a^2
*c^4*(44*n + 12*n^2 + n^3 + 48)) + (x*(6*n + n^2 + 6))/(a^3*c^4*(44*n + 12
*n^2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*((3*x)/a^2 - 1/a^3 + x^3 - (3
*x^2)/a))
```

3.372 $\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

3.372.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{5/2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{1}{2}(-5+n),-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

output
```
2/7*((a-1/x)/(a+1/x))^(-5/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(5/2)*
hypergeom([-7/2, -5/2+1/2*n],[-5/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

3.372.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)^3\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{1}{2}(\right.}{7a}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
(2*c^2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^3
*Sqrt[c - a*c*x]*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/(1 + a*x)])/(
7*a*(1 - 1/(a*x))^(n/2))
```

3.372.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{142}$$

$$\frac{2}{7}x(c-acx)^{5/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-5}{2}}\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}-\frac{5}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{n-5}{2},-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
(2*((a - x^(-1))/(a + x^(-1)))^((-5 + n)/2)*(1 - 1/(a*x))^(-5/2 + (5 - n)/
2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^(5/2)*Hypergeometric2F1[-7/2, (
-5 + n)/2, -5/2, 2/((a + x^(-1))*x)])/7
```

3.372.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.372.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}(-acx+c)^{\frac{5}{2}}\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

3.372.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\int (-acx+c)^{\frac{5}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x
- 1))^(1/2*n), x)
```

3.372.6 Sympy [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(5/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.372.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\int(-acx+c)^{\frac{5}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^(5/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.372.8 Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,6,1,0,0]%%%}+%%%{3,[0,4,1,1,0]%%%}+%%%{-3,[0,2,1,2,0
]%%%}+%%%
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{5/2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2), x)
```

3.373 $\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

3.373.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

output
```
2/5*((a-1/x)/(a+1/x))^(-3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(3/2)*
hypergeom([-5/2, -3/2+1/2*n],[-3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

3.373.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=$$
$$-\frac{2c\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)^2\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\right.}{5a}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(-2*c*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^2*
Sqrt[c - a*c*x]*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/(1 + a*x)])/(5
*a*(1 - 1/(a*x))^(n/2))
```

3.373.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{142}$$

$$\frac{2}{5}x(c-acx)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{n-3}{2},-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(2*((a - x^(-1))/(a + x^(-1)))^((-3 + n)/2)*(1 - 1/(a*x))^(-3/2 + (3 - n)/
2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^(3/2)*Hypergeometric2F1[-5/2, (
-3 + n)/2, -3/2, 2/((a + x^(-1))*x)])/5
```

3.373.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.373.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}(-acx+c)^{\frac{3}{2}}\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

3.373.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-acx+c)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*c*x - c)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.373.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-c(ax-1))^{\frac{3}{2}}\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1))**(3/2)*exp(n*acoth(a*x)), x)
```

3.373.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-acx+c)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.373.8 Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,4,1,0,0]%%%}+%%%{-2,[0,2,1,1,0]%%%}+%%%{1,[0,0,1,2,0]
%%%} / %%
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{3/2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2), x)
```

3.374 $\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

3.374.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

output

```
2/3*((a-1/x)/(a+1/x))^(-1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-3/2,
-1/2+1/2*n],[-1/2],2/(a+1/x)/x)*(-a*c*x+c)^(1/2)/((1-1/a/x)^(1/2*n))
```

3.374.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\right.}{3a}$$

input

```
Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)*Sqrt[
c - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/(1 + a*x)])/(3*a*(1
 - 1/(a*x))^(n/2))
```

3.374.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{142}$$

$$\frac{2}{3}x\sqrt{c-acx}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{n-1}{2},-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

input
```
Int[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*((a - x^(-1))/(a + x^(-1)))^((-1 + n)/2)*(1 - 1/(a*x))^(-1/2 + (1 - n)/
2)*(1 + 1/(a*x))^((2 + n)/2)*x*Sqrt[c - a*c*x]*Hypergeometric2F1[-3/2, (-1
 + n)/2, -1/2, 2/((a + x^(-1))*x)])/3
```

3.374.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.374.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}\sqrt{-acx+c}dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

3.374.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.374.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-c\,(ax-1)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*exp(n*acoth(a*x)), x)
```

3.374.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.374.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a\,c\,x}\,dx$$

input

```
int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2),x)
```

output

```
int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2), x)
```

3.375 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

3.375.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

output

```
2*((a-1/x)/(a+1/x))^(1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-1/2, 1/2
+1/2*n],[1/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(1/2)
```

3.375.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}(1+ax)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{1}{2},\frac{2}{1+ax}\right)}{a\sqrt{c-acx}}$$

input

```
Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
(2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*(1 + a*x)*Hyperg
eometric2F1[-1/2, (1 + n)/2, 1/2, 2/(1 + a*x)])/(a*(1 - 1/(a*x))^(n/2)*Sqr
t[c - a*c*x])
```

3.375.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{n/2}}{\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{142}$$

$$\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{n+1}{2},\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

input
```
Int[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
(2*((a - x^(-1))/(a + x^(-1)))^((1 + n)/2)*(1 - 1/(a*x))^(1/2 + (-1 - n)/2
)*(1 + 1/(a*x))^((2 + n)/2)*x*Hypergeometric2F1[-1/2, (1 + n)/2, 1/2, 2/((
a + x^(-1))*x)])/Sqrt[c - a*c*x]
```

3.375.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[p]
```

3.375.4 Maple [F]

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\sqrt{-acx+c}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

3.375.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

3.375.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{e^{n\operatorname{acoth}(ax)}}{\sqrt{-c\,(ax-1)}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(1/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)), x)
```

3.375.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

3.375.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a\,c\,x}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2), x)
```

3.376 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

3.376.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

output
```
-2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([1/2, 3/2
+1/2*n],[3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(3/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{1+ax}\right)}{ac\sqrt{c-acx}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
(2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F
1[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)])/(a*c*(1 - 1/(a*x))^(n/2)*Sqrt[c - a*c
*x])
```

3.376.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \; 6727$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{n/2}}{\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \; 142$$

$$-\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
(-2*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^(3/2 + (-3 - n)/
2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((
a + x^(-1))*x)])/(c - a*c*x)^(3/2)
```

3.376.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.376.4 Maple [F]

$$\int \frac{\mathrm{e}^{n \,\mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{3}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

3.376.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a
*c^2*x + c^2), x)
```

3.376.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(3/2), x)
```

3.376.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

3.376.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{3/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2), x)
```

3.377 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

3.377.1 Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(3+n)(c-acx)^{5/2}}$$

output
```
-a*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/(3+n)/(-a*c*x+c)^(5/2)+a*((
a-1/x)/(a+1/x))^(3/2+1/2*n)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2*hy
pergeom([1/2, 3/2+1/2*n],[3/2],2/(a+1/x)/x)/(3+n)/(-a*c*x+c)^(5/2)
```

3.377.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(-1-ax+(-1+ax)\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{ac^2(3+n)(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
((1 + 1/(a*x))^(n/2)*(-1 - a*x + (-1 + a*x)*((-1 + a*x)/(1 + a*x))^((1 + n
)/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)]))/(a*c^2*(3 + n)*
(1 - 1/(a*x))^(n/2)*(-1 + a*x)*Sqrt[c - a*c*x])
```

3.377.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6727, 105, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \; 6727$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{n/2}\sqrt{\frac{1}{x}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \; 105$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{a\sqrt{\frac{1}{x}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+3}-\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{n/2}}{\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2(n+3)}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \; 142$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{a\sqrt{\frac{1}{x}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+3}-\frac{a\sqrt{\frac{1}{x}}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{(a+}\right.}{n+3}\right.}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

3.377. $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$

output
```
-(((1 - 1/(a*x))^(5/2)*((a*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((2 +
n)/2)*Sqrt[x^(-1)])/(3 + n) - (a*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(
1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[x^(-1)]*Hypergeom
etric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(3 + n)))/((x^(-1))^(5/
2)*(c - a*c*x)^(5/2)))
```

3.377.3.1 Defintions of rubi rules used

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.377.4 Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{5}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

3.377.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*x^3 - 3*
a^2*c^3*x^2 + 3*a*c^3*x - c^3), x)
```

3.377.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(5/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(5/2), x)
```

3.377.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

3.377.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{5/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2), x)
```

3.378 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}} \, dx$

3.378.1 Optimal result

Integrand size $= 20$, antiderivative size $= 245$

$$\begin{aligned}\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}} \, dx = & -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} \\ & - \frac{3a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}} \left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x^3 \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2\left(15+8n+n^2\right)(c-acx)^{7/2}}\end{aligned}$$

output
```
-a*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/(5+n)/(-a*c*x+c)^(7/2)+3/2*
a^2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^3/(n^2+8*n+15)/(-a*c*x+c)^(7
/2)-3/2*a^2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1
+1/2*n)*x^3*hypergeom([1/2, 3/2+1/2*n],[3/2],2/(a+1/x)/x)/(n^2+8*n+15)/(-a
*c*x+c)^(7/2)
```

3.378.2 Mathematica [A] (verified)

Time $= 0.19$ (sec) , antiderivative size $= 138$, normalized size of antiderivative $= 0.56$

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{7/2}} \, dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} \left((9+2n-3ax)(1+ax)+3(-1+ax)^2 \left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}} \text{Hyperge}}{2ac^3(3+n)(5+n)(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
((1 + 1/(a*x))^(n/2)*((9 + 2*n - 3*a*x)*(1 + a*x) + 3*(-1 + a*x)^2*((-1 +
a*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 +
a*x)]))/(2*a*c^3*(3 + n)*(5 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x)^2*Sqrt[c -
 a*c*x])
```

3.378.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6727, 105, 105, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \textcolor{blue}{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-7)}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{3/2}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \textcolor{blue}{105}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{a\left(\frac{1}{x}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+5}-\frac{3a\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{n/2}\sqrt{\frac{1}{x}}d\frac{1}{x}}{2(n+5)}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \textcolor{blue}{105}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{a\left(\frac{1}{x}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+5}-\frac{3a\left(\frac{a\sqrt{\frac{1}{x}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+3}-\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{n/2}}{\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2(n+3)}\right)}{2(n+5)}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \textcolor{blue}{142}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{a\left(\frac{1}{x}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+5}-\frac{3a\left(\frac{a\sqrt{\frac{1}{x}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+3}-\frac{a\sqrt{\frac{1}{x}}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right.}{}\right.}{2(n+5)}\right.}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input

```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output

```
-(((1 - 1/(a*x))^(7/2)*((a*(1 - 1/(a*x))^((-5 - n)/2)*(1 + 1/(a*x))^((2 +
n)/2)*(x^(-1))^(3/2))/(5 + n) - (3*a*((a*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1
/(a*x))^((2 + n)/2)*Sqrt[x^(-1)])/(3 + n) - (a*((a - x^(-1))/(a + x^(-1)))
^((3 + n)/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[x^(
-1)]*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(3 + n)))
/(2*(5 + n))))/((x^(-1))^(7/2)*(c - a*c*x)^(7/2)))
```

3.378.3.1 Defintions of rubi rules used

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

3.378.4 Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{7}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

3.378.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^4*x^4 - 4*a
^3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4), x)
```

3.378.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(7/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.378.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

3.378.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{7/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2), x)
```

3.379 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.379.1 Optimal result

Integrand size = 20, antiderivative size = 114

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x-\frac{c^4\csc^{-1}(ax)}{2a}-\frac{3c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-1/3*c^4*(1-1/a^2/x^2)^(3/2)/a+c^4*(1-1/a^2/x^2)^(3/2)*x-1/2*c^4*arccsc(a*
x)/a-3*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+1/2*c^4*(6*a-1/x)*(1-1/a^2/x^2)^
(1/2)/a^2
```

3.379.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.54

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$

$$=\frac{c^4\left(-2+9ax-14a^2x^2-15a^3x^3+16a^4x^4+6a^5x^5+24a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+9a^4\sqrt{1-\frac{1}{a^2x^2}}\right.}{6a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

output
```
(c^4*(-2 + 9*a*x - 14*a^2*x^2 - 15*a^3*x^3 + 16*a^4*x^4 + 6*a^5*x^5 + 24*a
^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 9*a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 18*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

3.379.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {6731, 27, 540, 2340, 27, 535, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2}{a^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^4\left(a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\int \sqrt{1-\frac{1}{a^2x^2}}\left(3a^2-\frac{a}{x}+\frac{1}{x^2}\right)xd\frac{1}{x}\right)}{a^3}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^4\left(\frac{1}{3}a^2\int -\frac{3\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)x}{a}d\frac{1}{x}+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)\right)}{a^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\left(-a\int \sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)xd\frac{1}{x}+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)\right)}{a^3}$$

$$\Big\downarrow \text{535}$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\int \frac{\left(6a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)\right)}{a^3}$$

$$\Big\downarrow 538$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(6a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right.\right.\right.}{a^3}$$

$$\Big\downarrow 223$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(6a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)\right.\right.}{a^3}$$

$$\Big\downarrow 243$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(3a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right.\right.\right.}{a^3}$$

$$\Big\downarrow 73$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(-6a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\right.}{a^3}$$

$$\Big\downarrow 221$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(-6a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-(1\right.\right.}{a^3}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

output
```
-((c^4*((a^2*(1 - 1/(a^2*x^2))^(3/2))/3 - a^3*(1 - 1/(a^2*x^2))^(3/2)*x -
a*((Sqrt[1 - 1/(a^2*x^2)]*(6*a - x^(-1)))/2 + (-(a*ArcSin[1/(a*x)]) - 6*a*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/2)))/a^3)
```

3.379. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

3.379.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.379. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.379.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

method	result
risch	$\frac{(ax-1)(16a^2x^2-9ax+2)c^4}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{(ax-1)(ax+1)}}{a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^4\Bigl(-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+18(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+3a^3x^3\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+18\ln\Bigl(}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x-1)*(16*a^2*x^2-9*a*x+2)/x^3*c^4/a^4/((a*x-1)/(a*x+1))^(1/2)+(-3*a
^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-1/2*a^3*arctan(1/(a
^2*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/((a*x-1)/(a*x+1))^(1
/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.37

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{6\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^4x^4+22\,a^3\right.}{6\,a^4x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^4*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 7*a^2*c^4*x^2 - 7*a*c^4*x + 2
*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

3.379.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{c^4\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a}{x\sqrt{\frac{ax}{ax+1}}}\right.\right.}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**4,x)
```

output
```
c**4*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x*
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a/(x**3*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))), x) + Integral(6*a**2/(x**2*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x) + Integral(-4*a**3/(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)))
, x))/a**4
```

3.379.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(100) = 200$.

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.96

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{1}{3}\left(\frac{3\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17\,c^4\left(\frac{ax-1}{ax+1}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-}{(ax+}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/3*(3*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^2 + 9*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21
*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 37
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a
*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a
*x + 1)^4 + a^2))*a
```

3.379.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(100) = 200$.

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.18

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}+\frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^4}{a\mathrm{sgn}\left(ax+1\right)}$$
$$+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4+36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4-9\left(x|a|-\sqrt{a^2\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="giac")
```

output
```
c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + 3*c^4*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)
*c^4/(a*sgn(a*x + 1)) + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)
 + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4 + 36*(x*abs(a) - sqrt(a^2*x^2
 - 1))^2*a*c^4 - 9*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a) + 16*a*c^4)/(
((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

3.379.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.61

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{5\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{17\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^4*((a*x - 1)/(a*x + 1))^(3/2))/
3 + (17*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^4*((a*x - 1)/(a*x + 1))^(
7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6
*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.380 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.380.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{c^3\csc^{-1}(ax)}{2a}-\frac{2c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c^3*(1-1/a^2/x^2)^(3/2)*x+1/2*c^3*arccsc(a*x)/a-2*c^3*arctanh((1-1/a^2/x^2
)^(1/2))/a+1/2*c^3*(4*a+1/x)*(1-1/a^2/x^2)^(1/2)/a^2
```

3.380.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=\frac{c^3\left(1-4ax-3a^2x^2+4a^3x^3+2a^4x^4+2a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+2a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{1}{ax}\right)\right.}{2a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

output
```
(c^3*(1 - 4*a*x - 3*a^2*x^2 + 4*a^3*x^3 + 2*a^4*x^4 + 2*a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 2*a^3*Sqrt[1 - 1/(a^2*x^2)
]*x^3*ArcSin[1/(a*x)] - 4*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1
/(a^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

3.380.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6731, 27, 540, 535, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x^2}{a^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x^2d\frac{1}{x}}{a^2}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^3\left(a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\int \sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)xd\frac{1}{x}\right)}{a^2}$$

$$\Big\downarrow \text{535}$$

$$-\frac{c^3\left(-\frac{1}{2}\int \frac{\left(4a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c^3\left(\frac{1}{2}\left(-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c^3\left(\frac{1}{2}\left(-4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow 243$$

$$-\frac{c^3\left(\frac{1}{2}\left(-2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow 73$$

$$-\frac{c^3\left(\frac{1}{2}\left(4a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow 221$$

$$-\frac{c^3\left(\frac{1}{2}\left(4a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

input

```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

output

```
-((c^3*(-1/2*(Sqrt[1 - 1/(a^2*x^2)]*(4*a + x^(-1))) - a^2*(1 - 1/(a^2*x^2)
)^(3/2)*x + (-(a*ArcSin[1/(a*x)]) + 4*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/2)
)/a^2)
```

3.380.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.380. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.380.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.59

method	result
risch	$\frac{(ax-1)(2a^2x^2+4ax-1)c^3}{2x^2a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{2a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)c^3\sqrt{(ax-1)(ax+1)}}{a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^3\left(-4\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+4(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+4\ln\left(\frac{a^2x+\sqrt{a^2x^2-}}{\sqrt{a^2}}\right.\right.}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^3x^2\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-1)*(2*a^2*x^2+4*a*x-1)/x^2*c^3/a^3/((a*x-1)/(a*x+1))^(1/2)+(-2*a^
3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+1/2*a^2*arctan(1/(a^
2*x^2-1)^(1/2)))*c^3/a^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(
a*x+1)
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx =$$
$$-\frac{2\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^3x^3+6\,a^2c}{2\,a^3x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*a^2*c^3*x^2*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 4*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1
)) - 1) - (2*a^3*c^3*x^3 + 6*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*sqrt((a*x - 1)
/(a*x + 1)))/(a^3*x^2)
```

3.380.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{c^3\left(\int \frac{a^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int \frac{3a}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**3,x)
```

output
```
c**3*(Integral(a**3/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a/(x**2*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**2/(x*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))), x))/a**3
```

3.380.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(78) = 156$.

Time $= 0.29$ (sec) , antiderivative size $= 201$, normalized size of antiderivative $= 2.28$

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\left(\frac{c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-(c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c^3*log(sqrt((a*x - 1)/(a*
x + 1)) + 1)/a^2 - 2*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (3*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqr
t((a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x +
1)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

3.380.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(78) = 156$.

Time $= 0.29$ (sec) , antiderivative size $= 221$, normalized size of antiderivative $= 2.51$

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$= -\frac{c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}\left(ax+1\right)}+\frac{2\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^3}{a\text{sgn}\left(ax+1\right)}$$
$$+\frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|+4\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3-\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|a|+4\,ac^3}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^2a|a|\text{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="giac")
```

output
```
-c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + 2*c^3*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^3/(a*sgn(a*x + 1)) + ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a) + 4*
(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - (x*abs(a) - sqrt(a^2*x^2 - 1))*c^
3*abs(a) + 4*a*c^3)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a)*sgn
(a*x + 1))
```

3.380.9 Mupad [B] (verification not implemented)

Time $= 0.12$ (sec) , antiderivative size $= 163$, normalized size of antiderivative $= 1.85$

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-3\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,\left(a\,x-1\right)}{a\,x+1}-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 3
*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x -
 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^3*atan(((a*x - 1)/(
a*x + 1))^(1/2)))/a - (4*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.381 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$

3.381.1 Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+c^2*(a+1/x)*x*(1-1/a^
2/x^2)^(1/2)/a
```

3.381.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(62) = 124.

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.55

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$=\frac{c^2\left(-2-2ax+2a^2x^2+2a^3x^3-2a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{1}{ax}\right)-2a^2\sqrt{}\right.}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

output
```
(c^2*(-2 - 2*a*x + 2*a^2*x^2 + 2*a^3*x^3 - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2
*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[
1/(a*x)] - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
)/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

3.381.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6731, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \frac{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}{a}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 536}$$

$$-\frac{c^2\left(\int \frac{\left(-1-\frac{1}{ax}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-x\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)\right)}{a}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{c^2\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\left(a+\frac{1}{x}\right)\right)}{a}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{c^2\left(-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\left(a+\frac{1}{x}\right)-\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{c^2\left(-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\left(a+\frac{1}{x}\right)-\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

$$\Big\downarrow 73$$

$$-\frac{c^2\left(a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)-\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c^2\left(\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\left(a+\frac{1}{x}\right)-\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

output
```
-((c^2*(-(Sqrt[1 - 1/(a^2*x^2)]*(a + x^(-1))*x) - ArcSin[1/(a*x)] + ArcTan
h[Sqrt[1 - 1/(a^2*x^2)]]))/a)
```

3.381.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.381.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(58) = 116$.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(ax-1)c^{2}}{x\,a^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{a\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{\sqrt{a^{2}}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\right)c^{2}\sqrt{(ax-1)(ax+1)}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^{2}\left(-\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+(a^{2}x^{2}-1)^{\frac{3}{2}}\sqrt{a^{2}}-\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax-ax\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{2}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x\sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/x*c^2/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(-a*ln(a^2*x/(a^2)^(1/2)+(a^
2*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(
1/2)))*c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.381.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx =$$
$$-\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2c^2x^2+2\,ac^2x+c^2\right)\sqrt{\frac{a}{a}}}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
-(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c^2*x*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) - a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x
^2 + 2*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

3.381.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$=\frac{c^2\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{2a}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**2,x)
```

output
```
c**2*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a/(x*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))), x))/a**2
```

3.381.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.02

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*
c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c^2*log(sqrt((a*x - 1)/(a*x +
1)) + 1)/a^2 - c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

3.381.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(58) = 116$.

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.21

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = &-\frac{2\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}\,(ax+1)} \\ &+\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}c^2}{a\text{sgn}\,(ax+1)} \\ &+\frac{2\,c^2}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\text{sgn}\,(ax+1)}\end{aligned}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="giac")
```

output
```
-2*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + c^2*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^2/(a*sgn(a*x + 1)) + 2*c^2/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs
(a)*sgn(a*x + 1))
```

3.381.9 Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(4*c^2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2
*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^2*atanh(((a*x - 1)/(a*x +
 1))^(1/2)))/a
```

3.382 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.382.1 Optimal result

Integrand size = 18, antiderivative size = 27

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\csc^{-1}(ax)}{a}$$

output
```
c*arccsc(a*x)/a+c*x*(1-1/a^2/x^2)^(1/2)
```

3.382.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x+\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

output
```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x + ArcSin[1/(a*x)]))/a
```

3.382.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6731, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)e^{\coth^{-1}(ax)}dx$$
$$\Big\downarrow \text{6731}$$
$$-c\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{247}$$
$$-c\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}\right)$$
$$\Big\downarrow \text{223}$$
$$-c\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\arcsin\left(\frac{1}{ax}\right)}{a}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

output
```
-(c*(-(Sqrt[1 - 1/(a^2*x^2)]*x) - ArcSin[1/(a*x)]/a))
```

3.382.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.382.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{(ax-1)c\left(\sqrt{a^2x^2-1}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a}$	63

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/((a*x-1)*(a*x+1))^(1/2)*c/a*((a^2*x^2-1)
^(1/2)+arctan(1/(a^2*x^2-1)^(1/2)))
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="fricas")
```

output
```
-(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*c*x + c)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

3.382.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(\int\frac{a}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x),x)
```

output
```
c*(Integral(a/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))), x))/a
```

3.382.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arct
an(sqrt((a*x - 1)/(a*x + 1)))/a^2)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{c\arctan\left(\sqrt{a^2x^2-1}\right)-\sqrt{a^2x^2-1}c}{a\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="giac")
```

output
```
-(c*arctan(sqrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)*c)/(a*sgn(a*x + 1))
```

3.382.9 Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input

```
int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
(2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*ata
n(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.383 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.383.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
2*arctanh((1-1/a^2/x^2)^(1/2))/a/c-2*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(
1-1/a^2/x^2)^(1/2)/c
```

3.383.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-3+ax)+2(-1+ax)\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3 + a*x) + 2*(-1 + a*x)*Log[(1 + Sqrt[1 - 1/(
a^2*x^2)])*x])/(a*c*(-1 + a*x))
```

3.383.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6731, 27, 564, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^2\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{564}$$

$$-\frac{a^2\left(\frac{\int \frac{\left(a+\frac{2}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\left(\frac{\int \frac{\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^3}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{534}$$

$$-\frac{a^2\left(\frac{2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^3}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{243}$$

$$-\frac{a^2\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^3}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow 73$$

$$-\frac{a^2\left(\frac{-2a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^3}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow 221$$

$$-\frac{a^2\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{-2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^3}\right)}{c}$$

input

```
Int[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

output

```
-((a^2*((2*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (-(a*Sqrt[1 - 1/(a^
2*x^2)]*x) - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a^3))/c)
```

3.383.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.383. $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.383.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{2\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a\sqrt{a^{2}}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{a^{3}\left(x-\frac{1}{a}\right)}\right)a\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+2\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x^{2}-4\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{2}x-((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}-4}{a\sqrt{a^{2}}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-1}{ax+1}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(2/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1
)^(1/2))/(a^2)^(1/2)-2/a^3/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a/c/
((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.383.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-2\,ax-3)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

output
```
(2*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2*(a*x - 1)*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 2*a*x - 3)*sqrt((a*x - 1)/(a*x + 1)))/
(a^2*c*x - a*c)
```

3.383.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\int \frac{x}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

output
```
a*Integral(x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))), x)/c
```

3.383.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.66

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -2\,a\left(\frac{\frac{2\,(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*((2*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2
*c*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c)
 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

3.383.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

output
```
undef
```

3.383.9 Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\,a\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-6}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a*x + 8*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 6)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.384 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.384.1 Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
-4/3*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(3/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/
c^2+1/3*(-9*a-11/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^2
```

3.384.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{14-5ax-16a^2x^2+3a^3x^3+9a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

output
```
(14 - 5*a*x - 16*a^2*x^2 + 3*a^3*x^3 + 9*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a
*x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x*(-1
 + a*x))
```

3.384.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^3\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^3x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^3c^2}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(3a^3+\frac{9a^2}{x}+\frac{8a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^3+\frac{9a^2}{x}+\frac{8a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{3}\left(\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{3a^2\left(a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{1}{3}\left(3a^2\int \frac{\left(a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$$\Big\downarrow 534$$

$$-\frac{\frac{1}{3}\left(3a^2\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$$\Big\downarrow 243$$

$$-\frac{\frac{1}{3}\left(3a^2\left(\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$$\Big\downarrow 73$$

$$-\frac{\frac{1}{3}\left(3a^2\left(-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{1}{3}\left(3a^2\left(-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

input

```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

output

```
-(((4*a*(a + x^(-1)))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((a*(9*a + 11/x))/Sqrt
[1 - 1/(a^2*x^2)] + 3*a^2*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 3*ArcTanh[Sqrt[1
 - 1/(a^2*x^2)]]))/3)/(a^3*c^2))
```

3.384. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.384.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.384. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.384.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.76

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{3a^{5}\left(x-\frac{1}{a}\right)^{2}}-\frac{13\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{3a^{4}\left(x-\frac{1}{a}\right)}\right)a^{2}\sqrt{(ax-1)(ax+1)}}{c^{2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$9\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{4}x^{3}+9\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{3}x^{3}-27\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}-6\sqrt{a^{2}}\,((ax-1)(ax+1))$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-2/3/a^5/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-13/3/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^2/c^2/((a*x-1)/(a
*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.384.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= \frac{9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-16\,a^2x^2-5\,ax\right.}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x
^2 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a
*c^2)
```

3.384.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

output
```
a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c
**2
```

3.384.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= \frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^2))
```

3.384.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.384.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2} - \frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)} - \frac{6\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{1}{3}}{a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} - a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((11*(a*x - 1))/(3*(a*x +
 1)) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c^2*((a*x - 1)/(a*x + 1))^(3/
2) - a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.385 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

3.385.1 Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx = -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output
```
-8/5*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(5/2)-4/15*(5*a+8/x)/a^2/c^3/(1-1/a^2/x
^2)^(3/2)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c^3+1/15*(-60*a-79/x)/a^2/c^3/(
1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^3
```

3.385.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx = \frac{-94+128ax+73a^2x^2-134a^3x^3+15a^4x^4+60a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

output
```
(-94 + 128*a*x + 73*a^2*x^2 - 134*a^3*x^3 + 15*a^4*x^4 + 60*a*Sqrt[1 - 1/(
a^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^3*Sqrt[
1 - 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

3.385.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {6731, 27, 570, 532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{a^4\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^4\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^4x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^4c^3}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(5a^4+\frac{20a^3}{x}+\frac{27a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^4c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\int \frac{\left(5a^4+\frac{20a^3}{x}+\frac{27a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow\ 2336$$

$$-\frac{\frac{1}{5}\left(\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(15a^4+\frac{60a^3}{x}+\frac{64a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow\ 25$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(15a^4+\frac{60a^3}{x}+\frac{64a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow\ 2336$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{15a^3\left(a+\frac{4}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow\ 27$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^3\int \frac{\left(a+\frac{4}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow\ 534$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^3\left(4\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow\ 243$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^3\left(2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow\ 73$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^3\left(-4a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow\ 221$$

3.385. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

$$-\frac{\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{1}{5}\left(\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+15a^3\left(-4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)\right)\right)}{a^4c^3}$$

input

```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

output

```
-(((8*a^2*(a + x^(-1)))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((4*a^2*(5*a + 8/x))
/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((a^2*(60*a + 79/x))/Sqrt[1 - 1/(a^2*x^2)]
+ 15*a^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
)/3)/5)/(a^4*c^3))
```

3.385.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.385. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

3.385.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.63

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^7\left(x-\frac{1}{a}\right)^3}-\frac{31\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{15a^6\left(x-\frac{1}{a}\right)^2}-\frac{104\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{15a^5\left(x-\frac{1}{a}\right)}\right)a^3}{c^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-60\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^4x^4-60\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^5x^4+45\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+240\sqrt{a^2}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(4/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-2/5/a^7/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-31/15/a^6/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-104/15/a^5/(x-1/a
)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^3/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x
-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int\frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$=\frac{60\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-60\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/15*(60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
 1) - 60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
 1) + (15*a^4*x^4 - 134*a^3*x^3 + 73*a^2*x^2 + 128*a*x - 94)*sqrt((a*x - 1
)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

3.385.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3 \int \frac{x^3}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

output
```
a**3*Integral(x**3/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

3.385.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= \frac{1}{30}a\left(\frac{\frac{22\,(ax-1)}{ax+1}+\frac{155\,(ax-1)^2}{(ax+1)^2}-\frac{240\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
1/30*a*((22*(a*x - 1)/(a*x + 1) + 155*(a*x - 1)^2/(a*x + 1)^2 - 240*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*
x - 1)/(a*x + 1))^(5/2)) + 120*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3
) - 120*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

3.385.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^3|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac^3\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
-4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^3*abs(a)*sgn(a*x + 1)) + sqr
t(a^2*x^2 - 1)/(a*c^3*sgn(a*x + 1))
```

3.385.9 Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{31\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{16\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{22\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((31*(a*x - 1)^2)/(3*(a*x
 + 1)^2) - (16*(a*x - 1)^3)/(a*x + 1)^3 + (22*(a*x - 1))/(15*(a*x + 1)) +
1/5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^
(7/2))
```

3.386 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

3.386.1 Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}+\frac{5\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
-16/7*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-4/35*(7*a+17/x)/a^2/c^4/(1-1/a^2
/x^2)^(5/2)+1/105*(-175*a-307/x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)+5*arctanh((1-
1/a^2/x^2)^(1/2))/a/c^4+1/105*(-525*a-719/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+x
*(1-1/a^2/x^2)^(1/2)/c^4
```

3.386.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.65

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

$$= \frac{824-1947ax+485a^2x^2+1812a^3x^3-1339a^4x^4+105a^5x^5+525a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^3\operatorname{arctanh}\Big(\sqrt{1-}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^3}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

output
```
(824 - 1947*a*x + 485*a^2*x^2 + 1812*a^3*x^3 - 1339*a^4*x^4 + 105*a^5*x^5
+ 525*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]
])/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

3.386.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6731, 27, 570, 532, 25, 2336, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{a^5\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^5\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^5\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^5x^2}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^5c^4}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}\int -\frac{\left(7a^5+\frac{35a^4}{x}+\frac{61a^3}{x^2}-\frac{7a^2}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^5c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{7}\int \frac{\left(7a^5+\frac{35a^4}{x}+\frac{61a^3}{x^2}-\frac{7a^2}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \; 2336$$

$$-\frac{\frac{1}{7}\left(\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(35a^5+\frac{175a^4}{x}+\frac{272a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\int \frac{\left(35a^5+\frac{175a^4}{x}+\frac{272a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \; 2336$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(105a^5+\frac{525a^4}{x}+\frac{614a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(105a^5+\frac{525a^4}{x}+\frac{614a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \; 2336$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{105a^4\left(a+\frac{5}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a^4\int \frac{\left(a+\frac{5}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \; 534$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a^4\left(5\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3(}{7(1-\frac{1}{a^2}}}{a^5c^4}$$

$$\Big\downarrow \; 243$$

3.386. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a^4\left(\frac{5}{2}\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3}{7\left(1-\right.}}{a^5c^4}$$

$$\Big\downarrow 73$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a^4\left(-5a^2\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3(7}{5\left(1-\right.}}{a^5c^4}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{1}{7}\left(\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{1}{5}\left(\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(105a^4\left(-5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3(5}{\sqrt{}}\right.\right.\right.}{a^5c^4}$$

input

```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

output

```
-(((16*a^3*(a + x^(-1)))/(7*(1 - 1/(a^2*x^2))^(7/2)) + ((4*a^3*(7*a + 17/x
))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((a^3*(175*a + 307/x))/(3*(1 - 1/(a^2*x^2
))^(3/2)) + ((a^3*(525*a + 719/x))/Sqrt[1 - 1/(a^2*x^2)] + 105*a^4*(-(a*Sq
rt[1 - 1/(a^2*x^2)]*x) - 5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/3)/5)/7)/(a^5*
c^4))
```

3.386.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.386. $\int\frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.386. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.386.4 Maple [A] (verified)

Time $= 0.17$ (sec) , antiderivative size $= 265$, normalized size of antiderivative $= 1.55$

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{57\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{35a^8\left(x-\frac{1}{a}\right)^3}-\frac{446\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{105a^7\left(x-\frac{1}{a}\right)^2}-\frac{1024\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{105a^6\left(x-\frac{1}{a}\right)}-2\sqrt{(}\right.}{c^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5-525\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^6x^5+420((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+2625\sqrt{a^2}\,\sqrt{(ax-1)(}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(5/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-57/35/a^8/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(
1/2)-446/105/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1024/105/a^6/
(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-2/7/a^9/(x-1/a)^4*((x-1/a)^2*a^2
+2*(x-1/a)*a)^(1/2))*a^4/c^4/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/
2)/(a*x+1)
```

3.386.5 Fricas [A] (verification not implemented)

Time $= 0.25$ (sec) , antiderivative size $= 204$, normalized size of antiderivative $= 1.19$

$$\int\frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{525\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-525\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\Big(}{105\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/105*(525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1
)/(a*x + 1)) + 1) - 525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(
sqrt((a*x - 1)/(a*x + 1)) - 1) + (105*a^5*x^5 - 1339*a^4*x^4 + 1812*a^3*x^
3 + 485*a^2*x^2 - 1947*a*x + 824)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x^4
- 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

3.386.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{a^4\int \frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

output
```
a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1)) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))), x)/c**4
```

3.386.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{1}{420}a\left(\frac{\frac{111\,(ax-1)}{ax+1}+\frac{469\,(ax-1)^2}{(ax+1)^2}+\frac{2765\,(ax-1)^3}{(ax+1)^3}-\frac{4200\,(ax-1)^4}{(ax+1)^4}+15}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{2100\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{2100\log\left(\sqrt{\frac{ax}{ax}}\right.}{a^2c^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/420*a*((111*(a*x - 1)/(a*x + 1) + 469*(a*x - 1)^2/(a*x + 1)^2 + 2765*(a*
x - 1)^3/(a*x + 1)^3 - 4200*(a*x - 1)^4/(a*x + 1)^4 + 15)/(a^2*c^4*((a*x -
 1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 2100*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 2100*log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^4))
```

3.386.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.386.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4} - \frac{\frac{67\,(a\,x-1)^2}{15\,(a\,x+1)^2}+\frac{79\,(a\,x-1)^3}{3\,(a\,x+1)^3}-\frac{40\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{37\,(a\,x-1)}{35\,(a\,x+1)}+\frac{1}{7}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

input
```
int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((67*(a*x - 1)^2)/(15*(a
*x + 1)^2) + (79*(a*x - 1)^3)/(3*(a*x + 1)^3) - (40*(a*x - 1)^4)/(a*x + 1)
^4 + (37*(a*x - 1))/(35*(a*x + 1)) + 1/7)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(
7/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2))
```

3.387 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

3.387.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(x)}{a}$$

output
```
-1/4*c^5/a^5/x^4+c^5/a^4/x^3-c^5/a^3/x^2-2*c^5/a^2/x+c^5*x-3*c^5*ln(x)/a
```

3.387.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=-\frac{c^5\left(\frac{5a^4}{4}+\frac{1}{4x^4}-\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{2a^3}{x}-a^5x+3a^4\log(x)\right)}{a^5}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

output
```
-((c^5*((5*a^4)/4 + 1/(4*x^4) - a/x^3 + a^2/x^2 + (2*a^3)/x - a^5*x + 3*a^
4*Log[x]))/a^5)
```

3.387.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^5 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^5 e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^5}{a^5}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^5\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^5dx}{a^5}$$

$$\Big\downarrow \text{6681}$$

$$\frac{c^5\int \frac{e^{2\text{arctanh}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$

$$\Big\downarrow \text{6679}$$

$$\frac{c^5\int \frac{(1-ax)^4(ax+1)}{x^5}dx}{a^5}$$

$$\Big\downarrow \text{84}$$

$$\frac{c^5\int \left(a^5-\frac{3a^4}{x}+\frac{2a^3}{x^2}+\frac{2a^2}{x^3}-\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^5\left(a^5x-3a^4\log(x)-\frac{2a^3}{x}-\frac{a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a^5}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

output
```
(c^5*(-1/4*1/x^4 + a/x^3 - a^2/x^2 - (2*a^3)/x + a^5*x - 3*a^4*Log[x]))/a^
5
```

3.387.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.387. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

3.387.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
default	$\frac{c^5\left(a^5x-3a^4\ln(x)-\frac{1}{4x^4}+\frac{a}{x^3}-\frac{a^2}{x^2}-\frac{2a^3}{x}\right)}{a^5}$
risch	$c^5x+\frac{-2a^3c^5x^3-a^2c^5x^2+a\,c^5x-\frac{1}{4}c^5}{a^5x^4}-\frac{3c^5\ln(x)}{a}$
norman	$\frac{c^5x+a^4c^5x^5-\frac{c^5}{4a}-a\,c^5x^2-2c^5a^2x^3}{a^4x^4}-\frac{3c^5\ln(x)}{a}$
parallelrisch	$-\frac{-4a^5c^5x^5+12c^5\ln(x)a^4x^4+8a^3c^5x^3+4a^2c^5x^2-4a\,c^5x+c^5}{4a^5x^4}$
meijerg	$-\frac{c^5(-ax-\ln(-ax+1))}{a}-\frac{4c^5\ln(-ax+1)}{a}-\frac{5c^5(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{5c^5\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5/a^5*(a^5*x-3*a^4*ln(x)-1/4/x^4+a/x^3-a^2/x^2-2*a^3/x)
```

3.387.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{4\,a^5c^5x^5-12\,a^4c^5x^4\log{(x)}-8\,a^3c^5x^3-4\,a^2c^5x^2+4\,ac^5x-c^5}{4\,a^5x^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="fricas")
```

output
```
1/4*(4*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) - 8*a^3*c^5*x^3 - 4*a^2*c^5*x^2
 + 4*a*c^5*x - c^5)/(a^5*x^4)
```

3.387.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{a^5c^5x-3a^4c^5\log{(x)}+\frac{-8a^3c^5x^3-4a^2c^5x^2+4ac^5x-c^5}{4x^4}}{a^5}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**5,x)
```

output
```
(a**5*c**5*x - 3*a**4*c**5*log(x) + (-8*a**3*c**5*x**3 - 4*a**2*c**5*x**2
+ 4*a*c**5*x - c**5)/(4*x**4))/a**5
```

3.387.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=c^5x-\frac{3\,c^5\log{(x)}}{a}-\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="maxima")
```

output
```
c^5*x - 3*c^5*log(x)/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*x +
c^5)/(a^5*x^4)
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=c^5x-\frac{3\,c^5\log{(|x|)}}{a}-\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="giac")
```

output
```
c^5*x - 3*c^5*log(abs(x))/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5
*x + c^5)/(a^5*x^4)
```

3.387.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=-\frac{c^5\,\left(4\,a^2\,x^2-4\,a\,x+8\,a^3\,x^3-4\,a^5\,x^5+12\,a^4\,x^4\,\ln\left(x\right)+1\right)}{4\,a^5\,x^4}$$

input

```
int(((c - c/(a*x))^5*(a*x + 1))/(a*x - 1),x)
```

output

```
-(c^5*(4*a^2*x^2 - 4*a*x + 8*a^3*x^3 - 4*a^5*x^5 + 12*a^4*x^4*log(x) + 1))
/(4*a^5*x^4)
```

3.388 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.388.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(x)}{a}$$

output
```
1/3*c^4/a^4/x^3-c^4/a^3/x^2+c^4*x-2*c^4*ln(x)/a
```

3.388.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\left(\frac{4a^3}{3}-\frac{1}{3x^3}+\frac{a}{x^2}-a^4x+2a^3\log(x)\right)}{a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
-((c^4*((4*a^3)/3 - 1/(3*x^3) + a/x^2 - a^4*x + 2*a^3*Log[x]))/a^4)
```

3.388.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^4 e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^4dx}{a^4}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{c^4\int \frac{e^{2\text{arctanh}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{c^4\int \frac{(1-ax)^3(ax+1)}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{84}$$

$$-\frac{c^4\int \left(-a^4+\frac{2a^3}{x}-\frac{2a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4\left(a^4(-x)+2a^3\log(x)+\frac{a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
-((c^4*(-1/3*1/x^3 + a/x^2 - a^4*x + 2*a^3*Log[x]))/a^4)
```

3.388.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.388. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

3.388.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^{4}\left(a^{4}x-2a^{3}\ln\left(x\right)+\frac{1}{3x^{3}}-\frac{a}{x^{2}}\right)}{a^{4}}$
risch	$c^{4}x+\frac{-a\,c^{4}x+\frac{1}{3}c^{4}}{a^{4}x^{3}}-\frac{2c^{4}\ln\left(x\right)}{a}$
norman	$\frac{a^{3}c^{4}x^{4}+\frac{c^{4}}{3a}-c^{4}x}{a^{3}x^{3}}-\frac{2c^{4}\ln\left(x\right)}{a}$
parallelrisch	$-\frac{-3a^{4}c^{4}x^{4}+6c^{4}\ln\left(x\right)a^{3}x^{3}+3a\,c^{4}x-c^{4}}{3a^{4}x^{3}}$
meijerg	$-\frac{c^{4}\left(-ax-\ln\left(-ax+1\right)\right)}{a}-\frac{3c^{4}\ln\left(-ax+1\right)}{a}-\frac{2c^{4}\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)\right)}{a}+\frac{2c^{4}\left(\ln\left(-ax+1\right)-\ln\left(x\right)-\ln\left(-a\right)+\frac{1}{ax}\right)}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4/a^4*(a^4*x-2*a^3*ln(x)+1/3/x^3-a/x^2)
```

3.388.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{4}\,dx=\frac{3\,a^{4}c^{4}x^{4}-6\,a^{3}c^{4}x^{3}\log\left(x\right)-3\,ac^{4}x+c^{4}}{3\,a^{4}x^{3}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^4*x^4 - 6*a^3*c^4*x^3*log(x) - 3*a*c^4*x + c^4)/(a^4*x^3)
```

3.388.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{4}\,dx=\frac{a^{4}c^{4}x-2a^{3}c^{4}\log\left(x\right)+\frac{-3ac^{4}x+c^{4}}{3x^{3}}}{a^{4}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**4,x)
```

output
```
(a**4*c**4*x - 2*a**3*c**4*log(x) + (-3*a*c**4*x + c**4)/(3*x**3))/a**4
```

3.388.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = c^4x-\frac{2\,c^4\log\left(x\right)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
c^4*x - 2*c^4*log(x)/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

3.388.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = c^4x-\frac{2\,c^4\log\left(|x|\right)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="giac")
```

output
```
c^4*x - 2*c^4*log(abs(x))/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

3.388.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = -\frac{c^4\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln\left(x\right)-1\right)}{3\,a^4\,x^3}$$

input
```
int(((c - c/(a*x))^4*(a*x + 1))/(a*x - 1),x)
```

output
```
-(c^4*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

3.389 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.389.1 Optimal result

Integrand size = 22, antiderivative size = 39

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(x)}{a}$$

output
```
-1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x-c^3*ln(x)/a
```

3.389.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3\left(3a^2+\frac{1}{x^2}-\frac{2a}{x}-2a^3x+2a^2\log(x)\right)}{2a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
-1/2*(c^3*(3*a^2 + x^(-2) - (2*a)/x - 2*a^3*x + 2*a^2*Log[x]))/a^3
```

3.389.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^3 e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3}{a^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3dx}{a^3}$$

$$\Big\downarrow \text{6681}$$

$$\frac{c^3\int \frac{e^{2\text{arctanh}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{6679}$$

$$\frac{c^3\int \frac{(1-ax)^2(ax+1)}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{84}$$

$$\frac{c^3\int \left(a^3-\frac{a^2}{x}-\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(a^3x-a^2\log(x)+\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
(c^3*(-1/2*1/x^2 + a/x + a^3*x - a^2*Log[x]))/a^3
```

3.389.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.389. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.389.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^{3}\left(a^{3}x-a^{2}\ln(x)-\frac{1}{2x^{2}}+\frac{a}{x}\right)}{a^{3}}$
risch	$c^{3}x+\frac{a\,c^{3}x-\frac{1}{2}c^{3}}{a^{3}x^{2}}-\frac{c^{3}\ln(x)}{a}$
norman	$\frac{c^{3}x+a^{2}c^{3}x^{3}-\frac{c^{3}}{2a}}{a^{2}x^{2}}-\frac{c^{3}\ln(x)}{a}$
parallelrisch	$-\frac{-2a^{3}c^{3}x^{3}+2c^{3}\ln(x)a^{2}x^{2}-2a\,c^{3}x+c^{3}}{2a^{3}x^{2}}$
meijerg	$-\frac{c^{3}(-ax-\ln(-ax+1))}{a}-\frac{2c^{3}\ln(-ax+1)}{a}+\frac{2c^{3}\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)}{a}+\frac{c^{3}\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/a^3*(a^3*x-a^2*ln(x)-1/2/x^2+a/x)
```

3.389.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3}dx=\frac{2\,a^{3}c^{3}x^{3}-2\,a^{2}c^{3}x^{2}\log\left(x\right)+2\,ac^{3}x-c^{3}}{2\,a^{3}x^{2}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^3*c^3*x^3 - 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x - c^3)/(a^3*x^2)
```

3.389.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3}dx=\frac{a^{3}c^{3}x-a^{2}c^{3}\log\left(x\right)+\frac{2ac^{3}x-c^{3}}{2x^{2}}}{a^{3}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**3,x)
```

output
```
(a**3*c**3*x - a**2*c**3*log(x) + (2*a*c**3*x - c**3)/(2*x**2))/a**3
```

3.389.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=c^3x-\frac{c^3\log(x)}{a}+\frac{2\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
c^3*x - c^3*log(x)/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

3.389.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=c^3x-\frac{c^3\log(|x|)}{a}+\frac{2\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="giac")
```

output
```
c^3*x - c^3*log(abs(x))/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

3.389.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\,(2\,a\,x+2\,a^3\,x^3-2\,a^2\,x^2\,\ln(x)-1)}{2\,a^3\,x^2}$$

input
```
int(((c - c/(a*x))^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^3*(2*a*x + 2*a^3*x^3 - 2*a^2*x^2*log(x) - 1))/(2*a^3*x^2)
```

3.390 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.390.1 Optimal result

Integrand size = 22, antiderivative size = 16

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x$$

output
```
c^2/a^2/x+c^2*x
```

3.390.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
c^2/(a^2*x) + c^2*x
```

3.390.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6717, 27, 6681, 6679, 82, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6717$$

$$-\int \frac{c^2 e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2}{a^2}dx$$

$$\Big\downarrow \; 27$$

$$-\frac{c^2 \int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2dx}{a^2}$$

$$\Big\downarrow \; 6681$$

$$-\frac{c^2 \int \frac{e^{2\text{arctanh}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \; 6679$$

$$-\frac{c^2 \int \frac{(1-ax)(ax+1)}{x^2}dx}{a^2}$$

$$\Big\downarrow \; 82$$

$$-\frac{c^2 \int \frac{1-a^2x^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \; 244$$

$$-\frac{c^2 \int \left(\frac{1}{x^2}-a^2\right)dx}{a^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{c^2\left(a^2(-x)-\frac{1}{x}\right)}{a^2}$$

input

```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
-((c^2*(-x^(-1) - a^2*x))/a^2)
```

3.390.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.390. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.390.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	s
default	$\frac{c^2\left(a^2x+\frac{1}{x}\right)}{a^2}$	1
risch	$\frac{c^2}{a^2x}+c^2x$	1
gosper	$\frac{c^2\left(a^2x^2+1\right)}{x\,a^2}$	2
parallelrisch	$\frac{a^2c^2x^2+c^2}{a^2x}$	2
norman	$\frac{\frac{c^2}{a}+a\,c^2x^2}{ax}$	2
meijerg	$-\frac{c^2(-ax-\ln(-ax+1))}{a}-\frac{c^2\ln(-ax+1)}{a}+\frac{c^2(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{c^2\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)}{a}$	9

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/a^2*(a^2*x+1/x)
```

3.390.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2+c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
(a^2*c^2*x^2 + c^2)/(a^2*x)
```

3.390.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x+\frac{c^2}{x}}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**2,x)
```

output
```
(a**2*c**2*x + c**2/x)/a**2
```

3.390.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
c^2*x + c^2/(a^2*x)
```

3.390.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="giac")
```

output
```
c^2*x + c^2/(a^2*x)
```

3.390.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\,\left(a^2\,x^2+1\right)}{a^2\,x}$$

input
```
int(((c - c/(a*x))^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^2*(a^2*x^2 + 1))/(a^2*x)
```

3.391 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.391.1 Optimal result

Integrand size = 20, antiderivative size = 11

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}$$

output
```
c*x+c*ln(x)/a
```

3.391.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
c*x + (c*Log[x])/a
```

3.391.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6681, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{ce^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)}{a}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)dx}{a}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{c\int \frac{e^{2\text{arctanh}(ax)}(1-ax)}{x}dx}{a}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{c\int \frac{ax+1}{x}dx}{a}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)dx}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c(ax+\log(x))}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
(c*(a*x + Log[x]))/a
```

3.391.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.391.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{c(ax+\ln(x))}{a}$	12
norman	$cx+\frac{c\ln(x)}{a}$	12
risch	$cx+\frac{c\ln(x)}{a}$	12
parallelrisch	$\frac{acx+c\ln(x)}{a}$	14
meijerg	$-\frac{c(-ax-\ln(-ax+1))}{a}+\frac{c(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}$	43

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
c/a*(a*x+ln(x))
```

3.391.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+c\log\left(x\right)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="fricas")
```

output
```
(a*c*x + c*log(x))/a
```

3.391.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+c\log\left(x\right)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x)
```

output
```
(a*c*x + c*log(x))/a
```

3.391.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log\left(x\right)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="maxima")
```

output
```
c*x + c*log(x)/a
```

3.391.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log\left(|x|\right)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="giac")
```

output
```
c*x + c*log(abs(x))/a
```

3.391.9 Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(\ln\left(x\right)+a\,x\right)}{a}$$

input
```
int(((c - c/(a*x))*(a*x + 1))/(a*x - 1),x)
```

output
```
(c*(log(x) + a*x))/a
```

3.392 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.392.1 Optimal result

Integrand size = 22, antiderivative size = 37

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}+\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}$$

output
```
x/c+2/a/c/(-a*x+1)+3*ln(-a*x+1)/a/c
```

3.392.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{ax+\frac{2}{1-ax}+3\log(1-ax)}{ac}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
(a*x + 2/(1 - a*x) + 3*Log[1 - a*x])/(a*c)
```

3.392.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{ae^{2\text{arctanh}(ax)}}{c\left(a-\frac{1}{x}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{e^{2\text{arctanh}(ax)}}{a-\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{6681}$$

$$\frac{a\int \frac{e^{2\text{arctanh}(ax)}x}{1-ax}dx}{c}$$

$$\Big\downarrow \text{6679}$$

$$\frac{a\int \frac{x(ax+1)}{(1-ax)^2}dx}{c}$$

$$\Big\downarrow \text{86}$$

$$\frac{a\int \left(\frac{3}{(ax-1)a}+\frac{2}{(ax-1)^2a}+\frac{1}{a}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{2}{a^2(1-ax)}+\frac{3\log(1-ax)}{a^2}+\frac{x}{a}\right)}{c}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
(a*(x/a + 2/(a^2*(1 - a*x)) + (3*Log[1 - a*x])/a^2))/c
```

3.392. $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$

3.392.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.392. $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.392.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{a\left(\frac{x}{a}-\frac{2}{a^{2}(ax-1)}+\frac{3\ln(ax-1)}{a^{2}}\right)}{c}$	35
risch	$\frac{x}{c}-\frac{2}{ac(ax-1)}+\frac{3\ln(ax-1)}{ac}$	36
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{3x}{c}}{ax-1}+\frac{3\ln(ax-1)}{ac}$	39
parallelrisch	$\frac{a^{2}x^{2}+3a\ln(ax-1)x-3ax-3\ln(ax-1)}{c(ax-1)a}$	45

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
a/c*(x/a-2/a^2/(a*x-1)+3/a^2*ln(a*x-1))
```

3.392.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a^2x^2-ax+3\,(ax-1)\log\left(ax-1\right)-2}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="fricas")
```

output
```
(a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

3.392.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{2}{a^2cx-ac}+\frac{x}{c}+\frac{3\log\left(ax-1\right)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x)
```

output
```
-2/(a**2*c*x - a*c) + x/c + 3*log(a*x - 1)/(a*c)
```

3.392.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}-\frac{2}{a^2cx-ac}+\frac{3\log{(ax-1)}}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="maxima")
```

output
```
x/c - 2/(a^2*c*x - a*c) + 3*log(a*x - 1)/(a*c)
```

3.392.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}+\frac{3\log{(|ax-1|)}}{ac}-\frac{2}{(ax-1)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="giac")
```

output
```
x/c + 3*log(abs(a*x - 1))/(a*c) - 2/((a*x - 1)*a*c)
```

3.392.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}+\frac{2}{a\,(c-a\,c\,x)}+\frac{3\ln{(a\,x-1)}}{a\,c}$$

input
```
int((a*x + 1)/((c - c/(a*x))*(a*x - 1)),x)
```

output
```
x/c + 2/(a*(c - a*c*x)) + (3*log(a*x - 1))/(a*c)
```

3.393 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.393.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{1}{ac^2(1-ax)^2}+\frac{5}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

output
```
x/c^2-1/a/c^2/(-a*x+1)^2+5/a/c^2/(-a*x+1)+4*ln(-a*x+1)/a/c^2
```

3.393.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{a^2\left(-\frac{x}{a^2}+\frac{1}{a^3(1-ax)^2}-\frac{5}{a^3(1-ax)}-\frac{4\log(1-ax)}{a^3}\right)}{c^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
-((a^2*(-(x/a^2) + 1/(a^3*(1 - a*x)^2) - 5/(a^3*(1 - a*x)) - (4*Log[1 - a*
x])/a^3))/c^2)
```

3.393.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^2 e^{2\text{arctanh}(ax)}}{c^2\left(a-\frac{1}{x}\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{e^{2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^2\int \frac{e^{2\text{arctanh}(ax)}x^2}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^2\int \frac{x^2(ax+1)}{(1-ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^2\int \left(-\frac{1}{a^2}-\frac{4}{a^2(ax-1)}-\frac{5}{a^2(ax-1)^2}-\frac{2}{a^2(ax-1)^3}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(-\frac{5}{a^3(1-ax)}+\frac{1}{a^3(1-ax)^2}-\frac{4\log(1-ax)}{a^3}-\frac{x}{a^2}\right)}{c^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
-((a^2*(-(x/a^2) + 1/(a^3*(1 - a*x)^2) - 5/(a^3*(1 - a*x)) - (4*Log[1 - a*
x])/a^3))/c^2)
```

3.393. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$

3.393.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.393. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.393.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x}{c^{2}}+\frac{-5c^{2}x+\frac{4c^{2}}{a}}{c^{4}\left(ax-1\right)^{2}}+\frac{4\ln\left(ax-1\right)}{a\,c^{2}}$	47
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{1}{a^{3}\left(ax-1\right)^{2}}-\frac{5}{a^{3}\left(ax-1\right)}+\frac{4\ln\left(ax-1\right)}{a^{3}}\right)}{c^{2}}$	49
norman	$\frac{\frac{a^{2}x^{3}}{c}-\frac{6a\,x^{2}}{c}+\frac{4x}{c}}{c\left(ax-1\right)^{2}}+\frac{4\ln\left(ax-1\right)}{a\,c^{2}}$	53
parallelrisch	$\frac{a^{3}x^{3}+4a^{2}\ln\left(ax-1\right)x^{2}-6a^{2}x^{2}-8a\ln\left(ax-1\right)x+4ax+4\ln\left(ax-1\right)}{c^{2}\left(ax-1\right)^{2}a}$	67

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+(-5*c^2*x+4*c^2/a)/c^4/(a*x-1)^2+4/a/c^2*ln(a*x-1)
```

3.393.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{a^3x^3-2\,a^2x^2-4\,ax+4\,(a^2x^2-2\,ax+1)\log\left(ax-1\right)+4}{a^3c^2x^2-2\,a^2c^2x+ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
(a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(
a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

3.393.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{-5ax+4}{a^3c^2x^2-2a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\log\left(ax-1\right)}{ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**2,x)
```

output
```
(-5*a*x + 4)/(a**3*c**2*x**2 - 2*a**2*c**2*x + a*c**2) + x/c**2 + 4*log(a*
x - 1)/(a*c**2)
```

3.393.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{5\,ax-4}{a^3c^2x^2-2\,a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\,\log\left(ax-1\right)}{ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-(5*a*x - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 + 4*log(a*x - 1)/
(a*c^2)
```

3.393.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}+\frac{4\,\log\left(|ax-1|\right)}{ac^2}-\frac{5\,ax-4}{\left(ax-1\right)^2ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
x/c^2 + 4*log(abs(a*x - 1))/(a*c^2) - (5*a*x - 4)/((a*x - 1)^2*a*c^2)
```

3.393.9 Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{5\,x-\frac{4}{a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c^2}$$

input
```
int((a*x + 1)/((c - c/(a*x))^2*(a*x - 1)),x)
```

output
```
x/c^2 - (5*x - 4/a)/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (4*log(a*x - 1))/(a*
c^2)
```

3.394 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

3.394.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{2}{3ac^3(1-ax)^3}-\frac{7}{2ac^3(1-ax)^2}+\frac{9}{ac^3(1-ax)}+\frac{5\log(1-ax)}{ac^3}$$

output
```
x/c^3+2/3/a/c^3/(-a*x+1)^3-7/2/a/c^3/(-a*x+1)^2+9/a/c^3/(-a*x+1)+5*ln(-a*x
+1)/a/c^3
```

3.394.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-37+81ax-36a^2x^2-18a^3x^3+6a^4x^4+30(-1+ax)^3\log(1-ax)}{6ac^3(-1+ax)^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
(-37 + 81*a*x - 36*a^2*x^2 - 18*a^3*x^3 + 6*a^4*x^4 + 30*(-1 + a*x)^3*Log[
1 - a*x])/(6*a*c^3*(-1 + a*x)^3)
```

3.394.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^3e^{2\text{arctanh}(ax)}}{c^3\left(a-\frac{1}{x}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{e^{2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6681}$$

$$\frac{a^3\int \frac{e^{2\text{arctanh}(ax)}x^3}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{6679}$$

$$\frac{a^3\int \frac{x^3(ax+1)}{(1-ax)^4}dx}{c^3}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3\int \left(\frac{1}{a^3}+\frac{5}{a^3(ax-1)}+\frac{9}{a^3(ax-1)^2}+\frac{7}{a^3(ax-1)^3}+\frac{2}{a^3(ax-1)^4}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(\frac{9}{a^4(1-ax)}-\frac{7}{2a^4(1-ax)^2}+\frac{2}{3a^4(1-ax)^3}+\frac{5\log(1-ax)}{a^4}+\frac{x}{a^3}\right)}{c^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
(a^3*(x/a^3 + 2/(3*a^4*(1 - a*x)^3) - 7/(2*a^4*(1 - a*x)^2) + 9/(a^4*(1 -
a*x)) + (5*Log[1 - a*x])/a^4))/c^3
```

3.394. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$

3.394.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.394. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

3.394.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{x}{c^{3}}+\frac{-9 a \,c^{3} x^{2}+\frac{29 c^{3} x}{2}-\frac{37 c^{3}}{6 a}}{c^{6} \left(a x -1\right)^{3}}+\frac{5 \ln \left(a x -1\right)}{a \,c^{3}}$	56
default	$\frac{a^{3} \left(\frac{x}{a^{3}}-\frac{7}{2 a^{4} \left(a x -1\right)^{2}}-\frac{9}{a^{4} \left(a x -1\right)}-\frac{2}{3 a^{4} \left(a x -1\right)^{3}}+\frac{5 \ln \left(a x -1\right)}{a^{4}}\right)}{c^{3}}$	61
norman	$\frac{\frac{a^{3} x^{4}}{c}-\frac{5 x}{c}+\frac{25 a \,x^{2}}{2 c}-\frac{55 a^{2} x^{3}}{6 c}}{\left(a x -1\right)^{3} c^{2}}+\frac{5 \ln \left(a x -1\right)}{a \,c^{3}}$	64
parallelrisch	$\frac{6 a^{4} x^{4}+30 a^{3} \ln \left(a x -1\right) x^{3}-55 a^{3} x^{3}-90 a^{2} \ln \left(a x -1\right) x^{2}+75 a^{2} x^{2}+90 a \ln \left(a x -1\right) x -30 a x -30 \ln \left(a x -1\right)}{6 c^{3} \left(a x -1\right)^{3} a}$	91

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3+(-9*a*c^3*x^2+29/2*c^3*x-37/6*c^3/a)/c^6/(a*x-1)^3+5/a/c^3*ln(a*x-1)
```

3.394.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{6\,a^4x^4-18\,a^3x^3-36\,a^2x^2+81\,ax+30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log{(ax-1)}-37}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/6*(6*a^4*x^4 - 18*a^3*x^3 - 36*a^2*x^2 + 81*a*x + 30*(a^3*x^3 - 3*a^2*x^
2 + 3*a*x - 1)*log(a*x - 1) - 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)
```

3.394.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-54a^2x^2+87ax-37}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}+\frac{x}{c^3}+\frac{5\log(ax-1)}{ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**3,x)
```

output
```
(-54*a**2*x**2 + 87*a*x - 37)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a
**2*c**3*x - 6*a*c**3) + x/c**3 + 5*log(a*x - 1)/(a*c**3)
```

3.394.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{54\,a^2x^2-87\,ax+37}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}+\frac{x}{c^3}+\frac{5\,\log(ax-1)}{ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-1/6*(54*a^2*x^2 - 87*a*x + 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
 - a*c^3) + x/c^3 + 5*log(a*x - 1)/(a*c^3)
```

3.394.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{5\,\log(|ax-1|)}{ac^3}-\frac{54\,a^2x^2-87\,ax+37}{6\,(ax-1)^3ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
x/c^3 + 5*log(abs(a*x - 1))/(a*c^3) - 1/6*(54*a^2*x^2 - 87*a*x + 37)/((a*x
 - 1)^3*a*c^3)
```

3.394.9 Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{9\,a\,x^2-\frac{29\,x}{2}+\frac{37}{6\,a}}{-a^3\,c^3\,x^3+3\,a^2\,c^3\,x^2-3\,a\,c^3\,x+c^3}+\frac{x}{c^3}+\frac{5\,\ln\left(a\,x-1\right)}{a\,c^3}$$

input

```
int((a*x + 1)/((c - c/(a*x))^3*(a*x - 1)),x)
```

output

```
(9*a*x^2 - (29*x)/2 + 37/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c
^3*x) + x/c^3 + (5*log(a*x - 1))/(a*c^3)
```

3.395 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

3.395.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}-\frac{1}{2ac^4(1-ax)^4}+\frac{3}{ac^4(1-ax)^3} -\frac{8}{ac^4(1-ax)^2}+\frac{14}{ac^4(1-ax)}+\frac{6\log(1-ax)}{ac^4}$$

output
```
x/c^4-1/2/a/c^4/(-a*x+1)^4+3/a/c^4/(-a*x+1)^3-8/a/c^4/(-a*x+1)^2+14/a/c^4/
(-a*x+1)+6*ln(-a*x+1)/a/c^4
```

3.395.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{17-56ax+60a^2x^2-16a^3x^3-8a^4x^4+2a^5x^5+12(-1+ax)^4\log(1-ax)}{2ac^4(-1+ax)^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
(17 - 56*a*x + 60*a^2*x^2 - 16*a^3*x^3 - 8*a^4*x^4 + 2*a^5*x^5 + 12*(-1 +
a*x)^4*Log[1 - a*x])/(2*a*c^4*(-1 + a*x)^4)
```

3.395.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^4 e^{2\text{arctanh}(ax)}}{c^4\left(a-\frac{1}{x}\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}x^4}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^4\int \frac{x^4(ax+1)}{(1-ax)^5}dx}{c^4}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^4\int \left(-\frac{1}{a^4}-\frac{6}{a^4(ax-1)}-\frac{14}{a^4(ax-1)^2}-\frac{16}{a^4(ax-1)^3}-\frac{9}{a^4(ax-1)^4}-\frac{2}{a^4(ax-1)^5}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\left(-\frac{14}{a^5(1-ax)}+\frac{8}{a^5(1-ax)^2}-\frac{3}{a^5(1-ax)^3}+\frac{1}{2a^5(1-ax)^4}-\frac{6\log(1-ax)}{a^5}-\frac{x}{a^4}\right)}{c^4}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
-((a^4*(-(x/a^4) + 1/(2*a^5*(1 - a*x)^4) - 3/(a^5*(1 - a*x)^3) + 8/(a^5*(1
 - a*x)^2) - 14/(a^5*(1 - a*x)) - (6*Log[1 - a*x])/a^5))/c^4)
```

3.395.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.395. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} \, dx$

3.395.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x}{c^4}+\frac{-14a^2c^4x^3+34a\,c^4x^2-29c^4x+\frac{17c^4}{2a}}{c^8(ax-1)^4}+\frac{6\ln(ax-1)}{a\,c^4}$
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{8}{a^5(ax-1)^2}-\frac{1}{2a^5(ax-1)^4}-\frac{14}{a^5(ax-1)}-\frac{3}{a^5(ax-1)^3}+\frac{6\ln(ax-1)}{a^5}\right)}{c^4}$
norman	$\frac{\frac{a^4x^5}{c}+\frac{6x}{c}-\frac{21a\,x^2}{c}+\frac{26a^2x^3}{c}-\frac{25a^3x^4}{2c}}{(ax-1)^4c^3}+\frac{6\ln(ax-1)}{a\,c^4}$
parallelrisch	$\frac{2a^5x^5+12\ln(ax-1)x^4a^4-25a^4x^4-48a^3\ln(ax-1)x^3+52a^3x^3+72a^2\ln(ax-1)x^2-42a^2x^2-48a\ln(ax-1)x+12ax+12\ln(ax-1}{2c^4(ax-1)^4a}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
x/c^4+(-14*a^2*c^4*x^3+34*a*c^4*x^2-29*c^4*x+17/2*c^4/a)/c^8/(a*x-1)^4+6/a
/c^4*ln(a*x-1)
```

3.395.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{2\,a^5x^5-8\,a^4x^4-16\,a^3x^3+60\,a^2x^2-56\,ax+12\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log{(ax-1)}+17}{2\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/2*(2*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 60*a^2*x^2 - 56*a*x + 12*(a^4*x^
4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 17)/(a^5*c^4*x^4 - 4
*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

3.395.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-28a^3x^3+68a^2x^2-58ax+17}{2a^5c^4x^4-8a^4c^4x^3+12a^3c^4x^2-8a^2c^4x+2ac^4}+\frac{x}{c^4}+\frac{6\log(ax-1)}{ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**4,x)
```

output
```
(-28*a**3*x**3 + 68*a**2*x**2 - 58*a*x + 17)/(2*a**5*c**4*x**4 - 8*a**4*c*
*4*x**3 + 12*a**3*c**4*x**2 - 8*a**2*c**4*x + 2*a*c**4) + x/c**4 + 6*log(a
*x - 1)/(a*c**4)
```

3.395.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}+\frac{6\,\log(ax-1)}{ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
-1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x - 17)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3
+ 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4) + x/c^4 + 6*log(a*x - 1)/(a*c^4)
```

3.395.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}+\frac{6\,\log(|ax-1|)}{ac^4}-\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\,(ax-1)^4ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
x/c^4 + 6*log(abs(a*x - 1))/(a*c^4) - 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*
x - 17)/((a*x - 1)^4*a*c^4)
```

3.395.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4} - \frac{29\,x - 34\,a\,x^2 - \frac{17}{2\,a} + 14\,a^2\,x^3}{a^4\,c^4\,x^4 - 4\,a^3\,c^4\,x^3 + 6\,a^2\,c^4\,x^2 - 4\,a\,c^4\,x + c^4} + \frac{6\,\ln\left(a\,x-1\right)}{a\,c^4}$$

input
```
int((a*x + 1)/((c - c/(a*x))^4*(a*x - 1)),x)
```

output
```
x/c^4 - (29*x - 34*a*x^2 - 17/(2*a) + 14*a^2*x^3)/(c^4 + 6*a^2*c^4*x^2 - 4
*a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*c^4*x) + (6*log(a*x - 1))/(a*c^4)
```

3.396 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.396.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}-\frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
1/3*c^4*(1-1/a^2/x^2)^(3/2)*(3*a+1/x)*x/a+3/2*c^4*arccsc(a*x)/a-c^4*arctan
h((1-1/a^2/x^2)^(1/2))/a+1/2*c^4*(2*a+3/x)*(1-1/a^2/x^2)^(1/2)/a^2
```

3.396.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=$$
$$-\frac{c^4\left(-8+12ax+40a^2x^2+12a^3x^3-32a^4x^4-24a^5x^5+42a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-15a^4\sqrt{1}\right.}{24a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
-1/24*(c^4*(-8 + 12*a*x + 40*a^2*x^2 + 12*a^3*x^3 - 32*a^4*x^4 - 24*a^5*x^
5 + 42*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 1
5*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] + 24*a^4*Sqrt[1 - 1/(a^2*x
^2)]*x^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

3.396.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 27, 536, 535, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \color{blue}{6731}$$

$$-c^3\int \frac{c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2}{a}d\frac{1}{x}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{c^4\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \color{blue}{536}$$

$$-\frac{c^4\left(\int \sqrt{1-\frac{1}{a^2x^2}}\left(-1-\frac{3}{ax}\right)xd\frac{1}{x}-\frac{1}{3}x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)\right)}{a}$$

$$\Big\downarrow \color{blue}{535}$$

$$-\frac{c^4\left(\frac{1}{2}\int -\frac{\left(2a+\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{c^4\left(-\frac{1}{2}\int \frac{\left(2a+\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{c^4\left(-\frac{\int \frac{\left(2a+\frac{3}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow 538$$

$$-\frac{c^4\left(-\frac{3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow 223$$

$$-\frac{c^4\left(-\frac{2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+3a\arcsin\left(\frac{1}{ax}\right)}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow 243$$

$$-\frac{c^4\left(-\frac{a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+3a\arcsin\left(\frac{1}{ax}\right)}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow 73$$

$$-\frac{c^4\left(-\frac{3a\arcsin\left(\frac{1}{ax}\right)-2a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c^4\left(-\frac{3a\arcsin\left(\frac{1}{ax}\right)-2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
-((c^4*(-1/2*(Sqrt[1 - 1/(a^2*x^2)]*(2*a + 3/x))/a - ((1 - 1/(a^2*x^2))^(3
/2)*(3*a + x^(-1))*x)/3 - (3*a*ArcSin[1/(a*x)] - 2*a*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]])/(2*a)))/a)
```

3.396. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

3.396.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

3.396. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.396.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.52

method	result
risch	$\frac{(ax-1)(8a^2x^2+3ax-2)c^4}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{(ax-1)(ax+1)}}{a^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c^4\left(-6\sqrt{a^2x^2-1}\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-9\sqrt{a^2x^2-1}\sqrt{a^2}\,a^3x^3-9a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln\left(\frac{a^2x+\surd}{}\right.\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x-1)*(8*a^2*x^2+3*a*x-2)/x^3*c^4/a^4/((a*x-1)/(a*x+1))^(1/2)+(-a^4*
ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+3/2*a^3*arctan(1/(a^2*
x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/(a*x+1)/((a*x-1)/(a*x+1
))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.396.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx =$$

$$-\frac{18\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^4x^4+14\,a}{6\,a^4x^3}$$

---

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
-1/6*(18*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^4*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (6*a^4*c^4*x^4 + 14*a^3*c^4*x^3 + 11*a^2*c^4*x^2 + a*c^4*x - 2*
c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

3.396.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{c^4\left(\int\left(-\frac{4a}{\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^3}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-x\sqrt{}}\right.\right.}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**4,x)
```

output
```
c**4*(Integral(-4*a/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2/(a
*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**3/(a*x**2*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1)), x) + Integral(a**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1)), x))/a**4
```

3.396.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(91) = 182$.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.17

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx =$$
$$-\frac{1}{3}\left(\frac{9\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-}{(ax-}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
-1/3*(9*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^4*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 - 3*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3
*c^4*((a*x - 1)/(a*x + 1))^(7/2) + c^4*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^
4*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
- 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))*a
```

3.396.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(91) = 182$.

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$

$$= -\frac{3\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}+\frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^4}{a\mathrm{sgn}\left(ax+1\right)}$$

$$-\frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4-12\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4-3\left(x|a|-\sqrt{a^2\right.}}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="giac")
```

output
```
-3*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + c^4*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^4/(a*sgn(a*x + 1)) - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a
) - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4 - 12*(x*abs(a) - sqrt(a^2*x^
2 - 1))^2*a*c^4 - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a) - 8*a*c^4)/(
((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

3.396.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{5\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^4*((a*x - 1)/(a*x + 1))^(3/2))/
3 + (c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^4*((a*x - 1)/(a*x + 1))^(7/2))
/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x
 - 1)^4)/(a*x + 1)^4) - (3*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^
4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.397 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.397.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{3c^3\csc^{-1}(ax)}{2a}$$

output
```
c^3*(1-1/a^2/x^2)^(3/2)*x+3/2*c^3*arccsc(a*x)/a+3/2*c^3*(1-1/a^2/x^2)^(1/2
)/a^2/x
```

3.397.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(1+2a^2x^2)+3ax\arcsin\left(\frac{1}{ax}\right)\right)}{2a^2x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
(c^3*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a^2*x^2) + 3*a*x*ArcSin[1/(a*x)]))/(2*a
^2*x)
```

3.397.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6731, 247, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$-c^3\left(-\frac{3\int \sqrt{1-\frac{1}{a^2x^2}}d\frac{1}{x}}{a^2}-x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$-c^3\left(-\frac{3\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)}{a^2}-x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{223}$$

$$-c^3\left(-\frac{3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2x}+\frac{1}{2}a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}-x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
-(c^3*(-((1 - 1/(a^2*x^2))^(3/2)*x) - (3*(Sqrt[1 - 1/(a^2*x^2)]/(2*x) + (a
*ArcSin[1/(a*x)])/2))/a^2))
```

3.397.3.1 Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 6731

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.397.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.72

method	result	size
default	$-\frac{(ax-1)^{2} c^{3} \left(-3 \sqrt{a^{2} x^{2}-1}\, a^{2} x^{2}-3 a^{2} x^{2} \arctan\left(\frac{1}{\sqrt{a^{2} x^{2}-1}}\right)+\left(a^{2} x^{2}-1\right)^{\frac{3}{2}}\right)}{2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1) \sqrt{(ax-1)(ax+1)}\, a^{3} x^{2}}$	105
risch	$\frac{(ax-1) c^{3}}{2 x^{2} a^{3} \sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^{2} \sqrt{(ax-1)(ax+1)}+\frac{3 a^{2} \arctan\left(\frac{1}{\sqrt{a^{2} x^{2}-1}}\right)}{2}\right) c^{3} \sqrt{(ax-1)(ax+1)}}{a^{3} (ax+1) \sqrt{\frac{ax-1}{ax+1}}}$	110

input

```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*(a*x-1)^2*c^3*(-3*(a^2*x^2-1)^(1/2)*a^2*x^2-3*a^2*x^2*arctan(1/(a^2*x
^2-1)^(1/2))+(a^2*x^2-1)^(3/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(
a*x+1))^(1/2)/a^3/x^2
```

3.397.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = -\frac{6\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(2\,a^3c^3x^3+2\,a^2c^3x^2+ac^3x+c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^3x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^3*c^3*x^3 + 2
*a^2*c^3*x^2 + a*c^3*x + c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x^2)
```

3.397.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = \frac{c^3\left(\int\frac{3a}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^2}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-a}}{ax+1}}\right.}{a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**3,x)
```

output
```
c**3*(Integral(3*a/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2/(a
*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + I
ntegral(-1/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**3*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**3
```

3.397.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.48

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=-\left(\frac{3\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-(3*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - (3*c^3*((a*x - 1)/(a*x + 1
))^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((a*x - 1)/(a*x +
 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3
*a^2/(a*x + 1)^3 + a^2))*a
```

3.397.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{3\,a^4c^3\arctan\left(\sqrt{a^2x^2-1}\right)-2\,\sqrt{a^2x^2-1}a^4c^3-\frac{\sqrt{a^2x^2-1}a^2c^3}{x^2}}{2\,a^5\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="giac")
```

output
```
-1/2*(3*a^4*c^3*arctan(sqrt(a^2*x^2 - 1)) - 2*sqrt(a^2*x^2 - 1)*a^4*c^3 -
sqrt(a^2*x^2 - 1)*a^2*c^3/x^2)/(a^5*sgn(a*x + 1))
```

3.397.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+c^3\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,x}+\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^3\,x^2}$$

input

```
int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output

```
(c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1
/2)))/a + c^3*x*((a*x - 1)/(a*x + 1))^(1/2) + (c^3*((a*x - 1)/(a*x + 1))^(
1/2))/(2*a^2*x) + (c^3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a^3*x^2)
```

3.398 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.398.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}+\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c^2*arccsc(a*x)/a+c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+c^2*(a-1/x)*x*(1-1/a^
2/x^2)^(1/2)/a
```

3.398.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(63) = 126.

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.44

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=$$

$$-\frac{c^2\left(-1+ax+a^2x^2-a^3x^3+4a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{1}{ax}\right)-a^2\sqrt{1-}\right.}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
-((c^2*(-1 + a*x + a^2*x^2 - a^3*x^3 + 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*Arc
Sin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/(a
*x)] - a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^3
*Sqrt[1 - 1/(a^2*x^2)]*x^2))
```

3.398.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6731, 27, 566, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-ac^2\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{566}$$

$$-ac^2\int \sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{a}+\frac{1}{xa^2}\right)x^2d\frac{1}{x}$$

$$\Big\downarrow \text{536}$$

$$-ac^2\left(\int \frac{\left(\frac{1}{a^2}-\frac{1}{a^3x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \text{538}$$

$$-ac^2\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^3}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \text{223}$$

$$-ac^2\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}-\frac{\arcsin\left(\frac{1}{ax}\right)}{a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \textbf{243}$$

$$-ac^2\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}-\frac{\arcsin\left(\frac{1}{ax}\right)}{a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \textbf{73}$$

$$-ac^2\left(-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-\frac{\arcsin\left(\frac{1}{ax}\right)}{a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \textbf{221}$$

$$-ac^2\left(-\frac{\arcsin\left(\frac{1}{ax}\right)}{a^2}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
-(a*c^2*(-((Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))*x)/a^2) - ArcSin[1/(a*x)]/a
^2 - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2))
```

3.398.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.398. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.398. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$

3.398.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.08

method	result
risch	$-\frac{(ax-1)c^2}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{(ax-1)(ax+1)}}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)/x*c^2/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(a*ln(a^2*x/(a^2)^(1/2)+(a^
2*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(
1/2)))*c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.81

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2-c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
-(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c^2*x*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) + a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x
^2 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

3.398.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$
$$=\frac{c^2\left(\int\left(-\frac{2a}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-}}{ax+1}}\right.}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**2,x)
```

output
```
c**2*(Integral(-2*a/(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2/(a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/(a*x + 1)), x) + Integral(1/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**2
```

3.398.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(59) = 118$.

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.98

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=$$
$$-\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) +
2*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c^2*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

3.398.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(59) = 118.

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.19

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{2\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)} - \frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^2}{a\mathrm{sgn}\left(ax+1\right)} - \frac{2\,c^2}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="giac")
```

output
```
-2*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^2*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^2/(a*sgn(a*x + 1)) - 2*c^2/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs
(a)*sgn(a*x + 1))
```

3.398.9 Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{4\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}} - \frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} + \frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2
*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^2*atanh(((a*x - 1)/(a*x +
 1))^(1/2)))/a
```

3.399 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.399.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{c\csc^{-1}(ax)}{a}+\frac{2c\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-c*arccsc(a*x)/a+2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+c*x*(1-1/a^2/x^2)^(1/2
)
```

3.399.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x-2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\arcsin\left(\frac{1}{ax}\right)+2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Ar
cSin[1/(a*x)] + 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

3.399.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 27, 570, 540, 25, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c^3\int \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^2\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-a^2c\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)^2x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}$$

$$\Big\downarrow \text{ 540}$$

$$-\frac{c\left(a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int -\frac{\left(2a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{c\left(\int \frac{\left(2a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a^2x\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{c\left(2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^2}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{c\left(2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow 243$$

$$-\frac{c\left(a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow 73$$

$$-\frac{c\left(-2a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a^2x\sqrt{1-\frac{1}{a^2x^2}}+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(-2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
-((c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + a*ArcSin[1/(a*x)] - 2*a*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]))/a^2)
```

3.399.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.399. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.399.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(45) = 90.

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.96

method	result	size
default	$-\frac{(ax-1)^{2} c\left(\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}+\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{a^{2}}-2a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)-2\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^{2}}}$	145

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)^2*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a
^2)^(1/2)-2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))-
2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a
*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

3.399.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="fricas")
```

output
```
(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 2*c*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/a
```

3.399.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c\left(\int \frac{a}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \left(-\frac{1}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)\,dx\right)}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x),x)
```

output
```
c*(Integral(a/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**2*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1)), x))/a
```

3.399.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = -2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arct
an(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a
^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.399.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.86

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c}{a\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="giac")
```

output
```
2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 2*c*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c
/(a*sgn(a*x + 1))
```

3.399.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,\left(a\,x-1\right)}{a\,x+1}}$$

input
```
int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (4*c*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x +
 1))
```

3.400 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.400.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output

```
-8/3*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(3/2)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c-
4/3*(3*a+4/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c
```

3.400.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(19-26ax+3a^2x^2)}{(-1+ax)^2}+12\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{3ac}$$

input

```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output

```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 26*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + 12*L
og[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(3*a*c)
```

3.400.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c^3\int \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^4\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^4x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^4c}$$

$$\Big\downarrow \text{ 532}$$

$$-\frac{\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(3a^4+\frac{12a^3}{x}+\frac{13a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^4c}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^4+\frac{12a^3}{x}+\frac{13a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\frac{1}{3}\left(\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{3a^3\left(a+\frac{4}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{1}{3}\left(3a^3\int \frac{\left(a+\frac{4}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow 534$$

$$-\frac{\frac{1}{3}\left(3a^3\left(4\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow 243$$

$$-\frac{\frac{1}{3}\left(3a^3\left(2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow 73$$

$$-\frac{\frac{1}{3}\left(3a^3\left(-4a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+3a^3\left(-4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4c}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
-(((8*a^2*(a + x^(-1)))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((4*a^2*(3*a + 4/x))
/Sqrt[1 - 1/(a^2*x^2)] + 3*a^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 4*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]]))/3)/(a^4*c))
```

3.400.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.400. $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.400. $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.400.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.74

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{4\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a\sqrt{a^{2}}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{3a^{4}\left(x-\frac{1}{a}\right)^{2}}-\frac{20\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{3a^{3}\left(x-\frac{1}{a}\right)}\right)a\sqrt{(ax-1)(ax+1)}}{c(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{12\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{4}x^{3}+12\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{3}x^{3}-36\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}-9\sqrt{a^{2}}\,((ax-1)(ax+1}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(4/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1
)^(1/2))/(a^2)^(1/2)-4/3/a^4/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-2
0/3/a^3/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a/c/(a*x+1)/((a*x-1)/(a
*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.400.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$=\frac{12\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-23\,a^2x^2-7\,a}{3\,(a^3cx^2-2\,a^2cx+ac)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

output
```
1/3*(12*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*(a^2
*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 23*a^2
*x^2 - 7*a*x + 19)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c
)
```

3.400.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\int\frac{x}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

output
```
a*Integral(x/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1)), x)/c
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$=\frac{2}{3}\,a\left(\frac{\frac{8\,(ax-1)}{ax+1}-\frac{12\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

output
```
2/3*a*((8*(a*x - 1)/(a*x + 1) - 12*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a
*x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 6*log(sqrt
((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 6*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c))
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

output
```
-4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c*abs(a)*sgn(a*x + 1)) + sqrt(
a^2*x^2 - 1)/(a*c*sgn(a*x + 1))
```

3.400.9 Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}-\frac{\frac{16\,(a\,x-1)}{3\,(a\,x+1)}-\frac{8\,{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{2}{3}}{a\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-a\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

input
```
int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((16*(a*x - 1))/(3*(a*x + 1
)) - (8*(a*x - 1)^2)/(a*x + 1)^2 + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) -
 a*c*((a*x - 1)/(a*x + 1))^(5/2))
```

3.401 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.401.1 Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}+\frac{5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
-16/5*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(5/2)-4/15*(5*a+11/x)/a^2/c^2/(1-1/a^2
/x^2)^(3/2)+5*arctanh((1-1/a^2/x^2)^(1/2))/a/c^2+1/15*(-75*a-103/x)/a^2/c^
2/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^2
```

3.401.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-118+161ax+91a^2x^2-173a^3x^3+15a^4x^4+75a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
(-118 + 161*a*x + 91*a^2*x^2 - 173*a^3*x^3 + 15*a^4*x^4 + 75*a*Sqrt[1 - 1/
(a^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^2*Sqrt
[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

3.401.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6731, 27, 570, 532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^5\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^5\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^5x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^5c^2}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(5a^5+\frac{25a^4}{x}+\frac{39a^3}{x^2}-\frac{5a^2}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^5c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\int \frac{\left(5a^5+\frac{25a^4}{x}+\frac{39a^3}{x^2}-\frac{5a^2}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$$\Big\downarrow \ 2336$$

$$-\frac{\frac{1}{5}\left(\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(15a^5+\frac{75a^4}{x}+\frac{88a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(15a^5+\frac{75a^4}{x}+\frac{88a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$$\Big\downarrow \ 2336$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{15a^4\left(a+\frac{5}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^4\int \frac{\left(a+\frac{5}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$$\Big\downarrow \ 534$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^4\left(5\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$$\Big\downarrow \ 243$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^4\left(\frac{5}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$$\Big\downarrow \ 73$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^4\left(-5a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5}}}{a^5c^2}$$

$$\Big\downarrow \ 221$$

3.401. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

$$-\frac{\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{1}{5}\left(\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(15a^4\left(-5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)\right)}{a^5c^2}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
-(((16*a^3*(a + x^(-1)))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((4*a^3*(5*a + 11/x
))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((a^3*(75*a + 103/x))/Sqrt[1 - 1/(a^2*x^2
)] + 15*a^4*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]]))/3)/5)/(a^5*c^2))
```

3.401.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.401. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.401.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.63

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{5a^{6}\left(x-\frac{1}{a}\right)^{3}}-\frac{52\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{15a^{5}\left(x-\frac{1}{a}\right)^{2}}-\frac{143\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{15a^{4}\left(x-\frac{1}{a}\right)}\right)a^{2}\sqrt{(ax}}{c^{2}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-75\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{4}x^{4}-75\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{5}x^{4}+60\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}a^{2}x^{2}+300\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(5/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-4/5/a^6/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-52/15/a^5/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-143/15/a^4/(x-1/a
)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^2/c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1/
2)*((a*x-1)*(a*x+1))^(1/2)
```

3.401.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int\frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{2}}\,dx$$
$$=\frac{75\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-75\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a}{15\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
1/15*(75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
 1) - 75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
 1) + (15*a^4*x^4 - 173*a^3*x^3 + 91*a^2*x^2 + 161*a*x - 118)*sqrt((a*x -
1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

3.401.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

output
```
a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a
*x + 1)), x)/c**2
```

3.401.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$=\frac{1}{15}a\left(\frac{\frac{17(ax-1)}{ax+1}+\frac{100(ax-1)^2}{(ax+1)^2}-\frac{150(ax-1)^3}{(ax+1)^3}+3}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{75\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{75\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
1/15*a*((17*(a*x - 1)/(a*x + 1) + 100*(a*x - 1)^2/(a*x + 1)^2 - 150*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*
x - 1)/(a*x + 1))^(5/2)) + 75*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2)
 - 75*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

3.401.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{5\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^2|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac^2\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
-5*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^2*abs(a)*sgn(a*x + 1)) + sqr
t(a^2*x^2 - 1)/(a*c^2*sgn(a*x + 1))
```

3.401.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.87

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}-\frac{\frac{20\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{10\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{17\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((20*(a*x - 1)^2)/(3*(a*
x + 1)^2) - (10*(a*x - 1)^3)/(a*x + 1)^3 + (17*(a*x - 1))/(15*(a*x + 1)) +
 1/5)/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - a*c^2*((a*x - 1)/(a*x + 1))^(7/
2))
```

3.402 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

3.402.1 Optimal result

Integrand size = 22, antiderivative size = 165

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{6\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output
```
-32/7*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(7/2)-2/7*(7*a+13/x)/a^2/c^3/(1-1/a^2/
x^2)^(3/2)-16/7/a^2/c^3/(1-1/a^2/x^2)^(5/2)/x+6*arctanh((1-1/a^2/x^2)^(1/2
))/a/c^3+1/7*(-42*a-59/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2
)/c^3
```

3.402.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{66-156ax+39a^2x^2+145a^3x^3-109a^4x^4+7a^5x^5+42a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
(66 - 156*a*x + 39*a^2*x^2 + 145*a^3*x^3 - 109*a^4*x^4 + 7*a^5*x^5 + 42*a*
Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(7*a^
2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

3.402.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \ 6731$$

$$-c^3\int \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^6\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^6\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \ 570$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^6x^2}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^6c^3}$$

$$\Big\downarrow \ 532$$

$$-\frac{\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}\int -\frac{\left(7a^6+\frac{42a^5}{x}+\frac{80a^4}{x^2}-\frac{42a^3}{x^3}-\frac{7a^2}{x^4}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^6c^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{7}\int \frac{\left(7a^6+\frac{42a^5}{x}+\frac{80a^4}{x^2}-\frac{42a^3}{x^3}-\frac{7a^2}{x^4}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 2336$$

$$-\frac{\frac{1}{7}\left(\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{5\left(7a^6+\frac{42a^5}{x}+\frac{71a^4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{7}\left(\int \frac{\left(7a^6+\frac{42a^5}{x}+\frac{71a^4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 2336$$

$$-\frac{\frac{1}{7}\left(-\frac{1}{3}\int -\frac{3\left(7a^6+\frac{42a^5}{x}+\frac{52a^4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{7}\left(\int \frac{\left(7a^6+\frac{42a^5}{x}+\frac{52a^4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 2336$$

$$-\frac{\frac{1}{7}\left(-\int -\frac{7a^5\left(a+\frac{6}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{7}\left(7a^5\int \frac{\left(a+\frac{6}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 534$$

$$-\frac{\frac{1}{7}\left(7a^5\left(6\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 243$$

3.402. $\displaystyle\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

$$-\frac{\frac{1}{7}\left(7a^5\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow 73$$

$$-\frac{\frac{1}{7}\left(7a^5\left(-6a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32}{7\left(1-\right.}}{a^6c^3}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{1}{7}\left(7a^5\left(-6\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)}{a^6c^3}$$

input

```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output

```
-(((32*a^4*(a + x^(-1)))/(7*(1 - 1/(a^2*x^2))^(7/2)) + ((2*a^4*(7*a + 13/x
))/(1 - 1/(a^2*x^2))^(3/2) + (a^4*(42*a + 59/x))/Sqrt[1 - 1/(a^2*x^2)] + (
16*a^4)/((1 - 1/(a^2*x^2))^(5/2)*x) + 7*a^5*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x)
- 6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/7)/(a^6*c^3))
```

3.402.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.402. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.402. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.402.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.61

method	result
risch	$\frac{ax-1}{a\,c^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{6\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{3}\sqrt{a^{2}}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{7a^{8}\left(x-\frac{1}{a}\right)^{4}}-\frac{20\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{7a^{7}\left(x-\frac{1}{a}\right)^{3}}-\frac{45\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{7a^{6}\left(x-\frac{1}{a}\right)^{2}}-\frac{88\sqrt{\left(x-\right.}}{7}\right.}{c^{3}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-42\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{5}x^{5}-42\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{6}x^{5}+35((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}\,a^{3}x^{3}+210\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(6/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-4/7/a^8/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-20/7/a^7/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-45/7/a^6/(x-1/a)^2
*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-88/7/a^5/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/
a)*a)^(1/2))*a^3/c^3/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/
2)
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

$$\int\frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3}}\,dx$$
$$=\frac{42\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(\sqrt{\right.}}{7\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/7*(42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/(
a*x + 1)) + 1) - 42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt
((a*x - 1)/(a*x + 1)) - 1) + (7*a^5*x^5 - 109*a^4*x^4 + 145*a^3*x^3 + 39*a
^2*x^2 - 156*a*x + 66)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c^3
*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

3.402.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3\int \frac{x^3}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

output
```
a**3*Integral(x**3/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- 4*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**
3
```

3.402.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{1}{14}\,a\left(\frac{\frac{6\,(ax-1)}{ax+1}+\frac{21\,(ax-1)^2}{(ax+1)^2}+\frac{112\,(ax-1)^3}{(ax+1)^3}-\frac{168\,(ax-1)^4}{(ax+1)^4}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{84\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{84\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
1/14*a*((6*(a*x - 1)/(a*x + 1) + 21*(a*x - 1)^2/(a*x + 1)^2 + 112*(a*x - 1
)^3/(a*x + 1)^3 - 168*(a*x - 1)^4/(a*x + 1)^4 + 1)/(a^2*c^3*((a*x - 1)/(a*
x + 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 84*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/(a^2*c^3) - 84*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*
c^3))
```

3.402.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.402.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3} - \frac{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{16\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{24\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{6\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{7}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

input
```
int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(12*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((3*(a*x - 1)^2)/(a*x +
1)^2 + (16*(a*x - 1)^3)/(a*x + 1)^3 - (24*(a*x - 1)^4)/(a*x + 1)^4 + (6*(a
*x - 1))/(7*(a*x + 1)) + 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 2*a*c
^3*((a*x - 1)/(a*x + 1))^(9/2))
```

3.403 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

3.403.1 Optimal result

Integrand size = 22, antiderivative size = 204

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}\\ &\quad-\frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\\ &\quad-\frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}+\frac{7\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}\end{aligned}$$

output
```
16/63*(9*a-5/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-64/9*(a+1/x)/a^2/c^4/(1-1/a^2/
x^2)^(9/2)-8/105*(21*a+41/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/315*(-735*a-141
7/x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)+7*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4+1/31
5*(-2205*a-3149/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^4
```

3.403.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{-3464+11651ax-10232a^2x^2-5567a^3x^3+13241a^4x^4-6224a^5x^5+315a^6x^6+2205a\sqrt{1-\frac{1}{a^2x^2}}x(-1}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^4}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
(-3464 + 11651*a*x - 10232*a^2*x^2 - 5567*a^3*x^3 + 13241*a^4*x^4 - 6224*a
^5*x^5 + 315*a^6*x^6 + 2205*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4*ArcTanh
[Sqrt[1 - 1/(a^2*x^2)]])/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4
)
```

3.403.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 2336, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{a^7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^7\left(a-\frac{1}{x}\right)^7}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^7\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^7}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^7x^2}{\left(1-\frac{1}{a^2x^2}\right)^{11/2}}d\frac{1}{x}}{a^7c^4}$$

$$\Big\downarrow \ 532$$

$$-\frac{\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}\int -\frac{\left(9a^7+\frac{63a^6}{x}+\frac{134a^5}{x^2}-\frac{198a^4}{x^3}-\frac{63a^3}{x^4}-\frac{9a^2}{x^5}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^7c^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{9}\int \frac{\left(9a^7+\frac{63a^6}{x}+\frac{134a^5}{x^2}-\frac{198a^4}{x^3}-\frac{63a^3}{x^4}-\frac{9a^2}{x^5}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \ 2336$$

$$-\frac{\frac{1}{9}\left(-\frac{1}{7}\int -\frac{3\left(21a^7+\frac{147a^6}{x}+\frac{307a^5}{x^2}+\frac{21a^4}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\int \frac{\left(21a^7+\frac{147a^6}{x}+\frac{307a^5}{x^2}+\frac{21a^4}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \ 2336$$

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(105a^7+\frac{735a^6}{x}+\frac{1312a^5}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\int \frac{\left(105a^7+\frac{735a^6}{x}+\frac{1312a^5}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \ 2336$$

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(315a^7+\frac{2205a^6}{x}+\frac{2834a^5}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \ 25$$

3.403. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

$$-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\int \frac{\left(315a^7+\frac{2205a^6}{x}+\frac{2834a^5}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\Bigg)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\Bigg)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\Bigg)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{315a^6\left(a+\frac{7}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\Bigg)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\Bigg)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\Bigg)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\Bigg)+\frac{64a^5(}{9(1-}}{a^7c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(315a^6\int \frac{\left(a+\frac{7}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\Bigg)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\Bigg)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\Bigg)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\Bigg)+\frac{64a^5}{9(1-}}{a^7c^4}$$

$$\Big\downarrow \text{534}$$

$$-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(315a^6\Bigg(7\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\Bigg)+\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\Bigg)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\Bigg)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\Bigg)-}{a^7c^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(315a^6\Bigg(\frac{7}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\Bigg)+\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\Bigg)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\Bigg)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\Bigg)-}{a^7c^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(315a^6\Bigg(-7a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\Bigg)+\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\Bigg)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\Bigg)+}{a^7c^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{1}{5}\Bigg(\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\Bigg(315a^6\Bigg(-7\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\Bigg)+}{a^7c^4}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

3.403. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

output

```
-(((64*a^5*(a + x^(-1)))/(9*(1 - 1/(a^2*x^2))^(9/2)) + ((-16*a^5*(9*a - 5/
x))/(7*(1 - 1/(a^2*x^2))^(7/2)) + (3*((8*a^5*(21*a + 41/x))/(5*(1 - 1/(a^2
*x^2))^(5/2)) + ((a^5*(735*a + 1417/x))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((a^
5*(2205*a + 3149/x))/Sqrt[1 - 1/(a^2*x^2)] + 315*a^6*(-(a*Sqrt[1 - 1/(a^2*
x^2)]*x) - 7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/3)/5))/7)/9)/(a^7*c^4))
```

3.403.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.403. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.403.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.50

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{9a^{10}\left(x-\frac{1}{a}\right)^5}-\frac{164\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{63a^9\left(x-\frac{1}{a}\right)^4}-\frac{697\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{105a^8\left(x-\frac{1}{a}\right)^3}-\frac{3226\sqrt{(}}{}\right.}{c^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-2205\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^6x^6-2205\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^7x^6+1890((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+13230\sqrt{(ax-1)(}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

3.403. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

output
```
1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(7/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-4/9/a^10/(x-1/a)^5*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1
/2)-164/63/a^9/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-697/105/a^8/(x-
1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-3226/315/a^7/(x-1/a)^2*((x-1/a)^2
*a^2+2*(x-1/a)*a)^(1/2)-4964/315/a^6/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(
1/2))*a^4/c^4/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.403.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.18

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{2205\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2205\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-\right.}{315\left(a^6c^4x^5-5\,a^5c\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/315*(2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3
- 10*a^2*x^2 + 5*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^6*x^
6 - 6224*a^5*x^5 + 13241*a^4*x^4 - 5567*a^3*x^3 - 10232*a^2*x^2 + 11651*a*
x - 3464)*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4
*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

3.403.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

output
```
a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- 5*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x +
1) - 1/(a*x + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

3.403.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.91

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{1}{1260}a\left(\frac{\frac{235\,(ax-1)}{ax+1}+\frac{801\,(ax-1)^2}{(ax+1)^2}+\frac{2289\,(ax-1)^3}{(ax+1)^3}+\frac{11760\,(ax-1)^4}{(ax+1)^4}-\frac{17640\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{8820\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/1260*a*((235*(a*x - 1)/(a*x + 1) + 801*(a*x - 1)^2/(a*x + 1)^2 + 2289*(a
*x - 1)^3/(a*x + 1)^3 + 11760*(a*x - 1)^4/(a*x + 1)^4 - 17640*(a*x - 1)^5/
(a*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x -
1)/(a*x + 1))^(9/2)) + 8820*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) -
 8820*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.31

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=-\frac{7\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^4|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac^4\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
-7*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^4*abs(a)*sgn(a*x + 1)) + sqr
t(a^2*x^2 - 1)/(a*c^4*sgn(a*x + 1))
```

3.403.9 Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{14\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4} - \frac{\frac{89\,(a\,x-1)^2}{35\,(a\,x+1)^2}+\frac{109\,(a\,x-1)^3}{15\,(a\,x+1)^3}+\frac{112\,(a\,x-1)^4}{3\,(a\,x+1)^4}-\frac{56\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{47\,(a\,x-1)}{63\,(a\,x+1)}+\frac{1}{9}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}-4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}$$

input
```
int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(14*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((89*(a*x - 1)^2)/(35*(a
*x + 1)^2) + (109*(a*x - 1)^3)/(15*(a*x + 1)^3) + (112*(a*x - 1)^4)/(3*(a*
x + 1)^4) - (56*(a*x - 1)^5)/(a*x + 1)^5 + (47*(a*x - 1))/(63*(a*x + 1)) +
 1/9)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))
^(11/2))
```

3.404 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

3.404.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}$$

output
```
1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/a^3/x^2+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

3.404.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{c^5\left(\frac{1}{4x^4}-\frac{a}{3x^3}-\frac{a^2}{x^2}+\frac{2a^3}{x}+a^5x-a^4\log(x)\right)}{a^5}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

output
```
(c^5*(1/(4*x^4) - a/(3*x^3) - a^2/x^2 + (2*a^3)/x + a^5*x - a^4*Log[x]))/a
^5
```

3.404.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^5 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c^5\left(a-\frac{1}{x}\right)^5 e^{4\text{arctanh}(ax)}}{a^5}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^5\int e^{4\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^5dx}{a^5}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{c^5\int \frac{e^{4\text{arctanh}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{c^5\int \frac{(1-ax)^3(ax+1)^2}{x^5}dx}{a^5}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^5\int \left(-a^5+\frac{a^4}{x}+\frac{2a^3}{x^2}-\frac{2a^2}{x^3}-\frac{a}{x^4}+\frac{1}{x^5}\right)dx}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^5\left(a^5(-x)+a^4\log(x)-\frac{2a^3}{x}+\frac{a^2}{x^2}+\frac{a}{3x^3}-\frac{1}{4x^4}\right)}{a^5}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

output
```
-((c^5*(-1/4*1/x^4 + a/(3*x^3) + a^2/x^2 - (2*a^3)/x - a^5*x + a^4*Log[x])
)/a^5)
```

3.404.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.404. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

3.404.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

method	result
default	$\frac{c^5\left(a^5x-a^4\ln(x)+\frac{1}{4x^4}-\frac{a}{3x^3}-\frac{a^2}{x^2}+\frac{2a^3}{x}\right)}{a^5}$
risch	$c^5x+\frac{2a^3c^5x^3-a^2c^5x^2-\frac{1}{3}a\,c^5x+\frac{1}{4}c^5}{a^5x^4}-\frac{c^5\ln(x)}{a}$
parallelrisch	$-\frac{-12a^5c^5x^5+12c^5\ln(x)a^4x^4-24a^3c^5x^3+12a^2c^5x^2+4a\,c^5x-3c^5}{12a^5x^4}$
norman	$\frac{a^4c^5x^5+a^5c^5x^6-\frac{c^5}{4a}+\frac{7c^5x}{12}+\frac{2a\,c^5x^2}{3}-3c^5a^2x^3}{(ax-1)a^4x^4}-\frac{c^5\ln(x)}{a}$
meijerg	$-\frac{c^5\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{3c^5\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}+\frac{c^5x}{-ax+1}+\frac{5c^5\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5/a^5*(a^5*x-a^4*ln(x)+1/4/x^4-1/3*a/x^3-a^2/x^2+2*a^3/x)
```

3.404.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$$
$$=\frac{12\,a^5c^5x^5-12\,a^4c^5x^4\log(x)+24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="fricas")
```

output
```
1/12*(12*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) + 24*a^3*c^5*x^3 - 12*a^2*c^5
*x^2 - 4*a*c^5*x + 3*c^5)/(a^5*x^4)
```

3.404.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{a^5c^5x-a^4c^5\log\left(x\right)+\frac{24a^3c^5x^3-12a^2c^5x^2-4ac^5x+3c^5}{12x^4}}{a^5}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**5,x)
```

output
```
(a**5*c**5*x - a**4*c**5*log(x) + (24*a**3*c**5*x**3 - 12*a**2*c**5*x**2 -
 4*a*c**5*x + 3*c**5)/(12*x**4))/a**5
```

3.404.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=c^5x-\frac{c^5\log\left(x\right)}{a}+\frac{24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="maxima")
```

output
```
c^5*x - c^5*log(x)/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x +
 3*c^5)/(a^5*x^4)
```

3.404.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx&=\frac{c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}\\&\quad+\frac{\left(12\,c^5+\frac{37\,c^5}{ax-1}+\frac{52\,c^5}{(ax-1)^2}+\frac{42\,c^5}{(ax-1)^3}+\frac{12\,c^5}{(ax-1)^4}\right)(ax-1)}{12\,a\left(\frac{1}{ax-1}+1\right)^4}\end{aligned}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="giac")
```

output
```
c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c^5*log(abs(-1/(a*x - 1) -
1))/a + 1/12*(12*c^5 + 37*c^5/(a*x - 1) + 52*c^5/(a*x - 1)^2 + 42*c^5/(a*x
 - 1)^3 + 12*c^5/(a*x - 1)^4)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^4)
```

3.404.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=-\frac{c^5\,\left(4\,a\,x+12\,a^2\,x^2-24\,a^3\,x^3-12\,a^5\,x^5+12\,a^4\,x^4\,\ln\left(x\right)-3\right)}{12\,a^5\,x^4}$$

input
```
int(((c - c/(a*x))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
-(c^5*(4*a*x + 12*a^2*x^2 - 24*a^3*x^3 - 12*a^5*x^5 + 12*a^4*x^4*log(x) -
3))/(12*a^5*x^4)
```

3.405 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.405.1 Optimal result

Integrand size = 22, antiderivative size = 30

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

output
```
-1/3*c^4/a^4/x^3+2*c^4/a^2/x+c^4*x
```

3.405.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4\left(-\frac{1}{3x^3}+\frac{2a^2}{x}+a^4x\right)}{a^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
(c^4*(-1/3*1/x^3 + (2*a^2)/x + a^4*x))/a^4
```

3.405.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6717, 27, 6681, 6679, 82, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6717$$

$$\int \frac{c^4\left(a-\frac{1}{x}\right)^4 e^{4\text{arctanh}(ax)}}{a^4}dx$$

$$\Big\downarrow \; 27$$

$$\frac{c^4\int e^{4\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^4dx}{a^4}$$

$$\Big\downarrow \; 6681$$

$$\frac{c^4\int \frac{e^{4\text{arctanh}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$

$$\Big\downarrow \; 6679$$

$$\frac{c^4\int \frac{(1-ax)^2(ax+1)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \; 82$$

$$\frac{c^4\int \frac{\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \; 244$$

$$\frac{c^4\int \left(a^4-\frac{2a^2}{x^2}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^4\left(a^4x+\frac{2a^2}{x}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

3.405. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4dx$

output
```
(c^4*(-1/3*1/x^3 + (2*a^2)/x + a^4*x))/a^4
```

3.405.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.405. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.405.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^4\left(a^4x-\frac{1}{3x^3}+\frac{2a^2}{x}\right)}{a^4}$
gosper	$\frac{c^4(3a^4x^4+6a^2x^2-1)}{3x^3a^4}$
risch	$c^4x+\frac{2a^2c^4x^2-\frac{1}{3}c^4}{a^4x^3}$
parallelrisch	$\frac{3a^4c^4x^4+6a^2c^4x^2-c^4}{3a^4x^3}$
norman	$\frac{a^3c^4x^4+a^4c^4x^5+\frac{c^4}{3a}-\frac{c^4x}{3}-2a\,c^4x^2}{(ax-1)a^3x^3}$
meijerg	$-\frac{c^4\Bigl(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\Bigr)}{a}-\frac{2c^4\Bigl(\frac{ax}{-ax+1}+\ln(-ax+1)\Bigr)}{a}-\frac{c^4x}{-ax+1}+\frac{4c^4\Bigl(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4/a^4*(a^4*x-1/3/x^3+2*a^2/x)
```

3.405.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{3\,a^4c^4x^4+6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^4*x^4 + 6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

3.405.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{a^4c^4x+\frac{6a^2c^4x^2-c^4}{3x^3}}{a^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**4,x)
```

output
```
(a**4*c**4*x + (6*a**2*c**4*x**2 - c**4)/(3*x**3))/a**4
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=c^4x+\frac{6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

3.405.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{(ax-1)c^4}{a}-\frac{5\,c^4+\frac{9\,c^4}{ax-1}+\frac{3\,c^4}{(ax-1)^2}}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="giac")
```

output
```
(a*x - 1)*c^4/a - 1/3*(5*c^4 + 9*c^4/(a*x - 1) + 3*c^4/(a*x - 1)^2)/(a*(1/
(a*x - 1) + 1)^3)
```

3.405.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4\,\left(a^4\,x^4+2\,a^2\,x^2-\frac{1}{3}\right)}{a^4\,x^3}$$

input
```
int(((c - c/(a*x))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^4*(2*a^2*x^2 + a^4*x^4 - 1/3))/(a^4*x^3)
```

3.406 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.406.1 Optimal result

Integrand size = 22, antiderivative size = 38

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}$$

output
```
1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

3.406.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\left(\frac{3a^2}{2}+\frac{1}{2x^2}+\frac{a}{x}+a^3x+a^2\log(x)\right)}{a^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
(c^3*((3*a^2)/2 + 1/(2*x^2) + a/x + a^3*x + a^2*Log[x]))/a^3
```

3.406.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c^3\left(a-\frac{1}{x}\right)^3 e^{4\text{arctanh}(ax)}}{a^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\int e^{4\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3dx}{a^3}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{c^3\int \frac{e^{4\text{arctanh}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{c^3\int \frac{(1-ax)(ax+1)^2}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{84}$$

$$-\frac{c^3\int \left(-a^3-\frac{a^2}{x}+\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3\left(a^3(-x)-a^2\log(x)-\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
-((c^3*(-1/2*1/x^2 - a/x - a^3*x - a^2*Log[x]))/a^3)
```

3.406. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3dx$

3.406.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.406. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.406.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^3\left(a^3x+a^2\ln(x)+\frac{1}{2x^2}+\frac{a}{x}\right)}{a^3}$
risch	$c^3x+\frac{a\,c^3x+\frac{1}{2}c^3}{a^3x^2}+\frac{c^3\ln(x)}{a}$
parallelrisch	$\frac{2a^3c^3x^3+2c^3\ln(x)a^2x^2+2a\,c^3x+c^3}{2a^3x^2}$
norman	$\frac{a^3c^3x^4-\frac{c^3}{2a}-\frac{c^3x}{2}}{(ax-1)a^2x^2}+\frac{c^3\ln(x)}{a}$
meijerg	$-\frac{c^3\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{c^3\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{2c^3x}{-ax+1}+\frac{2c^3\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/a^3*(a^3*x+a^2*ln(x)+1/2/x^2+a/x)
```

3.406.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{2\,a^3c^3x^3+2\,a^2c^3x^2\log\left(x\right)+2\,ac^3x+c^3}{2\,a^3x^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^3*c^3*x^3 + 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x + c^3)/(a^3*x^2)
```

3.406.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{a^3c^3x+a^2c^3\log\left(x\right)+\frac{2ac^3x+c^3}{2x^2}}{a^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**3,x)
```

output
```
(a**3*c**3*x + a**2*c**3*log(x) + (2*a*c**3*x + c**3)/(2*x**2))/a**3
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=c^3x+\frac{c^3\log\left(x\right)}{a}+\frac{2\,ac^3x+c^3}{2\,a^3x^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
c^3*x + c^3*log(x)/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

3.406.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.58

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=&-\frac{c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}\\&+\frac{\left(2\,c^3+\frac{c^3}{ax-1}-\frac{2\,c^3}{(ax-1)^2}\right)(ax-1)}{2\,a\left(\frac{1}{ax-1}+1\right)^2}\end{aligned}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="giac")
```

output
```
-c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + c^3*log(abs(-1/(a*x - 1) -
 1))/a + 1/2*(2*c^3 + c^3/(a*x - 1) - 2*c^3/(a*x - 1)^2)*(a*x - 1)/(a*(1/(
a*x - 1) + 1)^2)
```

3.406.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\,\left(a\,x+a^3\,x^3+a^2\,x^2\,\ln\left(x\right)+\frac{1}{2}\right)}{a^3\,x^2}$$

input

```
int(((c - c/(a*x))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output

```
(c^3*(a*x + a^3*x^3 + a^2*x^2*log(x) + 1/2))/(a^3*x^2)
```

3.407 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.407.1 Optimal result

Integrand size = 22, antiderivative size = 27

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}$$

output
```
-c^2/a^2/x+c^2*x+2*c^2*ln(x)/a
```

3.407.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\left(-\frac{1}{x}+a^2x+2a\log(x)\right)}{a^2}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
(c^2*(-x^(-1) + a^2*x + 2*a*Log[x]))/a^2
```

3.407.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{c^2\left(a-\frac{1}{x}\right)^2 e^{4\text{arctanh}(ax)}}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^2\int e^{4\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{c^2\int \frac{e^{4\text{arctanh}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{c^2\int \frac{(ax+1)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c^2\int \left(a^2+\frac{2a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\left(a^2x+2a\log(x)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
(c^2*(-x^(-1) + a^2*x + 2*a*Log[x]))/a^2
```

3.407.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.407.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{c^{2}\left(a^{2}x+2a\ln\left(x\right)-\frac{1}{x}\right)}{a^{2}}$	24
risch	$-\frac{c^{2}}{a^{2}x}+c^{2}x+\frac{2c^{2}\ln\left(x\right)}{a}$	28
parallelrisch	$\frac{a^{2}c^{2}x^{2}+2c^{2}\ln\left(x\right)ax-c^{2}}{a^{2}x}$	33
norman	$\frac{\frac{c^{2}}{a}-2a\,c^{2}x^{2}+a^{2}c^{2}x^{3}}{\left(ax-1\right)ax}+\frac{2c^{2}\ln\left(x\right)}{a}$	53
meijerg	$-\frac{c^{2}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{2}x}{-ax+1}-\frac{c^{2}\left(-\frac{3ax}{-3ax+3}+2\ln\left(-ax+1\right)-1-2\ln\left(x\right)-2\ln\left(-a\right)+\frac{1}{ax}\right)}{a}$	100

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/a^2*(a^2*x+2*a*ln(x)-1/x)
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2+2\,ac^2x\log\left(x\right)-c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
(a^2*c^2*x^2 + 2*a*c^2*x*log(x) - c^2)/(a^2*x)
```

3.407.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x+2ac^2\log\left(x\right)-\frac{c^2}{x}}{a^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**2,x)
```

output
```
(a**2*c**2*x + 2*a*c**2*log(x) - c**2/x)/a**2
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{2\,c^2\log\left(x\right)}{a}-\frac{c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
c^2*x + 2*c^2*log(x)/a - c^2/(a^2*x)
```

3.407.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.48

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{2\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{2\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{c^2+\frac{2\,c^2}{ax-1}}{a^2\left(\frac{1}{(ax-1)a}+\frac{1}{(ax-1)^2a}\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="giac")
```

output
```
-2*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 2*c^2*log(abs(-1/(a*x -
1) - 1))/a + (c^2 + 2*c^2/(a*x - 1))/(a^2*(1/((a*x - 1)*a) + 1/((a*x - 1)^
2*a)))
```

3.407.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\,(a^2\,x^2+2\,a\,x\,\ln(x)-1)}{a^2\,x}$$

input
```
int(((c - c/(a*x))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^2*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

3.408 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.408.1 Optimal result

Integrand size = 20, antiderivative size = 25

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}$$

output
```
c*x-c*ln(x)/a+4*c*ln(-a*x+1)/a
```

3.408.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c(ax-\log(x)+4\log(1-ax))}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
(c*(a*x - Log[x] + 4*Log[1 - a*x]))/a
```

3.408.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6681, 6679, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{4\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c\left(a-\frac{1}{x}\right) e^{4\text{arctanh}(ax)}}{a}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int e^{4\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)dx}{a}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{c\int \frac{e^{4\text{arctanh}(ax)}(1-ax)}{x}dx}{a}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{c\int \frac{(ax+1)^2}{x(1-ax)}dx}{a}$$

$$\Big\downarrow \text{93}$$

$$-\frac{c\int \left(-\frac{4a}{ax-1}-a+\frac{1}{x}\right)dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(-ax-4\log(1-ax)+\log(x))}{a}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
-((c*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/a)
```

3.408.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.408.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result
default	$\frac{c(ax-\ln(x)+4\ln(ax-1))}{a}$
parallelrisch	$-\frac{-acx+c\ln(x)-4c\ln(ax-1)}{a}$
risch	$cx-\frac{c\ln(x)}{a}+\frac{4c\ln(-ax+1)}{a}$
norman	$\frac{ac\,x^{2}-cx}{ax-1}-\frac{c\ln(x)}{a}+\frac{4c\ln(ax-1)}{a}$
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{cx}{-ax+1}-\frac{c\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
c/a*(a*x-ln(x)+4*ln(a*x-1))
```

3.408.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+4\,c\log\left(ax-1\right)-c\log\left(x\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="fricas")
```

output
```
(a*c*x + 4*c*log(a*x - 1) - c*log(x))/a
```

3.408.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\left(-\log\left(x\right)+4\log\left(x-\frac{1}{a}\right)\right)}{a}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x),x)
```

output
```
c*x + c*(-log(x) + 4*log(x - 1/a))/a
```

3.408.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{4\,c\log\left(ax-1\right)}{a}-\frac{c\log\left(x\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="maxima")
```

output
```
c*x + 4*c*log(a*x - 1)/a - c*log(x)/a
```

3.408.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{(ax-1)c}{a}-\frac{3\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="giac")
```

output
```
(a*x - 1)*c/a - 3*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c*log(abs(-
1/(a*x - 1) - 1))/a
```

3.408.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\,x-\frac{c\,\ln(x)}{a}+\frac{4\,c\,\ln(a\,x-1)}{a}$$

input
```
int(((c - c/(a*x))*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c*x - (c*log(x))/a + (4*c*log(a*x - 1))/a
```

3.409 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.409.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}$$

output
```
x/c-2/a/c/(-a*x+1)^2+8/a/c/(-a*x+1)+5*ln(-a*x+1)/a/c
```

3.409.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{a\left(-\frac{x}{a}+\frac{2}{a^2(1-ax)^2}-\frac{8}{a^2(1-ax)}-\frac{5\log(1-ax)}{a^2}\right)}{c}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
-((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/
a^2))/c)
```

3.409.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{ae^{4\text{arctanh}(ax)}}{c\left(a-\frac{1}{x}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{e^{4\text{arctanh}(ax)}}{a-\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a\int \frac{e^{4\text{arctanh}(ax)}x}{1-ax}dx}{c}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a\int \frac{x(ax+1)^2}{(1-ax)^3}dx}{c}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a\int \left(-\frac{5}{(ax-1)a}-\frac{8}{(ax-1)^2a}-\frac{4}{(ax-1)^3a}-\frac{1}{a}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{8}{a^2(1-ax)}+\frac{2}{a^2(1-ax)^2}-\frac{5\log(1-ax)}{a^2}-\frac{x}{a}\right)}{c}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
-((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/
a^2))/c)
```

3.409.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.409. $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.409.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{c}+\frac{-8cx+\frac{6c}{a}}{c^{2}\left(ax-1\right)^{2}}+\frac{5\ln\left(ax-1\right)}{ac}$	43
default	$\frac{a\left(\frac{x}{a}-\frac{2}{a^{2}\left(ax-1\right)^{2}}-\frac{8}{a^{2}\left(ax-1\right)}+\frac{5\ln\left(ax-1\right)}{a^{2}}\right)}{c}$	47
norman	$\frac{\frac{a^{2}x^{3}}{c}-\frac{8a\,x^{2}}{c}+\frac{5x}{c}}{\left(ax-1\right)^{2}}+\frac{5\ln\left(ax-1\right)}{ac}$	50
parallelrisch	$\frac{a^{3}x^{3}+5a^{2}\ln\left(ax-1\right)x^{2}-8a^{2}x^{2}-10a\ln\left(ax-1\right)x+5ax+5\ln\left(ax-1\right)}{\left(ax-1\right)^{2}ca}$	67

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
x/c+(-8*c*x+6*c/a)/c^2/(a*x-1)^2+5/a/c*ln(a*x-1)
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a^{3}x^{3}-2\,a^{2}x^{2}-7\,ax+5\,(a^{2}x^{2}-2\,ax+1)\log\left(ax-1\right)+6}{a^{3}cx^{2}-2\,a^{2}cx+ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="fricas")
```

output
```
(a^3*x^3 - 2*a^2*x^2 - 7*a*x + 5*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 6)/(
a^3*c*x^2 - 2*a^2*c*x + a*c)
```

3.409.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{-8ax+6}{a^{3}cx^{2}-2a^{2}cx+ac}+\frac{x}{c}+\frac{5\log\left(ax-1\right)}{ac}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x),x)
```

output
```
(-8*a*x + 6)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

3.409.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{2\,(4\,ax-3)}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{5\,\log\left(ax-1\right)}{ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*(4*a*x - 3)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

3.409.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{ax-1}{ac}-\frac{5\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{2\left(\frac{4\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}\right)}{a^4c^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c) - 5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - 2*(4*a^
3*c/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

3.409.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}-\frac{8\,x-\frac{6}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{5\,\ln\left(a\,x-1\right)}{a\,c}$$

input
```
int((a*x + 1)^2/((c - c/(a*x))*(a*x - 1)^2),x)
```

output
```
x/c - (8*x - 6/a)/(c + a^2*c*x^2 - 2*a*c*x) + (5*log(a*x - 1))/(a*c)
```

3.410 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.410.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}$$

output
```
x/c^2+4/3/a/c^2/(-a*x+1)^3-6/a/c^2/(-a*x+1)^2+13/a/c^2/(-a*x+1)+6*ln(-a*x+
1)/a/c^2
```

3.410.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-25+57ax-30a^2x^2-9a^3x^3+3a^4x^4+18(-1+ax)^3\log(1-ax)}{3ac^2(-1+ax)^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
(-25 + 57*a*x - 30*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 18*(-1 + a*x)^3*Log[1
 - a*x])/(3*a*c^2*(-1 + a*x)^3)
```

3.410.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^2e^{4\text{arctanh}(ax)}}{c^2\left(a-\frac{1}{x}\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{6681}$$

$$\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}x^2}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{6679}$$

$$\frac{a^2\int \frac{x^2(ax+1)^2}{(1-ax)^4}dx}{c^2}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^2\int \left(\frac{1}{a^2}+\frac{6}{a^2(ax-1)}+\frac{13}{a^2(ax-1)^2}+\frac{12}{a^2(ax-1)^3}+\frac{4}{a^2(ax-1)^4}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(\frac{13}{a^3(1-ax)}-\frac{6}{a^3(1-ax)^2}+\frac{4}{3a^3(1-ax)^3}+\frac{6\log(1-ax)}{a^3}+\frac{x}{a^2}\right)}{c^2}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
(a^2*(x/a^2 + 4/(3*a^3*(1 - a*x)^3) - 6/(a^3*(1 - a*x)^2) + 13/(a^3*(1 - a
*x)) + (6*Log[1 - a*x])/a^3))/c^2
```

3.410.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.410.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{c^{2}}+\frac{-13 a\,c^{2} x^{2}+20 c^{2} x-\frac{25 c^{2}}{3 a}}{c^{4} \left(a x -1\right)^{3}}+\frac{6 \ln \left(a x -1\right)}{a\,c^{2}}$	56
default	$\frac{a^{2} \left(\frac{x}{a^{2}}-\frac{6}{a^{3} \left(a x -1\right)^{2}}-\frac{13}{a^{3} \left(a x -1\right)}-\frac{4}{3 a^{3} \left(a x -1\right)^{3}}+\frac{6 \ln \left(a x -1\right)}{a^{3}}\right)}{c^{2}}$	61
norman	$\frac{\frac{a^{3} x^{4}}{c}-\frac{6 x}{c}+\frac{15 a\,x^{2}}{c}-\frac{34 a^{2} x^{3}}{3 c}}{\left(a x -1\right)^{3} c}+\frac{6 \ln \left(a x -1\right)}{a\,c^{2}}$	64
parallelrisch	$\frac{3 a^{4} x^{4}+18 a^{3} \ln \left(a x -1\right) x^{3}-34 a^{3} x^{3}-54 a^{2} \ln \left(a x -1\right) x^{2}+45 a^{2} x^{2}+54 a \ln \left(a x -1\right) x -18 a x -18 \ln \left(a x -1\right)}{3 \left(a x -1\right)^{3} c^{2} a}$	91

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+(-13*a*c^2*x^2+20*c^2*x-25/3*c^2/a)/c^4/(a*x-1)^3+6/a/c^2*ln(a*x-1)
```

3.410.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{3\,a^4x^4-9\,a^3x^3-30\,a^2x^2+57\,ax+18\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-25}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*x^4 - 9*a^3*x^3 - 30*a^2*x^2 + 57*a*x + 18*(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)*log(a*x - 1) - 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*
x - a*c^2)
```

3.410.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-39a^2x^2+60ax-25}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}+\frac{x}{c^2}+\frac{6\log\left(ax-1\right)}{ac^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**2,x)
```

output
```
(-39*a**2*x**2 + 60*a*x - 25)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**
2*c**2*x - 3*a*c**2) + x/c**2 + 6*log(a*x - 1)/(a*c**2)
```

3.410.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{39\,a^2x^2-60\,ax+25}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}+\frac{x}{c^2}+\frac{6\,\log\left(ax-1\right)}{ac^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-1/3*(39*a^2*x^2 - 60*a*x + 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
 - a*c^2) + x/c^2 + 6*log(a*x - 1)/(a*c^2)
```

3.410.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{ax-1}{ac^2}-\frac{6\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{39\,a^5c^4}{ax-1}+\frac{18\,a^5c^4}{(ax-1)^2}+\frac{4\,a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^2) - 6*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3
*(39*a^5*c^4/(a*x - 1) + 18*a^5*c^4/(a*x - 1)^2 + 4*a^5*c^4/(a*x - 1)^3)/(
a^6*c^6)
```

3.410.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{13\,a\,x^2-20\,x+\frac{25}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{6\,\ln\left(a\,x-1\right)}{a\,c^2}$$

input
```
int((a*x + 1)^2/((c - c/(a*x))^2*(a*x - 1)^2),x)
```

output
```
(13*a*x^2 - 20*x + 25/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*
x) + x/c^2 + (6*log(a*x - 1))/(a*c^2)
```

3.411 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

3.411.1 Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}$$

output
```
x/c^3-1/a/c^3/(-a*x+1)^4+16/3/a/c^3/(-a*x+1)^3-25/2/a/c^3/(-a*x+1)^2+19/a/
c^3/(-a*x+1)+7*ln(-a*x+1)/a/c^3
```

3.411.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{65-218ax+243a^2x^2-78a^3x^3-24a^4x^4+6a^5x^5+42(-1+ax)^4\log(1-ax)}{6ac^3(-1+ax)^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
(65 - 218*a*x + 243*a^2*x^2 - 78*a^3*x^3 - 24*a^4*x^4 + 6*a^5*x^5 + 42*(-1
 + a*x)^4*Log[1 - a*x])/(6*a*c^3*(-1 + a*x)^4)
```

3.411.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^3e^{4\text{arctanh}(ax)}}{c^3\left(a-\frac{1}{x}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{e^{4\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^3\int \frac{e^{4\text{arctanh}(ax)}x^3}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^3\int \frac{x^3(ax+1)^2}{(1-ax)^5}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^3\int \left(-\frac{1}{a^3}-\frac{7}{a^3(ax-1)}-\frac{19}{a^3(ax-1)^2}-\frac{25}{a^3(ax-1)^3}-\frac{16}{a^3(ax-1)^4}-\frac{4}{a^3(ax-1)^5}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\left(-\frac{19}{a^4(1-ax)}+\frac{25}{2a^4(1-ax)^2}-\frac{16}{3a^4(1-ax)^3}+\frac{1}{a^4(1-ax)^4}-\frac{7\log(1-ax)}{a^4}-\frac{x}{a^3}\right)}{c^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
-((a^3*(-(x/a^3) + 1/(a^4*(1 - a*x)^4) - 16/(3*a^4*(1 - a*x)^3) + 25/(2*a^
4*(1 - a*x)^2) - 19/(a^4*(1 - a*x)) - (7*Log[1 - a*x])/a^4))/c^3)
```

3.411.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.411.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x}{c^{3}}+\frac{-19 a^{2} c^{3} x^{3}+\frac{89 a \,c^{3} x^{2}}{2}-\frac{112 c^{3} x}{3}+\frac{65 c^{3}}{6 a}}{c^{6} \left(a x -1\right)^{4}}+\frac{7 \ln \left(a x -1\right)}{a \,c^{3}}$
default	$\frac{a^{3} \left(\frac{x}{a^{3}}-\frac{25}{2 a^{4} \left(a x -1\right)^{2}}-\frac{1}{a^{4} \left(a x -1\right)^{4}}-\frac{19}{a^{4} \left(a x -1\right)}-\frac{16}{3 a^{4} \left(a x -1\right)^{3}}+\frac{7 \ln \left(a x -1\right)}{a^{4}}\right)}{c^{3}}$
norman	$\frac{\frac{a^{4} x^{5}}{c}+\frac{7 x}{c}-\frac{49 a \,x^{2}}{2 c}+\frac{91 a^{2} x^{3}}{3 c}-\frac{89 a^{3} x^{4}}{6 c}}{\left(a x -1\right)^{4} c^{2}}+\frac{7 \ln \left(a x -1\right)}{a \,c^{3}}$
parallelrisch	$\frac{6 a^{5} x^{5}+42 \ln \left(a x -1\right) x^{4} a^{4}-89 a^{4} x^{4}-168 a^{3} \ln \left(a x -1\right) x^{3}+182 a^{3} x^{3}+252 a^{2} \ln \left(a x -1\right) x^{2}-147 a^{2} x^{2}-168 a \ln \left(a x -1\right) x+42 a x+42 \ln}{6 \left(a x -1\right)^{4} c^{3} a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3+(-19*a^2*c^3*x^3+89/2*a*c^3*x^2-112/3*c^3*x+65/6*c^3/a)/c^6/(a*x-1)^
4+7/a/c^3*ln(a*x-1)
```

3.411.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.42

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$=\frac{6\,a^5x^5-24\,a^4x^4-78\,a^3x^3+243\,a^2x^2-218\,ax+42\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log{(ax-1)}+6}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/6*(6*a^5*x^5 - 24*a^4*x^4 - 78*a^3*x^3 + 243*a^2*x^2 - 218*a*x + 42*(a^4
*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 65)/(a^5*c^3*x^4
- 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

3.411.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{-114a^3x^3+267a^2x^2-224ax+65}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}+\frac{x}{c^3}+\frac{7\log{(ax-1)}}{ac^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**3,x)
```

output
```
(-114*a**3*x**3 + 267*a**2*x**2 - 224*a*x + 65)/(6*a**5*c**3*x**4 - 24*a**
4*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3) + x/c**3 + 7*
log(a*x - 1)/(a*c**3)
```

3.411.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{114\,a^3x^3-267\,a^2x^2+224\,ax-65}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}+\frac{x}{c^3}+\frac{7\,\log{(ax-1)}}{ac^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*a*x - 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x
^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 + 7*log(a*x - 1)/(a*c^3)
```

3.411.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.22

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{ax-1}{ac^3}-\frac{7\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3}-\frac{\frac{114\,a^7c^9}{ax-1}+\frac{75\,a^7c^9}{(ax-1)^2}+\frac{32\,a^7c^9}{(ax-1)^3}+\frac{6\,a^7c^9}{(ax-1)^4}}{6\,a^8c^{12}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^3) - 7*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/6
*(114*a^7*c^9/(a*x - 1) + 75*a^7*c^9/(a*x - 1)^2 + 32*a^7*c^9/(a*x - 1)^3
+ 6*a^7*c^9/(a*x - 1)^4)/(a^8*c^12)
```

3.411.9 Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}-\frac{\frac{112\,x}{3}-\frac{89\,a\,x^2}{2}-\frac{65}{6\,a}+19\,a^2\,x^3}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}+\frac{7\,\ln{(a\,x-1)}}{a\,c^3}$$

input
```
int((a*x + 1)^2/((c - c/(a*x))^3*(a*x - 1)^2),x)
```

output
```
x/c^3 - ((112*x)/3 - (89*a*x^2)/2 - 65/(6*a) + 19*a^2*x^3)/(c^3 + 6*a^2*c^
3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (7*log(a*x - 1))/(a*c^3
)
```

3.412 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

3.412.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3} -\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1-ax)}{ac^4}$$

output
```
x/c^4+4/5/a/c^4/(-a*x+1)^5-5/a/c^4/(-a*x+1)^4+41/3/a/c^4/(-a*x+1)^3-22/a/c
^4/(-a*x+1)^2+26/a/c^4/(-a*x+1)+8*ln(-a*x+1)/a/c^4
```

3.412.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-202+890ax-1480a^2x^2+1080a^3x^3-240a^4x^4-75a^5x^5+15a^6x^6+120(-1+ax)^5\log(1-ax)}{15ac^4(-1+ax)^5}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
(-202 + 890*a*x - 1480*a^2*x^2 + 1080*a^3*x^3 - 240*a^4*x^4 - 75*a^5*x^5 +
 15*a^6*x^6 + 120*(-1 + a*x)^5*Log[1 - a*x])/(15*a*c^4*(-1 + a*x)^5)
```

3.412.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{a^4e^{4\text{arctanh}(ax)}}{c^4\left(a-\frac{1}{x}\right)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^4\int \frac{e^{4\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{a^4\int \frac{e^{4\text{arctanh}(ax)}x^4}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{a^4\int \frac{x^4(ax+1)^2}{(1-ax)^6}dx}{c^4}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{a^4\int \left(\frac{1}{a^4}+\frac{8}{a^4(ax-1)}+\frac{26}{a^4(ax-1)^2}+\frac{44}{a^4(ax-1)^3}+\frac{41}{a^4(ax-1)^4}+\frac{20}{a^4(ax-1)^5}+\frac{4}{a^4(ax-1)^6}\right)dx}{c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^4\left(\frac{26}{a^5(1-ax)}-\frac{22}{a^5(1-ax)^2}+\frac{41}{3a^5(1-ax)^3}-\frac{5}{a^5(1-ax)^4}+\frac{4}{5a^5(1-ax)^5}+\frac{8\log(1-ax)}{a^5}+\frac{x}{a^4}\right)}{c^4}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
(a^4*(x/a^4 + 4/(5*a^5*(1 - a*x)^5) - 5/(a^5*(1 - a*x)^4) + 41/(3*a^5*(1 -
 a*x)^3) - 22/(a^5*(1 - a*x)^2) + 26/(a^5*(1 - a*x)) + (8*Log[1 - a*x])/a^
5))/c^4
```

3.412.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.412.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x}{c^{4}}+\frac{-26 a^{3} c^{4} x^{4}+82 a^{2} c^{4} x^{3}-\frac{311 a\,c^{4} x^{2}}{3}+\frac{181 c^{4} x}{3}-\frac{202 c^{4}}{15 a}}{c^{8} \left(a x -1\right)^{5}}+\frac{8 \ln \left(a x -1\right)}{a\,c^{4}}$
default	$\frac{a^{4} \left(\frac{x}{a^{4}}-\frac{4}{5 a^{5} \left(a x -1\right)^{5}}-\frac{22}{a^{5} \left(a x -1\right)^{2}}-\frac{5}{a^{5} \left(a x -1\right)^{4}}-\frac{26}{a^{5} \left(a x -1\right)}-\frac{41}{3 a^{5} \left(a x -1\right)^{3}}+\frac{8 \ln \left(a x -1\right)}{a^{5}}\right)}{c^{4}}$
norman	$\frac{\frac{a^{5} x^{6}}{c}-\frac{8 x}{c}+\frac{36 a\,x^{2}}{c}-\frac{188 a^{2} x^{3}}{3 c}+\frac{154 a^{3} x^{4}}{3 c}-\frac{277 a^{4} x^{5}}{15 c}}{\left(a x -1\right)^{5} c^{3}}+\frac{8 \ln \left(a x -1\right)}{a\,c^{4}}$
parallelrisch	$\frac{15 a^{6} x^{6}+120 \ln \left(a x -1\right) x^{5} a^{5}-277 a^{5} x^{5}-600 \ln \left(a x -1\right) x^{4} a^{4}+770 a^{4} x^{4}+1200 a^{3} \ln \left(a x -1\right) x^{3}-940 a^{3} x^{3}-1200 a^{2} \ln \left(a x -1\right) x^{2}+540}{15 \left(a x -1\right)^{5} c^{4} a}$

input

```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output

```
x/c^4+(-26*a^3*c^4*x^4+82*a^2*c^4*x^3-311/3*a*c^4*x^2+181/3*c^4*x-202/15*c
^4/a)/c^8/(a*x-1)^5+8/a/c^4*ln(a*x-1)
```

3.412.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.47

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{15\,a^{6}x^{6} - 75\,a^{5}x^{5} - 240\,a^{4}x^{4} + 1080\,a^{3}x^{3} - 1480\,a^{2}x^{2} + 890\,ax + 120\,(a^{5}x^{5} - 5\,a^{4}x^{4} + 10\,a^{3}x^{3} - 10\,a^{2}x^{2}}{15\,(a^{6}c^{4}x^{5} - 5\,a^{5}c^{4}x^{4} + 10\,a^{4}c^{4}x^{3} - 10\,a^{3}c^{4}x^{2} + 5\,a^{2}c^{4}x - ac^{4})}$$

input

```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="fricas")
```

output

```
1/15*(15*a^6*x^6 - 75*a^5*x^5 - 240*a^4*x^4 + 1080*a^3*x^3 - 1480*a^2*x^2
+ 890*a*x + 120*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1
)*log(a*x - 1) - 202)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a
^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

3.412.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-390a^4x^4+1230a^3x^3-1555a^2x^2+905ax-202}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4} + \frac{x}{c^4} + \frac{8\log(ax-1)}{ac^4}$$

input

```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**4,x)
```

output

```
(-390*a**4*x**4 + 1230*a**3*x**3 - 1555*a**2*x**2 + 905*a*x - 202)/(15*a**
6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**4*c**4*x**3 - 150*a**3*c**4*x**2
+ 75*a**2*c**4*x - 15*a*c**4) + x/c**4 + 8*log(a*x - 1)/(a*c**4)
```

3.412.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{390\,a^4x^4-1230\,a^3x^3+1555\,a^2x^2-905\,ax+202}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)} + \frac{x}{c^4} + \frac{8\,\log(ax-1)}{ac^4}$$

input

```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="maxima")
```

output

```
-1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 1555*a^2*x^2 - 905*a*x + 202)/(a^6*c^4
*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c
^4) + x/c^4 + 8*log(a*x - 1)/(a*c^4)
```

3.412.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{ax-1}{ac^4} - \frac{8\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4} - \frac{\frac{390\,a^9c^{16}}{ax-1} + \frac{330\,a^9c^{16}}{(ax-1)^2} + \frac{205\,a^9c^{16}}{(ax-1)^3} + \frac{75\,a^9c^{16}}{(ax-1)^4} + \frac{12\,a^9c^{16}}{(ax-1)^5}}{15\,a^{10}c^{20}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^4) - 8*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 1/1
5*(390*a^9*c^16/(a*x - 1) + 330*a^9*c^16/(a*x - 1)^2 + 205*a^9*c^16/(a*x -
 1)^3 + 75*a^9*c^16/(a*x - 1)^4 + 12*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

3.412.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4} + \frac{\frac{311\,a\,x^2}{3} - \frac{181\,x}{3} + \frac{202}{15\,a} - 82\,a^2\,x^3 + 26\,a^3\,x^4}{-a^5\,c^4\,x^5 + 5\,a^4\,c^4\,x^4 - 10\,a^3\,c^4\,x^3 + 10\,a^2\,c^4\,x^2 - 5\,a\,c^4\,x + c^4} + \frac{8\,\ln\left(a\,x-1\right)}{a\,c^4}$$

input
```
int((a*x + 1)^2/((c - c/(a*x))^4*(a*x - 1)^2),x)
```

output
```
x/c^4 + ((311*a*x^2)/3 - (181*x)/3 + 202/(15*a) - 82*a^2*x^3 + 26*a^3*x^4)
/(c^4 + 10*a^2*c^4*x^2 - 10*a^3*c^4*x^3 + 5*a^4*c^4*x^4 - a^5*c^4*x^5 - 5*
a*c^4*x) + (8*log(a*x - 1))/(a*c^4)
```

3.413 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.413.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx=-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^4\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{25c^4\csc^{-1}(ax)}{2a}-\frac{5c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-25/2*c^4*arccsc(a*x)/a-5*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a-32/3*c^4*(1-1
/a^2/x^2)^(1/2)/a-1/3*c^4*(1-1/a^2/x^2)^(1/2)/a^3/x^2+5/2*c^4*(1-1/a^2/x^2
)^(1/2)/a^2/x+c^4*x*(1-1/a^2/x^2)^(1/2)
```

3.413.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$=\frac{c^4\left(2-15ax+62a^2x^2+9a^3x^3-64a^4x^4+6a^5x^5+90a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-30a^4\sqrt{1-\frac{1}{a^2x^2}}\right.}{6a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

input
```
Integrate[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

output
```
(c^4*(2 - 15*a*x + 62*a^2*x^2 + 9*a^3*x^3 - 64*a^4*x^4 + 6*a^5*x^5 + 90*a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 30*a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 30*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

3.413.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6731, 27, 540, 2340, 25, 2340, 25, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^5 \left(a-\frac{1}{x}\right)^5 x^2}{a^5 \sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4 \int \frac{\left(a-\frac{1}{x}\right)^5 x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^5}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^4 \left(a^5 x \left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(5a^4-\frac{10a^3}{x}+\frac{10a^2}{x^2}-\frac{5a}{x^3}+\frac{1}{x^4}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^5}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^4 \left(\frac{1}{3}a^2 \int -\frac{\left(15a^2-\frac{30a}{x}+\frac{32}{x^2}-\frac{15}{x^3a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5 x \left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^4 \left(-\frac{1}{3}a^2 \int \frac{\left(15a^2-\frac{30a}{x}+\frac{32}{x^2}-\frac{15}{x^3a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5 x \left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$\Big\downarrow$ 2340

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int -\frac{\left(30-\frac{75}{ax}+\frac{64}{a^2x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$\Big\downarrow$ 25

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\int \frac{\left(30-\frac{75}{ax}+\frac{64}{a^2x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$\Big\downarrow$ 2340

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(a^2\left(-\int -\frac{15\left(2a-\frac{5}{x}\right)x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right.}{a^5}$$

$\Big\downarrow$ 27

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\int \frac{\left(2a-\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$\Big\downarrow$ 538

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-5\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{}\right.\right.}{a^5}$$

$\Big\downarrow$ 223

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-5a\arcsin\left(\frac{1}{ax}\right)\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-}\right.\right.}{a^5}$$

$\Big\downarrow$ 243

3.413. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-5a\arcsin\left(\frac{1}{ax}\right)\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-}\right.\right.}{a^5}$$

$$\Big\downarrow 73$$

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(-2a^3\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-5a\arcsin\left(\frac{1}{ax}\right)\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}\right.}{a^5}$$

$$\Big\downarrow 221$$

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(-2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-5a\arcsin\left(\frac{1}{ax}\right)\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\right.\right.}{a^5}$$

input
```
Int[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

output
```
-((c^4*((a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) - a^5*Sqrt[1 - 1/(a^2*x^2)]*x
- (a^2*((15*a*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (a^2*(-64*Sqrt[1 - 1/(a^2*x^2
)] + (15*(-5*a*ArcSin[1/(a*x)] - 2*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a))/
2))/3))/a^5)
```

3.413.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

---

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.413. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

rule 6731

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.413.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{(ax+1)\left(64a^2x^2-15ax+2\right)c^4\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{5a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{25a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{\frac{ax-1}{ax+1}}}{a^4(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-66\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+66\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-75\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-75a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+66\right.}{6\sqrt{(ax}$

input

```
int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/6*(a*x+1)*(64*a^2*x^2-15*a*x+2)/x^3*c^4/a^4*((a*x-1)/(a*x+1))^(1/2)+(-5
*a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-25/2*a^3*arctan(1
/(a^2*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/(a*x-1)*((a*x-1)/
(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.413.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{150\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^4x^4-58}{6\,a^4x^3}$$

input

```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output

```
1/6*(150*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 30*a^3*c^4*x^3*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) + 30*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (6*a^4*c^4*x^4 - 58*a^3*c^4*x^3 - 49*a^2*c^4*x^2 + 13*a*c^4*x
 - 2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

3.413.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= \frac{c^4\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{4a^3}{}\right.\right.}{a^4}$$

input
```
integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**4*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-4*a*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/x**3, x) + Integral(6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/x**2, x) + Integral(-4*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x, x))/a**4
```

3.413.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.65

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$=\frac{1}{3}\left(\frac{75\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{87\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+61\,c^4\left(\frac{ax}{ax}\right.}{\frac{2\,(ax-1)a^2}{ax+1}-2}\right.$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(75*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 15*c^4*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 +
(87*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 61*c^4*((a*x - 1)/(a*x + 1))^(5/2) -
 55*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 45*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2
*(a*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2
/(a*x + 1)^4 + a^2))*a
```

3.413.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(117) = 234$.

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.96

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{25\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{a} + \frac{5\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^4\mathrm{sgn}(ax+1)}{a} - \frac{15\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|\mathrm{sgn}(ax+1)+60\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4\mathrm{sgn}(ax+1)+132\left(x|a|-\sqrt{a^2x^2}\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+\right.}$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
25*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 5*c^4*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*
c^4*sgn(a*x + 1)/a - 1/3*(15*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*s
gn(a*x + 1) + 60*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 132
*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 15*(x*abs(a) - sqrt
(a^2*x^2 - 1))*c^4*abs(a)*sgn(a*x + 1) + 64*a*c^4*sgn(a*x + 1))/(((x*abs(a
) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

3.413.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.37

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{25\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{15\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{55\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{61\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}} - \frac{10\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
(25*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (15*c^4*((a*x - 1)/(a*x + 1
))^(1/2) + (55*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3 - (61*c^4*((a*x - 1)/(a*
x + 1))^(5/2))/3 - 29*c^4*((a*x - 1)/(a*x + 1))^(7/2))/(a + (2*a*(a*x - 1)
)/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1)^4)/(a*x + 1)^4)
 - (10*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.413. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.414 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.414.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}x-\frac{13c^3\csc^{-1}(ax)}{2a}-\frac{4c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-13/2*c^3*arccsc(a*x)/a-4*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a-4*c^3*(1-1/a^
2/x^2)^(1/2)/a+1/2*c^3*(1-1/a^2/x^2)^(1/2)/a^2/x+c^3*x*(1-1/a^2/x^2)^(1/2)
```

3.414.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.58

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=\frac{c^3\left(-1+8ax-a^2x^2-8a^3x^3+2a^4x^4+10a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-8a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{1}{ax}\right.\right.}{2a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

input
```
Integrate[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

output
```
(c^3*(-1 + 8*a*x - a^2*x^2 - 8*a^3*x^3 + 2*a^4*x^4 + 10*a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)
]*x^3*ArcSin[1/(a*x)] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1
/(a^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

3.414.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6731, 27, 540, 2340, 25, 2340, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^4\left(a-\frac{1}{x}\right)^4x^2}{a^4\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^3\left(a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(4a^3-\frac{6a^2}{x}+\frac{4a}{x^2}-\frac{1}{x^3}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^4}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^3\left(\frac{1}{2}a^2\int -\frac{\left(8a-\frac{13}{x}+\frac{8}{x^2a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\int \frac{\left(8a-\frac{13}{x}+\frac{8}{x^2a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(a^2\left(-\int -\frac{\left(8a-\frac{13}{x}\right)x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-8a\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(a^2\int \frac{\left(8a-\frac{13}{x}\right)x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(\int \frac{\left(8a-\frac{13}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(8a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-13\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(8a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\sqrt{1-\frac{1}{a^2x^2}}-13a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-8a\sqrt{1-\frac{1}{a^2x^2}}-13a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(-8a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-8a\sqrt{1-\frac{1}{a^2x^2}}-13a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\right.\right.}{a^4}$$

$$\Big\downarrow \text{221}$$

3.414. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(-8a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-8a\sqrt{1-\frac{1}{a^2x^2}}-13a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

input
```
Int[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

output
```
-((c^3*(-1/2*(a^2*Sqrt[1 - 1/(a^2*x^2)])/x - a^4*Sqrt[1 - 1/(a^2*x^2)]*x -
 (a^2*(-8*a*Sqrt[1 - 1/(a^2*x^2)] - 13*a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]))/2))/a^4)
```

3.414.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.414. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.414.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{(ax+1)(8ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{2x^2a^3}+\frac{\left(-\frac{4a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{13a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^2\sqrt{(ax-1)(ax+1)}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(}{a^3(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\left(-8\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+8\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,ax-13\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-13a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+8\ln\left(\right.\right.}{2\sqrt{(ax-1)(ax+1)}\,a^3x}$

input
```
int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+1)*(8*a*x-1)/x^2*c^3/a^3*((a*x-1)/(a*x+1))^(1/2)+(-4*a^3*ln(a^2*
x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-13/2*a^2*arctan(1/(a^2*x^2-1)
^(1/2))+a^2*((a*x-1)*(a*x+1))^(1/2))*c^3/a^3/(a*x-1)*((a*x-1)/(a*x+1))^(1/
2)*((a*x-1)*(a*x+1))^(1/2)
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.35

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$= \frac{26\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3c^3x^3-6\,a^2c^3\right.}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(26*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 8*a^2*c^3*x^2*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1
)) - 1) + (2*a^3*c^3*x^3 - 6*a^2*c^3*x^2 - 7*a*c^3*x + c^3)*sqrt((a*x - 1)
/(a*x + 1)))/(a^3*x^2)
```

3.414.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$= \frac{c^3\left(\int a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^3}$$

input
```
integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**3*(Integral(a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/x**3, x) + Integral(3*a*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/x**2, x) + Integral(-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/x, x))/a**3
```

3.414.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.90

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$=\left(\frac{13\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-}{(ax}}\right.$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
(13*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 4*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) + 1)/a^2 + 4*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (11*c^
3*((a*x - 1)/(a*x + 1))^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*
sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x
 + 1)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

3.414.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.19

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx=\frac{13\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}$$
$$+\frac{4\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^3\operatorname{sgn}(ax+1)}{a}$$
$$-\frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|\operatorname{sgn}(ax+1)+8\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3\operatorname{sgn}(ax+1)-\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^2a|a|}$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
13*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 4*c^3*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*
c^3*sgn(a*x + 1)/a - ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn(a*x
+ 1) + 8*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - (x*abs(a) -
 sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 8*a*c^3*sgn(a*x + 1))/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a))
```

3.414.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.54

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{2\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+11\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a+\frac{a\,(a\,x-1)}{a\,x+1}-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}}+\frac{13\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{8\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 1
1*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x
- 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (13*c^3*atan(((a*x -
1)/(a*x + 1))^(1/2)))/a - (8*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.415 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.415.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3c^2\csc^{-1}(ax)}{a}-\frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-3*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a-c^2*(1-1/a^2/x^2
)^(1/2)/a+c^2*x*(1-1/a^2/x^2)^(1/2)
```

3.415.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)-3\arcsin\left(\frac{1}{ax}\right)-3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

input
```
Integrate[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]))/a
```

3.415.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6731, 27, 540, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \textcolor{blue}{6731}$$

$$-\frac{\int \frac{c^3\left(a-\frac{1}{x}\right)^3 x^2}{a^3\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{c^2\int \frac{\left(a-\frac{1}{x}\right)^3 x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \textcolor{blue}{540}$$

$$-\frac{c^2\left(a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(3a^2-\frac{3a}{x}+\frac{1}{x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^3}$$

$$\Big\downarrow \textcolor{blue}{2340}$$

$$-\frac{c^2\left(a^2\int -\frac{3\left(a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{c^2\left(-3a\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \textcolor{blue}{538}$$

$$-\frac{c^2\left(-3a\left(a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \textcolor{blue}{223}$$

$$-\frac{c^2\left(-3a\left(a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow 243$$

$$-\frac{c^2\left(-3a\left(\frac{1}{2}a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow 73$$

$$-\frac{c^2\left(-3a\left(a^3\left(-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow 221$$

$$-\frac{c^2\left(-3a\left(-a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

input

```
Int[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

output

```
-((c^2*(a^2*Sqrt[1 - 1/(a^2*x^2)] - a^3*Sqrt[1 - 1/(a^2*x^2)]*x - 3*a*(-(a
*ArcSin[1/(a*x)]) - a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])))/a^3)
```

3.415.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.415. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.415. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.415.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(-\frac{3a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\Bigg(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x-3ax\sqrt{a^2}\,\arctan\Bigg(}{\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+1)/x*c^2/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(-3*a*ln(a^2*x/(a^2)^(1/2)+
(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)-3*arctan(1/(a^2*x^2
-1)^(1/2)))*c^2/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$
$$=\frac{6\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2c^2x^2-c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
(6*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c^2*x*log(sqrt((a*x - 1
)/(a*x + 1)) + 1) + 3*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c^
2*x^2 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

3.415.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^2}$$

input
```
integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**2*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**2, x) + Integral(-2*a*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/x, x))/a**2
```

3.415.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.64

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) -
6*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*
x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

3.415.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{6\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{a} + \frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^2\mathrm{sgn}(ax+1)}{a} - \frac{2\,c^2\mathrm{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
6*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^2*sgn(a*x + 1)/a - 2*c^2*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2
+ 1)*abs(a))
```

3.415.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{4\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}} + \frac{6\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) + (6
*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^2*atanh(((a*x - 1)/(a*x +
 1))^(1/2)))/a
```

3.416 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.416.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{c\csc^{-1}(ax)}{a}-\frac{2c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-c*arccsc(a*x)/a-2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+c*x*(1-1/a^2/x^2)^(1/2
)
```

3.416.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x-2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\arcsin\left(\frac{1}{ax}\right)-2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

input
```
Integrate[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

output
```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Ar
cSin[1/(a*x)] - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

3.416.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6731, 27, 540, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^2 \left(a-\frac{1}{x}\right)^2 x^2}{a^2 \sqrt{1-\frac{1}{a^2 x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\int \frac{\left(a-\frac{1}{x}\right)^2 x^2}{\sqrt{1-\frac{1}{a^2 x^2}}}d\frac{1}{x}}{a^2}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c\left(a^2 x\left(-\sqrt{1-\frac{1}{a^2 x^2}}\right)-\int \frac{\left(2a-\frac{1}{x}\right) x}{\sqrt{1-\frac{1}{a^2 x^2}}}d\frac{1}{x}\right)}{a^2}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c\left(-2a\int \frac{x}{\sqrt{1-\frac{1}{a^2 x^2}}}d\frac{1}{x}+\int \frac{1}{\sqrt{1-\frac{1}{a^2 x^2}}}d\frac{1}{x}+a^2 x\left(-\sqrt{1-\frac{1}{a^2 x^2}}\right)\right)}{a^2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c\left(-2a\int \frac{x}{\sqrt{1-\frac{1}{a^2 x^2}}}d\frac{1}{x}+a^2 x\left(-\sqrt{1-\frac{1}{a^2 x^2}}\right)+a \arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c\left(-a\int \frac{x}{\sqrt{1-\frac{1}{a^2 x^2}}}d\frac{1}{x^2}+a^2 x\left(-\sqrt{1-\frac{1}{a^2 x^2}}\right)+a \arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(2a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a^2x\sqrt{1-\frac{1}{a^2x^2}}+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

input
```
Int[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

output
```
-((c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + a*ArcSin[1/(a*x)] + 2*a*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]))/a^2)
```

3.416.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.416. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.416.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(45) = 90$.

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.78

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}+\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{a^{2}}+2a\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)-2\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\right)}{\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^{2}}}$	136

input
```
int((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1
/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1)
)^(1/2))/(a^2)^(1/2))-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/((a*x-1)*(a*x
+1))^(1/2)/a/(a^2)^(1/2)
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - 2*c*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/a
```

3.416.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c\left(\int a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)\,dx\right)}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c*(Integral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/x, x))/a
```

3.416.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = -2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arct
an(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a
^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.416.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 2*c*log(abs(-x*
abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn
(a*x + 1)/a
```

3.416.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}$$

input
```
int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (4*c*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x +
 1))
```

3.417 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.417.1 Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c
```

3.417.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

3.417.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6731, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \; {\color{blue}6731}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; {\color{blue}242}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

3.417.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.417.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

method	result	size
gosper	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
trager	$\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{ac}$	30

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

output
```
(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

3.417.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\int \frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

output
```
a*Integral(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

3.417.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{2\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c)
```

3.417.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

output
```
sqrt(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

3.417.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1))
```

3.418 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.418.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
arctanh((1-1/a^2/x^2)^(1/2))/a/c^2+2*x*(1-1/a^2/x^2)^(1/2)/c^2-a*x*(1-1/a^
2/x^2)^(1/2)/c^2/(a-1/x)
```

3.418.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{-2-ax+a^2x^2+a\sqrt{1-\frac{1}{a^2x^2}}\,x\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

output
```
(-2 - a*x + a^2*x^2 + a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.418.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6731, 27, 564, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{ax^2}{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{564}$$

$$-\frac{a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}-\int -\frac{\left(a+\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\left(\int \frac{\left(a+\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(\frac{\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \text{534}$$

$$-\frac{a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \; 243$$

$$-\frac{a\left(\frac{\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(\frac{a^2\left(-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(\frac{-\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

input

```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

output

```
-((a*(Sqrt[1 - 1/(a^2*x^2)]/(a*(a - x^(-1))) + (-(a*Sqrt[1 - 1/(a^2*x^2)]*
x) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a^2))/c^2)
```

3.418.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.418. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6731

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.418.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(67) = 134$.

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.97

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^2(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(-3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2-2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+6\sqrt{a^2}\,\sqrt{(ax-1)(}{2a\sqrt{(ax-1)(ax+1)}\,c^2(ax-}$

3.418. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*
a^2/c^2*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c
^2*x - a*c^2)
```

3.418.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

output
```
a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x +
1), x)/c**2
```

3.418.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -a\left(\frac{\frac{3\,(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2
*c^2*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*
c^2) + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

3.418.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
undef
```

3.418.9 Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^2,x)
```

output
```
(2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 4)/(2*a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

3.419 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

3.419.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output

```
-2/3*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/
c^3+1/3*(-6*a-7/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^3
```

3.419.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{10-4ax-11a^2x^2+3a^3x^3+6a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

input

```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

output

```
(10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a
*x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1
 + a*x))
```

3.419.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{a^2x^2}{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^2\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^2x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^2c^3}$$

$$\Big\downarrow \text{ 532}$$

$$-\frac{\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(3a^2+\frac{6a}{x}+\frac{4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^2c^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^2+\frac{6a}{x}+\frac{4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\frac{1}{3}\left(\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{3a\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{1}{3}\left(3a\int \frac{\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow 534$$

$$-\frac{\frac{1}{3}\left(3a\left(2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow 243$$

$$-\frac{\frac{1}{3}\left(3a\left(\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow 73$$

$$-\frac{\frac{1}{3}\left(3a\left(-2a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{1}{3}\left(3a\left(-2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

input

```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

output

```
-(((2*(a + x^(-1)))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((6*a + 7/x)/Sqrt[1 - 1/
(a^2*x^2)] + 3*a*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 2*ArcTanh[Sqrt[1 - 1/(a^2
*x^2)]]))/3)/(a^2*c^3))
```

3.419.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.419. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

3.419. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.419.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.76

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\left(\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^6\left(x-\frac{1}{a}\right)^2}-\frac{8\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^5\left(x-\frac{1}{a}\right)}\right)a^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^3(ax-1)}$
default	$-\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-27\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+15\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}ax+81\sqrt{a^2}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(2/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/3/a^6/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-8/3/a^5/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^3/c^3*((a*x-1)/(a*
x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= \frac{6\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-11\,a^2x^2-4\,ax\right.}{3\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/3*(6*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 11*a^2*x
^2 - 4*a*x + 10)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a
*c^3)
```

3.419.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3\int \frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-3a^2x^2+3ax-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

output
```
a**3*Integral(x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x
**2 + 3*a*x - 1), x)/c**3
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= \frac{1}{6}\,a\left(\frac{\frac{14\,(ax-1)}{ax+1}-\frac{27\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
1/6*a*((14*(a*x - 1)/(a*x + 1) - 27*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^3*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + 12*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 12*log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c^3))
```

3.419.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.419.9 Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3} - \frac{\frac{14\,(a\,x-1)}{3\,(a\,x+1)} - \frac{9\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{1}{3}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} - 2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^3,x)
```

output
```
(4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((14*(a*x - 1))/(3*(a*x +
 1)) - (9*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(
3/2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

3.420 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

3.420.1 Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
-4/5*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/5*(-5*a-7/x)/a^2/c^4/(1-1/a^2/x
^2)^(3/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4+1/5*(-15*a-19/x)/a^2/c^4/(1
-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^4
```

3.420.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = \frac{-24+33ax+18a^2x^2-34a^3x^3+5a^4x^4+15a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

output
```
(-24 + 33*a*x + 18*a^2*x^2 - 34*a^3*x^3 + 5*a^4*x^4 + 15*a*Sqrt[1 - 1/(a^2
*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(5*a^2*c^4*Sqrt[1 -
1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

3.420.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{a^3x^2}{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^3x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^3c^4}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(5a^3+\frac{15a^2}{x}+\frac{16a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^3c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\int \frac{\left(5a^3+\frac{15a^2}{x}+\frac{16a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{5}\left(\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{3\left(5a^3+\frac{15a^2}{x}+\frac{14a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{5}\left(\int \frac{\left(5a^3+\frac{15a^2}{x}+\frac{14a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{5}\left(-\int -\frac{5a^2\left(a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{5}\left(5a^2\int \frac{\left(a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{534}$$

$$-\frac{\frac{1}{5}\left(5a^2\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\frac{1}{5}\left(5a^2\left(\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{5}\left(5a^2\left(-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{1}{5}\left(5a^2\left(-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

3.420. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

output
```
-(((4*a*(a + x^(-1)))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((a*(5*a + 7/x))/(1 -
1/(a^2*x^2))^(3/2) + (a*(15*a + 19/x))/Sqrt[1 - 1/(a^2*x^2)] + 5*a^2*(-(a*
Sqrt[1 - 1/(a^2*x^2)]*x) - 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/5)/(a^3*c^4)
)
```

3.420.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.420.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.63

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^8\left(x-\frac{1}{a}\right)^3}-\frac{6\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^7\left(x-\frac{1}{a}\right)^2}-\frac{24\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^6\left(x-\frac{1}{a}\right)}\right)a^4\sqrt{\frac{ax}{ax}}}{c^4(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-125\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^4x^4-120\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^5x^4+85\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+500}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

3.420. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

output
```
1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(3/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/5/a^8/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-6/5/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-24/5/a^6/(x-1/a)*((
x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^4*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(
(a*x-1)*(a*x+1))^(1/2)
```

3.420.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{15\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(5\,a^{4}\right.}{5\left(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
1) + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a
*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

3.420.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

output
```
a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x
**3 + 6*a**2*x**2 - 4*a*x + 1), x)/c**4
```

3.420.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{1}{20}\,a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1
)^3/(a*x + 1)^3 + 1)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*((a*x
- 1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) -
 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

3.420.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.43

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\text{sgn}(ax+1)}{c^4|a|}+\frac{\sqrt{a^2x^2-1}\text{sgn}(ax+1)}{ac^4}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
-3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^4*abs(a)) + sqr
t(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^4)
```

3.420.9 Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}-\frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{25\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{9\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^4,x)
```

output
```
(6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((15*(a*x - 1)^2)/(a*x +
1)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(
4*a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

3.420. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

3.421 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.421.1 Optimal result

Integrand size = 22, antiderivative size = 65

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(1+ax)}{a}$$

output
```
1/3*c^4/a^4/x^3-3*c^4/a^3/x^2+16*c^4/a^2/x+c^4*x+26*c^4*ln(x)/a-32*c^4*ln(
a*x+1)/a
```

3.421.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx =-\frac{c^4\left(-\frac{1}{3x^3}+\frac{3a}{x^2}-\frac{16a^2}{x}-a^4x-26a^3\log(x)+32a^3\log(1+ax)\right)}{a^4}$$

input
```
Integrate[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^4*(-1/3*1/x^3 + (3*a)/x^2 - (16*a^2)/x - a^4*x - 26*a^3*Log[x] + 32*a
^3*Log[1 + a*x]))/a^4)
```

3.421.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \; 6717$$

$$-\int \frac{c^4 e^{-2\text{arctanh}(ax)} \left(a-\frac{1}{x}\right)^4}{a^4}dx$$

$$\Big\downarrow \; 27$$

$$-\frac{c^4 \int e^{-2\text{arctanh}(ax)} \left(a-\frac{1}{x}\right)^4 dx}{a^4}$$

$$\Big\downarrow \; 6681$$

$$-\frac{c^4 \int \frac{e^{-2\text{arctanh}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$

$$\Big\downarrow \; 6679$$

$$-\frac{c^4 \int \frac{(1-ax)^5}{x^4(ax+1)}dx}{a^4}$$

$$\Big\downarrow \; 99$$

$$-\frac{c^4 \int \left(\frac{32a^4}{ax+1}-a^4-\frac{26a^3}{x}+\frac{16a^2}{x^2}-\frac{6a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \; 2009$$

$$-\frac{c^4 \left(a^4(-x)-26a^3\log(x)+32a^3\log(ax+1)-\frac{16a^2}{x}+\frac{3a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^4*(-1/3*1/x^3 + (3*a)/x^2 - (16*a^2)/x - a^4*x - 26*a^3*Log[x] + 32*a
^3*Log[1 + a*x]))/a^4)
```

3.421. $\int e^{-2\coth^{-1}(ax)} \left(c-\frac{c}{ax}\right)^4 dx$

3.421.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.421. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

3.421.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

method	result
default	$\frac{c^4\left(-32a^3\ln(ax+1)+a^4x+\frac{1}{3x^3}-\frac{3a}{x^2}+\frac{16a^2}{x}+26a^3\ln(x)\right)}{a^4}$
risch	$c^4x+\frac{16a^2c^4x^2-3a\,c^4x+\frac{1}{3}c^4}{a^4x^3}+\frac{26c^4\ln(-x)}{a}-\frac{32c^4\ln(ax+1)}{a}$
norman	$\frac{a^3c^4x^4+\frac{c^4}{3a}-3c^4x+16a\,c^4x^2}{a^3x^3}+\frac{26c^4\ln(x)}{a}-\frac{32c^4\ln(ax+1)}{a}$
parallelrisch	$\frac{3a^4c^4x^4+78c^4\ln(x)a^3x^3-96c^4\ln(ax+1)a^3x^3+48a^2c^4x^2-9a\,c^4x+c^4}{3a^4x^3}$
meijerg	$\frac{c^4(ax-\ln(ax+1))}{a}-\frac{5c^4\ln(ax+1)}{a}+\frac{10c^4(-\ln(ax+1)+\ln(x)+\ln(a))}{a}-\frac{10c^4\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+ 5c^4\big(-\ln$

input
```
int((c-c/a/x)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^4/a^4*(-32*a^3*ln(a*x+1)+a^4*x+1/3/x^3-3*a/x^2+16*a^2/x+26*a^3*ln(x))
```

3.421.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{3\,a^4c^4x^4-96\,a^3c^4x^3\log\left(ax+1\right)+78\,a^3c^4x^3\log\left(x\right)+48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

input
```
integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^4*x^4 - 96*a^3*c^4*x^3*log(a*x + 1) + 78*a^3*c^4*x^3*log(x) +
 48*a^2*c^4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

3.421.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=c^4x+\frac{2c^4\cdot\left(13\log\left(x\right)-16\log\left(x+\frac{1}{a}\right)\right)}{a}+\frac{48a^2c^4x^2-9ac^4x+c^4}{3a^4x^3}$$

input
```
integrate((c-c/a/x)**4*(a*x-1)/(a*x+1),x)
```

output
```
c**4*x + 2*c**4*(13*log(x) - 16*log(x + 1/a))/a + (48*a**2*c**4*x**2 - 9*a
*c**4*x + c**4)/(3*a**4*x**3)
```

3.421.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=c^4x-\frac{32\,c^4\log\left(ax+1\right)}{a}+\frac{26\,c^4\log\left(x\right)}{a}+\frac{48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

input
```
integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^4*x - 32*c^4*log(a*x + 1)/a + 26*c^4*log(x)/a + 1/3*(48*a^2*c^4*x^2 - 9*
a*c^4*x + c^4)/(a^4*x^3)
```

3.421.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=c^4x-\frac{32\,c^4\log\left(|ax+1|\right)}{a}+\frac{26\,c^4\log\left(|x|\right)}{a}+\frac{48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

input
```
integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^4*x - 32*c^4*log(abs(a*x + 1))/a + 26*c^4*log(abs(x))/a + 1/3*(48*a^2*c^
4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

3.421.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = c^4\,x+\frac{16\,a^2\,c^4\,x^2-3\,a\,c^4\,x+\frac{c^4}{3}}{a^4\,x^3} + \frac{26\,c^4\,\ln\left(x\right)}{a}-\frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - c/(a*x))^4*(a*x - 1))/(a*x + 1),x)
```

output
```
c^4*x + (c^4/3 + 16*a^2*c^4*x^2 - 3*a*c^4*x)/(a^4*x^3) + (26*c^4*log(x))/a
 - (32*c^4*log(a*x + 1))/a
```

3.422 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.422.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(1+ax)}{a}$$

output
```
-1/2*c^3/a^3/x^2+5*c^3/a^2/x+c^3*x+11*c^3*ln(x)/a-16*c^3*ln(a*x+1)/a
```

3.422.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3\left(\frac{1}{2x^2}-\frac{5a}{x}-a^3x-11a^2\log(x)+16a^2\log(1+ax)\right)}{a^3}$$

input
```
Integrate[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^3*(1/(2*x^2) - (5*a)/x - a^3*x - 11*a^2*Log[x] + 16*a^2*Log[1 + a*x])
)/a^3)
```

3.422.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{-2\coth^{-1}(ax)}dx$$
$$\Big\downarrow \text{6717}$$
$$-\int \frac{c^3 e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3}{a^3}dx$$
$$\Big\downarrow \text{27}$$
$$-\frac{c^3\int e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3dx}{a^3}$$
$$\Big\downarrow \text{6681}$$
$$\frac{c^3\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$
$$\Big\downarrow \text{6679}$$
$$\frac{c^3\int \frac{(1-ax)^4}{x^3(ax+1)}dx}{a^3}$$
$$\Big\downarrow \text{99}$$
$$\frac{c^3\int \left(-\frac{16a^3}{ax+1}+a^3+\frac{11a^2}{x}-\frac{5a}{x^2}+\frac{1}{x^3}\right)dx}{a^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^3\left(a^3x+11a^2\log(x)-16a^2\log(ax+1)+\frac{5a}{x}-\frac{1}{2x^2}\right)}{a^3}$$

input
```
Int[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^3*(-1/2*1/x^2 + (5*a)/x + a^3*x + 11*a^2*Log[x] - 16*a^2*Log[1 + a*x]))
/a^3
```

3.422. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$

3.422.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.422. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$

3.422.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^3\left(-16a^2\ln(ax+1)+a^3x-\frac{1}{2x^2}+\frac{5a}{x}+11a^2\ln(x)\right)}{a^3}$
risch	$c^3x+\frac{5a\,c^3x-\frac{1}{2}c^3}{a^3x^2}+\frac{11c^3\ln(-x)}{a}-\frac{16c^3\ln(ax+1)}{a}$
norman	$\frac{a^2c^3x^3-\frac{c^3}{2a}+5c^3x}{a^2x^2}+\frac{11c^3\ln(x)}{a}-\frac{16c^3\ln(ax+1)}{a}$
parallelrisch	$\frac{2a^3c^3x^3+22c^3\ln(x)a^2x^2-32c^3\ln(ax+1)a^2x^2+10a\,c^3x-c^3}{2a^3x^2}$
meijerg	$\frac{c^3(ax-\ln(ax+1))}{a}-\frac{4c^3\ln(ax+1)}{a}+\frac{6c^3(-\ln(ax+1)+\ln(x)+\ln(a))}{a}-\frac{4c^3\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+$ $c^3\left(-\ln(ax\right.$

input
```
int((c-c/a/x)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^3/a^3*(-16*a^2*ln(a*x+1)+a^3*x-1/2/x^2+5*a/x+11*a^2*ln(x))
```

3.422.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx \\ =\frac{2\,a^3c^3x^3-32\,a^2c^3x^2\log(ax+1)+22\,a^2c^3x^2\log(x)+10\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/2*(2*a^3*c^3*x^3 - 32*a^2*c^3*x^2*log(a*x + 1) + 22*a^2*c^3*x^2*log(x) +
 10*a*c^3*x - c^3)/(a^3*x^2)
```

3.422.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = c^3x + \frac{c^3\cdot\left(11\log\left(x\right)-16\log\left(x+\frac{1}{a}\right)\right)}{a} + \frac{10ac^3x-c^3}{2a^3x^2}$$

input
```
integrate((c-c/a/x)**3*(a*x-1)/(a*x+1),x)
```

output
```
c**3*x + c**3*(11*log(x) - 16*log(x + 1/a))/a + (10*a*c**3*x - c**3)/(2*a*
*3*x**2)
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = c^3x - \frac{16\,c^3\log\left(ax+1\right)}{a} + \frac{11\,c^3\log\left(x\right)}{a} + \frac{10\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^3*x - 16*c^3*log(a*x + 1)/a + 11*c^3*log(x)/a + 1/2*(10*a*c^3*x - c^3)/(
a^3*x^2)
```

3.422.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = c^3x - \frac{16\,c^3\log\left(|ax+1|\right)}{a} + \frac{11\,c^3\log\left(|x|\right)}{a} + \frac{10\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^3*x - 16*c^3*log(abs(a*x + 1))/a + 11*c^3*log(abs(x))/a + 1/2*(10*a*c^3*
x - c^3)/(a^3*x^2)
```

3.422.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=c^3\,x-\frac{\frac{c^3}{2}-5\,a\,c^3\,x}{a^3\,x^2}+\frac{11\,c^3\,\ln\left(x\right)}{a}-\frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - c/(a*x))^3*(a*x - 1))/(a*x + 1),x)
```

output
```
c^3*x - (c^3/2 - 5*a*c^3*x)/(a^3*x^2) + (11*c^3*log(x))/a - (16*c^3*log(a*
x + 1))/a
```

3.423 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.423.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(1+ax)}{a}$$

output
```
c^2/a^2/x+c^2*x+4*c^2*ln(x)/a-8*c^2*ln(a*x+1)/a
```

3.423.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2\left(-\frac{1}{x}-a^2x-4a\log(x)+8a\log(1+ax)\right)}{a^2}$$

input
```
Integrate[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^2*(-x^(-1) - a^2*x - 4*a*Log[x] + 8*a*Log[1 + a*x]))/a^2)
```

3.423.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{c^2e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2}{a^2}dx$$

$$\Big\downarrow \ 27$$

$$-\frac{c^2\int e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2dx}{a^2}$$

$$\Big\downarrow \ 6681$$

$$-\frac{c^2\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \ 6679$$

$$-\frac{c^2\int \frac{(1-ax)^3}{x^2(ax+1)}dx}{a^2}$$

$$\Big\downarrow \ 99$$

$$-\frac{c^2\int \left(\frac{8a^2}{ax+1}-a^2-\frac{4a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{c^2\left(a^2(-x)-4a\log(x)+8a\log(ax+1)-\frac{1}{x}\right)}{a^2}$$

input

```
Int[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

output

```
-((c^2*(-x^(-1) - a^2*x - 4*a*Log[x] + 8*a*Log[1 + a*x]))/a^2)
```

3.423. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2dx$

3.423.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.423. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.423.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{c^2\left(-8a\ln(ax+1)+a^2x+\frac{1}{x}+4a\ln(x)\right)}{a^2}$	31
risch	$\frac{c^2}{a^2x}+c^2x+\frac{4c^2\ln(-x)}{a}-\frac{8c^2\ln(ax+1)}{a}$	43
parallelrisch	$\frac{a^2c^2x^2+4c^2\ln(x)ax-8c^2\ln(ax+1)ax+c^2}{a^2x}$	44
norman	$\frac{\frac{c^2}{a}+a\,c^2x^2}{ax}+\frac{4c^2\ln(x)}{a}-\frac{8c^2\ln(ax+1)}{a}$	49
meijerg	$\frac{c^2(ax-\ln(ax+1))}{a}-\frac{3c^2\ln(ax+1)}{a}+\frac{3c^2(-\ln(ax+1)+\ln(x)+\ln(a))}{a}-\frac{c^2\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}$	87

input
```
int((c-c/a/x)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^2/a^2*(-8*a*ln(a*x+1)+a^2*x+1/x+4*a*ln(x))
```

3.423.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2-8\,ac^2x\log\left(ax+1\right)+4\,ac^2x\log\left(x\right)+c^2}{a^2x}$$

input
```
integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
(a^2*c^2*x^2 - 8*a*c^2*x*log(a*x + 1) + 4*a*c^2*x*log(x) + c^2)/(a^2*x)
```

3.423.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{4c^2\left(\log\left(x\right)-2\log\left(x+\frac{1}{a}\right)\right)}{a}+\frac{c^2}{a^2x}$$

input
```
integrate((c-c/a/x)**2*(a*x-1)/(a*x+1),x)
```

output
```
c**2*x + 4*c**2*(log(x) - 2*log(x + 1/a))/a + c**2/(a**2*x)
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x-\frac{8\,c^2\log\left(ax+1\right)}{a}+\frac{4\,c^2\log\left(x\right)}{a}+\frac{c^2}{a^2x}$$

input
```
integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^2*x - 8*c^2*log(a*x + 1)/a + 4*c^2*log(x)/a + c^2/(a^2*x)
```

3.423.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x-\frac{8\,c^2\log\left(|ax+1|\right)}{a}+\frac{4\,c^2\log\left(|x|\right)}{a}+\frac{c^2}{a^2x}$$

input
```
integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^2*x - 8*c^2*log(abs(a*x + 1))/a + 4*c^2*log(abs(x))/a + c^2/(a^2*x)
```

3.423.9 Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2\,x+\frac{c^2}{a^2\,x}+\frac{4\,c^2\,\ln\left(x\right)}{a}-\frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - c/(a*x))^2*(a*x - 1))/(a*x + 1),x)
```

output
```
c^2*x + c^2/(a^2*x) + (4*c^2*log(x))/a - (8*c^2*log(a*x + 1))/a
```

3.424 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.424.1 Optimal result

Integrand size = 20, antiderivative size = 23

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

output
```
c*x+c*ln(x)/a-4*c*ln(a*x+1)/a
```

3.424.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

input
```
Integrate[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
```

output
```
c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

3.424.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6681, 6679, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{-2\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \textcolor{blue}{6717}$$
$$-\int \frac{ce^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)}{a}dx$$
$$\Big\downarrow \textcolor{blue}{27}$$
$$-\frac{c\int e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)dx}{a}$$
$$\Big\downarrow \textcolor{blue}{6681}$$
$$\frac{c\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)}{x}dx}{a}$$
$$\Big\downarrow \textcolor{blue}{6679}$$
$$\frac{c\int \frac{(1-ax)^2}{x(ax+1)}dx}{a}$$
$$\Big\downarrow \textcolor{blue}{93}$$
$$\frac{c\int \left(-\frac{4a}{ax+1}+a+\frac{1}{x}\right)dx}{a}$$
$$\Big\downarrow \textcolor{blue}{2009}$$
$$\frac{c(ax-4\log(ax+1)+\log(x))}{a}$$

input
```
Int[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*(a*x + Log[x] - 4*Log[1 + a*x]))/a
```

3.424.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.424.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{c(ax-4\ln(ax+1)+\ln(x))}{a}$	20
parallelrisch	$\frac{acx+c\ln(x)-4c\ln(ax+1)}{a}$	23
norman	$cx+\frac{c\ln(x)}{a}-\frac{4c\ln(ax+1)}{a}$	24
risch	$cx+\frac{c\ln(-x)}{a}-\frac{4c\ln(ax+1)}{a}$	26
meijerg	$\frac{c(ax-\ln(ax+1))}{a}-\frac{2c\ln(ax+1)}{a}+\frac{c(-\ln(ax+1)+\ln(x)+\ln(a))}{a}$	49

input
```
int((c-c/a/x)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c/a*(a*x-4*ln(a*x+1)+ln(x))
```

3.424.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx-4\,c\log\left(ax+1\right)+c\log\left(x\right)}{a}$$

input
```
integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
(a*c*x - 4*c*log(a*x + 1) + c*log(x))/a
```

3.424.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\left(\log\left(x\right)-4\log\left(x+\frac{1}{a}\right)\right)}{a}$$

input
```
integrate((c-c/a/x)*(a*x-1)/(a*x+1),x)
```

output
```
c*x + c*(log(x) - 4*log(x + 1/a))/a
```

3.424.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{4\,c\log\left(ax+1\right)}{a}+\frac{c\log\left(x\right)}{a}$$

input
```
integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c*x - 4*c*log(a*x + 1)/a + c*log(x)/a
```

3.424.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{4\,c\log\left(|ax+1|\right)}{a}+\frac{c\log\left(|x|\right)}{a}$$

input
```
integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c*x - 4*c*log(abs(a*x + 1))/a + c*log(abs(x))/a
```

3.424.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\,x+\frac{c\,\ln\left(x\right)}{a}-\frac{4\,c\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - c/(a*x))*(a*x - 1))/(a*x + 1),x)
```

output
```
c*x + (c*log(x))/a - (4*c*log(a*x + 1))/a
```

3.425 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.425.1 Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}-\frac{\log(1+ax)}{ac}$$

output
```
x/c-ln(a*x+1)/a/c
```

3.425.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\left(\frac{x}{a}-\frac{\log(1+ax)}{a^2}\right)}{c}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
```

output
```
(a*(x/a - Log[1 + a*x]/a^2))/c
```

3.425.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \textcolor{blue}{6717}$$

$$-\int \frac{ae^{-2\text{arctanh}(ax)}}{c\left(a-\frac{1}{x}\right)}dx$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{a\int \frac{e^{-2\text{arctanh}(ax)}}{a-\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \textcolor{blue}{6681}$$

$$\frac{a\int \frac{e^{-2\text{arctanh}(ax)}x}{1-ax}dx}{c}$$

$$\Big\downarrow \textcolor{blue}{6679}$$

$$\frac{a\int \frac{x}{ax+1}dx}{c}$$

$$\Big\downarrow \textcolor{blue}{49}$$

$$\frac{a\int \left(\frac{1}{a}-\frac{1}{a(ax+1)}\right)dx}{c}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{a\left(\frac{x}{a}-\frac{\log(ax+1)}{a^2}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
```

output
```
(a*(x/a - Log[1 + a*x]/a^2))/c
```

3.425. $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$

3.425.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.425.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{ax-\ln(ax+1)}{ac}$	20
norman	$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$	21
risch	$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$	21
default	$\frac{a\left(\frac{x}{a}-\frac{\ln(ax+1)}{a^{2}}\right)}{c}$	23

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
(a*x-ln(a*x+1))/a/c
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{ax-\log(ax+1)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="fricas")
```

output
```
(a*x - log(a*x + 1))/(a*c)
```

3.425.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = a\left(\frac{x}{ac}-\frac{\log(ax+1)}{a^2c}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x),x)
```

output
```
a*(x/(a*c) - log(a*x + 1)/(a**2*c))
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}-\frac{\log(ax+1)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="maxima")
```

output
```
x/c - log(a*x + 1)/(a*c)
```

3.425.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}-\frac{\log\left(|ax+1|\right)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="giac")
```

output
```
x/c - log(abs(a*x + 1))/(a*c)
```

3.425.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{\ln\left(a\,x+1\right)-a\,x}{a\,c}$$

input
```
int((a*x - 1)/((c - c/(a*x))*(a*x + 1)),x)
```

output
```
-(log(a*x + 1) - a*x)/(a*c)
```

3.426 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.426.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{\operatorname{arctanh}(ax)}{ac^2}$$

output
```
x/c^2-arctanh(a*x)/a/c^2
```

3.426.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{\operatorname{arctanh}(ax)}{ac^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

output
```
x/c^2 - ArcTanh[a*x]/(a*c^2)
```

3.426.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6717, 27, 6681, 6679, 82, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^2 e^{-2\text{arctanh}(ax)}}{c^2\left(a-\frac{1}{x}\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}x^2}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^2\int \frac{x^2}{(1-ax)(ax+1)}dx}{c^2}$$

$$\Big\downarrow \text{82}$$

$$-\frac{a^2\int \frac{x^2}{1-a^2x^2}dx}{c^2}$$

$$\Big\downarrow \text{262}$$

$$-\frac{a^2\left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{c^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^2\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{c^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

output
```
-((a^2*(-(x/a^2) + ArcTanh[a*x]/a^3))/c^2)
```

3.426.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.426. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.426.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

method	result	size
parallelrisch	$\frac{2ax+\ln(ax-1)-\ln(ax+1)}{2a\,c^{2}}$	28
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{\ln(ax+1)}{2a^{3}}+\frac{\ln(ax-1)}{2a^{3}}\right)}{c^{2}}$	36
risch	$\frac{x}{c^{2}}-\frac{\ln(ax+1)}{2a\,c^{2}}+\frac{\ln(-ax+1)}{2a\,c^{2}}$	36
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{x}{c}}{c(ax-1)}+\frac{\ln(ax-1)}{2a\,c^{2}}-\frac{\ln(ax+1)}{2a\,c^{2}}$	56

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a*x+ln(a*x-1)-ln(a*x+1))/a/c^2
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{2}}\,dx=\frac{2\,ax-\log\left(ax+1\right)+\log\left(ax-1\right)}{2\,ac^{2}}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*x - log(a*x + 1) + log(a*x - 1))/(a*c^2)
```

3.426.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{2}}\,dx=a^{2}\left(\frac{x}{a^{2}c^{2}}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{a^{3}c^{2}}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**2,x)
```

output
```
a**2*(x/(a**2*c**2) + (log(x - 1/a)/2 - log(x + 1/a)/2)/(a**3*c**2))
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2} - \frac{\log(ax+1)}{2\,ac^2} + \frac{\log(ax-1)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
x/c^2 - 1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

3.426.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2} - \frac{\log(|ax+1|)}{2\,ac^2} + \frac{\log(|ax-1|)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
x/c^2 - 1/2*log(abs(a*x + 1))/(a*c^2) + 1/2*log(abs(a*x - 1))/(a*c^2)
```

3.426.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\operatorname{atanh}(a\,x) - a\,x}{a\,c^2}$$

input
```
int((a*x - 1)/((c - c/(a*x))^2*(a*x + 1)),x)
```

output
```
-(atanh(a*x) - a*x)/(a*c^2)
```

3.427 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} \, dx$

3.427.1 Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} \, dx = \frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

output
```
x/c^3+1/2/a/c^3/(-a*x+1)+5/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

3.427.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} \, dx = \frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

output
```
x/c^3 + 1/(2*a*c^3*(1 - a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/
(4*a*c^3)
```

3.427.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^3e^{-2\text{arctanh}(ax)}}{c^3\left(a-\frac{1}{x}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6681}$$

$$\frac{a^3\int \frac{e^{-2\text{arctanh}(ax)}x^3}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{6679}$$

$$\frac{a^3\int \frac{x^3}{(1-ax)^2(ax+1)}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3\int \left(-\frac{1}{4a^3(ax+1)}+\frac{1}{a^3}+\frac{5}{4a^3(ax-1)}+\frac{1}{2a^3(ax-1)^2}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}+\frac{x}{a^3}\right)}{c^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

output
```
(a^3*(x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 - a*x])/(4*a^4) - Log[1 + a*x
]/(4*a^4)))/c^3
```

3.427. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$

3.427.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.427.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{a^{3}\left(-\frac{\ln\left(ax+1\right)}{4a^{4}}+\frac{x}{a^{3}}-\frac{1}{2a^{4}\left(ax-1\right)}+\frac{5\ln\left(ax-1\right)}{4a^{4}}\right)}{c^{3}}$	48
risch	$\frac{x}{c^{3}}-\frac{1}{2a\left(ax-1\right)c^{3}}+\frac{5\ln\left(-ax+1\right)}{4a\,c^{3}}-\frac{\ln\left(ax+1\right)}{4a\,c^{3}}$	51
parallelrisch	$\frac{4a^{2}x^{2}+5a\ln\left(ax-1\right)x-a\ln\left(ax+1\right)x-6ax-5\ln\left(ax-1\right)+\ln\left(ax+1\right)}{4c^{3}\left(ax-1\right)a}$	63
norman	$\frac{\frac{a^{2}x^{3}}{c}+\frac{3x}{2c}-\frac{5a\,x^{2}}{2c}}{c^{2}\left(ax-1\right)^{2}}+\frac{5\ln\left(ax-1\right)}{4a\,c^{3}}-\frac{\ln\left(ax+1\right)}{4a\,c^{3}}$	67

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/c^3*(-1/4*ln(a*x+1)/a^4+x/a^3-1/2/a^4/(a*x-1)+5/4/a^4*ln(a*x-1))
```

3.427.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{4\,a^2x^2-4\,ax-(ax-1)\log{(ax+1)}+5\,(ax-1)\log{(ax-1)}-2}{4\,(a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)/(a^2*c^3*x - a*c^3)
```

3.427.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=a^3\left(-\frac{1}{2a^5c^3x-2a^4c^3}+\frac{x}{a^3c^3}+\frac{\frac{5\log{\left(x-\frac{1}{a}\right)}}{4}-\frac{\log{\left(x+\frac{1}{a}\right)}}{4}}{a^4c^3}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**3,x)
```

output
```
a**3*(-1/(2*a**5*c**3*x - 2*a**4*c**3) + x/(a**3*c**3) + (5*log(x - 1/a)/4
 - log(x + 1/a)/4)/(a**4*c**3))
```

3.427.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{1}{2\left(a^2c^3x-ac^3\right)}+\frac{x}{c^3}-\frac{\log(ax+1)}{4\,ac^3}+\frac{5\,\log(ax-1)}{4\,ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-1/2/(a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 5/4*log(a*x
- 1)/(a*c^3)
```

3.427.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}-\frac{\log(|ax+1|)}{4\,ac^3}+\frac{5\,\log(|ax-1|)}{4\,ac^3}-\frac{1}{2\,(ax-1)ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 5/4*log(abs(a*x - 1))/(a*c^3) - 1/
2/((a*x - 1)*a*c^3)
```

3.427.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}+\frac{5\,\ln(a\,x-1)}{4\,a\,c^3}-\frac{\ln(a\,x+1)}{4\,a\,c^3}$$

input
```
int((a*x - 1)/((c - c/(a*x))^3*(a*x + 1)),x)
```

output
```
x/c^3 + 1/(2*a*(c^3 - a*c^3*x)) + (5*log(a*x - 1))/(4*a*c^3) - log(a*x + 1
)/(4*a*c^3)
```

3.428 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

3.428.1 Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{x}{c^4}-\frac{1}{4ac^4(1-ax)^2}+\frac{7}{4ac^4(1-ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}$$

output
```
x/c^4-1/4/a/c^4/(-a*x+1)^2+7/4/a/c^4/(-a*x+1)+17/8*ln(-a*x+1)/a/c^4-1/8*ln
(a*x+1)/a/c^4
```

3.428.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=-\frac{a^4\left(-\frac{x}{a^4}+\frac{1}{4a^5(1-ax)^2}-\frac{7}{4a^5(1-ax)}-\frac{17\log(1-ax)}{8a^5}+\frac{\log(1+ax)}{8a^5}\right)}{c^4}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

output
```
-((a^4*(-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1
 - a*x])/(8*a^5) + Log[1 + a*x]/(8*a^5)))/c^4)
```

3.428.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^4e^{-2\text{arctanh}(ax)}}{c^4\left(a-\frac{1}{x}\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}x^4}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^4\int \frac{x^4}{(1-ax)^3(ax+1)}dx}{c^4}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^4\int \left(\frac{1}{8a^4(ax+1)}-\frac{1}{a^4}-\frac{17}{8a^4(ax-1)}-\frac{7}{4a^4(ax-1)^2}-\frac{1}{2a^4(ax-1)^3}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\left(-\frac{7}{4a^5(1-ax)}+\frac{1}{4a^5(1-ax)^2}-\frac{17\log(1-ax)}{8a^5}+\frac{\log(ax+1)}{8a^5}-\frac{x}{a^4}\right)}{c^4}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

output
```
-((a^4*(-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1
 - a*x])/(8*a^5) + Log[1 + a*x]/(8*a^5)))/c^4)
```

3.428.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.428.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{a^{4} \left(-\frac{\ln \left(a x +1\right)}{8 a^{5}}+\frac{x}{a^{4}}-\frac{1}{4 a^{5} \left(a x -1\right)^{2}}-\frac{7}{4 a^{5} \left(a x -1\right)}+\frac{17 \ln \left(a x -1\right)}{8 a^{5}}\right)}{c^{4}}$	60
risch	$\frac{x}{c^{4}}+\frac{-\frac{7 c^{4} x}{4}+\frac{3 c^{4}}{2 a}}{c^{8} \left(a x -1\right)^{2}}-\frac{\ln \left(a x +1\right)}{8 a \,c^{4}}+\frac{17 \ln \left(-a x +1\right)}{8 a \,c^{4}}$	62
norman	$\frac{\frac{a^{3} x^{4}}{c}-\frac{9 x}{4 c}+\frac{23 a \,x^{2}}{4 c}-\frac{9 a^{2} x^{3}}{2 c}}{c^{3} \left(a x -1\right)^{3}}+\frac{17 \ln \left(a x -1\right)}{8 a \,c^{4}}-\frac{\ln \left(a x +1\right)}{8 a \,c^{4}}$	78
parallelrisch	$\frac{8 a^{3} x^{3}+17 a^{2} \ln \left(a x -1\right) x^{2}-a^{2} \ln \left(a x +1\right) x^{2}-28 a^{2} x^{2}-34 a \ln \left(a x -1\right) x +2 a \ln \left(a x +1\right) x +18 a x +17 \ln \left(a x -1\right)-\ln \left(a x +1\right)}{8 c^{4} \left(a x -1\right)^{2} a}$	101

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4/c^4*(-1/8*ln(a*x+1)/a^5+x/a^4-1/4/a^5/(a*x-1)^2-7/4/a^5/(a*x-1)+17/8/a
^5*ln(a*x-1))
```

3.428.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{8\,a^3x^3 - 16\,a^2x^2 - 6\,ax - (a^2x^2 - 2\,ax + 1)\log(ax+1) + 17\,(a^2x^2 - 2\,ax + 1)\log(ax-1) + 12}{8\,(a^3c^4x^2 - 2\,a^2c^4x + ac^4)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^4*x^2 - 2*a^2*c^4*x +
a*c^4)
```

3.428.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = a^4\left(\frac{-7ax+6}{4a^7c^4x^2 - 8a^6c^4x + 4a^5c^4} + \frac{x}{a^4c^4} + \frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8} - \frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^4}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**4,x)
```

output
```
a**4*((-7*a*x + 6)/(4*a**7*c**4*x**2 - 8*a**6*c**4*x + 4*a**5*c**4) + x/(a
**4*c**4) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**4))
```

3.428.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=-\frac{7\,ax-6}{4\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}+\frac{x}{c^4}-\frac{\log{(ax+1)}}{8\,ac^4}+\frac{17\,\log{(ax-1)}}{8\,ac^4}$$

input

```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

output

```
-1/4*(7*a*x - 6)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 1/8*log(a*x
 + 1)/(a*c^4) + 17/8*log(a*x - 1)/(a*c^4)
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{x}{c^4}-\frac{\log{(|ax+1|)}}{8\,ac^4}+\frac{17\,\log{(|ax-1|)}}{8\,ac^4}-\frac{7\,ax-6}{4\,(ax-1)^2ac^4}$$

input

```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

output

```
x/c^4 - 1/8*log(abs(a*x + 1))/(a*c^4) + 17/8*log(abs(a*x - 1))/(a*c^4) - 1
/4*(7*a*x - 6)/((a*x - 1)^2*a*c^4)
```

3.428.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{x}{c^4}-\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}+\frac{17\,\ln{(a\,x-1)}}{8\,a\,c^4}-\frac{\ln{(a\,x+1)}}{8\,a\,c^4}$$

input

```
int((a*x - 1)/((c - c/(a*x))^4*(a*x + 1)),x)
```

output

```
x/c^4 - ((7*x)/4 - 3/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) + (17*log(a*x
- 1))/(8*a*c^4) - log(a*x + 1)/(8*a*c^4)
```

3.429 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.429.1 Optimal result

Integrand size = 22, antiderivative size = 164

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^4\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{91c^4\csc^{-1}(ax)}{2a}-\frac{7c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
91/2*c^4*arccsc(a*x)/a-7*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+64*c^4*(a-1/x)
/a^2/(1-1/a^2/x^2)^(1/2)+68/3*c^4*(1-1/a^2/x^2)^(1/2)/a+1/3*c^4*(1-1/a^2/x
^2)^(1/2)/a^3/x^2-7/2*c^4*(1-1/a^2/x^2)^(1/2)/a^2/x+c^4*x*(1-1/a^2/x^2)^(1
/2)
```

3.429.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.33 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.46

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$

$$=\frac{c^4\left(2772\sqrt{2}a^3x^3(-1+ax)^3(1+ax)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+1980\sqrt{2}a^2x^2(-1+ax)^4(1\right.}{}$$

input

```
Integrate[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

output

```
(c^4*(2772*Sqrt[2]*a^3*x^3*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5
/2, 7/2, (1 - 1/(a*x))/2] + 1980*Sqrt[2]*a^2*x^2*(-1 + a*x)^4*(1 + a*x)*Hy
pergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] + 35*(-198*a^2*Sqrt[1 + 1/
(a*x)]*x^2 + 1716*a^3*Sqrt[1 + 1/(a*x)]*x^3 - 7425*a^4*Sqrt[1 + 1/(a*x)]*x
^4 + 26268*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 29403*a^6*Sqrt[1 + 1/(a*x)]*x^6 - 5
0160*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 396*a^8*Sqrt[1 + 1/(a*x)]*x^8 + 66726*a^6
*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 66726*a^7*Sqrt[
1 - 1/(a*x)]*x^7*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 1980*a^6*Sqrt[1 - 1/(
a*x)]*x^6*ArcSin[1/(a*x)] - 1980*a^7*Sqrt[1 - 1/(a*x)]*x^7*ArcSin[1/(a*x)]
 - 2772*a^7*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^7*ArcTanh[Sqrt[1 - 1
/(a^2*x^2)]] + 44*Sqrt[2]*a*x*(-1 + a*x)^5*(1 + a*x)*Hypergeometric2F1[3/2
, 9/2, 11/2, (1 - 1/(a*x))/2] + 36*Sqrt[2]*(-1 + a*x)^6*(1 + a*x)*Hypergeo
metric2F1[3/2, 11/2, 13/2, (1 - 1/(a*x))/2])))/(13860*a^7*Sqrt[1 - 1/(a*x)
]*x^6*(1 + a*x))
```

3.429.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6731, 27, 528, 2338, 2340, 25, 2340, 25, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^7\left(a-\frac{1}{x}\right)^7x^2}{a^7\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\int \frac{\left(a-\frac{1}{x}\right)^7x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^7}$$

$$\Big\downarrow \text{528}$$

$$-\frac{c^4\left(a^2\int\frac{\left(a^5-\frac{7a^4}{x}-\frac{42a^3}{x^2}+\frac{22a^2}{x^3}-\frac{7a}{x^4}+\frac{1}{x^5}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$$\Big\downarrow\ 2338$$

$$-\frac{c^4\left(a^2\left(a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int\frac{\left(7a^4+\frac{42a^3}{x}-\frac{22a^2}{x^2}+\frac{7a}{x^3}-\frac{1}{x^4}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$$\Big\downarrow\ 2340$$

$$-\frac{c^4\left(a^2\left(\frac{1}{3}a^2\int-\frac{\left(21a^2+\frac{126a}{x}-\frac{68}{x^2}+\frac{21}{x^3a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$$\Big\downarrow\ 25$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\int\frac{\left(21a^2+\frac{126a}{x}-\frac{68}{x^2}+\frac{21}{x^3a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$$\Big\downarrow\ 2340$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(-\frac{1}{2}a^2\int-\frac{\left(42+\frac{273}{ax}-\frac{136}{a^2x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$$\Big\downarrow\ 25$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\int\frac{\left(42+\frac{273}{ax}-\frac{136}{a^2x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$$\Big\downarrow\ 2340$$

$$-\frac{c^4\Bigg(a^2\Bigg(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(136\sqrt{1-\frac{1}{a^2x^2}}-a^2\int-\frac{21\left(2a+\frac{13}{x}\right)x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^7}$$

$$\Big\downarrow\ 27$$

3.429. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

$$-\frac{c^4 \left(a^2 \left(-\frac{1}{3}a^2 \left(\frac{1}{2}a^2 \left(\frac{21\int \frac{\left(2a+\frac{13}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a} + 136\sqrt{1-\frac{1}{a^2x^2}} \right) - \frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x} \right) - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} + a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right) \right) - \right.\right.}{a^7}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c^4 \left(a^2 \left(-\frac{1}{3}a^2 \left(\frac{1}{2}a^2 \left(\frac{21\left(13\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a} + 136\sqrt{1-\frac{1}{a^2x^2}} \right) - \frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x} \right) - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} + a^5 \right.\right.}{a^7}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c^4 \left(a^2 \left(-\frac{1}{3}a^2 \left(\frac{1}{2}a^2 \left(\frac{21\left(2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+13a\arcsin\left(\frac{1}{ax}\right)\right)}{a} + 136\sqrt{1-\frac{1}{a^2x^2}} \right) - \frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x} \right) - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} + a^5x \right.\right.}{a^7}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^4 \left(a^2 \left(-\frac{1}{3}a^2 \left(\frac{1}{2}a^2 \left(\frac{21\left(a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+13a\arcsin\left(\frac{1}{ax}\right)\right)}{a} + 136\sqrt{1-\frac{1}{a^2x^2}} \right) - \frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x} \right) - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} + a^5x \right.\right.}{a^7}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^4 \left(a^2 \left(-\frac{1}{3}a^2 \left(\frac{1}{2}a^2 \left(\frac{21\left(13a\arcsin\left(\frac{1}{ax}\right)-2a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)}{a} + 136\sqrt{1-\frac{1}{a^2x^2}} \right) - \frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x} \right) - \right.\right.}{a^7}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^4 \left(a^2 \left(-\frac{1}{3}a^2 \left(\frac{1}{2}a^2 \left(\frac{21\left(13a\arcsin\left(\frac{1}{ax}\right)-2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a} + 136\sqrt{1-\frac{1}{a^2x^2}} \right) - \frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x} \right) - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} + a \right.\right.}{a^7}$$

3.429. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

input

```
Int[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

output

```
-((c^4*((-64*a^5*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-1/3*(a^2*Sqrt
[1 - 1/(a^2*x^2)])/x^2 - a^5*Sqrt[1 - 1/(a^2*x^2)]*x - (a^2*((-21*a*Sqrt[1
 - 1/(a^2*x^2)])/(2*x) + (a^2*(136*Sqrt[1 - 1/(a^2*x^2)] + (21*(13*a*ArcSi
n[1/(a*x)] - 2*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a))/2))/3)))/a^7)
```

3.429.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.429. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.429. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

3.429.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16

method	result
risch	$\frac{(ax+1)\left(136a^2x^2-21ax+2\right)c^4\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{7a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{91a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}+\frac{64a^2\sqrt{a^2\left(x+\right.}}{x}\right.}{a^4(ax-1)}$
default	$-\frac{\left(-138\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+96\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5+138\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-549\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5-273\arctan\left(\frac{}{\sqrt{a^2}}\right.\right.}{}$

input
```
int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)*(136*a^2*x^2-21*a*x+2)/x^3*c^4/a^4*((a*x-1)/(a*x+1))^(1/2)+(-7
*a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+91/2*a^3*arctan(1
/(a^2*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2)+64*a^2/(x+1/a)*(a^2*(x+1/a
)^2-2*a*(x+1/a))^(1/2))*c^4/a^4/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(
a*x+1))^(1/2)
```

3.429.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = -\frac{546\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^4x^4+\right.}{6\,a^4x^3}$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(546*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 42*a^3*c^4*x^3*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) - 42*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (6*a^4*c^4*x^4 + 526*a^3*c^4*x^3 + 115*a^2*c^4*x^2 - 19*a*c^
4*x + 2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

3.429.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$=\frac{c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{5a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{10a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{10a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(\right.\right.}{a^4}$$

input
```
integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + In
tegral(5*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integra
l(-10*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integra
l(10*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-5
*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

3.429.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.50

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx =$$
$$-\frac{1}{3}\left(\frac{273\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{192\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2}+153\right.$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(273*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 21*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 - 21*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
- 192*c^4*sqrt((a*x - 1)/(a*x + 1))/a^2 + (153*c^4*((a*x - 1)/(a*x + 1))^(
7/2) + 91*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 169*c^4*((a*x - 1)/(a*x + 1))^
(3/2) - 123*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)*a^2/(a*x + 1) - 2*
(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 + a^2))*a
```

3.429.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\int\left(c-\frac{c}{ax}\right)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.429.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{41\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{169\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{91\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-51\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}$$
$$+\frac{64\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{91\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,14{}\mathrm{i}}{a}$$

input
```
int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(41*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (169*c^4*((a*x - 1)/(a*x + 1))^(3/2)
)/3 - (91*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 51*c^4*((a*x - 1)/(a*x + 1)
)^(7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 -
(a*(a*x - 1)^4)/(a*x + 1)^4) + (64*c^4*((a*x - 1)/(a*x + 1))^(1/2))/a - (9
1*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^4*atan(((a*x - 1)/(a*x + 1
))^(1/2)*1i)*14i)/a
```

3.430 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.430.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}x+\frac{33c^3\csc^{-1}(ax)}{2a}-\frac{6c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
33/2*c^3*arccsc(a*x)/a-6*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+32*c^3*(a-1/x)
/a^2/(1-1/a^2/x^2)^(1/2)+6*c^3*(1-1/a^2/x^2)^(1/2)/a-1/2*c^3*(1-1/a^2/x^2)
^(1/2)/a^2/x+c^3*x*(1-1/a^2/x^2)^(1/2)
```

3.430.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.91

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$

$$=\frac{c^3\left(420a^2\sqrt{1+\frac{1}{ax}}x^2-3465a^3\sqrt{1+\frac{1}{ax}}x^3+16800a^4\sqrt{1+\frac{1}{ax}}x^4+17955a^5\sqrt{1+\frac{1}{ax}}x^5-32340a^6\right.}{}$$

input
```
Integrate[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^3*(420*a^2*Sqrt[1 + 1/(a*x)]*x^2 - 3465*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 168
00*a^4*Sqrt[1 + 1/(a*x)]*x^4 + 17955*a^5*Sqrt[1 + 1/(a*x)]*x^5 - 32340*a^6
*Sqrt[1 + 1/(a*x)]*x^6 + 630*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 44730*a^5*Sqrt[1
- 1/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 44730*a^6*Sqrt[1 - 1/(a
*x)]*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2520*a^5*Sqrt[1 - 1/(a*x)]*x^
5*ArcSin[1/(a*x)] - 2520*a^6*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[1/(a*x)] - 3780*
a^6*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^6*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]] + 126*Sqrt[2]*a^2*x^2*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5
/2, 7/2, (1 - 1/(a*x))/2] + 90*Sqrt[2]*a*x*(-1 + a*x)^4*(1 + a*x)*Hypergeo
metric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] - 70*Sqrt[2]*Hypergeometric2F1[3
/2, 9/2, 11/2, (1 - 1/(a*x))/2] + 280*Sqrt[2]*a*x*Hypergeometric2F1[3/2, 9
/2, 11/2, (1 - 1/(a*x))/2] - 350*Sqrt[2]*a^2*x^2*Hypergeometric2F1[3/2, 9/
2, 11/2, (1 - 1/(a*x))/2] + 350*Sqrt[2]*a^4*x^4*Hypergeometric2F1[3/2, 9/2
, 11/2, (1 - 1/(a*x))/2] - 280*Sqrt[2]*a^5*x^5*Hypergeometric2F1[3/2, 9/2,
 11/2, (1 - 1/(a*x))/2] + 70*Sqrt[2]*a^6*x^6*Hypergeometric2F1[3/2, 9/2, 1
1/2, (1 - 1/(a*x))/2]))/(630*a^6*Sqrt[1 - 1/(a*x)]*x^5*(1 + a*x))
```

3.430.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6731, 27, 528, 2338, 2340, 27, 2340, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^6\left(a-\frac{1}{x}\right)^6x^2}{a^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int \frac{\left(a-\frac{1}{x}\right)^6x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^6}$$

$$\Big\downarrow \text{528}$$

$$-\frac{c^3\left(a^2\int\frac{\left(a^4-\frac{6a^3}{x}-\frac{16a^2}{x^2}+\frac{6a}{x^3}-\frac{1}{x^4}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \ 2338$$

$$-\frac{c^3\left(a^2\left(a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int\frac{\left(6a^3+\frac{16a^2}{x}-\frac{6a}{x^2}+\frac{1}{x^3}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \ 2340$$

$$-\frac{c^3\left(a^2\left(\frac{1}{2}a^2\int-\frac{3\left(4a+\frac{11}{x}-\frac{4}{x^2a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\int\frac{\left(4a+\frac{11}{x}-\frac{4}{x^2a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \ 2340$$

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(4a\sqrt{1-\frac{1}{a^2x^2}}-a^2\int-\frac{\left(4a+\frac{11}{x}\right)x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \ 25$$

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(a^2\int\frac{\left(4a+\frac{11}{x}\right)x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(\int\frac{\left(4a+\frac{11}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \ 538$$

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(4a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+11\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\right.}{a^6}$$

3.430. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

$$\Big\downarrow \; 223$$

$$-\frac{c^3\Bigg(a^2\Bigg(-\frac{3}{2}a^2\Bigg(4a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a\sqrt{1-\frac{1}{a^2x^2}}+11a\arcsin\left(\frac{1}{ax}\right)\Bigg)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\Bigg)-\frac{32a^4}{\sqrt{1-}}}{a^6}$$

$$\Big\downarrow \; 243$$

$$-\frac{c^3\Bigg(a^2\Bigg(-\frac{3}{2}a^2\Bigg(2a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+4a\sqrt{1-\frac{1}{a^2x^2}}+11a\arcsin\left(\frac{1}{ax}\right)\Bigg)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\Bigg)-\frac{32a}{\sqrt{1}}}{a^6}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^3\Bigg(a^2\Bigg(-\frac{3}{2}a^2\Bigg(-4a^3\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}+4a\sqrt{1-\frac{1}{a^2x^2}}+11a\arcsin\left(\frac{1}{ax}\right)\Bigg)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\Bigg(-\sqrt{1}}{a^6}$$

$$\Big\downarrow \; 221$$

$$-\frac{c^3\Bigg(a^2\Bigg(-\frac{3}{2}a^2\left(-4a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+4a\sqrt{1-\frac{1}{a^2x^2}}+11a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^6}$$

input
```
Int[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^3*((-32*a^4*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*((a^2*Sqrt[1 -
1/(a^2*x^2)])/(2*x) - a^4*Sqrt[1 - 1/(a^2*x^2)]*x - (3*a^2*(4*a*Sqrt[1 - 1
/(a^2*x^2)] + 11*a*ArcSin[1/(a*x)] - 4*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/
2)))/a^6)
```

3.430.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.430. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.430.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34

method	result
risch	$\frac{(ax+1)(12ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{2x^2a^3}+\frac{\left(-\frac{6a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{33a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^2\sqrt{(ax-1)(ax+1)}+\frac{32a\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\right.}}{x+\frac{1}{a}}\right.}{a^3(ax-1)}$
default	$-\frac{\left(-12\sqrt{a^2x^2-1}\sqrt{a^2}\,a^5x^5+12\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-57\sqrt{a^2x^2-1}\sqrt{a^2}\,a^4x^4-33\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^4x^4+12\ln\left(\frac{a^2x+\sqrt{a^2x^2}}{\sqrt{a^2}}\right.\right.}{}$

input
```
int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+1)*(12*a*x-1)/x^2*c^3/a^3*((a*x-1)/(a*x+1))^(1/2)+(-6*a^3*ln(a^2*
x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+33/2*a^2*arctan(1/(a^2*x^2-1)
^(1/2))+a^2*((a*x-1)*(a*x+1))^(1/2)+32*a/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a
))^(1/2))*c^3/a^3/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.430.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{66\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^3x^3+7}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(66*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 12*a^2*c^3*x^2*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) - (2*a^3*c^3*x^3 + 78*a^2*c^3*x^2 + 11*a*c^3*x - c^3)*sqrt((a*x
 - 1)/(a*x + 1)))/(a^3*x^2)
```

3.430.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^4x}{}\right.}{a^3}$$

input
```
integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Int
egral(-4*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integra
l(6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-4*
a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**3
```

3.430.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.67

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx =$$

$$-\Bigg(\frac{33\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{32\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{11\,c^3\bigl(\frac{ax-1}{ax+1}}{\frac{(ax-1)a^2}{ax+1}}$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-(33*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 6*c^3*log(sqrt((a*x - 1)/
(a*x + 1)) + 1)/a^2 - 6*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 32*c^
3*sqrt((a*x - 1)/(a*x + 1))/a^2 + (11*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 6*
c^3*((a*x - 1)/(a*x + 1))^(3/2) - 13*c^3*sqrt((a*x - 1)/(a*x + 1)))/((a*x
- 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^2/(a*x +
1)^3 + a^2))*a
```

3.430.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\int\left(c-\frac{c}{ax}\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.430.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{13\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,\left(a\,x-1\right)}{a\,x+1}-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{32\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{33\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,12{}\mathrm{i}}{a}$$

input
```
int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(13*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
11*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x
 - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (32*c^3*((a*x - 1)/(
a*x + 1))^(1/2))/a - (33*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^3*a
tan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*12i)/a
```

3.430. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

3.431 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$

3.431.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2\sqrt{1-\frac{1}{a^2x^2}}x+\frac{5c^2\csc^{-1}(ax)}{a}-\frac{5c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
5*c^2*arccsc(a*x)/a-5*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+16*c^2*(a-1/x)/a^
2/(1-1/a^2/x^2)^(1/2)+c^2*(1-1/a^2/x^2)^(1/2)/a+c^2*x*(1-1/a^2/x^2)^(1/2)
```

3.431.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 424, normalized size of antiderivative = 4.04

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$

$$=\frac{c^2\left(-35a^2\sqrt{1+\frac{1}{ax}}x^2+315a^3\sqrt{1+\frac{1}{ax}}x^3+280a^4\sqrt{1+\frac{1}{ax}}x^4-595a^5\sqrt{1+\frac{1}{ax}}x^5+35a^6\sqrt{1+\frac{1}{ax}}x^6+91\right.}{}$$

input
```
Integrate[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*(-35*a^2*Sqrt[1 + 1/(a*x)]*x^2 + 315*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 280*
a^4*Sqrt[1 + 1/(a*x)]*x^4 - 595*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 35*a^6*Sqrt[1
+ 1/(a*x)]*x^6 + 910*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sq
rt[2]] + 910*a^5*Sqrt[1 - 1/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] -
 105*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[1/(a*x)] - 105*a^5*Sqrt[1 - 1/(a*x)]
*x^5*ArcSin[1/(a*x)] - 175*a^5*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^5
*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]] + 7*Sqrt[2]*a*x*(-1 + a*x)^3*(1 + a*x)*Hyp
ergeometric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2] + 5*Sqrt[2]*(-1 + a*x)^4*(1
 + a*x)*Hypergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2]))/(35*a^5*Sqrt[1
 - 1/(a*x)]*x^4*(1 + a*x))
```

3.431.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6731, 27, 528, 2338, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^5\left(a-\frac{1}{x}\right)^5 x^2}{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{\left(a-\frac{1}{x}\right)^5 x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^5}$$

$$\Big\downarrow \text{528}$$

$$-\frac{c^2\left(a^2\int \frac{\left(a^3-\frac{5a^2}{x}-\frac{5a}{x^2}+\frac{1}{x^3}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{c^2\left(a^2\left(a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int\frac{\left(5a^2+\frac{5a}{x}-\frac{1}{x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$$\Big\downarrow 2340$$

$$-\frac{c^2\left(a^2\left(a^2\int-\frac{5\left(a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$$\Big\downarrow 27$$

$$-\frac{c^2\left(a^2\left(-5a\int\frac{\left(a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$$\Big\downarrow 538$$

$$-\frac{c^2\left(a^2\left(-5a\left(\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$$\Big\downarrow 223$$

$$-\frac{c^2\left(a^2\left(-5a\left(a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a\arcsin\left(\frac{1}{ax}\right)\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$$\Big\downarrow 243$$

$$-\frac{c^2\left(a^2\left(-5a\left(\frac{1}{2}a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+a\arcsin\left(\frac{1}{ax}\right)\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$$\Big\downarrow 73$$

$$-\frac{c^2\left(a^2\left(-5a\left(a\arcsin\left(\frac{1}{ax}\right)-a^3\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\right.}{\sqrt{1-\frac{1}{a^2}}}\right.}{a^5}$$

$$\Big\downarrow 221$$

$$-\frac{c^2\left(a^2\left(-5a\left(a\arcsin\left(\frac{1}{ax}\right)-a\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

3.431. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

input
```
Int[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^2*((-16*a^3*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-(a^2*Sqrt[1 -
 1/(a^2*x^2)]) - a^3*Sqrt[1 - 1/(a^2*x^2)]*x - 5*a*(a*ArcSin[1/(a*x)] - a*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))))/a^5)
```

3.431.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.431.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.61

method	result
risch	$\frac{(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(-\frac{5a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a\sqrt{(ax-1)(ax+1)}+5a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\frac{16\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)c^2}{a^2(ax-1)}$
default	$-\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-7\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-5a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.$

input
```
int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

3.431. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

output
```
(a*x+1)/x*c^2/a^2*((a*x-1)/(a*x+1))^(1/2)+(-5*a^2*ln(a^2*x/(a^2)^(1/2)+(a^
2*x^2-1)^(1/2))/(a^2)^(1/2)+a*((a*x-1)*(a*x+1))^(1/2)+5*a*arctan(1/(a^2*x^
2-1)^(1/2))+16/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*c^2/a^2/(a*x-1)*
((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.431.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\frac{10\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2+18\,ac^2x+c}{a^2x}$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-(10*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 5*a*c^2*x*log(sqrt((a*x -
 1)/(a*x + 1)) + 1) - 5*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*
c^2*x^2 + 18*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

3.431.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

input
```
integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + In
tegral(3*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-
3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

3.431.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.42

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{10\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{16\,c^2\sqrt{}}{a^2}\right.$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 10
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 5*c^2*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1)/a^2 - 5*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 16*c^2*sq
rt((a*x - 1)/(a*x + 1))/a^2)*a
```

3.431.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \int\left(c-\frac{c}{ax}\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.431.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \frac{16\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{4\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}} - \frac{10\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,10{}\mathrm{i}}{a}$$

input
```
int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(16*c^2*((a*x - 1)/(a*x + 1))^(1/2))/a + (4*c^2*((a*x - 1)/(a*x + 1))^(1/2
))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (10*c^2*atan(((a*x - 1)/(a*x + 1))^
(1/2)))/a + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*10i)/a
```

3.431. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

3.432 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.432.1 Optimal result

Integrand size = 20, antiderivative size = 75

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{c\csc^{-1}(ax)}{a}-\frac{4c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c*arccsc(a*x)/a-4*c*arctanh((1-1/a^2/x^2)^(1/2))/a+8*c*(a-1/x)/a^2/(1-1/a^
2/x^2)^(1/2)+c*x*(1-1/a^2/x^2)^(1/2)
```

3.432.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.12

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$

$$=\frac{5a^2cx^2\Bigg((1+ax)\Bigg(\sqrt{1+\frac{1}{ax}}(2-3ax+a^2x^2)+6a\sqrt{1-\frac{1}{ax}}x\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2a\sqrt{1-\frac{1}{ax}}x\arcsin\left(\frac{1}{ax}\right)}{5a^4}$$

input
```
Integrate[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

output
```
(5*a^2*c*x^2*((1 + a*x)*(Sqrt[1 + 1/(a*x)]*(2 - 3*a*x + a^2*x^2) + 6*a*Sqr
t[1 - 1/(a*x)]*x*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*a*Sqrt[1 - 1/(a*x)]
*x*ArcSin[1/(a*x)]) - 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^2*Ar
cTanh[Sqrt[1 - 1/(a^2*x^2)]]) + Sqrt[2]*c*(-1 + a*x)^3*(1 + a*x)*Hypergeom
etric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2])/(5*a^4*Sqrt[1 - 1/(a*x)]*x^3*(1
+ a*x))
```

3.432.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6731, 27, 528, 2338, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{-3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^4\left(a-\frac{1}{x}\right)^4x^2}{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^4}$$

$$\Big\downarrow \text{528}$$

$$-\frac{c\left(a^2\int \frac{\left(a^2-\frac{4a}{x}-\frac{1}{x^2}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{c\left(a^2\left(a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(4a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c\left(a^2\left(-4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{c\left(a^2\left(-4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{c\left(a^2\left(-2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{c\left(a^2\left(4a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a^2x\sqrt{1-\frac{1}{a^2x^2}}-a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{c\left(a^2\left(4a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

input
```
Int[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c*((-8*a^2*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-(a^2*Sqrt[1 - 1/
(a^2*x^2)]*x) - a*ArcSin[1/(a*x)] + 4*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])))/
a^4)
```

3.432. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.432.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.432. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.432.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(69) = 138$.

Time = 0.14 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.95

method	result
default	$\left(\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+a^{2}x^{2}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)-4\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+4\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x^{2}+2\sqrt{a^{2}x^{2}}$

input
```
int((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x
^2-1)^(1/2))-4*ln((a^2*x+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))
*a^3*x^2+4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^2*x^2+2*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*a*x+2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-8*ln((a^2*x
+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))*a^2*x-4*((a*x-1)*(a*x+1
))^(3/2)*(a^2)^(1/2)+8*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a*x+(a^2*x^2-1)
^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-4*a*ln((a^2*x+(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2))/(a^2)^(1/2))+4*(a^2)^(1/2)*((a*x-1)*(a
*x+1))^(1/2))/a*c*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1
/2)/(a*x-1)
```

3.432.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$= -\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(acx+9\,c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*c*log(sqrt((a*x - 1)/(a*x + 1)
) + 1) - 4*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a*c*x + 9*c)*sqrt((a*x
- 1)/(a*x + 1)))/a
```

3.432.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$= \frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^2x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(
-2*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a
```

3.432.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = -2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arct
an(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
/a^2 - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 4*c*sqrt((a*x - 1)/(a*
x + 1))/a^2)
```

3.432.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \int \left(c-\frac{c}{ax}\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.432.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,8\mathrm{i}}{a}$$

input
```
int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*ata
n(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i
)*8i)/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

3.432. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.433 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.433.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}-\frac{2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
-2*arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*
(1-1/a^2/x^2)^(1/2)/c
```

3.433.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(3+ax)-2(1+ax)\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a(c+acx)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(3 + a*x) - 2*(1 + a*x)*Log[(1 + Sqrt[1 - 1/(a^
2*x^2)])*x])/(a*(c + a*c*x))
```

3.433.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6731, 27, 528, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{c^2\left(a-\frac{1}{x}\right)^2x^2}{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^2c}$$

$$\Big\downarrow \text{ 528}$$

$$-\frac{a^2\int \frac{\left(a-\frac{2}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\int \frac{\left(a-\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \text{ 534}$$

$$-\frac{a\left(-2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{a\left(-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{a\left(2a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

output
```
-(((-2*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a*(-(a*Sqrt[1 - 1/(a^2*x^2)]*
x) + 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^2*c))
```

3.433.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.433. $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.433.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(66) = 132$.

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.90

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a\sqrt{a^2}}+\frac{2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^3\left(x+\frac{1}{a}\right)}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{(2\ln(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}})a^3x^2-2\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+4\ln(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}})a^2x+((ax-1)(ax+1))^{\frac{3}{2}}}{a\sqrt{a^2}\,c\sqrt{(ax-1)(ax+1)}\,(ax-}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-2/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-
1)^(1/2))/(a^2)^(1/2)+2/a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a/c
*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.433.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{(ax+3)\sqrt{\frac{ax-1}{ax+1}} - 2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

output
```
((a*x + 3)*sqrt((a*x - 1)/(a*x + 1)) - 2*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) + 2*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

3.433.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\left(\int\left(-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx + \int\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx\right)}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

output
```
a*(Integral(-x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Int
egral(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

3.433.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.67

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$= -2a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c} - \frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

3.433.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{c-\frac{c}{ax}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

output
```
undef
```

3.433.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}} + \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c} - \frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (2*((a
*x - 1)/(a*x + 1))^(1/2))/(a*c) - (4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(
a*c)
```

3.434 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

3.434.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{c^2}-\frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
-arctanh((1-1/a^2/x^2)^(1/2))/a/c^2-(a-1/x)*x/a/c^2/(1-1/a^2/x^2)^(1/2)+2*
x*(1-1/a^2/x^2)^(1/2)/c^2
```

3.434.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{-2+ax+a^2x^2-a\sqrt{1-\frac{1}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

output
```
(-2 + a*x + a^2*x^2 - a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.434.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6731, 27, 528, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c\left(a-\frac{1}{x}\right)x^2}{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(a-\frac{1}{x}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{ac^2}$$

$$\Big\downarrow \text{528}$$

$$-\frac{a^2\int \frac{\left(a-\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(a-\frac{1}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{ac^2}$$

$$\Big\downarrow \text{534}$$

$$-\frac{-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-ax\sqrt{1-\frac{1}{a^2x^2}}}{ac^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-ax\sqrt{1-\frac{1}{a^2x^2}}}{ac^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{ac^2}$$

$$\Big\downarrow \color{blue}{221}$$

$$-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-ax\sqrt{1-\frac{1}{a^2x^2}}}{ac^2}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

output
```
-((-((a - x^(-1))/(a*Sqrt[1 - 1/(a^2*x^2)])) - a*Sqrt[1 - 1/(a^2*x^2)]*x +
 ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^2))
```

3.434.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

---

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.434.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(68) = 136$.

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.86

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^2(ax-1)}$
default	$-\frac{(-3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2+2\ln(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}})a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-6\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,ax+4\ln(}{2a\sqrt{a^2}\,c^2\sqrt{(ax-1)(ax+1)}\,(ax}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+1/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))
*a^2/c^2/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.434.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)
+ log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c^2)
```

3.434.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{a^2\left(\int\left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx+\int\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx\right)}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

output
```
a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x
**2 - a*x + 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
*3*x**3 - a**2*x**2 - a*x + 1), x))/c**2
```

3.434.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.69

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^2) +
log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - log(sqrt((a*x - 1)/(a*x + 1
)) - 1)/(a^2*c^2) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

3.434.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
undef
```

3.434.9 Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^2,x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + ((
a*x - 1)/(a*x + 1))^(1/2)/(a*c^2) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))
/(a*c^2)
```

3.435 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

3.435.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=-\frac{2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-2/a^2/c^3/x/(1-1/a^2/x^2)^(1/2)+x/c^3/(1-1/a^2/x^2)^(1/2)
```

3.435.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{-2+a^2x^2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

output
```
(-2 + a^2*x^2)/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.435.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6731, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{245}$$

$$-\frac{\frac{2\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{208}$$

$$-\frac{\frac{2}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

output
```
-((2/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) - x/Sqrt[1 - 1/(a^2*x^2)])/c^3)
```

3.435.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.435.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

method	result	size
trager	$\frac{\left(a^{2} x^{2}-2\right) \sqrt{-\frac{-a x +1}{a x +1}}}{a \,c^{3} \left(a x -1\right)}$	41
gosper	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(a^{2} x^{2}-2\right) \left(a x +1\right)}{a \left(a x -1\right)^{2} c^{3}}$	44
default	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(a^{3} x^{3}+a^{2} x^{2}-2 a x -2\right)}{a \left(a x -1\right)^{2} c^{3}}$	50
risch	$\frac{\left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}{a \,c^{3}}-\frac{\sqrt{\frac{a x -1}{a x +1}}}{a \,c^{3} \left(a x -1\right)}$	59

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a/c^3*(a^2*x^2-2)/(a*x-1)*(-(-a*x+1)/(a*x+1))^(1/2)
```

3.435.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{(a^2x^2-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3x-ac^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
(a^2*x^2 - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*x - a*c^3)
```

3.435.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3\left(\int\left(-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx+\int\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx\right)}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

output
```
a**3*(Integral(-x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3
*x**3 + 2*a*x - 1), x) + Integral(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

3.435.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{1}{2}\,a\left(\frac{\frac{5\,(ax-1)}{ax+1}-1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-1/2*a*((5*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
 a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

3.435.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{\left(\frac{\sqrt{a^2x^2-1}}{c^3}-\frac{1}{\sqrt{a^2x^2-1}c^3}\right)\operatorname{sgn}(ax+1)}{a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
(sqrt(a^2*x^2 - 1)/c^3 - 1/(sqrt(a^2*x^2 - 1)*c^3))*sgn(a*x + 1)/a
```

3.435.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{a^2\,x^2-2}{\left(x\,a^2\,c^3+a\,c^3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^3,x)
```

output
```
(a^2*x^2 - 2)/((a*c^3 + a^2*c^3*x)*((a*x - 1)/(a*x + 1))^(1/2))
```

3.436 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

3.436.1 Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = \frac{8\sqrt{1-\frac{1}{a^2x^2}}x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
arctanh((1-1/a^2/x^2)^(1/2))/a/c^4-1/3*a*x/c^4/(a-1/x)/(1-1/a^2/x^2)^(1/2)
-1/3*(4*a+3/x)*x/a/c^4/(1-1/a^2/x^2)^(1/2)+8/3*x*(1-1/a^2/x^2)^(1/2)/c^4
```

3.436.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = \frac{8-5ax-7a^2x^2+3a^3x^3+3a\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

output
```
(8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x
)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

3.436.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6731, 27, 569, 25, 532, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{ax^2}{c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{569}$$

$$-\frac{a\left(\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{3a^2}\right)}{c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\left(\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \text{532}$$

$$-\frac{a\left(\frac{\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\left(\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \text{534}$$

$$-\frac{a\left(\frac{3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{a\left(\frac{\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(\frac{-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-4ax\sqrt{1-\frac{1}{a^2x^2}}+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(\frac{-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

3.436. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

output
```
-((a*(x/(3*Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))) + ((3*a + 4/x)/(a*Sqrt[1 -
1/(a^2*x^2)]) - 4*a*Sqrt[1 - 1/(a^2*x^2)]*x - 3*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(3*a^2)))/c^4)
```

3.436.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.436. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.436.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(97) = 194$.

Time = 0.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.96

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{4a^6\left(x+\frac{1}{a}\right)}-\frac{19\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{12a^6\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{6a^7\left(x-\frac{1}{a}\right)^2}\right)a^4\sqrt{\frac{ax-1}{ax+1}}}{c^4(ax-1)}$
default	$\left(24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^6x^5+45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5-24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^5x^4-21((ax-1)(ax+1))$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)+1/4/a^6/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)
-19/12/a^6/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1/6/a^7/(x-1/a)^2*((x
-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^4*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((
a*x-1)*(a*x+1))^(1/2)
```

3.436. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

3.436.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+\right.}{3\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^
2 - 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c
^4)
```

3.436.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx\right)}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

output
```
a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4
*x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x**5*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a*
*2*x**2 - 3*a*x + 1), x))/c**4
```

3.436.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^4
*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)) + 12*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 12*log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c^4) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4))
```

3.436.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4, x)
```

3.436.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^4}-\frac{\frac{17\,(a\,x-1)}{3\,(a\,x+1)}-\frac{14\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4,x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^4) - ((17*(a*x - 1))/(3*(a*x + 1)) - (1
4*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 4
*a*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/(a*c^4)
```

3.436. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

3.437 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

3.437.1 Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5}+\frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

output
```
-2/5*(a+1/x)/a^2/c^5/(1-1/a^2/x^2)^(5/2)+1/15*(-10*a-13/x)/a^2/c^5/(1-1/a^
2/x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c^5+1/15*(-30*a-41/x)/a^2/c^
5/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^5
```

3.437.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{-56+82ax+32a^2x^2-76a^3x^3+15a^4x^4+30a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

output
```
(-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*a*Sqrt[1 - 1/(a^
2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^5*Sqrt[1
- 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

3.437.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6731, 27, 570, 532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{a^2x^2}{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c^5}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^2x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^2c^5}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(5a^2+\frac{10a}{x}+\frac{8}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^2c^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\int \frac{\left(5a^2+\frac{10a}{x}+\frac{8}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$$\Big\downarrow \ 2336$$

$$-\frac{\frac{1}{5}\left(\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(15a^2+\frac{30a}{x}+\frac{26}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(15a^2+\frac{30a}{x}+\frac{26}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$$\Big\downarrow \ 2336$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{15a\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\int \frac{\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$$\Big\downarrow \ 534$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\left(2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$$\Big\downarrow \ 243$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\left(\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$$\Big\downarrow \ 73$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\left(-2a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$$\Big\downarrow \ 221$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\left(-2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

3.437. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

output
```
-(((2*(a + x^(-1)))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((10*a + 13/x)/(3*(1 - 1
/(a^2*x^2))^(3/2)) + ((30*a + 41/x)/Sqrt[1 - 1/(a^2*x^2)] + 15*a*(-(a*Sqrt
[1 - 1/(a^2*x^2)]*x) - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/3)/5)/(a^2*c^5))
```

3.437.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.437. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.437.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(122) = 244$.

Time = 0.24 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.88

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{5}}+\frac{\left(\frac{2\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{5}\sqrt{a^{2}}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{10a^{9}\left(x-\frac{1}{a}\right)^{3}}-\frac{41\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{60a^{8}\left(x-\frac{1}{a}\right)^{2}}-\frac{383\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{120a^{7}\left(x-\frac{1}{a}\right)}+\right.}{c^{5}(ax-1)}$
default	$-\frac{\left(-75\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{6}x^{6}-60\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{7}x^{6}+45((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}\,a^{4}x^{4}+150\sqrt{(ax-1)(ax+1)}\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

3.437. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5}}\,dx$

output
```
1/a*(a*x+1)/c^5*((a*x-1)/(a*x+1))^(1/2)+(2/a^5*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/10/a^9/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1
/2)-41/60/a^8/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-383/120/a^7/(x-1
/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)+1/8/a^7/(x+1/a)*(a^2*(x+1/a)^2-2*a*(
x+1/a))^(1/2))*a^5/c^5/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(
1/2)
```

3.437.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a}{15\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="fricas")
```

output
```
1/15*(30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
 1) - 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
 1) + (15*a^4*x^4 - 76*a^3*x^3 + 32*a^2*x^2 + 82*a*x - 56)*sqrt((a*x - 1)/
(a*x + 1)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

3.437.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{a^5\left(\int\left(-\frac{x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\right)dx+\int\frac{ax^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx\right)}{c^5}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**5,x)
```

output
```
a**5*(Integral(-x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5
*x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1), x) + Integral(a*x**6*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a*
*2*x**2 + 4*a*x - 1), x))/c**5
```

3.437.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$$
$$= \frac{1}{120}a\left(\frac{\frac{32\,(ax-1)}{ax+1}+\frac{310\,(ax-1)^2}{(ax+1)^2}-\frac{585\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^5}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^5}+\frac{15\sqrt{}}{a^2c^5}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="maxima")
```

output
```
1/120*a*((32*(a*x - 1)/(a*x + 1) + 310*(a*x - 1)^2/(a*x + 1)^2 - 585*(a*x
- 1)^3/(a*x + 1)^3 + 3)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^5*((a
*x - 1)/(a*x + 1))^(5/2)) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^
5) - 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^5) + 15*sqrt((a*x - 1)/
(a*x + 1))/(a^2*c^5))
```

3.437.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.437.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^5} - \frac{\frac{62\,(a\,x-1)^2}{3\,(a\,x+1)^2} - \frac{39\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{32\,(a\,x-1)}{15\,(a\,x+1)} + \frac{1}{5}}{8\,a\,c^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2} - 8\,a\,c^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}} + \frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^5}$$

input

```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^5,x)
```

output

```
((a*x - 1)/(a*x + 1))^(1/2)/(8*a*c^5) - ((62*(a*x - 1)^2)/(3*(a*x + 1)^2)
- (39*(a*x - 1)^3)/(a*x + 1)^3 + (32*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(8*a
*c^5*((a*x - 1)/(a*x + 1))^(5/2) - 8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2)) +
(4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^5)
```

3.438 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

3.438.1 Optimal result

Integrand size = 22, antiderivative size = 235

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{173c^5\sqrt{1-\frac{1}{a^2x^2}}}{105a\sqrt{c-\frac{c}{ax}}}+\frac{227c^4\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{105a}+\frac{59c^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35a}+\frac{9c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7a}+c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{7c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-7*c^(9/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+59/35*c^
3*(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/a+9/7*c^2*(c-c/a/x)^(5/2)*(1-1/a^2/x
^2)^(1/2)/a+c*(c-c/a/x)^(7/2)*x*(1-1/a^2/x^2)^(1/2)+173/105*c^5*(1-1/a^2/x
^2)^(1/2)/a/(c-c/a/x)^(1/2)+227/105*c^4*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2
)/a
```

3.438.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-30+162ax-356a^2x^2+292a^3x^3+105a^4x^4)-735a^3x^3\text{arctanh}\left(\sqrt{}\right.\right.}{105a^4\sqrt{1-\frac{1}{ax}}x^3}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

output
```
(c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-30 + 162*a*x - 356*a^2*x^2 + 2
92*a^3*x^3 + 105*a^4*x^4) - 735*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(105*
a^4*Sqrt[1 - 1/(a*x)]*x^3)
```

3.438.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.78, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {6731, 585, 27, 108, 27, 170, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{9/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \ 6731$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{7/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \ 585$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x^2}{a^4}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 108$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(a-\frac{1}{x}\right)^3\left(7a+\frac{9}{x}\right)x}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-x\left(a-\frac{1}{x}\right)^4\sqrt{\frac{1}{ax}+1}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\int \frac{\left(a-\frac{1}{x}\right)^3\left(7a+\frac{9}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 170$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{2}{7}a\int \frac{\left(a-\frac{1}{x}\right)^2\left(49a+\frac{59}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\int \frac{\left(a-\frac{1}{x}\right)^2\left(49a+\frac{59}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 170$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\left(\frac{2}{5}a\int \frac{\left(a-\frac{1}{x}\right)\left(245a+\frac{227}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\left(\frac{1}{5}a\int \frac{\left(a-\frac{1}{x}\right)\left(245a+\frac{227}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

3.438. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$

$$\Big\downarrow \; 164$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\Bigg(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\Big(\frac{1}{5}a\Big(245a^2\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\big(400a-\frac{227}{x}\big)\sqrt{\frac{1}{ax}+1}\Big)+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\big(a-\frac{1}{x}\big)^2\Big)+\frac{18}{7}a\sqrt{\frac{1}{ax}}}{2a}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\Bigg(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\Big(\frac{1}{5}a\Big(490a^3\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\big(400a-\frac{227}{x}\big)\sqrt{\frac{1}{ax}+1}\Big)+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\big(a-\frac{1}{x}\big)^2\Big)+\frac{18}{7}a}{2a}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{c^4\Bigg(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\Big(\frac{1}{5}a\Big(\frac{2}{3}a\big(400a-\frac{227}{x}\big)\sqrt{\frac{1}{ax}+1}-490a^2\text{arctanh}\Big(\sqrt{\frac{1}{ax}+1}\Big)\Big)+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\big(a-\frac{1}{x}\big)^2\Big)+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\big(a}{2a}}{a^4\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

output
```
-((c^4*Sqrt[c - c/(a*x)]*(-((a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*x) - ((18*a*(
a - x^(-1))^3*Sqrt[1 + 1/(a*x)])/7 + (a*((118*a*(a - x^(-1))^2*Sqrt[1 + 1/
(a*x)])/5 + (a*((2*a*(400*a - 227/x)*Sqrt[1 + 1/(a*x)])/3 - 490*a^2*ArcTan
h[Sqrt[1 + 1/(a*x)]]))/5))/7)/(2*a)))/(a^4*Sqrt[1 - 1/(a*x)]))
```

3.438. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$

3.438.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

3.438. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.438.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{4} \Big(210 a^{\frac{9}{2}} \sqrt{(ax+1)x}\, x^{4}+584 a^{\frac{7}{2}} x^{3} \sqrt{(ax+1)x}-735 \ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right) a^{4} x^{4}-712 a^{\frac{5}{2}} x^{2} \sqrt{(ax+1)x}+324 a^{\frac{3}{2}} x \sqrt{(a}}{210 \sqrt{\frac{ax-1}{ax+1}}\, x^{3} a^{\frac{9}{2}} \sqrt{(ax+1)x}}$
risch	$\frac{\left(105 a^{5} x^{5}+397 a^{4} x^{4}-64 a^{3} x^{3}-194 a^{2} x^{2}+132 a x-30\right) c^{4} \sqrt{\frac{c(ax-1)}{ax}}}{105 x^{3} a^{4} \sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{7 \ln\left(\frac{\frac{1}{2} a c+a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\, x^{2}+a c x}\right) c^{4} \sqrt{\frac{c(ax-1)}{ax}}\, \sqrt{(ax+1) a c x}}{2 \sqrt{a^{2} c}\, \sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/210/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^4*(210*a^(9/2)*((a*x
+1)*x)^(1/2)*x^4+584*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-735*ln(1/2*(2*((a*x+1)*
x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4-712*a^(5/2)*x^2*((a*x+1)*x)^(1/
2)+324*a^(3/2)*x*((a*x+1)*x)^(1/2)-60*((a*x+1)*x)^(1/2)*a^(1/2))/x^3/a^(9/
2)/((a*x+1)*x)^(1/2)
```

3.438.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.86

$$\int e^{\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{9/2} dx = \left[\frac{735\left(a^4c^4x^4 - a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right) + 4\,(105\,a^5}{420\,(a^5x^4 - a^4x}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

output
```
[1/420*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*
x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt
((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(105*a^5*c^4*x^5 + 397*a^4*c^4*x^4
 - 64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x - 30*c^4)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/210*(735*(a^4*
c^4*x^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(
105*a^5*c^4*x^5 + 397*a^4*c^4*x^4 - 64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132
*a*c^4*x - 30*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5
*x^4 - a^4*x^3)]
```

3.438.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{9/2} dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(9/2),x)
```

output
```
Timed out
```

3.438.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.438.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.439 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

3.439.1 Optimal result

Integrand size = 22, antiderivative size = 196

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{49c^4\sqrt{1-\frac{1}{a^2x^2}}}{15a\sqrt{c-\frac{c}{ax}}}+\frac{31c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{15a}+\frac{7c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5a}+c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{5c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-5*c^(7/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+7/5*c^2*
(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/a+c*(c-c/a/x)^(5/2)*x*(1-1/a^2/x^2)^(1
/2)+49/15*c^4*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+31/15*c^3*(1-1/a^2/x^2
)^(1/2)*(c-c/a/x)^(1/2)/a
```

3.439.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(6-28ax+56a^2x^2+15a^3x^3)-75a^2x^2\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{15a^3\sqrt{1-\frac{1}{ax}}x^2}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(6 - 28*a*x + 56*a^2*x^2 + 15*a^
3*x^3) - 75*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]
*x^2)
```

3.439.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6731, 585, 27, 108, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x^2}{a^3}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 108}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(a-\frac{1}{x}\right)^2\left(5a+\frac{7}{x}\right)x}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\int \frac{\left(a-\frac{1}{x}\right)^2\left(5a+\frac{7}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 170$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{2}{5}a\int \frac{\left(a-\frac{1}{x}\right)\left(25a+\frac{31}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{1}{5}a\int \frac{\left(a-\frac{1}{x}\right)\left(25a+\frac{31}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 164$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{1}{5}a\left(25a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{1}{5}a\left(50a^3\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 221$$

$$-\frac{c^3\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{1}{5}a\left(\frac{2}{3}a\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}-50a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)\sqrt{c-\frac{c}{ax}}}{a^3\sqrt{1-\frac{1}{ax}}}$$

3.439. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

output
```
-((c^3*Sqrt[c - c/(a*x)]*(-((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*x) - ((14*a*(
a - x^(-1))^2*Sqrt[1 + 1/(a*x)])/5 + (a*((2*a*(80*a - 31/x)*Sqrt[1 + 1/(a*
x)])/3 - 50*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/5)/(2*a)))/(a^3*Sqrt[1 - 1/(a
*x)]))
```

3.439.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.439.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{3}\left(30a^{\frac{7}{2}}x^{3}\sqrt{(ax+1)x}+112a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}-75\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{3}x^{3}-56a^{\frac{3}{2}}x\sqrt{(ax+1)x}+12\sqrt{(ax+1)x}\,\sqrt{a}\right)}{30\sqrt{\frac{ax-1}{ax+1}}\,x^{2}a^{\frac{7}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{\left(15a^{4}x^{4}+71a^{3}x^{3}+28a^{2}x^{2}-22ax+6\right)c^{3}\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}a^{3}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c^{3}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/30/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30*a^(7/2)*x^3*((a
*x+1)*x)^(1/2)+112*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-75*ln(1/2*(2*((a*x+1)*x)^
(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3-56*a^(3/2)*x*((a*x+1)*x)^(1/2)+12*
((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/((a*x+1)*x)^(1/2)
```

3.439.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.12

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\left[\frac{75\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15a^4c^3\right.}{60\left(a^4x^3-a^3x^2\right)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 + 2
8*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqr
t(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 71*a^3*c^
3*x^3 + 28*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

3.439.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(7/2),x)
```

output
```
Timed out
```

3.439.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.439.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.440 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.440.1 Optimal result

Integrand size $=22$, antiderivative size $=157$

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=-\frac{2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-2/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^4*(1-1/a^2/x^2)^(3/2)*x/(
c-c/a/x)^(3/2)-3*c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/
2))/a+3*c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

3.440.2 Mathematica [A] (verified)

Time $=0.08$ (sec) , antiderivative size $=89$, normalized size of antiderivative $=0.57$

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\left(-2+10ax+3a^2x^2\right)-9ax\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}\,x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 10*a*x + 3*a^2*x^2) - 9*a*
x*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

3.440.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6731, 585, 27, 100, 27, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}x^2}{a^2}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\int -\frac{1}{2}\left(3a-\frac{2}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(a^2x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{2}\int \left(3a-\frac{2}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 90$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{3}a\left(\frac{1}{ax}+1\right)^{3/2}-3a\int\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}\right)-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{3}a\left(\frac{1}{ax}+1\right)^{3/2}-3a\left(\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{3}a\left(\frac{1}{ax}+1\right)^{3/2}-3a\left(2a\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 221$$

$$-\frac{c^2\left(\frac{1}{2}\left(\frac{4}{3}a\left(\frac{1}{ax}+1\right)^{3/2}-3a\left(2\sqrt{\frac{1}{ax}+1}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

output
```
-((c^2*Sqrt[c - c/(a*x)]*(-(a^2*(1 + 1/(a*x))^(3/2)*x) + ((4*a*(1 + 1/(a*x
))^(3/2))/3 - 3*a*(2*Sqrt[1 + 1/(a*x)] - 2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/2)
)/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.440.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.440. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.440.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{2} \left(-6a^{\frac{5}{2}} x^{2} \sqrt{(ax+1)x}+9\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right) a^{2} x^{2}-20a^{\frac{3}{2}} x \sqrt{(ax+1)x}+4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{6\sqrt{\frac{ax-1}{ax+1}}\, x\, a^{\frac{5}{2}} \sqrt{(ax+1)x}}$	132
risch	$\frac{(3a^{3}x^{3}+13a^{2}x^{2}+8ax-2)c^{2}\sqrt{\frac{c(ax-1)}{ax}}}{3x\, a^{2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\, x^{2}+acx}\right)c^{2}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	168

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(-6*a^(5/
2)*x^2*((a*x+1)*x)^(1/2)+9*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*a^2*x^2-20*a^(3/2)*x*((a*x+1)*x)^(1/2)+4*((a*x+1)*x)^(1/2)*a^(1/2))
/((a*x+1)*x)^(1/2)
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.43

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\Bigg[\frac{9\,(a^{2}c^{2}x^{2}-ac^{2}x)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^{3}c^{2}x^{3}+}{12\,(a^{3}x^{2}-a^{2}x)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2
*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 -
a^2*x), 1/6*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a
*c*x - c)) + 2*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x - 2*c^2)*sqrt((
a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

3.440.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

3.440.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.440.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input

```
int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.441 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

3.441.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a/x)^(3/2)-c^(3/2)*arctanh(c^(1/2)*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

3.441.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2+ax)-\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + a*x) - ArcTanh[Sqrt[1 + 1/(a*
x)]]))/(a*Sqrt[1 - 1/(a*x)])
```

3.441.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6731, 580, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \textbf{6731}$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \textbf{580}$$

$$-c\left(-\frac{c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \textbf{576}$$

$$-c\left(-\frac{c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \textbf{573}$$

$$-c\left(-\frac{c\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \textbf{219}$$

$$-c\left(-\frac{c\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{c}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

output
```
-(c*(-((c^2*(1 - 1/(a^2*x^2))^(3/2)*x)/(c - c/(a*x))^(3/2)) - (c*((2*Sqrt[
1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - (2*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*
x^2)])/Sqrt[c - c/(a*x)]])/Sqrt[c]))/(2*a)))
```

3.441.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.441.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(-2a^{\frac{3}{2}} x\sqrt{(ax+1)x}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\, a^{\frac{3}{2}}\sqrt{(ax+1)x}}$	105
risch	$\frac{(a^2x^2+3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	151

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-2*a^(3/2)*x
*((a*x+1)*x)^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a
*x-4*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

3.441.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.68

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \left[\frac{(acx-c)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2+3acx+2\right.}{4(a^2x-a)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*((a*c*x - c)*sqrt(-c)*arctan(2*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2
*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

3.441.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

3.441.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.441.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.442 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.442.1 Optimal result

Integrand size = 22, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output

```
arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^
2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.442.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(1+ax+\sqrt{1+\frac{1}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

output

```
(Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]]
))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.442.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6731, 575, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{575}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2ac}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c\left(-\frac{\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

output
```
-(c*(-((Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) - ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])))
```

3.442.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.442.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)x}\,\sqrt{a}}$	87
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a
^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^
(1/2)/a^(1/2)
```

3.442.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$= \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4(a^2x-a)},\right.$$
$$\left.-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)-2(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(a^2x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sq
rt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a^2*x - a)]
```

3.442.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.442.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.442.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.443 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.443.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-\frac{c}{ax}}}+\frac{3\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

output
```
3*arctanh((1+1/a/x)^(1/2))*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-2*arctanh(1/2
*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)+x*(1-1
/a/x)^(1/2)*(1+1/a/x)^(1/2)/(c-c/a/x)^(1/2)
```

3.443.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(\sqrt{1+\frac{1}{ax}}x+\frac{3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
 - (2*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[c - c/(a*x)]
```

3.443.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6731, 585, 27, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a\sqrt{1+\frac{1}{ax}}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\int \frac{\sqrt{1+\frac{1}{ax}}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{110}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(3a+\frac{1}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(3a+\frac{1}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{174}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{4\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+3\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{8a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+6a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-6\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

output
```
-((a*Sqrt[1 - 1/(a*x)]*(-((Sqrt[1 + 1/(a*x)]*x)/a) + (-6*ArcTanh[Sqrt[1 +
1/(a*x)]] + 4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a^2)))/Sqrt[c
 - c/(a*x)])
```

3.443. $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.443.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.443. $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.443.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2\sqrt{(ax+1)x}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\, a^{\frac{3}{2}}\, c\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\right)\sqrt{(ax+1)c}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\, x}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a
^(3/2)*(1/a)^(1/2)-2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a
+3*a*x+1)/(a*x-1))*a^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*(1/a)^(1/2))/a^(3/2)/c/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.40

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$= \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,\left(a^{2}x^{2}+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}+-}{4\,\left(a^{2}cx-ac\right)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)) + 2*sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x -
4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(
a^2*c*x - a*c), 1/2*(2*sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^
2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/
(3*a^2*x^2 - 2*a*x - 1)) - 3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a
*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*c*x - a*c)]
```

3.443.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))), x)
```

3.443.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.443.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.444 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.444.1 Optimal result

Integrand size = 22, antiderivative size = 215

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=-\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}$$

output
```
5*(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-7/2*(1-1/a/x)
^(3/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)-2*(1
-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)+a*(1-1/a/x)^(3/2)*x*
(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)
```

3.444.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-2+ax)+10(-1+ax)\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-7\sqrt{2}(-1+ax)\right.}{2ac\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-2 + a*x) + 10*(-1 + a*x)*Arc
Tanh[Sqrt[1 + 1/(a*x)]] - 7*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/S
qrt[2]]))/(2*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

3.444.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6731, 585, 27, 110, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2\sqrt{1+\frac{1}{ax}}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{\sqrt{1+\frac{1}{ax}}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{110}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(5a+\frac{2}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(5a+\frac{2}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{7\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{14a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+10a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{7\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-10\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

3.444. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

output
```
-((a^2*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(a*(a - x^(-1))) + (-4*Sqr
t[1 + 1/(a*x)]*x + (-10*ArcTanh[Sqrt[1 + 1/(a*x)]] + 7*Sqrt[2]*ArcTanh[Sqr
t[1 + 1/(a*x)]/Sqrt[2]])/a)/(2*a^2)))/(c*Sqrt[c - c/(a*x)]))
```

3.444.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.444.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(4\sqrt{(ax+1)x}\, a^{\frac{5}{2}} \sqrt{\frac{1}{a}}\, x-7a^{\frac{3}{2}}\sqrt{2}\, \ln\left(\frac{2\sqrt{2}\, \sqrt{\frac{1}{a}}\, \sqrt{(ax+1)x}\, a+3ax+1}{ax-1}\right) x-8\sqrt{(ax+1)x}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+10\ln\left(\frac{2\sqrt{(ax+1)x}\, \sqrt{a}+2ax+1}{2\sqrt{a}}\right.\right.}{4\sqrt{\frac{ax-1}{ax+1}}\, (ax-1)a^{\frac{3}{2}}c^{2}\sqrt{(ax+1)x}\, \sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}\, \sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\, x^{2}+acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{a^{4}c\left(x-\frac{1}{a}\right)}-\frac{7\sqrt{2}\, \ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\, \sqrt{c}\, \sqrt{a^{2}c\left(x-\frac{1}{a}\right.}}{x-\frac{1}{a}}\right.}{4a^{3}\sqrt{c}}\right.}{c\sqrt{\frac{ax-1}{ax+1}}\, (ax+1)x\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a*x+1)*x)
^(1/2)*a^(5/2)*(1/a)^(1/2)*x-7*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(
(a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-8*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(
1/2)+10*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1
/2)*x-10*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/
2)+7*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1
))*a^(1/2))/a^(3/2)/c^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.76

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{7\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-}{ax}}}{a^3x^3-3\,a^2x^2+3\,ax-1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)) + 10*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4
*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2
), 1/4*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2
 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*
c*x^2 - 2*a*c*x - c)) - 10*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 4*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x - 1)/(a*x +
 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

3.444.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

3.444.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.444.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.444.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.445 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.445.1 Optimal result

Integrand size = 22, antiderivative size = 277

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx &= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^{2}\left(c-\frac{c}{ax}\right)^{5/2}} \\ &\quad - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^{2}\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^{2}\left(c-\frac{c}{ax}\right)^{5/2}} \\ &\quad + \frac{7\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{79\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{8\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}\end{aligned}$$

output
```
7*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-79/16*(1-1/a/
x)^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-3/
2*a*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)-23/8*(1-1/a/
x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a^2*(1-1/a/x)^(5/2)*x*(1+
1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)
```

3.445.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\Big(2a\sqrt{1+\frac{1}{ax}}x(23-35ax+8a^2x^2)+112(-1+ax)^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-79}{16ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(23 - 35*a*x + 8*a^2*x^2) + 11
2*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 79*Sqrt[2]*(-1 + a*x)^2*ArcTan
h[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(16*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

3.445.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.63, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {6731, 585, 27, 110, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^3\sqrt{1+\frac{1}{ax}}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\int \frac{\sqrt{1+\frac{1}{ax}}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 110}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}-\frac{\int -\frac{\left(6a+\frac{5}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(6a+\frac{5}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(46a+\frac{33}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(46a+\frac{33}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{-\frac{\int -\frac{\left(56a+\frac{23}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 25}$$

3.445. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{\int \frac{\left(56a+\frac{23}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textbf{174}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{79\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+56\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{158a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+112a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{79\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-112\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(2*a*(a - x^(-1))^2) + ((1
1*Sqrt[1 + 1/(a*x)]*x)/(2*(a - x^(-1))) + (-46*Sqrt[1 + 1/(a*x)]*x + (-112
*ArcTanh[Sqrt[1 + 1/(a*x)]] + 79*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]
])/a)/(4*a))/(4*a^2)))/(c^2*Sqrt[c - c/(a*x)]))
```

3.445. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.445.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.445. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.445.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.18

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{2a^6c\left(x-\frac{1}{a}\right)^2}-\frac{19\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{8a^5c\left(x-\frac{1}{a}\right)}-\frac{79\sqrt{2}\,\mathrm{l}}{}\right.}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x\sqrt{\frac{c(ax-}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2-79a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^2-140\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+112\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{}}{2\sqrt{c}}\right.\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(7/2/a^3*ln(
(1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/2
/a^6/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-19/8/a^5/c/(x-1
/a)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-79/32/a^4/c^(1/2)*2^(1/2)*ln
((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^
(1/2))/(x-1/a)))*a^2/c^2/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)
^(1/2)*((a*x+1)*a*c*x)^(1/2)*(a*x-1)
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.41

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[\frac{79\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{}}{a^3x^3-3\,a^2x^2+3\,ax-1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c
*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*
a^2*x^2 + 3*a*x - 1)) + 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(
-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(8*a^4*x^4 -
 27*a^3*x^3 - 12*a^2*x^2 + 23*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3), 1/32*(79*
sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^
2*c*x^2 - 2*a*c*x - c)) - 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*a
rctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(8*a^4*x^4 - 27*a^3*x^3 - 12*a^2*x
^2 + 23*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x
^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

3.445.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

3.445.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.445.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.446 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

3.446.1 Optimal result

Integrand size = 24, antiderivative size = 143

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{5c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
5/3*c^3*(c-c/a/x)^(3/2)/a+c^2*(c-c/a/x)^(5/2)/a+5/7*c*(c-c/a/x)^(7/2)/a+(c
-c/a/x)^(9/2)*x-5*c^(9/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+5*c^4*(c-c/a/
x)^(1/2)/a
```

3.446.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{c^4\sqrt{c-\frac{c}{ax}}\left(6-18ax+4a^2x^2+92a^3x^3+21a^4x^4\right)}{21a^4x^3}-\frac{5c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

output
```
(c^4*Sqrt[c - c/(a*x)]*(6 - 18*a*x + 4*a^2*x^2 + 92*a^3*x^3 + 21*a^4*x^4))
/(21*a^4*x^3) - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

3.446.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6683, 1035, 281, 899, 87, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{9/2} e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{9/2}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(-\frac{5}{2}\int \left(c-\frac{c}{ax}\right)^{7/2}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c\left(-\frac{5}{2}\left(c\int \left(c-\frac{c}{ax}\right)^{5/2}xd\frac{1}{x}+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right)}{a}$$

$$\Big\downarrow \; 60$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\int \left(c-\frac{c}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right)}{a}$$

$$\Big\downarrow \; 60$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\left(c\int \sqrt{c-\frac{c}{ax}}xd\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right)}{a}$$

$$\Big\downarrow \; 60$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\left(c\left(c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right.}{a}$$

$$\Big\downarrow \; 73$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\left(c\left(2\sqrt{c-\frac{c}{ax}}-2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right.}{c}\right.}{a}$$

$$\Big\downarrow \; 221$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\left(c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-ax\right.}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

output
```
-((c*(-((a*(c - c/(a*x))^(9/2)*x)/c) - (5*((2*(c - c/(a*x))^(7/2))/7 + c*(
(2*(c - c/(a*x))^(5/2))/5 + c*((2*(c - c/(a*x))^(3/2))/3 + c*(2*Sqrt[c - c
/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])))))/2))/a)
```

3.446. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

3.446.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.446. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.446.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\left(21 a^{5} x^{5}+71 a^{4} x^{4}-88 a^{3} x^{3}-22 a^{2} x^{2}+24 a x -6\right) c^{4} \sqrt{\frac{c \left(a x -1\right)}{a x}}}{21 x^{3} a^{4} \left(a x -1\right)}-\frac{5 \ln \left(\frac{-\frac{1}{2} a c +a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-a c x}\right) c^{4} \sqrt{\frac{c \left(a x -1\right)}{a x}}\, \sqrt{c \left(a x -1\right) a x}}{2 \sqrt{a^{2} c}\, \left(a x -1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a x -1\right)}{a x}}\, c^{4} \bigg(-210 a^{\frac{9}{2}} \sqrt{a \,x^{2}-x}\, x^{5}+105 \ln \left(\frac{2 \sqrt{a \,x^{2}-x}\, \sqrt{a}+2 a x -1}{2 \sqrt{a}}\right) a^{4} x^{5}+168 a^{\frac{7}{2}} \left(a \,x^{2}-x \right)^{\frac{3}{2}} x^{3}-16 a^{\frac{5}{2}} \left(a \,x^{2}-x \right)^{\frac{3}{2}} x^{2}-24 a^{\frac{3}{2}} (a \,x^{2}-}{42 x^{4} \sqrt{\left(a x -1\right) x}\, a^{\frac{9}{2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*(21*a^5*x^5+71*a^4*x^4-88*a^3*x^3-22*a^2*x^2+24*a*x-6)/x^3*c^4/a^4/(a
*x-1)*(c*(a*x-1)/a/x)^(1/2)-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c
*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)*c^4/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x
-1)*a*x)^(1/2)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.64

$$\int e^{2\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{9/2} dx = \left[\frac{105\,a^3c^{\frac{9}{2}}x^3\log\left(-2\,acx + 2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}} + c\right) + 2\,(21\,a^4c^4x^4 + 92\,a^3c^4x^3 + 4\,a^2c^4x^2 - 18\,a}{42\,a^4x^3}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

output
```
[1/42*(105*a^3*c^(9/2)*x^3*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(
a*x)) + c) + 2*(21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4
*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3), 1/21*(105*a^3*sqrt(-c)*c^4
*x^3*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (21*a^4*c^4*x^4 + 92*a^3
*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a
^4*x^3)]
```

3.446.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.54 (sec) , antiderivative size = 2222, normalized size of antiderivative = 15.54

$$\int e^{2\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{9/2} dx = \text{Too large to display}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(9/2),x)
```

output
```
c**4*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a
*x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1
) + I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a
*x + 1)), True)) + 2*c**4*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/
sqrt(-c) + 2*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a +
2*c**4*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15
*a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) -
15*a**(5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1)/(15*a**(7/2)*x**
(7/2) - 15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(a*x - 1)/(15*a**(
7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*sqrt(a*x - 1)/(15
*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)*sqrt(a*x - 1)/
(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*a**(11/2
)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9
/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*I*a
**5*sqrt(c)*x**3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/
2)) - 2*I*a**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(
5/2)*x**(5/2)) - 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) -
 15*a**(5/2)*x**(5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7/2)*x**(
7/2) - 15*a**(5/2)*x**(5/2)), True))/a**3 - c**4*Piecewise((-16*a**(19/2)*
sqrt(c)*x**(13/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + ...
```

3.446.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^(9/2)/(a*x - 1), x)
```

3.446.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{9/2}(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1), x)
```

3.447 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

3.447.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{3c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c/a/x)^(7/2)*x-3*c^(7/2)*
arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+3*c^3*(c-c/a/x)^(1/2)/a
```

3.447.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-2+4ax+8a^2x^2+5a^3x^3\right)}{5a^3x^2}-\frac{3c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(-2 + 4*a*x + 8*a^2*x^2 + 5*a^3*x^3))/(5*a^3*x^2) -
 (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

3.447.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6717, 6683, 1035, 281, 899, 87, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2} e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}dx}{a}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left(-\frac{3}{2}\int \left(c-\frac{c}{ax}\right)^{5/2}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(-\frac{3}{2}\left(c\int \left(c-\frac{c}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow 60$$

$$-\frac{c\left(-\frac{3}{2}\left(c\left(c\int \sqrt{c-\frac{c}{ax}}xd\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow 60$$

$$-\frac{c\left(-\frac{3}{2}\left(c\left(c\left(c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow 73$$

$$-\frac{c\left(-\frac{3}{2}\left(c\left(c\left(2\sqrt{c-\frac{c}{ax}}-2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(-\frac{3}{2}\left(c\left(c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

output
```
-((c*(-((a*(c - c/(a*x))^(7/2)*x)/c) - (3*((2*(c - c/(a*x))^(5/2))/5 + c*(
(2*(c - c/(a*x))^(3/2))/3 + c*(2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]]))))/2))/a)
```

3.447.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.447. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.447.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{3} \left(-30\sqrt{a\,x^{2}-x}\, a^{\frac{7}{2}} x^{4}+20a^{\frac{5}{2}} \left(a\,x^{2}-x\right)^{\frac{3}{2}} x^{2}+15\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a^{3} x^{4}+4a^{\frac{3}{2}} \left(a\,x^{2}-x\right)^{\frac{3}{2}} x-4\left(a\,x^{2}-x\right)^{\frac{3}{2}} \sqrt{a}\right)}{10x^{3}\sqrt{(ax-1)x}\, a^{\frac{7}{2}}}$
risch	$\frac{\left(5a^{4}x^{4}+3a^{3}x^{3}-4a^{2}x^{2}-6ax+2\right) c^{3} \sqrt{\frac{c(ax-1)}{ax}}}{5x^{2}a^{3}(ax-1)}-\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right) c^{3} \sqrt{\frac{c(ax-1)}{ax}}\, \sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3*(-30*(a*x^2-x)^(1/2)*a^(7/2)*x^4+20*a^
(5/2)*(a*x^2-x)^(3/2)*x^2+15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^
(1/2))*a^3*x^4+4*a^(3/2)*(a*x^2-x)^(3/2)*x-4*(a*x^2-x)^(3/2)*a^(1/2))/((a*
x-1)*x)^(1/2)/a^(7/2)
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.80

$$\int e^{2\coth^{-1}(ax)}\Big(c - \frac{c}{ax}\Big)^{7/2}\, dx = \left[\frac{15\,a^{2}c^{\frac{7}{2}}x^{2}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(5\,a^{3}c^{3}x^{3}+8\,a^{2}c^{3}x^{2}+4\,ac^{3}x-2\,c^{3}\right)}{10\,a^{3}x^{2}}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/10*(15*a^2*c^(7/2)*x^2*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) + 2*(5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*
c*x - c)/(a*x)))/(a^3*x^2), 1/5*(15*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*s
qrt((a*c*x - c)/(a*x))/c) + (5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2
*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]
```

3.447.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.54 (sec) , antiderivative size = 740, normalized size of antiderivative = 6.27

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx = c^{3}\left(\begin{cases} -\frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{\sqrt{c}\sqrt{x}\sqrt{ax-1}}{\sqrt{a}} & \text{for}\ |ax|>1 \\ -\frac{i\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \frac{c^{3}\left(\begin{cases} \frac{2c\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}} & \text{for}\ \frac{c}{a}\neq 0 \\ -\sqrt{c}\log\left(x\right) & \text{otherwise} \end{cases}\right)}{a} - \frac{c^{3}\left(\begin{cases} 0 & \text{for}\ c=0 \\ \frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right)}{a^{2}} + \frac{c^{3}\left(\begin{cases} -\frac{4a^{\frac{11}{2}}\sqrt{c}x^{\frac{7}{2}}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{4a^{\frac{9}{2}}\sqrt{c}x^{\frac{5}{2}}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{4a^{5}\sqrt{c}x^{3}\sqrt{ax-1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}-\frac{2a^{4}\sqrt{c}x^{2}\sqrt{ax-1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}-\frac{8a^{3}\sqrt{c}x\sqrt{ax-1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{6a^{2}\sqrt{c}\sqrt{ax-1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a} \\ -\frac{4a^{\frac{11}{2}}\sqrt{c}x^{\frac{7}{2}}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{4a^{\frac{9}{2}}\sqrt{c}x^{\frac{5}{2}}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{4ia^{5}\sqrt{c}x^{3}\sqrt{-ax+1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}-\frac{2ia^{4}\sqrt{c}x^{2}\sqrt{-ax+1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}-\frac{8ia^{3}\sqrt{c}x\sqrt{-ax+1}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a^{\frac{5}{2}}x^{\frac{5}{2}}}+\frac{6ia^{2}\sqrt{c}\sqrt{-ax}}{15a^{\frac{7}{2}}x^{\frac{7}{2}}-15a} \end{cases}\right.}{a^{3}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(7/2),x)
```

output
```
c**3*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a
*x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1
) + I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a
*x + 1)), True)) + c**3*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sq
rt(-c) + 2*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a - c*
*3*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)/(3*c), True))/a**2 +
 c**3*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*
a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 1
5*a**(5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1)/(15*a**(7/2)*x**(
7/2) - 15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(a*x - 1)/(15*a**(7
/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*sqrt(a*x - 1)/(15*
a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)*sqrt(a*x - 1)/(
15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*a**(11/2)
*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9/
2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*I*a*
*5*sqrt(c)*x**3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2
)) - 2*I*a**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5
/2)*x**(5/2)) - 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) -
15*a**(5/2)*x**(5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7
/2) - 15*a**(5/2)*x**(5/2)), True))/a**3
```

3.447.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^(7/2)/(a*x - 1), x)
```

3.447.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.448 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2} dx$

3.448.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2} dx=\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-c^(5/2)*arctanh((c-c/a/x)^(1/2)/
c^(1/2))/a+c^2*(c-c/a/x)^(1/2)/a
```

3.448.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2} dx=\frac{c^2\sqrt{c-\frac{c}{ax}}(2-2ax+3a^2x^2)-3ac^{5/2}x\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{3a^2x}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(2 - 2*a*x + 3*a^2*x^2) - 3*a*c^(5/2)*x*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/(3*a^2*x)
```

3.448.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}dx}{a}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left(-\frac{1}{2}\int \left(c-\frac{c}{ax}\right)^{3/2}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(\frac{1}{2}\left(-c\int \sqrt{c-\frac{c}{ax}}xd\frac{1}{x}-\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(\frac{1}{2}\left(-c\left(c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)-\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

$$\Big\downarrow 73$$

$$-\frac{c\left(\frac{1}{2}\left(-c\left(2\sqrt{c-\frac{c}{ax}}-2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)-\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(\frac{1}{2}\left(-c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)\right)-\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

output
```
-((c*(-((a*(c - c/(a*x))^(5/2)*x)/c) + ((-2*(c - c/(a*x))^(3/2))/3 - c*(2*
Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]))/2))/a)
```

3.448.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.448. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.448. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.448.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{2} \left(-6\sqrt{a\,x^{2}-x}\, a^{\frac{5}{2}} x^{3}+3\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a^{2} x^{3}+4\left(a\,x^{2}-x\right)^{\frac{3}{2}} \sqrt{a}\right)}{6x^{2}\sqrt{(ax-1)x}\, a^{\frac{5}{2}}}$	108
risch	$\frac{\left(3a^{3}x^{3}-5a^{2}x^{2}+4ax-2\right)c^{2}\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^{2}(ax-1)}-\frac{\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)c^{2}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	139

input

```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3+3*ln(1/
2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^3+4*(a*x^2-x)^(3/2)*a
^(1/2))/((a*x-1)*x)^(1/2)/a^(5/2)
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{3\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^{2}c^{2}x^{2}-2\,ac^{2}x+2\,c^{2}\right)\sqrt{\frac{acx-c}{ax}}}{6\,a^{2}x},\ \frac{3\,a\sqrt{-c}c^{2}}{}\right.$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output

```
[1/6*(3*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
 c) + 2*(3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*
x), 1/3*(3*a*sqrt(-c)*c^2*x*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (
3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

3.448.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.52

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = c^2\left(\begin{cases} -\frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{\sqrt{c}\sqrt{x}\sqrt{ax-1}}{\sqrt{a}} & \text{for } |ax|>1 \\ -\frac{i\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right) - \frac{c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(5/2),x)
```

output
```
c**2*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a
*x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1
) + I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a
*x + 1)), True)) - c**2*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)
/(3*c), True))/a**2
```

3.448.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^(5/2)/(a*x - 1), x)
```

3.448.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.448.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.449 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$

3.449.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
(c-c/a/x)^(3/2)*x+c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-c*(c-c/a/x)^(
1/2)/a
```

3.449.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{ax}}(-2+ax)+c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(-2 + a*x) + c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]])/a
```

3.449.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6717, 6683, 1035, 281, 899, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}dx}{a}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left(\frac{1}{2}\int \sqrt{c-\frac{c}{ax}}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^{3/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(\frac{1}{2}\left(c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{3/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{c\left(\frac{1}{2}\left(2\sqrt{c-\frac{c}{ax}}-2a\int\frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{3/2}}{c}\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(\frac{1}{2}\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{3/2}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
-((c*(-((a*(c - c/(a*x))^(3/2)*x)/c) + (2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/2))/a)
```

3.449.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.449. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.449.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(-2\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}x^{2}+4\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}+\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^{2}\right)}{2x\sqrt{(ax-1)x}\,a^{\frac{3}{2}}}$	103
risch	$\frac{(a^{2}x^{2}-3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	122

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(a*x-1)/a/x)^(1/2)/x*c*(-2*(a*x^2-x)^(1/2)*a^(3/2)*x^2+4*(a*x^2-x)^
(3/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*x^2)/(
(a*x-1)*x)^(1/2)/a^(3/2)
```

3.449.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.96

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\left[\frac{c^{\frac{3}{2}}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},\ -\frac{\sqrt{-c}c\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(c^(3/2)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) +
2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(sqrt(-c)*c*arctan(sqrt(-c)*s
qrt((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a]
```

3.449.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.43 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.47

$$\int e^{2\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{3/2} dx = c\left(\begin{cases} -\frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} + \frac{\sqrt{c}\sqrt{x}\sqrt{ax-1}}{\sqrt{a}} & \text{for } |ax| > 1 \\ -\frac{i\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{-ax+1}} + \frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} + \frac{i\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) - \frac{c\left(\begin{cases} \frac{2c\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{c-\frac{c}{ax}} & \text{for } \frac{c}{a} \neq 0 \\ -\sqrt{c}\log(x) & \text{otherwise} \end{cases}\right)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(3/2),x)
```

output
```
c*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a*x
- 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1) +
 I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a*x
+ 1)), True)) - c*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c)
 + 2*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a
```

3.449.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{3/2} dx = \int \frac{(ax+1)\left(c - \frac{c}{ax}\right)^{\frac{3}{2}}}{ax-1}\, dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^(3/2)/(a*x - 1), x)
```

3.449.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

3.450 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.450.1 Optimal result

Integrand size = 24, antiderivative size = 50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}\,x+\frac{3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

3.450.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}\,x+\frac{3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

3.450.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{6717, 6683, 1035, 281, 899, 87, 73, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left(\frac{3}{2}\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{c\left(-\frac{3a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(-\frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((c*(-((a*Sqrt[c - c/(a*x)]*x)/c) - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
)/Sqrt[c]))/a)
```

3.450.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.450.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

method	result	size
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^{2}-x}\,\sqrt{a}-4\sqrt{(ax-1)x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{(ax-1)x}\,\sqrt{a}}$	120

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(c*(a*x-1)/a/x)^(1/2)+3/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2
-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)
^(1/2)
```

3.450. $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.450.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.48

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\frac{ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(
-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

3.450.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.450.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

3.450.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{3\sqrt{c}\log\left(|a||c|\right)\operatorname{sgn}(x)}{2a} - \frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{2\,a\operatorname{sgn}(x)} + \frac{\sqrt{a^2cx^2-acx}|a|}{a^2\operatorname{sgn}(x)}$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output

```
3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(
a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input

```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output

```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.451 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.451.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=-\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

output
```
5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)-5/a/(c-c/a/x)^(1/2)+x/(c-c/a/
x)^(1/2)
```

3.451.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{ax-5\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
(a*x - 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*Sqrt[c - c/(a*x)
])
```

3.451.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6717, 6683, 1035, 281, 899, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{ax+1}{\sqrt{c-\frac{c}{ax}}(1-ax)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{a+\frac{1}{x}}{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{5}{2}\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax}{c\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

$$\Big\downarrow \text{61}$$

$$-\frac{c\left(\frac{5}{2}\left(\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2}{c\sqrt{c-\frac{c}{ax}}}\right)-\frac{ax}{c\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

$$\Big\downarrow 73$$

$$-\frac{c\left(\frac{5}{2}\left(\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c^2}\right)-\frac{ax}{c\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(\frac{5}{2}\left(\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{c^{3/2}}\right)-\frac{ax}{c\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
-((c*(-((a*x)/(c*Sqrt[c - c/(a*x)])) + (5*(2/(c*Sqrt[c - c/(a*x)]) - (2*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/c^(3/2)))/2))/a)
```

3.451.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.451. $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.451. $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.451.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(60) = 120$.

Time = 0.54 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.14

method	result
risch	$\frac{ax-1}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a\sqrt{a^{2}c}}-\frac{4\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(x-\frac{1}{a}\right)ac}}{a^{3}c\left(x-\frac{1}{a}\right)}\right)\sqrt{c(ax-1)ax}}{\sqrt{\frac{c(ax-1)}{ax}}\,x}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(10a^{\frac{5}{2}}\sqrt{(ax-1)x}\,x^{2}+5\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^{2}x^{2}-8a^{\frac{3}{2}}\left((ax-1)x\right)^{\frac{3}{2}}-20\sqrt{(ax-1)x}\,a^{\frac{3}{2}}x-10\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2}{2\sqrt{a}}\right.}{2\sqrt{(ax-1)x}\,c(ax-1)^{2}\sqrt{a}}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/(c*(a*x-1)/a/x)^(1/2)+(5/2/a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/
2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/a^3/c/(x-1/a)*(a^2*c*(x-1/a)^2
+(x-1/a)*a*c)^(1/2))/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)/x
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.51

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{5\,(ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(a^{2}x^{2}-5\,ax)\sqrt{\frac{acx-c}{ax}}}{2\,(a^{2}cx-ac)},\right.$$
$$\left.-\frac{5\,(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(a^{2}x^{2}-5\,ax)\sqrt{\frac{acx-c}{ax}}}{a^{2}cx-ac}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(5*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) + 2*(a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c),
-(5*(a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a^2*x
^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

3.451.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(1/2),x)
```

output
```
Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)), x)
```

3.451.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a*x))), x)
```

3.451.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(60) = 120$.

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.43

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{5\,\log\left(c^2|a|\right)\operatorname{sgn}(x)}{6\,a\sqrt{c}} - \frac{5\,\log\Bigg(\Bigg|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3\sqrt{c}|a|-5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^2ac+4\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-}\right.}{6\,a\sqrt{c}} + \frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c}$$

---

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
5/6*log(c^2*abs(a))*sgn(x)/(a*sqrt(c)) - 5/6*log(abs(2*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - a*c*x))^3*sqrt(c)*abs(a) - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - a*c*x))^2*a*c + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(3/2)*ab
s(a) - a*c^2))*sgn(x)/(a*sqrt(c)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/
(a^2*c)
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)), x)
```

3.452 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.452.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

output
```
-7/3/a/(c-c/a/x)^(3/2)+x/(c-c/a/x)^(3/2)+7*arctanh((c-c/a/x)^(1/2)/c^(1/2)
)/a/c^(3/2)-7/a/c/(c-c/a/x)^(1/2)
```

3.452.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{x\left(3ax-7\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\frac{1}{ax}\right)\right)}{3c\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
(x*(3*a*x - 7*Hypergeometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c*Sqrt[c
- c/(a*x)]*(-1 + a*x))
```

3.452.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{ax+1}{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{a+\frac{1}{x}}{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{7}{2}\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{61}$$

$$-\frac{c\left(\frac{7}{2}\left(\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

$$\Big\downarrow 61$$

$$-\frac{c\left(\frac{7}{2}\left(\frac{\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

$$\Big\downarrow 73$$

$$-\frac{c\left(\frac{7}{2}\left(\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c^2}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(\frac{7}{2}\left(\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{c^{3/2}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
-((c*(-((a*x)/(c*(c - c/(a*x))^(3/2))) + (7*(2/(3*c*(c - c/(a*x))^(3/2)) +
 (2/(c*Sqrt[c - c/(a*x)]) - (2*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/c^(3/2)
)/c))/2))/a)
```

3.452.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.452. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.452.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(81) = 162$.

Time = 0.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.12

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^2\sqrt{a^2c}}-\frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{3a^5c\left(x-\frac{1}{a}\right)^2}-\frac{22\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{3a^4c\left(x-\frac{1}{a}\right)}\right)a\sqrt{c(ax-1)ax}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\sqrt{\frac{c(ax-1)}{ax}}\,x\left(42a^{\frac{7}{2}}\sqrt{(ax-1)x}\,x^3+21\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^3-36a^{\frac{5}{2}}((ax-1)x)^{\frac{3}{2}}x-126a^{\frac{5}{2}}\sqrt{(ax-1)x}\,x^2-63\ln\left(\frac{2\sqrt{(ax-1)}}{2}\right.\right.$ $6\sqrt{(a}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/(c*(a*x-1)/a/x)^(1/2)+(7/2/a^2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)
^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/3/a^5/c/(x-1/a)^2*(a^2*c*(
x-1/a)^2+(x-1/a)*a*c)^(1/2)-22/3/a^4/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*
c)^(1/2))*a/c/x/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.51

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\left[\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^3x^3-28\,a^2x^2+21\right.}{6\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right.$$

$$\left.-\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-\left(3\,a^3x^3-28\,a^2x^2+21\,ax\right)\sqrt{\frac{acx-c}{ax}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

---

output
```
[1/6*(21*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((
a*c*x - c)/(a*x)) + c) + 2*(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x -
 c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(21*(a^2*x^2 - 2*a*x
 + 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^3*x^3 - 2
8*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x +
a*c^2)]
```

3.452.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(3/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

3.452.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(3/2)), x)
```

3.452.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(81) = 162$.

Time = 0.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{7\,\log\left(c^2|a||c|\right)\operatorname{sgn}(x)}{10\,ac^{\frac{3}{2}}}$$

$$-\frac{7\,\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5\sqrt{c}|a|-9\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4ac+16\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-a}\right.\right.\right.}{10\,ac^{\frac{3}{2}}}$$

$$+\frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
7/10*log(c^2*abs(a)*abs(c))*sgn(x)/(a*c^(3/2)) - 7/10*log(abs(2*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*sqrt(c)*abs(a) - 9*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^4*a*c + 16*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^
3*c^(3/2)*abs(a) - 14*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a*c^2 +
6*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(5/2)*abs(a) - a*c^3))*sgn(x
)/(a*c^(3/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/(a^2*c^2)
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)), x)
```

3.453 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.453.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{9\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

output
```
-9/5/a/(c-c/a/x)^(5/2)-3/a/c/(c-c/a/x)^(3/2)+x/(c-c/a/x)^(5/2)+9*arctanh((
c-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)-9/a/c^2/(c-c/a/x)^(1/2)
```

3.453.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\frac{1}{ax}\right)}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

output
```
x/(c - c/(a*x))^(5/2) - (9*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)])/
(5*a*(c - c/(a*x))^(5/2))
```

3.453.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6683, 1035, 281, 899, 87, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{ax+1}{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{a+\frac{1}{x}}{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{9}{2}\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow 61$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow 61$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow 61$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\frac{\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow 73$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c^2}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{c^{3/2}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

input

```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

3.453. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

output
```
-((c*(-((a*x)/(c*(c - c/(a*x))^(5/2))) + (9*(2/(5*c*(c - c/(a*x))^(5/2)) +
 (2/(3*c*(c - c/(a*x))^(3/2)) + (2/(c*Sqrt[c - c/(a*x)]) - (2*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/c^(3/2))/c)/c))/2))/a)
```

3.453.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.453. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.453.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(102) = 204$.

Time = 0.54 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.09

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{9\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^3\sqrt{a^2c}}-\frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{5a^7c\left(x-\frac{1}{a}\right)^3}-\frac{18\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{5a^6c\left(x-\frac{1}{a}\right)^2}-\frac{54\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\right.}}{5a^5c\left(x-\frac{1}{a}\right)}\right.}{c^2x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(90a^{\frac{9}{2}}\sqrt{(ax-1)x}\,x^4+45\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^4x^4-80a^{\frac{7}{2}}((ax-1)x)^{\frac{3}{2}}x^2-360a^{\frac{7}{2}}\sqrt{(ax-1)x}\,x^3-180\ln\left(\frac{2\sqrt{(ax-}}{}\right.\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/(c*(a*x-1)/a/x)^(1/2)+(9/2/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*
c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/5/a^7/c/(x-1/a)^3*(a^2*c
*(x-1/a)^2+(x-1/a)*a*c)^(1/2)-18/5/a^6/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+(x-1/a
)*a*c)^(1/2)-54/5/a^5/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2))*a^2/c
^2/x/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.453.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.49

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[\frac{45\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(5\,a^4x^4-69\,a\right.}{10\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}\right.$$

$$\left.-\frac{45\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-\left(5\,a^4x^4-69\,a^3x^3+105\,a^2x^2-45\,ax\right)\sqrt{\frac{acx-c}{ax}}}{5\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/10*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqr
t(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*
x^2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^
2*c^3*x - a*c^3), -1/5*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arct
an(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (5*a^4*x^4 - 69*a^3*x^3 + 105*a^2
*x^2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a
^2*c^3*x - a*c^3)]
```

3.453.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(5/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

3.453.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(5/2)), x)
```

3.453.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(102) = 204$.

Time = 0.48 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{9\log\left(c^4|a|\right)\operatorname{sgn}(x)}{14\,ac^{\frac{5}{2}}}$$

$$-\frac{9\log\Bigg(\Bigg|2\Big(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\Big)^7\sqrt{c}|a|-13\Big(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\Big)^6ac+36\Big(\sqrt{a^2c}x-\sqrt{a^2cx^2-}}{}$$

$$+\frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
9/14*log(c^4*abs(a))*sgn(x)/(a*c^(5/2)) - 9/14*log(abs(2*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - a*c*x))^7*sqrt(c)*abs(a) - 13*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - a*c*x))^6*a*c + 36*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*c^(3
/2)*abs(a) - 55*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a*c^2 + 50*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*c^(5/2)*abs(a) - 27*(sqrt(a^2*c)*
x - sqrt(a^2*c*x^2 - a*c*x))^2*a*c^3 + 8*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 a*c*x))*c^(7/2)*abs(a) - a*c^4))*sgn(x)/(a*c^(5/2)) + sqrt(a^2*c*x^2 - a*
c*x)*abs(a)*sgn(x)/(a^2*c^3)
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{a\,x+1}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,\left(a\,x-1\right)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)), x)
```

3.454 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

3.454.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}$$

output
```
-11/7/a/(c-c/a/x)^(7/2)-11/5/a/c/(c-c/a/x)^(5/2)-11/3/a/c^2/(c-c/a/x)^(3/2
)+x/(c-c/a/x)^(7/2)+11*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(7/2)-11/a/c^3
/(c-c/a/x)^(1/2)
```

3.454.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{7x-\frac{11\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\frac{1}{ax}\right)}{a}}{7\left(c-\frac{c}{ax}\right)^{7/2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

output
```
(7*x - (11*Hypergeometric2F1[-7/2, 1, -5/2, 1 - 1/(a*x)])/a)/(7*(c - c/(a*
x))^(7/2))
```

3.454.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6683, 1035, 281, 899, 87, 61, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{ax+1}{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{a+\frac{1}{x}}{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{9/2}}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\left(c-\frac{c}{ax}\right)^{9/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{11}{2}\int \frac{x}{\left(c-\frac{c}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$$\Big\downarrow \text{61}$$

$$-\frac{c\left(\frac{11}{2}\left(\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$$\Big\downarrow \text{61}$$

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$$\Big\downarrow \text{61}$$

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$$\Big\downarrow \text{61}$$

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\frac{\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c^2}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

3.454. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

$$\Big\downarrow \color{blue}{221}$$

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{c^{3/2}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

output
```
-((c*(-((a*x)/(c*(c - c/(a*x))^(7/2))) + (11*(2/(7*c*(c - c/(a*x))^(7/2))
+ (2/(5*c*(c - c/(a*x))^(5/2)) + (2/(3*c*(c - c/(a*x))^(3/2)) + (2/(c*Sqrt
[c - c/(a*x)]) - (2*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/c^(3/2))/c)/c)/c))
/2))/a)
```

3.454.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.454. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.454. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

3.454.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(123) = 246$.

Time = 0.54 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.01

method	result
risch	$\frac{ax-1}{a\,c^{3}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{11\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a^{4}\sqrt{a^{2}c}}-\frac{4\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(x-\frac{1}{a}\right)ac}}{7a^{9}c\left(x-\frac{1}{a}\right)^{4}}-\frac{102\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(x-\frac{1}{a}\right)ac}}{35a^{8}c\left(x-\frac{1}{a}\right)^{3}}-\frac{712\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(\right.}}{105a^{7}c\left(x-\frac{1}{a}\right.}\right.}{c^{3}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2310a^{\frac{11}{2}}\sqrt{(ax-1)x}\,x^{5}+1155\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^{5}x^{5}-2100a^{\frac{9}{2}}\left((ax-1)x\right)^{\frac{3}{2}}x^{3}-11550a^{\frac{9}{2}}\sqrt{(ax-1)x}\,x^{4}-5775\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/(c*(a*x-1)/a/x)^(1/2)+(11/2/a^4*ln((-1/2*a*c+a^2*c*x)/(a^2
*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/7/a^9/c/(x-1/a)^4*(a^2*
c*(x-1/a)^2+(x-1/a)*a*c)^(1/2)-102/35/a^8/c/(x-1/a)^3*(a^2*c*(x-1/a)^2+(x-
1/a)*a*c)^(1/2)-712/105/a^7/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2
)-1516/105/a^6/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2))/c^3*a^3/x/(c
*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.39

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \left[\frac{1155\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(1}{210\left(a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}+6\,a^{3}c^{4}x^{2}-\right.}\right.$$

$$-\frac{1155\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-\left(105\,a^{5}x^{5}-1936\,a^{4}x^{4}+4466\,a^{3}x^{3}\right.}{105\left(a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}+6\,a^{3}c^{4}x^{2}-4\,a^{2}c^{4}x+ac^{4}\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/210*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log(-2*
a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(105*a^5*x^5 - 1936
*a^4*x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x))
)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/
105*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(sq
rt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (105*a^5*x^5 - 1936*a^4*x^4 + 4466*a^3
*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^5*c^4*x^4 - 4*
a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

3.454.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(7/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

3.454.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(7/2)), x)
```

3.454.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(123) = 246$.

Time = 0.63 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{11\log\left(c^4|a||c|\right)\operatorname{sgn}(x)}{18\,ac^{\frac{7}{2}}}$$

$$-\,11\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^9\sqrt{c}|a|-17\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^8ac+64\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-\right.\right.\right.$$

$$+\frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
11/18*log(c^4*abs(a)*abs(c))*sgn(x)/(a*c^(7/2)) - 11/18*log(abs(2*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*sqrt(c)*abs(a) - 17*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - a*c*x))^8*a*c + 64*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))^7*c^(3/2)*abs(a) - 140*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a*c^
2 + 196*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*c^(5/2)*abs(a) - 182*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a*c^3 + 112*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - a*c*x))^3*c^(7/2)*abs(a) - 44*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - a*c*x))^2*a*c^4 + 10*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(9/2
)*abs(a) - a*c^5))*sgn(x)/(a*c^(7/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn
(x)/(a^2*c^4)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)), x)
```

3.455 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

3.455.1 Optimal result

Integrand size = 24, antiderivative size = 268

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx &= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}} \\ &\quad+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} \\ &\quad+\frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{3\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}\end{aligned}$$

output

```
3/35*(28*a-17/x)*(1+1/a/x)^(3/2)*(c-c/a/x)^(9/2)/a^2/(1-1/a/x)^(9/2)+9/7*(
a-1/x)^2*(1+1/a/x)^(3/2)*(c-c/a/x)^(9/2)/a^3/(1-1/a/x)^(9/2)+(a-1/x)^3*(1+
1/a/x)^(3/2)*(c-c/a/x)^(9/2)*x/a^3/(1-1/a/x)^(9/2)-3*(c-c/a/x)^(9/2)*arcta
nh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(9/2)+3*(c-c/a/x)^(9/2)*(1+1/a/x)^(1/2)/a/
(1-1/a/x)^(9/2)
```

3.455.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\bigg(c-\frac{c}{ax}\bigg)^{9/2}dx=\frac{c^4\sqrt{c-\frac{c}{ax}}\bigg(\sqrt{1+\frac{1}{ax}}(10-26ax-12a^2x^2+164a^3x^3+35a^4x^4)-105a^3x^3\operatorname{arctanh}\bigg(\sqrt{1+\frac{1}{ax}}}{35a^4\sqrt{1-\frac{1}{ax}}x^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

output
```
(c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(10 - 26*a*x - 12*a^2*x^2 + 164*
a^3*x^3 + 35*a^4*x^4) - 105*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(35*a^4*S
qrt[1 - 1/(a*x)]*x^3)
```

3.455.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.63, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6731, 585, 27, 108, 27, 170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{9/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a^3}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 108$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\int -\frac{3\left(a-\frac{1}{x}\right)^2\left(a+\frac{3}{x}\right)\sqrt{1+\frac{1}{ax}}x}{2a}d\frac{1}{x}-x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\int\left(a-\frac{1}{x}\right)^2\left(a+\frac{3}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 170$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{2}{7}a\int\frac{1}{2}\left(a-\frac{1}{x}\right)\left(7a+\frac{17}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^2\right)}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\int\left(a-\frac{1}{x}\right)\left(7a+\frac{17}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^2\right)}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 164$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\left(7a^2\int\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{2}{5}a\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^2\right)}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 60$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\Big(\frac{1}{7}a\Big(7a^2\Big(\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2\sqrt{\frac{1}{ax}+1}\Big)+\frac{2}{5}a\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\Big)+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}(a-}{2a}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 73$$

3.455. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\left(7a^2\left(2a\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+2\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{5}a\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3}\right.}{2a}\right.}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{221}$$

$$-\frac{c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\left(7a^2\left(2\sqrt{\frac{1}{ax}+1}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{2}{5}a\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^2\right)\right.}{2a}\right.}{a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

output
```
-((c^4*Sqrt[c - c/(a*x)]*(-((a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*x) - (3*((6
*a*(a - x^(-1))^2*(1 + 1/(a*x))^(3/2))/7 + (a*((2*a*(28*a - 17/x)*(1 + 1/(
a*x))^(3/2))/5 + 7*a^2*(2*Sqrt[1 + 1/(a*x)] - 2*ArcTanh[Sqrt[1 + 1/(a*x)]]
)))/7))/(2*a)))/(a^3*Sqrt[1 - 1/(a*x)]))
```

3.455.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

3.455. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.455.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.66

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c^4 \bigg(70a^{\frac{9}{2}}\sqrt{(ax+1)x}\, x^4+328a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-105\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^4x^4-24a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-52a^{\frac{3}{2}}x}{70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^3a^{\frac{9}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(35a^5x^5+199a^4x^4+152a^3x^3-38a^2x^2-16ax+10)c^4\sqrt{\frac{c(ax-1)}{ax}}}{35x^3a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^4\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/70/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^4*(70
*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+328*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-105*ln(1/
2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4-24*a^(5/2)*x^2*((
a*x+1)*x)^(1/2)-52*a^(3/2)*x*((a*x+1)*x)^(1/2)+20*((a*x+1)*x)^(1/2)*a^(1/2
))/x^3/a^(9/2)/((a*x+1)*x)^(1/2)
```

3.455.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.63

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\Bigg[\frac{105\,(a^4c^4x^4-a^3c^4x^3)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(35\,a^5c}{140\,(a^5x^4-a^4x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

output
```
[1/140*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*
x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt
((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(35*a^5*c^4*x^5 + 199*a^4*c^4*x^4
+ 152*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10*c^4)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/70*(105*(a^4*c^4
*x^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(35*
a^5*c^4*x^5 + 199*a^4*c^4*x^4 + 152*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^
4*x + 10*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4
- a^4*x^3)]
```

3.455.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(9/2),x)
```

output
```
Timed out
```

3.455.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.455.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.456 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

3.456.1 Optimal result

Integrand size = 24, antiderivative size = 237

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

output
```
1/3*(1+1/a/x)^(3/2)*(c-c/a/x)^(7/2)/a/(1-1/a/x)^(7/2)-2/5*(1+1/a/x)^(5/2)*
(c-c/a/x)^(7/2)/a/(1-1/a/x)^(7/2)+(1+1/a/x)^(5/2)*(c-c/a/x)^(7/2)*x/(1-1/a
/x)^(7/2)-(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)+(c-c/
a/x)^(7/2)*(1+1/a/x)^(1/2)/a/(1-1/a/x)^(7/2)
```

3.456.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.43

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\left(-6+8ax+44a^2x^2+15a^3x^3\right)-15a^2x^2\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{15a^3\sqrt{1-\frac{1}{ax}}\,x^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-6 + 8*a*x + 44*a^2*x^2 + 15*a^
3*x^3) - 15*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]
*x^2)
```

3.456.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.56, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6731, 585, 27, 100, 27, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2} e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a^2}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\int -\frac{1}{2}\left(a-\frac{2}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \left(a-\frac{2}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 90$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\int\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 60$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\left(\int\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{2}{3}\left(\frac{1}{ax}+1\right)^{3/2}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 60$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\left(\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}\left(\frac{1}{ax}+1\right)^{3/2}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\left(2a\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}\left(\frac{1}{ax}+1\right)^{3/2}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 221$$

$$-\frac{c^3\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\left(-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{3}\left(\frac{1}{ax}+1\right)^{3/2}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

output

```
-((c^3*Sqrt[c - c/(a*x)]*(-(a^2*(1 + 1/(a*x))^(5/2)*x) + ((4*a*(1 + 1/(a*x
))^(5/2))/5 - a*(2*Sqrt[1 + 1/(a*x)] + (2*(1 + 1/(a*x))^(3/2))/3 - 2*ArcTa
nh[Sqrt[1 + 1/(a*x)]]))/2))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.456. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

3.456.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.456. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.456.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,c^{3}\Bigg(30a^{\frac{7}{2}}x^{3}\sqrt{(ax+1)x}+88a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}-15\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{3}x^{3}+16a^{\frac{3}{2}}x\sqrt{(ax+1)x}-12\sqrt{(ax+1)}}{30\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^{2}a^{\frac{7}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(15a^{4}x^{4}+59a^{3}x^{3}+52a^{2}x^{2}+2ax-6)c^{3}\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}a^{3}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c^{3}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/30/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30
*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+88*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-15*ln(1/2*
(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+16*a^(3/2)*x*((a*x+
1)*x)^(1/2)-12*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/((a*x+1)*x)^(1/2)
```

3.456.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.75

$$\int e^{3\coth^{-1}(ax)}\Big(c-\frac{c}{ax}\Big)^{7/2}\,dx=\Bigg[\frac{15\,(a^{3}c^{3}x^{3}-a^{2}c^{3}x^{2})\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(15\,a^{4}c^{3}}{60\,(a^{4}x^{3}-a^{3}x^{2})}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 + 5
2*a^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt
(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 59*a^3*c^3
*x^3 + 52*a^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

3.456.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(7/2),x)
```

output
```
Timed out
```

3.456.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.456.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.457 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.457.1 Optimal result

Integrand size = 24, antiderivative size = 156

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-1/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^5*(1-1/a^2/x^2)^(5/2)*x/(
c-c/a/x)^(5/2)+c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
)/a-c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

3.457.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2+2ax+3a^2x^2)+3ax\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + 2*a*x + 3*a^2*x^2) + 3*a*x*
ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

3.457.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6731, 580, 576, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 580}$$

$$-c^3\left(\frac{c\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \text{ 576}$$

$$-c^3\left(\frac{c\left(\frac{\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \text{ 576}$$

$$-c^3\left(\frac{c\left(\frac{\frac{\int \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \; 573$$

$$-c^3\left(\frac{c\left(\frac{\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \; 219$$

$$-c^3\left(\frac{c\left(\frac{\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{c}}}{c}+\frac{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

input

```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

output

```
-(c^3*(-((c^2*(1 - 1/(a^2*x^2))^(5/2)*x)/(c - c/(a*x))^(5/2)) + (c*((2*(1
- 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + ((2*Sqrt[1 - 1/(a^2*x^2)])
/Sqrt[c - c/(a*x)] - (2*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c
/(a*x)]])/Sqrt[c])/c))/(2*a)))
```

3.457. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.457.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.457. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.457.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,c^{2}\left(6a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}x^{2}+4a^{\frac{3}{2}}x\sqrt{(ax+1)x}+4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x\,a^{\frac{5}{2}}\sqrt{(ax+1)x}}$	144
risch	$\frac{(3a^{3}x^{3}+5a^{2}x^{2}+4ax+2)c^{2}\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^{2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c^{2}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	168

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^
(5/2)*(6*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2))*a^2*x^2+4*a^(3/2)*x*((a*x+1)*x)^(1/2)+4*((a*x+1)*x)^(
1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

3.457.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.44

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{3\,(a^{2}c^{2}x^{2}-ac^{2}x)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^{3}c^{2}x^{3}+}{12\,(a^{3}x^{2}-a^{2}x)}\right.$$

$$-\frac{3\,(a^{2}c^{2}x^{2}-ac^{2}x)\sqrt{-c}\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)-2\,(3\,a^{3}c^{2}x^{3}+5\,a^{2}c^{2}x^{2}+4\,ac^{2}x+2\,c^{2})\sqrt{\frac{ax-}{ax+}}}{6\,(a^{3}x^{2}-a^{2}x)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*
x + 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a
^2*x), -1/6*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a
*c*x - c)) - 2*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x + 2*c^2)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

3.457.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

3.457.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.457.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.458 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

3.458.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a/x)^(3/2)+3*c^(3/2)*arctanh(c^(1/2)*(1-1/a
^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a-3*c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/
2)
```

3.458.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1+\frac{1}{ax}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
(-2*(1 + 1/(a*x))^(5/2)*(c - c/(a*x))^(3/2)*Hypergeometric2F1[2, 5/2, 7/2,
 1 + 1/(a*x)])/(5*a*(1 - 1/(a*x))^(3/2))
```

3.458.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6731, 575, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{575}$$

$$-c^3\left(\frac{3\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2ac}-\frac{x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{576}$$

$$-c^3\left(\frac{3\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2ac}-\frac{x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c^3\left(\frac{3\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2ac}-\frac{x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c^3\left(\frac{3\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{c}}\right)}{2ac}-\frac{x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
-(c^3*(-(((1 - 1/(a^2*x^2))^(3/2)*x)/(c - c/(a*x))^(3/2)) + (3*((2*Sqrt[1
- 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - (2*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^
2)])/Sqrt[c - c/(a*x)]])/Sqrt[c]))/(2*a*c)))
```

3.458.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.458.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{3}{2}}\sqrt{(ax+1)x}}$	118
risch	$\frac{\left(a^{2}x^{2}-ax-2\right)c\sqrt{\frac{c(ax-1)}{ax}}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}+\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	151

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2
)*(2*a^(3/2)*x*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a
*x+1)/a^(1/2))*a*x-4*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

3.458.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.67

$$\int e^{3\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{3/2} dx = \left[\frac{3\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2-acx-2}{4\,(a^2x-a)} - \frac{3\,(acx-c)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(a^2cx^2-acx-2\,c)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output

```
[1/4*(3*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), -1/2*(3*(a*c*x - c)*sqrt(-c)*arctan(2*
(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

3.458.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{3/2} dx = \text{Timed out}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(3/2),x)
```

output

```
Timed out
```

3.458.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.458.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.458.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.459 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.459.1 Optimal result

Integrand size = 24, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

output
```
5*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(1/2
*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+x*(1+1
/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

3.459.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x+\frac{5\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (5*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
 - (4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[1 - 1/(a*x)]
```

3.459.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6731, 585, 27, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(5a+\frac{3}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(5a+\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textbf{174}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{16a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+10a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-10\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*(-((Sqrt[1 + 1/(a*x)]*x)/a) + (-10*ArcTanh[Sqrt[1
 + 1/(a*x)]] + 8*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a^2)))/Sqr
t[c - c/(a*x)])
```

3.459. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.459.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.459. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.459.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a
*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2
*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1
)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/((a*x+1)*x)^(1/2)/a^(3/2)/(1/a)^(1
/2)
```

3.459.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.37

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{4\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+5\,(ax-1)\sqrt{c}\log\left(\right.}{4\,(a^{2}x-a)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*
c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*
(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x
- 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2
*c*x^2 - 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c
*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a)]
```

3.459.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

3.459.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.459.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.459.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.460 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.460.1 Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

output
```
7*arctanh((1+1/a/x)^(1/2))*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-5*arctanh(1/2
*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-3*(1-1
/a/x)^(1/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(1/2)+a*x*(1-1/a/x)^(1/2)*(1
+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(1/2)
```

3.460.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.53

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(a\sqrt{1+\frac{1}{ax}}x(-3+ax)+7(-1+ax)\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-5\sqrt{2}(-1+ax)\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x*(-3 + a*x) + 7*(-1 + a*x)*ArcTan
h[Sqrt[1 + 1/(a*x)]] - 5*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt
[2]]))/(a*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

3.460.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.63, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6731, 585, 27, 109, 25, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(3a+\frac{2}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(3a+\frac{2}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(3a+\frac{2}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 168$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(7a+\frac{3}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(7a+\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 174$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{10\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{20a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+14a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 221$$

3.460. $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{10\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-14\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
-((a^2*Sqrt[1 - 1/(a*x)]*((2*Sqrt[1 + 1/(a*x)]*x)/(a*(a - x^(-1))) + (-3*S
qrt[1 + 1/(a*x)]*x + (-14*ArcTanh[Sqrt[1 + 1/(a*x)]] + 10*Sqrt[2]*ArcTanh[
Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a))/a^2))/Sqrt[c - c/(a*x)])
```

3.460.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

3.460. $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.460. $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.460.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-5a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x-6\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{3}{2}}c\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}-\frac{2\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{a^{3}c\left(x-\frac{1}{a}\right)}-\frac{5\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right.}}{x-\frac{1}{a}}\right.}{2a^{2}\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)
^(1/2)*a^(5/2)*(1/a)^(1/2)*x-5*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(
(a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-6*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(
1/2)+7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/
2)*x-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)
+5*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))
*a^(1/2))/a^(3/2)/c/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.70

$$\int\frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$=\left[\frac{7\left(a^{2}x^{2}-2\,ax+1\right)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^{3}x^{3}-2\,a^{2}x^{2}-3\,ax\right)}{4\left(a^{3}cx^{2}-2\,a^{2}cx+\right.}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(7*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a
^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)) - c)/(a*x - 1)) + 4*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) + 5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*
log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 +
 a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/sqrt(c) - 1)/(a^3*
x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c), 1/2*
(5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2
+ a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2
*x^2 - 2*a*x - 1)) - 7*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x
^2 - a*c*x - c)) + 2*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

3.460.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

3.460.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.460.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.461 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.461.1 Optimal result

Integrand size = 24, antiderivative size = 275

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx &= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^{2}\left(c-\frac{c}{ax}\right)^{3/2}} \\ &\quad -\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a^{2}\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^{2}\left(c-\frac{c}{ax}\right)^{3/2}} \\ &\quad +\frac{9\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{51\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{4\sqrt{2}a\left(c-\frac{c}{ax}\right)^{3/2}}\end{aligned}$$

output
```
9*(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-51/8*(1-1/a/x
)^(3/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)-2*a
*(1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(3/2)-15/4*(1-1/a/x)^
(3/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)+a^2*(1-1/a/x)^(3/2)*x*(1+1/a
/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(3/2)
```

3.461.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(15-23ax+4a^2x^2)+72(-1+ax)^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-51\surd\right.}{8ac\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(15 - 23*a*x + 4*a^2*x^2) + 72
*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 51*Sqrt[2]*(-1 + a*x)^2*ArcTanh
[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(8*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

3.461.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.63, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6731, 585, 27, 109, 25, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{9/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^3\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 109$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}-\frac{\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{3\left(10a+\frac{7}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\int \frac{\left(10a+\frac{7}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

3.461. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(-\frac{\int -\frac{\left(12a+\frac{5}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{\int \frac{\left(12a+\frac{5}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 174$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{17\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+12\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{34a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+24a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 221$$

3.461. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{17\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-24\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(a*(a - x^(-1))^2) + ((7*S
qrt[1 + 1/(a*x)]*x)/(2*(a - x^(-1))) + (3*(-10*Sqrt[1 + 1/(a*x)]*x + (-24*
ArcTanh[Sqrt[1 + 1/(a*x)]] + 17*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]
)/a))/(4*a))/(2*a^2)))/(c*Sqrt[c - c/(a*x)]))
```

3.461.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.461. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.461.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.19

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{9\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{a^{5}c\left(x-\frac{1}{a}\right)^{2}}-\frac{15\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^{4}c\left(x-\frac{1}{a}\right)}-\frac{51\sqrt{2}\,\ln}{}\right.}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^{2}-51a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{2}-92\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+72\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}-}{2\sqrt{a}}\right.\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(9/2/a^2*ln((1
/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/a^5
/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-15/4/a^4/c/(x-1/a)*
(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-51/16/a^3/c^(1/2)*2^(1/2)*ln((4*
c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2
))/(x-1/a)))*a/c/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)^(1/2)*(
(a*x+1)*a*c*x)^(1/2)*(a*x-1)
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\left[\frac{51\,\sqrt{2}(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

---

output
```
[1/32*(51*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c
*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*
a^2*x^2 + 3*a*x - 1)) + 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-
(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^4*x^4 -
19*a^3*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2), 1/16*(51*sq
rt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2
 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*
c*x^2 - 2*a*c*x - c)) - 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(4*a^4*x^4 - 19*a^3*x^3 - 8*a^2*x^2 +
 15*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^2*x^3 -
 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

3.461.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

3.461.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.461.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.462 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.462.1 Optimal result

Integrand size = 24, antiderivative size = 335

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{249\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{16\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

output

```
11*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-249/32*(1-1/
a/x)^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-
5/3*a^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^3/(c-c/a/x)^(5/2)-29/12*a*
(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)-73/16*(1-1/a/x)^
(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a^3*(1-1/a/x)^(5/2)*x*(1+1/a
/x)^(1/2)/(a-1/x)^3/(c-c/a/x)^(5/2)
```

3.462.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-219+554ax-415a^2x^2+48a^3x^3)+1056(-1+ax)^3\text{arctanh}\right.}{96ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-219 + 554*a*x - 415*a^2*x^2
+ 48*a^3*x^3) + 1056*(-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]] - 747*Sqrt[2]
*(-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(96*a*c^2*Sqrt[c - c/(a
*x)]*(-1 + a*x)^3)
```

3.462.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.63, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6731, 585, 27, 109, 25, 27, 168, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{11/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^4\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 109}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}-\frac{\int -\frac{\left(5a+\frac{4}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(5a+\frac{4}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(5a+\frac{4}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{\int -\frac{\left(58a+\frac{45}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(58a+\frac{45}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 168}$$

3.462. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{3\left(146a+\frac{103}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\int \frac{\left(146a+\frac{103}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 168$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(-\frac{\int -\frac{\left(176a+\frac{73}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 25$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{\int \frac{\left(176a+\frac{73}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

3.462. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

$$\Big\downarrow 174$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{249\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+176\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{498a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+352a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 221$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{249\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-352\,\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

3.462. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

output
```
-((a^4*Sqrt[1 - 1/(a*x)]*((2*Sqrt[1 + 1/(a*x)]*x)/(3*a*(a - x^(-1))^3) + (
(9*Sqrt[1 + 1/(a*x)]*x)/(4*(a - x^(-1))^2) + ((103*Sqrt[1 + 1/(a*x)]*x)/(2
*(a - x^(-1))) + (3*(-146*Sqrt[1 + 1/(a*x)]*x + (-352*ArcTanh[Sqrt[1 + 1/(
a*x)]] + 249*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/(4*a))/(8*a))
/(3*a^2)))/(c^2*Sqrt[c - c/(a*x)]))
```

3.462.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.462. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.462.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.12

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{11\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{2\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{3a^7c\left(x-\frac{1}{a}\right)^3}-\frac{11\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^6c\left(x-\frac{1}{a}\right)^2}-\frac{271\sqrt{}}{}\right.}{c^2\sqrt{\frac{ax-}{ax+}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(192\sqrt{(ax+1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^3-747a^{\frac{7}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^3-1660\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2+1056\ln\left(\frac{2\sqrt{(ax}}{}\right.\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(11/2/a^3*ln
((1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-2/
3/a^7/c/(x-1/a)^3*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-11/4/a^6/c/(x-
1/a)^2*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-271/48/a^5/c/(x-1/a)*(a^2
*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-249/64/a^4/c^(1/2)*2^(1/2)*ln((4*c+3
*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2))/
(x-1/a)))*a^2/c^2/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)^(1/2)*
((a*x+1)*a*c*x)^(1/2)*(a*x-1)
```

3.462.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.20

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[747\sqrt{2}(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a}{a^3x^3-3\,a^2x^2+3\,a}\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(747*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*
log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^
2 - 4*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) + 8*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^3*x^3 + 335*a^2*x^2 - 219
*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*
a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), 1/192*(747*sqrt(2)*(a^
4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a
^2*c*x^2 - 2*a*c*x - c)) - 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x +
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(48*a^5*x^5 - 367*a^
4*x^4 + 139*a^3*x^3 + 335*a^2*x^2 - 219*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2
*c^3*x + a*c^3)]
```

3.462.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

3.462.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.462.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.463 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

3.463.1 Optimal result

Integrand size = 24, antiderivative size = 221

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{9\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

output
```
-9*(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)-1/5*(80*a-7/
x)*(c-c/a/x)^(7/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(7/2)+3/5*(a-1/x)^2*(c-c/
a/x)^(7/2)*(1+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)+(a-1/x)^3*(c-c/a/x)^(7/2)*x
*(1+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)
```

3.463.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.46

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+16ax-92a^2x^2+5a^3x^3)-45a^2x^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{5a^3\sqrt{1-\frac{1}{ax}}x^2}$$

input
```
Integrate[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 16*a*x - 92*a^2*x^2 + 5*a^
3*x^3) - 45*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(5*a^3*Sqrt[1 - 1/(a*x)]*
x^2)
```

3.463.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.67, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6731, 585, 27, 109, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{a^4\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 109$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\int \frac{3\left(a-\frac{1}{x}\right)^2\left(3a+\frac{1}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\int \frac{\left(a-\frac{1}{x}\right)^2\left(3a+\frac{1}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 170$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\left(\frac{2}{5}a\int \frac{\left(15a-\frac{7}{x}\right)\left(a-\frac{1}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\left(\frac{1}{5}a\int \frac{\left(15a-\frac{7}{x}\right)\left(a-\frac{1}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 164$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\left(\frac{1}{5}a\left(15a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2}{3}a\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right.\right.}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\left(\frac{1}{5}a\left(30a^3\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}-\frac{2}{3}a\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(a\right.\right.}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 221$$

3.463. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

$$-\frac{c^3 \Big(-\frac{3}{2}\Big(\frac{1}{5}a\Big(-30a^2\text{arctanh}\Big(\sqrt{\frac{1}{ax}+1}\Big)-\frac{2}{3}a(80a-\frac{7}{x})\sqrt{\frac{1}{ax}+1}\Big)+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}(a-\frac{1}{x})^2\Big)-ax\sqrt{\frac{1}{ax}+1}(a-\frac{1}{x}}{a^4\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

output
```
-((c^3*Sqrt[c - c/(a*x)]*(-(a*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*x) - (3*((2
*a*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)])/5 + (a*((-2*a*(80*a - 7/x)*Sqrt[1 + 1
/(a*x)])/3 - 30*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/5))/2))/(a^4*Sqrt[1 - 1/(
a*x)]))
```

3.463.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.463. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

3.463.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\Bigg(10a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-184a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-45\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3+32a^{\frac{3}{2}}x\sqrt{(ax+1)x}-4}{10x^2a^{\frac{7}{2}}(ax-1)\sqrt{(ax+1)x}}$
risch	$\frac{(5a^4x^4-87a^3x^3-76a^2x^2+14ax-2)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{5x^2a^3(ax-1)}-\frac{9\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$

input
```
int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(10*a^(7/2)
*x^3*((a*x+1)*x)^(1/2)-184*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-45*ln(1/2*(2*((a*
x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+32*a^(3/2)*x*((a*x+1)*x)^(
1/2)-4*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

3.463.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.88

$$\int e^{-\coth^{-1}(ax)}\Big(c-\frac{c}{ax}\Big)^{7/2}\,dx=\Bigg[\frac{45\,(a^3c^3x^3-a^2c^3x^2)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(5\,a^4c^3x}{20\,(a^4x^3-a^3x^2)}$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/20*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 76
*a^2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/10*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt
(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(5*a^4*c^3*x^4 - 87*a^3*c^3*
x^3 - 76*a^2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

3.463.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.463.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.463.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\left(c-\frac{c}{a\,x}\right)^{7/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input

```
int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.464 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.464.1 Optimal result

Integrand size = 24, antiderivative size = 161

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

output

```
-7*(c-c/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(5/2)-1/3*(16*a+1/
x)*(c-c/a/x)^(5/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)+(a-1/x)^2*(c-c/a/x)
^(5/2)*x*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)
```

3.464.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2-22ax+3a^2x^2)-21ax\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}x}$$

input
```
Integrate[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 - 22*a*x + 3*a^2*x^2) - 21*a*
x*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

3.464.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6731, 585, 27, 109, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3x^2}{a^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-\int \frac{\left(a-\frac{1}{x}\right)\left(7a+\frac{1}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(a-\frac{1}{x}\right)\left(7a+\frac{1}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{164}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{2}{3}a\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}-7a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-ax\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{2}{3}a\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}-14a^3\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}\right)-ax\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{c^2\left(\frac{1}{2}\left(14a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{3}a\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\right)-ax\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{a^3\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

output

```
-((c^2*Sqrt[c - c/(a*x)]*(-(a*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*x) + ((2*a*
(16*a + x^(-1))*Sqrt[1 + 1/(a*x)])/3 + 14*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]])/
2))/(a^3*Sqrt[1 - 1/(a*x)]))
```

3.464.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.464. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.464. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.464.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^{2}\left(6a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}-44a^{\frac{3}{2}}x\sqrt{(ax+1)x}-21\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}x^{2}+4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{6x\,a^{\frac{5}{2}}\,(ax-1)\sqrt{(ax+1)x}}$	144
risch	$\frac{(3a^{3}x^{3}-19a^{2}x^{2}-20ax+2)c^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^{2}(ax-1)}-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,(ax-1)}$	168

input
```
int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x
^2*((a*x+1)*x)^(1/2)-44*a^(3/2)*x*((a*x+1)*x)^(1/2)-21*ln(1/2*(2*((a*x+1)*
x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2+4*((a*x+1)*x)^(1/2)*a^(1/2))/x/
a^(5/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.37

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{21\,(a^{2}c^{2}x^{2}-ac^{2}x)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^{3}c^{2}x^{3}}{12\,(a^{3}x^{2}-a^{2}x)}\right.$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*
(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c
^2*x + 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2
- a^2*x), 1/6*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 2*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*x + 2*c^2)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

3.464.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.464.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.464.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int \left(c-\frac{c}{a\,x}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input

```
int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.465 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

3.465.1 Optimal result

Integrand size = 24, antiderivative size = 140

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=-\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

output

```
-5*(c-c/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(3/2)-2*(c-c/a/x)^
(3/2)*(1+1/a/x)^(1/2)/a/(1-1/a/x)^(3/2)+(c-c/a/x)^(3/2)*x*(1+1/a/x)^(1/2)/
(1-1/a/x)^(3/2)
```

3.465.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+ax)-5\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

input

```
Integrate[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

output

```
(c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 5*ArcTanh[Sqrt[1 + 1/
(a*x)]]))/(a*Sqrt[1 - 1/(a*x)])
```

3.465.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6731, 585, 27, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(5a-\frac{2}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a^2x\sqrt{\frac{1}{ax}+1}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(a^2x\left(-\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\int \frac{\left(5a-\frac{2}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{90}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(4a\sqrt{\frac{1}{ax}+1}-5a\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-a^2x\sqrt{\frac{1}{ax}+1}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(4a\sqrt{\frac{1}{ax}+1}-10a^2\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}\right)-a^2x\sqrt{\frac{1}{ax}+1}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(\frac{1}{2}\left(10a\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)+4a\sqrt{\frac{1}{ax}+1}\right)-a^2x\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

output

```
-((c*Sqrt[c - c/(a*x)]*(-(a^2*Sqrt[1 + 1/(a*x)]*x) + (4*a*Sqrt[1 + 1/(a*x)
] + 10*a*ArcTanh[Sqrt[1 + 1/(a*x)]])/2))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.465.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.465.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+4\sqrt{(ax+1)x}\,\sqrt{a}\right)}{2a^{\frac{3}{2}}(ax-1)\sqrt{(ax+1)x}}$	118
risch	$\frac{(a^2x^2-ax-2)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	151

input
```
int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-2*a
^(3/2)*x*((a*x+1)*x)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*x+4*((a*x+1)*x)^(1/2)*a^(1/2))/(a*x-1)/((a*x+1)*x)^(1/2)
```

3.465.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.25

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{5\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2-acx-2}{4\,(a^2x-a)}\right.$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(5*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(5*(a*c*x - c)*sqrt(-c)*arctan(2*(
a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/
(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

3.465.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.465.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int \left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.465.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.465.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int \left(c-\frac{c}{a\,x}\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.466 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.466.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1
/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.466.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x-\frac{3\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
))/Sqrt[1 - 1/(a*x)]
```

3.466.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6731, 580, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 580}$$

$$-\frac{-\frac{3c\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 573}$$

$$-\frac{\frac{3c^2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

output
```
-((-((c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (3*c^(3/2)*ArcTanh
[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a)/c)
```

3.466. $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.466.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.466.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$	101
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	139

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)
^(1/2)*a^(1/2)-3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a
*x-1)/((a*x+1)*x)^(1/2)/a^(1/2)
```

3.466.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\right.$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a)]
```

3.466.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

3.466.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.466.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.467 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.467.1 Optimal result

Integrand size $=24$, antiderivative size $=78$

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

output
```
-arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)+x*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.467.2 Mathematica [A] (verified)

Time $=0.05$ (sec) , antiderivative size $=65$, normalized size of antiderivative $=0.83$

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x-\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - ArcTanh[Sqrt[1 + 1/(a*x)]]/a))/S
qrt[c - c/(a*x)]
```

3.467.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{6731, 579, 573, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \; 6731$$

$$-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 579$$

$$-\frac{-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \; 573$$

$$-\frac{\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

output
```
-((-((c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sq
rt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a)/c)
```

3.467.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.467.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.31

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{a}\,c(ax-1)\sqrt{(ax+1)x}}$	102
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{\frac{c(ax-1)}{ax}}}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{2a\sqrt{a^{2}c}\,\sqrt{\frac{c(ax-1)}{ax}}\,x}$	133

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c*(-2
*((a*x+1)*x)^(1/2)*a^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2)))/(a*x-1)/((a*x+1)*x)^(1/2)
```

3.467.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.83

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2cx-ac\right)},\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^2*c*x - a*c), 1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c*x - a*c)]
```

3.467.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))), x)
```

3.467.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

3.467.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2), x)
```

3.468 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.468.1 Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

output
```
(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-(1-1/a/x)^(3/2)
*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)+(1-1/a/x)^
(3/2)*x*(1+1/a/x)^(1/2)/(c-c/a/x)^(3/2)
```

3.468.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\left(a\sqrt{1+\frac{1}{ax}}\,x+\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

output
```
((1 - 1/(a*x))^(3/2)*(a*Sqrt[1 + 1/(a*x)]*x + ArcTanh[Sqrt[1 + 1/(a*x)]] -
 Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*(c - c/(a*x))^(3/2))
```

3.468.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6731, 585, 27, 114, 27, 35, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{ax^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(a+\frac{1}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(a+\frac{1}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 35$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\sqrt{1+\frac{1}{ax}}x}{a-\frac{1}{x}}d\frac{1}{x}}{2a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 94$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}+\frac{\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}}{2a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{4\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+2\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 221$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a}-\frac{2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}}{2a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

output
```
-((a*Sqrt[1 - 1/(a*x)]*(-((Sqrt[1 + 1/(a*x)]*x)/a) + ((-2*ArcTanh[Sqrt[1 +
 1/(a*x)]])/a + (2*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a)/(2*a)))/
(c*Sqrt[c - c/(a*x)]))
```

3.468. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.468.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

3.468. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.468.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}\right)}{2a^{\frac{3}{2}}c^{2}(ax-1)\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{2a^{3}\sqrt{c}}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(2
*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1
)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/(a*x-1)/((a*x+1)*x)^(1/2)/(1/a)^(1
/2)
```

3.468.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.46

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+ax)\sqrt{}}{4\,(a^{2}c^{2}x-}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)) + sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sq
rt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*
c^2*x - a*c^2), 1/2*(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*
x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3
*a^2*x^2 - 2*a*x - 1)) - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(
-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x
 - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*c^2*x - a*c^2)]
```

3.468.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

3.468.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

3.468.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

3.468.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2), x)
```

3.469 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.469.1 Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=-\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{2\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

output
```
3*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-9/4*(1-1/a/x)
^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-3/2*
(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a*(1-1/a/x)^(5/2)*
x*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)
```

3.469.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.56

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-3+2ax)+12(-1+ax)\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-9\sqrt{2}(-1+ax\right.}{4ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-3 + 2*a*x) + 12*(-1 + a*x)*A
rcTanh[Sqrt[1 + 1/(a*x)]] - 9*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]
/Sqrt[2]]))/(4*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

3.469.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6731, 585, 27, 114, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{3\left(2a+\frac{1}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\int \frac{\left(2a+\frac{1}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(-\frac{\int -\frac{\left(2a+\frac{1}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{\int \frac{\left(2a+\frac{1}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{3\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{6a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+4a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 221$$

3.469. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-4\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

output
```
-((a^2*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(2*a*(a - x^(-1))) + (3*(-
2*Sqrt[1 + 1/(a*x)]*x + (-4*ArcTanh[Sqrt[1 + 1/(a*x)]] + 3*Sqrt[2]*ArcTanh
[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/(4*a^2)))/(c^2*Sqrt[c - c/(a*x)]))
```

3.469.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.469. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.469. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.469.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigl(8\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-9a^{\frac{3}{2}}\sqrt{2}\,\ln\Bigl(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\Bigr)x-12\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+12\ln\Bigl(\frac{2\sqrt{(a}{}}{8a^{\frac{3}{2}}c^{3}(ax-1)^{2}\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{2}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\Biggl(\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{3}\sqrt{a^{2}c}}-\frac{9\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{8a^{4}\sqrt{c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+}}{2a^{5}c(}}{c^{2}x\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(8*((a*x+1)*x)
^(1/2)*a^(5/2)*(1/a)^(1/2)*x-9*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(
(a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-12*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^
(1/2)+12*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(
1/2)*x-12*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1
/2)+9*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-
1))*a^(1/2))/a^(3/2)/c^3/(a*x-1)^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

3.469.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.72

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \Biggl[\frac{9\sqrt{2}(a^{2}x^{2}-2\,ax+1)\sqrt{c}\log\Bigl(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-}{ax}}}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a
*x - 1)) + 12*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x +
4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*
c^3), 1/8*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a
^2*c*x^2 - 2*a*c*x - c)) - 12*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*
a^2*c*x^2 - a*c*x - c)) + 4*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

3.469.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

3.469.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(5/2), x)
```

3.469.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2), x)
```

3.470 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

3.470.1 Optimal result

Integrand size = 24, antiderivative size = 277

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{115\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{16\sqrt{2}a\left(c-\frac{c}{ax}\right)^{7/2}}$$

output

```
5*(1-1/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(7/2)-115/32*(1-1/a
/x)^(7/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(7/2)*2^(1/2)-5
/4*a*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(7/2)-35/16*(1-1/
a/x)^(7/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(7/2)+a^2*(1-1/a/x)^(7/2)*x*(
1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(7/2)
```

3.470.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(35-55ax+16a^2x^2)+160(-1+ax)^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-1\right.}{32ac^3\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(35 - 55*a*x + 16*a^2*x^2) + 1
60*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 115*Sqrt[2]*(-1 + a*x)^2*ArcT
anh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(32*a*c^3*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

3.470.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.63, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 585, 27, 114, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^3x^2}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 114$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}-\frac{\int -\frac{5\left(2a+\frac{1}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\int \frac{\left(2a+\frac{1}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(14a+\frac{9}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\int \frac{\left(14a+\frac{9}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{-\frac{\int -\frac{\left(16a+\frac{7}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

3.470. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

$$\Big\downarrow 25$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\frac{\int \frac{\left(16a+\frac{7}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 174$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\frac{23\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+16\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\frac{46a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+32a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 221$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\frac{23\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-32\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

3.470. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

output

```
-((a^3*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(4*a*(a - x^(-1))^2) + (5*
((3*Sqrt[1 + 1/(a*x)]*x)/(2*(a - x^(-1))) + (-14*Sqrt[1 + 1/(a*x)]*x + (-3
2*ArcTanh[Sqrt[1 + 1/(a*x)]] + 23*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2
]])/a)/(4*a)))/(8*a^2)))/(c^3*Sqrt[c - c/(a*x)]))
```

3.470.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.470. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.470.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{3}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{4}\sqrt{a^{2}c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^{7}c\left(x-\frac{1}{a}\right)^{2}}-\frac{23\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{16a^{6}c\left(x-\frac{1}{a}\right)}-\frac{115\sqrt{2}\,\ln\left(\frac{4c+}{}\right.}{}\right.}{c^{3}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(64\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^{2}-115a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{2}-220\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+16\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)+(5/2/a^4*ln(
(1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/4
/a^7/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-23/16/a^6/c/(x-
1/a)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c)^(1/2)-115/64/a^5/c^(1/2)*2^(1/2)*
ln((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c
)^(1/2))/(x-1/a)))/c^3*a^3*((a*x-1)/(a*x+1))^(1/2)/x/(c*(a*x-1)/a/x)^(1/2)
*((a*x+1)*a*c*x)^(1/2)
```

3.470.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.41

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \left[\frac{115\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}}{a^3x^3-3a^2x^2+3ax-1}\right.}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/128*(115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3
*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 -
3*a^2*x^2 + 3*a*x - 1)) + 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*lo
g(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((
a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(16*a^4*x^
4 - 39*a^3*x^3 - 20*a^2*x^2 + 35*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4), 1/64*(
115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(
3*a^2*c*x^2 - 2*a*c*x - c)) - 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-
c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(16*a^4*x^4 - 39*a^3*x^3 - 20*
a^2*x^2 + 35*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*
c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

3.470.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(7/2),x)
```

output
```
Timed out
```

3.470.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

3.470.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

3.470.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2), x)
```

3.471 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

3.471.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{11c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-5/3*c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c/a/x)^(7/2)*x-11*c^
(7/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+32*c^(7/2)*arctanh(1/2*(c-c/a/x)^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-21*c^3*(c-c/a/x)^(1/2)/a
```

3.471.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\,(-6+52ax-376a^2x^2+15a^3x^3)}{15a^3x^2}-\frac{11c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

input
```
Integrate[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(-6 + 52*a*x - 376*a^2*x^2 + 15*a^3*x^3))/(15*a^3*x
^2) - (11*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7
/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

3.471.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 171, 27, 171, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2} e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(11a+\frac{3}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}x}{2a\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{a}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow 27$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(11a+\frac{3}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}x}{a+\frac{1}{x}}d\frac{1}{x}}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow 171$$

$$-\frac{a\left(-\frac{c^2\left(\frac{2}{5}\int \frac{5c\left(11a-\frac{5}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x}{2\left(a+\frac{1}{x}\right)}d\frac{1}{x}+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow 27$$

$$-\frac{a\left(-\frac{c^2\left(c\int \frac{\left(11a-\frac{5}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}d\frac{1}{x}+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow 171$$

$$-\frac{a\left(-\frac{c^2\left(c\left(\frac{2}{3}\int \frac{3c\left(11a-\frac{21}{x}\right)\sqrt{c-\frac{c}{ax}}x}{2\left(a+\frac{1}{x}\right)}d\frac{1}{x}-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow 27$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\int \frac{\left(11a-\frac{21}{x}\right)\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}d\frac{1}{x}-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow 171$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(2\int \frac{c\left(11a-\frac{53}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow 27$$

3.471. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(c\int \frac{\left(11a-\frac{53}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(c\left(11\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-64\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(c\left(\frac{128a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{22a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(c\left(\frac{64\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{22\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

input

```
Int[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output

```
-((a*(-((c*(c - c/(a*x))^(7/2)*x)/a) - (c^2*((6*(c - c/(a*x))^(5/2))/5 + c
*((-10*(c - c/(a*x))^(3/2))/3 + c*(-42*Sqrt[c - c/(a*x)] + c*((-22*ArcTanh
[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] + (64*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x
)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])))))/(2*a^2)))/c)
```

3.471. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

3.471.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.471. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.471.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.37

method	result
risch	$\frac{\left(15a^4x^4-391a^3x^3+428a^2x^2-58ax+6\right)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3(ax-1)}+\frac{\left(-\frac{11a^3\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{16a^2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}}{}\right.}{\sqrt{c}}\right.}{a^3(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(480\sqrt{(ax-1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4-1110a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,x^4-480a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)x^4+660a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,(a\,x^2\right.}{30x^3a}$

3.471. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

input
```
int((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/15*(15*a^4*x^4-391*a^3*x^3+428*a^2*x^2-58*a*x+6)/x^2*c^3/a^3/(a*x-1)*(c*
(a*x-1)/a/x)^(1/2)+(-11/2*a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x
^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-16*a^2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*
c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*c
^3/a^3/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.471.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\left[\frac{480\,\sqrt{2}a^2c^{\frac{7}{2}}x^2\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+165\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+\right.}{30\,a^3x^2}\right.$$

$$-\frac{480\,\sqrt{2}a^2\sqrt{-c}c^3x^2\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-165\,a^2\sqrt{-c}c^3x^2\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(15\,a^3c^3x^3-376\,a^2c^3}{15\,a^3x^2}$$

input
```
integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/30*(480*sqrt(2)*a^2*c^(7/2)*x^2*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 165*a^2*c^(7/2)*x^2*log(-2*a*c*x
+ 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(15*a^3*c^3*x^3 - 376*a^2
*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), -1/15*(
480*sqrt(2)*a^2*sqrt(-c)*c^3*x^2*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
 c)/(a*x))/c) - 165*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt((a*c*x - c)/
(a*x))/c) - (15*a^3*c^3*x^3 - 376*a^2*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((
a*c*x - c)/(a*x)))/(a^3*x^2)]
```

3.471.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(7/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)/(a*x + 1), x)
```

3.471.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)
```

3.471.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.471.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

3.472 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.472.1 Optimal result

Integrand size = 24, antiderivative size = 138

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{9c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-9*c^(5/2)*arctanh((c-c/a/x)^(1/2
)/c^(1/2))/a+16*c^(5/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/
2)/a-7*c^2*(c-c/a/x)^(1/2)/a
```

3.472.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{ax}}(2-26ax+3a^2x^2)-27ac^{5/2}x\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+48\sqrt{2}ac^{5/2}x\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{3a^2x}$$

input
```
Integrate[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(2 - 26*a*x + 3*a^2*x^2) - 27*a*c^(5/2)*x*ArcTanh[S
qrt[c - c/(a*x)]/Sqrt[c]] + 48*Sqrt[2]*a*c^(5/2)*x*ArcTanh[Sqrt[c - c/(a*x
)]/(Sqrt[2]*Sqrt[c])])/(3*a^2*x)
```

3.472.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 171, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(9a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x}{2a\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{a}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(9a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}d\frac{1}{x}}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 171$$

$$-\frac{a\left(-\frac{c^2\left(\frac{2}{3}\int \frac{3c\left(9a-\frac{7}{x}\right)\sqrt{c-\frac{c}{ax}}x}{2\left(a+\frac{1}{x}\right)}d\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(-\frac{c^2\left(c\int \frac{\left(9a-\frac{7}{x}\right)\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}d\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 171$$

$$-\frac{a\left(-\frac{c^2\left(c\left(2\int \frac{c\left(9a-\frac{23}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\int \frac{\left(9a-\frac{23}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(9\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-32\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \; 73$$

3.472. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(\frac{64a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{18a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{18\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

input
```
Int[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-((c*(c - c/(a*x))^(5/2)*x)/a) - (c^2*((2*(c - c/(a*x))^(3/2))/3 + c
*(-14*Sqrt[c - c/(a*x)] + c*((-18*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt
[c] + (32*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c]))
)/(2*a^2)))/c)
```

3.472.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.472. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.472. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2} dx$

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.472.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.54

method	result
risch	$\frac{(3a^3x^3-29a^2x^2+28ax-2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}+\frac{\left(-\frac{9a^2\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{8a\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3}}{x+\frac{1}{a}}\right.}{\sqrt{c}}\right.}{a^2(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(48\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3-90\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3+48a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+45\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^2x^3-48a\right.}{6x^2a^{\frac{5}{2}}\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*a^3*x^3-29*a^2*x^2+28*a*x-2)/x*c^2/a^2/(a*x-1)*(c*(a*x-1)/a/x)^(1/2
)+(-9/2*a^2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(
a^2*c)^(1/2)-8*a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(
a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*c^2/a^2/(a*x-1)*(c*(a*
x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.07

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{48\sqrt{2}ac^{\frac{5}{2}}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+27ac^{\frac{5}{2}}x\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+}{6a^2x}\right.$$

$$-\frac{48\sqrt{2}a\sqrt{-c}c^2x\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2c}\right)-27a\sqrt{-c}c^2x\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(3a^2c^2x^2-26ac^2x+2c^2)}{3a^2x}$$

input
```
integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/6*(48*sqrt(2)*a*c^(5/2)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/
(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 27*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(
c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)
*sqrt((a*c*x - c)/(a*x)))/(a^2*x), -1/3*(48*sqrt(2)*a*sqrt(-c)*c^2*x*arcta
n(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 27*a*sqrt(-c)*c^2*x*ar
ctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^2*c^2*x^2 - 26*a*c^2*x + 2
*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

3.472.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)/(a*x + 1), x)
```

3.472.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

3.472.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.472.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

3.473 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

3.473.1 Optimal result

Integrand size $= 24$, antiderivative size $= 113$

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
(c-c/a/x)^(3/2)*x-7*c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+8*c^(3/2)*a
rctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-c*(c-c/a/x)^(1/2)/a
```

3.473.2 Mathematica [A] (verified)

Time $= 0.09$ (sec) , antiderivative size $= 95$, normalized size of antiderivative $= 0.84$

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{ax}}(-2+ax)-7c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+8\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

input
```
Integrate[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(-2 + a*x) - 7*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt
[c]] + 8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

3.473.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(7a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x}{2a\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{a}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(7a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}d\frac{1}{x}}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$$\Big\downarrow 171$$

$$-\frac{a\left(-\frac{c^2\left(2\int \frac{c\left(7a-\frac{9}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$$\Big\downarrow 27$$

$$-\frac{a\left(-\frac{c^2\left(c\int \frac{\left(7a-\frac{9}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$$\Big\downarrow 174$$

$$-\frac{a\left(-\frac{c^2\left(c\left(7\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-16\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$$\Big\downarrow 73$$

$$-\frac{a\left(-\frac{c^2\left(c\left(\frac{32a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{14a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$$\Big\downarrow 221$$

$$-\frac{a\left(-\frac{c^2\left(c\left(\frac{16\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{14\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

input

```
Int[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output

```
-((a*(-((c*(c - c/(a*x))^(3/2)*x)/a) - (c^2*(-2*Sqrt[c - c/(a*x)] + c*((-1
4*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] + (16*Sqrt[2]*ArcTanh[Sqrt[c
 - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])))/(2*a^2)))/c)
```

3.473. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$

3.473.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.473. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.473.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(94) = 188$.

Time = 0.50 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.71

method	result
risch	$\frac{(a^2x^2-3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\Bigg(-\frac{7a\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{4\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{\sqrt{c}}}{a(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c\Bigg(8\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}\,a^{\frac{3}{2}}x^2-10\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}x^2+4(a\,x^2-x)^{\frac{3}{2}}\sqrt{a}\,\sqrt{\frac{1}{a}}+5\sqrt{\frac{1}{a}}\,\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^2-8\ln\Bigg(\frac{2\sqrt{2}}{}}{2x\,a^{\frac{3}{2}}\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}}$

3.473. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

input

```
int((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output

```
(a^2*x^2-3*a*x+2)*c/a*(c*(a*x-1)/a/x)^(1/2)/(a*x-1)+(-7/2*a*ln((-1/2*a*c+a
^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4*2^(1/2)/c^(
1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*
c+2*c)^(1/2))/(x+1/a)))*c/a*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)/(a
*x-1)
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{8\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+7\,c^{\frac{3}{2}}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-}{2\,a}\right.$$

$$\left.-\frac{8\sqrt{2}\sqrt{-c}c\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-7\sqrt{-c}c\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

input

```
integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output

```
[1/2*(8*sqrt(2)*c^(3/2)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)
) + 3*a*c*x - c)/(a*x + 1)) + 7*c^(3/2)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt(
(a*c*x - c)/(a*x)) + c) + 2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(8*
sqrt(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c)
- 7*sqrt(-c)*c*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*
sqrt((a*c*x - c)/(a*x)))/a]
```

3.473.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)/(a*x + 1), x)
```

3.473.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

3.473.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.473.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.474 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.474.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+4*arctanh(1/2*(c-c/a/x)^(1/2
)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

3.474.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

output
```
Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (
4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

3.474.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(5a-\frac{3}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(5a-\frac{3}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a\left(-\frac{c^2\left(5\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-8\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(-\frac{c^2\left(\frac{16a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{10a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(-\frac{c^2\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

input

```
Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

output

```
-((a*(-((c*Sqrt[c - c/(a*x)]*x)/a) - (c^2*((-10*ArcTanh[Sqrt[c - c/(a*x)]/
Sqrt[c]])/Sqrt[c] + (8*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
])/Sqrt[c]))/(2*a^2)))/c)
```

3.474. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.474.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.474. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.474.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(-\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}-}{2\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
x*(c*(a*x-1)/a/x)^(1/2)+(-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x
^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2
*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x
-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.474. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.474.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.38

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\sqrt{c}\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ ax\sqrt{\ }\right.$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*
sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*lo
g(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c
*x - c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*
x - c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/
a]
```

3.474.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.474.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

3.474.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.475 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.475.1 Optimal result

Integrand size $= 24$, antiderivative size $= 95$

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

output

```
-3*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)+2*arctanh(1/2*(c-c/a/x)^(1/2
)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)+x*(c-c/a/x)^(1/2)/c
```

3.475.2 Mathematica [A] (verified)

Time $= 0.06$ (sec) , antiderivative size $= 95$, normalized size of antiderivative $= 1.00$

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

input

```
Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

output

```
(Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c
]) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

3.475.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6717, 6683, 1035, 281, 899, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \ 6683$$

$$-\int \frac{1-ax}{\sqrt{c-\frac{c}{ax}}(ax+1)}dx$$

$$\Big\downarrow \ 1035$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \ 281$$

$$\frac{a\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \ 899$$

$$-\frac{a\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \ 110$$

$$-\frac{a\left(\frac{\int -\frac{c\left(3a-\frac{1}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(-\frac{c\int \frac{\left(3a-\frac{1}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(-\frac{c\left(3\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-4\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{2a^2}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(-\frac{c\left(\frac{8a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{6a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)}{2a^2}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(-\frac{c\left(\frac{4\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{6\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2a^2}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

input

```
Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

output

```
-((a*(-((Sqrt[c - c/(a*x)]*x)/a) - (c*((-6*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[
c]])/Sqrt[c] + (4*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sq
rt[c]))/(2*a^2)))/c)
```

3.475. $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.475.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.475. $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.475.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(2\sqrt{(ax-1)x}\, a^{\frac{3}{2}} \sqrt{\frac{1}{a}}-2\sqrt{2}\, \ln\left(\frac{2\sqrt{2}\, \sqrt{\frac{1}{a}}\, \sqrt{(ax-1)x}\, a-3ax+1}{ax+1}\right)\sqrt{a}-3\ln\left(\frac{2\sqrt{(ax-1)x}\, \sqrt{a}+2ax-1}{2\sqrt{a}}\right) a\sqrt{\frac{1}{a}}\right)}{2\sqrt{(ax-1)x}\, c\, a^{\frac{3}{2}} \sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a\sqrt{a^{2}c}}-\frac{\sqrt{2}\, \ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\, \sqrt{c}\, \sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\right)\sqrt{c(ax-1)ax}}{\sqrt{\frac{c(ax-1)}{ax}}\, x}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-2*2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/
2)-3*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2))/
((a*x-1)*x)^(1/2)/c/a^(3/2)/(1/a)^(1/2)
```

3.475. $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\, dx$

3.475.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.46

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}} + 2\,\sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\,\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right) + 3\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac},\right.$$
$$\left.-\frac{2\,\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}ax\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right) - ax\sqrt{\frac{acx-c}{ax}} - 3\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{ac}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 2*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*
x*sqrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + 3*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c), -(2*sqrt(2)*
c*sqrt(-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x -
1)) - a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x
 - c)/(a*x))/c))/(a*c)]
```

3.475.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(1/2),x)
```

output
```
Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)), x)
```

3.475.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

3.475.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.475.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{a\,x-1}{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)), x)
```

3.476 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.476.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

output

```
-arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(3/2)+arctanh(1/2*(c-c/a/x)^(1/2)*2^
(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)+x*(c-c/a/x)^(1/2)/c^2
```

3.476.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

input

```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

output

```
(Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
 + (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

3.476.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{3/2}(ax+1)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{x^2}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{2\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}d\frac{1}{x}}{2ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

$$\Big\downarrow 94$$

$$-\frac{a\left(-\frac{\frac{c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{2c\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}}{2ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

$$\Big\downarrow 73$$

$$-\frac{a\left(-\frac{4\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}-2\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{2ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

$$\Big\downarrow 221$$

$$-\frac{a\left(-\frac{\frac{2\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}}{2ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

output
```
-((a*(-((Sqrt[c - c/(a*x)]*x)/(a*c)) - ((-2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*
x)]/Sqrt[c]])/a + (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sq
rt[c])])/a)/(2*a*c)))/c)
```

3.476.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.476. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.476.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(2\sqrt{(ax-1)x}\, a^{\frac{3}{2}} \sqrt{\frac{1}{a}}-\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a\sqrt{\frac{1}{a}}-\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}\right)}{2a^{\frac{3}{2}}\sqrt{(ax-1)x}\,c^{2}\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{2a^{3}\sqrt{c}}\right)a\sqrt{c(ax-1)ax}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/
2)-ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/
2))/((a*x-1)*x)^(1/2)/c^2/(1/a)^(1/2)
```

3.476.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.46

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+\sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\,\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right)+\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}\right.}{2\,ac^{2}}\right.$$

$$\left.-\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}ax\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right)-ax\sqrt{\frac{acx-c}{ax}}-\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{ac^{2}}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x*
sqrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + sqrt(c)*log(-2*a
*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c^2), -(sqrt(2)*c*sq
rt(-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1))
- a*x*sqrt((a*c*x - c)/(a*x)) - sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/
(a*x))/c))/(a*c^2)]
```

3.476.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(3/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

3.476.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

3.476.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)), x)
```

3.477 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.477.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

output
```
arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)+1/2*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)-2/a/c^2/(c-c/a/x)^(1/2)+x/c^2/(c-c/a/x
)^(1/2)
```

3.477.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{ax-\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right)-\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)}{ac^2\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

output
```
(a*x - Hypergeometric2F1[-1/2, 1, 1/2, (a - x^(-1))/(2*a)] - Hypergeometri
c2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*c^2*Sqrt[c - c/(a*x)])
```

3.477.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 169, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{5/2}(ax+1)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{x^2}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a\left(-\frac{\int -\frac{c\left(a+\frac{3}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{ac}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(\frac{\int \frac{\left(a+\frac{3}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \ 169$$

$$-\frac{a\left(\frac{\frac{4}{c\sqrt{c-\frac{c}{ax}}}-\frac{\int -\frac{c\left(a+\frac{2}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^2}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \ 25$$

$$-\frac{a\left(\frac{\frac{\int \frac{c\left(a+\frac{2}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^2}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(\frac{\frac{\int \frac{\left(a+\frac{2}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \ 174$$

$$-\frac{a\left(\frac{\frac{\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \ 73$$

$$-\frac{a\left(\frac{\frac{-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{2a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{c}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

3.477. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

$$\Big\downarrow 221$$

$$-\frac{a\left(\frac{\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}}{c}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

output
```
-((a*(-(x/(a*c*Sqrt[c - c/(a*x)])) + (4/(c*Sqrt[c - c/(a*x)]) + ((-2*ArcTa
nh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)
]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])/c)/(2*a^2)))/c)
```

3.477.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.477. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.477. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.477.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(98) = 196$.

Time = 0.50 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\Bigg(\frac{\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^3\sqrt{a^2c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{4a^4\sqrt{c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+(}}{a^5c\left(x-\frac{1}{a}\right.}}{c^2x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigg(-8\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}\,a^{\frac{7}{2}}x^2+4((ax-1)x)^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,a^{\frac{5}{2}}+\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)a^{\frac{5}{2}}x^2-2\sqrt{\frac{1}{a}}\,\ln\Bigg(\frac{2\sqrt{(ax-1)x}}{2\sqrt{}}}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/(c*(a*x-1)/a/x)^(1/2)+(1/2/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*
c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-1/4/a^4*2^(1/2)/c^(1/2)*ln
((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^
(1/2))/(x+1/a))-1/a^5/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2))*a^2/c
^2/x/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.477.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.47

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\Bigg[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\,(ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\right.}{4\,(a^2c^3x-ac^3)}$$

$$-\frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+2\,(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-2\,(a^2x^2-2\,ax)\sqrt{\frac{acx-c}{ax}}}{2\,(a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 2*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2
*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(a^2*x^2 - 2*a*x)*sqrt((a*c*
x - c)/(a*x)))/(a^2*c^3*x - a*c^3), -1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arcta
n(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*(a*x - 1)*sqrt(-c)*a
rctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(a^2*x^2 - 2*a*x)*sqrt((a*c*
x - c)/(a*x)))/(a^2*c^3*x - a*c^3)]
```

3.477.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(5/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

3.477.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

3.477.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.477.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)), x)
```

3.478 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

3.478.1 Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

output
```
-4/3/a/c^2/(c-c/a/x)^(3/2)+x/c^2/(c-c/a/x)^(3/2)+3*arctanh((c-c/a/x)^(1/2)
/c^(1/2))/a/c^(7/2)+1/4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a/c^(
7/2)*2^(1/2)-7/2/a/c^3/(c-c/a/x)^(1/2)
```

3.478.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.54

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{x\Big(3ax-\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right)-3\,\text{Hypergeometric2F1}\big(-\frac{3}{2},1,-\frac{1}{2},1}{3c^3\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

output
```
(x*(3*a*x - Hypergeometric2F1[-3/2, 1, -1/2, (a - x^(-1))/(2*a)] - 3*Hyper
geometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c^3*Sqrt[c - c/(a*x)]*(-1 +
a*x))
```

3.478.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{7/2}(ax+1)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{x^2}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a\left(-\frac{\int -\frac{c\left(3a+\frac{5}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{ac}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow 27$$

$$-\frac{a\left(\frac{\int \frac{\left(3a+\frac{5}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow 169$$

$$-\frac{a\left(\frac{\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\int -\frac{3c\left(3a+\frac{4}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{3c^2}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow 27$$

$$-\frac{a\left(\frac{\frac{\int \frac{\left(3a+\frac{4}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow 169$$

$$-\frac{a\left(\frac{\frac{\frac{7}{c\sqrt{c-\frac{c}{ax}}}-\frac{\int -\frac{c\left(6a+\frac{7}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^2}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow 27$$

3.478. $\displaystyle\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$

$$-\frac{a\left(\frac{\frac{\frac{\int \frac{\left(6a+\frac{7}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2c}+\frac{7}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac(c-\frac{c}{ax})^{3/2}}\right)}{c}$$

$$\Big\downarrow \ 174$$

$$-\frac{a\left(\frac{\frac{\frac{\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+6\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2c}+\frac{7}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac(c-\frac{c}{ax})^{3/2}}\right)}{c}$$

$$\Big\downarrow \ 73$$

$$-\frac{a\left(\frac{\frac{\frac{-\frac{12a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{2a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2c}+\frac{7}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac(c-\frac{c}{ax})^{3/2}}\right)}{c}$$

$$\Big\downarrow \ 221$$

$$-\frac{a\left(\frac{\frac{\frac{-\frac{12\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}}{2c}+\frac{7}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac(c-\frac{c}{ax})^{3/2}}\right)}{c}$$

input

```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

3.478. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

output
```
-((a*(-(x/(a*c*(c - c/(a*x))^(3/2))) + (8/(3*c*(c - c/(a*x))^(3/2)) + (7/(
c*Sqrt[c - c/(a*x)]) + ((-12*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] -
 (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])/(2*c))/c)
/(2*a^2)))/c)
```

3.478.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

3.478. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.478. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

3.478.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(120) = 240$.

Time = 0.52 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.84

method	result
risch	$\frac{ax-1}{a\,c^{3}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a^{4}\sqrt{a^{2}c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(x-\frac{1}{a}\right)ac}}{3a^{7}c\left(x-\frac{1}{a}\right)^{2}}-\frac{17\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(x-\frac{1}{a}\right)ac}}{6a^{6}c\left(x-\frac{1}{a}\right)}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+}{}\right.}{}\right.}{c^{3}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-84\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a^{\frac{9}{2}}x^{3}+60\sqrt{\frac{1}{a}}\,((ax-1)x)^{\frac{3}{2}}a^{\frac{7}{2}}x-36\sqrt{\frac{1}{a}}\,\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^{4}x^{3}+3\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1}{ax+1}\right.\right.}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/(c*(a*x-1)/a/x)^(1/2)+(3/2/a^4*ln((-1/2*a*c+a^2*c*x)/(a^2*
c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-1/3/a^7/c/(x-1/a)^2*(a^2*c
*(x-1/a)^2+(x-1/a)*a*c)^(1/2)-17/6/a^6/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*
a*c)^(1/2)-1/8/a^5*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)
*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/c^3*a^3/x/(c*(a*x-1)
/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.478.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.44

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\left[\frac{3\sqrt{2}(a^{2}x^{2}-2\,ax+1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+36\,(a^{2}x^{2}-2\,ax+1)\sqrt{c}\,\mathrm{lo}}{24\,(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x}\right.$$

$$-\frac{3\sqrt{2}(a^{2}x^{2}-2\,ax+1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+36\,(a^{2}x^{2}-2\,ax+1)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-2\,(}{12\,(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4})}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x
*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 36*(a^2*x^2 - 2*a*x +
 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*
(6*a^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 -
2*a^2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arct
an(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 36*(a^2*x^2 - 2*a*x +
 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(6*a^3*x^3 - 2
9*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x +
a*c^4)]
```

3.478.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(7/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

3.478.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

3.478.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.478.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)), x)
```

3.479 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

3.479.1 Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

output
```
-6/5/a/c^2/(c-c/a/x)^(5/2)-11/6/a/c^3/(c-c/a/x)^(3/2)+x/c^2/(c-c/a/x)^(5/2
)+5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(9/2)+1/8*arctanh(1/2*(c-c/a/x)^(
1/2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)-21/4/a/c^4/(c-c/a/x)^(1/2)
```

3.479.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.48

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\frac{ax^2\left(5ax-\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{a-\frac{1}{x}}{2a}\right)-5\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2}\right.\right.}{5c^4\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

output
```
(a*x^2*(5*a*x - Hypergeometric2F1[-5/2, 1, -3/2, (a - x^(-1))/(2*a)] - 5*H
ypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]))/(5*c^4*Sqrt[c - c/(a*x)]*(-
1 + a*x)^2)
```

3.479.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 169, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{9/2}(ax+1)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{9/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{x^2}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a\left(-\frac{\int -\frac{c\left(5a+\frac{7}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{ac}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(\frac{\int \frac{\left(5a+\frac{7}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \ 169$$

$$-\frac{a\left(\frac{\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\int -\frac{5c\left(5a+\frac{6}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{5c^2}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(\frac{\frac{\int \frac{\left(5a+\frac{6}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \ 169$$

$$-\frac{a\left(\frac{\frac{\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\int -\frac{3c\left(10a+\frac{11}{x}\right)x}{2\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{3c^2}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \ 27$$

3.479. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

$$-\frac{a\left(\frac{\frac{\frac{\int \frac{\left(10a+\frac{11}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \; 169$$

$$-\frac{a\left(\frac{\frac{\frac{\frac{21}{c\sqrt{c-\frac{c}{ax}}}-\frac{\int -\frac{c\left(20a+\frac{21}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^2}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{\frac{\frac{\frac{\int \frac{\left(20a+\frac{21}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2c}+\frac{21}{c\sqrt{c-\frac{c}{ax}}}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(\frac{\frac{\frac{\frac{\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+20\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2c}+\frac{21}{c\sqrt{c-\frac{c}{ax}}}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \; 73$$

3.479. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

$$-\frac{a\left(\frac{\frac{\frac{-\frac{40a\int\frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{2a\int\frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2c}+\frac{21}{c\sqrt{c-\frac{c}{ax}}}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow 221$$

$$-\frac{a\left(\frac{\frac{\frac{-\frac{40\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}}{2c}+\frac{21}{c\sqrt{c-\frac{c}{ax}}}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

output
```
-((a*(-(x/(a*c*(c - c/(a*x))^(5/2))) + (12/(5*c*(c - c/(a*x))^(5/2)) + (11
/(3*c*(c - c/(a*x))^(3/2)) + (21/(c*Sqrt[c - c/(a*x)]) + ((-40*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqr
t[2]*Sqrt[c])])/Sqrt[c])/(2*c))/(2*c))/c)/(2*a^2)))/c)
```

3.479. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

3.479.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.479. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.479.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(141) = 282.

Time = 0.49 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.83

method	result
risch	$\frac{ax-1}{a\,c^{4}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\Bigg(\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a^{5}\sqrt{a^{2}c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(x-\frac{1}{a}\right)ac}}{5a^{9}c\left(x-\frac{1}{a}\right)^{3}}-\frac{37\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(x-\frac{1}{a}\right)ac}}{30a^{8}c\left(x-\frac{1}{a}\right)^{2}}-\frac{317\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+\left(x-\right.}}{60a^{7}c\left(x-\frac{1}{a}\right)}}{c^{4}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigg(-1260\sqrt{(ax-1)x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^{4}+1020((ax-1)x)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^{2}-600\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^{5}x^{4}+15a^{\frac{9}{2}}\sqrt{2}\,\ln\Bigg(}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^4/(c*(a*x-1)/a/x)^(1/2)+(5/2/a^5*ln((-1/2*a*c+a^2*c*x)/(a^2*
c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-1/5/a^9/c/(x-1/a)^3*(a^2*c
*(x-1/a)^2+(x-1/a)*a*c)^(1/2)-37/30/a^8/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+(x-1/
a)*a*c)^(1/2)-317/60/a^7/c/(x-1/a)*(a^2*c*(x-1/a)^2+(x-1/a)*a*c)^(1/2)-1/1
6/a^6*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/
a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*a^4/c^4/x/(c*(a*x-1)/a/x)^(1/2)*(
c*(a*x-1)*a*x)^(1/2)
```

3.479.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.51

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \left[\frac{15\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+600\,(a^3x^3-3\,a}{240\,(a^4c^5}\right.$$

$$-\frac{15\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+600\,(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\,\mathrm{arcta}}{120\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="fricas")
```

output
```
[1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(2*sqrt(
2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 600*(a^
3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((
a*c*x - c)/(a*x)) + c) + 4*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a
*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x -
a*c^5), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arct
an(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 600*(a^3*x^3 - 3*a^2*
x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(
60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*sqrt((a*c*x - c)/(a*x)))
/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

3.479.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{9}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(9/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(9/2)*(a*x + 1)), x)
```

3.479.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(9/2)), x)
```

3.479.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.479.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{9/2}\,\left(a\,x+1\right)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)), x)
```

3.480 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

3.480.1 Optimal result

Integrand size = 24, antiderivative size = 335

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{15\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

output
```
-15*(c-c/a/x)^(9/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(9/2)+10*(a-1/x)^
4*(c-c/a/x)^(9/2)/a^5/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)+(a-1/x)^5*(c-c/a/x)^
(9/2)*x/a^5/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)+5/7*(304*a-65/x)*(c-c/a/x)^(9/
2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(9/2)+135/7*(a-1/x)^2*(c-c/a/x)^(9/2)*(1+
1/a/x)^(1/2)/a^3/(1-1/a/x)^(9/2)+65/7*(a-1/x)^3*(c-c/a/x)^(9/2)*(1+1/a/x)^
(1/2)/a^4/(1-1/a/x)^(9/2)
```

3.480.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{c^4\sqrt{c-\frac{c}{ax}}\,(-2+20ax-110a^2x^2+720a^3x^3+1685a^4x^4+7a^5x^5-35a^4\sqrt{1+\frac{1}{ax}}\,x^4\text{arctanh}}{7a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

input
```
Integrate[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*Sqrt[c - c/(a*x)]*(-2 + 20*a*x - 110*a^2*x^2 + 720*a^3*x^3 + 1685*a^4
*x^4 + 7*a^5*x^5 - 35*a^4*Sqrt[1 + 1/(a*x)]*x^4*ArcTanh[Sqrt[1 + 1/(a*x)]]
 + 70*a^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(7*a^5*Sqrt[1
 - 1/(a^2*x^2)]*x^4)
```

3.480.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.61, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6731, 585, 27, 109, 27, 167, 27, 170, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{9/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{15/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^6x^2}{a^6\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^6 x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 109$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-\int \frac{5\left(a-\frac{1}{x}\right)^4\left(3a+\frac{1}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^5}{\sqrt{\frac{1}{ax}+1}}\right)}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-\frac{5}{2}\int \frac{\left(a-\frac{1}{x}\right)^4\left(3a+\frac{1}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^5}{\sqrt{\frac{1}{ax}+1}}\right)}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 167$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-\frac{5}{2}\left(\frac{4a\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}-2a\int -\frac{\left(a-\frac{1}{x}\right)^3\left(3a+\frac{13}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{ax\left(a-\frac{1}{x}\right)^5}{\sqrt{\frac{1}{ax}+1}}\right)}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-\frac{5}{2}\left(a\int \frac{\left(a-\frac{1}{x}\right)^3\left(3a+\frac{13}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4a\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^5}{\sqrt{\frac{1}{ax}+1}}\right)}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 170$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-\frac{5}{2}\left(a\left(\frac{2}{7}a\int \frac{3\left(a-\frac{1}{x}\right)^2\left(7a+\frac{45}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{26}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)+\frac{4a\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^5}{\sqrt{\frac{1}{ax}+1}}\right)}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-\frac{5}{2}\left(a\left(\frac{3}{7}a\int \frac{\left(a-\frac{1}{x}\right)^2\left(7a+\frac{45}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{26}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)+\frac{4a\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^5}{\sqrt{\frac{1}{ax}+1}}\right)}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 170$$

3.480. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\Bigg(-\frac{5}{2}\Bigg(a\Bigg(\frac{3}{7}a\Bigg(\frac{2}{5}a\int\frac{5\left(a-\frac{1}{x}\right)\left(7a+\frac{65}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+18a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\Bigg)+\frac{26}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\Bigg)+\frac{4a\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\Bigg(-\frac{5}{2}\Bigg(a\Bigg(\frac{3}{7}a\Bigg(a\int\frac{\left(a-\frac{1}{x}\right)\left(7a+\frac{65}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+18a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\Bigg)+\frac{26}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\Bigg)+\frac{4a\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\Bigg)}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{164}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\Bigg(-\frac{5}{2}\Bigg(a\Bigg(\frac{3}{7}a\Bigg(a\Bigg(7a^2\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\left(304a-\frac{65}{x}\right)\sqrt{\frac{1}{ax}+1}\Bigg)+18a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\Bigg)+\frac{26}{7}a\sqrt{\frac{1}{ax}}}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{73}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\Bigg(-\frac{5}{2}\Bigg(a\left(\frac{3}{7}a\left(a\left(14a^3\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\left(304a-\frac{65}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+18a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{26}{7}a\sqrt{}}{a^6\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \color{blue}{221}$$

$$-\frac{c^4\Bigg(-\frac{5}{2}\Bigg(a\left(\frac{3}{7}a\left(a\left(\frac{2}{3}a\left(304a-\frac{65}{x}\right)\sqrt{\frac{1}{ax}+1}-14a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+18a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{26}{7}a\sqrt{\frac{1}{ax}+1}}{a^6\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output

```
-((c^4*Sqrt[c - c/(a*x)]*(-((a*(a - x^(-1))^5*x)/Sqrt[1 + 1/(a*x)]) - (5*(
(4*a*(a - x^(-1))^4)/Sqrt[1 + 1/(a*x)] + a*((26*a*(a - x^(-1))^3*Sqrt[1 +
1/(a*x)])/7 + (3*a*(18*a*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)] + a*((2*a*(304*a
 - 65/x)*Sqrt[1 + 1/(a*x)])/3 - 14*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]])))/7)))/
2))/(a^6*Sqrt[1 - 1/(a*x)]))
```

3.480. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

3.480.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

3.480. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.480.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.68

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(14\sqrt{(ax+1)x}\,a^{\frac{11}{2}}x^5+3510a^{\frac{9}{2}}\sqrt{(ax+1)x}\,x^4+1440a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-105\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right.}{14(ax-1)^2x^3a^{\frac{9}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{\left(7a^5x^5+859a^4x^4+720a^3x^3-110a^2x^2+20ax-2\right)c^4\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{7x^3a^4(ax-1)}+\frac{\left(-\frac{15a^4\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{128a^2\sqrt{a^2c\left(x+\frac{1}{a}\right)}}{c\left(x+\frac{1}{a}\right)}\right.}{a^4(ax-1)}$

input
```
int((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/14*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^4*(
14*((a*x+1)*x)^(1/2)*a^(11/2)*x^5+3510*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+1440*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)-105*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*
x+1)/a^(1/2))*a^5*x^5-105*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(
1/2))*a^4*x^4-220*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+40*a^(3/2)*x*((a*x+1)*x)^(
1/2)-4*((a*x+1)*x)^(1/2)*a^(1/2))/x^3/a^(9/2)/((a*x+1)*x)^(1/2)
```

3.480.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.30

$$\int e^{-3\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{9/2} dx = \left[\frac{105\,(a^4c^4x^4 - a^3c^4x^3)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right) + 4\,(7\,a^5c^4}{28\,(a^5x^4 - a^4x^3}\right.$$

input
```
integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/28*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 +
 720*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/14*(105*(a^4*c^4*
x^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x -
1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(7*a^
5*c^4*x^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^
4*x - 2*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 -
 a^4*x^3)]
```

3.480.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.480.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.480.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.480.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\left(c-\frac{c}{a\,x}\right)^{9/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.481 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

3.481.1 Optimal result

Integrand size = 24, antiderivative size = 277

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx &= \frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}\\ &+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}\\ &+\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{13\left(c-\frac{c}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}\end{aligned}$$

output

```
-13*(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)+10*(a-1/x)^
3*(c-c/a/x)^(7/2)/a^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+(a-1/x)^4*(c-c/a/x)^
(7/2)*x/a^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+1/15*(1360*a-311/x)*(c-c/a/x)^
(7/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(7/2)+47/5*(a-1/x)^2*(c-c/a/x)^(7/2)*(
1+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)
```

3.481.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}\Bigg(c-\frac{c}{ax}\Bigg)^{7/2}\,dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(6-62ax+548a^2x^2+1441a^3x^3+15a^4x^4-45a^3\sqrt{1+\frac{1}{ax}}x^3\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right.}{15a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

input
```
Integrate[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(6 - 62*a*x + 548*a^2*x^2 + 1441*a^3*x^3 + 15*a^4*x
^4 - 45*a^3*Sqrt[1 + 1/(a*x)]*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]] + 150*a^3*x^3
*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(15*a^4*Sqrt[1 - 1/(a^2*x^
2)]*x^3)
```

3.481.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.64, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 585, 27, 109, 27, 167, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{13/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^5x^2}{a^5\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^5 x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 109$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\int \frac{\left(a-\frac{1}{x}\right)^3\left(13a+\frac{3}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(a-\frac{1}{x}\right)^3\left(13a+\frac{3}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 167$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(2a\int -\frac{\left(a-\frac{1}{x}\right)^2\left(13a+\frac{47}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\int \frac{\left(a-\frac{1}{x}\right)^2\left(13a+\frac{47}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 170$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(\frac{2}{5}a\int \frac{\left(a-\frac{1}{x}\right)\left(65a+\frac{311}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(\frac{1}{5}a\int \frac{\left(a-\frac{1}{x}\right)\left(65a+\frac{311}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

3.481. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

$$\Big\downarrow \textcolor{blue}{164}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\Bigg(\frac{1}{2}\Bigg(-a\Bigg(\frac{1}{5}a\Bigg(65a^2\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\Bigg)-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\Bigg(\frac{1}{2}\Bigg(-a\Big(\frac{1}{5}a\Big(130a^3\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\Big)+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\Big)-\frac{20a(a}{}}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$-\frac{c^3\Bigg(\frac{1}{2}\Bigg(-a\Bigg(\frac{1}{5}a\Bigg(\frac{2}{3}a\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}-130a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\Bigg)\Bigg)+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\Bigg)-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\Bigg)}{a^5\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^3*Sqrt[c - c/(a*x)]*(-((a*(a - x^(-1))^4*x)/Sqrt[1 + 1/(a*x)]) + ((-2
0*a*(a - x^(-1))^3)/Sqrt[1 + 1/(a*x)] - a*((94*a*(a - x^(-1))^2*Sqrt[1 + 1
/(a*x)])/5 + (a*((2*a*(1360*a - 311/x)*Sqrt[1 + 1/(a*x)])/3 - 130*a^2*ArcT
anh[Sqrt[1 + 1/(a*x)]]))/5))/2))/(a^5*Sqrt[1 - 1/(a*x)]))
```

3.481.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.481. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.481. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.481.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\Bigg(30a^{\frac{9}{2}}\sqrt{(ax+1)x}\,x^4+3182a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-195\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^4x^4+1096a^{\frac{5}{2}}x^2\sqrt{(ax+}}{30(ax-1)^2x^2a^{\frac{7}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(15a^4x^4+631a^3x^3+548a^2x^2-62ax+6)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3(ax-1)}+\frac{\Bigg(-\frac{13a^3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{64a\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)a}}{c\left(x+\frac{1}{a}\right)}}{a^3(ax-1)}$

input
```
int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^3*(
30*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+3182*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-195*ln
(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4+1096*a^(5/2)*x
^2*((a*x+1)*x)^(1/2)-195*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1
/2))*a^3*x^3-124*a^(3/2)*x*((a*x+1)*x)^(1/2)+12*((a*x+1)*x)^(1/2)*a^(1/2))
/x^2/a^(7/2)/((a*x+1)*x)^(1/2)
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.50

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\left[\frac{195\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15a^4c\right.}{60\left(a^4x^3-a^3x^2\right)}\right.$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/60*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3
+ 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(195*(a^3*c^3*x^3 - a^2*c^3*x^2
)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 1591
*a^3*c^3*x^3 + 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x +
 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

3.481.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.481.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.481.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.481.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\left(c-\frac{c}{a\,x}\right)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

input
```
int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.482 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.482.1 Optimal result

Integrand size = 24, antiderivative size = 219

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx&=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}\\&+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}\\&-\frac{11\left(c-\frac{c}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}\end{aligned}$$

output
```
-11*(c-c/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(5/2)+10*(a-1/x)^
2*(c-c/a/x)^(5/2)/a^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+(a-1/x)^3*(c-c/a/x)^
(5/2)*x/a^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+1/3*(112*a-29/x)*(c-c/a/x)^(5/
2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)
```

3.482.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int e^{-3\coth^{-1}(ax)}\Big(c-\frac{c}{ax}\Big)^{5/2}dx = \frac{c^2\sqrt{c-\frac{c}{ax}}\Big(-2+32ax+103a^2x^2+3a^3x^3-3a^2\sqrt{1+\frac{1}{ax}}x^2\text{arctanh}\Big(\sqrt{1+\frac{1}{ax}}\Big)+30a^2x^2\,\text{Hy}}{3a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

input
```
Integrate[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(-2 + 32*a*x + 103*a^2*x^2 + 3*a^3*x^3 - 3*a^2*Sqrt
[1 + 1/(a*x)]*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]] + 30*a^2*x^2*Hypergeometric2F
1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(3*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

3.482.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.66, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6731, 585, 27, 109, 27, 167, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{11/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{a^4\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 109$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-\int \frac{\left(a-\frac{1}{x}\right)^2\left(11a+\frac{1}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(a-\frac{1}{x}\right)^2\left(11a+\frac{1}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 167$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(2a\int -\frac{\left(a-\frac{1}{x}\right)\left(11a+\frac{29}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\int \frac{\left(a-\frac{1}{x}\right)\left(11a+\frac{29}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 164$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(11a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\right)-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(22a^3\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\right)-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 221$$

3.482. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$

$$-\frac{c^2\left(\frac{1}{2}\left(-a\left(\frac{2}{3}a\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}-22a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^4\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output

```
-((c^2*Sqrt[c - c/(a*x)]*(-((a*(a - x^(-1))^3*x)/Sqrt[1 + 1/(a*x)]) + ((-2
0*a*(a - x^(-1))^2)/Sqrt[1 + 1/(a*x)] - a*((2*a*(112*a - 29/x)*Sqrt[1 + 1/
(a*x)])/3 - 22*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/2))/(a^4*Sqrt[1 - 1/(a*x)]
))
```

3.482.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.482. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

3.482.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(6a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}+266a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-33\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3+64a^{\frac{3}{2}}x\sqrt{(ax+1)x}-3}{6(ax-1)^2x\,a^{\frac{5}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(3a^3x^3+37a^2x^2+32ax-2)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}+\frac{\left(-\frac{11a^2\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{32\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{c\left(x+\frac{1}{a}\right)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2(ax-1)}$

input
```
int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^2*(6
*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+266*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-33*ln(1/2
*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+64*a^(3/2)*x*((a*x
+1)*x)^(1/2)-33*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*
x^2-4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(5/2)/((a*x+1)*x)^(1/2)
```

3.482.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.74

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{33\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3\right.}{12\left(a^3x^2-a^2x\right)}\right.$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*
(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*
c^2*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2
 - a^2*x), 1/6*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
 - a*c*x - c)) + 2*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

3.482.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.482.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.482.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int \left(c-\frac{c}{a\,x}\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.483 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

3.483.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

output

```
-9*(c-c/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(3/2)+(21*a+1/x)*(
c-c/a/x)^(3/2)/a^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+(a-1/x)^2*(c-c/a/x)^(3/
2)*x/a^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)
```

3.483.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.45

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{ax}}\left(2+10ax+a^2x^2+9ax\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{1}{ax}\right)\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(2 + 10*a*x + a^2*x^2 + 9*a*x*Hypergeometric2F1[-1/2,
 1, 1/2, 1 + 1/(a*x)]))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.483.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6731, 585, 27, 109, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3x^2}{a^3\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(-\int \frac{\left(a-\frac{1}{x}\right)\left(9a-\frac{1}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(a-\frac{1}{x}\right)\left(9a-\frac{1}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 163$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-9a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2a\left(21a+\frac{1}{x}\right)}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-18a^3\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}-\frac{2a\left(21a+\frac{1}{x}\right)}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(\frac{1}{2}\left(18a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\frac{2a\left(21a+\frac{1}{x}\right)}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c*Sqrt[c - c/(a*x)]*(-((a*(a - x^(-1))^2*x)/Sqrt[1 + 1/(a*x)]) + ((-2*a
*(21*a + x^(-1)))/Sqrt[1 + 1/(a*x)] + 18*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]])/2
))/(a^3*Sqrt[1 - 1/(a*x)]))
```

3.483.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.483. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

3.483.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(2a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}+38a^{\frac{3}{2}}x\sqrt{(ax+1)x}-9\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}x^{2}-9\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+}{2\sqrt{a}}\right.\right.}{2(ax-1)^{2}a^{\frac{3}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(a^{2}x^{2}+3ax+2)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\left(-\frac{9a\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{16\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{ac\left(x+\frac{1}{a}\right)}\right)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(ax}}{a(ax-1)}$

input
```
int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c*(2*a
^(5/2)*x^2*((a*x+1)*x)^(1/2)+38*a^(3/2)*x*((a*x+1)*x)^(1/2)-9*ln(1/2*(2*((
a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2-9*ln(1/2*(2*((a*x+1)*x)^
(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+4*((a*x+1)*x)^(1/2)*a^(1/2))/a^(3/2)/(
(a*x+1)*x)^(1/2)
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.99

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{9\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}cx^{2}+19\,acx+}{4\,(a^{2}x-a)}\right.$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(9*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))
*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(9*(a*c*x - c)*sqrt(-c)*arctan(
2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

3.483.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.483.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.483.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a\,x}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.484 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.484.1 Optimal result

Integrand size = 24, antiderivative size = 140

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

output
```
-7*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+9*(c-c/a/x)^
(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+x*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/
(1+1/a/x)^(1/2)
```

3.484.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(9+ax-7\sqrt{1+\frac{1}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)
]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.484.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6731, 585, 27, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(7a-\frac{2}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(7a-\frac{2}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-7a\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-14a^2\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 221$$

$$-\frac{\left(\frac{1}{2}\left(14a\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

output

```
-((Sqrt[c - c/(a*x)]*(-((a^2*x)/Sqrt[1 + 1/(a*x)]) + ((-18*a)/Sqrt[1 + 1/(
a*x)] + 14*a*ArcTanh[Sqrt[1 + 1/(a*x)]])/2))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.484.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.484. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.484.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)x}-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+18\sqrt{(ax+1)x}\,\sqrt{a}-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)^{2}\sqrt{a}\,\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{8\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{2}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a
^(3/2)*x*((a*x+1)*x)^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*x+18*((a*x+1)*x)^(1/2)*a^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/((a*x+1)*x)^(1/2)
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.14

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,\left(a^{2}x^{2}+9\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}x-a)},\frac{7}{}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
 - a*c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^2*x - a)]
```

3.484.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.484.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.484.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.484.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

input
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.485 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.485.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

output

```
-5*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)+5*(c-c/a
/x)^(1/2)/a/c/(1-1/a^2/x^2)^(1/2)+x*(c-c/a/x)^(1/2)/c/(1-1/a^2/x^2)^(1/2)
```

3.485.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(ax+5\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{1}{ax}\right)\right)}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}$$

input

```
Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

output

```
(Sqrt[1 - 1/(a*x)]*(a*x + 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))
/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])
```

3.485.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6731, 580, 578, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{580}$$

$$-\frac{-\frac{5c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{c^2x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{578}$$

$$-\frac{-\frac{5c\left(c\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2c\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{2a}-\frac{c^2x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{573}$$

$$-\frac{-\frac{5c\left(\frac{2c\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}-2c^2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2a}-\frac{c^2x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{5c\left(\frac{2c\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}-2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)\right)}{2a}-\frac{c^2x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

3.485. $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$

output
```
-((-((c^2*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a^2*x^2)]) - (5*c*((2*c*Sqrt[c
- c/(a*x)])/Sqrt[1 - 1/(a^2*x^2)] - 2*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/
(a^2*x^2)])/Sqrt[c - c/(a*x)]]))/(2*a))/c^3)
```

3.485.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 578
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-c)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)
/(a*e*(p + 1))), x] + Simp[c*((m - n + 2)/(a*(p + 1)))   Int[(e*x)^m*(c + d
*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] &&
EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[p, -1] && RationalQ[m]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.485. $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

3.485.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-10\sqrt{(ax+1)x}\,\sqrt{a}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+}{2\sqrt{a}}\right.\right.}{2(ax-1)^{2}\sqrt{a}\,c\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}+\frac{4\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{3}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{\sqrt{\frac{c(ax-1)}{ax}}\,x}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(
1/2)/c*(-2*a^(3/2)*x*((a*x+1)*x)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2))*a*x-10*((a*x+1)*x)^(1/2)*a^(1/2)+5*ln(1/2*(2*((a*x+1)
*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$= \left[\frac{5\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+5\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}cx-ac)}\ 5, -\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(5*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^2*c*x - a*c), 1/2*(5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2
+ a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c
*x^2 - a*c*x - c)) + 2*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

3.485.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

3.485.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

3.485.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

3.485.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2), x)
```

3.486 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.486.1 Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

output

```
-3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(3/2)+3*x*(1-1
/a^2/x^2)^(1/2)/c/(c-c/a/x)^(1/2)-2*x*(c-c/a/x)^(1/2)/c^2/(1-1/a^2/x^2)^(1
/2)
```

3.486.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1+\frac{1}{ax}\right)}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}$$

input

```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

output

```
(2*(1 - 1/(a*x))^(3/2)*Hypergeometric2F1[-1/2, 2, 1/2, 1 + 1/(a*x)])/(a*Sq
rt[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))
```

3.486.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6731, 578, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{578}$$

$$-\frac{3c\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2cx\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{579}$$

$$-\frac{3c\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)+\frac{2cx\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{573}$$

$$-\frac{3c\left(\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)+\frac{2cx\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)+\frac{2cx\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

output
```
-(((2*c*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a^2*x^2)] + 3*c*(-((c*Sqrt[1 - 1/
(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)])/Sqrt[c - c/(a*x)]])/a))/c^3)
```

3.486.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 578
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-c)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)
/(a*e*(p + 1))), x] + Simp[c*((m - n + 2)/(a*(p + 1)))   Int[(e*x)^m*(c + d
*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] &&
EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[p, -1] && RationalQ[m]
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.486.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)x}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-6\sqrt{(ax+1)x}\,\sqrt{a}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right.\right.}{2(ax-1)^2\sqrt{a}\,c^2\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^2\sqrt{a^2c}}+\frac{2\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^4c\left(x+\frac{1}{a}\right)}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(
1/2)/c^2*(-2*a^(3/2)*x*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(
1/2)+2*a*x+1)/a^(1/2))*a*x-6*((a*x+1)*x)^(1/2)*a^(1/2)+3*ln(1/2*(2*((a*x+1
)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

3.486.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.66

$$\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+3\,ax}{4\,(a^2c^2x-ac^2)}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^2*c^2*x - a*c^2), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) + 2*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^2*c^2*x - a*c^2)]
```

3.486.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

3.486.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

3.486.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

3.486.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

3.487 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.487.1 Optimal result

Integrand size = 24, antiderivative size = 199

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{2}a\left(c-\frac{c}{ax}\right)^{5/2}}$$

output

```
-(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-1/2*(1-1/a/x)^
(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)+2*(1-
1/a/x)^(5/2)/a/(c-c/a/x)^(5/2)/(1+1/a/x)^(1/2)+(1-1/a/x)^(5/2)*x/(c-c/a/x)
^(5/2)/(1+1/a/x)^(1/2)
```

3.487.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(ax+\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+\frac{1}{x}}{2a}\right)+\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\right.\right.}{ac^2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(a*x + Hypergeometric2F1[-1/2, 1, 1/2, (a + x^(-1))/(2*
a)] + Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(a*c^2*Sqrt[1 + 1/(a*
x)]*Sqrt[c - c/(a*x)])
```

3.487.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6731, 585, 27, 114, 27, 169, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \textcolor{blue}{6731}$$

$$-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \textcolor{blue}{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{ax^2}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{114}$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{\int \frac{\left(a-\frac{3}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{\int \frac{\left(a-\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{169}$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{\int \frac{\left(a-\frac{2}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{\sqrt{\frac{1}{ax}+1}}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{174}$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{-\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{\sqrt{\frac{1}{ax}+1}}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{-2a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{4}{\sqrt{\frac{1}{ax}+1}}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{a\left(-\frac{-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)+\frac{4}{\sqrt{\frac{1}{ax}+1}}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{c^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

output
```
-((a*Sqrt[c - c/(a*x)]*(-(x/(a*Sqrt[1 + 1/(a*x)])) - (4/Sqrt[1 + 1/(a*x)]
- 2*ArcTanh[Sqrt[1 + 1/(a*x)]] - Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]
])/(2*a^2)))/(c^3*Sqrt[1 - 1/(a*x)]))
```

3.487. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

3.487.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.487. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.487.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.30

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{2}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\Bigg(-\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{3}\sqrt{a^{2}c}}+\frac{\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{5}c\left(x+\frac{1}{a}\right)}-\frac{\sqrt{2}\,\ln\Bigg(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\Big(x-\frac{1}{a}}}{x-\frac{1}{a}}}{4a^{4}\sqrt{c}}}{c^{2}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigg(4\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x+8\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\ln\Bigg(\frac{2\sqrt{(ax-}}{}}{4(ax-1)^{2}a^{\frac{3}{2}}c^{3}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)+(-1/2/a^3*ln
((1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)+1/
a^5/c/(x+1/a)*(a^2*c*(x+1/a)^2-(x+1/a)*a*c)^(1/2)-1/4/a^4/c^(1/2)*2^(1/2)*
ln((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c
)^(1/2))/(x-1/a)))*a^2/c^2*((a*x-1)/(a*x+1))^(1/2)/x/(c*(a*x-1)/a/x)^(1/2)
*((a*x+1)*a*c*x)^(1/2)
```

3.487.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.63

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*
x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(a
*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a
*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) + 8*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*c^3*x - a*c^3), 1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(
a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/
(3*a^2*c*x^2 - 2*a*c*x - c)) + 2*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
 - a*c*x - c)) + 4*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^2*c^3*x - a*c^3)]
```

3.487.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

3.487.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

3.487.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.487.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

3.488 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

3.488.1 Optimal result

Integrand size = 24, antiderivative size = 267

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx &= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} \\ &- \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} \\ &+ \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{4\sqrt{2}a\left(c-\frac{c}{ax}\right)^{7/2}}\end{aligned}$$

output
```
(1-1/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(7/2)-11/8*(1-1/a/x)^
(7/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(7/2)*2^(1/2)+7/4*(
1-1/a/x)^(7/2)/a/(c-c/a/x)^(7/2)/(1+1/a/x)^(1/2)-3/2*(1-1/a/x)^(7/2)/(a-1/
x)/(c-c/a/x)^(7/2)/(1+1/a/x)^(1/2)+a*(1-1/a/x)^(7/2)*x/(a-1/x)/(c-c/a/x)^(
7/2)/(1+1/a/x)^(1/2)
```

3.488.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2ax(-3+2ax)+11(-1+ax)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+\frac{1}{x}}{2a}\right)+(4-\right.}{4ac^3\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*x*(-3 + 2*a*x) + 11*(-1 + a*x)*Hypergeometric2F1[-
1/2, 1, 1/2, (a + x^(-1))/(2*a)] + (4 - 4*a*x)*Hypergeometric2F1[-1/2, 1,
1/2, 1 + 1/(a*x)]))/(4*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*(-1 + a*x
))
```

3.488.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.60, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 585, 27, 114, 27, 168, 25, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2x^2}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 114$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{\int -\frac{\left(6a+\frac{5}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{2a}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(6a+\frac{5}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 168$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(2a+\frac{9}{x}\right)x}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(2a+\frac{9}{x}\right)x}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 169$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(4a+\frac{7}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 27$$

3.488. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{1}{2}\int \frac{\left(4a+\frac{7}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 174$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{1}{2}\left(11\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+4\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{1}{2}\left(22a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+8a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}\right)-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 221$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{1}{2}\left(11\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-8\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

output

```
-((a^2*Sqrt[1 - 1/(a*x)]*(x/(2*a*(a - x^(-1))*Sqrt[1 + 1/(a*x)]) + ((-6*x)
/Sqrt[1 + 1/(a*x)] + (-7/Sqrt[1 + 1/(a*x)] + (-8*ArcTanh[Sqrt[1 + 1/(a*x)]
] + 11*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/2)/a)/(4*a^2)))/(c^3*Sq
rt[c - c/(a*x)]))
```

3.488. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

3.488.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

3.488. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.488.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.09

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\,(16\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^{2}-11a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{2}+4\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+8\ln(}{16(ax-1)^{3}a}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{3}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{(\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{4}\sqrt{a^{2}c}}-\frac{\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^{6}c\left(x-\frac{1}{a}\right)}-\frac{11\sqrt{2}\,\ln(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3(}{x-\frac{1}{a}}}{16a^{5}\sqrt{c}}}{c^{3}x\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^3*(c*(a*x-1)/a/x)^(1/2)*x*(16
*((a*x+1)*x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2-11*a^(5/2)*2^(1/2)*ln((2*2^(1/2
)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+4*((a*x+1)*x)^(1/2
)*a^(5/2)*(1/a)^(1/2)*x+8*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(
1/2))*a^3*(1/a)^(1/2)*x^2-28*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-8*ln(1/
2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+11*2^(1/2)*
ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/a
^(3/2)/c^4/(1/a)^(1/2)/((a*x+1)*x)^(1/2)
```

3.488.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.22

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \left[\frac{11\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{ac}}{a^3x^3-3\,a^2x^2+3\,ax-1}\right.}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/32*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2
*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((
a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*
a*x - 1)) + 8*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x +
4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*
c^4), 1/16*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3
*a^2*c*x^2 - 2*a*c*x - c)) - 8*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2
*a^2*c*x^2 - a*c*x - c)) + 4*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

3.488.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(7/2),x)
```

output
```
Timed out
```

3.488.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

3.488.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

3.489 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$

3.489.1 Optimal result

Integrand size = 25, antiderivative size = 60

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-m,-m,-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

output
```
x^(1+m)*hypergeom([-1/2, -1-m],[-m],-1/a/x)*(c-c/a/x)^(1/2)/(1+m)/(1-1/a/x
)^(1/2)
```

3.489.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\frac{\sqrt{c-\frac{c}{ax}}x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-m,-m,-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

output
```
(Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x)
)])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

3.489.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6736, 6735, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6736}$$

$$\frac{\sqrt{c-\frac{c}{ax}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^m dx}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{6735}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m\sqrt{c-\frac{c}{ax}}\int\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-2}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-1,-m,-\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

output
```
(Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x)
)])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

3.489.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 6735
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/
2)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p
}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
 0]) &&  !IntegerQ[m]
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.489.4 Maple [F]

$$\int \frac{x^m\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

3.489.5 Fricas [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

output
```
integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(
a*x - 1), x)
```

3.489.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

3.489.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.489.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx=\int \frac{x^m\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x^m*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x^m*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.490 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.490.1 Optimal result

Integrand size = 25, antiderivative size = 164

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=-\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

output
```
1/8*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^3-1/8*c
*x*(1-1/a^2/x^2)^(1/2)/a^2/(c-c/a/x)^(1/2)+1/12*c*x^2*(1-1/a^2/x^2)^(1/2)/
a/(c-c/a/x)^(1/2)+1/3*c*x^3*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.490.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-3+2ax+8a^2x^2)}{-1+ax}-3\sqrt{c}\log(1-ax)+3\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax\right.}{48a^3}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3 + 2*a*x + 8*a^2*x^
2))/(-1 + a*x) - 3*Sqrt[c]*Log[1 - a*x] + 3*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt
[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a
^3)
```

3.490.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6733, 575, 579, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^4}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 575}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{ 579}$$

$$-c\left(\frac{-\frac{3\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{ 579}$$

$$-c\left(\frac{-\frac{3\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow 573$$

$$-c\left(\frac{-\frac{3\left(\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow 219$$

$$-c\left(\frac{-\frac{3\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

input

```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

output

```
-(c*(-1/3*(Sqrt[1 - 1/(a^2*x^2)]*x^3)/Sqrt[c - c/(a*x)] + (-1/2*(c*Sqrt[1
- 1/(a^2*x^2)]*x^2)/Sqrt[c - c/(a*x)] - (3*(-((c*Sqrt[1 - 1/(a^2*x^2)]*x)/
Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt
[c - c/(a*x)]])/a))/(4*a))/(6*a*c)))
```

3.490. $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.490.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.490. $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

3.490.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(16 a^{\frac{5}{2}} x^{2} \sqrt{(ax+1)x}+4 a^{\frac{3}{2}} x \sqrt{(ax+1)x}-6 \sqrt{(ax+1)x}\, \sqrt{a}+3 \ln\left(\frac{2 \sqrt{(ax+1)x}\, \sqrt{a}+2ax+1}{2 \sqrt{a}}\right)\right)}{48 \sqrt{\frac{ax-1}{ax+1}}\, a^{\frac{5}{2}} \sqrt{(ax+1)x}}$	121
risch	$\frac{(8a^2x^2+2ax-3)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{16a^2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	148

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(5/2)*(16*a^(5/2)*x
^2*((a*x+1)*x)^(1/2)+4*a^(3/2)*x*((a*x+1)*x)^(1/2)-6*((a*x+1)*x)^(1/2)*a^(
1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^
(1/2)
```

3.490.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.05

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$= \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax}{96\,(a^4x-a^3)}\right.$$

$$-\frac{3\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-}{ax}}}{48\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

output

```
[1/96*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x -
1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), -1/48*(3*(a*x - 1)*s
qrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(8*a^4*x^4 + 10*a^3*x^3 -
 a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*
x - a^3)]
```

3.490.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a/x)**(1/2),x)
```

output

```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.490.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

output

```
integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.490.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.491 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

3.491.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

output
```
-1/4*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^2+1/4*
c*x*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/2*c*x^2*(1-1/a^2/x^2)^(1/2)/(c
-c/a/x)^(1/2)
```

3.491.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(1+2ax)+\sqrt{c}(-1+ax)\log(1-ax)+\sqrt{c}(1-ax)\log\Bigg(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-}}{8a^2(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

output
```
(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(1 + 2*a*x) + Sqrt[c]*(
-1 + a*x)*Log[1 - a*x] + Sqrt[c]*(1 - a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2*(-1 + a
*x))
```

3.491.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6733, 575, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6733$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 575$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4ac}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \; 579$$

$$-c\left(\frac{-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{4ac}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \; 573$$

$$-c\left(\frac{\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{4ac}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow 219$$

$$-c\left(\frac{\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{4ac}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

output
```
-(c*(-1/2*(Sqrt[1 - 1/(a^2*x^2)]*x^2)/Sqrt[c - c/(a*x)] + (-((c*Sqrt[1 - 1
/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)])/Sqrt[c - c/(a*x)]])/a)/(4*a*c)))
```

3.491.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

3.491. $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.491.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-4a^{\frac{3}{2}}x\sqrt{(ax+1)x}-2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{8\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{3}{2}}\sqrt{(ax+1)x}}$	102
risch	$\frac{(2ax+1)x\sqrt{\frac{c(ax-1)}{ax}}}{4a\sqrt{\frac{ax-1}{ax+1}}}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{8a\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	140

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-4*a^(3/2)*x
*((a*x+1)*x)^(1/2)-2*((a*x+1)*x)^(1/2)*a^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)
*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

3.491.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.56

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$= \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}}{16\,(a^3x-a^2)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

output
```
[1/16*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*((a*x - 1)*sqrt(-c)*arctan(2*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(
2*a^2*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

3.491.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a/x)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.491.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.491.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.492 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.492.1 Optimal result

Integrand size = 22, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^
2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.492.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(1+ax+\sqrt{1+\frac{1}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]]
))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.492.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6731, 575, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{575}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2ac}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c\left(-\frac{\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

output
```
-(c*(-((Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) - ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])))
```

3.492.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.492.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(2\sqrt{(ax+1)x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)x}\,\sqrt{a}}$	87
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a
^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^
(1/2)/a^(1/2)
```

3.492.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}x-a)},\right.$$
$$\left.-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)-2\,(a^{2}x^{2}+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^{2}x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sq
rt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a^2*x - a)]
```

3.492.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.492.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.492.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.493 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.493.1 Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
2*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)-2*c*(1-1/a^
2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.493.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$=\frac{-2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x+\sqrt{c}(1-ax)\log(1-ax)+\sqrt{c}(-1+ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c\right.}{-1+ax}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1
- a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c
 - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

3.493.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6733, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{576}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{c}}\right)$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

output
```
-(c*((2*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - (2*ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/Sqrt[c]))
```

3.493. $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}dx$

3.493.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.493.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\left(\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-2\sqrt{(ax+1)x}\,\sqrt{a}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)x}\,\sqrt{a}}$	88
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{a\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

3.493. $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

output
```
1/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*((a*x+1)*x)^(1/
2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-2*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^
(1/2)/a^(1/2)
```

3.493.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(64) = 128$.

Time = 0.27 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.62

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas
")
```

output
```
[1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) - 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) +
2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

3.493.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.493.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.493.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.494 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

3.494.1 Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

output
```
-2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)
```

3.494.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1+ax)}{-3+3ax}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + a*x))/(-3 + 3*a*x)
```

3.494.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6733, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{458}$$

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
(-2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2))
```

3.494.3.1 Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.494.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
gosper	$-\frac{2(ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	41
default	$-\frac{2(ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	41
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(a^2x^2+2ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x}$	56

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a*x+1)/x/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)
```

3.494.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fric
as")
```

output
```
-2/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
))/(a*x^2 - x)
```

3.494.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.494.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.494.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.494.9 Mupad [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{2\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x\,\left(a\,x-1\right)}$$

input
```
int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
-(2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*x*(a*x
 - 1))
```

3.494. $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

3.495 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.495.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}$$

output
```
-2/15*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)+2/5*a^2*c*(1-1/a^2/x^2)^
(3/2)/(c-c/a/x)^(1/2)
```

3.495.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-3-ax+2a^2x^2)}{15x(-1+ax)}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-3 - a*x + 2*a^2*x^2))/(15*x
*(-1 + a*x))
```

3.495.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6733, 572, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \textbf{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}x}d\frac{1}{x}$$

$$\Big\downarrow \textbf{572}$$

$$-c\left(\frac{1}{5}a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \textbf{458}$$

$$-c\left(\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
-(c*((2*a^2*c*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) - (2*a^2*(
1 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])))
```

3.495.3.1 Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.495.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}\sqrt{\frac{ax-1}{ax+1}}}$	47
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(2a^{3}x^{3}+a^{2}x^{2}-4ax-3)}{15\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^{2}}$	64

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*(a*x+1)*(2*a*x-3)*(c*(a*x-1)/a/x)^(1/2)/x^2/((a*x-1)/(a*x+1))^(1/2)
```

3.495.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^{3}}\,dx=\frac{2\left(2\,a^{3}x^{3}+a^{2}x^{2}-4\,ax-3\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^{3}-x^{2}\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fric
as")
```

output
```
2/15*(2*a^3*x^3 + a^2*x^2 - 4*a*x - 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(a*x^3 - x^2)
```

3.495.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**3,x)
```

output
```
Timed out
```

3.495.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.495.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.495.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)^2\,\left(2\,a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,x^2\,\left(a\,x-1\right)}$$

input
```
int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2)
)/(15*x^2*(a*x - 1))
```

3.496 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.496.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

output
```
8/105*a^3*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-2/7*a*c^2*(1-1/a^2/x^2)^
(3/2)/(c-c/a/x)^(3/2)/x^2-8/35*a^3*c*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(1/2)
```

3.496.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(15+3ax-4a^2x^2+8a^3x^3)}{105x^2(-1+ax)}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(15 + 3*a*x - 4*a^2*x^2 + 8*
a^3*x^3))/(105*x^2*(-1 + a*x))
```

3.496.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6733, 581, 27, 672, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{581}$$

$$-c\left(\frac{2a^2\int \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{6}{x}\right)}{2a\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{7c^2}+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{7}a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{6}{x}\right)}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7c}\right)$$

$$\Big\downarrow \text{672}$$

$$-c\left(\frac{1}{7}a\left(\frac{11}{5}a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{12a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}\right)+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7c}\right)$$

$$\Big\downarrow \text{458}$$

$$-c\left(\frac{1}{7}a\left(\frac{22a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{12a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}\right)+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7c}\right)$$

input

```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

3.496. $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx$

output
```
-(c*((a*((22*a^2*c*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) - (12
*a^2*(1 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])))/7 + (2*a^3*(1 - 1/(a
^2*x^2))^(3/2)*Sqrt[c - c/(a*x)])/(7*c)))
```

3.496.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.496. $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.496.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(ax+1)(8a^2x^2-12ax+15)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3\sqrt{\frac{ax-1}{ax+1}}}$	55
default	$-\frac{2(ax+1)(8a^2x^2-12ax+15)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3\sqrt{\frac{ax-1}{ax+1}}}$	55
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(8a^4x^4+4a^3x^3-a^2x^2+18ax+15)}{105\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^3}$	73

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/105*(a*x+1)*(8*a^2*x^2-12*a*x+15)*(c*(a*x-1)/a/x)^(1/2)/x^3/((a*x-1)/(a
*x+1))^(1/2)
```

3.496.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2\,(8\,a^4x^4+4\,a^3x^3-a^2x^2+18\,ax+15)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\,(ax^4-x^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fric
as")
```

output
```
-2/105*(8*a^4*x^4 + 4*a^3*x^3 - a^2*x^2 + 18*a*x + 15)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

3.496.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**4,x)
```

output
```
Timed out
```

3.496.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.496.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.496.9 Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=-\frac{2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(8\,a^3\,x^3+12\,a^2\,x^2+11\,a\,x+29\right)\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3}-\frac{88\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

input
```
int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
- (2*((a*x - 1)/(a*x + 1))^(1/2)*(11*a*x + 12*a^2*x^2 + 8*a^3*x^3 + 29)*((
c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (88*((a*x - 1)/(a*x + 1))^(1/2)*((c
*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.496. $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.497 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.497.1 Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3}+\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

output
```
-16/315*a^4*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-2/9*a*c^2*(1-1/a^2/x^2
)^(3/2)/(c-c/a/x)^(3/2)/x^3+4/21*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/
2)/x^2+16/105*a^4*c*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(1/2)
```

3.497.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.47

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(-35-5ax+6a^2x^2-8a^3x^3+16a^4x^4\right)}{315x^3(-1+ax)}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-35 - 5*a*x + 6*a^2*x^2 - 8*
a^3*x^3 + 16*a^4*x^4))/(315*x^3*(-1 + a*x))
```

3.497.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6733, 581, 27, 2170, 27, 672, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{581}$$

$$-c\left(-\frac{2a^3\int \frac{3\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{c^3}{ax}-\frac{5c^3}{a^2x^2}+c^3\right)}{2\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{a^3\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{c^3}{ax}-\frac{5c^3}{a^2x^2}+c^3\right)}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{3c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right)$$

$$\Big\downarrow \text{2170}$$

$$-c\left(-\frac{a^3\left(-\frac{2a^4\int -\frac{c^5\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{23}{x}\right)}{2a^5\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{7c^2}-\frac{10}{7}ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\right)}{3c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{a^3\left(\frac{c^3\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{23}{x}\right)}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{7a}-\frac{10}{7}ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\right)}{3c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right)$$

$$\Big\downarrow \text{672}$$

$$-c\left(-\frac{a^3\left(\frac{c^3\left(\frac{46a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{13}{5}a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{7a}-\frac{10}{7}ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\right)}{3c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right.$$

$$\Big\downarrow \text{458}$$

$$-c\left(-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}-\frac{a^3\left(\frac{c^3\left(\frac{46a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{26a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{7a}-\frac{10}{7}ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\right)}{3c^3}\right.$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
-(c*(-1/3*(a^3*((c^3*((-26*a^2*c*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x)
)^(3/2)) + (46*a^2*(1 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])))/(7*a)
- (10*a*c^2*(1 - 1/(a^2*x^2))^(3/2)*Sqrt[c - c/(a*x)])/7))/c^3 - (2*a^4*(1
 - 1/(a^2*x^2))^(3/2)*(c - c/(a*x))^(3/2))/(9*c^2)))
```

3.497. $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.497.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.497. $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.497.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

method	result	size
gosper	$\frac{2(ax+1)(16a^3x^3-24a^2x^2+30ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	63
default	$\frac{2(ax+1)(16a^3x^3-24a^2x^2+30ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	63
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(16a^5x^5+8a^4x^4-2a^3x^3+a^2x^2-40ax-35)}{315\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^4}$	80

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/315*(a*x+1)*(16*a^3*x^3-24*a^2*x^2+30*a*x-35)*(c*(a*x-1)/a/x)^(1/2)/x^4/
((a*x-1)/(a*x+1))^(1/2)
```

3.497.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\left(16\,a^5x^5+8\,a^4x^4-2\,a^3x^3+a^2x^2-40\,ax-35\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{315\left(ax^5-x^4\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fric
as")
```

output
```
2/315*(16*a^5*x^5 + 8*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 40*a*x - 35)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

3.497.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**5,x)
```

output
```
Timed out
```

3.497.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.497.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.497.9 Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}\,\left(16\,a^4\,x^4+24\,a^3\,x^3+22\,a^2\,x^2+23\,a\,x-17\right)}{315\,x^4} - \frac{104\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{315\,x^4\,\left(a\,x-1\right)}$$

input

```
int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output

```
(2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(23*a*x + 22*a^
2*x^2 + 24*a^3*x^3 + 16*a^4*x^4 - 17))/(315*x^4) - (104*((a*x - 1)/(a*x +
1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(315*x^4*(a*x - 1))
```

3.498 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

3.498.1 Optimal result

Integrand size = 27, antiderivative size = 130

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{75\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}$$

output
```
75/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4+75/64*x*(c-c/a/x)^(1/2)
/a^3+25/32*x^2*(c-c/a/x)^(1/2)/a^2+5/8*x^3*(c-c/a/x)^(1/2)/a+1/4*x^4*(c-c/
a/x)^(1/2)
```

3.498.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(a^4x^4+15\,\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\frac{1}{ax}\right)\right)}{4a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

output
```
(Sqrt[c - c/(a*x)]*(a^4*x^4 + 15*Hypergeometric2F1[1/2, 4, 3/2, 1 - 1/(a*x
)]))/(4*a^4)
```

3.498.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6683, 1070, 281, 948, 87, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^3dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x^3(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x^3}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^5}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left(\frac{15}{8}\int \frac{x^4}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\int \frac{x^3}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \text{52}$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\left(\frac{3\int \frac{x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \text{52}$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\left(\frac{3\left(\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

3.498. $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\left(\frac{3\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

input

```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

output

```
-((c*(-1/4*(a*Sqrt[c - c/(a*x)]*x^4)/c + (15*(-1/3*(Sqrt[c - c/(a*x)]*x^3)
/c + (5*(-1/2*(Sqrt[c - c/(a*x)]*x^2)/c + (3*(-((Sqrt[c - c/(a*x)]*x)/c) -
 ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*Sqrt[c])))/(4*a)))/(6*a)))/8))/a)
```

3.498.3.1 Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.498. $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

3.498.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(16a^3x^3+40a^2x^2+50ax+75)x\sqrt{\frac{c(ax-1)}{ax}}}{64a^3}+\frac{75\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{128a^3\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\Big(32x(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}+112(a\,x^2-x)^{\frac{3}{2}}a^{\frac{5}{2}}+212\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-106\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+256a^{\frac{3}{2}}\sqrt{(ax-1)x}+128a\ln\Big(}{128\sqrt{(ax-1)x}\,a^{\frac{9}{2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*a^3*x^3+40*a^2*x^2+50*a*x+75)/a^3*x*(c*(a*x-1)/a/x)^(1/2)+75/128/
a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(
1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.498.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\left[\frac{2\left(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax\right)\sqrt{\frac{acx-c}{ax}}+75\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{128\,a^4},\frac{(16\,a^4x^4+}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x - c)
/(a*x)) + 75*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
+ c))/a^4, 1/64*((16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c
*x - c)/(a*x)) - 75*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a
^4]
```

3.498.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a/x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.498.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^3}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))*x^3/(a*x - 1), x)
```

3.498.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09

$$\begin{aligned}&\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx\\&=\frac{1}{64}\sqrt{a^2cx^2-acx}\left(2\left(4x\left(\frac{2x|a|}{a^2\mathrm{sgn}(x)}+\frac{5|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{25|a|}{a^4\mathrm{sgn}(x)}\right)x+\frac{75|a|}{a^5\mathrm{sgn}(x)}\right)\\&\quad+\frac{75\sqrt{c}\log(|a||c|)\,\mathrm{sgn}(x)}{128a^4}-\frac{75\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{128a^4\mathrm{sgn}(x)}\end{aligned}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
1/64*sqrt(a^2*c*x^2 - a*c*x)*(2*(4*x*(2*x*abs(a)/(a^2*sgn(x)) + 5*abs(a)/(
a^3*sgn(x))) + 25*abs(a)/(a^4*sgn(x)))*x + 75*abs(a)/(a^5*sgn(x))) + 75/12
8*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^4 - 75/128*sqrt(c)*log(abs(-2*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^4*sgn(x))
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.499 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.499.1 Optimal result

Integrand size = 27, antiderivative size = 105

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}+\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3+\frac{11\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}$$

output
```
11/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3+11/8*x*(c-c/a/x)^(1/2)/a
^2+11/12*x^2*(c-c/a/x)^(1/2)/a+1/3*x^3*(c-c/a/x)^(1/2)
```

3.499.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(a^3x^3+11\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{1}{ax}\right)\right)}{3a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
(Sqrt[c - c/(a*x)]*(a^3*x^3 + 11*Hypergeometric2F1[1/2, 3, 3/2, 1 - 1/(a*x
)]))/(3*a^3)
```

3.499.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = $\{6717, 6683, 1070, 281, 948, 87, 52, 52, 73, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^2dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x^2(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x^2}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^4}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{11}{6}\int \frac{x^3}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

$$\Big\downarrow \text{52}$$

$$-\frac{c\left(\frac{11}{6}\left(\frac{3\int \frac{x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

$$\Big\downarrow \; 52$$

$$-\frac{c\left(\frac{11}{6}\left(\frac{3\left(\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

$$\Big\downarrow \; 73$$

$$-\frac{c\left(\frac{11}{6}\left(\frac{3\left(-\frac{\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$-\frac{c\left(\frac{11}{6}\left(\frac{3\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

input

```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

output

```
-((c*(-1/3*(a*Sqrt[c - c/(a*x)]*x^3)/c + (11*(-1/2*(Sqrt[c - c/(a*x)]*x^2)
/c + (3*(-((Sqrt[c - c/(a*x)]*x)/c) - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(
a*Sqrt[c])))/(4*a)))/6))/a)
```

3.499.3.1 Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

3.499. $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.499. $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.499.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

method	result
risch	$\frac{\left(8 a^{2} x^{2}+22 a x+33\right) x \sqrt{\frac{c \left(a x-1\right)}{a x}}}{24 a^{2}}+\frac{11 \ln \left(\frac{-\frac{1}{2} a c+a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-a c x}\right) \sqrt{\frac{c \left(a x-1\right)}{a x}}\, \sqrt{c \left(a x-1\right) a x}}{16 a^{2} \sqrt{a^{2} c}\, \left(a x-1\right)}$
default	$\frac{\sqrt{\frac{c \left(a x-1\right)}{a x}}\, x \left(16 \left(a \,x^{2}-x \right)^{\frac{3}{2}} a^{\frac{5}{2}}+60 \sqrt{a \,x^{2}-x}\, a^{\frac{5}{2}} x-30 \sqrt{a \,x^{2}-x}\, a^{\frac{3}{2}}+96 a^{\frac{3}{2}} \sqrt{\left(a x-1\right) x}+48 a \ln \left(\frac{2 \sqrt{\left(a x-1\right) x}\, \sqrt{a}+2 a x-1}{2 \sqrt{a}}\right)-15 \ln \left(2 \sqrt{a \,x^{2}}\right.\right.}{48 \sqrt{\left(a x-1\right) x}\, a^{\frac{7}{2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2+22*a*x+33)/a^2*x*(c*(a*x-1)/a/x)^(1/2)+11/16/a^2*ln((-1/2*
a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*
(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.499.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.55

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$=\left[\frac{2\,\left(8\,a^{3}x^{3}+22\,a^{2}x^{2}+33\,ax\right)\sqrt{\frac{acx-c}{ax}}+33\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{48\,a^{3}},\ \frac{\left(8\,a^{3}x^{3}+22\,a^{2}x^{2}+33\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) + 33*sq
rt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^3, 1/24
*((8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sqrt(-c)*
arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

3.499.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a/x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.499.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^2}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/(a*x - 1), x)
```

3.499.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx\\ &=\frac{1}{24}\sqrt{a^2cx^2-acx}\left(2x\left(\frac{4x|a|}{a^2\mathrm{sgn}(x)}+\frac{11|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{33|a|}{a^4\mathrm{sgn}(x)}\right)\\ &\quad+\frac{11\sqrt{c}\log(|a||c|)\,\mathrm{sgn}(x)}{16a^3}\\ &\quad-\frac{11\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{16a^3\mathrm{sgn}(x)}\end{aligned}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a^2*c*x^2 - a*c*x)*(2*x*(4*x*abs(a)/(a^2*sgn(x)) + 11*abs(a)/(a^
3*sgn(x))) + 33*abs(a)/(a^4*sgn(x))) + 11/16*sqrt(c)*log(abs(a)*abs(c))*sg
n(x)/a^3 - 11/16*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*sqrt(c)*abs(a) + a*c))/(a^3*sgn(x))
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.500 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

3.500.1 Optimal result

Integrand size = 25, antiderivative size = 80

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{7\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{7\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

output
```
7/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2+7/4*x*(c-c/a/x)^(1/2)/a+1
/2*x^2*(c-c/a/x)^(1/2)
```

3.500.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(a\sqrt{1-\frac{1}{ax}}x(7+2ax)+7\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\right)\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

output
```
(Sqrt[c - c/(a*x)]*(a*Sqrt[1 - 1/(a*x)]*x*(7 + 2*a*x) + 7*ArcTanh[Sqrt[1 -
 1/(a*x)]]))/(4*a^2*Sqrt[1 - 1/(a*x)])
```

3.500.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {6717, 6683, 1070, 281, 948, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6717$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}xdx$$

$$\Big\downarrow \; 6683$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x(ax+1)}{1-ax}dx$$

$$\Big\downarrow \; 1070$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \; 281$$

$$\frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$

$$\Big\downarrow \; 948$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \; 87$$

$$-\frac{c\left(\frac{7}{4}\int \frac{x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{a}$$

$$\Big\downarrow \; 52$$

$$-\frac{c\left(\frac{7}{4}\left(\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)-\frac{ax^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(\frac{7}{4}\left(-\frac{\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)-\frac{ax^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(\frac{7}{4}\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)-\frac{ax^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{a}$$

input

```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

output

```
-((c*(-1/2*(a*Sqrt[c - c/(a*x)]*x^2)/c + (7*(-((Sqrt[c - c/(a*x)]*x)/c) -
ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*Sqrt[c])))/4))/a)
```

3.500.3.1 Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.500.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39

method	result	si
risch	$\frac{(2ax+7)x\sqrt{\frac{c(ax-1)}{ax}}}{4a}+\frac{7\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{8a\sqrt{a^{2}c}\,(ax-1)}$	11
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{a\,x^{2}-x}\,a^{\frac{5}{2}}x-2\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}+16a^{\frac{3}{2}}\sqrt{(ax-1)x}+8a\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\right)}{8\sqrt{(ax-1)x}\,a^{\frac{5}{2}}}$	13

input
```
int(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x+7)/a*x*(c*(a*x-1)/a/x)^(1/2)+7/8/a*ln((-1/2*a*c+a^2*c*x)/(a^2*c
)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/
2)*(c*(a*x-1)*a*x)^(1/2)
```

3.500.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.84

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\left[\frac{2\,(2\,a^2x^2+7\,ax)\sqrt{\frac{acx-c}{ax}}+7\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^2},\frac{(2\,a^2x^2+7\,ax)\sqrt{\frac{acx-c}{ax}}-7\,\sqrt{-c}\,\mathrm{ar}}{4\,a^2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*(2*a^2*x^2 + 7*a*x)*sqrt((a*c*x - c)/(a*x)) + 7*sqrt(c)*log(-2*a*c
*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2, 1/4*((2*a^2*x^2 + 7*
a*x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)
/(a*x))/c))/a^2]
```

3.500.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.500.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}x}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))*x/(a*x - 1), x)
```

3.500.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{1}{4}\sqrt{a^2cx^2-acx}\left(\frac{2\,x|a|}{a^2\mathrm{sgn}\,(x)}+\frac{7\,|a|}{a^3\mathrm{sgn}\,(x)}\right)+\frac{7\sqrt{c}\log\left(|a||c|\right)\mathrm{sgn}(x)}{8\,a^2}-\frac{7\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{8\,a^2\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a^2*c*x^2 - a*c*x)*(2*x*abs(a)/(a^2*sgn(x)) + 7*abs(a)/(a^3*sgn(x
))) + 7/8*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^2 - 7/8*sqrt(c)*log(abs(-2*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^2*sgn(x
))
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.501 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.501.1 Optimal result

Integrand size = 24, antiderivative size = 50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}\,x+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

3.501.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}\,x+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

3.501.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6683, 1035, 281, 899, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{3}{2}\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(-\frac{3a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow 221$$

$$-\frac{c\left(-\frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((c*(-((a*Sqrt[c - c/(a*x)]*x)/c) - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
)/Sqrt[c]))/a)
```

3.501.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.501.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

method	result	size
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^{2}-x}\,\sqrt{a}-4\sqrt{(ax-1)x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{(ax-1)x}\,\sqrt{a}}$	120

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(c*(a*x-1)/a/x)^(1/2)+3/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2
-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)
^(1/2)
```

3.501.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.48

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}} + 3\,\sqrt{c}\log\left(-2\,acx - 2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}} + c\right)}{2\,a}, \frac{ax\sqrt{\frac{acx-c}{ax}} - 3\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(
-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

3.501.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.501.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

3.501.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{3\sqrt{c}\log\left(|a||c|\right)\operatorname{sgn}(x)}{2a} - \frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{2\,a\operatorname{sgn}\left(x\right)} + \frac{\sqrt{a^2cx^2-acx}|a|}{a^2\operatorname{sgn}\left(x\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(
a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.502 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.502.1 Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

output
```
2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)+2*(c-c/a/x)^(1/2)
```

3.502.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

output
```
2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

3.502.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6717, 6683, 1070, 281, 948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x(1-ax)}dx$$

$$\Big\downarrow \text{ 1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x}dx}{a}$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 90}$$

$$-\frac{c\left(a\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{2a\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{c\left(-\frac{2a^2\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{2a\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$-\frac{c\left(-\frac{2a\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2a\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

output
```
-((c*((-2*a*Sqrt[c - c/(a*x)])/c - (2*a*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
)/Sqrt[c]))/a)
```

3.502.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.502. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.502.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

method	result	size
risch	$2\sqrt{\frac{c(ax-1)}{ax}}+\frac{a\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{\sqrt{a^{2}c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-2\sqrt{(ax-1)x}\,a^{\frac{3}{2}}x^{2}+2\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}-\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^{2}\right)}{x\sqrt{(ax-1)x}\,\sqrt{a}}$	99

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(c*(a*x-1)/a/x)^(1/2)+a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a
*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(
1/2)
```

3.502. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.502.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\sqrt{c}\log\left(-2\,acx - 2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}} + c\right) + 2\sqrt{\frac{acx-c}{ax}},\; -2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right) + 2\sqrt{\frac{acx-c}{ax}}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*sqr
t((a*c*x - c)/(a*x)), -2*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/
c) + 2*sqrt((a*c*x - c)/(a*x))]
```

3.502.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.

Time = 4.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.02

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \begin{cases} -\frac{2a\left(\frac{c^2\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c} & \text{for } \frac{c}{a}\neq 0 \\ -\frac{3a\sqrt{c}\left(\frac{\log\left(\frac{2}{x}\right)}{a} - \frac{\log\left(2a-\frac{2}{x}\right)}{a}\right)}{2} + \frac{\sqrt{c}\log\left(\frac{a}{x}-\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x,x)
```

output
```
Piecewise((-2*a*(c**2*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - c*sq
rt(c - c/(a*x))/a)/c, Ne(c/a, 0)), (-3*a*sqrt(c)*(log(2/x)/a - log(2*a - 2
/x)/a)/2 + sqrt(c)*log(a/x - 1/x**2)/2, True))
```

3.502.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x), x)
```

3.502.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

3.503 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

3.503.1 Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = 4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

output
```
-2/3*a*(c-c/a/x)^(3/2)/c+4*a*(c-c/a/x)^(1/2)
```

3.503.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(1+5ax)}{3x}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(1 + 5*a*x))/(3*x)
```

3.503.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6717, 6683, 1070, 281, 946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x^2(1-ax)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x^2}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^2}dx}{a}$$

$$\Big\downarrow \text{946}$$

$$-\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{53}$$

$$-\frac{c\int \left(\frac{2a}{\sqrt{c-\frac{c}{ax}}}-\frac{a\sqrt{c-\frac{c}{ax}}}{c}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c^2}-\frac{4a^2\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
-((c*((-4*a^2*Sqrt[c - c/(a*x)])/c + (2*a^2*(c - c/(a*x))^(3/2))/(3*c^2)))
/a)
```

3.503.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.503. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.503.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result
gosper	$\frac{2(5ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x}$
trager	$\frac{2(5ax+1)\sqrt{-\frac{-acx+c}{ax}}}{3x}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(5a^2x^2-4ax-1)}{3(ax-1)x}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-6\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x^3-6a^{\frac{5}{2}}\sqrt{(ax-1)x}\,x^3+12a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x+3\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2x^3-3\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2}{2\sqrt{a}}\right.\right.}{3x^2\sqrt{(ax-1)x}\,\sqrt{a}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(5*a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x
```

3.503.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\,(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\,x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
2/3*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x
```

3.503.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

3.503.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^2}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^2), x)
```

3.503.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{6,[2,1,5]%%%}+%%%{-6,[1,1,4]%%%}+%%%{-6,[0,1,3]%%%},[4
]%%%}+%%%
```

3.503.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,(5\,a\,x+1)}{3\,x}$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

output
```
(2*(c - c/(a*x))^(1/2)*(5*a*x + 1))/(3*x)
```

3.504 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.504.1 Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=4a^2\sqrt{c-\frac{c}{ax}}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}$$

output
```
-2*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2+4*a^2*(c-c/a/x)^(1/2)
```

3.504.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}(1+3ax+6a^2x^2)}{5x^2}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(1 + 3*a*x + 6*a^2*x^2))/(5*x^2)
```

3.504.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6717, 6683, 1070, 281, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x^3(1-ax)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x^3}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^3}dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{86}$$

$$-\frac{c\int \left(\frac{\left(c-\frac{c}{ax}\right)^{3/2}a^2}{c^2}-\frac{3\sqrt{c-\frac{c}{ax}}a^2}{c}+\frac{2a^2}{\sqrt{c-\frac{c}{ax}}}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(-\frac{2a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^3}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{c^2}-\frac{4a^3\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
-((c*((-4*a^3*Sqrt[c - c/(a*x)])/c + (2*a^3*(c - c/(a*x))^(3/2))/c^2 - (2*
a^3*(c - c/(a*x))^(5/2))/(5*c^3)))/a)
```

3.504.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.504. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.504.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result
gosper	$\frac{2(6a^2x^2+3ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{5x^2}$
trager	$\frac{2(6a^2x^2+3ax+1)\sqrt{-\frac{-acx+c}{ax}}}{5x^2}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(6a^3x^3-3a^2x^2-2ax-1)}{5(ax-1)x^2}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-10\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}x^4-10a^{\frac{7}{2}}\sqrt{(ax-1)x}\,x^4+20a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2+5\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^4-5\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}}{2\sqrt{a}}\right.\right.}{5x^3\sqrt{(ax-1)x}\,\sqrt{a}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(6*a^2*x^2+3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^2
```

3.504.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{\frac{acx-c}{ax}}}{5\,x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x^2
```

3.504.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^3\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

3.504.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^3}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^3), x)
```

3.504.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{%%{[-5,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,5]%%%},[6]%%%}+%%
%{%%{[%%%
```

3.504.9 Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{a\,x}}\,\left(6\,a^2\,x^2+3\,a\,x+1\right)}{5\,x^2}$$

input

```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

output

```
(2*(c - c/(a*x))^(1/2)*(3*a*x + 6*a^2*x^2 + 1))/(5*x^2)
```

3.505 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.505.1 Optimal result

Integrand size = 27, antiderivative size = 96

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=4a^3\sqrt{c-\frac{c}{ax}}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$

output
```
-10/3*a^3*(c-c/a/x)^(3/2)/c+8/5*a^3*(c-c/a/x)^(5/2)/c^2-2/7*a^3*(c-c/a/x)^
(7/2)/c^3+4*a^3*(c-c/a/x)^(1/2)
```

3.505.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}(15+39ax+52a^2x^2+104a^3x^3)}{105x^3}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*x^3)
```

3.505.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6717, 6683, 1070, 281, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x^4(1-ax)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x^4}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^4}dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{86}$$

$$-\frac{c\int \left(-\frac{\left(c-\frac{c}{ax}\right)^{5/2}a^3}{c^3}+\frac{4\left(c-\frac{c}{ax}\right)^{3/2}a^3}{c^2}-\frac{5\sqrt{c-\frac{c}{ax}}a^3}{c}+\frac{2a^3}{\sqrt{c-\frac{c}{ax}}}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^4}-\frac{8a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^3}+\frac{10a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c^2}-\frac{4a^4\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input

```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

output

```
-((c*((-4*a^4*Sqrt[c - c/(a*x)])/c + (10*a^4*(c - c/(a*x))^(3/2))/(3*c^2)
- (8*a^4*(c - c/(a*x))^(5/2))/(5*c^3) + (2*a^4*(c - c/(a*x))^(7/2))/(7*c^4
)))/a)
```

3.505.3.1 Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 281

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.505. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.505.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.45

method	result
gosper	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(104a^3x^3+52a^2x^2+39ax+15)}{105x^3}$
trager	$\frac{2(104a^3x^3+52a^2x^2+39ax+15)\sqrt{-\frac{-acx+c}{ax}}}{105x^3}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(104a^4x^4-52a^3x^3-13a^2x^2-24ax-15)}{105(ax-1)x^3}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-210a^{\frac{9}{2}}\sqrt{a\,x^2-x}\,x^5-210a^{\frac{9}{2}}\sqrt{(ax-1)x}\,x^5+420a^{\frac{7}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^3+105\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^4x^5-105\ln\left(\right.\right.}{105x^4\sqrt{(ax-1)x}\,\sqrt{a}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/105*(c*(a*x-1)/a/x)^(1/2)*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3
```

3.505.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{2\,(104\,a^3x^3+52\,a^2x^2+39\,ax+15)\sqrt{\frac{acx-c}{ax}}}{105\,x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt((a*c*x - c)/(a*x))/x^3
```

3.505.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^4\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

3.505.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^4}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^4), x)
```

3.505.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{210,[2,1,9]%%%}+%%%{-210,[1,1,8]%%%}+%%%{-210,[0,1,7]%
%%},[8]%%
```

3.505.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{208\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{105}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{7\,x^3}+\frac{26\,a\,\sqrt{c-\frac{c}{a\,x}}}{35\,x^2}+\frac{104\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x}$$

input

```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

output

```
(208*a^3*(c - c/(a*x))^(1/2))/105 + (2*(c - c/(a*x))^(1/2))/(7*x^3) + (26*
a*(c - c/(a*x))^(1/2))/(35*x^2) + (104*a^2*(c - c/(a*x))^(1/2))/(105*x)
```

3.506 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.506.1 Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=4a^4\sqrt{c-\frac{c}{ax}}-\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

output
```
-14/3*a^4*(c-c/a/x)^(3/2)/c+18/5*a^4*(c-c/a/x)^(5/2)/c^2-10/7*a^4*(c-c/a/x
)^(7/2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4+4*a^4*(c-c/a/x)^(1/2)
```

3.506.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}(35+85ax+102a^2x^2+136a^3x^3+272a^4x^4)}{315x^4}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*a^4*x^
4))/(315*x^4)
```

3.506.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6717, 6683, 1070, 281, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x^5(1-ax)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x^5}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^5}dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^3}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{86}$$

$$-\frac{c\int \left(\frac{\left(c-\frac{c}{ax}\right)^{7/2}a^4}{c^4}-\frac{5\left(c-\frac{c}{ax}\right)^{5/2}a^4}{c^3}+\frac{9\left(c-\frac{c}{ax}\right)^{3/2}a^4}{c^2}-\frac{7\sqrt{c-\frac{c}{ax}}a^4}{c}+\frac{2a^4}{\sqrt{c-\frac{c}{ax}}}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(-\frac{2a^5\left(c-\frac{c}{ax}\right)^{9/2}}{9c^5}+\frac{10a^5\left(c-\frac{c}{ax}\right)^{7/2}}{7c^4}-\frac{18a^5\left(c-\frac{c}{ax}\right)^{5/2}}{5c^3}+\frac{14a^5\left(c-\frac{c}{ax}\right)^{3/2}}{3c^2}-\frac{4a^5\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

3.506. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}dx$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
-((c*((-4*a^5*Sqrt[c - c/(a*x)])/c + (14*a^5*(c - c/(a*x))^(3/2))/(3*c^2)
- (18*a^5*(c - c/(a*x))^(5/2))/(5*c^3) + (10*a^5*(c - c/(a*x))^(7/2))/(7*c
^4) - (2*a^5*(c - c/(a*x))^(9/2))/(9*c^5)))/a)
```

3.506.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.506.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

method	result
gosper	$\frac{2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4}$
trager	$\frac{2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)\sqrt{-\frac{-acx+c}{ax}}}{315x^4}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(272a^5x^5-136a^4x^4-34a^3x^3-17a^2x^2-50ax-35)}{315(ax-1)x^4}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\Bigg(-630\sqrt{a\,x^2-x}\,a^{\frac{11}{2}}x^6-630a^{\frac{11}{2}}\sqrt{(ax-1)x}\,x^6+1260(a\,x^2-x)^{\frac{3}{2}}a^{\frac{9}{2}}x^4+315\ln\Big(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\Big)a^5x^6-315\ln\Big(2}{315x^5\sqrt{(ax-1)x}\,\sqrt{}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/315*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35)*(c*(a*x-1)/a/x)^(1/2
)/x^4
```

3.506.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\frac{2\,(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35)\sqrt{\frac{acx-c}{ax}}}{315\,x^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt((a*c*x
- c)/(a*x))/x^4
```

3.506.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^5\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

3.506.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^5}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^5), x)
```

3.506.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{%%{[-315,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9]%%%},[10]%%%}
+%%%{%%{[
```

3.506.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{544\,a^4\,\sqrt{c-\frac{c}{a\,x}}}{315}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{9\,x^4}+\frac{34\,a\,\sqrt{c-\frac{c}{a\,x}}}{63\,x^3}+\frac{68\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x^2}+\frac{272\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{315\,x}$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

output
```
(544*a^4*(c - c/(a*x))^(1/2))/315 + (2*(c - c/(a*x))^(1/2))/(9*x^4) + (34*
a*(c - c/(a*x))^(1/2))/(63*x^3) + (68*a^2*(c - c/(a*x))^(1/2))/(105*x^2) +
 (272*a^3*(c - c/(a*x))^(1/2))/(315*x)
```

3.507 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

3.507.1 Optimal result

Integrand size = 27, antiderivative size = 313

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx &= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}}\\ &\quad+\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}}\\ &\quad+\frac{363\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}\\ &\quad-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output
```
363/64*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)-4*arct
anh(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/
2)+149/64*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+107/96*x^2
*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+17/24*x^3*(1+1/a/x)^(
1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/4*x^4*(1+1/a/x)^(1/2)*(c-c/a/x)^(
1/2)/(1-1/a/x)^(1/2)
```

3.507.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.81

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$= \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(447+214ax+136a^2x^2+48a^3x^3)}{-1+ax} - 1089\sqrt{c}\log(1-ax) + 768\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right) + 1089\sqrt{c}}{}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(447 + 214*a*x + 136*a
^2*x^2 + 48*a^3*x^3))/(-1 + a*x) - 1089*Sqrt[c]*Log[1 - a*x] + 768*Sqrt[2]
*Sqrt[c]*Log[(-1 + a*x)^2] + 1089*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^
2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 768*Sqrt[2]*Sq
rt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^
2 + c*(-1 - 2*a*x + 3*a^2*x^2)])/(384*a^4)
```

3.507.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.60, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {6733, 585, 27, 109, 27, 168, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6733}$$
$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$
$$\Big\downarrow \text{585}$$
$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^5}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$
$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^5}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 109$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(17a+\frac{15}{x}\right)x^4}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(17a+\frac{15}{x}\right)x^4}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 168$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(107a+\frac{85}{x}\right)x^3}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(107a+\frac{85}{x}\right)x^3}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 168$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{-\frac{\int -\frac{3\left(149a+\frac{107}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

3.507. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\int \frac{\left(149a+\frac{107}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 168$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(-\frac{\int -\frac{\left(363a+\frac{149}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{\int \frac{\left(363a+\frac{149}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 174$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{512\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+363\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 73$$

3.507. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{1024a\int\frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+726a\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{512\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-726\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*(-1/4*(Sqrt[1 + 1/(a*x)]*x^4)/a + ((-17*Sqrt[1 +
1/(a*x)]*x^3)/3 + ((-107*Sqrt[1 + 1/(a*x)]*x^2)/2 + (3*(-149*Sqrt[1 + 1/(a
*x)]*x + (-726*ArcTanh[Sqrt[1 + 1/(a*x)]] + 512*Sqrt[2]*ArcTanh[Sqrt[1 + 1
/(a*x)]/Sqrt[2]])/(2*a)))/(4*a))/(6*a))/(8*a^2)))/Sqrt[c - c/(a*x)])
```

3.507.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.507. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

3.507.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.72

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(96\sqrt{(ax+1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^{3}+272\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^{2}+428\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+894\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-768\sqrt{2}\right.}{384\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{9}{2}}\sqrt{(ax+1)x}\sqrt{\frac{1}{a}}}$
risch	$\frac{\left(48a^{3}x^{3}+136a^{2}x^{2}+214ax+447\right)x\sqrt{\frac{c(ax-1)}{ax}}}{192a^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{363\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{128a^{3}\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3}}{x-\frac{1}{a}}\right.}{a^{4}\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(96*
((a*x+1)*x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x^3+272*((a*x+1)*x)^(1/2)*a^(7/2)*(1
/a)^(1/2)*x^2+428*((a*x+1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x+894*((a*x+1)*x)^
(1/2)*a^(3/2)*(1/a)^(1/2)-768*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+1089*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(9/2)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.81

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^{3}\,dx$$
$$=\left[\frac{768\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+1089\,(ax-1)\sqrt{c}}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

output
```
[1/768*(768*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 1
3*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))
+ 1089*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(48*a^5*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 4
47*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4),
1/384*(768*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a
*c*x - c)) - 1089*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
+ 2*(48*a^5*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

3.507.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

3.507.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.507.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{x^3\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.508 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.508.1 Optimal result

Integrand size = 27, antiderivative size = 261

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}}+\frac{45\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

output
```
45/8*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)-4*arctan
h(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)
+19/8*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+13/12*x^2*(1+1
/a/x)^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/3*x^3*(1+1/a/x)^(1/2)*(c-c
/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

3.508.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(57+26ax+8a^2x^2)}{-1+ax}-135\sqrt{c}\log(1-ax)+96\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right)+135\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{}\right.}{}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(57 + 26*a*x + 8*a^2*x
^2))/(-1 + a*x) - 135*Sqrt[c]*Log[1 - a*x] + 96*Sqrt[2]*Sqrt[c]*Log[(-1 +
a*x)^2] + 135*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(
a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 96*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a
^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3
*a^2*x^2)])/(48*a^3)
```

3.508.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.61, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6733, 585, 27, 109, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^4}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^4}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 109$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(13a+\frac{11}{x}\right)x^3}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(13a+\frac{11}{x}\right)x^3}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 168$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{3\left(19a+\frac{13}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\int \frac{\left(19a+\frac{13}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 168$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(-\frac{\int -\frac{\left(45a+\frac{19}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

3.508. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{\int \frac{\left(45a+\frac{19}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 174$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{64\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+45\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{128a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+90a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 221$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{64\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-90\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

3.508. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*(-1/3*(Sqrt[1 + 1/(a*x)]*x^3)/a + ((-13*Sqrt[1 +
1/(a*x)]*x^2)/2 + (3*(-19*Sqrt[1 + 1/(a*x)]*x + (-90*ArcTanh[Sqrt[1 + 1/(a
*x)]] + 64*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a)))/(4*a))/(6*a
^2)))/Sqrt[c - c/(a*x)])
```

3.508.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

3.508. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.508.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.77

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{(ax+1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^{2}+52\sqrt{(ax+1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+114\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-96\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+}{ax-1}\right.\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{7}{2}}\sqrt{(ax+1)x}\sqrt{\frac{1}{a}}}$
risch	$\frac{(8a^{2}x^{2}+26ax+57)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{45\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{16a^{2}\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a^{3}\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*(
(a*x+1)*x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2+52*((a*x+1)*x)^(1/2)*a^(5/2)*(1/a
)^(1/2)*x+114*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-96*2^(1/2)*ln((2*2^(1/
2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+135*ln(1/2*(2
*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(7/2)/((a*x+
1)*x)^(1/2)/(1/a)^(1/2)
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.11

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{96\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+135\,(ax-1)\sqrt{c}\,\mathrm{lo}}{96\,(}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

output
```
[1/96*(96*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*
a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) +
135*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x
^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(8*a^4*x^4 + 34*a^3*x^3 + 83*a^2*x^2 + 57*a*x)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(96*sqrt(2)*(a*x
 - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 135*(a*x -
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 + 34*a^3*
x^3 + 83*a^2*x^2 + 57*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^4*x - a^3)]
```

3.508.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a/x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.508.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.508.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.508.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int \frac{x^2\sqrt{c-\frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.509 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

3.509.1 Optimal result

Integrand size = 25, antiderivative size = 209

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=&\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}}\\&+\frac{23\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}\\&-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output
```
23/4*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)-4*arctan
h(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)
+9/4*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/2*x^2*(1+1/a/x)
^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

3.509.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(9+2ax)}{-1+ax}-23\sqrt{c}\log(1-ax)+16\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right)+23\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\right.}{}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(9 + 2*a*x))/(-1 + a*x
) - 23*Sqrt[c]*Log[1 - a*x] + 16*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + 23*Sq
rt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-
1 - a*x + 2*a^2*x^2)] - 16*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)])/(8*a
^2)
```

3.509.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6733, 585, 27, 109, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^3}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^3}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ \mathbf{109}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(9a+\frac{7}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ \mathbf{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(9a+\frac{7}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ \mathbf{168}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(23a+\frac{9}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ \mathbf{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(23a+\frac{9}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ \mathbf{174}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{32\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+23\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ \mathbf{73}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{64a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+46a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

3.509. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

$$\Big\downarrow \text{221}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-46\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*(-1/2*(Sqrt[1 + 1/(a*x)]*x^2)/a + (-9*Sqrt[1 + 1/
(a*x)]*x + (-46*ArcTanh[Sqrt[1 + 1/(a*x)]] + 32*Sqrt[2]*ArcTanh[Sqrt[1 + 1
/(a*x)]/Sqrt[2]])/(2*a))/(4*a^2)))/Sqrt[c - c/(a*x)])
```

3.509.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.509. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

3.509.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, x \left(4\sqrt{(ax+1)x}\, a^{\frac{5}{2}} \sqrt{\frac{1}{a}}\, x+18\sqrt{(ax+1)x}\, a^{\frac{3}{2}} \sqrt{\frac{1}{a}}-16\sqrt{2}\, \ln\left(\frac{2\sqrt{2}\, \sqrt{\frac{1}{a}}\, \sqrt{(ax+1)x}\, a+3ax+1}{ax-1}\right)\sqrt{a}+23\ln(\frac{2\sqrt{(ax+1)x}\,\sqrt{}}{2\sqrt{a}}\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{5}{2}}\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{(2ax+9)x\sqrt{\frac{c(ax-1)}{ax}}}{4a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{23\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{8a\sqrt{a^2c}}-\frac{2\sqrt{2}\, \ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a^2\sqrt{c}}\right)\sqrt{\frac{c(ax-}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a
*x+1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x+18*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1
/2)-16*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x
-1))*a^(1/2)+23*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1
/a)^(1/2))/a^(5/2)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

3.509.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.56

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$

$$=\left[\frac{16\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+23\,(ax-1)\sqrt{c}\log}{16\,(a^3x-}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

output
```
[1/16*(16*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*
a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) +
23*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^
2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a
*x - 1)) + 4*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*(16*sqrt(2)*(a*x - 1)*sqrt(-c)*a
rctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 23*(a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

3.509.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

3.509.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.509.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.510 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.510.1 Optimal result

Integrand size = 24, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\,\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

output
```
5*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(1/2
*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+x*(1+1
/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

3.510.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x+\frac{5\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (5*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
 - (4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[1 - 1/(a*x)]
```

3.510.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6731, 585, 27, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(5a+\frac{3}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(5a+\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 174$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{16a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+10a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-10\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output

```
-((a*c*Sqrt[1 - 1/(a*x)]*(-((Sqrt[1 + 1/(a*x)]*x)/a) + (-10*ArcTanh[Sqrt[1
 + 1/(a*x)]] + 8*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a^2)))/Sqr
t[c - c/(a*x)])
```

3.510. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.510.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.510. $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.510.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a
*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2
*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1
)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/((a*x+1)*x)^(1/2)/a^(3/2)/(1/a)^(1
/2)
```

3.510.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.37

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$=\left[\frac{4\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+5\,(ax-1)\sqrt{c}\log}{4\,(a^2x-a)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*
c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*
(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x
- 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2
*c*x^2 - 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c
*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a)]
```

3.510.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

3.510.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.510.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.511 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.511.1 Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
2*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(1/2*(
1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2*(1+1/a/x
)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

3.511.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.49

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x}{-1+ax}-\sqrt{c}\log(1-ax) \\ +2\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right) \\ +\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax+2a^2x^2)\right) \\ -2\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 \right. \\ \left. +c(-1-2ax+3a^2x^2)\right)$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x)/(-1 + a*x) - Sqrt[c]*Log[1
 - a*x] + 2*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + Sqrt[c]*Log[2*a^2*Sqrt[c]*
Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] -
2*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
 c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

3.511.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6733, 585, 27, 95, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow 585$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 95$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\int -\frac{\left(a+\frac{3}{x}\right)x}{a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 25$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\int \frac{\left(a+\frac{3}{x}\right)x}{a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(a+\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 174$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{4\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 73$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow 221$$

3.511. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}dx$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

output

```
-((a*c*Sqrt[1 - 1/(a*x)]*((-2*Sqrt[1 + 1/(a*x)])/a + (-2*ArcTanh[Sqrt[1 +
1/(a*x)]] + 4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[c - c/(
a*x)])
```

3.511.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

3.511. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.511.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,ax-2\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x+2\sqrt{(ax+1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2
*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*(1/a)^(1/2)*a*x-2*a^(1/2)*2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+2*(
(a*x+1)*x)^(1/2)*a^(1/2)*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2
)
```

3.511.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.36

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$

$$= \left[\frac{2\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+(ax-1)\sqrt{c}\log\Bigg(-}{2\,(ax-1)}\right.$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas
")
```

output

```
[1/2*(2*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*
c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + (a
*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a
*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) + 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x
- 1), (2*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(
-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c
*x - c)) - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

3.511.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Timed out}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x,x)
```

output

```
Timed out
```

3.511.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.511.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.512 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

3.512.1 Optimal result

Integrand size = 27, antiderivative size = 125

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-4*a*arctanh(1/2*c^(1/2)*(1-1
/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)*c^(1/2)+4*a*c*(1-1/a^2/x^
2)^(1/2)/(c-c/a/x)^(1/2)
```

3.512.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.24

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2a\left(\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1+7ax)+3\sqrt{2}\sqrt{c}(-1+ax)\log\left((-1+ax)^2\right)-3\sqrt{2}\sqrt{c}(-1+ax)\log\left(2\sqrt{2}a^2\right.\right.}{-3+3ax}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
(2*a*(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + 7*a*x) + 3*Sqrt[2]*Sqrt
[c]*(-1 + a*x)*Log[(-1 + a*x)^2] - 3*Sqrt[2]*Sqrt[c]*(-1 + a*x)*Log[2*Sqrt
[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*
x + 3*a^2*x^2)]))/(-3 + 3*a*x)
```

3.512.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6733, 466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{466}$$

$$-c^3\left(\frac{2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{466}$$

$$-c^3\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{471}$$

$$-c^3\left(\frac{2\left(-\frac{4\int \frac{1}{\frac{c^2}{a^2x^2}-\frac{2c}{a^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow 221$$

$$-c^3\left(\frac{2\left(\frac{2\sqrt{2}a\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{c^{3/2}}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

input

```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

output

```
-(c^3*((-2*a*(1 - 1/(a^2*x^2))^(3/2))/(3*c*(c - c/(a*x))^(3/2)) + (2*((-2*
a*Sqrt[1 - 1/(a^2*x^2)])/(c*Sqrt[c - c/(a*x)]) + (2*Sqrt[2]*a*ArcTanh[(Sqr
t[c]*Sqrt[1 - 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])])/c^(3/2)))/c))
```

3.512.3.1 Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

3.512. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.512.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-3a\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{2}+7x\sqrt{(ax+1)x}\,a\sqrt{\frac{1}{a}}+\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$	140
risch	$\frac{2\left(7a^{2}x^{2}+8ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	180

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-3*a*2^
(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+
7*x*((a*x+1)*x)^(1/2)*a*(1/a)^(1/2)+((a*x+1)*x)^(1/2)*(1/a)^(1/2))/x/((a*x
+1)*x)^(1/2)/(1/a)^(1/2)
```

3.512.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.82

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$

$$= \left[\frac{3\sqrt{2}(a^2x^2-ax)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(7\,a^2x^2+8\,ax}{3\,(ax^2-x)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fric
as")
```

output
```
[1/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 -
 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
) + 2*(7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a*x^2 - x), 2/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*arctan(2*sqrt(
2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x)]
```

3.512.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**2,x)
```

output
```
Timed out
```

3.512.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.512.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.512.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.513 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.513.1 Optimal result

Integrand size = 27, antiderivative size = 170

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
2/5*a^2*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^2*c^2*(1-1/a^2/x^2)^
(3/2)/(c-c/a/x)^(3/2)-4*a^2*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2
)/(c-c/a/x)^(1/2))*2^(1/2)*c^(1/2)+4*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(
1/2)
```

3.513.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = {} & \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(3+11ax+38a^2x^2)}{15x(-1+ax)} \\ & + 2\sqrt{2}a^2\sqrt{c}\log\left((-1+ax)^2\right) \\ & - 2\sqrt{2}a^2\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2\right. \\ & \left. + c\left(-1-2ax+3a^2x^2\right)\right)\end{aligned}$$

input

```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

output

```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 11*a*x + 38*a^2*x^2))/(1
5*x*(-1 + a*x)) + 2*Sqrt[2]*a^2*Sqrt[c]*Log[(-1 + a*x)^2] - 2*Sqrt[2]*a^2*
Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*
x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

3.513.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6733, 572, 466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}x}d\frac{1}{x}$$

$$\Big\downarrow \text{572}$$

$$-c^3\left(a\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \text{466}$$

$$-c^3\left(a\left(\frac{2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \text{466}$$

$$-c^3\left(a\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \text{471}$$

$$-c^3\left(a\left(\frac{2\left(-\frac{4\int \frac{1}{\frac{c^2}{a^2x^2}-\frac{2c}{a^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$-c^3\left(a\left(\frac{2\left(\frac{2\sqrt{2}a\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{c^{3/2}}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
-(c^3*((-2*a^2*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) + a*((-2*a
*(1 - 1/(a^2*x^2))^(3/2))/(3*c*(c - c/(a*x))^(3/2)) + (2*((-2*a*Sqrt[1 - 1
/(a^2*x^2)])/(c*Sqrt[c - c/(a*x)]) + (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Sqrt[1
- 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])])/c^(3/2)))/c)))
```

3.513. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.513.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.513.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

method	result
default	$-\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(15a^2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^3-38a^2\sqrt{\frac{1}{a}}\,x^2\sqrt{(ax+1)x}-11x\sqrt{(ax+1)x}\,a\sqrt{\frac{1}{a}}-3\sqrt{(ax+1)x}\,\sqrt{}\right.}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^2\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2\left(38a^3x^3+49a^2x^2+14ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)ac}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

3.513. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^2*(1
5*a^2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-
1))*x^3-38*a^2*(1/a)^(1/2)*x^2*((a*x+1)*x)^(1/2)-11*x*((a*x+1)*x)^(1/2)*a*
(1/a)^(1/2)-3*((a*x+1)*x)^(1/2)*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

3.513.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.24

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$

$$= \left[\frac{15\sqrt{2}(a^3x^3-a^2x^2)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(38\,a^3x^3+}{15\,(ax^3-x^2)} \right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fric
as")
```

output
```
[1/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)) + 2*(38*a^3*x^3 + 49*a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a*x^3 - x^2), 2/15*(15*sqrt(2)*(a^3*x^3 - a^2
*x^2)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (38*a^3*x^
3 + 49*a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a*x^3 - x^2)]
```

3.513.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**3,x)
```

output
```
Timed out
```

3.513.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.513.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.513.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.514 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.514.1 Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
4/7*a^3*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^3*c^2*(1-1/a^2/x^2)^
(3/2)/(c-c/a/x)^(3/2)-2/7*a^3*c^2*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(3/2)-4*a^
3*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)
*c^(1/2)+4*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.514.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(3+9ax+16a^2x^2+52a^3x^3)}{21x^2(-1+ax)} + 2\sqrt{2}a^3\sqrt{c}\log\left((-1+ax)^2\right) - 2\sqrt{2}a^3\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c\left(-1-2ax+3a^2x^2\right)\right)$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 9*a*x + 16*a^2*x^2 + 52*
a^3*x^3))/(21*x^2*(-1 + a*x)) + 2*Sqrt[2]*a^3*Sqrt[c]*Log[(-1 + a*x)^2] -
2*Sqrt[2]*a^3*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt
[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

3.514.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6733, 581, 27, 672, 466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{581}$$

$$-c^3\left(\frac{2a^2\int -\frac{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a-\frac{10}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{7c^2}+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow 27$$

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\int\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a-\frac{10}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}\right)$$

$$\Big\downarrow 672$$

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\int\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}\right)\right)$$

$$\Big\downarrow 466$$

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\left(\frac{2\int\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)\right)$$

$$\Big\downarrow 466$$

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right.\right.\right.$$

$$\Big\downarrow 471$$

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\left(\frac{2\left(-\frac{4\int\frac{1}{\frac{c^2}{a^2x^2}-\frac{2c}{a^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3}}{3c\left(c-\frac{c}{ax}\right)^{3/}}\right.\right.\right.$$

$$\Big\downarrow 221$$

3.514. $\int\frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx$

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\left(\frac{2\left(\frac{2\sqrt{2}a\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{c^{3/2}}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{}{a^2}\right.}{3c\left(c-\right.}\right.\right.\right.$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
-(c^3*((2*a^3*(1 - 1/(a^2*x^2))^(5/2))/(7*c*(c - c/(a*x))^(3/2)) - (a*((4*
a^2*(1 - 1/(a^2*x^2))^(5/2))/(c - c/(a*x))^(5/2) - 7*a*((-2*a*(1 - 1/(a^2*
x^2))^(3/2))/(3*c*(c - c/(a*x))^(3/2)) + (2*((-2*a*Sqrt[1 - 1/(a^2*x^2)])/
(c*Sqrt[c - c/(a*x)]) + (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)
])/(Sqrt[2]*Sqrt[c - c/(a*x)])])/c^(3/2)))/c)))/7))
```

3.514.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

---

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.514.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89

method	result
default	$-\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\Bigg(21a^3\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^4-52a^3\sqrt{\frac{1}{a}}\,x^3\sqrt{(ax+1)x}-16a^2\sqrt{\frac{1}{a}}\,x^2\sqrt{(ax+1)x}-9x\sqrt{(ax+1)}}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^3\sqrt{(ax+1)x}\sqrt{\frac{1}{a}}}$
risch	$\frac{2(52a^4x^4+68a^3x^3+25a^2x^2+12ax+3)\sqrt{\frac{c(ax-1)}{ax}}}{21x^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^3\sqrt{2}\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(}}{\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/21/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^3*(2
1*a^3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-
1))*x^4-52*a^3*(1/a)^(1/2)*x^3*((a*x+1)*x)^(1/2)-16*a^2*(1/a)^(1/2)*x^2*((
a*x+1)*x)^(1/2)-9*x*((a*x+1)*x)^(1/2)*a*(1/a)^(1/2)-3*((a*x+1)*x)^(1/2)*(1
/a)^(1/2))/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.90

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= \Bigg[\frac{21\sqrt{2}(a^4x^4-a^3x^3)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(52\,a^4x^4+}{21\,(ax^4-x^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fric
as")
```

output
```
[1/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)) + 2*(52*a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3), 2/21*(21*sqrt(2)*(
a^4*x^4 - a^3*x^3)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c))
 + (52*a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3)]
```

3.514.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**4,x)
```

output
```
Timed out
```

3.514.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.514.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.515 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.515.1 Optimal result

Integrand size = 27, antiderivative size = 303

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^4\sqrt{c-\frac{c}{ax}}\,\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
2/3*a^4*(1+1/a/x)^(3/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2/5*a^4*(1+1/a/x)^
(5/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-2/7*a^4*(1+1/a/x)^(7/2)*(c-c/a/x)^(1
/2)/(1-1/a/x)^(1/2)+2/9*a^4*(1+1/a/x)^(9/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2
)-4*a^4*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/(1-1/
a/x)^(1/2)+4*a^4*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

3.515.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(35+95ax+138a^2x^2+236a^3x^3+788a^4x^4)}{315x^3(-1+ax)}$$
$$+2\sqrt{2}a^4\sqrt{c}\log\left((-1+ax)^2\right)$$
$$-2\sqrt{2}a^4\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-2ax+3a^2x^2)\right)$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(35 + 95*a*x + 138*a^2*x^2 +
236*a^3*x^3 + 788*a^4*x^4))/(315*x^3*(-1 + a*x)) + 2*Sqrt[2]*a^4*Sqrt[c]*L
og[(-1 + a*x)^2] - 2*Sqrt[2]*a^4*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1
- 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

3.515.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6733, 581, 27, 2170, 27, 672, 466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x^5}\,dx$$
$$\Big\downarrow \text{6733}$$
$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}x^3}d\frac{1}{x}$$
$$\Big\downarrow \text{581}$$

$$-c^3\left(-\frac{2a^3\int -\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(-\frac{11c^3}{ax}+\frac{19c^3}{a^2x^2}+c^3\right)}{2\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \ 27$$

$$-c^3\left(\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(-\frac{11c^3}{ax}+\frac{19c^3}{a^2x^2}+c^3\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \ 2170$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a^4\int \frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(50a-\frac{113}{x}\right)}{2a^5\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{7c^2}\right)}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \ 27$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(50a-\frac{113}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{7a}\right)}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \ 672$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\left(\frac{226a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}-63a\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}\right)}{7a}\right)}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

3.515. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

$$\Big\downarrow \text{466}$$

$$-c^3 \left(\frac{a^3 \left(\frac{38 a c^2 \left(1-\frac{1}{a^2 x^2}\right)^{5/2}}{7 \left(c-\frac{c}{a x}\right)^{3/2}}-\frac{c^3 \left(\frac{226 a^2 \left(1-\frac{1}{a^2 x^2}\right)^{5/2}}{5 \left(c-\frac{c}{a x}\right)^{5/2}}-63 a \left(\frac{2 \int \frac{\sqrt{1-\frac{1}{a^2 x^2}}}{\left(c-\frac{c}{a x}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{2 a \left(1-\frac{1}{a^2 x^2}\right)^{3/2}}{3 c \left(c-\frac{c}{a x}\right)^{3/2}}\right)\right)}{7 a}\right)}{9 c^3}-\frac{2 a^4 \left(1-\frac{1}{a^2 x^2}\right)^{5/2}}{9 c^2 \sqrt{c-\frac{c}{a x}}}\right)$$

$$\Big\downarrow \text{466}$$

$$-c^3 \left(\frac{a^3 \left(\frac{38 a c^2 \left(1-\frac{1}{a^2 x^2}\right)^{5/2}}{7 \left(c-\frac{c}{a x}\right)^{3/2}}-\frac{c^3 \left(\frac{226 a^2 \left(1-\frac{1}{a^2 x^2}\right)^{5/2}}{5 \left(c-\frac{c}{a x}\right)^{5/2}}-63 a \left(\frac{2 \left(\frac{2 \int \frac{1}{\sqrt{1-\frac{1}{a^2 x^2}} \sqrt{c-\frac{c}{a x}}}d\frac{1}{x}}{c}-\frac{2 a \sqrt{1-\frac{1}{a^2 x^2}}}{c \sqrt{c-\frac{c}{a x}}}\right)}{c}-\frac{2 a \left(1-\frac{1}{a^2 x^2}\right)^{3/2}}{3 c \left(c-\frac{c}{a x}\right)^{3/2}}\right)\right)}{7 a}\right)}{9 c^3}-2a\right.$$

3.515. $\int \frac{e^{3 \coth^{-1}(a x)} \sqrt{c-\frac{c}{a x}}}{x^5} \, dx$

$$\Big\downarrow \ 471$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\left(\frac{226a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}-63a\left(\frac{2\left(-\frac{4\int\frac{1}{\frac{c^2}{a^2x^2}-\frac{2c}{a^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)}{7a}\right)}{9c^3}-\right.$$

$$\Big\downarrow \ 221$$

3.515. $\displaystyle\int\frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\left(\frac{226a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}-63a\left(\frac{2\left(\frac{2\sqrt{2}a\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{c^{3/2}}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)}{7a}\right)}{9c^3}\right.$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

3.515. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

output
```
-(c^3*((-2*a^4*(1 - 1/(a^2*x^2))^(5/2))/(9*c^2*Sqrt[c - c/(a*x)]) + (a^3*(
(38*a*c^2*(1 - 1/(a^2*x^2))^(5/2))/(7*(c - c/(a*x))^(3/2)) - (c^3*((226*a^
2*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) - 63*a*((-2*a*(1 - 1/(a
^2*x^2))^(3/2))/(3*c*(c - c/(a*x))^(3/2)) + (2*((-2*a*Sqrt[1 - 1/(a^2*x^2)
])/(c*Sqrt[c - c/(a*x)]) + (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x
^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])])/c^(3/2)))/c)))/(7*a)))/(9*c^3)))
```

3.515.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

3.515. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.515.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.68

method	result
risch	$\frac{2\left(788a^5x^5+1024a^4x^4+374a^3x^3+233a^2x^2+130ax+35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^4\sqrt{2}\,\ln\bigg(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2}}{x-\frac{1}{a}}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\left(-315a^4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^5+788x^4\sqrt{(ax+1)x}\,a^4\sqrt{\frac{1}{a}}+236a^3\sqrt{\frac{1}{a}}\,x^3\sqrt{(ax+1)x}+138a^2\sqrt{\frac{1}{a}}\right.}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^4\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

3.515. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

output
```
2/315*(788*a^5*x^5+1024*a^4*x^4+374*a^3*x^3+233*a^2*x^2+130*a*x+35)/x^4/((
a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)-2*a^4*2^(1/2)/c^(1/2)*
ln((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x-1/a)^2+3*(x-1/a)*a*c+2*c
)^(1/2))/(x-1/a))/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*((
a*x+1)*a*c*x)^(1/2)
```

3.515.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.36

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$

$$= \left[\frac{315\sqrt{2}(a^5x^5-a^4x^4)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(788\,a^5x^5}{315\,(ax^5-x^4)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fric
as")
```

output
```
[1/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2
*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((
a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*
a*x - 1)) + 2*(788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2 + 13
0*a*x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5 - x^
4), 2/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a
^2*c*x^2 - 2*a*c*x - c)) + (788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233
*a^2*x^2 + 130*a*x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
)/(a*x^5 - x^4)]
```

3.515.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**5,x)
```

output
```
Timed out
```

3.515.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.515.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.516 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx$

3.516.1 Optimal result

Integrand size = 27, antiderivative size = 126

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}}-\frac{(3+4m)\sqrt{c-\frac{c}{ax}}x^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

output
```
-1/2*(3+4*m)*x^m*hypergeom([1/2, -m],[1-m],-1/a/x)*(c-c/a/x)^(1/2)/a/m/(1+
m)/(1-1/a/x)^(1/2)+x^(1+m)*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1+m)/(1-1/a/x)
^(1/2)
```

3.516.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \frac{\sqrt{c-\frac{c}{ax}}x^m\left(2am\sqrt{1+\frac{1}{ax}}x-(3+4m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a*x)]*x^m*(2*a*m*Sqrt[1 + 1/(a*x)]*x - (3 + 4*m)*Hypergeometr
ic2F1[1/2, -m, 1 - m, -(1/(a*x))]))/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

3.516.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6736, 6735, 27, 88, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \textit{6736}$$

$$\frac{\sqrt{c-\frac{c}{ax}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^mdx}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textit{6735}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textit{27}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textit{88}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m\sqrt{c-\frac{c}{ax}}\left(-\frac{(4m+3)\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2(m+1)}-\frac{a\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-m-1}}{m+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textit{74}$$

$$-\frac{\left(\frac{1}{x}\right)^m x^m \sqrt{c-\frac{c}{ax}}\left(\frac{(4m+3)\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)}{2m(m+1)}-\frac{a\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-m-1}}{m+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

output

```
-((Sqrt[c - c/(a*x)]*(x^(-1))^m*x^m*(-((a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(-1 -
 m))/(1 + m)) + ((3 + 4*m)*Hypergeometric2F1[1/2, -m, 1 - m, -(1/(a*x))])/
(2*m*(1 + m)*(x^(-1))^m)))/(a*Sqrt[1 - 1/(a*x)]))
```

3.516.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 88

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 6735

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/
2)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p
}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
 0]) &&  !IntegerQ[m]
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.516.4 Maple [F]

$$\int x^m \sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

3.516.5 Fricas [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \int \sqrt{c-\frac{c}{ax}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
integral(x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)), x)
```

3.516.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx = \text{Timed out}$$

input
```
integrate(x**m*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.516.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int \sqrt{c-\frac{c}{ax}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.516.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int \sqrt{c-\frac{c}{ax}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^m\,dx=\int x^m\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x^m*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^m*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.517 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.517.1 Optimal result

Integrand size = 27, antiderivative size = 164

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

output
```
-11/8*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^3+11/
8*c*x*(1-1/a^2/x^2)^(1/2)/a^2/(c-c/a/x)^(1/2)-11/12*c*x^2*(1-1/a^2/x^2)^(1
/2)/a/(c-c/a/x)^(1/2)+1/3*c*x^3*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.517.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(33-22ax+8a^2x^2)}{-1+ax}+33\sqrt{c}\log(1-ax)-33\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-\right.}{48a^3}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(33 - 22*a*x + 8*a^2*x
^2))/(-1 + a*x) + 33*Sqrt[c]*Log[1 - a*x] - 33*Sqrt[c]*Log[2*a^2*Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(4
8*a^3)
```

3.517.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6733, 580, 579, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{c-\frac{c}{ax}} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2} x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{580}$$

$$-\frac{-\frac{11c\int \frac{\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{579}$$

$$-\frac{-\frac{11c\left(-\frac{3\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{579}$$

$$-\frac{-\frac{11c\left(-\frac{3\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \ 573$$

$$-\frac{-\frac{11c\left(-\frac{3\left(\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{11c\left(-\frac{3\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

output
```
-((-1/3*(c^2*Sqrt[1 - 1/(a^2*x^2)]*x^3)/Sqrt[c - c/(a*x)] - (11*c*(-1/2*(c
*Sqrt[1 - 1/(a^2*x^2)]*x^2)/Sqrt[c - c/(a*x)] - (3*(-((c*Sqrt[1 - 1/(a^2*x
^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2
)])/Sqrt[c - c/(a*x)]])/a))/(4*a)))/(6*a))/c)
```

3.517. $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.517.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.517. $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.517.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{5}{2}}x^{2}\sqrt{(ax+1)x}-44a^{\frac{3}{2}}x\sqrt{(ax+1)x}+66\sqrt{(ax+1)x}\,\sqrt{a}-33\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{48a^{\frac{5}{2}}(ax-1)\sqrt{(ax+1)x}}$	133
risch	$\frac{(8a^2x^2-22ax+33)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{24a^2(ax-1)}-\frac{11\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{16a^2\sqrt{a^2c}\,(ax-1)}$	160

input
```
int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^(5/2)*x
^2*((a*x+1)*x)^(1/2)-44*a^(3/2)*x*((a*x+1)*x)^(1/2)+66*((a*x+1)*x)^(1/2)*a
^(1/2)-33*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(5/2)/(
a*x-1)/((a*x+1)*x)^(1/2)
```

3.517.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.05

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{33\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4-14\,a^3x^3+11\,a^2x^2+}{96\,(a^4x-a^3)}\right.$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
[1/96*(33*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(8*a^4*x^4 - 14*a^3*x^3 + 11*a^2*x^2 + 33*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(33*(a*x -
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 14*a^3*
x^3 + 11*a^2*x^2 + 33*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^4*x - a^3)]
```

3.517.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.517.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\sqrt{c-\frac{c}{ax}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.517.8 Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int x^2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.518 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

3.518.1 Optimal result

Integrand size = 25, antiderivative size = 124

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

output
```
7/4*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^2-7/4*c
*x*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/2*c*x^2*(1-1/a^2/x^2)^(1/2)/(c-
c/a/x)^(1/2)
```

3.518.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12

$$\begin{aligned}&\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx\\&=\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-7+2ax)}{-4+4ax}-\frac{7\sqrt{c}\log(1-ax)}{8a^2}\\&\quad+\frac{7\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax+2a^2x^2)\right)}{8a^2}\end{aligned}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

output

```
(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-7 + 2*a*x))/(-4 + 4*a*x) -
(7*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (7*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2)
```

3.518.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6733, 580, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{580}$$

$$-\frac{-\frac{7c\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a}-\frac{c^2x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{579}$$

$$-\frac{-\frac{7c\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{c^2x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{573}$$

$$-\frac{-\frac{7c\left(\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{c^2x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow 219$$

$$-\frac{-\frac{7c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{c^2x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{c}$$

input

```
Int[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

output

```
-((-1/2*(c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/Sqrt[c - c/(a*x)] - (7*c*(-((c*Sqr
t[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[
1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a))/(4*a))/c)
```

3.518.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573

```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 579

```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.518.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{3}{2}}x\sqrt{(ax+1)x}-14\sqrt{(ax+1)x}\,\sqrt{a}+7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{8a^{\frac{3}{2}}(ax-1)\sqrt{(ax+1)x}}$	116
risch	$\frac{(2ax-7)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{4a(ax-1)}+\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{8a\sqrt{a^{2}c}\,(ax-1)}$	152

input
```
int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(4*a^(
3/2)*x*((a*x+1)*x)^(1/2)-14*((a*x+1)*x)^(1/2)*a^(1/2)+7*ln(1/2*(2*((a*x+1)
*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)^(1/2)
```

3.518.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^3x^3-5\,a^2x^2-7\,ax)\sqrt{\frac{ax-1}{ax+1}}}{16\,(a^3x-a^2)}\right.$$
$$\left.-\frac{7\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(2\,a^3x^3-5\,a^2x^2-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8\,(a^3x-a^2)}\right]$$

input
```
integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(7*(a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

3.518.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

3.518.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\sqrt{c-\frac{c}{ax}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.518.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\sqrt{c-\frac{c}{ax}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.518.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int x\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.519 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.519.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1
/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.519.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x-\frac{3\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
))/Sqrt[1 - 1/(a*x)]
```

3.519.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6731, 580, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{580}$$

$$-\frac{-\frac{3c\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{573}$$

$$-\frac{\frac{3c^2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

output
```
-((-((c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (3*c^(3/2)*ArcTanh
[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a)/c)
```

3.519.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.519.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$	101
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,(ax-1)}$	139

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)
^(1/2)*a^(1/2)-3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a
*x-1)/((a*x+1)*x)^(1/2)/a^(1/2)
```

3.519.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\right.$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a)]
```

3.519.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

3.519.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.519.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.520 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.520.1 Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
2*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)+2*c*(1-1/a^
2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

3.520.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x+\sqrt{c}(1-ax)\log(1-ax)+\sqrt{c}(-1+ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-\right.}{-1+ax}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
 a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c
- c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

3.520.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6733, 574, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{574}$$

$$-\frac{c\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{573}$$

$$-\frac{-2c^2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

output
```
-(((-2*c^2*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - 2*c^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/c)
```

3.520. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}dx$

3.520.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 574
```
Int[((e_.)*(x_))^(n_)*((c_) + (d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^2*(e*x)^(n + 1)*(c + d*x)^(m - 2)*((a + b*x^2)^(p + 1)/
(b*e*(n + p + 2))), x] + Simp[c*((2*n + p + 3)/(n + p + 2))   Int[(e*x)^n*(
c + d*x)^(m - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x
] && EqQ[b*c^2 + a*d^2, 0] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && IntegerQ
[2*p]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.520.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+2\sqrt{(ax+1)x}\,\sqrt{a}\right)}{(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$	100
risch	$\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{a\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{\sqrt{a^{2}c}\,(ax-1)}$	139

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*((a*x+1)*
x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+2*((a*x+1)*x)^(1/2)*a^(1/2))/(a*x-1
)/((a*x+1)*x)^(1/2)/a^(1/2)
```

3.520.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.62

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) -
2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

3.520.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x, x)
```

3.520.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.520.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

output
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.521 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

3.521.1 Optimal result

Integrand size = 27, antiderivative size = 70

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}$$

output
```
-8/3*a*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-2/3*a*(1-1/a^2/x^2)^(1/2)*(c-
c/a/x)^(1/2)
```

3.521.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-1+5ax)}{-3+3ax}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 5*a*x))/(-3 + 3*a*x)
```

3.521.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6733, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{459}$$

$$-\frac{\frac{4}{3}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{c}$$

$$\Big\downarrow \text{458}$$

$$-\frac{\frac{8ac^2\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
-(((8*a*c^2*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) + (2*a*c*Sqrt[1 -
 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3)/c)
```

3.521.3.1 Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

---

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.521.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(ax+1)(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)x}$	54
default	$-\frac{2(ax+1)(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)x}$	54
risch	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,(5a^2x^2+4ax-1)}{3(ax-1)x}$	57

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a*x+1)*(5*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)/x
```

3.521.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\,(5\,a^2x^2+4\,ax-1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\,(ax^2-x)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas
")
```

output
```
-2/3*(5*a^2*x^2 + 4*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x))/(a*x^2 - x)
```

3.521.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x^2}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x**2, x)
```

3.521.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.521.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.521.9 Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^2\,x^2+4\,a\,x-1\right)}{3\,x\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

output
```
-(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + 5*a^2*x^2 - 1
))/(3*x*(a*x - 1))
```

3.522 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.522.1 Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}$$

output
```
2/5*a^2*(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/c+8/5*a^2*c*(1-1/a^2/x^2)^(1/2
)/(c-c/a/x)^(1/2)+2/5*a^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

3.522.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1-3ax+6a^2x^2)}{5x(-1+ax)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 - 3*a*x + 6*a^2*x^2))/(5*x
*(-1 + a*x))
```

3.522.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6733, 572, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{572}$$

$$-\frac{-\frac{3}{5}a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{c}$$

$$\Big\downarrow \text{459}$$

$$-\frac{-\frac{3}{5}a\left(\frac{4}{3}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)-\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{c}$$

$$\Big\downarrow \text{458}$$

$$-\frac{-\frac{3}{5}a\left(\frac{8ac^2\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)-\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

output
```
-(((-3*a*((8*a*c^2*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) + (2*a*c*S
qrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3))/5 - (2*a^2*Sqrt[1 - 1/(a^2*x^2
)]*(c - c/(a*x))^(3/2))/5)/c)
```

3.522.3.1 Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.522.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{2(ax+1)\left(6a^2x^2-3ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5(ax-1)x^2}$	62
default	$\frac{2(ax+1)\left(6a^2x^2-3ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5(ax-1)x^2}$	62
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\left(6a^3x^3+3a^2x^2-2ax+1\right)}{5(ax-1)x^2}$	65

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(a*x+1)*(6*a^2*x^2-3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1
/2)/(a*x-1)/x^2
```

3.522.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\,(6\,a^3x^3+3\,a^2x^2-2\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{5\,(ax^3-x^2)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas
")
```

output
```
2/5*(6*a^3*x^3 + 3*a^2*x^2 - 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x))/(a*x^3 - x^2)
```

3.522.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

output
```
Timed out
```

3.522.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

3.522.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

3.522.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(6\,a^3\,x^3+3\,a^2\,x^2-2\,a\,x+1)}{5\,x^2\,(a\,x-1)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^3,x)
```

output
```
(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 2*a*x + 6*
a^3*x^3 + 1))/(5*x^2*(a*x - 1))
```

3.523 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.523.1 Optimal result

Integrand size = 27, antiderivative size = 149

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} - \frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}x^2}$$

output
```
-104/105*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2/7*c*(1-1/a^2/x^2)^(1/
2)/x^3/(c-c/a/x)^(1/2)-26/35*a*c*(1-1/a^2/x^2)^(1/2)/x^2/(c-c/a/x)^(1/2)-1
04/105*a^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

3.523.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-15+39ax-52a^2x^2+104a^3x^3)}{105x^2(-1+ax)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 39*a*x - 52*a^2*x^2 +
 104*a^3*x^3))/(105*x^2*(-1 + a*x))
```

3.523.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6733, 574, 581, 27, 672, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{574}$$

$$-\frac{\frac{13}{7}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{581}$$

$$-\frac{\frac{13}{7}c\left(\frac{2a^2\int \frac{c^2\left(3a+\frac{2}{x}\right)\sqrt{c-\frac{c}{ax}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{5c^2}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{13}{7}c\left(\frac{1}{5}a\int \frac{\left(3a+\frac{2}{x}\right)\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{672}$$

$$-\frac{\frac{13}{7}c\left(\frac{1}{5}a\left(\frac{7}{3}a\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4}{3}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{458}$$

$$-\frac{\frac{13}{7}c\left(\frac{1}{5}a\left(\frac{14a^2c\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{4}{3}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

3.523. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx$

input
```
Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

output
```
-(((13*c*((a*((14*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) - (4*
a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3))/5 + (2*a^3*Sqrt[1 - 1/(a^
2*x^2)]*(c - c/(a*x))^(3/2))/(5*c)))/7 - (2*c^2*Sqrt[1 - 1/(a^2*x^2)])/(7*
Sqrt[c - c/(a*x)]*x^3))/c)
```

3.523.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 574
```
Int[((e_.)*(x_))^(n_)*((c_) + (d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^2*(e*x)^(n + 1)*(c + d*x)^(m - 2)*((a + b*x^2)^(p + 1)/
(b*e*(n + p + 2))), x] + Simp[c*((2*n + p + 3)/(n + p + 2))   Int[(e*x)^n*(
c + d*x)^(m - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x
] && EqQ[b*c^2 + a*d^2, 0] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && IntegerQ
[2*p]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

3.523. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.523.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(ax+1)\left(104a^{3}x^{3}-52a^{2}x^{2}+39ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105(ax-1)x^{3}}$	70
default	$-\frac{2(ax+1)\left(104a^{3}x^{3}-52a^{2}x^{2}+39ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105(ax-1)x^{3}}$	70
risch	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\left(104a^{4}x^{4}+52a^{3}x^{3}-13a^{2}x^{2}+24ax-15\right)}{105(ax-1)x^{3}}$	73

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/105*(a*x+1)*(104*a^3*x^3-52*a^2*x^2+39*a*x-15)*(c*(a*x-1)/a/x)^(1/2)*((
a*x-1)/(a*x+1))^(1/2)/(a*x-1)/x^3
```

3.523.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^{4}}\,dx=-\frac{2\left(104\,a^{4}x^{4}+52\,a^{3}x^{3}-13\,a^{2}x^{2}+24\,ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^{4}-x^{3}\right)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas
")
```

output
```
-2/105*(104*a^4*x^4 + 52*a^3*x^3 - 13*a^2*x^2 + 24*a*x - 15)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

3.523.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

output
```
Timed out
```

3.523.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

3.523.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

3.523. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.523.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(104\,a^3\,x^3+156\,a^2\,x^2+143\,a\,x+167\right)\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3} - \frac{304\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^4,x)
```

output
```
- (2*((a*x - 1)/(a*x + 1))^(1/2)*(143*a*x + 156*a^2*x^2 + 104*a^3*x^3 + 16
7)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (304*((a*x - 1)/(a*x + 1))^(1/
2)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.524 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

3.524.1 Optimal result

Integrand size $=27$, antiderivative size $=172$

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=-\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2}-\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{363\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

output
```
363/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4-4*arctanh(1/2*(c-c/a/x
)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4-149/64*x*(c-c/a/x)^(1/2)/a^3+
107/96*x^2*(c-c/a/x)^(1/2)/a^2-17/24*x^3*(c-c/a/x)^(1/2)/a+1/4*x^4*(c-c/a/
x)^(1/2)
```

3.524.2 Mathematica [A] (verified)

Time $=0.17$ (sec) , antiderivative size $=116$, normalized size of antiderivative $=0.67$

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$

$$=\frac{a\sqrt{c-\frac{c}{ax}}x(-447+214ax-136a^2x^2+48a^3x^3)+1089\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-768\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right.}{192a^4}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

output
```
(a*Sqrt[c - c/(a*x)]*x*(-447 + 214*a*x - 136*a^2*x^2 + 48*a^3*x^3) + 1089*
Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 768*Sqrt[2]*Sqrt[c]*ArcTanh[S
qrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(192*a^4)
```

3.524.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {6717, 6683, 1070, 281, 948, 109, 27, 168, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^3dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x^3(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}x^3}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^5}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(17a-\frac{15}{x}\right)x^4}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(17a-\frac{15}{x}\right)x^4}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \ 168$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{c\left(107a-\frac{85}{x}\right)x^3}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{3ac}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{\left(107a-\frac{85}{x}\right)x^3}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \ 168$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{\int \frac{3c\left(149a-\frac{107}{x}\right)x^2}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2ac}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \ 27$$

3.524. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\int \frac{\left(149a-\frac{107}{x}\right)x^2}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \; 168$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{\int \frac{c\left(363a-\frac{149}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{ac}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{\int \frac{\left(363a-\frac{149}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \; 174$$

3.524. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{363\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-512\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow 73$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{\frac{1024a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{726a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2a}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow 221$$

3.524. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{\frac{512\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{726\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

input

```
Int[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

output

```
-((a*(-1/4*(c*Sqrt[c - c/(a*x)]*x^4)/a - (c^2*((-17*Sqrt[c - c/(a*x)]*x^3)
/(3*c) - ((-107*Sqrt[c - c/(a*x)]*x^2)/(2*c) - (3*((-149*Sqrt[c - c/(a*x)]
*x)/c - ((-726*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] + (512*Sqrt[2]*
ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])/(2*a)))/(4*a))/(6*a
)))/(8*a^2)))/c)
```

3.524.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.524. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.524.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(48a^3x^3-136a^2x^2+214ax-447)x\sqrt{\frac{c(ax-1)}{ax}}}{192a^3}+\frac{\Bigg(\frac{363\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{128a^3\sqrt{a^2c}}+\frac{2\sqrt{2}\,\ln\Bigg(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2}}{x+\frac{1}{a}}}{a^4\sqrt{c}}}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigg(-96x(a\,x^2-x)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}+176(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}-252\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+768\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+126\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}}{384\sqrt{(ax-1}}$

input
```
int(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*a^3*x^3-136*a^2*x^2+214*a*x-447)/a^3*x*(c*(a*x-1)/a/x)^(1/2)+(36
3/128/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^
2*c)^(1/2)+2/a^4*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(
a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x
)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.58

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\left[\frac{768\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+2\left(48\,a^4x^4-136\,a^3x^3+214\,a^2x^2-447\,ax\right)\sqrt{\frac{acx-c}{ax}}+1089\,\sqrt{c}}{384\,a^4}\right.$$

input

```
integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output

```
[1/384*(768*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) - 3*a*c*x + c)/(a*x + 1)) + 2*(48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2
 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) + 1089*sqrt(c)*log(-2*a*c*x - 2*a*sqrt
(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^4, 1/192*(768*sqrt(2)*sqrt(-c)*arcta
n(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (48*a^4*x^4 - 136*a^3*
x^3 + 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) - 1089*sqrt(-c)*arcta
n(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

3.524.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input

```
integrate(x**3*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

output

```
Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.524.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x^3}{ax+1}\,dx$$

input
```
integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))*x^3/(a*x + 1), x)
```

3.524.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{x^3\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.525 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.525.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2}-\frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3-\frac{45\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

output
```
-45/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3+4*arctanh(1/2*(c-c/a/x)
^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3+19/8*x*(c-c/a/x)^(1/2)/a^2-13/
12*x^2*(c-c/a/x)^(1/2)/a+1/3*x^3*(c-c/a/x)^(1/2)
```

3.525.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$=\frac{a\sqrt{c-\frac{c}{ax}}x(57-26ax+8a^2x^2)-135\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+96\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{24a^3}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(a*Sqrt[c - c/(a*x)]*x*(57 - 26*a*x + 8*a^2*x^2) - 135*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]] + 96*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sq
rt[2]*Sqrt[c])])/(24*a^3)
```

3.525.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6717, 6683, 1070, 281, 948, 109, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow 6717$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^2dx$$

$$\Big\downarrow 6683$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x^2(1-ax)}{ax+1}dx$$

$$\Big\downarrow 1070$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}x^2}{a+\frac{1}{x}}dx$$

$$\Big\downarrow 281$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow 948$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow 109$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(13a-\frac{11}{x}\right)x^3}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{3a}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\int\frac{\left(13a-\frac{11}{x}\right)x^3}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int\frac{3c\left(19a-\frac{13}{x}\right)x^2}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2ac}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\int\frac{\left(19a-\frac{13}{x}\right)x^2}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{\int\frac{c\left(45a-\frac{19}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{ac}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

3.525. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{\int \frac{\left(45a-\frac{19}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{45\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-64\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{\frac{128a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{90a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2a}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \text{221}$$

3.525. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{\frac{64\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{90\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-1/3*(c*Sqrt[c - c/(a*x)]*x^3)/a - (c^2*((-13*Sqrt[c - c/(a*x)]*x^2)
/(2*c) - (3*((-19*Sqrt[c - c/(a*x)]*x)/c - ((-90*ArcTanh[Sqrt[c - c/(a*x)]
/Sqrt[c]])/Sqrt[c] + (64*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c
])])/Sqrt[c])/(2*a)))/(4*a)))/(6*a^2)))/c)
```

3.525.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.525. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.525.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.29

method	result
risch	$\frac{(8a^2x^2-26ax+57)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2}+\frac{\left(-\frac{45\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{16a^2\sqrt{a^2c}}-\frac{2\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+}}{x+\frac{1}{a}}\right.}{a^3\sqrt{c}}\right.}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}-36\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+96\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+18\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}-96a^{\frac{3}{2}}\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax-}}{ax+}\right.\right.}{48\sqrt{(ax-1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}}$

input
```
int(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2-26*a*x+57)/a^2*x*(c*(a*x-1)/a/x)^(1/2)+(-45/16/a^2*ln((-1/
2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a^3*
2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3
*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1
)*a*x)^(1/2)
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.76

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{96\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}+135\sqrt{c}\log\left(-2\,acx+\right.}{48\,a^3}\right.$$
$$-\frac{96\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}-135\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right.}{24\,a^3}$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(96*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + 3*a*c*x - c)/(a*x + 1)) + 2*(8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((
a*c*x - c)/(a*x)) + 135*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + c))/a^3, -1/24*(96*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-
c)*sqrt((a*c*x - c)/(a*x))/c) - (8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*
c*x - c)/(a*x)) - 135*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))
/a^3]
```

3.525.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

input
```
integrate(x**2*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.525.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x^2}{ax+1}\,dx$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

3.525.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.525.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.526 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

3.526.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{9\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{23\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

output
```
23/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2-4*arctanh(1/2*(c-c/a/x)^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2-9/4*x*(c-c/a/x)^(1/2)/a+1/2*x^2
*(c-c/a/x)^(1/2)
```

3.526.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{a\sqrt{c-\frac{c}{ax}}x(-9+2ax)+23\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-16\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4a^2}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
(a*Sqrt[c - c/(a*x)]*x*(-9 + 2*a*x) + 23*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]
/Sqrt[c]] - 16*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
])/(4*a^2)
```

3.526.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {6717, 6683, 1070, 281, 948, 109, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}xdx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(9a-\frac{7}{x}\right)x^2}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow 27$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(9a-\frac{7}{x}\right)x^2}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow 168$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{c\left(23a-\frac{9}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{ac}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow 27$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{\left(23a-\frac{9}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow 174$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{23\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-32\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow 73$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\frac{64a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{46a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2a}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow 221$$

3.526. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{46\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-1/2*(c*Sqrt[c - c/(a*x)]*x^2)/a - (c^2*((-9*Sqrt[c - c/(a*x)]*x)/c
- ((-46*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] + (32*Sqrt[2]*ArcTanh[
Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])/(2*a)))/(4*a^2)))/c)
```

3.526.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.526. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.526.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(2ax-9)x\sqrt{\frac{c(ax-1)}{ax}}}{4a}+\frac{\left(\frac{23\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{8a\sqrt{a^{2}c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\right)\sqrt{\frac{c(ax}{a}}}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-4\sqrt{a\,x^{2}-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+16\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+2\sqrt{a\,x^{2}-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}-16a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)-24a^{2}\right.}{8\sqrt{(ax-1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}}$

input
```
int(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x-9)/a*x*(c*(a*x-1)/a/x)^(1/2)+(23/8/a*ln((-1/2*a*c+a^2*c*x)/(a^2
*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)+2/a^2*2^(1/2)/c^(1/2)*ln(
(4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(
1/2))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.526.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.96

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$

$$=\left[\frac{16\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+2\left(2\,a^{2}x^{2}-9\,ax\right)\sqrt{\frac{acx-c}{ax}}+23\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}\right.}{8\,a^{2}}\right.$$

input
```
integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/8*(16*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)
) - 3*a*c*x + c)/(a*x + 1)) + 2*(2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)
) + 23*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/
a^2, 1/4*(16*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)
/(a*x))/c) + (2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)) - 23*sqrt(-c)*arc
tan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^2]
```

3.526.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.526.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x}{ax+1}\,dx$$

input
```
integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

3.526.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.526.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.527 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.527.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+4*arctanh(1/2*(c-c/a/x)^(1/2
)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

3.527.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

output
```
Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (
4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

3.527.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6717$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \; 6683$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \; 1035$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \; 281$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \; 899$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 109$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(5a-\frac{3}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(5a-\frac{3}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(-\frac{c^2\left(5\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-8\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(-\frac{c^2\left(\frac{16a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{10a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(-\frac{c^2\left(\frac{8\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-((c*Sqrt[c - c/(a*x)]*x)/a) - (c^2*((-10*ArcTanh[Sqrt[c - c/(a*x)]/
Sqrt[c]])/Sqrt[c] + (8*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
])/Sqrt[c]))/(2*a^2)))/c)
```

3.527. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.527.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.527. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.527.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(-\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}-\right.}{2\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
x*(c*(a*x-1)/a/x)^(1/2)+(-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x
^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2
*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x
-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.38

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\sqrt{c}\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{}}{}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*
sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*lo
g(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c
*x - c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*
x - c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/
a]
```

3.527.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.527.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

3.527.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.527.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.528 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.528.1 Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(c-c/a/x)^(1/2)*2
^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(c-c/a/x)^(1/2)
```

3.528.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqr
t[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

3.528.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6683, 1070, 281, 948, 95, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x(ax+1)}dx$$

$$\Big\downarrow \text{ 1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}dx}{c}$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 95}$$

$$-\frac{a\left(\int \frac{c^2\left(a-\frac{3}{x}\right)x}{a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\left(\frac{c^2\int \frac{\left(a-\frac{3}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow\ 174$$

$$-\frac{a\left(\frac{c^2\left(\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-4\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{a}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow\ 73$$

$$-\frac{a\left(\frac{c^2\left(\frac{8a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow\ 221$$

$$-\frac{a\left(\frac{c^2\left(\frac{4\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{a}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-((a*((-2*c*Sqrt[c - c/(a*x)])/a + (c^2*((-2*ArcTanh[Sqrt[c - c/(a*x)]/Sqr
t[c]])/Sqrt[c] + (4*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/
Sqrt[c]))/a))/c)
```

3.528. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.528.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.528. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.528.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(69) = 138.

Time = 0.52 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

method	result
risch	$2\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(\frac{a\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{\sqrt{a^{2}c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{\sqrt{c}}\right)\sqrt{c(ax-1)ax}}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}\,a^{\frac{3}{2}}x^{2}-4\sqrt{\frac{1}{a}}\,\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}x^{2}-3\sqrt{\frac{1}{a}}\,\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^{2}-2\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax}{ax+1}\right.\right.}{x\sqrt{(ax-1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(c*(a*x-1)/a/x)^(1/2)+(a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-
a*c*x)^(1/2))/(a^2*c)^(1/2)+2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1
/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*(c*(a*x-1
)*a*x)^(1/2)*(c*(a*x-1)/a/x)^(1/2)/(a*x-1)
```

3.528. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.528.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.36

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[2\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)\right.$$
$$+\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)$$
$$+2\sqrt{\frac{acx-c}{ax}},4\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)$$
$$\left.-2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)+2\sqrt{\frac{acx-c}{ax}}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
[2*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*
a*c*x + c)/(a*x + 1)) + sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + c) + 2*sqrt((a*c*x - c)/(a*x)), 4*sqrt(2)*sqrt(-c)*arctan(1/2
*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*sqrt(-c)*arctan(sqrt(-c)*
sqrt((a*c*x - c)/(a*x))/c) + 2*sqrt((a*c*x - c)/(a*x))]
```

3.528.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(70) = 140$.

Time = 4.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.64

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \begin{cases} -\frac{2a\left(\frac{c^2\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\frac{c}{ax}}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c} & \text{for}\ \frac{c}{a}\neq 0 \\ -\frac{3a\sqrt{c}\left(\frac{\log\left(-\frac{2}{x}\right)}{a}-\frac{\log\left(2a+\frac{2}{x}\right)}{a}\right)}{2}+\frac{\sqrt{c}\log\left(\frac{a}{x}+\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
Piecewise((-2*a*(c**2*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - 2*sq
rt(2)*c**2*atan(sqrt(2)*sqrt(c - c/(a*x))/(2*sqrt(-c)))/(a*sqrt(-c)) - c*s
qrt(c - c/(a*x))/a)/c, Ne(c/a, 0)), (-3*a*sqrt(c)*(log(-2/x)/a - log(2*a +
 2/x)/a)/2 + sqrt(c)*log(a/x + x**(-2))/2, True))
```

3.528.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

3.528.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.528.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.528. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.529 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

3.529.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4\sqrt{2}a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
-2/3*a*(c-c/a/x)^(3/2)/c+4*a*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*
2^(1/2)*c^(1/2)-4*a*(c-c/a/x)^(1/2)
```

3.529.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}(1-7ax)}{3x}+4\sqrt{2}a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(1 - 7*a*x))/(3*x) + 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

3.529.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6683, 1070, 281, 946, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^2(ax+1)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x^2}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}dx}{c}$$

$$\Big\downarrow \text{946}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\left(2c\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}d\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\left(2c\left(2c\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(2c\left(2\sqrt{c-\frac{c}{ax}}-4a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(2c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
-((a*((2*(c - c/(a*x))^(3/2))/3 + 2*c*(2*Sqrt[c - c/(a*x)] - 2*Sqrt[2]*Sqr
t[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])))/c)
```

3.529.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.529. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.529.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(67) = 134$.

Time = 0.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.73

method	result
risch	$-\frac{2\left(7a^{2}x^{2}-8ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}}{3x(ax-1)}-\frac{2a\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{\sqrt{c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(6\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^{3}-18\sqrt{a\,x^{2}-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^{3}+12a^{\frac{3}{2}}\left(a\,x^{2}-x\right)^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+9\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^{2}x^{3}-6a^{\frac{3}{2}}\sqrt{2}\right.}{3x^{2}\sqrt{(ax-1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(7*a^2*x^2-8*a*x+1)/x/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)-2*a*2^(1/2)/c^(1/
2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+
2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.529. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^{2}}\,dx$

3.529.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.96

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \left[\frac{2\left(3\sqrt{2}a\sqrt{c}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x},\; -\frac{2\left(6\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
[2/3*(3*sqrt(2)*a*sqrt(c)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(
a*x)) + 3*a*c*x - c)/(a*x + 1)) - (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x,
-2/3*(6*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/
(a*x))/c) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x]
```

3.529.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{x^2\,(ax+1)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

3.529.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

3.529.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.529.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^2\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.530 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.530.1 Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=4a^2\sqrt{c-\frac{c}{ax}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
2/3*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2-4*a^2*arctanh(1/2*(c
-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^2*(c-c/a/x)^(1/2)
```

3.530.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}(3-11ax+38a^2x^2)}{15x^2}-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(3 - 11*a*x + 38*a^2*x^2))/(15*x^2) - 4*Sqrt[2]*a^2*S
qrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

3.530.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6683, 1070, 281, 948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^3(ax+1)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x^3}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{90}$$

$$-\frac{a\left(-a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}d\frac{1}{x}-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\left(-a\left(2c\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}d\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\left(-a\left(2c\left(2c\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

$$\Big\downarrow 73$$

$$-\frac{a\left(-a\left(2c\left(2\sqrt{c-\frac{c}{ax}}-4a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

$$\Big\downarrow 221$$

$$-\frac{a\left(-a\left(2c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
-((a*((-2*a*(c - c/(a*x))^(5/2))/(5*c) - a*((2*(c - c/(a*x))^(3/2))/3 + 2*
c*(2*Sqrt[c - c/(a*x)] - 2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt
[2]*Sqrt[c])]))))/c)
```

3.530.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.530. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.530.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

method	result
risch	$\frac{2(38a^3x^3-49a^2x^2+14ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2(ax-1)}+\frac{2a^2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)a}}{\sqrt{c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(30\sqrt{(ax-1)x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4-90a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,x^4+60a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,(a\,x^2-x)^{\frac{3}{2}}x^2+45\sqrt{\frac{1}{a}}\,\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^4-3\right.}{15x^3\sqrt{(ax}}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*(38*a^3*x^3-49*a^2*x^2+14*a*x-3)/x^2/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)+2*
a^2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)
^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*
x-1)*a*x)^(1/2)
```

3.530.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.60

$$\int\frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$
$$=\left[\frac{2\left(15\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+(38\,a^2x^2-11\,ax+3)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2},\frac{2\left(30\sqrt{2}a^2\sqrt{-c}x^2\,\mathrm{arc}\right.}{}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
[2/15*(15*sqrt(2)*a^2*sqrt(c)*x^2*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c
*x - c)/(a*x)))/x^2, 2/15*(30*sqrt(2)*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*
sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*
x - c)/(a*x)))/x^2]
```

3.530.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

3.530.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^3}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

3.530.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(94) = 188.

Time = 0.64 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.46

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{4\sqrt{2}a^3c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\operatorname{sgn}(x)} + \frac{2\left(60\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4a^5c-45\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3a^4c^{\frac{3}{2}}|a|+35\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-}\right.\right.}{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5a^2|a|\operatorname{sgn}(x)}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
-4*sqrt(2)*a^3*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*
c*x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/15*(6
0*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c - 45*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - a*c*x))^3*a^4*c^(3/2)*abs(a) + 35*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - a*c*x))^2*a^5*c^2 - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x)
)*a^4*c^(5/2)*abs(a) + 3*a^5*c^3)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))^5*a^2*abs(a)*sgn(x))
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^3\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.531 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.531.1 Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=-4a^3\sqrt{c-\frac{c}{ax}}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+4\sqrt{2}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
-2/3*a^3*(c-c/a/x)^(3/2)/c-2/7*a^3*(c-c/a/x)^(7/2)/c^3+4*a^3*arctanh(1/2*(
c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-4*a^3*(c-c/a/x)^(1/2)
```

3.531.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}(3-9ax+16a^2x^2-52a^3x^3)}{21x^3}+4\sqrt{2}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(3 - 9*a*x + 16*a^2*x^2 - 52*a^3*x^3))/(21*x^3) + 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

3.531.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6717, 6683, 1070, 281, 948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^4(ax+1)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x^4}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^4}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a\int \left(\frac{a^2\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}-\frac{a\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-4\sqrt{2}a^2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)+\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{7c^2}+\frac{2}{3}a^2\left(c-\frac{c}{ax}\right)^{3/2}+4a^2c\sqrt{c-\frac{c}{ax}}\right)}{c}$$

---

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
-((a*(4*a^2*c*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/3 + (2*a^2*(
c - c/(a*x))^(7/2))/(7*c^2) - 4*Sqrt[2]*a^2*c^(3/2)*ArcTanh[Sqrt[c - c/(a*
x)]/(Sqrt[2]*Sqrt[c])]))/c)
```

3.531.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

3.531. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.531.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{2\left(52 a^{4} x^{4}-68 a^{3} x^{3}+25 a^{2} x^{2}-12 a x +3\right) \sqrt{\frac{c \left(a x -1\right)}{a x}}}{21 x^{3} \left(a x -1\right)}-\frac{2 a^{3} \sqrt{2}\, \ln\left(\frac{4 c -3 \left(x +\frac{1}{a}\right) a c +2 \sqrt{2}\, \sqrt{c}\, \sqrt{a^{2} c \left(x +\frac{1}{a}\right)^{2}-3 \left(x +\frac{1}{a}\right) a c +2 c}}{x +\frac{1}{a}}\right) \sqrt{\frac{c \left(a x -1\right)}{a x}}}{\sqrt{c}\, \left(a x -1\right)}$
default	$\frac{\sqrt{\frac{c \left(a x -1\right)}{a x}}\, \left(42 \sqrt{\left(a x -1\right) x}\, a^{\frac{9}{2}} \sqrt{\frac{1}{a}}\, x^{5}-126 \sqrt{a \,x^{2}-x}\, a^{\frac{9}{2}} \sqrt{\frac{1}{a}}\, x^{5}+84 \left(a \,x^{2}-x \right)^{\frac{3}{2}} a^{\frac{7}{2}} \sqrt{\frac{1}{a}}\, x^{3}+63 \ln\left(\frac{2 \sqrt{a \,x^{2}-x}\, \sqrt{a}+2 a x -1}{2 \sqrt{a}}\right) \sqrt{\frac{1}{a}}\, a^{4} x^{5}-42\right.}{}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/21*(52*a^4*x^4-68*a^3*x^3+25*a^2*x^2-12*a*x+3)/x^3/(a*x-1)*(c*(a*x-1)/a
/x)^(1/2)-2*a^3*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(a
^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^
(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.531.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= \left[\frac{2\left(21\,\sqrt{2}a^3\sqrt{c}x^3\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-\left(52\,a^3x^3-16\,a^2x^2+9\,ax-3\right)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3},\right.$$
$$\left.-\frac{2\left(42\,\sqrt{2}a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+\left(52\,a^3x^3-16\,a^2x^2+9\,ax-3\right)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
[2/21*(21*sqrt(2)*a^3*sqrt(c)*x^3*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (52*a^3*x^3 - 16*a^2*x^2 + 9*a*x -
 3)*sqrt((a*c*x - c)/(a*x)))/x^3, -2/21*(42*sqrt(2)*a^3*sqrt(-c)*x^3*arcta
n(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (52*a^3*x^3 - 16*a^2*x
^2 + 9*a*x - 3)*sqrt((a*c*x - c)/(a*x)))/x^3]
```

3.531.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

3.531.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

3.531.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(94) = 188.

Time = 0.74 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.15

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{4\sqrt{2}a^4c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\left(x\right)}-\frac{2\left(84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^6a^7c-84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5a^6c^{\frac{3}{2}}|a|+112\left(\sqrt{a^2c}x-\sqrt{a^2cx^2}\right.\right.}{21}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
4*sqrt(2)*a^4*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c
*x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/21*(84
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a^7*c - 84*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - a*c*x))^5*a^6*c^(3/2)*abs(a) + 112*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - a*c*x))^4*a^7*c^2 - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))^3*a^6*c^(5/2)*abs(a) + 63*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a
^7*c^3 - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^6*c^(7/2)*abs(a) +
 3*a^7*c^4)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^3*abs(a)*sgn(x)
)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^4\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.532 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.532.1 Optimal result

Integrand size = 27, antiderivative size = 163

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = 4a^4\sqrt{c-\frac{c}{ax}}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
2/3*a^4*(c-c/a/x)^(3/2)/c+2/5*a^4*(c-c/a/x)^(5/2)/c^2-2/7*a^4*(c-c/a/x)^(7
/2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4-4*a^4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1
/2)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^4*(c-c/a/x)^(1/2)
```

3.532.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}\left(35-95ax+138a^2x^2-236a^3x^3+788a^4x^4\right)}{315x^4}-4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(35 - 95*a*x + 138*a^2*x^2 - 236*a^3*x^3 + 788*a^4*x^
4))/(315*x^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*S
qrt[c])]
```

3.532.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6717, 6683, 1070, 281, 948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^5(ax+1)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x^5}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^5}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a\int \left(\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^2}-\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}}{c}+a^2\left(c-\frac{c}{ax}\right)^{3/2}-\frac{a^3\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\right)d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(4\sqrt{2}a^3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)-\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{9c^3}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^2}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c}-\frac{2}{3}a^3\left(c-\frac{c}{ax}\right)^{3/2}-4a^3c\sqrt{c-\frac{c}{ax}}\right)}{c}$$

input

```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output

```
-((a*(-4*a^3*c*Sqrt[c - c/(a*x)] - (2*a^3*(c - c/(a*x))^(3/2))/3 - (2*a^3*
(c - c/(a*x))^(5/2))/(5*c) + (2*a^3*(c - c/(a*x))^(7/2))/(7*c^2) - (2*a^3*
(c - c/(a*x))^(9/2))/(9*c^3) + 4*Sqrt[2]*a^3*c^(3/2)*ArcTanh[Sqrt[c - c/(a
*x)]/(Sqrt[2]*Sqrt[c])]))/c)
```

3.532.3.1 Defintions of rubi rules used

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 281

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.532.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03

method	result
risch	$\frac{2\left(788a^5x^5-1024a^4x^4+374a^3x^3-233a^2x^2+130ax-35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4(ax-1)}+\frac{2a^4\sqrt{2}\,\ln\Bigg(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2}}{x+\frac{1}{a}}}{\sqrt{c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\Bigg(630\sqrt{(ax-1)x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^6-1890\sqrt{a\,x^2-x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^6+1260\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^4+945\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}}{}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/315*(788*a^5*x^5-1024*a^4*x^4+374*a^3*x^3-233*a^2*x^2+130*a*x-35)/x^4/(a
*x-1)*(c*(a*x-1)/a/x)^(1/2)+2*a^4*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*
2^(1/2)*c^(1/2)*(a^2*c*(x+1/a)^2-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1
)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

3.532. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.532.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$

$$= \left[\frac{2\left(315\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35\right)\sqrt{\frac{acx-c}{ax}}\right.}{315\,x^4}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

output
```
[2/315*(315*sqrt(2)*a^4*sqrt(c)*x^4*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (788*a^4*x^4 - 236*a^3*x^3 + 138*
a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4, 2/315*(630*sqrt(2)*a^
4*sqrt(-c)*x^4*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (7
88*a^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*
x)))/x^4]
```

3.532.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{x^5\,(ax+1)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

3.532.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^5}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^5), x)
```

3.532.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(136) = 272$.

Time = 0.88 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.66

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{4\sqrt{2}a^5c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\,(x)} + \frac{2\left(1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^8a^9c-1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^7a^8c^{\frac{3}{2}}|a|+2100\left(\sqrt{a^2c}x-\sqrt{}\right.\right.}{}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

output
```
-4*sqrt(2)*a^5*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*
c*x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/315*(
1260*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^8*a^9*c - 1260*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^8*c^(3/2)*abs(a) + 2100*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - a*c*x))^6*a^9*c^2 - 3150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - a*c*x))^5*a^8*c^(5/2)*abs(a) + 3528*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a
*c*x))^4*a^9*c^3 - 2625*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^8*c^
(7/2)*abs(a) + 1215*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^9*c^4 -
315*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^8*c^(9/2)*abs(a) + 35*a^9*
c^5)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*a^4*abs(a)*sgn(x))
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.533 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

3.533.1 Optimal result

Integrand size = 27, antiderivative size = 303

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx= & -\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{1115\sqrt{c-\frac{c}{ax}}\,x}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}\\ & +\frac{223\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{25\sqrt{c-\frac{c}{ax}}\,x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}\\ & +\frac{\sqrt{c-\frac{c}{ax}}\,x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1115\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output
```
1115/64*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)-1115/
64*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1115/192*x*(c-c/a/x
)^(1/2)/a^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+223/96*x^2*(c-c/a/x)^(1/2)/a^2
/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-25/24*x^3*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2)+1/4*x^4*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
```

3.533.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$= \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-3345-1115ax+446a^2x^2-200a^3x^3+48a^4x^4)}{-1+a^2x^2} - 3345\sqrt{c}\log(1-ax) + 3345\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-}\right.}{384a^4}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3345 - 1115*a*x + 44
6*a^2*x^2 - 200*a^3*x^3 + 48*a^4*x^4))/(-1 + a^2*x^2) - 3345*Sqrt[c]*Log[1
 - a*x] + 3345*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/
(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(384*a^4)
```

3.533.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.57, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6733, 585, 27, 100, 27, 87, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6733}$$
$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^5}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$
$$\Big\downarrow \text{585}$$
$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^5}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$
$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^5}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{4}\int -\frac{\left(25a-\frac{8}{x}\right)x^4}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{8}\int \frac{\left(25a-\frac{8}{x}\right)x^4}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\int \frac{x^3}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\int \frac{x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\left(-\frac{3\int \frac{x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{61}$$

3.533. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\left(-\frac{3\left(\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\left(-\frac{3\left(2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$-\frac{\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\left(-\frac{3\left(\frac{2}{\sqrt{\frac{1}{ax}+1}}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a*x)]*(-1/4*(a^2*x^4)/Sqrt[1 + 1/(a*x)] + ((25*a*x^3)/(3*Sq
rt[1 + 1/(a*x)]) + (223*(-1/2*x^2/Sqrt[1 + 1/(a*x)] - (5*(-(x/Sqrt[1 + 1/(
a*x)]) - (3*(2/Sqrt[1 + 1/(a*x)] - 2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(2*a)))/
(4*a))/6)/8))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.533. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

3.533.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

3.533. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.533.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.65

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\,(96a^{\frac{9}{2}}\sqrt{(ax+1)x}\,x^4-400a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}+892a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-2230a^{\frac{3}{2}}x\sqrt{(ax+1)x}+3345\ln(}{384(ax-1)^2a^{\frac{7}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(48a^3x^3-248a^2x^2+694ax-1809)(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{192a^3(ax-1)}+\frac{\left(\frac{1115\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{128a^3\sqrt{a^2c}}-\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^5c\left(x+\frac{1}{a}\right)}\right)}{ax-1}$

input
```
int(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(9
6*a^(9/2)*((a*x+1)*x)^(1/2)*x^4-400*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+892*a^(5
/2)*x^2*((a*x+1)*x)^(1/2)-2230*a^(3/2)*x*((a*x+1)*x)^(1/2)+3345*ln(1/2*(2*
((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-6690*((a*x+1)*x)^(1/2)*a^(
1/2)+3345*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(7/2)/(
(a*x+1)*x)^(1/2)
```

3.533.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.17

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$

$$= \left[\frac{3345\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(48\,a^5x^5-200\,a^4x^4+446\,a}{768\,(a^5x-a^4)}\right.$$

$$-\frac{3345\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-1115\,a^2x}{384\,(a^5x-a^4)}$$

input
```
integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
[1/768*(3345*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3
+ 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)) - c)/(a*x - 1)) + 4*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*
x^2 - 3345*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x
- a^4), -1/384*(3345*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c
)) - 2*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2 - 3345*a*x)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

3.533.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\text{Timed out}$$

input
```
integrate(x**3*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.533.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int \sqrt{c-\frac{c}{ax}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.533.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int x^3\,\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.534 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.534.1 Optimal result

Integrand size = 27, antiderivative size = 251

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx &= \frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{c-\frac{c}{ax}}x}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}\\ &\quad-\frac{19\sqrt{c-\frac{c}{ax}}x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}\\ &\quad-\frac{119\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output

```
-119/8*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+119/8*
(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+119/24*x*(c-c/a/x)^(1/
2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-19/12*x^2*(c-c/a/x)^(1/2)/a/(1-1/a/
x)^(1/2)/(1+1/a/x)^(1/2)+1/3*x^3*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)
^(1/2)
```

3.534.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.63

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(357+119ax-38a^2x^2+8a^3x^3)}{-1+a^2x^2}+357\sqrt{c}\log(1-ax)-357\sqrt{c}\log\Big(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2}{48a^3}$$

input

```
Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

output

```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(357 + 119*a*x - 38*a^
2*x^2 + 8*a^3*x^3))/(-1 + a^2*x^2) + 357*Sqrt[c]*Log[1 - a*x] - 357*Sqrt[c
]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 -
a*x + 2*a^2*x^2)])/(48*a^3)
```

3.534.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6733, 585, 27, 100, 27, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6733$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^4}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \; 585$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^4}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^4}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 100$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{3}\int -\frac{\left(19a-\frac{6}{x}\right)x^3}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{6}\int \frac{\left(19a-\frac{6}{x}\right)x^3}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 87$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{6}\left(\frac{119}{4}\int \frac{x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 52$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3\int \frac{x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 61$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3\left(\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3\left(2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 221$$

3.534. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

$$-\frac{\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3\left(\frac{2}{\sqrt{\frac{1}{ax}+1}}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a*x)]*(-1/3*(a^2*x^3)/Sqrt[1 + 1/(a*x)] + ((19*a*x^2)/(2*Sq
rt[1 + 1/(a*x)]) + (119*(-(x/Sqrt[1 + 1/(a*x)]) - (3*(2/Sqrt[1 + 1/(a*x)]
- 2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(2*a)))/4)/6))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.534.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.534. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

3.534.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-76a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}+238a^{\frac{3}{2}}x\sqrt{(ax+1)x}-357\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+71\right.}{48(ax-1)^2a^{\frac{5}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(8a^2x^2-46ax+165)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{24a^2(ax-1)}+\frac{\left(-\frac{119\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{16a^2\sqrt{a^2c}}+\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^4c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(c}{}}}{ax-1}$

input
```
int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(16
*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-76*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+238*a^(3/2
)*x*((a*x+1)*x)^(1/2)-357*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(
1/2))*a*x+714*((a*x+1)*x)^(1/2)*a^(1/2)-357*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2)))/a^(5/2)/((a*x+1)*x)^(1/2)
```

3.534.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.34

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$=\left[\frac{357\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4-38\,a^3x^3+119\,a^2x^2}{96\,(a^4x-a^3)}\right.$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
[1/96*(357*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 +
3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) + 4*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(357*(a
*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 38*
a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^4*x - a^3)]
```

3.534.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.534.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\sqrt{c-\frac{c}{ax}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.534.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int x^2\,\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.535 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

3.535.1 Optimal result

Integrand size = 25, antiderivative size = 199

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = -\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{47\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

output
```
47/4*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)-47/4*(c-
c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-13/4*x*(c-c/a/x)^(1/2)/a/
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1/2*x^2*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)
```

3.535.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx\\&=\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-47-13ax+2a^2x^2)}{-4+4a^2x^2}-\frac{47\sqrt{c}\log(1-ax)}{8a^2}\\&\quad+\frac{47\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax+2a^2x^2)\right)}{8a^2}\end{aligned}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-47 - 13*a*x + 2*a^2*x^2))/(
-4 + 4*a^2*x^2) - (47*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (47*Sqrt[c]*Log[2*a^
2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^
2*x^2)])/(8*a^2)
```

3.535.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6733, 585, 27, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^3}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^3}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^3}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\int -\frac{\left(13a-\frac{4}{x}\right)x^2}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{4}\int \frac{\left(13a-\frac{4}{x}\right)x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 87$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{4}\left(\frac{47}{2}\int \frac{x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{13ax}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 61$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{4}\left(\frac{47}{2}\left(\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{13ax}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{4}\left(\frac{47}{2}\left(2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{13ax}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(\frac{1}{4}\left(\frac{47}{2}\left(\frac{2}{\sqrt{\frac{1}{ax}+1}}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{13ax}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

output

```
-((Sqrt[c - c/(a*x)]*(-1/2*(a^2*x^2)/Sqrt[1 + 1/(a*x)] + ((13*a*x)/Sqrt[1
+ 1/(a*x)] + (47*(2/Sqrt[1 + 1/(a*x)] - 2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/2)/
4))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.535. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

3.535.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.535. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.535.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.82

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigg(4a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-26a^{\frac{3}{2}}x\sqrt{(ax+1)x}+47\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-94\sqrt{(ax+1)x}\,\sqrt{a}+47\ln\Bigg(\frac{2}{}}{8(ax-1)^2a^{\frac{3}{2}}\sqrt{(ax+1)x}}$
risch	$\frac{(2ax-15)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{4a(ax-1)}+\frac{\left(\frac{47\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{8a\sqrt{a^2c}}-\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^3c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+}{ax-1}$

input
```
int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(4*a
^(5/2)*x^2*((a*x+1)*x)^(1/2)-26*a^(3/2)*x*((a*x+1)*x)^(1/2)+47*ln(1/2*(2*(
(a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-94*((a*x+1)*x)^(1/2)*a^(1/2
)+47*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(3/2)/((a*x+
1)*x)^(1/2)
```

3.535.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.61

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\left[\frac{47\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^3x^3-13\,a^2x^2-47\,ax)\sqrt{\frac{a}{a}}}{16\,(a^3x-a^2)}\right.$$
$$\left.-\frac{47\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(2\,a^3x^3-13\,a^2x^2-47\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8\,(a^3x-a^2)}\right]$$

input
```
integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(47*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(47*(a*x - 1)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)
*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

3.535.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Timed out}$$

input
```
integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.535.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int \sqrt{c-\frac{c}{ax}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.535.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int x\,\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.536 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.536.1 Optimal result

Integrand size = 24, antiderivative size = 140

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

output
```
-7*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+9*(c-c/a/x)^
(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+x*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/
(1+1/a/x)^(1/2)
```

3.536.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(9+ax-7\sqrt{1+\frac{1}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)
]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.536.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6731, 585, 27, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(7a-\frac{2}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(7a-\frac{2}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-7a\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-14a^2\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left(\frac{1}{2}\left(14a\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a*x)]*(-((a^2*x)/Sqrt[1 + 1/(a*x)]) + ((-18*a)/Sqrt[1 + 1/(
a*x)] + 14*a*ArcTanh[Sqrt[1 + 1/(a*x)]])/2))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.536.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

3.536. $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.536.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)x}-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+18\sqrt{(ax+1)x}\,\sqrt{a}-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)^{2}\sqrt{a}\,\sqrt{(ax+1)x}}$
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{8\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{2}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a
^(3/2)*x*((a*x+1)*x)^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*x+18*((a*x+1)*x)^(1/2)*a^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/((a*x+1)*x)^(1/2)
```

3.536.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.14

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}x-a)},\ 7\right.$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
 - a*c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^2*x - a)]
```

3.536.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.536.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.536.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.536.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.537 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.537.1 Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{c-\frac{c}{ax}}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output

```
2*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-8*(c-c/a/x)^(1/
2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/
a/x)^(1/2)
```

3.537.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x(1+5ax)}{-1+a^2x^2}-\sqrt{c}\log(1-ax) + \sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2 + c\left(-1-ax+2a^2x^2\right)\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x*(1 + 5*a*x))/(-1 + a^2*x^2
) - Sqrt[c]*Log[1 - a*x] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]
*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)]
```

3.537.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6733, 585, 27, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{98}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \left(\frac{xa^2}{\sqrt{1+\frac{1}{ax}}}+\frac{a}{\sqrt{1+\frac{1}{ax}}}-\frac{4a}{\left(1+\frac{1}{ax}\right)^{3/2}}\right)d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-2a^2\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)+2a^2\sqrt{\frac{1}{ax}+1}+\frac{8a^2}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
-((Sqrt[c - c/(a*x)]*((8*a^2)/Sqrt[1 + 1/(a*x)] + 2*a^2*Sqrt[1 + 1/(a*x)]
- 2*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(a^2*Sqrt[1 - 1/(a*x)]))
```

3.537.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.537. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

3.537.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(10a^{\frac{3}{2}}x\sqrt{(ax+1)x}-\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2-\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+2\sqrt{(ax+1)x}\right.}{(ax-1)^2\sqrt{a}\,\sqrt{(ax+1)x}}$
risch	$-\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{\sqrt{a^2c}}-\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{ac\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*(10*a^(3/
2)*x*((a*x+1)*x)^(1/2)-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2
))*a^2*x^2-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+2*((a
*x+1)*x)^(1/2)*a^(1/2))/a^(1/2)/((a*x+1)*x)^(1/2)
```

3.537.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$

$$=\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\,(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)},\right.$$

$$\left.-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) - 4*(5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x)))/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
+ 2*(5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x -
1)]
```

3.537.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Timed out
```

3.537.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.537.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

output
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.538 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

3.538.1 Optimal result

Integrand size = 27, antiderivative size = 109

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-16/3*a*(c-c/a/x)^(3/2)/c/(1-1/a^2/x^2)^(1/2)-2/3*a*(c-c/a/x)^(5/2)/c^2/(1
-1/a^2/x^2)^(1/2)+64/3*a*(c-c/a/x)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.538.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-1+10ax+23a^2x^2)}{-3+3a^2x^2}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 10*a*x + 23*a^2*x^2))/(
-3 + 3*a^2*x^2)
```

3.538.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6733, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \textbf{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \textbf{459}$$

$$-\frac{\frac{8}{3}c\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \textbf{459}$$

$$-\frac{\frac{8}{3}c\left(4c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2ac\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \textbf{458}$$

$$-\frac{\frac{8}{3}c\left(\frac{2ac\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8ac^2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input

```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output

```
-(((8*c*((-8*a*c^2*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a^2*x^2)] + (2*a*c*(c -
c/(a*x))^(3/2))/Sqrt[1 - 1/(a^2*x^2)]))/3 + (2*a*c*(c - c/(a*x))^(5/2))/(3
*Sqrt[1 - 1/(a^2*x^2)]))/c^3)
```

3.538.3.1 Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.538.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(ax+1)(23a^2x^2+10ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	62
default	$\frac{2(ax+1)(23a^2x^2+10ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	62
risch	$\frac{2(11a^2x^2+10ax-1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{3x(ax-1)}+\frac{8a^2x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	101

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x+1)*(23*a^2*x^2+10*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^
(3/2)/x/(a*x-1)^2
```

3.538. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

3.538.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\left(23\,a^2x^2+10\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas
")
```

output
```
2/3*(23*a^2*x^2 + 10*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x))/(a*x^2 - x)
```

3.538.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Timed out
```

3.538.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.538.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.538.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(23\,a^2\,x^2+10\,a\,x-1)}{3\,x\,(a\,x-1)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

output
```
(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(10*a*x + 23*a^2*x^2 -
1))/(3*x*(a*x - 1))
```

3.539 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.539.1 Optimal result

Integrand size = 27, antiderivative size = 150

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}} - \frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
-a^2*(c-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)-7/5*a^2*(c-c/a/x)^(3/2)*(1-1/
a^2/x^2)^(1/2)/c-224/15*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-56/15*a^
2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

3.539.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(3-16ax+79a^2x^2+158a^3x^3\right)}{15x\left(-1+a^2x^2\right)}$$

input

```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output

```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 - 16*a*x + 79*a^2*x^2 + 1
58*a^3*x^3))/(15*x*(-1 + a^2*x^2))
```

3.539.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6733, 572, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{572}$$

$$-\frac{-\frac{7}{5}a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{5\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{-\frac{7}{5}a\left(\frac{8}{3}c\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}\right)-\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{5\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{-\frac{7}{5}a\left(\frac{8}{3}c\left(4c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2ac\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}\right)-\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{5\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{458}$$

$$-\frac{-\frac{7}{5}a\left(\frac{8}{3}c\left(\frac{2ac\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8ac^2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}\right)-\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{5\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

3.539. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}dx$

output
```
-(((-7*a*((8*c*((-8*a*c^2*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a^2*x^2)] + (2*a*
c*(c - c/(a*x))^(3/2))/Sqrt[1 - 1/(a^2*x^2)]))/3 + (2*a*c*(c - c/(a*x))^(5
/2))/(3*Sqrt[1 - 1/(a^2*x^2)])))/5 - (2*a^2*(c - c/(a*x))^(7/2))/(5*Sqrt[1
 - 1/(a^2*x^2)]))/c^3)
```

3.539.3.1 Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.539. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

3.539.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(ax+1)\left(158a^{3}x^{3}+79a^{2}x^{2}-16ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^{2}(ax-1)^{2}}$	70
default	$-\frac{2(ax+1)\left(158a^{3}x^{3}+79a^{2}x^{2}-16ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^{2}(ax-1)^{2}}$	70
risch	$-\frac{2\left(98a^{3}x^{3}+79a^{2}x^{2}-16ax+3\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}(ax-1)}-\frac{8a^{3}x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	109

input

```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output

```
-2/15*(a*x+1)*(158*a^3*x^3+79*a^2*x^2-16*a*x+3)*(c*(a*x-1)/a/x)^(1/2)*((a*
x-1)/(a*x+1))^(3/2)/x^2/(a*x-1)^2
```

3.539.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^{3}}\,dx=-\frac{2\left(158\,a^{3}x^{3}+79\,a^{2}x^{2}-16\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^{3}-x^{2}\right)}$$

input

```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas
")
```

output

```
-2/15*(158*a^3*x^3 + 79*a^2*x^2 - 16*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x))/(a*x^3 - x^2)
```

3.539.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Timed out
```

3.539.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.539.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.539.9 Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=-\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(158\,a^3\,x^3+79\,a^2\,x^2-16\,a\,x+3\right)}{15\,x^2\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

output
```
-(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a^2*x^2 - 16*a*x +
 158*a^3*x^3 + 3))/(15*x^2*(a*x - 1))
```

3.540 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.540.1 Optimal result

Integrand size = 27, antiderivative size = 188

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= \frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\\ &\quad+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}\\ &\quad+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}+\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
a^3*(c-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)+59/35*a^3*(c-c/a/x)^(3/2)*(1-1
/a^2/x^2)^(1/2)/c+2/7*a^3*(c-c/a/x)^(5/2)*(1-1/a^2/x^2)^(1/2)/c^2+1888/105
*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+472/105*a^3*(1-1/a^2/x^2)^(1/2)
*(c-c/a/x)^(1/2)
```

3.540.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-15+66ax-167a^2x^2+668a^3x^3+1336a^4x^4)}{105x^2\left(-1+a^2x^2\right)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 66*a*x - 167*a^2*x^2 +
 668*a^3*x^3 + 1336*a^4*x^4))/(105*x^2*(-1 + a^2*x^2))
```

3.540.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6733, 581, 27, 669, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{581}$$

$$-\frac{\frac{2a^2\int \frac{c^2\left(9a-\frac{2}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{7c^2}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{7c\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{7}a\int \frac{\left(9a-\frac{2}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{7c\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{669}$$

$$-\frac{\frac{1}{7}a\left(-\frac{59}{2}ac\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{11a^2\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{7c\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{\frac{1}{7}a\left(-\frac{59}{2}ac\left(\frac{8}{5}c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{11a^2\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{7c\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{\frac{1}{7}a\left(-\frac{59}{2}ac\left(\frac{8}{5}c\left(\frac{4}{3}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{11a^2\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+}{c^3}$$

$$\Big\downarrow \text{458}$$

$$-\frac{\frac{1}{7}a\left(-\frac{59}{2}ac\left(\frac{8}{5}c\left(\frac{8ac^2\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{11a^2\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+}{c^3}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
-(((a*((-59*a*c*((8*c*((8*a*c^2*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)
]) + (2*a*c*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3))/5 + (2*a*c*Sqrt[1
 - 1/(a^2*x^2)]*(c - c/(a*x))^(3/2))/5))/2 - (11*a^2*(c - c/(a*x))^(7/2))/
Sqrt[1 - 1/(a^2*x^2)]))/7 + (2*a^3*(c - c/(a*x))^(9/2))/(7*c*Sqrt[1 - 1/(a
^2*x^2)]))/c^3)
```

3.540. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx$

3.540.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.540. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

3.540.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{2(ax+1)(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	78
default	$\frac{2(ax+1)(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	78
risch	$\frac{2(916a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{105x^3(ax-1)}+\frac{8a^4x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	117

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/105*(a*x+1)*(1336*a^4*x^4+668*a^3*x^3-167*a^2*x^2+66*a*x-15)*(c*(a*x-1)/
a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

3.540.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$=\frac{2\left(1336\,a^4x^4+668\,a^3x^3-167\,a^2x^2+66\,ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas
")
```

output
```
2/105*(1336*a^4*x^4 + 668*a^3*x^3 - 167*a^2*x^2 + 66*a*x - 15)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

3.540.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Timed out
```

3.540.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.540.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.540.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{\frac{ax-1}{ax+1}}\,(1336\,a^3\,x^3+2004\,a^2\,x^2+1837\,a\,x+1903)\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3} + \frac{3776\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3\,(a\,x-1)}$$

input

```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

output

```
(2*((a*x - 1)/(a*x + 1))^(1/2)*(1837*a*x + 2004*a^2*x^2 + 1336*a^3*x^3 + 1
903)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) + (3776*((a*x - 1)/(a*x + 1))^
(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.541 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.541.1 Optimal result

Integrand size = 27, antiderivative size = 289

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = & -\frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{32a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} \\ & +\frac{50a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} \\ & +\frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output
```
50/3*a^4*(1+1/a/x)^(3/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-38/5*a^4*(1+1/a/x
)^(5/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2*a^4*(1+1/a/x)^(7/2)*(c-c/a/x)^(1
/2)/(1-1/a/x)^(1/2)-2/9*a^4*(1+1/a/x)^(9/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2
)-8*a^4*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-32*a^4*(1+1/a/x)^(
1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

3.541.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$= -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(5-20ax+41a^2x^2-82a^3x^3+328a^4x^4+656a^5x^5)}{45x^3\left(-1+a^2x^2\right)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(5 - 20*a*x + 41*a^2*x^2 - 8
2*a^3*x^3 + 328*a^4*x^4 + 656*a^5*x^5))/(45*x^3*(-1 + a^2*x^2))
```

3.541.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6733, 581, 27, 2166, 27, 672, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x^5}\,dx$$
$$\Big\downarrow \text{6733}$$
$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}d\frac{1}{x}}{c^3}$$
$$\Big\downarrow \text{581}$$
$$-\frac{-\frac{2a^3\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}\left(-\frac{13c^3}{ax}-\frac{7c^3}{a^2x^2}+11c^3\right)}{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$
$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{a^3\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}\left(-\frac{13c^3}{ax}-\frac{7c^3}{a^2x^2}+11c^3\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{-\frac{a^3\left(-c\int \frac{c^3\left(97a+\frac{14}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{a^3\left(-\frac{c^4\int \frac{\left(97a+\frac{14}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{672}$$

$$-\frac{-\frac{a^3\left(-\frac{c^4\left(87a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{-\frac{a^3\left(-\frac{c^4\left(87a\left(\frac{8}{5}c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

3.541. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

$$-\frac{-\frac{a^3\left(-\frac{c^4\left(87a\left(\frac{8}{5}c\left(\frac{4}{3}c\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-}{c^3}$$

$$\Big\downarrow \text{458}$$

$$-\frac{-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{a^3\left(-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^4\left(87a\left(\frac{8}{5}c\left(\frac{8ac^2\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c\right.\right.}{2a}\right.}{9c^3}}{c^3}$$

input

```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output

```
-((-1/9*(a^3*(-1/2*(c^4*(87*a*((8*c*((8*a*c^2*Sqrt[1 - 1/(a^2*x^2)])/(3*Sq
rt[c - c/(a*x)]) + (2*a*c*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3))/5 +
 (2*a*c*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(3/2))/5) - 4*a^2*Sqrt[1 - 1/(
a^2*x^2)]*(c - c/(a*x))^(5/2)))/a - (17*a*c^3*(c - c/(a*x))^(7/2))/Sqrt[1
- 1/(a^2*x^2)]))/c^3 - (2*a^4*(c - c/(a*x))^(11/2))/(9*c^2*Sqrt[1 - 1/(a^2
*x^2)]))/c^3)
```

3.541.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

3.541. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

3.541. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.541.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.30

method	result	size
gosper	$-\frac{2(ax+1)\left(656a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45x^{4}(ax-1)^{2}}$	86
default	$-\frac{2(ax+1)\left(656a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45x^{4}(ax-1)^{2}}$	86
risch	$-\frac{2\left(476a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{45x^{4}(ax-1)}-\frac{8a^{5}x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	125

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/45*(a*x+1)*(656*a^5*x^5+328*a^4*x^4-82*a^3*x^3+41*a^2*x^2-20*a*x+5)*(c*
(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

3.541.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.29

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$=-\frac{2\left(656\,a^{5}x^{5}+328\,a^{4}x^{4}-82\,a^{3}x^{3}+41\,a^{2}x^{2}-20\,ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{45\left(ax^{5}-x^{4}\right)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas
")
```

output
```
-2/45*(656*a^5*x^5 + 328*a^4*x^4 - 82*a^3*x^3 + 41*a^2*x^2 - 20*a*x + 5)*s
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

3.541.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Timed out
```

3.541.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.541.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.541. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.541.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.37

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$=-\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}\,(656\,a^4\,x^4+984\,a^3\,x^3+902\,a^2\,x^2+943\,a\,x+923)}{45\,x^4}$$
$$-\frac{1856\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{45\,x^4\,(a\,x-1)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

output
```
- (2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(943*a*x + 90
2*a^2*x^2 + 984*a^3*x^3 + 656*a^4*x^4 + 923))/(45*x^4) - (1856*((a*x - 1)/
(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(45*x^4*(a*x - 1))
```

3.541. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

3.542 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

3.542.1 Optimal result

Integrand size = 20, antiderivative size = 185

$$\begin{aligned}
&\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx\\
&=c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x\\
&\quad-\frac{2c(1-n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{an}\\
&\quad-\frac{2^{n/2}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}
\end{aligned}$$

output
```
c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x-2*c*(1-n)*(1+1/a/x)^(1/2*n)*hype
rgeom([1, 1/2*n],[1+1/2*n],(a+1/x)/(a-1/x))/a/n/((1-1/a/x)^(1/2*n))-2^(1/2
*n)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([1-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/
x)/a)/a/(2-n)
```

3.542.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{ce^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}(-1+n)n}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
(c*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2
, 2 + n/2, -E^(2*ArcCoth[a*x])]) + E^(2*ArcCoth[a*x])*(-1 + n)*n*Hypergeom
etric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n*x + Hyper
geometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])] + (-1 + n)*Hypergeometr
ic2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*n*(2 + n))
```

3.542.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6732, 138, 79, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)e^{n\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{ 6732}$$
$$-c\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{ 138}$$
$$-c\left(\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x^2d\frac{1}{x}-\frac{\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}d\frac{1}{x}}{a^2}\right)$$
$$\Big\downarrow \text{ 79}$$

$$-c\left(\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x^2d\frac{1}{x}+\frac{2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}\right)$$

$$\Big\downarrow \text{107}$$

$$-c\left(-\frac{(1-n)\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{a}+\frac{2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}\right.$$

$$\Big\downarrow \text{141}$$

$$-c\left(\frac{2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}+\frac{2(1-n)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hyperg}}{an}\right.$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
-(c*(-((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x) + (2*(1 - n)*(1 + 1/
(a*x))^(n/2)*Hypergeometric2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1)
)])/(a*n*(1 - 1/(a*x))^(n/2)) + (2^(n/2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeom
etric2F1[(2 - n)/2, 1 - n/2, 2 - n/2, (a - x^(-1))/(2*a)])/(a*(2 - n))))
```

3.542.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 138
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_))^2, x_] :> Simp[b*(d/f^2)   Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1), x],
 x] + Simp[(b*e - a*f)*((d*e - c*f)/f^2)   Int[(a + b*x)^(m - 1)*((c + d*x)
^(n - 1)/(e + f*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ
[m + n, 0] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

3.542.4 Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

3.542.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="fricas")
```

output
```
integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*x), x)
```

3.542.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(\int ae^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-\frac{e^{n\,\mathrm{acoth}\,(ax)}}{x}\right)\,dx\right)}{a}$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a/x),x)
```

output
```
c*(Integral(a*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x, x))/a
```

3.542.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.542.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a*x)),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a*x)), x)
```

3.543 $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

3.543.1 Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c} - \frac{2(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

output

```
(1+1/a/x)^(1+1/2*n)*x/c/((1-1/a/x)^(1/2*n))-2*(1+n)*(1+1/a/x)^(1/2*n)*hype
rgeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n/((1-1/a/x)^(1/2*n))
```

3.543.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{e^{n\coth^{-1}(ax)}\Big(e^{2\coth^{-1}(ax)}n(1+n)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\Big(-1+anx}{acn(2+n)}$$

input

```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output

```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(1 + n)*Hypergeometric2F1[1, 1 +
 n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + (1 + n)*Hyperge
ometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

3.543.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6732, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6732}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{107}$$

$$-\frac{\frac{(n+1)\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{a}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}$$

$$\Big\downarrow \text{141}$$

$$-\frac{\frac{2(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{an}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}$$

input

```
Int[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output

```
-((-(((1 + 1/(a*x))^((2 + n)/2)*x)/(1 - 1/(a*x))^(n/2)) + (2*(1 + n)*(1 +
1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/(a + x^(
-1))])/(a*n*(1 - 1/(a*x))^(n/2)))/c)
```

3.543.3.1 Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

3.543.4 Maple [F]

$$\int \frac{\mathrm{e}^{n \,\mathrm{arccoth}(ax)}}{c-\frac{c}{ax}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

3.543.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="fricas")
```

output
```
integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

3.543.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a/x),x)
```

output
```
a*Integral(x*exp(n*acoth(a*x))/(a*x - 1), x)/c
```

3.543.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

3.543.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a\,x}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a*x)),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a*x)), x)
```

3.544 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} \, dx$

3.544.1 Optimal result

Integrand size = 22, antiderivative size = 166

$$\begin{aligned}\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} \, dx = & -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x}{c^2} \\ & - \frac{2(2+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2 n}\end{aligned}$$

output
```
-(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)+(1-1/a/x)^(-1-
1/2*n)*(1+1/a/x)^(1+1/2*n)*x/c^2-2*(2+n)*(1+1/a/x)^(1/2*n)*hypergeom([1, -
1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c^2/n/((1-1/a/x)^(1/2*n))
```

3.544.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} \, dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(n(1+ax)(-3+2ax+n(-1+ax))-2(2+n)^2(-1+ax)\,\text{Hypergeometric2F}\right.}{ac^2n(2+n)(-1+ax)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
((1 + 1/(a*x))^(n/2)*(n*(1 + a*x)*(-3 + 2*a*x + n*(-1 + a*x)) - 2*(2 + n)^
2*(-1 + a*x)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (-1 + a*x)/(1 + a*x)]))
/(a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x))
```

3.544.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6732, 114, 25, 27, 172, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6732}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{114}$$

$$-\frac{x\left(-\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}-\int -\frac{\left(a(n+2)+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}x}{a^2}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(a(n+2)+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \left(a(n+2)+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow \text{172}$$

$$-\frac{\frac{\frac{a(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+2}-\frac{a\int -(n+2)^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{a\int (n+2)^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{a(n+2)\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow 141$$

$$-\frac{\frac{\frac{2a(n+2)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+}{2}}}{c^2}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
-((-((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x) + ((a*(3 + n)*(
1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(2 + n) + (2*a*(2 + n)*
(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/(a
+ x^(-1))])/(n*(1 - 1/(a*x))^(n/2)))/a^2)/c^2)
```

3.544.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.544. $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

3.544.4 Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

3.544.5 Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*c^2*x +
c^2), x)
```

3.544.6 Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{a^2\int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-2ax+1}\,dx}{c^2}$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a/x)**2,x)
```

output
```
a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

3.544.7 Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

3.544.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^2}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^2,x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^2, x)
```

3.545 $\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{ax}\right)^{3/2} \, dx$

3.545.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{ax}\right)^{3/2} \, dx = -\frac{2^{\frac{5}{2}-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} \left(c-\frac{c}{ax}\right)^{3/2} \operatorname{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(-3+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(1-\frac{1}{ax}\right)^{3/2}}$$

output

```
-2^(5/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(c-c/a/x)^(3/2)*AppellF1(1+1/2*n,-3/2+1
/2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/(1-1/a/x)^(3/2)
```

3.545.2 Mathematica [F(-1)]

Timed out.

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{ax}\right)^{3/2} \, dx = \$\text{Aborted}$$

input

```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output

```
$Aborted
```

3.545.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6736}$$

$$\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{ 6732}$$

$$-\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int \left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{ 153}$$

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{AppellF1}\left(\frac{n+2}{2},\frac{n-3}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
-((2^(5/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^(3/2)*AppellF1[(2
 + n)/2, (-3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2
 + n)*(1 - 1/(a*x))^(3/2)))
```

3.545.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.545.4 Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

3.545.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))
/(a*x), x)
```

3.545.6 Sympy [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

3.545.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.545.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^{3/2}\,dx$$

input

```
int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2),x)
```

output

```
int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2), x)
```

3.546 $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

3.546.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\frac{2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{ax}}\,\text{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(-1+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{1-\frac{1}{ax}}}$$

output
```
-2^(3/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,-1/2+1/2*n,2,2+1/2*n,1
/2*(a+1/x)/a,1+1/a/x)*(c-c/a/x)^(1/2)/a/(2+n)/(1-1/a/x)^(1/2)
```

3.546.2 Mathematica [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \$\text{Aborted}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
$Aborted
```

3.546.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6736$$

$$\frac{\sqrt{c-\frac{c}{ax}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}dx}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 6732$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 153$$

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{AppellF1}\left(\frac{n+2}{2},\frac{n-1}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((2^(3/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[c - c/(a*x)]*AppellF1[(2 +
 n)/2, (-1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*Sqrt[1 - 1/(a*x)]))
```

3.546.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.546.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

3.546.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x)), x)
```

3.546.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{-c\left(-1+\frac{1}{ax}\right)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

3.546.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.546.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-\frac{c}{a\,x}}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2), x)
```

3.547 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} \, dx$

3.547.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} \, dx = -\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} \operatorname{AppellF1}\left(\frac{2+n}{2}, \frac{1+n}{2}, 2, \frac{4+n}{2}, \frac{a+\frac{1}{x}}{2a}, 1+\frac{1}{ax}\right)}{a(2+n)\sqrt{c-\frac{c}{ax}}}$$

output
```
-2^(1/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,1/2+1/2*n,2,2+1/2*n,1/
2*(a+1/x)/a,1+1/a/x)*(1-1/a/x)^(1/2)/a/(2+n)/(c-c/a/x)^(1/2)
```

3.547.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} \, dx = \$\text{Aborted}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
$Aborted
```

3.547.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \color{blue}{6736}$$

$$\frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}dx}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \color{blue}{6732}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \color{blue}{153}$$

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{AppellF1}\left(\frac{n+2}{2},\frac{n+1}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
-((2^(1/2 - n/2)*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 +
 n)/2, (1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
n)*Sqrt[c - c/(a*x)]))
```

3.547.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.547.4 Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
```

3.547.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*c*x
- c), x)
```

3.547.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a/x)**(1/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))), x)
```

3.547.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

3.547.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2), x)
```

3.548 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

3.548.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\frac{2^{-\frac{1}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\operatorname{AppellF1}\left(\frac{2+n}{2},\frac{3+n}{2},2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(c-\frac{c}{ax}\right)^{3/2}}$$

output
```
-2^(-1/2-1/2*n)*(1-1/a/x)^(3/2)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,3/2+1
/2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/(c-c/a/x)^(3/2)
```

3.548.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \$\text{Aborted}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
$Aborted
```

3.548.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6736}$$

$$\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{6732}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{-\frac{n}{2}-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\operatorname{AppellF1}\left(\frac{n+2}{2},\frac{n+3}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(c-\frac{c}{ax}\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
-((2^(-1/2 - n/2)*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(
2 + n)/2, (3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2
 + n)*(c - c/(a*x))^(3/2)))
```

3.548.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.548.4 Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
```

3.548.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a^
2*c^2*x^2 - 2*a*c^2*x + c^2), x)
```

3.548.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a/x)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(-1 + 1/(a*x)))**(3/2), x)
```

3.548.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

3.548.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

3.548.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2), x)
```

3.549 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

3.549.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^p\text{AppellF1}\left(\frac{2+n}{2},\frac{1}{2}(n-2p),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}$$

output

```
-2^(1-1/2*n+p)*(1+1/a/x)^(1+1/2*n)*(c-c/a/x)^p*AppellF1(1+1/2*n,1/2*n-p,2,
2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/((1-1/a/x)^p)
```

3.549.2 Mathematica [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

input

```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output

```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p, x]
```

3.549.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^p e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$\Big\downarrow \text{6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int \left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{-\frac{n}{2}+p+1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p \text{AppellF1}\left(\frac{n+2}{2},\frac{1}{2}(n-2p),2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
-((2^(1 - n/2 + p)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^p*AppellF1[(2 +
 n)/2, (n - 2*p)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2
+ n)*(1 - 1/(a*x))^p))
```

3.549.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.549.4 Maple [F]

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
```

3.549.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx=\int\left(c-\frac{c}{ax}\right)^{p}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a*c*x - c)/(a*x))^p, x)
```

3.549.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a/x)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

3.549.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.549.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^p,x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.550 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

3.550.1 Optimal result

Integrand size $=23$, antiderivative size $=67$

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{1}{ax}\right)}{a(1+p)}$$

output
```
-(1+1/a/x)^(p+1)*(c-c/a/x)^p*hypergeom([2, p+1],[2+p],1+1/a/x)/a/(p+1)/((1
-1/a/x)^p)
```

3.550.2 Mathematica [A] (verified)

Time $=0.03$ (sec) , antiderivative size $=67$, normalized size of antiderivative $=1.00$

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{1}{ax}\right)}{a(1+p)}$$

input
```
Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
-(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

3.550.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6736, 6732, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^p e^{2p\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$\Big\downarrow \text{6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int \left(1+\frac{1}{ax}\right)^p x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left(2,p+1,p+2,1+\frac{1}{ax}\right)}{a(p+1)}$$

input
```
Int[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
-(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

3.550.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Int
egerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

3.550.4 Maple [F]

$$\int e^{2p\,\operatorname{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx$$

input
```
int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

output
```
int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

3.550.5 Fricas [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx=\int\left(c-\frac{c}{ax}\right)^{p}\left(\frac{ax+1}{ax-1}\right)^{p}dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^p*((a*c*x - c)/(a*x))^p, x)
```

3.550.6 Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p}e^{2p\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(2*p*acoth(a*x))*(c-c/a/x)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

3.550.7 Maxima [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.550.8 Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

input
```
int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p,x)
```

output
```
int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.551 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

3.551.1 Optimal result

Integrand size = 23, antiderivative size = 93

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1-p}\left(c-\frac{c}{ax}\right)^p\operatorname{AppellF1}\left(1-p,-2p,2,2-p,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(1-p)}$$

output
```
-4^p*(1+1/a/x)^(1-p)*(c-c/a/x)^p*AppellF1(1-p,-2*p,2,2-p,1/2*(a+1/x)/a,1+1
/a/x)/a/(1-p)/((1-1/a/x)^p)
```

3.551.2 Mathematica [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

input
```
Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]), x]
```

3.551.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^p e^{-2p\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$\Big\downarrow \text{6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int \left(1-\frac{1}{ax}\right)^{2p}\left(1+\frac{1}{ax}\right)^{-p}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{1-p}\text{AppellF1}\left(1-p,-2p,2,2-p,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a(1-p)}$$

input
```
Int[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
-((4^p*(1 + 1/(a*x))^(1 - p)*(c - c/(a*x))^p*AppellF1[1 - p, -2*p, 2, 2 -
p, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - p)*(1 - 1/(a*x))^p))
```

3.551.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.551.4 Maple [F]

$$\int \left(c-\frac{c}{ax}\right)^{p} \mathrm{e}^{-2p\,\mathrm{arccoth}(ax)}dx$$

input
```
int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
```

output
```
int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
```

3.551.5 Fricas [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}\,dx=\int \frac{\left(c-\frac{c}{ax}\right)^{p}}{\left(\frac{ax+1}{ax-1}\right)^{p}}\,dx$$

input
```
integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

output
```
integral(((a*c*x - c)/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.551.6 Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\right)^pe^{-2p\operatorname{acoth}(ax)}\,dx$$

input
```
integrate((c-c/a/x)**p/exp(2*p*acoth(a*x)),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

3.551.7 Maxima [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.551.8 Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \mathrm{e}^{-2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

input
```
int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p,x)
```

output
```
int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.552 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

3.552.1 Optimal result

Integrand size = 22, antiderivative size = 57

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\left(c-\frac{c}{ax}\right)^p x+\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{1}{ax}\right)}{ap}$$

output
```
(c-c/a/x)^p*x+(2-p)*(c-c/a/x)^p*hypergeom([1, p],[p+1],1-1/a/x)/a/p
```

3.552.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\left(c-\frac{c}{ax}\right)^p\left(apx-(-2+p)\,\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{1}{ax}\right)\right)}{ap}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
((c - c/(a*x))^p*(a*p*x - (-2 + p)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a
*x)]))/(a*p)
```

3.552.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6717, 6683, 1035, 281, 899, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^p(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^p}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{p-1}dx}{a}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{p-1}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left((2-p)\int \left(c-\frac{c}{ax}\right)^{p-1}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^p}{c}\right)}{a}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{c\left(-\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)}{cp}-\frac{ax\left(c-\frac{c}{ax}\right)^p}{c}\right)}{a}$$

input

```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
-((c*(-((a*(c - c/(a*x))^p*x)/c) - ((2 - p)*(c - c/(a*x))^p*Hypergeometric
2F1[1, p, 1 + p, 1 - 1/(a*x)])/(c*p)))/a)
```

3.552.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.552.4 Maple [F]

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}dx$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)
```

output
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)
```

3.552.5 Fricas [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

3.552.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.90 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.72

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$= a\left(\begin{cases} \frac{0^px}{a}+\frac{0^p\log{(ax-1)}}{a^2}-\frac{a^{-p}c^ppx^{2-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{for}\ |ax|>1\\ \frac{0^px}{a}+\frac{0^p\log{(-ax+1)}}{a^2}-\frac{a^{-p}c^ppx^{2-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{otherwise}\end{cases}\right)$$
$$+\begin{cases} \frac{0^p\log{(ax-1)}}{a}-\frac{a^{-p}c^ppx^{1-p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|ax\right)}{\Gamma(2-p)\Gamma(p+1)} & \text{for}\ |ax|>1\\ \frac{0^p\log{(-ax+1)}}{a}-\frac{a^{-p}c^ppx^{1-p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|ax\right)}{\Gamma(2-p)\Gamma(p+1)} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**p,x)
```

output
```
a*Piecewise((0**p*x/a + 0**p*log(a*x - 1)/a**2 - c**p*p*x**(2 - p)*exp(I*p
i*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(a**p*gamm
a(3 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*x/a + 0**p*log(-a*x + 1)/a**2
 - c**p*p*x**(2 - p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p
), (3 - p,), a*x)/(a**p*gamma(3 - p)*gamma(p + 1)), True)) + Piecewise((0*
*p*log(a*x - 1)/a - c**p*p*x**(1 - p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hy
per((1 - p, 1 - p), (2 - p,), a*x)/(a**p*gamma(2 - p)*gamma(p + 1)), Abs(a
*x) > 1), (0**p*log(-a*x + 1)/a - c**p*p*x**(1 - p)*exp(I*pi*p)*gamma(p)*g
amma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(a**p*gamma(2 - p)*gamma(
p + 1)), True))
```

3.552.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

3.552.8 Giac [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

3.552.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1), x)
```

3.553 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

3.553.1 Optimal result

Integrand size = 20, antiderivative size = 90

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=-\frac{2^{\frac{1}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^p\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-p,2,\frac{5}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{3a}$$

output
```
-1/3*2^(1/2+p)*(1+1/a/x)^(3/2)*(c-c/a/x)^p*AppellF1(3/2,1/2-p,2,5/2,1/2*(a
+1/x)/a,1+1/a/x)/a/((1-1/a/x)^p)
```

3.553.2 Mathematica [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

output
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p, x]
```

3.553.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

$$\Big\downarrow \text{6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p\,dx$$

$$\Big\downarrow \text{6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-p,2,\frac{5}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{3a}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

output
```
-1/3*(2^(1/2 + p)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^p*AppellF1[3/2, 1/2 -
p, 2, 5/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p)
```

3.553.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.553.4 Maple [F]

$$\int \frac{\left(c-\frac{c}{ax}\right)^{p}}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

3.553.5 Fricas [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}\,dx=\int \frac{\left(c-\frac{c}{ax}\right)^{p}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1))/(a*x -
1), x)
```

3.553.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.553.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.553.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.554 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

3.554.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{2^{\frac{3}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^p \operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-p,2,\frac{3}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a}$$

output

```
-2^(3/2+p)*(c-c/a/x)^p*AppellF1(1/2,-1/2-p,2,3/2,1/2*(a+1/x)/a,1+1/a/x)*(1
+1/a/x)^(1/2)/a/((1-1/a/x)^p)
```

3.554.2 Mathematica [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

input

```
Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

output

```
Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x], x]
```

3.554.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

$$\Big\downarrow \ 6736$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$\Big\downarrow \ 6732$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int \frac{\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \ 153$$

$$-\frac{2^{p+\frac{3}{2}}\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p-\frac{1}{2},2,\frac{3}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a}$$

input
```
Int[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

output
```
-((2^(3/2 + p)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^p*AppellF1[1/2, -1/2 - p, 2
, 3/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p))
```

3.554.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

3.554.4 Maple [F]

$$\int \left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

3.554.5 Fricas [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
integral(((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.554.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(-1+\frac{1}{ax}\right)\right)^p\,dx$$

input
```
integrate((c-c/a/x)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(-1 + 1/(a*x)))**p, x)
```

3.554.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.554.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(c-\frac{c}{a\,x}\right)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input

```
int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.555 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

3.555.1 Optimal result

Integrand size = 22, antiderivative size = 114

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}\\ &\quad+\frac{\left(c-\frac{c}{ax}\right)^{2+p}\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)}\\ &\quad-\frac{\left(c-\frac{c}{ax}\right)^{2+p}\text{Hypergeometric2F1}\left(1,2+p,3+p,1-\frac{1}{ax}\right)}{ac^2}\end{aligned}$$

output
```
(c-c/a/x)^(2+p)*x/c^2+1/2*(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1/2*(a-
1/x)/a)/a/c^2/(2+p)-(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1-1/a/x)/a/c^
2
```

3.555.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=\frac{\left(c-\frac{c}{ax}\right)^p(-1+ax)^2\Big(\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{a-\frac{1}{x}}{2a}\right)+2(2+p)\big(ax-\text{Hypergeometric2F1}\,(}{2a^3(2+p)x^2}$$

input
```
Integrate[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

output
```
((c - c/(a*x))^p*(-1 + a*x)^2*(Hypergeometric2F1[1, 2 + p, 3 + p, (a - x^(
-1))/(2*a)] + 2*(2 + p)*(a*x - Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a
*x)])))/(2*a^3*(2 + p)*x^2)
```

3.555.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^p(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^p}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a\left(-\frac{\int \frac{c\left(c-\frac{c}{ax}\right)^{p+1}\left(\frac{p+1}{x}+a(p+2)\right)x}{a\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{ac}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{a\left(-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}\left(\frac{p+1}{x}+a(p+2)\right)x}{a+\frac{1}{x}}d\frac{1}{x}}{a^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

$$\Big\downarrow \textbf{174}$$

$$-\frac{a\left(-\frac{(p+2)\int \left(c-\frac{c}{ax}\right)^{p+1}xd\frac{1}{x}-\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}}{a+\frac{1}{x}}d\frac{1}{x}}{a^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

$$\Big\downarrow \textbf{75}$$

$$-\frac{a\left(-\frac{-\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}}{a+\frac{1}{x}}d\frac{1}{x}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,1-\frac{1}{ax}\right)}{c}}{a^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

$$\Big\downarrow \textbf{78}$$

$$-\frac{a\left(-\frac{\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{a-\frac{1}{x}}{2a}\right)}{2c(p+2)}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,1-\frac{1}{ax}\right)}{c}}{a^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

input
```
Int[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-(((c - c/(a*x))^(2 + p)*x)/(a*c)) - (((c - c/(a*x))^(2 + p)*Hyperge
ometric2F1[1, 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*c*(2 + p)) - ((c - c/(
a*x))^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/c)/a^2))/c)
```

3.555. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

3.555.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

3.555. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.555.4 Maple [F]

$$\int \frac{\left(c-\frac{c}{ax}\right)^{p} (ax-1)}{ax+1}dx$$

input
```
int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
```

output
```
int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
```

3.555.5 Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p} dx = \int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\, dx$$

input
```
integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

3.555.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**p*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p*(a*x - 1)/(a*x + 1), x)
```

3.555.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

3.555.8 Giac [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

3.555.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1), x)
```

3.556 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.556.1 Optimal result

Integrand size = 20, antiderivative size = 393

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/}$$

output
```
5/72*a^6*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)*x^7-7/72*a^7*c^4*(1-1/a/x)^(
5/2)*(1+1/a/x)^(11/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(11/2)*x^9
+35/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+35/384*a*c^4*(1+1/a
/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+7/192*a^2*c^4*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^
(1/2)+1/64*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)+1/144*a^4*c^4*(1+1/
a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)-5/144*a^5*c^4*(1+1/a/x)^(11/2)*x^6*(1-1/a/x
)^(1/2)+35/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.556.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.28

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$

$$=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(128-837ax-512a^2x^2+978a^3x^3+768a^4x^4-600a^5x^5-512a^6x^6+144a^7x^7+128a^8\right.}{1152a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

output
```
(c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 - 837*a*x - 512*a^2*x^2 + 978*a^3*x^3
 + 768*a^4*x^4 - 600*a^5*x^5 - 512*a^6*x^6 + 144*a^7*x^7 + 128*a^8*x^8) +
315*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

3.556.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^4 e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8 dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^4 dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8}{a^8} dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8 dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^4 dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8}{a^8} dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8 dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^4 dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8}{a^8} dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8 dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^4 dx$$

$$\Big\downarrow \text{6745}$$

3.556. $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^4 \, dx$

$$a^8c^4\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \ 27$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \ 2005$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \ 6745$$

$$a^8c^4\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \ 27$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \ 2005$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \ 6745$$

$$a^8c^4\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \ 27$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \ 2005$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \ 6745$$

$$a^8c^4\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \ 27$$

$$c^4\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

3.556. $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^4 dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^4 dx$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

output
```
$Aborted
```

3.556.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.556.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

method	result
risch	$\frac{(128a^8x^8+144a^7x^7-512a^6x^6-600a^5x^5+768a^4x^4+978a^3x^3-512a^2x^2-837ax+128)(ax-1)c^4}{1152a\sqrt{\frac{ax-1}{ax+1}}}+\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^4\sqrt{(ax}}{128\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+}$
default	$-\frac{(ax-1)c^4\left(-128(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-144(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+384(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+456(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-384(a^2x\right.}{1152a\sqrt{}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/1152*(128*a^8*x^8+144*a^7*x^7-512*a^6*x^6-600*a^5*x^5+768*a^4*x^4+978*a^
3*x^3-512*a^2*x^2-837*a*x+128)*(a*x-1)/a*c^4/((a*x-1)/(a*x+1))^(1/2)+35/12
8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4/((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.556.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.43

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(128\,a^9c^4x^9+272\,a^8c^4x^8-368\,a^7c^4x^7-1112\,a^6c^4x}{1152\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas"
)
```

output
```
1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (128*a^9*c^4*x^9 + 272*a^8*c^4*x^8 - 368*a^7*c^4*x
^7 - 1112*a^6*c^4*x^6 + 168*a^5*c^4*x^5 + 1746*a^4*c^4*x^4 + 466*a^3*c^4*x
^3 - 1349*a^2*c^4*x^2 - 709*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/
a
```

3.556.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\left(\int\left(-\frac{4a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^8x^8}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**4,x)
```

output
```
c**4*(Integral(-4*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integr
al(6*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**6*x**
*6/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**8*x**8/sqrt(a*x/(a*
x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)),
x))
```

3.556.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+104\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax-}{(ax+}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima"
)
```

output
```
1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) -
2730*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(1
3/2) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 32768*c^4*((a*x - 1)/(a*x
+ 1))^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)
/(a*x + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a
*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x +
 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 +
 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*
x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2
/(a*x + 1)^9 - a^2))*a
```

3.556.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.55

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = -\frac{35\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{128\,|a|\mathrm{sgn}\left(ax+1\right)}$$
$$-\frac{1}{1152}\sqrt{a^2x^2-1}\Bigg(\Bigg(\frac{837\,c^4}{\mathrm{sgn}\left(ax+1\right)}+2\Bigg(\frac{256\,ac^4}{\mathrm{sgn}\left(ax+1\right)}-\Bigg(\frac{489\,a^2c^4}{\mathrm{sgn}\left(ax+1\right)}+4\Bigg(\frac{96\,a^3c^4}{\mathrm{sgn}\left(ax+1\right)}-\Bigg(\frac{75\,a^4c^4}{\mathrm{sgn}\left(ax+\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
-35/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
- 1/1152*sqrt(a^2*x^2 - 1)*((837*c^4/sgn(a*x + 1) + 2*(256*a*c^4/sgn(a*x +
 1) - (489*a^2*c^4/sgn(a*x + 1) + 4*(96*a^3*c^4/sgn(a*x + 1) - (75*a^4*c^4
/sgn(a*x + 1) + 2*(32*a^5*c^4/sgn(a*x + 1) - (8*a^7*c^4*x/sgn(a*x + 1) + 9
*a^6*c^4/sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x - 128*c^4/(a*sgn(a*x + 1)))
```

3.556.9 Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{\frac{455\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96}-\frac{35\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{581\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32}+\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{32}+\frac{512\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{9}-\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{32}+\frac{581\,c^4\,\left(\right.}{}}{a-\frac{9\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{36\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{84\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{126\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{126\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{84\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{36\,a\,\left(a\,x\right.}{\left(a\,x+\right.}}$$
$$+\frac{35\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

input
```
int((c - a^2*c*x^2)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((455*c^4*((a*x - 1)/(a*x + 1))^(3/2))/96 - (35*c^4*((a*x - 1)/(a*x + 1))^
(1/2))/64 - (581*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (1289*c^4*((a*x - 1
)/(a*x + 1))^(7/2))/32 + (512*c^4*((a*x - 1)/(a*x + 1))^(9/2))/9 - (1289*c
^4*((a*x - 1)/(a*x + 1))^(11/2))/32 + (581*c^4*((a*x - 1)/(a*x + 1))^(13/2
))/32 - (455*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 + (35*c^4*((a*x - 1)/(a*
x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a
*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)
^4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (3
6*a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1
)^9)/(a*x + 1)^9) + (35*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

3.556. $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.557 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

3.557.1 Optimal result

Integrand size = 20, antiderivative size = 313

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}$$

output

```
5/42*a^5*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x^6-1/7*a^6*c^3*(1-1/a/x)^(5/
2)*(1+1/a/x)^(9/2)*x^7+5/16*c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a
+5/48*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/24*a^2*c^3*(1+1/a/x)^(5/
2)*x^3*(1-1/a/x)^(1/2)+1/56*a^3*c^3*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)-1/
14*a^4*c^3*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)+5/16*c^3*x*(1-1/a/x)^(1/2)*
(1+1/a/x)^(1/2)
```

3.557.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$$
$$=\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(48-231ax-144a^2x^2+182a^3x^3+144a^4x^4-56a^5x^5-48a^6x^6)+105\log\left(\left(1+\sqrt{1-}\right.\right.\right.}{336a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

output
```
(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 - 231*a*x - 144*a^2*x^2 + 182*a^3*x^3
+ 144*a^4*x^4 - 56*a^5*x^5 - 48*a^6*x^6) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^
2)])*x]))/(336*a)
```

3.557.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{\coth^{-1}(ax)}dx$$
$$\Big\downarrow \; 6745$$
$$-a^6c^3\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$
$$\Big\downarrow \; 27$$
$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$
$$\Big\downarrow \; 2005$$
$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$
$$\Big\downarrow \; 6745$$
$$-a^6c^3\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$
$$\Big\downarrow \; 27$$

$$-c^3 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3 \int \frac{e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3 \int \frac{e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3 \int \frac{e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3 \int e^{\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3 \int e^{\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \text{ 6745}$$

3.557. $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^3 \, dx$

$$-a^6c^3\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow 27$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow 2005$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow 6745$$

$$-a^6c^3\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow 27$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow 2005$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow 6745$$

$$-a^6c^3\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow 27$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow 2005$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow 6745$$

$$-a^6c^3\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow 27$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

3.557. $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.557.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.557. $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.557.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{\left(48 a^{6} x^{6}+56 a^{5} x^{5}-144 a^{4} x^{4}-182 a^{3} x^{3}+144 a^{2} x^{2}+231 a x -48\right) \left(a x -1\right) c^{3}}{336 a \sqrt{\frac{a x -1}{a x +1}}}+\frac{5 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{3} \sqrt{\left(a x -1\right) \left(a x +1\right)}}{16 \sqrt{a^{2}}\, \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$
default	$\frac{\left(a x -1\right) c^{3} \left(-48 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{4} x^{4}-56 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{3} x^{3}+96 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{2} x^{2}+126 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a x +64 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \right.}{336 a \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/336*(48*a^6*x^6+56*a^5*x^5-144*a^4*x^4-182*a^3*x^3+144*a^2*x^2+231*a*x-
48)*(a*x-1)/a*c^3/((a*x-1)/(a*x+1))^(1/2)+5/16*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x
+1))^(1/2)
```

3.557.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{105\,c^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(48\,a^{7}c^{3}x^{7}+104\,a^{6}c^{3}x^{6}-88\,a^{5}c^{3}x^{5}-326\,a^{4}c^{3}x^{4}-}{336\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

output
```
1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) - (48*a^7*c^3*x^7 + 104*a^6*c^3*x^6 - 88*a^5*c^3*x^5
- 326*a^4*c^3*x^4 - 38*a^3*c^3*x^3 + 375*a^2*c^3*x^2 + 183*a*c^3*x - 48*c^
3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.557.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-c^3\left(\int\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**3,x)
```

output
```
-c**3*(Integral(3*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integr
al(-3*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6
/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) -
1/(a*x + 1)), x))
```

3.557.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$$
$$=\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\,c\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+\frac{35\,(}{(}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

output
```
1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 7
00*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2)
 - 3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(
5/2) + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x +
 1)))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*
x - 1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5
*a^2/(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x +
1)^7 - a^2))*a
```

3.557.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{16\,|a|\text{sgn}\,(ax+1)}$$

$$-\frac{1}{336}\sqrt{a^2x^2-1}\Bigg(\Bigg(2\left(\frac{72\,ac^3}{\text{sgn}\,(ax+1)}-\Bigg(\frac{91\,a^2c^3}{\text{sgn}\,(ax+1)}+4\left(\frac{18\,a^3c^3}{\text{sgn}\,(ax+1)}-\Bigg(\frac{6\,a^5c^3x}{\text{sgn}\,(ax+1)}+\frac{7\,a^4c^3}{\text{sgn}\,(ax+}\right.\right.\Bigg.\Bigg.\Bigg.\Bigg.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-5/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) -
1/336*sqrt(a^2*x^2 - 1)*((2*(72*a*c^3/sgn(a*x + 1) - (91*a^2*c^3/sgn(a*x +
 1) + 4*(18*a^3*c^3/sgn(a*x + 1) - (6*a^5*c^3*x/sgn(a*x + 1) + 7*a^4*c^3/s
gn(a*x + 1))*x)*x)*x)*x + 231*c^3/sgn(a*x + 1))*x - 48*c^3/(a*sgn(a*x + 1)
))
```

3.557.9 Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$

$$-\frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6}+\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{24}+\frac{128\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{7}-\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{24}+\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{6}-\frac{5\,c^3\,\left(\frac{a\,x-}{a\,x+}\right.}{8}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}$$

input
```
int((c - a^2*c*x^2)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - ((5*c^3*((a*x - 1)/(a*x
 + 1))^(1/2))/8 - (25*c^3*((a*x - 1)/(a*x + 1))^(3/2))/6 + (283*c^3*((a*x
- 1)/(a*x + 1))^(5/2))/24 + (128*c^3*((a*x - 1)/(a*x + 1))^(7/2))/7 - (283
*c^3*((a*x - 1)/(a*x + 1))^(9/2))/24 + (25*c^3*((a*x - 1)/(a*x + 1))^(11/2
))/6 - (5*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (7*a*(a*x - 1))/(a*x +
 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)^3)/(a*x + 1)^3 + (3
5*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x + 1)^5 + (7*a*(a*x
- 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7)
```

3.558 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

3.558.1 Optimal result

Integrand size = 20, antiderivative size = 233

$$\begin{aligned}
&\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx\\
&=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2\\
&\quad+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4\\
&\quad+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a}
\end{aligned}$$

output

```
1/5*a^4*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5+3/8*c^2*arctanh((1-1/a/x)^
(1/2)*(1+1/a/x)^(1/2))/a+1/8*a*c^2*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/2
0*a^2*c^2*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-3/20*a^3*c^2*(1+1/a/x)^(7/2)
*x^4*(1-1/a/x)^(1/2)+3/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.558.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(8-25ax-16a^2x^2+10a^3x^3+8a^4x^4)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 25*a*x - 16*a^2*x^2 + 10*a^3*x^3 + 8*
a^4*x^4) + 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

3.558.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2e^{\coth^{-1}(ax)}dx$$
$$\Big\downarrow \textbf{6745}$$
$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$
$$\Big\downarrow \textbf{27}$$
$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$
$$\Big\downarrow \textbf{2005}$$
$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$
$$\Big\downarrow \textbf{6745}$$
$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$
$$\Big\downarrow \textbf{27}$$
$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

3.558. $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$\Big\downarrow 27$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow 2005$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow 6745$$

$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow 27$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow 2005$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow 6745$$

$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow 27$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow 2005$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow 6745$$

$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow 27$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow 2005$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

3.558. $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.558.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.558.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(8a^4x^4+10a^3x^3-16a^2x^2-25ax+8)(ax-1)c^2}{40a\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-45\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-40((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+45\right.}{120a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/40*(8*a^4*x^4+10*a^3*x^3-16*a^2*x^2-25*a*x+8)*(a*x-1)/a*c^2/((a*x-1)/(a*
x+1))^(1/2)+3/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/((
a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.558.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.54

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(8\,a^5c^2x^5+18\,a^4c^2x^4-6\,a^3c^2x^3-41\,a^2c^2x^2-17\,ac^2x}{40\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

output
```
1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) + (8*a^5*c^2*x^5 + 18*a^4*c^2*x^4 - 6*a^3*c^2*x^3 - 41*a
^2*c^2*x^2 - 17*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.558.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\left(\int\left(-\frac{2a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx + \int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**2,x)
```

output
```
c**2*(Integral(-2*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integr
al(a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(
a*x + 1) - 1/(a*x + 1)), x))
```

3.558.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{1}{40}a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+128\,c^2\left(\frac{ax-1}{ax+1}\right.\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

output
```
1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c
^2*((a*x - 1)/(a*x + 1))^(7/2) + 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*
x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2
/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5
 - a^2))
```

3.558.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{1}{40}\sqrt{a^2x^2-1}\Bigg(\Bigg(2\Bigg(\Bigg(\frac{4\,a^3c^2x}{\operatorname{sgn}(ax+1)}+\frac{5\,a^2c^2}{\operatorname{sgn}(ax+1)}\Bigg)x-\frac{8\,ac^2}{\operatorname{sgn}(ax+1)}\Bigg)x-\frac{25\,c^2}{\operatorname{sgn}(ax+1)}\Bigg)x+\frac{8\,c^2}{a\operatorname{sgn}(ax+}$$
$$-\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/40*sqrt(a^2*x^2 - 1)*((2*((4*a^3*c^2*x/sgn(a*x + 1) + 5*a^2*c^2/sgn(a*x
+ 1))*x - 8*a*c^2/sgn(a*x + 1))*x - 25*c^2/sgn(a*x + 1))*x + 8*c^2/(a*sgn(
a*x + 1))) - 3/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a
*x + 1))
```

3.558.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{\frac{7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2}-\frac{3\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{32\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{5}-\frac{7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{2}+\frac{3\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}$$
$$+\frac{3\,c^2\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a^2*c*x^2)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((7*c^2*((a*x - 1)/(a*x + 1))^(3/2))/2 - (3*c^2*((a*x - 1)/(a*x + 1))^(1/2
))/4 + (32*c^2*((a*x - 1)/(a*x + 1))^(5/2))/5 - (7*c^2*((a*x - 1)/(a*x + 1
))^(7/2))/2 + (3*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/
(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (3*c^2*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.559 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

3.559.1 Optimal result

Integrand size = 18, antiderivative size = 145

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{c\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

output
```
1/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+1/6*a*c*(1+1/a/x)^(3/2)*x
^2*(1-1/a/x)^(1/2)-1/3*a^2*c*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+1/2*c*x*(
1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.559.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(2-3ax-2a^2x^2)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input

```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
```

output

```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 2*a^2*x^2) + 3*Log[(1 + Sqrt[1
- 1/(a^2*x^2)])*x]))/(6*a)
```

3.559.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

3.559. $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

3.559. $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

$$-c\int e^{\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
```

output
```
$Aborted
```

3.559.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.559.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{(2a^2x^2+3ax-2)(ax-1)c}{6a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	108
default	$-\frac{(ax-1)c\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}$	119

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*a^2*x^2+3*a*x-2)*(a*x-1)/a*c/((a*x-1)/(a*x+1))^(1/2)+1/2*ln(a^2*x/
(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)*(a*x+1))^(1/2)/(a*x+
1)/((a*x-1)/(a*x+1))^(1/2)
```

3.559.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3cx^3+5\,a^2cx^2+acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) - (2*a^3*c*x^3 + 5*a^2*c*x^2 + a*c*x - 2*c)*sqrt((a*x - 1)/(a*x
 + 1)))/a
```

3.559.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -c\left(\int\frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c),x)
```

output
```
-c*(Integral(a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1
/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.559.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{1}{6}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) - 8*c*((a*x - 1)/(a*x + 1))^(3/
2) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.559.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= -\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,acx}{\operatorname{sgn}(ax+1)}+\frac{3\,c}{\operatorname{sgn}(ax+1)}\right)x-\frac{2\,c}{a\operatorname{sgn}(ax+1)}\right)\\ &\quad -\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}(ax+1)}\end{aligned}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-1/6*sqrt(a^2*x^2 - 1)*((2*a*c*x/sgn(a*x + 1) + 3*c/sgn(a*x + 1))*x - 2*c/
(a*sgn(a*x + 1))) - 1/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*
sgn(a*x + 1))
```

3.559.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}$$

input
```
int((c - a^2*c*x^2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2)
+ (8*c*((a*x - 1)/(a*x + 1))^(3/2))/3 - c*((a*x - 1)/(a*x + 1))^(5/2))/(a
- (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)
^3)/(a*x + 1)^3)
```

3.560 $\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

3.560.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{\coth^{-1}(ax)}}{ac}$$

output
```
1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

3.560.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{\coth^{-1}(ax)}}{ac}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
```

output
```
E^ArcCoth[a*x]/(a*c)
```

3.560.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx \\ \Big\downarrow \text{6737} \\ \frac{e^{\coth^{-1}(ax)}}{ac}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
```

output
```
E^ArcCoth[a*x]/(a*c)
```

3.560.3.1 Defintions of rubi rules used

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

3.560.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

method	result	size
gosper	$\frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,ac}$	23
default	$\frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,ac}$	23
trager	$\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{ac(ax-1)}$	37

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

3.560.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.62

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

3.560.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
```

output
```
-Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))), x)/c
```

3.560.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{ac\sqrt{\frac{ax-1}{ax+1}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/(a*c*sqrt((a*x - 1)/(a*x + 1)))
```

3.560.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \int -\frac{1}{(a^2cx^2-c)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
undef
```

3.560.9 Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
1/(a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.561 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.561.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2(1-a^2x^2)}$$

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)/a/c^2-1/3/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a
/c^2/(-a^2*x^2+1)
```

3.561.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-1-2ax+2a^2x^2)}{3c^2(-1+ax)^2(1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^2*(-1 + a*x)^2*(1
+ a*x))
```

3.561.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6739, 27, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{2\int \frac{e^{\coth^{-1}(ax)}}{c(1-a^2x^2)}dx}{3c}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2\left(1-a^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{\coth^{-1}(ax)}}{1-a^2x^2}dx}{3c^2}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2\left(1-a^2x^2\right)}$$

$$\Big\downarrow \text{6737}$$

$$\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2\left(1-a^2x^2\right)}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

output
```
(2*E^ArcCoth[a*x])/(3*a*c^2) - (E^ArcCoth[a*x]*(1 - 2*a*x))/(3*a*c^2*(1 -
a^2*x^2))
```

3.561.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.561.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

method	result	size
trager	$\frac{(2a^2x^2-2ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^2(ax-1)^2}$	47
gosper	$\frac{2a^2x^2-2ax-1}{3(a^2x^2-1)c^2\sqrt{\frac{ax-1}{ax+1}}\,a}$	49
default	$\frac{2a^2x^2-2ax-1}{3\sqrt{\frac{ax-1}{ax+1}}\,c^2(ax-1)a(ax+1)}$	52

input

```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output

```
1/3/a/c^2*(2*a^2*x^2-2*a*x-1)/(a*x-1)^2*(-(-a*x+1)/(a*x+1))^(1/2)
```

3.561.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\frac{\left(2\,a^2x^2-2\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\,\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

output

```
1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2
*c^2*x + a*c^2)
```

3.561.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
```

output
```
Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x**2*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**
2
```

3.561.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{12}\,a\left(\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2} + \frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

output
```
1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2) + (6*(a*x - 1)/(a*x + 1) - 1
)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)))
```

3.561.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \int \frac{1}{(a^2cx^2-c)^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 - c)^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.561.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{-2\,a^2\,x^2+2\,a\,x+1}{\left(3\,a\,c^2-3\,a^3\,c^2\,x^2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a*x - 2*a^2*x^2 + 1)/((3*a*c^2 - 3*a^3*c^2*x^2)*((a*x - 1)/(a*x + 1))^(
1/2))
```

3.562 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.562.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{8e^{\coth^{-1}(ax)}}{15ac^3} - \frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3\,(1-a^2x^2)^2} - \frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3\,(1-a^2x^2)}$$

output
```
8/15/((a*x-1)/(a*x+1))^(1/2)/a/c^3-1/15/((a*x-1)/(a*x+1))^(1/2)*(-4*a*x+1)
/a/c^3/(-a^2*x^2+1)^2-4/15/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/c^3/(-a^2*
x^2+1)
```

3.562.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(3+12ax-12a^2x^2-8a^3x^3+8a^4x^4)}{15c^3(-1+ax)^3(1+ax)^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(3 + 12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4)
)/(15*c^3*(-1 + a*x)^3*(1 + a*x)^2)
```

3.562.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6739, 27, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{4\int \frac{e^{\coth^{-1}(ax)}}{c^2(1-a^2x^2)^2}dx}{5c}-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{e^{\coth^{-1}(ax)}}{(1-a^2x^2)^2}dx}{5c^3}-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{6739}$$

$$\frac{4\left(\frac{2}{3}\int \frac{e^{\coth^{-1}(ax)}}{1-a^2x^2}dx-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3a(1-a^2x^2)}\right)}{5c^3}-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{6737}$$

$$\frac{4\left(\frac{2e^{\coth^{-1}(ax)}}{3a}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3a(1-a^2x^2)}\right)}{5c^3}-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}$$

input

```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
```

output

```
-1/15*(E^ArcCoth[a*x]*(1 - 4*a*x))/(a*c^3*(1 - a^2*x^2)^2) + (4*((2*E^ArcC
oth[a*x])/(3*a) - (E^ArcCoth[a*x]*(1 - 2*a*x))/(3*a*(1 - a^2*x^2))))/(5*c^
3)
```

3.562.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.562.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{8a^4x^4-8a^3x^3-12a^2x^2+12ax+3}{15(a^2x^2-1)^2c^3\sqrt{\frac{ax-1}{ax+1}}\,a}$	65
default	$\frac{8a^4x^4-8a^3x^3-12a^2x^2+12ax+3}{15\sqrt{\frac{ax-1}{ax+1}}\,c^3(ax-1)^2a(ax+1)^2}$	68
trager	$\frac{(8a^4x^4-8a^3x^3-12a^2x^2+12ax+3)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^3(ax+1)(ax-1)^3}$	70

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*(8*a^4*x^4-8*a^3*x^3-12*a^2*x^2+12*a*x+3)/(a^2*x^2-1)^2/c^3/((a*x-1)/
(a*x+1))^(1/2)/a
```

3.562.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{(8\,a^4x^4-8\,a^3x^3-12\,a^2x^2+12\,ax+3)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

output
```
1/15*(8*a^4*x^4 - 8*a^3*x^3 - 12*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

3.562.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\int \frac{1}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
```

output
```
-Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x**4*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x +
 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

3.562.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{240}\,a\left(\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-12\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{\frac{20\,(ax-1)}{ax+1}-\frac{90\,(ax-1)^2}{(ax+1)^2}-3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

output
```
-1/240*a*(5*(((a*x - 1)/(a*x + 1))^(3/2) - 12*sqrt((a*x - 1)/(a*x + 1)))/(
a^2*c^3) + (20*(a*x - 1)/(a*x + 1) - 90*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a^2*
c^3*((a*x - 1)/(a*x + 1))^(5/2)))
```

3.562.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{1}{(a^2cx^2-c)^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*c*x^2 - c)^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.562.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{8\,a^4\,x^4-8\,a^3\,x^3-12\,a^2\,x^2+12\,a\,x+3}{15\,a\,c^3\,(a\,x+1)^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4 + 3)/(15*a*c^3*(a*x + 1)^4*((
a*x - 1)/(a*x + 1))^(5/2))
```

3.563 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

3.563.1 Optimal result

Integrand size = 20, antiderivative size = 119

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{16e^{\coth^{-1}(ax)}}{35ac^4} - \frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4(1-a^2x^2)^3} - \frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4(1-a^2x^2)^2} - \frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4(1-a^2x^2)}$$

output
```
16/35/((a*x-1)/(a*x+1))^(1/2)/a/c^4-1/35/((a*x-1)/(a*x+1))^(1/2)*(-6*a*x+1
)/a/c^4/(-a^2*x^2+1)^3-2/35/((a*x-1)/(a*x+1))^(1/2)*(-4*a*x+1)/a/c^4/(-a^2
*x^2+1)^2-8/35/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/c^4/(-a^2*x^2+1)
```

3.563.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-5-30ax+30a^2x^2+40a^3x^3-40a^4x^4-16a^5x^5+16a^6x^6)}{35c^4(-1+ax)^4(1+ax)^3}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-5 - 30*a*x + 30*a^2*x^2 + 40*a^3*x^3 - 40*a^4*x
^4 - 16*a^5*x^5 + 16*a^6*x^6))/(35*c^4*(-1 + a*x)^4*(1 + a*x)^3)
```

3.563.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6739, 27, 6739, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}dx$$

$$\Big\downarrow \textcolor{blue}{6739}$$

$$\frac{6\int \frac{e^{\coth^{-1}(ax)}}{c^3\left(1-a^2x^2\right)^3}dx}{7c}-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{6\int \frac{e^{\coth^{-1}(ax)}}{\left(1-a^2x^2\right)^3}dx}{7c^4}-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}$$

$$\Big\downarrow \textcolor{blue}{6739}$$

$$\frac{6\left(\frac{4}{5}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-a^2x^2\right)^2}dx-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15a\left(1-a^2x^2\right)^2}\right)}{7c^4}-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}$$

$$\Big\downarrow \textcolor{blue}{6739}$$

$$\frac{6\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{e^{\coth^{-1}(ax)}}{1-a^2x^2}dx-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3a\left(1-a^2x^2\right)}\right)-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15a\left(1-a^2x^2\right)^2}\right)}{7c^4}-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}$$

$$\Big\downarrow \textcolor{blue}{6737}$$

$$\frac{6\left(\frac{4}{5}\left(\frac{2e^{\coth^{-1}(ax)}}{3a}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3a\left(1-a^2x^2\right)}\right)-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15a\left(1-a^2x^2\right)^2}\right)}{7c^4}-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```

3.563. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}dx$

output
```
-1/35*(E^ArcCoth[a*x]*(1 - 6*a*x))/(a*c^4*(1 - a^2*x^2)^3) + (6*(-1/15*(E^
ArcCoth[a*x]*(1 - 4*a*x))/(a*(1 - a^2*x^2)^2) + (4*((2*E^ArcCoth[a*x])/(3*
a) - (E^ArcCoth[a*x]*(1 - 2*a*x))/(3*a*(1 - a^2*x^2))))/5))/(7*c^4)
```

3.563.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.563.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5}{35(a^2x^2-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	81
default	$\frac{16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5}{35\sqrt{\frac{ax-1}{ax+1}}\,c^4(ax-1)^3(ax+1)^3a}$	84
trager	$\frac{\left(16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5\right)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^4(ax+1)^2(ax-1)^4}$	86

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/35*(16*a^6*x^6-16*a^5*x^5-40*a^4*x^4+40*a^3*x^3+30*a^2*x^2-30*a*x-5)/(a^
2*x^2-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)/a
```

3.563.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{(16\,a^6x^6-16\,a^5x^5-40\,a^4x^4+40\,a^3x^3+30\,a^2x^2-30\,ax-5)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas"
)
```

output
```
1/35*(16*a^6*x^6 - 16*a^5*x^5 - 40*a^4*x^4 + 40*a^3*x^3 + 30*a^2*x^2 - 30*
a*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*
x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

3.563.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\int \frac{1}{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

output
```
Integral(1/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x**6*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))), x)/c**4
```

3.563.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{1}{2240}\,a\left(\frac{7\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+75\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{42\,(ax-1)}{ax+1}-\frac{175\,(ax-1)^2}{(ax+1)^2}+\frac{700\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima"
)
```

output
```
1/2240*a*(7*(((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2)
+ 75*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + (42*(a*x - 1)/(a*x + 1) - 175*
(a*x - 1)^2/(a*x + 1)^2 + 700*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^4*((a*x
- 1)/(a*x + 1))^(7/2)))
```

3.563.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{1}{(a^2cx^2-c)^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 - c)^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.563.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{32\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}-\frac{\frac{5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{20\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{6\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(1/((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(15*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((a*x - 1)/(a*x + 1))^(3/2)/
(32*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - ((5*(a*x - 1)^2)/(a
*x + 1)^2 - (20*(a*x - 1)^3)/(a*x + 1)^3 - (6*(a*x - 1))/(5*(a*x + 1)) + 1
/7)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

3.563. $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

3.564 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

3.564.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{16c^5(1+ax)^7}{7a}+\frac{4c^5(1+ax)^8}{a}-\frac{8c^5(1+ax)^9}{3a}+\frac{4c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

output
```
-16/7*c^5*(a*x+1)^7/a+4*c^5*(a*x+1)^8/a-8/3*c^5*(a*x+1)^9/a+4/5*c^5*(a*x+1
)^10/a-1/11*c^5*(a*x+1)^11/a
```

3.564.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.56

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{c^5(1+ax)^7\left(281-812ax+938a^2x^2-504a^3x^3+105a^4x^4\right)}{1155a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

output
```
-1/1155*(c^5*(1 + a*x)^7*(281 - 812*a*x + 938*a^2*x^2 - 504*a^3*x^3 + 105*
a^4*x^4))/a
```

3.564.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^5 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^5e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^5dx$$

$$\Big\downarrow \text{27}$$

$$-c^5\int e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^5dx$$

$$\Big\downarrow \text{6690}$$

$$-c^5\int (1-ax)^4(ax+1)^6dx$$

$$\Big\downarrow \text{49}$$

$$-c^5\int \left((ax+1)^{10}-8(ax+1)^9+24(ax+1)^8-32(ax+1)^7+16(ax+1)^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^5\left(\frac{(ax+1)^{11}}{11a}-\frac{4(ax+1)^{10}}{5a}+\frac{8(ax+1)^9}{3a}-\frac{4(ax+1)^8}{a}+\frac{16(ax+1)^7}{7a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

output
```
-(c^5*((16*(1 + a*x)^7)/(7*a) - (4*(1 + a*x)^8)/a + (8*(1 + a*x)^9)/(3*a)
- (4*(1 + a*x)^10)/(5*a) + (1 + a*x)^11/(11*a)))
```

3.564.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.564.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

method	result
gosper	$-\frac{c^{5} x \left(105 a^{10} x^{10}+231 a^{9} x^{9}-385 a^{8} x^{8}-1155 a^{7} x^{7}+330 a^{6} x^{6}+2310 a^{5} x^{5}+462 a^{4} x^{4}-2310 a^{3} x^{3}-1155 a^{2} x^{2}+1155 a x+1155\right)}{1155}$
default	$c^{5} \left(-\frac{1}{11} a^{10} x^{11}-\frac{1}{5} a^{9} x^{10}+\frac{1}{3} a^{8} x^{9}+a^{7} x^{8}-\frac{2}{7} a^{6} x^{7}-2 a^{5} x^{6}-\frac{2}{5} a^{4} x^{5}+2 a^{3} x^{4}+a^{2} x^{3}-a x^{2}-\right.$
norman	$a^{7} c^{5} x^{8}+c^{5} a^{2} x^{3}-c^{5} x-a c^{5} x^{2}+2 a^{3} c^{5} x^{4}-\frac{2}{5} a^{4} c^{5} x^{5}-2 a^{5} c^{5} x^{6}-\frac{2}{7} a^{6} c^{5} x^{7}+\frac{1}{3} a^{8} c^{5} x^{9}-\frac{1}{5} a^{9}$
risch	$a^{7} c^{5} x^{8}+c^{5} a^{2} x^{3}-c^{5} x-a c^{5} x^{2}+2 a^{3} c^{5} x^{4}-\frac{2}{5} a^{4} c^{5} x^{5}-2 a^{5} c^{5} x^{6}-\frac{2}{7} a^{6} c^{5} x^{7}+\frac{1}{3} a^{8} c^{5} x^{9}-\frac{1}{5} a^{9}$
parallelrisch	$a^{7} c^{5} x^{8}+c^{5} a^{2} x^{3}-c^{5} x-a c^{5} x^{2}+2 a^{3} c^{5} x^{4}-\frac{2}{5} a^{4} c^{5} x^{5}-2 a^{5} c^{5} x^{6}-\frac{2}{7} a^{6} c^{5} x^{7}+\frac{1}{3} a^{8} c^{5} x^{9}-\frac{1}{5} a^{9}$
meijerg	$\frac{c^{5} \left(-\frac{x a \left(2520 a^{10} x^{10}+2772 a^{9} x^{9}+3080 a^{8} x^{8}+3465 a^{7} x^{7}+3960 a^{6} x^{6}+4620 a^{5} x^{5}+5544 a^{4} x^{4}+6930 a^{3} x^{3}+9240 a^{2} x^{2}+13860 a x+27720\right)}{27720}-\ln(-\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/1155*c^5*x*(105*a^10*x^10+231*a^9*x^9-385*a^8*x^8-1155*a^7*x^7+330*a^6*
x^6+2310*a^5*x^5+462*a^4*x^4-2310*a^3*x^3-1155*a^2*x^2+1155*a*x+1155)
```

3.564.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

output
```
-1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2
/7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5
*x^3 - a*c^5*x^2 - c^5*x
```

3.564.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}c^5x^{11}}{11}-\frac{a^9c^5x^{10}}{5}+\frac{a^8c^5x^9}{3}+a^7c^5x^8-\frac{2a^6c^5x^7}{7}-2a^5c^5x^6-\frac{2a^4c^5x^5}{5}+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**5,x)
```

output
```
-a**10*c**5*x**11/11 - a**9*c**5*x**10/5 + a**8*c**5*x**9/3 + a**7*c**5*x*
*8 - 2*a**6*c**5*x**7/7 - 2*a**5*c**5*x**6 - 2*a**4*c**5*x**5/5 + 2*a**3*c
**5*x**4 + a**2*c**5*x**3 - a*c**5*x**2 - c**5*x
```

3.564.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

output
```
-1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2
/7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5
*x^3 - a*c^5*x^2 - c^5*x
```

3.564.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

output
```
-1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2
/7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5
*x^3 - a*c^5*x^2 - c^5*x
```

3.564.9 Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{a^9\,c^5\,x^{10}}{5}+\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8-\frac{2\,a^6\,c^5\,x^7}{7}-2\,a^5\,c^5\,x^6-\frac{2\,a^4\,c^5\,x^5}{5}+2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3-a\,c^5\,x^2-c^5\,x$$

input
```
int(((c - a^2*c*x^2)^5*(a*x + 1))/(a*x - 1),x)
```

output
```
a^2*c^5*x^3 - a*c^5*x^2 - c^5*x + 2*a^3*c^5*x^4 - (2*a^4*c^5*x^5)/5 - 2*a^
5*c^5*x^6 - (2*a^6*c^5*x^7)/7 + a^7*c^5*x^8 + (a^8*c^5*x^9)/3 - (a^9*c^5*x
^10)/5 - (a^10*c^5*x^11)/11
```

3.564. $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

3.565 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.565.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx=-\frac{4c^4(1+ax)^6}{3a}+\frac{12c^4(1+ax)^7}{7a}-\frac{3c^4(1+ax)^8}{4a}+\frac{c^4(1+ax)^9}{9a}$$

output
```
-4/3*c^4*(a*x+1)^6/a+12/7*c^4*(a*x+1)^7/a-3/4*c^4*(a*x+1)^8/a+1/9*c^4*(a*x
+1)^9/a
```

3.565.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx=\frac{c^4(1+ax)^6(-65+138ax-105a^2x^2+28a^3x^3)}{252a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
(c^4*(1 + a*x)^6*(-65 + 138*a*x - 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

3.565.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^4 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^4e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4dx$$

$$\Big\downarrow \text{27}$$

$$-c^4\int e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4dx$$

$$\Big\downarrow \text{6690}$$

$$-c^4\int (1-ax)^3(ax+1)^5dx$$

$$\Big\downarrow \text{49}$$

$$-c^4\int \left(-(ax+1)^8+6(ax+1)^7-12(ax+1)^6+8(ax+1)^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^4\left(-\frac{(ax+1)^9}{9a}+\frac{3(ax+1)^8}{4a}-\frac{12(ax+1)^7}{7a}+\frac{4(ax+1)^6}{3a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
-(c^4*((4*(1 + a*x)^6)/(3*a) - (12*(1 + a*x)^7)/(7*a) + (3*(1 + a*x)^8)/(4
*a) - (1 + a*x)^9/(9*a)))
```

3.565.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.565.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{c^4x\left(28a^8x^8+63a^7x^7-72a^6x^6-252a^5x^5+378a^3x^3+168a^2x^2-252ax-252\right)}{252}$
default	$c^4\left(\frac{1}{9}a^8x^9+\frac{1}{4}a^7x^8-\frac{2}{7}a^6x^7-a^5x^6+\frac{3}{2}a^3x^4+\frac{2}{3}a^2x^3-a\,x^2-x\right)$
norman	$-c^4x-a\,c^4x^2+\frac{2}{3}a^2c^4x^3+\frac{3}{2}a^3c^4x^4-a^5c^4x^6-\frac{2}{7}a^6c^4x^7+\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
risch	$-c^4x-a\,c^4x^2+\frac{2}{3}a^2c^4x^3+\frac{3}{2}a^3c^4x^4-a^5c^4x^6-\frac{2}{7}a^6c^4x^7+\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
parallelrisch	$-c^4x-a\,c^4x^2+\frac{2}{3}a^2c^4x^3+\frac{3}{2}a^3c^4x^4-a^5c^4x^6-\frac{2}{7}a^6c^4x^7+\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
meijerg	$-\frac{c^4\left(-\frac{ax\left(280a^8x^8+315a^7x^7+360a^6x^6+420a^5x^5+504a^4x^4+630a^3x^3+840a^2x^2+1260ax+2520\right)}{2520}-\ln(-ax+1)\right)}{a}+\frac{4c^4\left(-\frac{ax\left(120a^6\right.}{}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/252*c^4*x*(28*a^8*x^8+63*a^7*x^7-72*a^6*x^6-252*a^5*x^5+378*a^3*x^3+168*
a^2*x^2-252*a*x-252)
```

3.565.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

3.565.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}-a^5c^4x^6+\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}-ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**4,x)
```

output
```
a**8*c**4*x**9/9 + a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 - a**5*c**4*x**6
+ 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 - a*c**4*x**2 - c**4*x
```

3.565.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}a^8c^4x^9+\frac{1}{4}a^7c^4x^8-\frac{2}{7}a^6c^4x^7-a^5c^4x^6+\frac{3}{2}a^3c^4x^4+\frac{2}{3}a^2c^4x^3-ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

3.565.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}a^8c^4x^9+\frac{1}{4}a^7c^4x^8-\frac{2}{7}a^6c^4x^7-a^5c^4x^6+\frac{3}{2}a^3c^4x^4+\frac{2}{3}a^2c^4x^3-ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

3.565.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{4}-\frac{2\,a^6\,c^4\,x^7}{7}-a^5\,c^4\,x^6+\frac{3\,a^3\,c^4\,x^4}{2}+\frac{2\,a^2\,c^4\,x^3}{3}-a\,c^4\,x^2-c^4\,x$$

input
```
int(((c - a^2*c*x^2)^4*(a*x + 1))/(a*x - 1),x)
```

output
```
(2*a^2*c^4*x^3)/3 - a*c^4*x^2 - c^4*x + (3*a^3*c^4*x^4)/2 - a^5*c^4*x^6 -
(2*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

3.565. $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.566 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.566.1 Optimal result

Integrand size = 22, antiderivative size = 52

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{4c^3(1+ax)^5}{5a}+\frac{2c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

output
```
-4/5*c^3*(a*x+1)^5/a+2/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

3.566.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(1+ax)^5\left(29-40ax+15a^2x^2\right)}{105a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
-1/105*(c^3*(1 + a*x)^5*(29 - 40*a*x + 15*a^2*x^2))/a
```

3.566.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6717$$

$$-\int c^3 e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \; 27$$

$$-c^3\int e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \; 6690$$

$$-c^3\int (1-ax)^2(ax+1)^4dx$$

$$\Big\downarrow \; 49$$

$$-c^3\int \left((ax+1)^6-4(ax+1)^5+4(ax+1)^4\right)dx$$

$$\Big\downarrow \; 2009$$

$$-c^3\left(\frac{(ax+1)^7}{7a}-\frac{2(ax+1)^6}{3a}+\frac{4(ax+1)^5}{5a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
-(c^3*((4*(1 + a*x)^5)/(5*a) - (2*(1 + a*x)^6)/(3*a) + (1 + a*x)^7/(7*a)))
```

3.566.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.566.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result
gosper	$-\frac{c^3x(15a^6x^6+35a^5x^5-21a^4x^4-105a^3x^3-35a^2x^2+105ax+105)}{105}$
default	$c^3\left(-\frac{1}{7}a^6x^7-\frac{1}{3}a^5x^6+\frac{1}{5}a^4x^5+a^3x^4+\frac{1}{3}a^2x^3-a\,x^2-x\right)$
norman	$a^3c^3x^4-c^3x-a\,c^3x^2+\frac{1}{3}a^2c^3x^3+\frac{1}{5}a^4c^3x^5-\frac{1}{3}a^5c^3x^6-\frac{1}{7}a^6c^3x^7$
risch	$a^3c^3x^4-c^3x-a\,c^3x^2+\frac{1}{3}a^2c^3x^3+\frac{1}{5}a^4c^3x^5-\frac{1}{3}a^5c^3x^6-\frac{1}{7}a^6c^3x^7$
parallelrisch	$a^3c^3x^4-c^3x-a\,c^3x^2+\frac{1}{3}a^2c^3x^3+\frac{1}{5}a^4c^3x^5-\frac{1}{3}a^5c^3x^6-\frac{1}{7}a^6c^3x^7$
meijerg	$\frac{c^3\left(-\frac{ax\left(120a^6x^6+140a^5x^5+168a^4x^4+210a^3x^3+280a^2x^2+420ax+840\right)}{840}-\ln(-ax+1)\right)}{a}-\frac{3c^3(-\frac{ax(12a^4x^4+15a^3x^3+20a^2x^2+30a}{60}}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/105*c^3*x*(15*a^6*x^6+35*a^5*x^5-21*a^4*x^4-105*a^3*x^3-35*a^2*x^2+105*
a*x+105)
```

3.566.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

3.566.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6c^3x^7}{7}-\frac{a^5c^3x^6}{3}+\frac{a^4c^3x^5}{5}+a^3c^3x^4+\frac{a^2c^3x^3}{3}-ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**3,x)
```

output
```
-a**6*c**3*x**7/7 - a**5*c**3*x**6/3 + a**4*c**3*x**5/5 + a**3*c**3*x**4 +
 a**2*c**3*x**3/3 - a*c**3*x**2 - c**3*x
```

3.566.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

3.566.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

3.566.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6\,c^3\,x^7}{7}-\frac{a^5\,c^3\,x^6}{3}+\frac{a^4\,c^3\,x^5}{5}+a^3\,c^3\,x^4+\frac{a^2\,c^3\,x^3}{3}-a\,c^3\,x^2-c^3\,x$$

input
```
int(((c - a^2*c*x^2)^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(a^2*c^3*x^3)/3 - a*c^3*x^2 - c^3*x + a^3*c^3*x^4 + (a^4*c^3*x^5)/5 - (a^5
*c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

3.566. $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.567 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

3.567.1 Optimal result

Integrand size = 22, antiderivative size = 35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=-\frac{c^2(1+ax)^4}{2a}+\frac{c^2(1+ax)^5}{5a}$$

output
```
-1/2*c^2*(a*x+1)^4/a+1/5*c^2*(a*x+1)^5/a
```

3.567.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{c^2(1+ax)^4(-3+2ax)}{10a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
(c^2*(1 + a*x)^4*(-3 + 2*a*x))/(10*a)
```

3.567.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^2e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^2dx$$

$$\Big\downarrow \text{27}$$

$$-c^2\int e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^2dx$$

$$\Big\downarrow \text{6690}$$

$$-c^2\int (1-ax)(ax+1)^3dx$$

$$\Big\downarrow \text{49}$$

$$-c^2\int \left(2(ax+1)^3-(ax+1)^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^2\left(\frac{(ax+1)^4}{2a}-\frac{(ax+1)^5}{5a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
-(c^2*((1 + a*x)^4/(2*a) - (1 + a*x)^5/(5*a)))
```

3.567.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.567.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result
gosper	$\frac{c^{2} x \left(2 a^{4} x^{4}+5 a^{3} x^{3}-10 a x -10\right)}{10}$
default	$c^{2} \left(\frac{1}{5} a^{4} x^{5}+\frac{1}{2} a^{3} x^{4}-a \,x^{2}-x \right)$
norman	$-c^{2} x -a \,c^{2} x^{2}+\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
risch	$-c^{2} x -a \,c^{2} x^{2}+\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
parallelrisch	$-c^{2} x -a \,c^{2} x^{2}+\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
meijerg	$-\frac{c^{2} \left(-\frac{a x \left(12 a^{4} x^{4}+15 a^{3} x^{3}+20 a^{2} x^{2}+30 a x +60\right)}{60}-\ln \left(-a x +1\right)\right)}{a}+\frac{2 c^{2} \left(-\frac{a x \left(4 a^{2} x^{2}+6 a x +12\right)}{12}-\ln \left(-a x +1\right)\right)}{a}-\frac{c^{2} (-a x -\ln (-}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^2*x*(2*a^4*x^4+5*a^3*x^3-10*a*x-10)
```

3.567.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

3.567.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}+\frac{a^3c^2x^4}{2}-ac^2x^2-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**2,x)
```

output
```
a**4*c**2*x**5/5 + a**3*c**2*x**4/2 - a*c**2*x**2 - c**2*x
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

3.567.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4\,c^2\,x^5}{5}+\frac{a^3\,c^2\,x^4}{2}-a\,c^2\,x^2-c^2\,x$$

input
```
int(((c - a^2*c*x^2)^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(a^3*c^2*x^4)/2 - a*c^2*x^2 - c^2*x + (a^4*c^2*x^5)/5
```

3.568 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

3.568.1 Optimal result

Integrand size = 20, antiderivative size = 15

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c(1+ax)^3}{3a}$$

output
```
-1/3*c*(a*x+1)^3/a
```

3.568.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-c\left(x+ax^2+\frac{a^2x^3}{3}\right)$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
-(c*(x + a*x^2 + (a^2*x^3)/3))
```

3.568.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int ce^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)dx$$

$$\Big\downarrow \text{6690}$$

$$-c\int (ax+1)^2dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{c(ax+1)^3}{3a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
-1/3*(c*(1 + a*x)^3)/a
```

3.568.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.568.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{c(ax+1)^{3}}{3a}$	14
gosper	$-\frac{cx\left(a^{2}x^{2}+3ax+3\right)}{3}$	18
norman	$-cx-ac\,x^{2}-\frac{1}{3}a^{2}c\,x^{3}$	22
parallelrisch	$-cx-ac\,x^{2}-\frac{1}{3}a^{2}c\,x^{3}$	22
risch	$-\frac{a^{2}c\,x^{3}}{3}-ac\,x^{2}-cx-\frac{c}{3a}$	28
meijerg	$\frac{c\left(-\frac{ax\left(4a^{2}x^{2}+6ax+12\right)}{12}-\ln\left(-ax+1\right)\right)}{a}-\frac{c\left(-ax-\ln\left(-ax+1\right)\right)}{a}-\frac{c\left(\frac{ax\left(3ax+6\right)}{6}+\ln\left(-ax+1\right)\right)}{a}+\frac{c\ln\left(-ax+1\right)}{a}$	91

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(a*x+1)^3/a
```

3.568.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)\,dx=-\frac{1}{3}\,a^{2}cx^{3}-acx^{2}-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

3.568.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}-acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c),x)
```

output
```
-a**2*c*x**3/3 - a*c*x**2 - c*x
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

3.568.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

input

```
int(((c - a^2*c*x^2)*(a*x + 1))/(a*x - 1),x)
```

output

```
-(c*x*(3*a*x + a^2*x^2 + 3))/3
```

3.569 $\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

3.569.1 Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{1}{ac(1-ax)}$$

output
```
-1/a/c/(-a*x+1)
```

3.569.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{2\coth^{-1}(ax)}}{2ac}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
E^(2*ArcCoth[a*x])/(2*a*c)
```

3.569.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}dx$$

$$\Big\downarrow \; 6717$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c\left(1-a^2x^2\right)}dx$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \; 6690$$

$$-\frac{\int \frac{1}{(1-ax)^2}dx}{c}$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{ac(1-ax)}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
-(1/(a*c*(1 - a*x)))
```

3.569.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.569.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{x}{c(ax-1)}$	13
parallelrisch	$\frac{x}{c(ax-1)}$	13
gosper	$\frac{1}{ac(ax-1)}$	15
default	$\frac{1}{ac(ax-1)}$	15
risch	$\frac{1}{ac(ax-1)}$	15

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
x/c/(a*x-1)
```

3.569.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/(a^2*c*x - a*c)
```

3.569.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c),x)
```

output
```
1/(a**2*c*x - a*c)
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/(a^2*c*x - a*c)
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{(ax-1)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/((a*x - 1)*a*c)
```

3.569.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{1}{a\,\left(c-a\,c\,x\right)}$$

input

```
int((a*x + 1)/((c - a^2*c*x^2)*(a*x - 1)),x)
```

output

```
-1/(a*(c - a*c*x))
```

3.570 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.570.1 Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{4ac^2(1-ax)^2} - \frac{1}{4ac^2(1-ax)} - \frac{\operatorname{arctanh}(ax)}{4ac^2}$$

output
```
-1/4/a/c^2/(-a*x+1)^2-1/4/a/c^2/(-a*x+1)-1/4*arctanh(a*x)/a/c^2
```

3.570.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{-2+ax-(-1+ax)^2\operatorname{arctanh}(ax)}{4ac^2(-1+ax)^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
(-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^2*(-1 + a*x)^2)
```

3.570.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = $\{6717, 27, 6690, 54, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \textcolor{blue}{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \textcolor{blue}{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^3(ax+1)}dx}{c^2}$$

$$\Big\downarrow \textcolor{blue}{54}$$

$$-\frac{\int \left(\frac{1}{4(ax-1)^2}-\frac{1}{2(ax-1)^3}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^2}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}+\frac{1}{4a(1-ax)}+\frac{1}{4a(1-ax)^2}}{c^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
-((1/(4*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)) + ArcTanh[a*x]/(4*a))/c^2)
```

3.570. $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$

3.570.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.570.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\frac{x}{4}-\frac{1}{2a}}{(ax-1)^2c^2}+\frac{\ln(-ax+1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	51
default	$\frac{-\frac{\ln(ax+1)}{8a}-\frac{1}{4(ax-1)^2a}+\frac{1}{4a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^2}$	52
norman	$\frac{-\frac{3}{4ac}+\frac{a\,x^2}{2c}-\frac{a^2x^3}{4c}}{c(ax+1)(ax-1)^2}+\frac{\ln(ax-1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	77
parallelrisch	$\frac{a^2\ln(ax-1)x^2-a^2\ln(ax+1)x^2+4a^2x^2-2a\ln(ax-1)x+2a\ln(ax+1)x-6ax+\ln(ax-1)-\ln(ax+1)}{8c^2(ax-1)^2a}$	90

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x-1/2/a)/(a*x-1)^2/c^2+1/8*ln(-a*x+1)/a/c^2-1/8*ln(a*x+1)/a/c^2
```

3.570. $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.570.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+(a^2x^2-2\,ax+1)\log(ax-1)-4}{8\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

3.570.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax-2}{4a^3c^2x^2-8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

output
```
(a*x - 2)/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8
 - log(x + 1/a)/8)/(a*c**2)
```

3.570.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax-2}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}-\frac{\log(ax+1)}{8\,ac^2}+\frac{\log(ax-1)}{8\,ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/4*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^
2) + 1/8*log(a*x - 1)/(a*c^2)
```

3.570.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}+\frac{ax-2}{4\,(ax-1)^2ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x
- 2)/((a*x - 1)^2*a*c^2)
```

3.570.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}-\frac{\operatorname{atanh}(a\,x)}{4\,a\,c^2}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^2*(a*x - 1)),x)
```

output
```
(x/4 - 1/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

3.571 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.571.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2} -\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^3}$$

output
```
-1/12/a/c^3/(-a*x+1)^3-1/8/a/c^3/(-a*x+1)^2-3/16/a/c^3/(-a*x+1)+1/16/a/c^3
/(a*x+1)-1/4*arctanh(a*x)/a/c^3
```

3.571.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{4+ax-6a^2x^2+3a^3x^3-3(-1+ax)^3(1+ax)\operatorname{arctanh}(ax)}{12ac^3(-1+ax)^3(1+ax)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
(4 + a*x - 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x])/
(12*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

3.571.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^3(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^4(ax+1)^2}dx}{c^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{3}{16(ax-1)^2}+\frac{1}{16(ax+1)^2}-\frac{1}{4(ax-1)^3}+\frac{1}{4(ax-1)^4}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}+\frac{3}{16a(1-ax)}-\frac{1}{16a(ax+1)}+\frac{1}{8a(1-ax)^2}+\frac{1}{12a(1-ax)^3}}{c^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
-((1/(12*a*(1 - a*x)^3) + 1/(8*a*(1 - a*x)^2) + 3/(16*a*(1 - a*x)) - 1/(16
*a*(1 + a*x)) + ArcTanh[a*x]/(4*a))/c^3)
```

3.571. $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$

3.571.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.571.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{1}{16a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{1}{12a(ax-1)^{3}}-\frac{1}{8(ax-1)^{2}a}+\frac{3}{16a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^{3}}$
risch	$\frac{\frac{a^{2}x^{3}}{4}-\frac{a\,x^{2}}{2}+\frac{x}{12}+\frac{1}{3a}}{(ax-1)^{2}(a^{2}x^{2}-1)c^{3}}+\frac{\ln(-ax+1)}{8a\,c^{3}}-\frac{\ln(ax+1)}{8a\,c^{3}}$
norman	$\frac{\frac{3x}{4c}+\frac{a\,x^{2}}{4c}-\frac{11a^{2}x^{3}}{12c}-\frac{a^{3}x^{4}}{12c}+\frac{a^{4}x^{5}}{3c}}{(ax+1)^{2}(ax-1)^{3}c^{2}}+\frac{\ln(ax-1)}{8a\,c^{3}}-\frac{\ln(ax+1)}{8a\,c^{3}}$
parallelrisch	$\frac{3\ln(ax-1)x^{4}a^{4}-3\ln(ax+1)x^{4}a^{4}+8a^{4}x^{4}-6a^{3}\ln(ax-1)x^{3}+6a^{3}\ln(ax+1)x^{3}-10a^{3}x^{3}-12a^{2}x^{2}+6a\ln(ax-1)x-6a\ln(ax+1)x}{24c^{3}(ax-1)^{2}(a^{2}x^{2}-1)a}$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/16/a/(a*x+1)-1/8*ln(a*x+1)/a+1/12/a/(a*x-1)^3-1/8/(a*x-1)^2/a+3/1
6/a/(a*x-1)+1/8/a*ln(a*x-1))
```

3.571. $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^{2}cx^{2})^{3}}\,dx$

3.571.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$$
$$= \frac{6\,a^3x^3 - 12\,a^2x^2 + 2\,ax - 3\,(a^4x^4 - 2\,a^3x^3 + 2\,ax - 1)\log{(ax+1)} + 3\,(a^4x^4 - 2\,a^3x^3 + 2\,ax - 1)\log{(ax}}{24\,(a^5c^3x^4 - 2\,a^4c^3x^3 + 2\,a^2c^3x - ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/24*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)
*log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^5
*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

3.571.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{-3a^3x^3 + 6a^2x^2 - ax - 4}{12a^5c^3x^4 - 24a^4c^3x^3 + 24a^2c^3x - 12ac^3} - \frac{-\frac{\log{\left(x-\frac{1}{a}\right)}}{8} + \frac{\log{\left(x+\frac{1}{a}\right)}}{8}}{ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

output
```
-(-3*a**3*x**3 + 6*a**2*x**2 - a*x - 4)/(12*a**5*c**3*x**4 - 24*a**4*c**3*
x**3 + 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a
*c**3)
```

3.571.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{3\,a^3x^3 - 6\,a^2x^2 + ax + 4}{12\,(a^5c^3x^4 - 2\,a^4c^3x^3 + 2\,a^2c^3x - ac^3)} - \frac{\log{(ax+1)}}{8\,ac^3} + \frac{\log{(ax-1)}}{8\,ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/12*(3*a^3*x^3 - 6*a^2*x^2 + a*x + 4)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^
2*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

3.571.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\log\left(|ax+1|\right)}{8\,ac^3} + \frac{\log\left(|ax-1|\right)}{8\,ac^3} + \frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(ax+1)(ax-1)^3ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a
^3*x^3 - 6*a^2*x^2 + a*x + 4)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

3.571.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\frac{x}{12}-\frac{a\,x^2}{2}+\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3} - \frac{\mathrm{atanh}(a\,x)}{4\,a\,c^3}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^3*(a*x - 1)),x)
```

output
```
- (x/12 - (a*x^2)/2 + 1/(3*a) + (a^2*x^3)/4)/(c^3 + 2*a^3*c^3*x^3 - a^4*c^
3*x^4 - 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

3.572 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

3.572.1 Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2} -\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+\frac{5}{64ac^4(1+ax)}-\frac{15\operatorname{arctanh}(ax)}{64ac^4}$$

output
```
-1/32/a/c^4/(-a*x+1)^4-1/16/a/c^4/(-a*x+1)^3-3/32/a/c^4/(-a*x+1)^2-5/32/a/
c^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2+5/64/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/
c^4
```

3.572.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{16+17ax-50a^2x^2+10a^3x^3+30a^4x^4-15a^5x^5+15(-1+ax)^4(1+ax)^2\operatorname{arctanh}(ax)}{64ac^4(-1+ax)^4(1+ax)^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
-1/64*(16 + 17*a*x - 50*a^2*x^2 + 10*a^3*x^3 + 30*a^4*x^4 - 15*a^5*x^5 + 1
5*(-1 + a*x)^4*(1 + a*x)^2*ArcTanh[a*x])/(a*c^4*(-1 + a*x)^4*(1 + a*x)^2)
```

3.572.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^4\left(1-a^2x^2\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{\left(1-a^2x^2\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^5(ax+1)^3}dx}{c^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{5}{32(ax-1)^2}+\frac{5}{64(ax+1)^2}-\frac{3}{16(ax-1)^3}+\frac{1}{32(ax+1)^3}+\frac{3}{16(ax-1)^4}-\frac{1}{8(ax-1)^5}-\frac{15}{64\left(a^2x^2-1\right)}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{15\text{arctanh}(ax)}{64a}+\frac{5}{32a(1-ax)}-\frac{5}{64a(ax+1)}+\frac{3}{32a(1-ax)^2}-\frac{1}{64a(ax+1)^2}+\frac{1}{16a(1-ax)^3}+\frac{1}{32a(1-ax)^4}}{c^4}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
-((1/(32*a*(1 - a*x)^4) + 1/(16*a*(1 - a*x)^3) + 3/(32*a*(1 - a*x)^2) + 5/
(32*a*(1 - a*x)) - 1/(64*a*(1 + a*x)^2) - 5/(64*a*(1 + a*x)) + (15*ArcTanh
[a*x])/(64*a))/c^4)
```

3.572.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.572.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result
risch	$\frac{\frac{15a^4x^5}{64}-\frac{15a^3x^4}{32}-\frac{5a^2x^3}{32}+\frac{25a\,x^2}{32}-\frac{17x}{64}-\frac{1}{4a}}{(ax-1)^2(a^2x^2-1)^2c^4}+\frac{15\ln(-ax+1)}{128a\,c^4}-\frac{15\ln(ax+1)}{128a\,c^4}$
default	$\frac{\frac{1}{64a(ax+1)^2}+\frac{5}{64a(ax+1)}-\frac{15\ln(ax+1)}{128a}-\frac{1}{32a(ax-1)^4}+\frac{1}{16a(ax-1)^3}-\frac{3}{32(ax-1)^2a}+\frac{5}{32a(ax-1)}+\frac{15\ln(ax-1)}{128a}}{c^4}$
norman	$\frac{-\frac{49x}{64c}-\frac{15a\,x^2}{64c}+\frac{11a^2x^3}{8c}+\frac{a^3x^4}{8c}-\frac{63a^4x^5}{64c}-\frac{a^5x^6}{64c}+\frac{a^6x^7}{4c}}{(ax+1)^3c^3(ax-1)^4}+\frac{15\ln(ax-1)}{128a\,c^4}-\frac{15\ln(ax+1)}{128a\,c^4}$
parallelrisch	$30a\ln(ax+1)x+15a^2\ln(ax+1)x^2-34a^5x^5+108a^3x^3+30\ln(ax+1)x^5a^5-15\ln(ax+1)x^6a^6+15\ln(ax+1)x^4a^4+15\ln(ax-1)x^6$

---

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/64*a^4*x^5-15/32*a^3*x^4-5/32*a^2*x^3+25/32*a*x^2-17/64*x-1/4/a)/(a*x-
1)^2/(a^2*x^2-1)^2/c^4+15/128*ln(-a*x+1)/a/c^4-15/128*ln(a*x+1)/a/c^4
```

3.572.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(103) = 206$.

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{30\,a^5x^5 - 60\,a^4x^4 - 20\,a^3x^3 + 100\,a^2x^2 - 34\,ax - 15\,(a^6x^6 - 2\,a^5x^5 - a^4x^4 + 4\,a^3x^3 - a^2x^2 - 2\,ax + 1)\,\mathrm{lo}}{128\,(a^7c^4x^6 - 2\,a^6c^4x^5 - a^5c^4x^4 + 4\,a^4c^4x^3 -}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/128*(30*a^5*x^5 - 60*a^4*x^4 - 20*a^3*x^3 + 100*a^2*x^2 - 34*a*x - 15*(a
^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x +
1) + 15*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*
log(a*x - 1) - 32)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*
x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

3.572.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{15a^5x^5 - 30a^4x^4 - 10a^3x^3 + 50a^2x^2 - 17ax - 16}{64a^7c^4x^6 - 128a^6c^4x^5 - 64a^5c^4x^4 + 256a^4c^4x^3 - 64a^3c^4x^2 - 128a^2c^4x + 64ac^4} + \frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128} - \frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

output
```
(15*a**5*x**5 - 30*a**4*x**4 - 10*a**3*x**3 + 50*a**2*x**2 - 17*a*x - 16)/
(64*a**7*c**4*x**6 - 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 + 256*a**4*c**
4*x**3 - 64*a**3*c**4*x**2 - 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/
a)/128 - 15*log(x + 1/a)/128)/(a*c**4)
```

3.572.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)} - \frac{15\,\log(ax+1)}{128\,ac^4} + \frac{15\,\log(ax-1)}{128\,ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/64*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/(a^
7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*
a^2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*
c^4)
```

3.572.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,\log(|ax+1|)}{128\,ac^4} + \frac{15\,\log(|ax-1|)}{128\,ac^4} + \frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\,(ax+1)^2(ax-1)^4ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
-15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/6
4*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/((a*x
+ 1)^2*(a*x - 1)^4*a*c^4)
```

3.572.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\frac{17\,x}{64}-\frac{25\,a\,x^2}{32}+\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}+\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5+a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+a^2\,c^4\,x^2+2\,a\,c^4\,x-c^4} - \frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

input

```
int((a*x + 1)/((c - a^2*c*x^2)^4*(a*x - 1)),x)
```

output

```
((17*x)/64 - (25*a*x^2)/32 + 1/(4*a) + (5*a^2*x^3)/32 + (15*a^3*x^4)/32 -
(15*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 + 2*a^5*
c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

3.573 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

3.573.1 Optimal result

Integrand size = 22, antiderivative size = 393

$$\begin{aligned}
&\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx\\
&=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2\\
&\quad-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4\\
&\quad-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008}-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}\\
&\quad+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)
\end{aligned}$$

output

```
-5/72*a^7*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(13/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(
5/2)*(1+1/a/x)^(13/2)*x^9-55/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/
2))/a-55/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-11/192*a^2*c^4*(1+1
/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-11/448*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/
x)^(1/2)-11/1008*a^4*c^4*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)-5/1008*a^5*c^
4*(1+1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2)+5/168*a^6*c^4*(1+1/a/x)^(13/2)*x^7*
(1-1/a/x)^(1/2)-55/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.573.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = \frac{c^4\bigl(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-3712+4599ax+10240a^2x^2+3066a^3x^3-8448a^4x^4-7224a^5x^5+1024a^6x^6+3024a}{8064a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
(c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 + 4599*a*x + 10240*a^2*x^2 + 3066*a
^3*x^3 - 8448*a^4*x^4 - 7224*a^5*x^5 + 1024*a^6*x^6 + 3024*a^7*x^7 + 896*a
^8*x^8) - 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

3.573.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^4 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{ 6745}$$

3.573. $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \; 27$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \; 2005$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \; 6745$$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \; 27$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \; 2005$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \; 6745$$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \; 27$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \; 2005$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \; 6745$$

$$a^8c^4\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \; 27$$

$$c^4\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

3.573. $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{3\coth^{-1}(ax)} \left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{3\coth^{-1}(ax)} \left(a^2x^2-1\right)^4dx$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
$Aborted
```

3.573.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.573. $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.573.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

method	result
risch	$\frac{(896a^8x^8+3024a^7x^7+1024a^6x^6-7224a^5x^5-8448a^4x^4+3066a^3x^3+10240a^2x^2+4599ax-3712)(ax-1)c^4}{8064a\sqrt{\frac{ax-1}{ax+1}}} - \frac{55\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-\right.}{128\sqrt{a^2}\sqrt{\frac{c}{a}}}$
default	$\frac{(ax-1)^2c^4\left(896(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+3024(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+1920(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-4200(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-6528(a^2\right.}{8064a\left(\frac{ax-}{ax+}\right.}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8064*(896*a^8*x^8+3024*a^7*x^7+1024*a^6*x^6-7224*a^5*x^5-8448*a^4*x^4+30
66*a^3*x^3+10240*a^2*x^2+4599*a*x-3712)*(a*x-1)/a*c^4/((a*x-1)/(a*x+1))^(1
/2)-55/128*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4/((a*x-1
)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.573.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.43

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx =$$

$$-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(896\,a^9c^4x^9+3920\,a^8c^4x^8+4048\,a^7c^4x^7-620}{8064}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas"
)
```

output
```
-1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) - 1) - (896*a^9*c^4*x^9 + 3920*a^8*c^4*x^8 + 4048*a^7*
c^4*x^7 - 6200*a^6*c^4*x^6 - 15672*a^5*c^4*x^5 - 5382*a^4*c^4*x^4 + 13306*
a^3*c^4*x^3 + 14839*a^2*c^4*x^2 + 887*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(
a*x + 1)))/a
```

3.573.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=c^4\left(\int\left(-\frac{4a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^6x^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^8x^8}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**4,x)
```

output
```
c**4*(Integral(-4*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4*x**
*4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6*x**6/(a*x*sqrt(a*x/(a*x +
1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1))
, x) + Integral(a**8*x**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
 - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/(a*x + 1)), x))
```

3.573.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx =$$

$$-\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}\right.}{\frac{9\,(ax-1)a^2}{ax+1}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima"
)
```

output
```
-1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sq
rt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2
) - 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x +
1))^(13/2) - 255222*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 360448*c^4*((a*x -
1)/(a*x + 1))^(9/2) - 334602*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*
((a*x - 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465
*c^4*sqrt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^
2*a^2/(a*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(
a*x + 1)^4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1
)^6 + 36*(a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*
x - 1)^9*a^2/(a*x + 1)^9 - a^2))*a
```

3.573.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.55

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = \frac{55\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{128\,|a|\mathrm{sgn}\left(ax+1\right)}$$

$$+\frac{1}{8064}\sqrt{a^2x^2-1}\left(\left(\left(\frac{4599\,c^4}{\mathrm{sgn}\left(ax+1\right)}+2\left(\frac{5120\,ac^4}{\mathrm{sgn}\left(ax+1\right)}+\left(\frac{1533\,a^2c^4}{\mathrm{sgn}\left(ax+1\right)}-4\left(\frac{1056\,a^3c^4}{\mathrm{sgn}\left(ax+1\right)}+\left(\frac{903\,a^4c^4}{\mathrm{sgn}\left(ax+\right.}\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
55/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) +
 1/8064*sqrt(a^2*x^2 - 1)*((4599*c^4/sgn(a*x + 1) + 2*(5120*a*c^4/sgn(a*x
+ 1) + (1533*a^2*c^4/sgn(a*x + 1) - 4*(1056*a^3*c^4/sgn(a*x + 1) + (903*a^
4*c^4/sgn(a*x + 1) - 2*(64*a^5*c^4/sgn(a*x + 1) + 7*(8*a^7*c^4*x/sgn(a*x +
 1) + 27*a^6*c^4/sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x - 3712*c^4/(a*sgn(a*x +
 1)))
```

3.573.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.92

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$

$$=\frac{\frac{55\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{715\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96}+\frac{913\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32}+\frac{18589\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224}-\frac{5632\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{63}+\frac{14179\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{224}-913}{a-\frac{9\,a\,(a\,x-1)}{a\,x+1}+\frac{36\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{84\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{126\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{126\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{84\,a\,(a\,x-1)^6}{(a\,x+1)^6}-36\,a\,(a}$$

$$-\frac{55\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

input
```
int((c - a^2*c*x^2)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((55*c^4*((a*x - 1)/(a*x + 1))^(1/2))/64 - (715*c^4*((a*x - 1)/(a*x + 1))^
(3/2))/96 + (913*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (18589*c^4*((a*x -
1)/(a*x + 1))^(7/2))/224 - (5632*c^4*((a*x - 1)/(a*x + 1))^(9/2))/63 + (14
179*c^4*((a*x - 1)/(a*x + 1))^(11/2))/224 - (913*c^4*((a*x - 1)/(a*x + 1))
^(13/2))/32 + (715*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 - (55*c^4*((a*x -
1)/(a*x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)
^2)/(a*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*
x + 1)^4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^
6 - (36*a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a
*x - 1)^9)/(a*x + 1)^9) - (55*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*
a)
```

3.574 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.574.1 Optimal result

Integrand size = 22, antiderivative size = 313

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x$$

output
```
-1/7*a^6*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)*x^7-9/16*c^3*arctanh((1-1/a/
x)^(1/2)*(1+1/a/x)^(1/2))/a-3/16*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)
-3/40*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-9/280*a^3*c^3*(1+1/a/x)^
(7/2)*x^4*(1-1/a/x)^(1/2)-1/70*a^4*c^3*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)
+1/14*a^5*c^3*(1+1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2)-9/16*c^3*x*(1-1/a/x)^(1
/2)*(1+1/a/x)^(1/2)
```

3.574.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx =$$

$$-\frac{c^3\Big(a\sqrt{1-\frac{1}{a^2x^2}}x(368-245ax-656a^2x^2-350a^3x^3+208a^4x^4+280a^5x^5+80a^6x^6)+315\log\Big(\Big(1+}{560a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
-1/560*(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 - 245*a*x - 656*a^2*x^2 - 350*
a^3*x^3 + 208*a^4*x^4 + 280*a^5*x^5 + 80*a^6*x^6) + 315*Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x]))/a
```

3.574.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{ 6745}$$

3.574. $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

3.574. $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^3\ dx$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.574.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.574. $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.574.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{(80a^6x^6+280a^5x^5+208a^4x^4-350a^3x^3-656a^2x^2-245ax+368)(ax-1)c^3}{560a\sqrt{\frac{ax-1}{ax+1}}}-\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax-1)(ax+1)}}{16\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c^3\Bigg(80(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+280(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+288(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-70(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+192(a^2x^2-1)}{560a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/560*(80*a^6*x^6+280*a^5*x^5+208*a^4*x^4-350*a^3*x^3-656*a^2*x^2-245*a*x
+368)*(a*x-1)/a*c^3/((a*x-1)/(a*x+1))^(1/2)-9/16*ln(a^2*x/(a^2)^(1/2)+(a^2
*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a
*x+1))^(1/2)
```

3.574.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx =$$
$$-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(80\,a^7c^3x^7+360\,a^6c^3x^6+488\,a^5c^3x^5-142\,a^4c^3x}{560\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

output
```
-1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (80*a^7*c^3*x^7 + 360*a^6*c^3*x^6 + 488*a^5*c^3*x^
5 - 142*a^4*c^3*x^4 - 1006*a^3*c^3*x^3 - 901*a^2*c^3*x^2 + 123*a*c^3*x + 3
68*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.574.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-c^3\left(\int\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^6x^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**3,x)
```

output
```
-c**3*(Integral(3*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4*x
**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**6*x**6/(a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x) + Integral(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

3.574.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=$$

$$-\frac{1}{560}\Bigg(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\Big(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+59}{\frac{7(ax-1)a^2}{ax+1}-\frac{21(ax-1)^2a^2}{(ax+1)^2}+}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

output
```
-1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) -
2100*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 5943*c^3*((a*x - 1)/(a*x + 1))^(9/
2) - 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 8393*c^3*((a*x - 1)/(a*x + 1))
^(5/2) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*
x + 1)))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*
(a*x - 1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1
)^5*a^2/(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x
 + 1)^7 - a^2))*a
```

3.574.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.57

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{9\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{16\,|a|\mathrm{sgn}\,(ax+1)}$$
$$+\frac{1}{560}\,\sqrt{a^2x^2-1}\Bigg(\Bigg(2\Bigg(\frac{328\,ac^3}{\mathrm{sgn}\,(ax+1)}+\Bigg(\frac{175\,a^2c^3}{\mathrm{sgn}\,(ax+1)}-4\Bigg(\frac{26\,a^3c^3}{\mathrm{sgn}\,(ax+1)}+5\Bigg(\frac{2\,a^5c^3x}{\mathrm{sgn}\,(ax+1)}+\frac{7\,a^4c^3}{\mathrm{sgn}\,(ax+}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
9/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + 1
/560*sqrt(a^2*x^2 - 1)*((2*(328*a*c^3/sgn(a*x + 1) + (175*a^2*c^3/sgn(a*x
+ 1) - 4*(26*a^3*c^3/sgn(a*x + 1) + 5*(2*a^5*c^3*x/sgn(a*x + 1) + 7*a^4*c^
3/sgn(a*x + 1))*x)*x)*x)*x + 245*c^3/sgn(a*x + 1))*x - 368*c^3/(a*sgn(a*x
+ 1)))
```

3.574.9 Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=$$
$$-\frac{\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}-\frac{9\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{1199\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}-\frac{1152\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{35}+\frac{849\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{40}-\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{2}+\frac{9\,c^3\,(a}{a}}{a-\frac{7\,a\,(a\,x-1)}{a\,x+1}+\frac{21\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{35\,a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}+\frac{35\,a\,{(a\,x-1)}^4}{{(a\,x+1)}^4}-\frac{21\,a\,{(a\,x-1)}^5}{{(a\,x+1)}^5}+\frac{7\,a\,{(a\,x-1)}^6}{{(a\,x+1)}^6}-\frac{a\,{(a\,x-1)}^7}{{(a\,x+1)}^7}}$$
$$-\frac{9\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$

input
```
int((c - a^2*c*x^2)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- ((15*c^3*((a*x - 1)/(a*x + 1))^(3/2))/2 - (9*c^3*((a*x - 1)/(a*x + 1))^(
1/2))/8 + (1199*c^3*((a*x - 1)/(a*x + 1))^(5/2))/40 - (1152*c^3*((a*x - 1)
/(a*x + 1))^(7/2))/35 + (849*c^3*((a*x - 1)/(a*x + 1))^(9/2))/40 - (15*c^3
*((a*x - 1)/(a*x + 1))^(11/2))/2 + (9*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)
/(a - (7*a*(a*x - 1))/(a*x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(
a*x - 1)^3)/(a*x + 1)^3 + (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)
^5)/(a*x + 1)^5 + (7*a*(a*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1
)^7) - (9*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a)
```

3.574. $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.575 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

3.575.1 Optimal result

Integrand size = 22, antiderivative size = 233

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx &= -\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2\\ &\quad-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4\\ &\quad+\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{7c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{8a}\end{aligned}$$

output
```
-7/8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/24*a*c^2*(1+1/a/x)^(
3/2)*x^2*(1-1/a/x)^(1/2)-7/60*a^2*c^2*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-
1/20*a^3*c^2*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)+1/5*a^4*c^2*(1+1/a/x)^(9/
2)*x^5*(1-1/a/x)^(1/2)-7/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.575.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx =\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x\left(-136+15ax+112a^2x^2+90a^3x^3+24a^4x^4\right)-105\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 + 15*a*x + 112*a^2*x^2 + 90*a^3*x^3
+ 24*a^4*x^4) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

3.575.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

3.575. $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$\Big\downarrow \text{27}$$

$$c^2 \int e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^2 x^4 dx$$

$$\Big\downarrow \text{2005}$$

$$c^2 \int e^{3\coth^{-1}(ax)} \left(a^2x^2-1\right)^2 dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2 \int \frac{e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^2 x^4}{a^4} dx$$

$$\Big\downarrow \text{27}$$

$$c^2 \int e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^2 x^4 dx$$

$$\Big\downarrow \text{2005}$$

$$c^2 \int e^{3\coth^{-1}(ax)} \left(a^2x^2-1\right)^2 dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2 \int \frac{e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^2 x^4}{a^4} dx$$

$$\Big\downarrow \text{27}$$

$$c^2 \int e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^2 x^4 dx$$

$$\Big\downarrow \text{2005}$$

$$c^2 \int e^{3\coth^{-1}(ax)} \left(a^2x^2-1\right)^2 dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2 \int \frac{e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^2 x^4}{a^4} dx$$

$$\Big\downarrow \text{27}$$

$$c^2 \int e^{3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^2 x^4 dx$$

$$\Big\downarrow \text{2005}$$

$$c^2 \int e^{3\coth^{-1}(ax)} \left(a^2x^2-1\right)^2 dx$$

3.575. $\int e^{3\coth^{-1}(ax)} \left(c-a^2cx^2\right)^2 \, dx$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.575.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.575. $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

3.575.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(24a^4x^4+90a^3x^3+112a^2x^2+15ax-136)(ax-1)c^2}{120a\sqrt{\frac{ax-1}{ax+1}}}-\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+90(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+105\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+120((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\right.}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*a^4*x^4+90*a^3*x^3+112*a^2*x^2+15*a*x-136)*(a*x-1)/a*c^2/((a*x-1
)/(a*x+1))^(1/2)-7/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c
^2/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.575.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx =$$

$$-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(24\,a^5c^2x^5+114\,a^4c^2x^4+202\,a^3c^2x^3+127\,a^2c^2x}{120\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

output
```
-1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) - (24*a^5*c^2*x^5 + 114*a^4*c^2*x^4 + 202*a^3*c^2*x^
3 + 127*a^2*c^2*x^2 - 121*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.575.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=c^2\left(\int\left(-\frac{2a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**2,x)
```

output
```
c**2*(Integral(-2*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4*x**4
/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

3.575.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=$$

$$-\frac{1}{120}\,a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-490\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+896\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

output
```
-1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqr
t((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) -
 490*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 896*c^2*((a*x - 1)/(a*x + 1))^(5/2)
 - 790*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1))
)/(5*(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x -
1)^3*a^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*
x + 1)^5 - a^2))
```

3.575.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.59

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{1}{120}\sqrt{a^2x^2-1}\left(\left(2\left(3\left(\frac{4\,a^3c^2x}{\operatorname{sgn}\left(ax+1\right)}+\frac{15\,a^2c^2}{\operatorname{sgn}\left(ax+1\right)}\right)x+\frac{56\,ac^2}{\operatorname{sgn}\left(ax+1\right)}\right)x+\frac{15\,c^2}{\operatorname{sgn}\left(ax+1\right)}\right)x-\frac{136}{a\operatorname{sgn}\left(a\right.}\right.$$
$$+\frac{7\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\operatorname{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/120*sqrt(a^2*x^2 - 1)*((2*(3*(4*a^3*c^2*x/sgn(a*x + 1) + 15*a^2*c^2/sgn(
a*x + 1))*x + 56*a*c^2/sgn(a*x + 1))*x + 15*c^2/sgn(a*x + 1))*x - 136*c^2/
(a*sgn(a*x + 1))) + 7/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a
)*sgn(a*x + 1))
```

3.575.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{\frac{7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{79\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6}-\frac{224\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{15}+\frac{49\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{6}-\frac{7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}$$
$$-\frac{7\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a^2*c*x^2)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((7*c^2*((a*x - 1)/(a*x + 1))^(1/2))/4 + (79*c^2*((a*x - 1)/(a*x + 1))^(3/
2))/6 - (224*c^2*((a*x - 1)/(a*x + 1))^(5/2))/15 + (49*c^2*((a*x - 1)/(a*x
 + 1))^(7/2))/6 - (7*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x -
1))/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x +
 1)^3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (7*
c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.575. $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

3.576 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

3.576.1 Optimal result

Integrand size = 20, antiderivative size = 145

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\,dx = {}& -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2\\ &-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3\\ &-\frac{5c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}\end{aligned}$$

output
```
-5/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-5/6*a*c*(1+1/a/x)^(3/2)*
x^2*(1-1/a/x)^(1/2)-1/3*a^2*c*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-5/2*c*x*
(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.576.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\begin{aligned}&\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\,dx\\ &=-\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(22+9ax+2a^2x^2)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}\end{aligned}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
-1/6*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 + 9*a*x + 2*a^2*x^2) + 15*Log[(1 +
Sqrt[1 - 1/(a^2*x^2)])*x]))/a
```

3.576.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

3.576. $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\ dx$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

3.576. $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\ dx$

$$-c\int e^{3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
$Aborted
```

3.576.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.576.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\left(2a^{2}x^{2}+9ax+22\right)\left(ax-1\right)c}{6a\sqrt{\frac{ax-1}{ax+1}}}-\frac{5\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)c\sqrt{\left(ax-1\right)\left(ax+1\right)}}{2\sqrt{a^{2}}\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{\left(ax-1\right)^{2}c\left(9\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax+2\left(\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{a^{2}}-9\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a+24\sqrt{a^{2}}\,\sqrt{\left(ax-1\right)\left(ax+1\right)}+24a\ln\left(\frac{a^{2}x+\sqrt{a}}{}\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)\sqrt{\left(ax-1\right)\left(ax+1\right)}\,a\sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output

```
-1/6*(2*a^2*x^2+9*a*x+22)*(a*x-1)/a*c/((a*x-1)/(a*x+1))^(1/2)-5/2*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.576.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3cx^3+11\,a^2cx^2+31\,acx+22\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

output

```
-1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) + (2*a^3*c*x^3 + 11*a^2*c*x^2 + 31*a*c*x + 22*c)*sqrt((a*x -
 1)/(a*x + 1)))/a
```

3.576.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -c\left(\int \frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int \left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c),x)
```

output

```
-c*(Integral(a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1)), x))
```

3.576.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$$
$$=\frac{1}{6}\,a\left(\frac{2\left(15\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+33\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/6*a*(2*(15*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3
/2) + 33*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x
- 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 15*c*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2)
```

3.576.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$$
$$=-\frac{1}{6}\,\sqrt{a^2x^2-1}\left(\left(\frac{2\,acx}{\operatorname{sgn}\left(ax+1\right)}+\frac{9\,c}{\operatorname{sgn}\left(ax+1\right)}\right)x+\frac{22\,c}{a\operatorname{sgn}\left(ax+1\right)}\right)$$
$$+\frac{5\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-1/6*sqrt(a^2*x^2 - 1)*((2*a*c*x/sgn(a*x + 1) + 9*c/sgn(a*x + 1))*x + 22*c
/(a*sgn(a*x + 1))) + 5/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)
*sgn(a*x + 1))
```

3.576.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{11\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+5\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{5\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input

```
int((c - a^2*c*x^2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output

```
- (11*c*((a*x - 1)/(a*x + 1))^(1/2) - (40*c*((a*x - 1)/(a*x + 1))^(3/2))/3
 + 5*c*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*
(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c*atanh(((a*x
 - 1)/(a*x + 1))^(1/2)))/a
```

3.577 $\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

3.577.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

output
```
1/3/((a*x-1)/(a*x+1))^(3/2)/a/c
```

3.577.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
E^(3*ArcCoth[a*x])/(3*a*c)
```

3.577.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$$

$$\Big\downarrow \; 6737$$

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

input

```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output

```
E^(3*ArcCoth[a*x])/(3*a*c)
```

3.577.3.1 Defintions of rubi rules used

rule 6737

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

3.577.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

method	result	size
gosper	$\frac{1}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} ac}$	24
default	$\frac{1}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} ac}$	24
trager	$\frac{(ax+1)^2\sqrt{-\frac{-ax+1}{ax+1}}}{3ac(ax-1)^2}$	40

input

```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*x-1)/(a*x+1))^(3/2)/a/c
```

3.577.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(23) = 46$.

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c*x^2 - 2*a^2*c*x
 + a*c)
```

3.577.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

output
```
-Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**2*
x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x
)/c
```

3.577.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{3\,ac\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3/(a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

3.577.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)
^3*a*c)
```

3.577.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{3\,a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

input
```
int(1/((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
1/(3*a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

3.578 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.578.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{2e^{3\coth^{-1}(ax)}}{15ac^2} + \frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)}$$

output
```
-2/15/((a*x-1)/(a*x+1))^(3/2)/a/c^2+1/5/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)
/a/c^2/(-a^2*x^2+1)
```

3.578.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(7-6ax+2a^2x^2)}{15c^2(-1+ax)^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
-1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^2*(-1 + a*x)^3)
```

3.578.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6739, 27, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}-\frac{2\int \frac{e^{3\coth^{-1}(ax)}}{c(1-a^2x^2)}dx}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}-\frac{2\int \frac{e^{3\coth^{-1}(ax)}}{1-a^2x^2}dx}{5c^2}$$

$$\Big\downarrow \text{6737}$$

$$\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
(-2*E^(3*ArcCoth[a*x]))/(15*a*c^2) + (E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(5*a
*c^2*(1 - a^2*x^2))
```

3.578.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.578.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2a^{2}x^{2}-6ax+7}{15\left(a^{2}x^{2}-1\right)c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	49
default	$-\frac{2a^{2}x^{2}-6ax+7}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)c^{2}\left(ax-1\right)a}$	52
trager	$-\frac{\left(ax+1\right)\left(2a^{2}x^{2}-6ax+7\right)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^{2}\left(ax-1\right)^{3}}$	52

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(2*a^2*x^2-6*a*x+7)/(a^2*x^2-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

3.578.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^{2}cx^{2}\right)^{2}}\,dx=-\frac{\left(2\,a^{3}x^{3}-4\,a^{2}x^{2}+ax+7\right)\sqrt{\frac{ax-1}{ax+1}}}{15\,\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

output
```
-1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2
*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

3.578.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

output
```
Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**4*x
**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

3.578.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-3}{60\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

output
```
1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^2*((a*
x - 1)/(a*x + 1))^(5/2))
```

3.578.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)
/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^2)
```

3.578.9 Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{3\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^2*((
a*x - 1)/(a*x + 1))^(5/2))
```

3.579 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.579.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=-\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}-\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3(1-a^2x^2)^2}+\frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3(1-a^2x^2)}$$

output
```
-8/35/((a*x-1)/(a*x+1))^(3/2)/a/c^3-1/7/((a*x-1)/(a*x+1))^(3/2)*(-4*a*x+3)
/a/c^3/(-a^2*x^2+1)^2+12/35/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/a/c^3/(-a^2
*x^2+1)
```

3.579.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-13+4ax+20a^2x^2-24a^3x^3+8a^4x^4)}{35c^3(-1+ax)^4(1+ax)}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
-1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*
a^4*x^4))/(c^3*(-1 + a*x)^4*(1 + a*x))
```

3.579.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6739, 27, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$$

$$\Big\downarrow \text{ 6739}$$

$$\frac{12\int \frac{e^{3\coth^{-1}(ax)}}{c^2(1-a^2x^2)^2}dx}{7c}-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{12\int \frac{e^{3\coth^{-1}(ax)}}{(1-a^2x^2)^2}dx}{7c^3}-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{ 6739}$$

$$\frac{12\left(\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5a(1-a^2x^2)}-\frac{2}{5}\int \frac{e^{3\coth^{-1}(ax)}}{1-a^2x^2}dx\right)}{7c^3}-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{ 6737}$$

$$\frac{12\left(\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5a(1-a^2x^2)}-\frac{2e^{3\coth^{-1}(ax)}}{15a}\right)}{7c^3}-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3(1-a^2x^2)^2}$$

input

```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output

```
-1/7*(E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(a*c^3*(1 - a^2*x^2)^2) + (12*((-2*E
^(3*ArcCoth[a*x]))/(15*a) + (E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(5*a*(1 - a^2
*x^2))))/(7*c^3)
```

3.579.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.579.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

method	result	size
trager	$-\frac{\left(8 a^{4} x^{4}-24 a^{3} x^{3}+20 a^{2} x^{2}+4 a x -13\right) \sqrt{-\frac{-a x +1}{a x +1}}}{35 a \,c^{3} \left(a x -1\right)^{4}}$	63
gosper	$-\frac{8 a^{4} x^{4}-24 a^{3} x^{3}+20 a^{2} x^{2}+4 a x -13}{35 \left(a^{2} x^{2}-1\right)^{2} c^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} a}$	65
default	$-\frac{8 a^{4} x^{4}-24 a^{3} x^{3}+20 a^{2} x^{2}+4 a x -13}{35 \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(a x +1\right)^{2} c^{3} a \left(a x -1\right)^{2}}$	68

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/35/a/c^3*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a*x-1)^4*(-(-a*x+1
)/(a*x+1))^(1/2)
```

3.579.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

output
```
-1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a
*x + 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^
3)
```

3.579.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\int \frac{1}{\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-ax\sqrt{}}}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

output
```
-Integral(1/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**6*
x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
 - 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
x + 1)), x)/c**3
```

3.579.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{560}\,a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

output
```
-1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + (28*(a*x - 1)/(a*x + 1)
 - 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^3*
((a*x - 1)/(a*x + 1))^(7/2)))
```

3.579.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{1}{(a^2cx^2-c)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*c*x^2 - c)^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.579.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{8\,a^4\,x^4-24\,a^3\,x^3+20\,a^2\,x^2+4\,a\,x-13}{35\,a\,c^3\,(a\,x+1)^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(1/((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-(4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^3*(a*x + 1)^4*
((a*x - 1)/(a*x + 1))^(7/2))
```

3.580 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

3.580.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{16e^{3\coth^{-1}(ax)}}{63ac^4} - \frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} - \frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4(1-a^2x^2)^2} + \frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4(1-a^2x^2)}$$

output

```
-16/63/((a*x-1)/(a*x+1))^(3/2)/a/c^4-1/9/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+1
)/a/c^4/(-a^2*x^2+1)^3-10/63/((a*x-1)/(a*x+1))^(3/2)*(-4*a*x+3)/a/c^4/(-a^
2*x^2+1)^2+8/21/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/a/c^4/(-a^2*x^2+1)
```

3.580.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(19+6ax-66a^2x^2+56a^3x^3+24a^4x^4-48a^5x^5+16a^6x^6)}{63c^4(-1+ax)^5(1+ax)^2}$$

input

```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
-1/63*(Sqrt[1 - 1/(a^2*x^2)]*x*(19 + 6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*
a^4*x^4 - 48*a^5*x^5 + 16*a^6*x^6))/(c^4*(-1 + a*x)^5*(1 + a*x)^2)
```

3.580.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6739, 27, 6739, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{10\int \frac{e^{3\coth^{-1}(ax)}}{c^3(1-a^2x^2)^3}dx}{9c}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{10\int \frac{e^{3\coth^{-1}(ax)}}{(1-a^2x^2)^3}dx}{9c^4}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6739}$$

$$\frac{10\left(\frac{12}{7}\int \frac{e^{3\coth^{-1}(ax)}}{(1-a^2x^2)^2}dx-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7a(1-a^2x^2)^2}\right)}{9c^4}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6739}$$

$$\frac{10\left(\frac{12}{7}\left(\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5a(1-a^2x^2)}-\frac{2}{5}\int \frac{e^{3\coth^{-1}(ax)}}{1-a^2x^2}dx\right)-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7a(1-a^2x^2)^2}\right)}{9c^4}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6737}$$

$$\frac{10\left(\frac{12}{7}\left(\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5a(1-a^2x^2)}-\frac{2e^{3\coth^{-1}(ax)}}{15a}\right)-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7a(1-a^2x^2)^2}\right)}{9c^4}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

3.580. $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$

output
```
-1/9*(E^(3*ArcCoth[a*x])*(1 - 2*a*x))/(a*c^4*(1 - a^2*x^2)^3) + (10*(-1/7*
(E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(a*(1 - a^2*x^2)^2) + (12*((-2*E^(3*ArcCo
th[a*x]))/(15*a) + (E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(5*a*(1 - a^2*x^2))))/
7))/(9*c^4)
```

3.580.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.580.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{16a^{6}x^{6}-48a^{5}x^{5}+24a^{4}x^{4}+56a^{3}x^{3}-66a^{2}x^{2}+6ax+19}{63\left(a^{2}x^{2}-1\right)^{3}c^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	81
default	$-\frac{16a^{6}x^{6}-48a^{5}x^{5}+24a^{4}x^{4}+56a^{3}x^{3}-66a^{2}x^{2}+6ax+19}{63\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)^{3}c^{4}\left(ax-1\right)^{3}a}$	84
trager	$-\frac{\left(16a^{6}x^{6}-48a^{5}x^{5}+24a^{4}x^{4}+56a^{3}x^{3}-66a^{2}x^{2}+6ax+19\right)\sqrt{-\frac{-ax+1}{ax+1}}}{63a\,c^{4}\left(ax+1\right)\left(ax-1\right)^{5}}$	86

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/63*(16*a^6*x^6-48*a^5*x^5+24*a^4*x^4+56*a^3*x^3-66*a^2*x^2+6*a*x+19)/(a
^2*x^2-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)/a
```

3.580.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{\left(16\,a^6x^6-48\,a^5x^5+24\,a^4x^4+56\,a^3x^3-66\,a^2x^2+6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\,\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas"
)
```

output
```
-1/63*(16*a^6*x^6 - 48*a^5*x^5 + 24*a^4*x^4 + 56*a^3*x^3 - 66*a^2*x^2 + 6*
a*x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c
^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

3.580.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

output
```
Timed out
```

3.580.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx\\&=\frac{1}{4032}\,a\left(\frac{21\,\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-18\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{54\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{420\,(ax-1)^3}{(ax+1)^3}-\frac{945\,(ax-1)^4}{(ax+1)^4}-7}{a^2c^4\,\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}\right)\end{aligned}$$

---

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima"
)
```

output
```
1/4032*a*(21*(((a*x - 1)/(a*x + 1))^(3/2) - 18*sqrt((a*x - 1)/(a*x + 1)))/
(a^2*c^4) + (54*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 420*(a
*x - 1)^3/(a*x + 1)^3 - 945*(a*x - 1)^4/(a*x + 1)^4 - 7)/(a^2*c^4*((a*x -
1)/(a*x + 1))^(9/2)))
```

3.580.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{1}{(a^2cx^2-c)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 - c)^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.580.9 Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{16\,a^6\,x^6-48\,a^5\,x^5+24\,a^4\,x^4+56\,a^3\,x^3-66\,a^2\,x^2+6\,a\,x+19}{63\,a\,c^4\,(a\,x+1)^6\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

input
```
int(1/((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-(6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a^4*x^4 - 48*a^5*x^5 + 16*a^6*x^6 +
 19)/(63*a*c^4*(a*x + 1)^6*((a*x - 1)/(a*x + 1))^(9/2))
```

3.581 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^5\,dx$

3.581.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=\frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

output
```
c^5*(a*x+1)^8/a-4/3*c^5*(a*x+1)^9/a+3/5*c^5*(a*x+1)^10/a-1/11*c^5*(a*x+1)^
11/a
```

3.581.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{c^5(1+ax)^8\left(-29+67ax-54a^2x^2+15a^3x^3\right)}{165a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

output
```
-1/165*(c^5*(1 + a*x)^8*(-29 + 67*a*x - 54*a^2*x^2 + 15*a^3*x^3))/a
```

3.581.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^5 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^5\left(1-a^2x^2\right)^5 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^5\int e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^5dx$$

$$\Big\downarrow \text{6690}$$

$$c^5\int (1-ax)^3(ax+1)^7dx$$

$$\Big\downarrow \text{49}$$

$$c^5\int \left(-(ax+1)^{10}+6(ax+1)^9-12(ax+1)^8+8(ax+1)^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^5\left(-\frac{(ax+1)^{11}}{11a}+\frac{3(ax+1)^{10}}{5a}-\frac{4(ax+1)^9}{3a}+\frac{(ax+1)^8}{a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

output
```
c^5*((1 + a*x)^8/a - (4*(1 + a*x)^9)/(3*a) + (3*(1 + a*x)^10)/(5*a) - (1 +
 a*x)^11/(11*a))
```

3.581.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.581.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

method	result
default	$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{2}{5}a^9x^{10}-\frac{1}{3}a^8x^9+a^7x^8+2a^6x^7-\frac{14}{5}a^4x^5-2a^3x^4+a^2x^3+2a\,x^2+x\right)$
gosper	$-\frac{c^5x\left(15a^{10}x^{10}+66a^9x^9+55a^8x^8-165a^7x^7-330a^6x^6+462a^4x^4+330a^3x^3-165a^2x^2-330ax-165\right)}{165}$
risch	$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+c^5a^2x^3+2a$
parallelrisch	$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+c^5a^2x^3+2a$
norman	$\frac{-c^5x+a^7c^5x^8+c^5a^2x^3-a\,c^5x^2+3a^3c^5x^4+\frac{4}{5}a^4c^5x^5-\frac{14}{5}a^5c^5x^6-2a^6c^5x^7+\frac{4}{3}a^8c^5x^9+\frac{1}{15}a^9c^5x^{10}-\frac{17}{55}a^{10}c^5x^{11}-\frac{1}{11}a^{11}c^5x^{12}}{ax-1}$
meijerg	$\frac{c^5\Bigg(-\frac{xa\big(-2730x^{11}a^{11}-3276a^{10}x^{10}-4004a^9x^9-5005a^8x^8-6435a^7x^7-8580a^6x^6-12012a^5x^5-18018a^4x^4-30030a^3x^3-60060a^2x^2-1801}{30030(-ax+1)}}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5*(-1/11*a^10*x^11-2/5*a^9*x^10-1/3*a^8*x^9+a^7*x^8+2*a^6*x^7-14/5*a^4*x
^5-2*a^3*x^4+a^2*x^3+2*a*x^2+x)
```

3.581.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

output
```
-1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2
*a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^
2 + c^5*x
```

3.581.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(54) = 108$.

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.65

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}c^5x^{11}}{11}-\frac{2a^9c^5x^{10}}{5}-\frac{a^8c^5x^9}{3}+a^7c^5x^8+2a^6c^5x^7-\frac{14a^4c^5x^5}{5}-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**5,x)
```

output
```
-a**10*c**5*x**11/11 - 2*a**9*c**5*x**10/5 - a**8*c**5*x**9/3 + a**7*c**5*
x**8 + 2*a**6*c**5*x**7 - 14*a**4*c**5*x**5/5 - 2*a**3*c**5*x**4 + a**2*c*
*5*x**3 + 2*a*c**5*x**2 + c**5*x
```

3.581.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

output
```
-1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2
*a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^
2 + c^5*x
```

3.581.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{\left(15\,c^5+\frac{231\,c^5}{ax-1}+\frac{1540\,c^5}{(ax-1)^2}+\frac{5775\,c^5}{(ax-1)^3}+\frac{13200\,c^5}{(ax-1)^4}+\frac{18480\,c^5}{(ax-1)^5}+\frac{14784\,c^5}{(ax-1)^6}+\frac{5280\,c^5}{(ax-1)^7}\right)(ax-1)^{11}}{165\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

output
```
-1/165*(15*c^5 + 231*c^5/(a*x - 1) + 1540*c^5/(a*x - 1)^2 + 5775*c^5/(a*x
- 1)^3 + 13200*c^5/(a*x - 1)^4 + 18480*c^5/(a*x - 1)^5 + 14784*c^5/(a*x -
1)^6 + 5280*c^5/(a*x - 1)^7)*(a*x - 1)^11/a
```

3.581.9 Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{2\,a^9\,c^5\,x^{10}}{5}-\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8+2\,a^6\,c^5\,x^7-\frac{14\,a^4\,c^5\,x^5}{5}-2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3+2\,a\,c^5\,x^2+c^5\,x$$

input
```
int(((c - a^2*c*x^2)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^5*x + 2*a*c^5*x^2 + a^2*c^5*x^3 - 2*a^3*c^5*x^4 - (14*a^4*c^5*x^5)/5 + 2
*a^6*c^5*x^7 + a^7*c^5*x^8 - (a^8*c^5*x^9)/3 - (2*a^9*c^5*x^10)/5 - (a^10*
c^5*x^11)/11
```

3.581. $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

3.582 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

3.582.1 Optimal result

Integrand size = 22, antiderivative size = 52

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}$$

output
```
4/7*c^4*(a*x+1)^7/a-1/2*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+1)^9/a
```

3.582.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{c^4(1+ax)^7\left(23-35ax+14a^2x^2\right)}{126a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
(c^4*(1 + a*x)^7*(23 - 35*a*x + 14*a^2*x^2))/(126*a)
```

3.582.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^4 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^4\left(1-a^2x^2\right)^4 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^4\int e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^4dx$$

$$\Big\downarrow \text{6690}$$

$$c^4\int (1-ax)^2(ax+1)^6dx$$

$$\Big\downarrow \text{49}$$

$$c^4\int \left((ax+1)^8-4(ax+1)^7+4(ax+1)^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^4\left(\frac{(ax+1)^9}{9a}-\frac{(ax+1)^8}{2a}+\frac{4(ax+1)^7}{7a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
c^4*((4*(1 + a*x)^7)/(7*a) - (1 + a*x)^8/(2*a) + (1 + a*x)^9/(9*a))
```

3.582.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.582.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result
gosper	$\frac{c^{4} x \left(14 a^{8} x^{8}+63 a^{7} x^{7}+72 a^{6} x^{6}-84 a^{5} x^{5}-252 a^{4} x^{4}-126 a^{3} x^{3}+168 a^{2} x^{2}+252 a x +126\right)}{126}$
default	$c^{4} \left(\frac{1}{9} a^{8} x^{9}+\frac{1}{2} a^{7} x^{8}+\frac{4}{7} a^{6} x^{7}-\frac{2}{3} a^{5} x^{6}-2 a^{4} x^{5}-a^{3} x^{4}+\frac{4}{3} a^{2} x^{3}+2 a \,x^{2}+x \right)$
risch	$\frac{1}{9} a^{8} c^{4} x^{9}+\frac{1}{2} a^{7} c^{4} x^{8}+\frac{4}{7} a^{6} c^{4} x^{7}-\frac{2}{3} a^{5} c^{4} x^{6}-2 a^{4} c^{4} x^{5}-a^{3} c^{4} x^{4}+\frac{4}{3} a^{2} c^{4} x^{3}+2 a \,c^{4} x^{2}+c^{4} x$
parallelrisch	$\frac{1}{9} a^{8} c^{4} x^{9}+\frac{1}{2} a^{7} c^{4} x^{8}+\frac{4}{7} a^{6} c^{4} x^{7}-\frac{2}{3} a^{5} c^{4} x^{6}-2 a^{4} c^{4} x^{5}-a^{3} c^{4} x^{4}+\frac{4}{3} a^{2} c^{4} x^{3}+2 a \,c^{4} x^{2}+c^{4} x$
norman	$\frac{-c^{4} x +a^{4} c^{4} x^{5}-a \,c^{4} x^{2}+\frac{2}{3} a^{2} c^{4} x^{3}+\frac{7}{3} a^{3} c^{4} x^{4}-\frac{4}{3} a^{5} c^{4} x^{6}-\frac{26}{21} a^{6} c^{4} x^{7}+\frac{1}{14} a^{7} c^{4} x^{8}+\frac{7}{18} a^{8} c^{4} x^{9}+\frac{1}{9} a^{9} c^{4} x^{10}}{a x -1}$
meijerg	$-\frac{c^{4} \left(-\frac{x a \left(-308 a^{9} x^{9}-385 a^{8} x^{8}-495 a^{7} x^{7}-660 a^{6} x^{6}-924 a^{5} x^{5}-1386 a^{4} x^{4}-2310 a^{3} x^{3}-4620 a^{2} x^{2}-13860 a x +27720\right)}{2772 \left(-a x +1\right)}-10 \ln \left(-a x +1\right)\right)}{a} -$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/126*c^4*x*(14*a^8*x^8+63*a^7*x^7+72*a^6*x^6-84*a^5*x^5-252*a^4*x^4-126*a
^3*x^3+168*a^2*x^2+252*a*x+126)
```

3.582.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*
a^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

3.582.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.92

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{2}+\frac{4a^6c^4x^7}{7}-\frac{2a^5c^4x^6}{3}-2a^4c^4x^5-a^3c^4x^4+\frac{4a^2c^4x^3}{3}+2ac^4x^2+c^4x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**4,x)
```

output
```
a**8*c**4*x**9/9 + a**7*c**4*x**8/2 + 4*a**6*c**4*x**7/7 - 2*a**5*c**4*x**
6/3 - 2*a**4*c**4*x**5 - a**3*c**4*x**4 + 4*a**2*c**4*x**3/3 + 2*a*c**4*x*
*2 + c**4*x
```

3.582.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*
a^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

3.582.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{\left(14\,c^4+\frac{189\,c^4}{ax-1}+\frac{1080\,c^4}{(ax-1)^2}+\frac{3360\,c^4}{(ax-1)^3}+\frac{6048\,c^4}{(ax-1)^4}+\frac{6048\,c^4}{(ax-1)^5}+\frac{2688\,c^4}{(ax-1)^6}\right)(ax-1)^9}{126\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
1/126*(14*c^4 + 189*c^4/(a*x - 1) + 1080*c^4/(a*x - 1)^2 + 3360*c^4/(a*x -
 1)^3 + 6048*c^4/(a*x - 1)^4 + 6048*c^4/(a*x - 1)^5 + 2688*c^4/(a*x - 1)^6
)*(a*x - 1)^9/a
```

3.582.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{2}+\frac{4\,a^6\,c^4\,x^7}{7}-\frac{2\,a^5\,c^4\,x^6}{3}-2\,a^4\,c^4\,x^5-a^3\,c^4\,x^4+\frac{4\,a^2\,c^4\,x^3}{3}+2\,a\,c^4\,x^2+c^4\,x$$

input
```
int(((c - a^2*c*x^2)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^4*x + 2*a*c^4*x^2 + (4*a^2*c^4*x^3)/3 - a^3*c^4*x^4 - 2*a^4*c^4*x^5 - (2
*a^5*c^4*x^6)/3 + (4*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/2 + (a^8*c^4*x^9)/9
```

3.582. $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.583 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

3.583.1 Optimal result

Integrand size = 22, antiderivative size = 35

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx=\frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

output
```
1/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

3.583.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx=-\frac{c^3(1+ax)^6(-4+3ax)}{21a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
-1/21*(c^3*(1 + a*x)^6*(-4 + 3*a*x))/a
```

3.583.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^3\left(1-a^2x^2\right)^3 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^3\int e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \text{6690}$$

$$c^3\int (1-ax)(ax+1)^5dx$$

$$\Big\downarrow \text{49}$$

$$c^3\int \left(2(ax+1)^5-(ax+1)^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{(ax+1)^6}{3a}-\frac{(ax+1)^7}{7a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
c^3*((1 + a*x)^6/(3*a) - (1 + a*x)^7/(7*a))
```

3.583.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.583.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

method	result
gosper	$-\frac{c^{3} x \left(3 a^{6} x^{6}+14 a^{5} x^{5}+21 a^{4} x^{4}-35 a^{2} x^{2}-42 a x -21\right)}{21}$
default	$c^{3} \left(-\frac{1}{7} a^{6} x^{7}-\frac{2}{3} a^{5} x^{6}-a^{4} x^{5}+\frac{5}{3} a^{2} x^{3}+2 a \,x^{2}+x \right)$
risch	$-\frac{1}{7} a^{6} c^{3} x^{7}-\frac{2}{3} a^{5} c^{3} x^{6}-a^{4} c^{3} x^{5}+\frac{5}{3} a^{2} c^{3} x^{3}+2 a \,c^{3} x^{2}+c^{3} x$
parallelrisch	$-\frac{1}{7} a^{6} c^{3} x^{7}-\frac{2}{3} a^{5} c^{3} x^{6}-a^{4} c^{3} x^{5}+\frac{5}{3} a^{2} c^{3} x^{3}+2 a \,c^{3} x^{2}+c^{3} x$
norman	$\frac{-c^{3} x +a^{4} c^{3} x^{5}-a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}+\frac{5}{3} a^{3} c^{3} x^{4}-\frac{1}{3} a^{5} c^{3} x^{6}-\frac{11}{21} a^{6} c^{3} x^{7}-\frac{1}{7} a^{7} c^{3} x^{8}}{a x -1}$
meijerg	$\frac{c^{3} \left(-\frac{a x \left(-45 a^{7} x^{7}-60 a^{6} x^{6}-84 a^{5} x^{5}-126 a^{4} x^{4}-210 a^{3} x^{3}-420 a^{2} x^{2}-1260 a x +2520\right)}{315 \left(-a x +1\right)}-8 \ln \left(-a x +1\right)\right)}{a}-\frac{2 c^{3} \left(-\frac{a x \left(-14 a^{5} x^{5}-21 a^{4} x^{4}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/21*c^3*x*(3*a^6*x^6+14*a^5*x^5+21*a^4*x^4-35*a^2*x^2-42*a*x-21)
```

3.583.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c
^3*x^2 + c^3*x
```

3.583.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6c^3x^7}{7}-\frac{2a^5c^3x^6}{3}-a^4c^3x^5+\frac{5a^2c^3x^3}{3}+2ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**3,x)
```

output
```
-a**6*c**3*x**7/7 - 2*a**5*c**3*x**6/3 - a**4*c**3*x**5 + 5*a**2*c**3*x**3
/3 + 2*a*c**3*x**2 + c**3*x
```

3.583.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c
^3*x^2 + c^3*x
```

3.583.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{\left(3\,c^3+\frac{35\,c^3}{ax-1}+\frac{168\,c^3}{(ax-1)^2}+\frac{420\,c^3}{(ax-1)^3}+\frac{560\,c^3}{(ax-1)^4}+\frac{336\,c^3}{(ax-1)^5}\right)(ax-1)^7}{21\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/21*(3*c^3 + 35*c^3/(a*x - 1) + 168*c^3/(a*x - 1)^2 + 420*c^3/(a*x - 1)^
3 + 560*c^3/(a*x - 1)^4 + 336*c^3/(a*x - 1)^5)*(a*x - 1)^7/a
```

3.583.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{a^6\,c^3\,x^7}{7}-\frac{2\,a^5\,c^3\,x^6}{3}-a^4\,c^3\,x^5+\frac{5\,a^2\,c^3\,x^3}{3}+2\,a\,c^3\,x^2+c^3\,x$$

input
```
int(((c - a^2*c*x^2)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^3*x + 2*a*c^3*x^2 + (5*a^2*c^3*x^3)/3 - a^4*c^3*x^5 - (2*a^5*c^3*x^6)/3
- (a^6*c^3*x^7)/7
```

3.584 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

3.584.1 Optimal result

Integrand size = 22, antiderivative size = 17

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{c^2(1+ax)^5}{5a}$$

output
```
1/5*c^2*(a*x+1)^5/a
```

3.584.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=c^2\left(x+2ax^2+2a^2x^3+a^3x^4+\frac{a^4x^5}{5}\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
c^2*(x + 2*a*x^2 + 2*a^2*x^3 + a^3*x^4 + (a^4*x^5)/5)
```

3.584.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6717$$

$$\int c^2\left(1-a^2x^2\right)^2 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \; 27$$

$$c^2\int e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^2 dx$$

$$\Big\downarrow \; 6690$$

$$c^2\int (ax+1)^4dx$$

$$\Big\downarrow \; 17$$

$$\frac{c^2(ax+1)^5}{5a}$$

input

```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output

```
(c^2*(1 + a*x)^5)/(5*a)
```

3.584.3.1 Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.584.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^{2} \left(a x +1\right)^{5}}{5 a}$
gosper	$\frac{c^{2} x \left(a^{4} x^{4}+5 a^{3} x^{3}+10 a^{2} x^{2}+10 a x +5\right)}{5}$
parallelrisch	$\frac{1}{5} a^{4} c^{2} x^{5}+a^{3} c^{2} x^{4}+2 a^{2} c^{2} x^{3}+2 a \,c^{2} x^{2}+c^{2} x$
risch	$\frac{a^{4} c^{2} x^{5}}{5}+a^{3} c^{2} x^{4}+2 a^{2} c^{2} x^{3}+2 a \,c^{2} x^{2}+c^{2} x +\frac{c^{2}}{5 a}$
norman	$\frac{-c^{2} x +a^{3} c^{2} x^{4}-a \,c^{2} x^{2}+\frac{4}{5} a^{4} c^{2} x^{5}+\frac{1}{5} a^{5} c^{2} x^{6}}{a x -1}$
meijerg	$-\frac{c^{2} \left(-\frac{a x \left(-14 a^{5} x^{5}-21 a^{4} x^{4}-35 a^{3} x^{3}-70 a^{2} x^{2}-210 a x +420\right)}{70 \left(-a x +1\right)}-6 \ln \left(-a x +1\right)\right)}{a}+\frac{c^{2} (-\frac{a x \left(-5 a^{3} x^{3}-10 a^{2} x^{2}-30 a x +60\right)}{15 \left(-a x +1\right)}-4 \ln (-a x$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*c^2*(a*x+1)^5/a
```

3.584.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)} \left(c-a^2cx^2\right)^2 \, dx = \frac{1}{5} \, a^{4} c^{2} x^{5} + a^{3} c^{2} x^{4} + 2 \, a^{2} c^{2} x^{3} + 2 \, a c^{2} x^{2} + c^{2} x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

3.584.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}+a^3c^2x^4+2a^2c^2x^3+2ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**2,x)
```

output
```
a**4*c**2*x**5/5 + a**3*c**2*x**4 + 2*a**2*c**2*x**3 + 2*a*c**2*x**2 + c**
2*x
```

3.584.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+a^3c^2x^4+2\,a^2c^2x^3+2\,ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

3.584.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{\left(c^2+\frac{10\,c^2}{ax-1}+\frac{40\,c^2}{(ax-1)^2}+\frac{80\,c^2}{(ax-1)^3}+\frac{80\,c^2}{(ax-1)^4}\right)(ax-1)^5}{5\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/5*(c^2 + 10*c^2/(a*x - 1) + 40*c^2/(a*x - 1)^2 + 80*c^2/(a*x - 1)^3 + 80
*c^2/(a*x - 1)^4)*(a*x - 1)^5/a
```

3.584.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4\,c^2\,x^5}{5}+a^3\,c^2\,x^4+2\,a^2\,c^2\,x^3+2\,a\,c^2\,x^2+c^2\,x$$

input
```
int(((c - a^2*c*x^2)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^2*x + 2*a*c^2*x^2 + 2*a^2*c^2*x^3 + a^3*c^2*x^4 + (a^4*c^2*x^5)/5
```

3.585 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

3.585.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)\,dx=-4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}$$

output
```
-4*c*x-c*(a*x+1)^2/a-1/3*c*(a*x+1)^3/a-8*c*ln(-a*x+1)/a
```

3.585.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)\,dx=-\frac{c(4+21ax+6a^2x^2+a^3x^3+24\log(1-ax))}{3a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
-1/3*(c*(4 + 21*a*x + 6*a^2*x^2 + a^3*x^3 + 24*Log[1 - a*x]))/a
```

3.585.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int c\left(1-a^2x^2\right)e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c\int e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)dx$$

$$\Big\downarrow \text{6690}$$

$$c\int \frac{(ax+1)^3}{1-ax}dx$$

$$\Big\downarrow \text{49}$$

$$c\int \left(-(ax+1)^2-2(ax+1)+\frac{8}{1-ax}-4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c\left(-\frac{(ax+1)^3}{3a}-\frac{(ax+1)^2}{a}-\frac{8\log(1-ax)}{a}-4x\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
c*(-4*x - (1 + a*x)^2/a - (1 + a*x)^3/(3*a) - (8*Log[1 - a*x])/a)
```

3.585.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.585.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

method	result
default	$c\left(-\frac{a^{2}x^{3}}{3}-2a\,x^{2}-7x-\frac{8\ln\left(ax-1\right)}{a}\right)$
risch	$-\frac{a^{2}c\,x^{3}}{3}-2ac\,x^{2}-7cx-\frac{8c\ln\left(ax-1\right)}{a}$
parallelrisch	$-\frac{a^{3}c\,x^{3}+6a^{2}c\,x^{2}+21acx+24c\ln\left(ax-1\right)}{3a}$
norman	$\frac{7cx-5ac\,x^{2}-\frac{5}{3}a^{2}c\,x^{3}-\frac{1}{3}a^{3}c\,x^{4}}{ax-1}-\frac{8c\ln\left(ax-1\right)}{a}$
meijerg	$\frac{c\left(-\frac{ax\left(-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{15\left(-ax+1\right)}-4\ln\left(-ax+1\right)\right)}{a}-\frac{2c\left(\frac{ax\left(-2a^{2}x^{2}-6ax+12\right)}{-4ax+4}+3\ln\left(-ax+1\right)\right)}{a}+\frac{2c\left(\frac{ax}{-ax+1}+\ln(-ax+1\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
c*(-1/3*a^2*x^3-2*a*x^2-7*x-8/a*ln(a*x-1))
```

3.585.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^3cx^3+6\,a^2cx^2+21\,acx+24\,c\log\left(ax-1\right)}{3\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/3*(a^3*c*x^3 + 6*a^2*c*x^2 + 21*a*c*x + 24*c*log(a*x - 1))/a
```

3.585.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}-2acx^2-7cx-\frac{8c\log\left(ax-1\right)}{a}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c),x)
```

output
```
-a**2*c*x**3/3 - 2*a*c*x**2 - 7*c*x - 8*c*log(a*x - 1)/a
```

3.585.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-2\,acx^2-7\,cx-\frac{8\,c\log\left(ax-1\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/3*a^2*c*x^3 - 2*a*c*x^2 - 7*c*x - 8*c*log(a*x - 1)/a
```

3.585.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{(ax-1)^3\left(c+\frac{9\,c}{ax-1}+\frac{36\,c}{(ax-1)^2}\right)}{3\,a}+\frac{8\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-1/3*(a*x - 1)^3*(c + 9*c/(a*x - 1) + 36*c/(a*x - 1)^2)/a + 8*c*log(abs(a*
x - 1)/((a*x - 1)^2*abs(a)))/a
```

3.585.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-7\,c\,x-\frac{a^2\,c\,x^3}{3}-\frac{8\,c\,\ln\left(a\,x-1\right)}{a}-2\,a\,c\,x^2$$

input
```
int(((c - a^2*c*x^2)*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
- 7*c*x - (a^2*c*x^3)/3 - (8*c*log(a*x - 1))/a - 2*a*c*x^2
```

3.586 $\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

3.586.1 Optimal result

Integrand size = 22, antiderivative size = 13

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{c(1-ax)^2}$$

output
```
x/c/(-a*x+1)^2
```

3.586.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{(1+ax)^2}{4ac(1-ax)^2}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
(1 + a*x)^2/(4*a*c*(1 - a*x)^2)
```

3.586.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6717, 27, 6690, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{tikzcd}
\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}dx \arrow[d, "6717"] \\
\int \frac{e^{4\text{arctanh}(ax)}}{c\left(1-a^2x^2\right)}dx \arrow[d, "27"] \\
\frac{\int \frac{e^{4\text{arctanh}(ax)}}{1-a^2x^2}dx}{c} \arrow[d, "6690"] \\
\frac{\int \frac{ax+1}{(1-ax)^3}dx}{c} \arrow[d, "38"] \\
\frac{x}{c(1-ax)^2}
\end{tikzcd}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
x/(c*(1 - a*x)^2)
```

3.586.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.586.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x}{c(ax-1)^2}$	13
norman	$\frac{x}{c(ax-1)^2}$	13
risch	$\frac{x}{c(ax-1)^2}$	13
parallelrisch	$\frac{x}{c(ax-1)^2}$	13
default	$\frac{\frac{1}{(ax-1)^2 a}+\frac{1}{a(ax-1)}}{c}$	28

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
x/c/(a*x-1)^2
```

3.586.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{a^2cx^2-2\,acx+c}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
x/(a^2*c*x^2 - 2*a*c*x + c)
```

3.586.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{x}{a^2cx^2-2acx+c}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c),x)
```

output
```
x/(a**2*c*x**2 - 2*a*c*x + c)
```

3.586.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{x}{a^2cx^2-2\,acx+c}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
x/(a^2*c*x^2 - 2*a*c*x + c)
```

3.586.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{\frac{1}{(ax-1)a}+\frac{1}{(ax-1)^2a}}{c}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
(1/((a*x - 1)*a) + 1/((a*x - 1)^2*a))/c
```

3.586.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{c\,{\left(a\,x-1\right)}^2}$$

input
```
int((a*x + 1)^2/((c - a^2*c*x^2)*(a*x - 1)^2),x)
```

output
```
x/(c*(a*x - 1)^2)
```

3.587 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.587.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{1}{3ac^2(1-ax)^3}$$

output
```
1/3/a/c^2/(-a*x+1)^3
```

3.587.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{1}{3ac^2(1-ax)^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
1/(3*a*c^2*(1 - a*x)^3)
```

3.587.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{6690}$$

$$\frac{\int \frac{1}{(1-ax)^4}dx}{c^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3ac^2(1-ax)^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
1/(3*a*c^2*(1 - a*x)^3)
```

3.587.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.587.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{1}{3c^2(ax-1)^3a}$	16
default	$-\frac{1}{3c^2(ax-1)^3a}$	16
risch	$-\frac{1}{3c^2(ax-1)^3a}$	16
parallelrisch	$\frac{-a^2x^3+3a\,x^2-3x}{3(ax-1)^3c^2}$	31
norman	$\frac{-\frac{x}{c}-\frac{a^3x^4}{3c}+\frac{2a^2x^3}{3c}}{(ax-1)^3(ax+1)c}$	48

input

```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/3/c^2/(a*x-1)^3/a
```

3.587.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

input

```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output

```
-1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

3.587.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3a^4c^2x^3 - 9a^3c^2x^2 + 9a^2c^2x - 3ac^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**2,x)
```

output
```
-1/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3*a*c**2)
```

3.587.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3\,(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

3.587.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3\,(ax-1)^3ac^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/3/((a*x - 1)^3*a*c^2)
```

3.587.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.22

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{-3\,a^4\,c^2\,x^3+9\,a^3\,c^2\,x^2-9\,a^2\,c^2\,x+3\,a\,c^2}$$

input

```
int((a*x + 1)^2/((c - a^2*c*x^2)^2*(a*x - 1)^2),x)
```

output

```
1/(3*a*c^2 - 9*a^2*c^2*x + 9*a^3*c^2*x^2 - 3*a^4*c^2*x^3)
```

3.588 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.588.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=\frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\operatorname{arctanh}(ax)}{16ac^3}$$

output
```
1/8/a/c^3/(-a*x+1)^4+1/12/a/c^3/(-a*x+1)^3+1/16/a/c^3/(-a*x+1)^2+1/16/a/c^
3/(-a*x+1)+1/16*arctanh(a*x)/a/c^3
```

3.588.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=\frac{16-19ax+12a^2x^2-3a^3x^3+3(-1+ax)^4\operatorname{arctanh}(ax)}{48ac^3(-1+ax)^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
(16 - 19*a*x + 12*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^4*ArcTanh[a*x])/(48*a
*c^3*(-1 + a*x)^4)
```

3.588.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$$

$$\Big\downarrow 6717$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^3(1-a^2x^2)^3}dx$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow 6690$$

$$\frac{\int \frac{1}{(1-ax)^5(ax+1)}dx}{c^3}$$

$$\Big\downarrow 54$$

$$\frac{\int \left(\frac{1}{16(ax-1)^2}-\frac{1}{8(ax-1)^3}+\frac{1}{4(ax-1)^4}-\frac{1}{2(ax-1)^5}-\frac{1}{16(a^2x^2-1)}\right)dx}{c^3}$$

$$\Big\downarrow 2009$$

$$\frac{\frac{\text{arctanh}(ax)}{16a}+\frac{1}{16a(1-ax)}+\frac{1}{16a(1-ax)^2}+\frac{1}{12a(1-ax)^3}+\frac{1}{8a(1-ax)^4}}{c^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
(1/(8*a*(1 - a*x)^4) + 1/(12*a*(1 - a*x)^3) + 1/(16*a*(1 - a*x)^2) + 1/(16
*a*(1 - a*x)) + ArcTanh[a*x]/(16*a))/c^3
```

3.588. $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.588.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.588.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

method	result
risch	$\frac{-\frac{a^{2} x^{3}}{16}+\frac{a\,x^{2}}{4}-\frac{19 x}{48}+\frac{1}{3 a}}{c^{3} \left(a x -1\right)^{4}}+\frac{\ln \left(-a x -1\right)}{32 a\,c^{3}}-\frac{\ln \left(a x -1\right)}{32 a\,c^{3}}$
default	$\frac{\frac{\ln \left(a x +1\right)}{32 a}+\frac{1}{8 a \left(a x -1\right)^{4}}-\frac{1}{12 a \left(a x -1\right)^{3}}+\frac{1}{16 \left(a x -1\right)^{2} a}-\frac{1}{16 a \left(a x -1\right)}-\frac{\ln \left(a x -1\right)}{32 a}}{c^{3}}$
norman	$\frac{\frac{15 x}{16 c}+\frac{a\,x^{2}}{8 c}-\frac{31 a^{2} x^{3}}{24 c}+\frac{11 a^{3} x^{4}}{24 c}+\frac{29 a^{4} x^{5}}{48 c}-\frac{a^{5} x^{6}}{3 c}}{\left(a x +1\right)^{2} \left(a x -1\right)^{4} c^{2}}-\frac{\ln \left(a x -1\right)}{32 a\,c^{3}}+\frac{\ln \left(a x +1\right)}{32 a\,c^{3}}$
parallelrisch	$\frac{-3 \ln \left(a x -1\right) x^{4} a^{4}+3 \ln \left(a x +1\right) x^{4} a^{4}-32 a^{4} x^{4}+12 a^{3} \ln \left(a x -1\right) x^{3}-12 a^{3} \ln \left(a x +1\right) x^{3}+122 a^{3} x^{3}-18 a^{2} \ln \left(a x -1\right) x^{2}+18 a^{2} \ln (a x +}{96 \left(a x -1\right)^{4} c^{3} a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/16*a^2*x^3+1/4*a*x^2-19/48*x+1/3/a)/c^3/(a*x-1)^4+1/32/a/c^3*ln(-a*x-1
)-1/32/a/c^3*ln(a*x-1)
```

3.588.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(73) = 146.

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.69

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \\ -\frac{6\,a^3x^3-24\,a^2x^2+38\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax+1\right)+3\left(a^4x^4-4\,a^3x^3+6\,a^2}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

input

```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output

```
-1/96*(6*a^3*x^3 - 24*a^2*x^2 + 38*a*x - 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^
2 - 4*a*x + 1)*log(a*x + 1) + 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x +
 1)*log(a*x - 1) - 32)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^
2*c^3*x + a*c^3)
```

3.588.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\frac{3a^3x^3-12a^2x^2+19ax-16}{48a^5c^3x^4-192a^4c^3x^3+288a^3c^3x^2-192a^2c^3x+48ac^3} \\ -\frac{\frac{\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{ac^3}$$

input

```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**3,x)
```

output

```
-(3*a**3*x**3 - 12*a**2*x**2 + 19*a*x - 16)/(48*a**5*c**3*x**4 - 192*a**4*
c**3*x**3 + 288*a**3*c**3*x**2 - 192*a**2*c**3*x + 48*a*c**3) - (log(x - 1
/a)/32 - log(x + 1/a)/32)/(a*c**3)
```

3.588.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{3\,a^3x^3-12\,a^2x^2+19\,ax-16}{48\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)} + \frac{\log\left(ax+1\right)}{32\,ac^3} - \frac{\log\left(ax-1\right)}{32\,ac^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/48*(3*a^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3
+ 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + 1/32*log(a*x + 1)/(a*c^3) - 1/32*
log(a*x - 1)/(a*c^3)
```

3.588.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3} - \frac{\frac{3\,a^3c^9}{ax-1}-\frac{3\,a^3c^9}{(ax-1)^2}+\frac{4\,a^3c^9}{(ax-1)^3}-\frac{6\,a^3c^9}{(ax-1)^4}}{48\,a^4c^{12}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
1/32*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(3*a^3*c^9/(a*x - 1) - 3*a^
3*c^9/(a*x - 1)^2 + 4*a^3*c^9/(a*x - 1)^3 - 6*a^3*c^9/(a*x - 1)^4)/(a^4*c^
12)
```

3.588.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\mathrm{atanh}(a\,x)}{16\,a\,c^3} - \frac{\frac{19\,x}{48}-\frac{a\,x^2}{4}-\frac{1}{3\,a}+\frac{a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}$$

input
```
int((a*x + 1)^2/((c - a^2*c*x^2)^3*(a*x - 1)^2),x)
```

output
```
atanh(a*x)/(16*a*c^3) - ((19*x)/48 - (a*x^2)/4 - 1/(3*a) + (a^2*x^3)/16)/(
c^3 + 6*a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x)
```

3.589 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

3.589.1 Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{3\text{arctanh}(ax)}{32ac^4}$$

output
```
1/20/a/c^4/(-a*x+1)^5+1/16/a/c^4/(-a*x+1)^4+1/16/a/c^4/(-a*x+1)^3+1/16/a/c
^4/(-a*x+1)^2+5/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+3/32*arctanh(a*x)/a/c
^4
```

3.589.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{-48+47ax+20a^2x^2-80a^3x^3+60a^4x^4-15a^5x^5+15(-1+ax)^5(1+ax)\text{arctanh}(ax)}{160ac^4(-1+ax)^5(1+ax)}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
(-48 + 47*a*x + 20*a^2*x^2 - 80*a^3*x^3 + 60*a^4*x^4 - 15*a^5*x^5 + 15*(-1
 + a*x)^5*(1 + a*x)*ArcTanh[a*x])/(160*a*c^4*(-1 + a*x)^5*(1 + a*x))
```

3.589.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^4(1-a^2x^2)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6690}$$

$$\frac{\int \frac{1}{(1-ax)^6(ax+1)^2}dx}{c^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{5}{64(ax-1)^2}+\frac{1}{64(ax+1)^2}-\frac{1}{8(ax-1)^3}+\frac{3}{16(ax-1)^4}-\frac{1}{4(ax-1)^5}+\frac{1}{4(ax-1)^6}-\frac{3}{32(a^2x^2-1)}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\text{arctanh}(ax)}{32a}+\frac{5}{64a(1-ax)}-\frac{1}{64a(ax+1)}+\frac{1}{16a(1-ax)^2}+\frac{1}{16a(1-ax)^3}+\frac{1}{16a(1-ax)^4}+\frac{1}{20a(1-ax)^5}}{c^4}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
(1/(20*a*(1 - a*x)^5) + 1/(16*a*(1 - a*x)^4) + 1/(16*a*(1 - a*x)^3) + 1/(1
6*a*(1 - a*x)^2) + 5/(64*a*(1 - a*x)) - 1/(64*a*(1 + a*x)) + (3*ArcTanh[a*
x])/(32*a))/c^4
```

3.589. $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$

3.589.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.589.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

method	result
risch	$\frac{-\frac{3a^4x^5}{32}+\frac{3a^3x^4}{8}-\frac{a^2x^3}{2}+\frac{a\,x^2}{8}+\frac{47x}{160}-\frac{3}{10a}}{c^4(ax-1)^4(a^2x^2-1)}-\frac{3\ln(ax-1)}{64a\,c^4}+\frac{3\ln(-ax-1)}{64a\,c^4}$
default	$\frac{-\frac{1}{64a(ax+1)}+\frac{3\ln(ax+1)}{64a}-\frac{1}{20a(ax-1)^5}+\frac{1}{16a(ax-1)^4}-\frac{1}{16a(ax-1)^3}+\frac{1}{16(ax-1)^2a}-\frac{5}{64a(ax-1)}-\frac{3\ln(ax-1)}{64a}}{c^4}$
norman	$\frac{-\frac{a^3x^4}{2c}-\frac{29x}{32c}-\frac{3a\,x^2}{16c}+\frac{59a^2x^3}{32c}-\frac{263a^4x^5}{160c}+\frac{63a^5x^6}{80c}+\frac{81a^6x^7}{160c}-\frac{3a^7x^8}{10c}}{(ax-1)^5(ax+1)^3c^3}-\frac{3\ln(ax-1)}{64a\,c^4}+\frac{3\ln(ax+1)}{64a\,c^4}$
parallelrisch	$\frac{60a\ln(ax+1)x-75a^2\ln(ax+1)x^2+354a^5x^5-160a^3x^3-60\ln(ax+1)x^5a^5+15\ln(ax+1)x^6a^6+75\ln(ax+1)x^4a^4-15\ln(ax-1)x}{320(}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(-3/32*a^4*x^5+3/8*a^3*x^4-1/2*a^2*x^3+1/8*a*x^2+47/160*x-3/10/a)/c^4/(a*x
-1)^4/(a^2*x^2-1)-3/64/a/c^4*ln(a*x-1)+3/64/a/c^4*ln(-a*x-1)
```

3.589.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.57

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx =$$
$$-\frac{30\,a^5x^5-120\,a^4x^4+160\,a^3x^3-40\,a^2x^2-94\,ax-15\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log}{320\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^{}\right.}$$

input

```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output

```
-1/320*(30*a^5*x^5 - 120*a^4*x^4 + 160*a^3*x^3 - 40*a^2*x^2 - 94*a*x - 15*
(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 1
5*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) +
 96)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*
c^4*x - a*c^4)
```

3.589.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax-48}{160a^7c^4x^6-640a^6c^4x^5+800a^5c^4x^4-800a^3c^4x^2+640a^2c^4x-160ac^4} + \frac{-\frac{3\log\left(x-\frac{1}{a}\right)}{64}+\frac{3\log\left(x+\frac{1}{a}\right)}{64}}{ac^4}$$

input

```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**4,x)
```

output

```
(-15*a**5*x**5 + 60*a**4*x**4 - 80*a**3*x**3 + 20*a**2*x**2 + 47*a*x - 48)
/(160*a**7*c**4*x**6 - 640*a**6*c**4*x**5 + 800*a**5*c**4*x**4 - 800*a**3*
c**4*x**2 + 640*a**2*c**4*x - 160*a*c**4) + (-3*log(x - 1/a)/64 + 3*log(x
+ 1/a)/64)/(a*c**4)
```

3.589.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,a^5x^5-60\,a^4x^4+80\,a^3x^3-20\,a^2x^2-47\,ax+48}{160\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)} + \frac{3\,\log(ax+1)}{64\,ac^4} - \frac{3\,\log(ax-1)}{64\,ac^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
-1/160*(15*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/(
a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x
- a*c^4) + 3/64*log(a*x + 1)/(a*c^4) - 3/64*log(a*x - 1)/(a*c^4)
```

3.589.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{3\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4} + \frac{1}{128\,ac^4\left(\frac{2}{ax-1}+1\right)} - \frac{\frac{25\,a^9c^{16}}{ax-1} - \frac{20\,a^9c^{16}}{(ax-1)^2} + \frac{20\,a^9c^{16}}{(ax-1)^3} - \frac{20\,a^9c^{16}}{(ax-1)^4} + \frac{16\,a^9c^{16}}{(ax-1)^5}}{320\,a^{10}c^{20}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
3/64*log(abs(-2/(a*x - 1) - 1))/(a*c^4) + 1/128/(a*c^4*(2/(a*x - 1) + 1))
- 1/320*(25*a^9*c^16/(a*x - 1) - 20*a^9*c^16/(a*x - 1)^2 + 20*a^9*c^16/(a*
x - 1)^3 - 20*a^9*c^16/(a*x - 1)^4 + 16*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

3.589.9 Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$=\frac{3\operatorname{atanh}(a\,x)}{32\,a\,c^4}-\frac{\frac{47\,x}{160}+\frac{a\,x^2}{8}-\frac{3}{10\,a}-\frac{a^2\,x^3}{2}+\frac{3\,a^3\,x^4}{8}-\frac{3\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

input
```
int((a*x + 1)^2/((c - a^2*c*x^2)^4*(a*x - 1)^2),x)
```

output
```
(3*atanh(a*x))/(32*a*c^4) - ((47*x)/160 + (a*x^2)/8 - 3/(10*a) - (a^2*x^3)
/2 + (3*a^3*x^4)/8 - (3*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4
+ 4*a^5*c^4*x^5 - a^6*c^4*x^6 - 4*a*c^4*x)
```

3.590 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.590.1 Optimal result

Integrand size = 22, antiderivative size = 393

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx &= -\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2\\ &\quad-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4\\ &\quad+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}\end{aligned}$$

output
```
-5/48*a^5*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x^6+1/8*a^6*c^4*(1-1/a/x)^(5
/2)*(1+1/a/x)^(9/2)*x^7-1/8*a^7*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(9/2)*x^8+1/
9*a^8*c^4*(1-1/a/x)^(9/2)*(1+1/a/x)^(9/2)*x^9-35/128*c^4*arctanh((1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2))/a-35/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)
-7/192*a^2*c^4*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-1/64*a^3*c^4*(1+1/a/x)^
(7/2)*x^4*(1-1/a/x)^(1/2)+1/16*a^4*c^4*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)
-35/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.590.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.28

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(128+837ax-512a^2x^2-978a^3x^3+768a^4x^4+600a^5x^5-512a^6x^6-144a^7x^7+128a^8}{1152a}$$

input
```
Integrate[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```

output
```
(c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 + 837*a*x - 512*a^2*x^2 - 978*a^3*x^3
 + 768*a^4*x^4 + 600*a^5*x^5 - 512*a^6*x^6 - 144*a^7*x^7 + 128*a^8*x^8) -
315*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

3.590.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^4 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{-\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{-\coth^{-1}(ax)} \left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{-\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{-\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{-\coth^{-1}(ax)} \left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{-\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{-\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{-\coth^{-1}(ax)} \left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{-\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{-\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^4 x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{-\coth^{-1}(ax)} \left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

3.590. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

$$\begin{tikzcd}
a^8c^4\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx \arrow[d, "27"] \\
c^4\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx \arrow[d, "2005"] \\
c^4\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx \arrow[d, "6745"] \\
a^8c^4\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx \arrow[d, "27"] \\
c^4\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx \arrow[d, "2005"] \\
c^4\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx \arrow[d, "6745"] \\
a^8c^4\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx \arrow[d, "27"] \\
c^4\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx \arrow[d, "2005"] \\
c^4\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx \arrow[d, "6745"] \\
a^8c^4\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx \arrow[d, "27"] \\
c^4\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx
\end{tikzcd}$$

3.590. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

$$\Big\downarrow \text{2005}$$

$$c^4\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

input
```
Int[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```

output
```
$Aborted
```

3.590.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.590. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

3.590.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\left(128a^8x^8-144a^7x^7-512a^6x^6+600a^5x^5+768a^4x^4-978a^3x^3-512a^2x^2+837ax+128\right)(ax+1)c^4\sqrt{\frac{ax-1}{ax+1}}}{1152a}-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{128\sqrt{}}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-128(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+144(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+384(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-456(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-\right.}{1}$

input
```
int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1152*(128*a^8*x^8-144*a^7*x^7-512*a^6*x^6+600*a^5*x^5+768*a^4*x^4-978*a^
3*x^3-512*a^2*x^2+837*a*x+128)*(a*x+1)/a*c^4*((a*x-1)/(a*x+1))^(1/2)-35/12
8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4*((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.590.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.43

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx =$$

$$-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(128\,a^9c^4x^9-16\,a^8c^4x^8-656\,a^7c^4x^7+88\,a^6c^4x^6}{1152\,a}$$

input
```
integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) - (128*a^9*c^4*x^9 - 16*a^8*c^4*x^8 - 656*a^7*c^4*x
^7 + 88*a^6*c^4*x^6 + 1368*a^5*c^4*x^5 - 210*a^4*c^4*x^4 - 1490*a^3*c^4*x^
3 + 325*a^2*c^4*x^2 + 965*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.590.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\left(\int\left(-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int 6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**4*(Integral(-4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integr
al(6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**6*x*
*6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**8*x**8*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x)
)
```

3.590.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx =$$

$$-\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+1\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(a}{(a}}\right.$$

input
```
integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) -
 2730*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(
13/2) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 32768*c^4*((a*x - 1)/(a*x
 + 1))^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1
)/(a*x + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((
a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4
+ 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a
*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^
2/(a*x + 1)^9 - a^2))*a
```

3.590.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.50

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{35\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{128\,|a|}$$

$$+\frac{1}{1152}\sqrt{a^2x^2-1}\Bigg(\frac{128\,c^4\operatorname{sgn}(ax+1)}{a}+(837\,c^4\operatorname{sgn}(ax+1)-2\,(256\,ac^4\operatorname{sgn}(ax+1)+(489\,a^2c^4\operatorname{sgn}(ax$$

input
```
integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
35/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1
/1152*sqrt(a^2*x^2 - 1)*(128*c^4*sgn(a*x + 1)/a + (837*c^4*sgn(a*x + 1) -
2*(256*a*c^4*sgn(a*x + 1) + (489*a^2*c^4*sgn(a*x + 1) - 4*(96*a^3*c^4*sgn(
a*x + 1) + (75*a^4*c^4*sgn(a*x + 1) - 2*(32*a^5*c^4*sgn(a*x + 1) - (8*a^7*
c^4*x*sgn(a*x + 1) - 9*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)
```

3.590.9 Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$

$$=\frac{\frac{35\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{455\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96}+\frac{581\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32}-\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{32}+\frac{512\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{9}+\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{32}-\frac{581\,c^4}{}}{a-\frac{9\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{36\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{84\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{126\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{126\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{84\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{36\,a\,(a\,x}{(a\,x+}}$$

$$-\frac{35\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

input
```
int((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((35*c^4*((a*x - 1)/(a*x + 1))^(1/2))/64 - (455*c^4*((a*x - 1)/(a*x + 1))^
(3/2))/96 + (581*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 - (1289*c^4*((a*x - 1
)/(a*x + 1))^(7/2))/32 + (512*c^4*((a*x - 1)/(a*x + 1))^(9/2))/9 + (1289*c
^4*((a*x - 1)/(a*x + 1))^(11/2))/32 - (581*c^4*((a*x - 1)/(a*x + 1))^(13/2
))/32 + (455*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 - (35*c^4*((a*x - 1)/(a*
x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a
*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)
^4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (3
6*a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1
)^9)/(a*x + 1)^9) - (35*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

3.590. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

3.591 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

3.591.1 Optimal result

Integrand size = 22, antiderivative size = 313

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$

$$-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$-\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/}$$

output
```
-1/6*a^4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5+1/6*a^5*c^3*(1-1/a/x)^(5/
2)*(1+1/a/x)^(7/2)*x^6-1/7*a^6*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)*x^7-5/1
6*c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-5/48*a*c^3*(1+1/a/x)^(3/2
)*x^2*(1-1/a/x)^(1/2)-1/24*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+1/8
*a^3*c^3*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)-5/16*c^3*x*(1-1/a/x)^(1/2)*(1
+1/a/x)^(1/2)
```

3.591.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(48+231ax-144a^2x^2-182a^3x^3+144a^4x^4+56a^5x^5-48a^6x^6)-105\log\left(\left(1+\sqrt{1-}\right.\right.\right.}{336a}$$

input

```
Integrate[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

output

```
(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 231*a*x - 144*a^2*x^2 - 182*a^3*x^3
+ 144*a^4*x^4 + 56*a^5*x^5 - 48*a^6*x^6) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^
2)])*x]))/(336*a)
```

3.591.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6745$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \; 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \; 2005$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \; 6745$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \; 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow 2005$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow 6745$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow 2005$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow 6745$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow 2005$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow 6745$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow 2005$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow 6745$$

3.591. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \; 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \; 2005$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \; 6745$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \; 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \; 2005$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \; 6745$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \; 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \; 2005$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \; 6745$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \; 27$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

3.591. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

input
```
Int[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

output
```
$Aborted
```

3.591.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.591. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.591.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{\left(48 a^{6} x^{6}-56 a^{5} x^{5}-144 a^{4} x^{4}+182 a^{3} x^{3}+144 a^{2} x^{2}-231 a x -48\right) \left(a x +1\right) c^{3} \sqrt{\frac{a x -1}{a x +1}}}{336 a}-\frac{5 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{3} \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) (a}}{16 \sqrt{a^{2}}\, \left(a x -1\right)}$
default	$-\frac{\sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) c^{3} \left(48 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{4} x^{4}-56 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{3} x^{3}-96 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{2} x^{2}+126 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a x -64 (a^{2} x^{2}}{336 a \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$

input
```
int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/336*(48*a^6*x^6-56*a^5*x^5-144*a^4*x^4+182*a^3*x^3+144*a^2*x^2-231*a*x-
48)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1/2)-5/16*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)*c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/
2)/(a*x-1)
```

3.591.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(48\,a^7c^3x^7-8\,a^6c^3x^6-200\,a^5c^3x^5+38\,a^4c^3x^4+}{336\,a}$$

input
```
integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (48*a^7*c^3*x^7 - 8*a^6*c^3*x^6 - 200*a^5*c^3*x^5
+ 38*a^4*c^3*x^4 + 326*a^3*c^3*x^3 - 87*a^2*c^3*x^2 - 279*a*c^3*x - 48*c^3
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.591.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -c^3\left(\int 3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

input

```
integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output

```
-c**3*(Integral(3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integr
al(-3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/
(a*x + 1)), x))
```

3.591.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx =$$

$$-\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+198\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+}\right.$$

input

```
integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) -
700*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2
) + 3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^
(5/2) + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x
+ 1)))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a
*x - 1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^
5*a^2/(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x +
 1)^7 - a^2))*a
```

3.591.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.51

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{16\,|a|}$$
$$+\frac{1}{336}\,\sqrt{a^2x^2-1}\left(\frac{48\,c^3\operatorname{sgn}(ax+1)}{a}+\left(231\,c^3\operatorname{sgn}(ax+1)-2\left(72\,ac^3\operatorname{sgn}(ax+1)+\left(91\,a^2c^3\operatorname{sgn}(ax+1)\right.\right.\right.\right.$$

input
```
integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
5/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/3
36*sqrt(a^2*x^2 - 1)*(48*c^3*sgn(a*x + 1)/a + (231*c^3*sgn(a*x + 1) - 2*(7
2*a*c^3*sgn(a*x + 1) + (91*a^2*c^3*sgn(a*x + 1) - 4*(18*a^3*c^3*sgn(a*x +
1) - (6*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

3.591.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=$$
$$-\frac{\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6}-\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{24}+\frac{128\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{7}+\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{24}-\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{6}+\frac{5\,c^3\,\left(\frac{a\,x-}{a\,x+}\right.}{8}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}$$
$$-\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$

input

```
int((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
- ((25*c^3*((a*x - 1)/(a*x + 1))^(3/2))/6 - (5*c^3*((a*x - 1)/(a*x + 1))^(
1/2))/8 - (283*c^3*((a*x - 1)/(a*x + 1))^(5/2))/24 + (128*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/7 + (283*c^3*((a*x - 1)/(a*x + 1))^(9/2))/24 - (25*c^3*((
a*x - 1)/(a*x + 1))^(11/2))/6 + (5*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a
 - (7*a*(a*x - 1))/(a*x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x
 - 1)^3)/(a*x + 1)^3 + (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)
/(a*x + 1)^5 + (7*a*(a*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7
) - (5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a)
```

3.591. $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

3.592 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

3.592.1 Optimal result

Integrand size = 22, antiderivative size = 233

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x$$
$$-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3$$
$$-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5-\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right.}{8a}$$

output
```
-1/4*a^3*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x^4+1/5*a^4*c^2*(1-1/a/x)^(5/
2)*(1+1/a/x)^(5/2)*x^5-3/8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-
1/8*a*c^2*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/4*a^2*c^2*(1+1/a/x)^(5/2)*
x^3*(1-1/a/x)^(1/2)-3/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.592.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(8+25ax-16a^2x^2-10a^3x^3+8a^4x^4)-15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

input
```
Integrate[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*
a^4*x^4) - 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

3.592.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2 e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{ 6745}$$

3.592. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$\begin{tikzcd}
a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx \arrow[d, "27"] \\
c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx \arrow[d, "2005"] \\
c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx \arrow[d, "6745"] \\
a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx \arrow[d, "27"] \\
c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx \arrow[d, "2005"] \\
c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx \arrow[d, "6745"] \\
a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx \arrow[d, "27"] \\
c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx \arrow[d, "2005"] \\
c^2\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx \arrow[d, "6745"] \\
a^4c^2\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx \arrow[d, "27"] \\
c^2\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx
\end{tikzcd}$$

3.592. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$\Big\downarrow \text{2005}$$

$$c^2 \int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)^2 dx$$

input
```
Int[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```

output
```
$Aborted
```

3.592.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.592.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.55

method	result
risch	$\frac{\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{40a}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-30\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,ax+16\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}+45\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-40((ax-1)(ax+1))^{\frac{3}{2}}\right.}{120a\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

input
```
int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*(8*a^4*x^4-10*a^3*x^3-16*a^2*x^2+25*a*x+8)*(a*x+1)/a*c^2*((a*x-1)/(a*
x+1))^(1/2)-3/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2*((
a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.592.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx =$$

$$-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(8\,a^5c^2x^5-2\,a^4c^2x^4-26\,a^3c^2x^3+9\,a^2c^2x^2+33\,ac^2}{40\,a}$$

input
```
integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (8*a^5*c^2*x^5 - 2*a^4*c^2*x^4 - 26*a^3*c^2*x^3 + 9*a
^2*c^2*x^2 + 33*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.592.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\left(\int\left(-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**2*(Integral(-2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integr
al(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*
x + 1) - 1/(a*x + 1)), x))
```

3.592.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx =$$

$$-\frac{1}{40}\,a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^2\left(\frac{ax-}{ax-}\right.\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}}\right.$$

input
```
integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*
c^2*((a*x - 1)/(a*x + 1))^(7/2) - 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70
*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a
*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^
2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^
5 - a^2))
```

3.592.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{8\,|a|}$$

$$+\frac{1}{40}\sqrt{a^2x^2-1}\Big(\left(25\,c^2\operatorname{sgn}(ax+1)-2\left(8\,ac^2\operatorname{sgn}(ax+1)-\left(4\,a^3c^2x\operatorname{sgn}(ax+1)-5\,a^2c^2\operatorname{sgn}(ax+1)\right)x\right)\right.$$

input
```
integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
3/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/40
*sqrt(a^2*x^2 - 1)*((25*c^2*sgn(a*x + 1) - 2*(8*a*c^2*sgn(a*x + 1) - (4*a^
3*c^2*x*sgn(a*x + 1) - 5*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 8*c^2*sgn(a*x + 1
)/a)
```

3.592.9 Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{\frac{3\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}+\frac{32\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{5}+\frac{7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{2}-\frac{3\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}$$
$$-\frac{3\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((3*c^2*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^2*((a*x - 1)/(a*x + 1))^(3/2
))/2 + (32*c^2*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^2*((a*x - 1)/(a*x + 1
))^(7/2))/2 - (3*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/
(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^2*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.593 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

3.593.1 Optimal result

Integrand size = 20, antiderivative size = 145

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

output
```
-1/3*a^2*c*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x^3-1/2*c*arctanh((1-1/a/x)^(1/
2)*(1+1/a/x)^(1/2))/a+1/2*a*c*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-1/2*c*x*
(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.593.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(2+3ax-2a^2x^2)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input
```
Integrate[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
```

output
```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 2*a^2*x^2) - 3*Log[(1 + Sqrt[1
- 1/(a^2*x^2)])*x]))/(6*a)
```

3.593.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

3.593. $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)\ dx$

$$-a^2 c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2 c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2 c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2 c\int \frac{e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

3.593. $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

$$-c\int e^{-\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

input
```
Int[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
```

output
```
$Aborted
```

3.593.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.593.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\left(2a^2x^2-3ax-2\right)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{6a}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$	108
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a}$	119

input
```
int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*a^2*x^2-3*a*x-2)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)-1/2*ln(a^2*x/
(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)/(a*x+1))^(1/2)*((a*x
-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.593.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)\,dx$$
$$= -\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2\,a^3cx^3-a^2cx^2-5\,acx-2\,c)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) + (2*a^3*c*x^3 - a^2*c*x^2 - 5*a*c*x - 2*c)*sqrt((a*x - 1)/(a*
x + 1)))/a
```

3.593.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)\,dx = -c\left(\int a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

input
```
integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
-c*(Integral(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-s
qrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.593.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{1}{6}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) + 8*c*((a*x - 1)/(a*x + 1))^(3/2
) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1
)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.593.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= \frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}\\ &\quad-\frac{1}{6}\,\sqrt{a^2x^2-1}\left((2\,acx\operatorname{sgn}(ax+1)-3\,c\operatorname{sgn}(ax+1))x-\frac{2\,c\operatorname{sgn}(ax+1)}{a}\right)\end{aligned}$$

input
```
integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
1/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sq
rt(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 3*c*sgn(a*x + 1))*x - 2*c*sgn(a*x
 + 1)/a)
```

3.593.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{\frac{8\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,\left(a\,x-1\right)^2}{\left(a\,x+1\right)^2}-\frac{a\,\left(a\,x-1\right)^3}{\left(a\,x+1\right)^3}}-\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input

```
int((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
- ((8*c*((a*x - 1)/(a*x + 1))^(3/2))/3 - c*((a*x - 1)/(a*x + 1))^(1/2) + c
*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x -
 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c*atanh(((a*x - 1)/(a
*x + 1))^(1/2)))/a
```

3.594 $\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

3.594.1 Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-\coth^{-1}(ax)}}{ac}$$

output
```
-1/a/c*((a*x-1)/(a*x+1))^(1/2)
```

3.594.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-\coth^{-1}(ax)}}{ac}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]
```

output
```
-(1/(a*c*E^ArcCoth[a*x]))
```

3.594.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$$

$$\Big\downarrow \; 6737$$

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]
```

output
```
-(1/(a*c*E^ArcCoth[a*x]))
```

3.594.3.1 Defintions of rubi rules used

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

3.594.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$	24
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$	24
trager	$-\frac{\sqrt{-\frac{-ax+1}{ax+1}}}{ac}$	26

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/a/c*((a*x-1)/(a*x+1))^(1/2)
```

3.594.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

3.594.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
```

output
```
-Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

3.594.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

3.594.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\int -\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2cx^2-c}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
undef
```

3.594.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2),x)
```

output
```
-((a*x - 1)/(a*x + 1))^(1/2)/(a*c)
```

3.595 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.595.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=-\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}+\frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2\left(1-a^2x^2\right)}$$

output
```
-2/3/a/c^2*((a*x-1)/(a*x+1))^(1/2)+1/3*(2*a*x+1)/a/c^2*((a*x-1)/(a*x+1))^(
1/2)/(-a^2*x^2+1)
```

3.595.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-1+2ax+2a^2x^2\right)}{3(-1+ax)(c+acx)^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
```

output
```
-1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x + 2*a^2*x^2))/((-1 + a*x)*(c + a
*c*x)^2)
```

3.595.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6739, 27, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{2\int \frac{e^{-\coth^{-1}(ax)}}{c(1-a^2x^2)}dx}{3c}+\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{-\coth^{-1}(ax)}}{1-a^2x^2}dx}{3c^2}+\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}$$

$$\Big\downarrow \text{6737}$$

$$\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}-\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
```

output
```
-2/(3*a*c^2*E^ArcCoth[a*x]) + (1 + 2*a*x)/(3*a*c^2*E^ArcCoth[a*x]*(1 - a^2
*x^2))
```

3.595.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.595.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2+2ax-1)}{3(a^2x^2-1)a\,c^2}$	49
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2+2ax-1)}{3c^2(ax+1)(ax-1)a}$	52
trager	$-\frac{(2a^2x^2+2ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^2(ax-1)(ax+1)}$	54

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2+2*a*x-1)/(a^2*x^2-1)/a/c^2
```

3.595.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{(2\,a^2x^2+2\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-ac^2)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*a^2*x^2 + 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - a*c^
2)
```

3.595.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
```

output
```
Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x**2 + 1),
x)/c**2
```

3.595.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{12}a\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-6\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}-\frac{3}{a^2c^2\sqrt{\frac{ax-1}{ax+1}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/12*a*((((a*x - 1)/(a*x + 1))^(3/2) - 6*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c
^2) - 3/(a^2*c^2*sqrt((a*x - 1)/(a*x + 1))))
```

3.595.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(a^2cx^2-c)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^2, x)
```

3.595.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=-\frac{\frac{6\,(a\,x-1)}{a\,x+1}-\frac{{(a\,x-1)}^2}{{(a\,x+1)}^2}+3}{12\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^2,x)
```

output
```
-((6*(a*x - 1))/(a*x + 1) - (a*x - 1)^2/(a*x + 1)^2 + 3)/(12*a*c^2*((a*x -
 1)/(a*x + 1))^(1/2))
```

3.596 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.596.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}+\frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3(1-a^2x^2)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3(1-a^2x^2)}$$

output
```
-8/15/a/c^3*((a*x-1)/(a*x+1))^(1/2)+1/15*(4*a*x+1)/a/c^3*((a*x-1)/(a*x+1))
^(1/2)/(-a^2*x^2+1)^2+4/15*(2*a*x+1)/a/c^3*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x
^2+1)
```

3.596.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(3-12ax-12a^2x^2+8a^3x^3+8a^4x^4)}{15(-1+ax)^2(c+acx)^3}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```

output
```
-1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(3 - 12*a*x - 12*a^2*x^2 + 8*a^3*x^3 + 8*a^
4*x^4))/((-1 + a*x)^2*(c + a*c*x)^3)
```

3.596.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6739, 27, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{4\int \frac{e^{-\coth^{-1}(ax)}}{c^2(1-a^2x^2)^2}dx}{5c}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{e^{-\coth^{-1}(ax)}}{(1-a^2x^2)^2}dx}{5c^3}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{6739}$$

$$\frac{4\left(\frac{2}{3}\int \frac{e^{-\coth^{-1}(ax)}}{1-a^2x^2}dx+\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3a(1-a^2x^2)}\right)}{5c^3}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{6737}$$

$$\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4\left(\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3a(1-a^2x^2)}-\frac{2e^{-\coth^{-1}(ax)}}{3a}\right)}{5c^3}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```

output
```
(1 + 4*a*x)/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^2*x^2)^2) + (4*(-2/(3*a*E^ArcC
oth[a*x]) + (1 + 2*a*x)/(3*a*E^ArcCoth[a*x]*(1 - a^2*x^2))))/(5*c^3)
```

3.596.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.596.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)}{15(a^2x^2-1)^2c^3a}$	65
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)}{15c^3(ax-1)^2a(ax+1)^2}$	68
trager	$-\frac{(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^3(ax-1)^2(ax+1)^2}$	70

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*((a*x-1)/(a*x+1))^(1/2)*(8*a^4*x^4+8*a^3*x^3-12*a^2*x^2-12*a*x+3)/(a
^2*x^2-1)^2/c^3/a
```

3.596. $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.596.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\frac{\left(8\,a^4x^4+8\,a^3x^3-12\,a^2x^2-12\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/15*(8*a^4*x^4 + 8*a^3*x^3 - 12*a^2*x^2 - 12*a*x + 3)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

3.596.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
```

output
```
-Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x**4 + 3*a
**2*x**2 - 1), x)/c**3
```

3.596.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\frac{1}{240}\,a\left(\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-20\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+90\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{5\left(\frac{12\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/240*a*((3*((a*x - 1)/(a*x + 1))^(5/2) - 20*((a*x - 1)/(a*x + 1))^(3/2)
+ 90*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 5*(12*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)))
```

3.596.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=\int -\frac{\sqrt{\frac{ax-1}{ax+1}}}{(a^2cx^2-c)^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(-sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^3, x)
```

3.596.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12\,a\,c^3}-\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{80\,a\,c^3}-\frac{\frac{4\,(a\,x-1)}{a\,x+1}-\frac{1}{3}}{16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^3,x)
```

output
```
((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^3) - (3*((a*x - 1)/(a*x + 1))^(1/2))/(
8*a*c^3) - ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) - ((4*(a*x - 1))/(a*x +
1) - 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

3.597 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

3.597.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx=-\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}+\frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3}+\frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2}+\frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4(1-a^2x^2)}$$

output
```
-16/35/a/c^4*((a*x-1)/(a*x+1))^(1/2)+1/35*(6*a*x+1)/a/c^4*((a*x-1)/(a*x+1)
)^(1/2)/(-a^2*x^2+1)^3+2/35*(4*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(1/2)/(-a^2*
x^2+1)^2+8/35*(2*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^2+1)
```

3.597.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-5+30ax+30a^2x^2-40a^3x^3-40a^4x^4+16a^5x^5+16a^6x^6)}{35(-1+ax)^3(c+acx)^4}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

output
```
-1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + 30*a*x + 30*a^2*x^2 - 40*a^3*x^3 - 40
*a^4*x^4 + 16*a^5*x^5 + 16*a^6*x^6))/((-1 + a*x)^3*(c + a*c*x)^4)
```

3.597.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6739, 27, 6739, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$$

$$\Big\downarrow \text{ 6739}$$

$$\frac{6\int \frac{e^{-\coth^{-1}(ax)}}{c^3(1-a^2x^2)^3}dx}{7c}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6\int \frac{e^{-\coth^{-1}(ax)}}{(1-a^2x^2)^3}dx}{7c^4}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{ 6739}$$

$$\frac{6\left(\frac{4}{5}\int \frac{e^{-\coth^{-1}(ax)}}{(1-a^2x^2)^2}dx+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15a(1-a^2x^2)^2}\right)}{7c^4}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{ 6739}$$

$$\frac{6\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{e^{-\coth^{-1}(ax)}}{1-a^2x^2}dx+\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3a(1-a^2x^2)}\right)+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15a(1-a^2x^2)^2}\right)}{7c^4}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{ 6737}$$

$$\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4(1-a^2x^2)^3}+\frac{6\left(\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15a(1-a^2x^2)^2}+\frac{4}{5}\left(\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3a(1-a^2x^2)}-\frac{2e^{-\coth^{-1}(ax)}}{3a}\right)\right)}{7c^4}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

output
```
(1 + 6*a*x)/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2)^3) + (6*((1 + 4*a*x)/(1
5*a*E^ArcCoth[a*x]*(1 - a^2*x^2)^2) + (4*(-2/(3*a*E^ArcCoth[a*x]) + (1 + 2
*a*x)/(3*a*E^ArcCoth[a*x]*(1 - a^2*x^2))))/5))/(7*c^4)
```

3.597.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.597.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(16a^{6}x^{6}+16a^{5}x^{5}-40a^{4}x^{4}-40a^{3}x^{3}+30a^{2}x^{2}+30ax-5\right)}{35\left(a^{2}x^{2}-1\right)^{3}c^{4}a}$	81
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(16a^{6}x^{6}+16a^{5}x^{5}-40a^{4}x^{4}-40a^{3}x^{3}+30a^{2}x^{2}+30ax-5\right)}{35c^{4}\left(ax+1\right)^{3}\left(ax-1\right)^{3}a}$	84
trager	$-\frac{\left(16a^{6}x^{6}+16a^{5}x^{5}-40a^{4}x^{4}-40a^{3}x^{3}+30a^{2}x^{2}+30ax-5\right)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^{4}\left(ax-1\right)^{3}\left(ax+1\right)^{3}}$	86

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/35*((a*x-1)/(a*x+1))^(1/2)*(16*a^6*x^6+16*a^5*x^5-40*a^4*x^4-40*a^3*x^3
+30*a^2*x^2+30*a*x-5)/(a^2*x^2-1)^3/c^4/a
```

3.597.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{(16\,a^6x^6+16\,a^5x^5-40\,a^4x^4-40\,a^3x^3+30\,a^2x^2+30\,ax-5)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4)}$$

input

```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output

```
-1/35*(16*a^6*x^6 + 16*a^5*x^5 - 40*a^4*x^4 - 40*a^3*x^3 + 30*a^2*x^2 + 30
*a*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c
^4*x^2 - a*c^4)
```

3.597.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}\,dx}{c^4}$$

input

```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

output

```
Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**8*x**8 - 4*a**6*x**6 + 6*a*
*4*x**4 - 4*a**2*x**2 + 1), x)/c**4
```

3.597.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx\\&=\frac{1}{2240}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+175\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-700\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{7\left(\frac{10\,(ax-1)}{ax+1}-\frac{75\,(ax-1)^2}{(ax+1)^2}-1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)\end{aligned}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/2240*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 42*((a*x - 1)/(a*x + 1))^(5/2)
+ 175*((a*x - 1)/(a*x + 1))^(3/2) - 700*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^
4) + 7*(10*(a*x - 1)/(a*x + 1) - 75*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^4*
((a*x - 1)/(a*x + 1))^(5/2)))
```

3.597.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(a^2cx^2-c)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^4, x)
```

3.597.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx=\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{64\,a\,c^4}-\frac{5\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{160\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{448\,a\,c^4}-\frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{a\,x+1}+\frac{1}{5}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^4,x)
```

output
```
(5*((a*x - 1)/(a*x + 1))^(3/2))/(64*a*c^4) - (5*((a*x - 1)/(a*x + 1))^(1/2
))/(16*a*c^4) - (3*((a*x - 1)/(a*x + 1))^(5/2))/(160*a*c^4) + ((a*x - 1)/(
a*x + 1))^(7/2)/(448*a*c^4) - ((15*(a*x - 1)^2)/(a*x + 1)^2 - (2*(a*x - 1)
)/(a*x + 1) + 1/5)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2))
```

3.598 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.598.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{4c^4(1-ax)^6}{3a}-\frac{12c^4(1-ax)^7}{7a}+\frac{3c^4(1-ax)^8}{4a}-\frac{c^4(1-ax)^9}{9a}$$

output
```
4/3*c^4*(-a*x+1)^6/a-12/7*c^4*(-a*x+1)^7/a+3/4*c^4*(-a*x+1)^8/a-1/9*c^4*(-
a*x+1)^9/a
```

3.598.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{c^4(-1+ax)^6\left(65+138ax+105a^2x^2+28a^3x^3\right)}{252a}$$

input
```
Integrate[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^4*(-1 + a*x)^6*(65 + 138*a*x + 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

3.598.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^4 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6717$$

$$-\int c^4e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4dx$$

$$\Big\downarrow \; 27$$

$$-c^4\int e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4dx$$

$$\Big\downarrow \; 6690$$

$$-c^4\int (1-ax)^5(ax+1)^3dx$$

$$\Big\downarrow \; 49$$

$$-c^4\int \left(-(1-ax)^8+6(1-ax)^7-12(1-ax)^6+8(1-ax)^5\right)dx$$

$$\Big\downarrow \; 2009$$

$$-c^4\left(\frac{(1-ax)^9}{9a}-\frac{3(1-ax)^8}{4a}+\frac{12(1-ax)^7}{7a}-\frac{4(1-ax)^6}{3a}\right)$$

input
```
Int[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^4*((-4*(1 - a*x)^6)/(3*a) + (12*(1 - a*x)^7)/(7*a) - (3*(1 - a*x)^8)/(
4*a) + (1 - a*x)^9/(9*a)))
```

3.598.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.598.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{c^4x\left(28a^8x^8-63a^7x^7-72a^6x^6+252a^5x^5-378a^3x^3+168a^2x^2+252ax-252\right)}{252}$
default	$c^4\left(\frac{1}{9}a^8x^9-\frac{1}{4}a^7x^8-\frac{2}{7}a^6x^7+a^5x^6-\frac{3}{2}a^3x^4+\frac{2}{3}a^2x^3+a\,x^2-x\right)$
norman	$a\,c^4x^2+a^5c^4x^6-c^4x+\frac{2}{3}a^2c^4x^3-\frac{3}{2}a^3c^4x^4-\frac{2}{7}a^6c^4x^7-\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
risch	$a\,c^4x^2+a^5c^4x^6-c^4x+\frac{2}{3}a^2c^4x^3-\frac{3}{2}a^3c^4x^4-\frac{2}{7}a^6c^4x^7-\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
parallelrisch	$a\,c^4x^2+a^5c^4x^6-c^4x+\frac{2}{3}a^2c^4x^3-\frac{3}{2}a^3c^4x^4-\frac{2}{7}a^6c^4x^7-\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
meijerg	$\frac{c^4\left(\frac{xa\left(280a^8x^8-315a^7x^7+360a^6x^6-420a^5x^5+504a^4x^4-630a^3x^3+840a^2x^2-1260ax+2520\right)}{2520}-\ln(ax+1)\right)}{a}-\frac{c^4\left(-\frac{ax\left(-315a^7x^7+36\right.}{}\right.}{}$

input
```
int((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/252*c^4*x*(28*a^8*x^8-63*a^7*x^7-72*a^6*x^6+252*a^5*x^5-378*a^3*x^3+168*
a^2*x^2+252*a*x-252)
```

3.598.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

input
```
integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

3.598.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}-\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}+a^5c^4x^6-\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}+ac^4x^2-c^4x$$

input
```
integrate((-a**2*c*x**2+c)**4*(a*x-1)/(a*x+1),x)
```

output
```
a**8*c**4*x**9/9 - a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 + a**5*c**4*x**6
- 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 + a*c**4*x**2 - c**4*x
```

3.598.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

input
```
integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

3.598.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

input
```
integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

3.598.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}-\frac{a^7\,c^4\,x^8}{4}-\frac{2\,a^6\,c^4\,x^7}{7}+a^5\,c^4\,x^6-\frac{3\,a^3\,c^4\,x^4}{2}+\frac{2\,a^2\,c^4\,x^3}{3}+a\,c^4\,x^2-c^4\,x$$

input
```
int(((c - a^2*c*x^2)^4*(a*x - 1))/(a*x + 1),x)
```

output
```
a*c^4*x^2 - c^4*x + (2*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + a^5*c^4*x^6 -
(2*a^6*c^4*x^7)/7 - (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

3.598. $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

3.599 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

3.599.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{4c^3(1-ax)^5}{5a}-\frac{2c^3(1-ax)^6}{3a}+\frac{c^3(1-ax)^7}{7a}$$

output
```
4/5*c^3*(-a*x+1)^5/a-2/3*c^3*(-a*x+1)^6/a+1/7*c^3*(-a*x+1)^7/a
```

3.599.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(-1+ax)^5\left(29+40ax+15a^2x^2\right)}{105a}$$

input
```
Integrate[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-1/105*(c^3*(-1 + a*x)^5*(29 + 40*a*x + 15*a^2*x^2))/a
```

3.599.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^3e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \text{6690}$$

$$-c^3\int (1-ax)^4(ax+1)^2dx$$

$$\Big\downarrow \text{49}$$

$$-c^3\int \left((1-ax)^6-4(1-ax)^5+4(1-ax)^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^3\left(-\frac{(1-ax)^7}{7a}+\frac{2(1-ax)^6}{3a}-\frac{4(1-ax)^5}{5a}\right)$$

input
```
Int[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^3*((-4*(1 - a*x)^5)/(5*a) + (2*(1 - a*x)^6)/(3*a) - (1 - a*x)^7/(7*a))
)
```

3.599.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.599.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result
gosper	$-\frac{c^{3} x \left(15 a^{6} x^{6}-35 a^{5} x^{5}-21 a^{4} x^{4}+105 a^{3} x^{3}-35 a^{2} x^{2}-105 a x +105\right)}{105}$
default	$c^{3} \left(-\frac{1}{7} a^{6} x^{7}+\frac{1}{3} a^{5} x^{6}+\frac{1}{5} a^{4} x^{5}-a^{3} x^{4}+\frac{1}{3} a^{2} x^{3}+a \,x^{2}-x \right)$
norman	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{5} a^{4} c^{3} x^{5}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
risch	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{5} a^{4} c^{3} x^{5}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
parallelrisch	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{5} a^{4} c^{3} x^{5}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}$
meijerg	$-\frac{c^{3} \left(\frac{a x \left(120 a^{6} x^{6}-140 a^{5} x^{5}+168 a^{4} x^{4}-210 a^{3} x^{3}+280 a^{2} x^{2}-420 a x +840\right)}{840}-\ln \left(a x +1\right)\right)}{a}+\frac{c^{3} \left(-\frac{a x \left(-70 a^{5} x^{5}+84 a^{4} x^{4}-105 a^{3} x^{3}+140$

input
```
int((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/105*c^3*x*(15*a^6*x^6-35*a^5*x^5-21*a^4*x^4+105*a^3*x^3-35*a^2*x^2-105*
a*x+105)
```

3.599.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

input
```
integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
-1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

3.599.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6c^3x^7}{7}+\frac{a^5c^3x^6}{3}+\frac{a^4c^3x^5}{5}-a^3c^3x^4+\frac{a^2c^3x^3}{3}+ac^3x^2-c^3x$$

input
```
integrate((-a**2*c*x**2+c)**3*(a*x-1)/(a*x+1),x)
```

output
```
-a**6*c**3*x**7/7 + a**5*c**3*x**6/3 + a**4*c**3*x**5/5 - a**3*c**3*x**4 +
 a**2*c**3*x**3/3 + a*c**3*x**2 - c**3*x
```

3.599.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

input
```
integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

3.599.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

input
```
integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

3.599.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6\,c^3\,x^7}{7}+\frac{a^5\,c^3\,x^6}{3}+\frac{a^4\,c^3\,x^5}{5}-a^3\,c^3\,x^4+\frac{a^2\,c^3\,x^3}{3}+a\,c^3\,x^2-c^3\,x$$

input
```
int(((c - a^2*c*x^2)^3*(a*x - 1))/(a*x + 1),x)
```

output
```
a*c^3*x^2 - c^3*x + (a^2*c^3*x^3)/3 - a^3*c^3*x^4 + (a^4*c^3*x^5)/5 + (a^5
*c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

3.599. $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

3.600 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

3.600.1 Optimal result

Integrand size = 22, antiderivative size = 37

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

output
```
1/2*c^2*(-a*x+1)^4/a-1/5*c^2*(-a*x+1)^5/a
```

3.600.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{10}c^2x\left(-10+10ax-5a^3x^3+2a^4x^4\right)$$

input
```
Integrate[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

3.600.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^2e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^2dx$$

$$\Big\downarrow \text{27}$$

$$-c^2\int e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^2dx$$

$$\Big\downarrow \text{6690}$$

$$-c^2\int (1-ax)^3(ax+1)dx$$

$$\Big\downarrow \text{49}$$

$$-c^2\int \left(2(1-ax)^3-(1-ax)^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^2\left(\frac{(1-ax)^5}{5a}-\frac{(1-ax)^4}{2a}\right)$$

input
```
Int[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^2*(-1/2*(1 - a*x)^4/a + (1 - a*x)^5/(5*a)))
```

3.600. $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

3.600.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.600.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{c^2 x\left(2 a^4 x^4-5 a^3 x^3+10 a x-10\right)}{10}$
default	$c^2\left(\frac{1}{5} a^4 x^5-\frac{1}{2} a^3 x^4+a\, x^2-x\right)$
norman	$a\, c^2 x^2-c^2 x-\frac{1}{2} a^3 c^2 x^4+\frac{1}{5} a^4 c^2 x^5$
risch	$a\, c^2 x^2-c^2 x-\frac{1}{2} a^3 c^2 x^4+\frac{1}{5} a^4 c^2 x^5$
parallelrisch	$a\, c^2 x^2-c^2 x-\frac{1}{2} a^3 c^2 x^4+\frac{1}{5} a^4 c^2 x^5$
meijerg	$\frac{c^2\left(\frac{a x\left(12 a^4 x^4-15 a^3 x^3+20 a^2 x^2-30 a x+60\right)}{60}-\ln(a x+1)\right)}{a}-\frac{c^2\left(-\frac{a x\left(-15 a^3 x^3+20 a^2 x^2-30 a x+60\right)}{60}+\ln(a x+1)\right)}{a}-$

input
```
int((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/10*c^2*x*(2*a^4*x^4-5*a^3*x^3+10*a*x-10)
```

3.600.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

input
```
integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

3.600.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}-\frac{a^3c^2x^4}{2}+ac^2x^2-c^2x$$

input
```
integrate((-a**2*c*x**2+c)**2*(a*x-1)/(a*x+1),x)
```

output
```
a**4*c**2*x**5/5 - a**3*c**2*x**4/2 + a*c**2*x**2 - c**2*x
```

3.600.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

input
```
integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

3.600.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

input
```
integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

3.600.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4\,c^2\,x^5}{5}-\frac{a^3\,c^2\,x^4}{2}+a\,c^2\,x^2-c^2\,x$$

input
```
int(((c - a^2*c*x^2)^2*(a*x - 1))/(a*x + 1),x)
```

output
```
a*c^2*x^2 - c^2*x - (a^3*c^2*x^4)/2 + (a^4*c^2*x^5)/5
```

3.601 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

3.601.1 Optimal result

Integrand size = 20, antiderivative size = 16

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c(1-ax)^3}{3a}$$

output
```
1/3*c*(-a*x+1)^3/a
```

3.601.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-c\left(x-ax^2+\frac{a^2x^3}{3}\right)$$

input
```
Integrate[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(x - a*x^2 + (a^2*x^3)/3))
```

3.601.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int ce^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)dx$$

$$\Big\downarrow \text{ 6690}$$

$$-c\int (1-ax)^2dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{c(1-ax)^3}{3a}$$

input
```
Int[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*(1 - a*x)^3)/(3*a)
```

3.601.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.601.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{c(ax-1)^3}{3a}$	14
gosper	$-\frac{cx(a^2x^2-3ax+3)}{3}$	18
norman	$ac\,x^2-cx-\frac{1}{3}a^2c\,x^3$	21
parallelrisch	$ac\,x^2-cx-\frac{1}{3}a^2c\,x^3$	21
risch	$-\frac{a^2c\,x^3}{3}+ac\,x^2-cx+\frac{c}{3a}$	27
meijerg	$-\frac{c\left(\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)\right)}{a}+\frac{c\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a}+\frac{c(ax-\ln(ax+1))}{a}-\frac{c\ln(ax+1)}{a}$	86

input
```
int((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(a*x-1)^3/a
```

3.601.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

input
```
integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
-1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

3.601.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}+acx^2-cx$$

input
```
integrate((-a**2*c*x**2+c)*(a*x-1)/(a*x+1),x)
```

output
```
-a**2*c*x**3/3 + a*c*x**2 - c*x
```

3.601.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

input
```
integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

3.601.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

input
```
integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

3.601.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c\,x\,\left(a^2\,x^2-3\,a\,x+3\right)}{3}$$

input
```
int(((c - a^2*c*x^2)*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c*x*(a^2*x^2 - 3*a*x + 3))/3
```

3.602 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

3.602.1 Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{ac(1+ax)}$$

output
```
1/a/c/(a*x+1)
```

3.602.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-2\coth^{-1}(ax)}}{2ac}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

output
```
-1/2*1/(a*c*E^(2*ArcCoth[a*x]))
```

3.602.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c(1-a^2x^2)}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{ 6690}$$

$$-\frac{\int \frac{1}{(ax+1)^2}dx}{c}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{ac(ax+1)}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

output
```
1/(a*c*(1 + a*x))
```

3.602.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.602.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
norman	$-\frac{x}{c(ax+1)}$	14
parallelrisch	$-\frac{x}{c(ax+1)}$	14
gosper	$\frac{1}{ac(ax+1)}$	15
default	$\frac{1}{ac(ax+1)}$	15
risch	$\frac{1}{ac(ax+1)}$	15

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-x/c/(a*x+1)
```

3.602.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a^2cx+ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/(a^2*c*x + a*c)
```

3.602.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a^2cx+ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c),x)
```

output
```
1/(a**2*c*x + a*c)
```

3.602.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a^2cx+ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/(a^2*c*x + a*c)
```

3.602.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{(ax+1)ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/((a*x + 1)*a*c)
```

3.602.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a\,(c+a\,c\,x)}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)*(a*x + 1)),x)
```

output
```
1/(a*(c + a*c*x))
```

3.603 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.603.1 Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{4ac^2(1+ax)^2} + \frac{1}{4ac^2(1+ax)} - \frac{\operatorname{arctanh}(ax)}{4ac^2}$$

output
```
1/4/a/c^2/(a*x+1)^2+1/4/a/c^2/(a*x+1)-1/4*arctanh(a*x)/a/c^2
```

3.603.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2+ax-(1+ax)^2\operatorname{arctanh}(ax)}{4a(c+acx)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

output
```
(2 + a*x - (1 + a*x)^2*ArcTanh[a*x])/(4*a*(c + a*c*x)^2)
```

3.603.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)(ax+1)^3}dx}{c^2}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{4(ax+1)^2}+\frac{1}{2(ax+1)^3}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}-\frac{1}{4a(ax+1)}-\frac{1}{4a(ax+1)^2}}{c^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

output
```
-((-1/4*1/(a*(1 + a*x)^2) - 1/(4*a*(1 + a*x)) + ArcTanh[a*x]/(4*a))/c^2)
```

3.603. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$

3.603.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.603.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{\frac{x}{4}+\frac{1}{2a}}{(ax+1)^{2} c^{2}}+\frac{\ln(-ax+1)}{8a\,c^{2}}-\frac{\ln(ax+1)}{8a\,c^{2}}$	51
default	$\frac{\frac{1}{4a(ax+1)^{2}}+\frac{1}{4a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{\ln(ax-1)}{8a}}{c^{2}}$	52
norman	$\frac{-\frac{3}{4ac}+\frac{a\,x^{2}}{2c}+\frac{a^{2}x^{3}}{4c}}{c(ax+1)^{2}(ax-1)}+\frac{\ln(ax-1)}{8a\,c^{2}}-\frac{\ln(ax+1)}{8a\,c^{2}}$	77
parallelrisch	$\frac{a^{2}\ln(ax-1)x^{2}-a^{2}\ln(ax+1)x^{2}-4a^{2}x^{2}+2a\ln(ax-1)x-2a\ln(ax+1)x-6ax+\ln(ax-1)-\ln(ax+1)}{8c^{2}(ax+1)^{2}a}$	90

input

```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output

```
(1/4*x+1/2/a)/(a*x+1)^2/c^2+1/8*ln(-a*x+1)/a/c^2-1/8*ln(a*x+1)/a/c^2
```

3.603. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.603.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = \frac{2\,ax-\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2+2\,ax+1\right)\log\left(ax-1\right)+4}{8\,\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

input

```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output

```
1/8*(2*a*x - (a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 + 2*a*x + 1)*lo
g(a*x - 1) + 4)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

3.603.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = \frac{ax+2}{4a^3c^2x^2+8a^2c^2x+4ac^2} + \frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

input

```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

output

```
(a*x + 2)/(4*a**3*c**2*x**2 + 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**2)
```

3.603.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = \frac{ax+2}{4\,\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)} - \frac{\log\left(ax+1\right)}{8\,ac^2} + \frac{\log\left(ax-1\right)}{8\,ac^2}$$

input

```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output

```
1/4*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^
2) + 1/8*log(a*x - 1)/(a*c^2)
```

3.603.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}+\frac{ax+2}{4\left(ax+1\right)^2ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x
+ 2)/((a*x + 1)^2*a*c^2)
```

3.603.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\frac{\frac{x}{4}+\frac{1}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2}-\frac{\mathrm{atanh}(a\,x)}{4\,a\,c^2}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^2*(a*x + 1)),x)
```

output
```
(x/4 + 1/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

3.604 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.604.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^3}$$

output
```
-1/16/a/c^3/(-a*x+1)+1/12/a/c^3/(a*x+1)^3+1/8/a/c^3/(a*x+1)^2+3/16/a/c^3/(
a*x+1)-1/4*arctanh(a*x)/a/c^3
```

3.604.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{-4+ax+6a^2x^2+3a^3x^3-3(-1+ax)(1+ax)^3\operatorname{arctanh}(ax)}{12a(-1+ax)(c+acx)^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

output
```
(-4 + a*x + 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)*(1 + a*x)^3*ArcTanh[a*x])
/(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

3.604.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^3(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^2(ax+1)^4}dx}{c^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{16(ax-1)^2}+\frac{3}{16(ax+1)^2}+\frac{1}{4(ax+1)^3}+\frac{1}{4(ax+1)^4}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}+\frac{1}{16a(1-ax)}-\frac{3}{16a(ax+1)}-\frac{1}{8a(ax+1)^2}-\frac{1}{12a(ax+1)^3}}{c^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

output
```
-((1/(16*a*(1 - a*x)) - 1/(12*a*(1 + a*x)^3) - 1/(8*a*(1 + a*x)^2) - 3/(16
*a*(1 + a*x)) + ArcTanh[a*x]/(4*a))/c^3)
```

3.604. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$

3.604.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.604.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{1}{12a(ax+1)^{3}}+\frac{1}{8a(ax+1)^{2}}+\frac{3}{16a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{1}{16a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^{3}}$
risch	$\frac{\frac{a^{2}x^{3}}{4}+\frac{a\,x^{2}}{2}+\frac{x}{12}-\frac{1}{3a}}{(ax+1)^{2}(a^{2}x^{2}-1)c^{3}}+\frac{\ln(-ax+1)}{8a\,c^{3}}-\frac{\ln(ax+1)}{8a\,c^{3}}$
norman	$\frac{-\frac{3x}{4c}+\frac{a\,x^{2}}{4c}+\frac{11a^{2}x^{3}}{12c}-\frac{a^{3}x^{4}}{12c}-\frac{a^{4}x^{5}}{3c}}{(ax+1)^{3}c^{2}(ax-1)^{2}}+\frac{\ln(ax-1)}{8a\,c^{3}}-\frac{\ln(ax+1)}{8a\,c^{3}}$
parallelrisch	$\frac{3\ln(ax-1)x^{4}a^{4}-3\ln(ax+1)x^{4}a^{4}-8a^{4}x^{4}+6a^{3}\ln(ax-1)x^{3}-6a^{3}\ln(ax+1)x^{3}-10a^{3}x^{3}+12a^{2}x^{2}-6a\ln(ax-1)x+6a\ln(ax+1)x}{24c^{3}(ax+1)^{2}(a^{2}x^{2}-1)a}$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/12/a/(a*x+1)^3+1/8/a/(a*x+1)^2+3/16/a/(a*x+1)-1/8*ln(a*x+1)/a+1/1
6/a/(a*x-1)+1/8/a*ln(a*x-1))
```

3.604.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$$
$$= \frac{6\,a^3x^3+12\,a^2x^2+2\,ax-3\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log(ax+1)+3\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log(ax}{24\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/24*(6*a^3*x^3 + 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)
*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) - 8)/(a^5
*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

3.604.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{-3a^3x^3-6a^2x^2-ax+4}{12a^5c^3x^4+24a^4c^3x^3-24a^2c^3x-12ac^3} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

output
```
-(-3*a**3*x**3 - 6*a**2*x**2 - a*x + 4)/(12*a**5*c**3*x**4 + 24*a**4*c**3*
x**3 - 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a
*c**3)
```

3.604.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)} - \frac{\log(ax+1)}{8\,ac^3} + \frac{\log(ax-1)}{8\,ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/12*(3*a^3*x^3 + 6*a^2*x^2 + a*x - 4)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^
2*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

3.604.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\log\left(|ax+1|\right)}{8\,ac^3} + \frac{\log\left(|ax-1|\right)}{8\,ac^3} + \frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(ax+1)^3(ax-1)ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a
^3*x^3 + 6*a^2*x^2 + a*x - 4)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

3.604.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\frac{x}{12}+\frac{a\,x^2}{2}-\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3} - \frac{\operatorname{atanh}(a\,x)}{4\,a\,c^3}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^3*(a*x + 1)),x)
```

output
```
- (x/12 + (a*x^2)/2 - 1/(3*a) + (a^2*x^3)/4)/(c^3 - 2*a^3*c^3*x^3 - a^4*c^
3*x^4 + 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

3.605 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

3.605.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+ax)^2}+\frac{5}{32ac^4(1+ax)}-\frac{15\text{arctanh}(ax)}{64ac^4}$$

output
```
-1/64/a/c^4/(-a*x+1)^2-5/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4+1/16/a/c^4
/(a*x+1)^3+3/32/a/c^4/(a*x+1)^2+5/32/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/c^
4
```

3.605.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{16-17ax-50a^2x^2-10a^3x^3+30a^4x^4+15a^5x^5-15(-1+ax)^2(1+ax)^4\text{arctanh}(ax)}{64a(-1+ax)^2(c+acx)^4}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

output
```
(16 - 17*a*x - 50*a^2*x^2 - 10*a^3*x^3 + 30*a^4*x^4 + 15*a^5*x^5 - 15*(-1
+ a*x)^2*(1 + a*x)^4*ArcTanh[a*x])/(64*a*(-1 + a*x)^2*(c + a*c*x)^4)
```

3.605.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^4\left(1-a^2x^2\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^3(ax+1)^5}dx}{c^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{5}{64(ax-1)^2}+\frac{5}{32(ax+1)^2}-\frac{1}{32(ax-1)^3}+\frac{3}{16(ax+1)^3}+\frac{3}{16(ax+1)^4}+\frac{1}{8(ax+1)^5}-\frac{15}{64(a^2x^2-1)}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{15\text{arctanh}(ax)}{64a}+\frac{5}{64a(1-ax)}-\frac{5}{32a(ax+1)}+\frac{1}{64a(1-ax)^2}-\frac{3}{32a(ax+1)^2}-\frac{1}{16a(ax+1)^3}-\frac{1}{32a(ax+1)^4}}{c^4}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

output
```
-((1/(64*a*(1 - a*x)^2) + 5/(64*a*(1 - a*x)) - 1/(32*a*(1 + a*x)^4) - 1/(1
6*a*(1 + a*x)^3) - 3/(32*a*(1 + a*x)^2) - 5/(32*a*(1 + a*x)) + (15*ArcTanh
[a*x])/(64*a))/c^4)
```

3.605.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.605.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\frac{15 a^{4} x^{5}}{64}+\frac{15 a^{3} x^{4}}{32}-\frac{5 a^{2} x^{3}}{32}-\frac{25 a \,x^{2}}{32}-\frac{17 x}{64}+\frac{1}{4 a}}{\left(a x +1\right)^{2} \left(a^{2} x^{2}-1\right)^{2} c^{4}}-\frac{15 \ln \left(a x +1\right)}{128 a \,c^{4}}+\frac{15 \ln \left(-a x +1\right)}{128 a \,c^{4}}$
default	$\frac{\frac{1}{32 a \left(a x +1\right)^{4}}+\frac{1}{16 a \left(a x +1\right)^{3}}+\frac{3}{32 a \left(a x +1\right)^{2}}+\frac{5}{32 a \left(a x +1\right)}-\frac{15 \ln \left(a x +1\right)}{128 a}-\frac{1}{64 \left(a x -1\right)^{2} a}+\frac{5}{64 a \left(a x -1\right)}+\frac{15 \ln \left(a x -1\right)}{128 a}}{c^{4}}$
norman	$\frac{\frac{49 x}{64 c}-\frac{15 a \,x^{2}}{64 c}-\frac{11 a^{2} x^{3}}{8 c}+\frac{a^{3} x^{4}}{8 c}+\frac{63 a^{4} x^{5}}{64 c}-\frac{a^{5} x^{6}}{64 c}-\frac{a^{6} x^{7}}{4 c}}{\left(a x +1\right)^{4} c^{3} \left(a x -1\right)^{3}}+\frac{15 \ln \left(a x -1\right)}{128 a \,c^{4}}-\frac{15 \ln \left(a x +1\right)}{128 a \,c^{4}}$
parallelrisch	$\frac{-30 a \ln \left(a x +1\right) x +15 a^{2} \ln \left(a x +1\right) x^{2}-34 a^{5} x^{5}+108 a^{3} x^{3}-30 \ln \left(a x +1\right) x^{5} a^{5}-15 \ln \left(a x +1\right) x^{6} a^{6}+15 \ln \left(a x +1\right) x^{4} a^{4}+15 \ln \left(a x -1\right)}{}$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/64*a^4*x^5+15/32*a^3*x^4-5/32*a^2*x^3-25/32*a*x^2-17/64*x+1/4/a)/(a*x+
1)^2/(a^2*x^2-1)^2/c^4-15/128*ln(a*x+1)/a/c^4+15/128*ln(-a*x+1)/a/c^4
```

3.605.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(103) = 206$.

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.82

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{30\,a^5x^5 + 60\,a^4x^4 - 20\,a^3x^3 - 100\,a^2x^2 - 34\,ax - 15\left(a^6x^6 + 2\,a^5x^5 - a^4x^4 - 4\,a^3x^3 - a^2x^2 + 2\,ax + 1\right)\mathrm{lo}}{128\left(a^7c^4x^6 + 2\,a^6c^4x^5 - a^5c^4x^4 - 4\,a^4c^4x^3 -\right.}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/128*(30*a^5*x^5 + 60*a^4*x^4 - 20*a^3*x^3 - 100*a^2*x^2 - 34*a*x - 15*(a
^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x +
1) + 15*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*
log(a*x - 1) + 32)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*
x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

3.605.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{15a^5x^5 + 30a^4x^4 - 10a^3x^3 - 50a^2x^2 - 17ax + 16}{64a^7c^4x^6 + 128a^6c^4x^5 - 64a^5c^4x^4 - 256a^4c^4x^3 - 64a^3c^4x^2 + 128a^2c^4x + 64ac^4}$$
$$+ \frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128} - \frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

output
```
(15*a**5*x**5 + 30*a**4*x**4 - 10*a**3*x**3 - 50*a**2*x**2 - 17*a*x + 16)/
(64*a**7*c**4*x**6 + 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 - 256*a**4*c**
4*x**3 - 64*a**3*c**4*x**2 + 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/
a)/128 - 15*log(x + 1/a)/128)/(a*c**4)
```

3.605.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)} - \frac{15\,\log(ax+1)}{128\,ac^4} + \frac{15\,\log(ax-1)}{128\,ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/64*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/(a^
7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*
a^2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*
c^4)
```

3.605.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,\log(|ax+1|)}{128\,ac^4} + \frac{15\,\log(|ax-1|)}{128\,ac^4} + \frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(ax+1)^4(ax-1)^2ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
-15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/6
4*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/((a*x
+ 1)^4*(a*x - 1)^2*a*c^4)
```

3.605.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{\frac{17\,x}{64}+\frac{25\,a\,x^2}{32}-\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}-\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4} - \frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

input

```
int((a*x - 1)/((c - a^2*c*x^2)^4*(a*x + 1)),x)
```

output

```
- ((17*x)/64 + (25*a*x^2)/32 - 1/(4*a) + (5*a^2*x^3)/32 - (15*a^3*x^4)/32
- (15*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^4*x^4 + 2*a^
5*c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

3.606 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

3.606.1 Optimal result

Integrand size = 22, antiderivative size = 393

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x$$
$$+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2$$
$$+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$
$$+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}(1+\frac{1}{ax}$$

output
```
11/48*a^4*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5-11/48*a^5*c^4*(1-1/a/x)^
(5/2)*(1+1/a/x)^(7/2)*x^6+11/56*a^6*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)*x^
7-11/72*a^7*c^4*(1-1/a/x)^(9/2)*(1+1/a/x)^(7/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^
(11/2)*(1+1/a/x)^(7/2)*x^9+55/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1
/2))/a+55/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+11/192*a^2*c^4*(1+
1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-11/64*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/
x)^(1/2)+55/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.606.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-3712-4599ax+10240a^2x^2-3066a^3x^3-8448a^4x^4+7224a^5x^5+1024a^6x^6-3024a}{8064a}$$

input
```
Integrate[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 - 4599*a*x + 10240*a^2*x^2 - 3066*a
^3*x^3 - 8448*a^4*x^4 + 7224*a^5*x^5 + 1024*a^6*x^6 - 3024*a^7*x^7 + 896*a
^8*x^8) + 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

3.606.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^4 e^{-3\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6745}$$
$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$
$$\Big\downarrow \text{27}$$
$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$
$$\Big\downarrow \text{2005}$$
$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$
$$\Big\downarrow \text{6745}$$
$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$
$$\Big\downarrow \text{27}$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{ 6745}$$

3.606. $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow 27$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow 2005$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow 6745$$

$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow 27$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow 2005$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow 6745$$

$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow 27$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow 2005$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow 6745$$

$$a^8c^4\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow 27$$

$$c^4\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

3.606. $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

$$\Big\downarrow \text{6745}$$

$$a^8c^4 \int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$c^4 \int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^4x^8dx$$

$$\Big\downarrow \text{2005}$$

$$c^4 \int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^4dx$$

input
```
Int[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```

output
```
$Aborted
```

3.606.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.606. $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

3.606.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\left(896a^{8}x^{8}-3024a^{7}x^{7}+1024a^{6}x^{6}+7224a^{5}x^{5}-8448a^{4}x^{4}-3066a^{3}x^{3}+10240a^{2}x^{2}-4599ax-3712\right)(ax+1)c^{4}\sqrt{\frac{ax-1}{ax+1}}}{8064a}+\frac{55\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\right)}{}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^{2}c^{4}\left(896\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{6}x^{6}-3024\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{5}x^{5}+1920\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{4}x^{4}+4200\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{3}x^{3}\right.}{8}$

input
```
int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8064*(896*a^8*x^8-3024*a^7*x^7+1024*a^6*x^6+7224*a^5*x^5-8448*a^4*x^4-30
66*a^3*x^3+10240*a^2*x^2-4599*a*x-3712)*(a*x+1)/a*c^4*((a*x-1)/(a*x+1))^(1
/2)+55/128*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4*((a*x-1
)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.606.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.43

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{4}\,dx$$
$$=\frac{3465\,c^{4}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^{4}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(896\,a^{9}c^{4}x^{9}-2128\,a^{8}c^{4}x^{8}-2000\,a^{7}c^{4}x^{7}+8248\,a\right.}{8064\,a}$$

input
```
integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1) + (896*a^9*c^4*x^9 - 2128*a^8*c^4*x^8 - 2000*a^7*c
^4*x^7 + 8248*a^6*c^4*x^6 - 1224*a^5*c^4*x^5 - 11514*a^4*c^4*x^4 + 7174*a^
3*c^4*x^3 + 5641*a^2*c^4*x^2 - 8311*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*
x + 1)))/a
```

3.606.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right.$$
$$+\int\frac{4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx$$
$$+\int\left(-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx$$
$$+\int\left(-\frac{6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx$$
$$+\int\frac{6a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx$$
$$+\int\left(-\frac{4a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx$$
$$+\int\left(-\frac{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx$$
$$\left.+\int\frac{a^9x^9\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral
(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(4*a**2*x**
2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**3*x**3*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-6*a**4*x**4*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(6*a**5*x**5*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(4*a**6*x**6*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**7*x**7*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**8*x**8*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**9*x**9*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1), x))
```

3.606.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+\right.}{\frac{9\,(ax-1)a^2}{ax+1}-}\right.$$

input
```
integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqr
t((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2)
 - 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 334602*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 360448*c^4*((a*x - 1
)/(a*x + 1))^(9/2) + 255222*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*(
(a*x - 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*
c^4*sqrt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2
*a^2/(a*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a
*x + 1)^4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)
^6 + 36*(a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x
 - 1)^9*a^2/(a*x + 1)^9 - a^2))*a
```

3.606.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=-\frac{55\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{128\,|a|}$$
$$-\frac{1}{8064}\sqrt{a^2x^2-1}\left(\frac{3712\,c^4\operatorname{sgn}(ax+1)}{a}+\left(4599\,c^4\operatorname{sgn}(ax+1)-2\left(5120\,ac^4\operatorname{sgn}(ax+1)-\left(1533\,a^2c^4\operatorname{sgn}\right.\right.\right.\right.$$

input
```
integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
-55/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) -
1/8064*sqrt(a^2*x^2 - 1)*(3712*c^4*sgn(a*x + 1)/a + (4599*c^4*sgn(a*x + 1)
 - 2*(5120*a*c^4*sgn(a*x + 1) - (1533*a^2*c^4*sgn(a*x + 1) + 4*(1056*a^3*c
^4*sgn(a*x + 1) - (903*a^4*c^4*sgn(a*x + 1) + 2*(64*a^5*c^4*sgn(a*x + 1) +
 7*(8*a^7*c^4*x*sgn(a*x + 1) - 27*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*
x)
```

3.606.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$

$$=\frac{\frac{715\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96}-\frac{55\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{913\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32}+\frac{14179\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224}-\frac{5632\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{63}+\frac{18589\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{224}+913}{a-\frac{9\,a\,(a\,x-1)}{a\,x+1}+\frac{36\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{84\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{126\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{126\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{84\,a\,(a\,x-1)^6}{(a\,x+1)^6}-36\,a\,(a}$$

$$+\frac{55\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

input
```
int((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((715*c^4*((a*x - 1)/(a*x + 1))^(3/2))/96 - (55*c^4*((a*x - 1)/(a*x + 1))^
(1/2))/64 - (913*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (14179*c^4*((a*x -
1)/(a*x + 1))^(7/2))/224 - (5632*c^4*((a*x - 1)/(a*x + 1))^(9/2))/63 + (18
589*c^4*((a*x - 1)/(a*x + 1))^(11/2))/224 + (913*c^4*((a*x - 1)/(a*x + 1))
^(13/2))/32 - (715*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 + (55*c^4*((a*x -
1)/(a*x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)
^2)/(a*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*
x + 1)^4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^
6 - (36*a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a
*x - 1)^9)/(a*x + 1)^9) + (55*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*
a)
```

3.607 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.607.1 Optimal result

Integrand size = 22, antiderivative size = 313

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x$$
$$+\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3$$
$$+\frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4-\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5+\frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)$$

output
```
3/8*a^3*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x^4-3/10*a^4*c^3*(1-1/a/x)^(5/
2)*(1+1/a/x)^(5/2)*x^5+3/14*a^5*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^6-1/
7*a^6*c^3*(1-1/a/x)^(9/2)*(1+1/a/x)^(5/2)*x^7+9/16*c^3*arctanh((1-1/a/x)^(
1/2)*(1+1/a/x)^(1/2))/a+3/16*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-3/8
*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+9/16*c^3*x*(1-1/a/x)^(1/2)*(1
+1/a/x)^(1/2)
```

3.607.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx =$$

$$-\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(368+245ax-656a^2x^2+350a^3x^3+208a^4x^4-280a^5x^5+80a^6x^6)-315\log\left(\left(1+\right.\right.\right.}{560a}$$

input
```
Integrate[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

output
```
-1/560*(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 + 245*a*x - 656*a^2*x^2 + 350*
a^3*x^3 + 208*a^4*x^4 - 280*a^5*x^5 + 80*a^6*x^6) - 315*Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x]))/a
```

3.607.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6 dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3 \int \frac{e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6} dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6 dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3 \int \frac{e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6} dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6 dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^6c^3 \int \frac{e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6} dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6 dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \text{ 6745}$$

3.607. $\int e^{-3\coth^{-1}(ax)} \left(c-a^2cx^2\right)^3 \, dx$

$$-a^6c^3\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

3.607. $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3 \, dx$

$$\Big\downarrow \color{blue}{2005}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

$$\Big\downarrow \color{blue}{6745}$$

$$-a^6c^3 \int \frac{e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6} dx$$

$$\Big\downarrow \color{blue}{27}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6 dx$$

$$\Big\downarrow \color{blue}{2005}$$

$$-c^3 \int e^{-3\coth^{-1}(ax)} \left(a^2x^2-1\right)^3 dx$$

input

```
Int[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

output

```
$Aborted
```

3.607.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005

```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.607. $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.607.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{(80a^6x^6-280a^5x^5+208a^4x^4+350a^3x^3-656a^2x^2+245ax+368)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{560a}+\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)}}{16\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(80(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-280(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+288(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+70(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,ax+192\right.}{560a(ax-1)\sqrt{(ax-1)(ax+1)}\sqrt{}}$

input
```
int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/560*(80*a^6*x^6-280*a^5*x^5+208*a^4*x^4+350*a^3*x^3-656*a^2*x^2+245*a*x
+368)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1/2)+9/16*ln(a^2*x/(a^2)^(1/2)+(a^2
*x^2-1)^(1/2))/(a^2)^(1/2)*c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(
1/2)/(a*x-1)
```

3.607.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$$

$$=\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(80\,a^7c^3x^7-200\,a^6c^3x^6-72\,a^5c^3x^5+558\,a^4c^3x^4-}{560\,a}$$

input
```
integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) - (80*a^7*c^3*x^7 - 200*a^6*c^3*x^6 - 72*a^5*c^3*x^5
+ 558*a^4*c^3*x^4 - 306*a^3*c^3*x^3 - 411*a^2*c^3*x^2 + 613*a*c^3*x + 368*
c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.607.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
-c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral
(-a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(3*a**3*x**3
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(3*a**4*x**4*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**5*x**5*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**6*x**6*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**7*x**7*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.607.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-8393\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+35}\right.$$

input
```
integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 2
100*c^3*((a*x - 1)/(a*x + 1))^(11/2) - 8393*c^3*((a*x - 1)/(a*x + 1))^(9/2
) + 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 5943*c^3*((a*x - 1)/(a*x + 1))^
(5/2) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x
 + 1)))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(
a*x - 1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)
^5*a^2/(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x
+ 1)^7 - a^2))*a
```

3.607.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{9\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{16\,|a|} - \frac{1}{560}\sqrt{a^2x^2-1}\left(\frac{368\,c^3\operatorname{sgn}(ax+1)}{a}+\left(245\,c^3\operatorname{sgn}(ax+1)-2\left(328\,ac^3\operatorname{sgn}(ax+1)-\left(175\,a^2c^3\operatorname{sgn}(ax+\right.\right.\right.\right.$$

input
```
integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
-9/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/
560*sqrt(a^2*x^2 - 1)*(368*c^3*sgn(a*x + 1)/a + (245*c^3*sgn(a*x + 1) - 2*
(328*a*c^3*sgn(a*x + 1) - (175*a^2*c^3*sgn(a*x + 1) + 4*(26*a^3*c^3*sgn(a*
x + 1) + 5*(2*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)
*x)
```

3.607.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{9\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$

$$-\frac{\frac{9\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{15\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2}+\frac{849\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}-\frac{1152\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{35}+\frac{1199\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{40}+\frac{15\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{2}-9\,c^3}{a-\frac{7\,a\,(a\,x-1)}{a\,x+1}+\frac{21\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{35\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{35\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{21\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{7\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{a\,(a\,x-1)^7}{(a\,x+1)^7}}$$

input
```
int((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(9*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - ((9*c^3*((a*x - 1)/(a*x
 + 1))^(1/2))/8 - (15*c^3*((a*x - 1)/(a*x + 1))^(3/2))/2 + (849*c^3*((a*x
- 1)/(a*x + 1))^(5/2))/40 - (1152*c^3*((a*x - 1)/(a*x + 1))^(7/2))/35 + (1
199*c^3*((a*x - 1)/(a*x + 1))^(9/2))/40 + (15*c^3*((a*x - 1)/(a*x + 1))^(1
1/2))/2 - (9*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (7*a*(a*x - 1))/(a*
x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)^3)/(a*x + 1)^3 +
 (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x + 1)^5 + (7*a*(a
*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7)
```

3.608 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

3.608.1 Optimal result

Integrand size = 22, antiderivative size = 233

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x$$
$$-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3$$
$$-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5+\frac{7c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{a}}\right.}{8a}$$

output
```
7/12*a^2*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x^3-7/20*a^3*c^2*(1-1/a/x)^(5
/2)*(1+1/a/x)^(3/2)*x^4+1/5*a^4*c^2*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^5+7/
8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/8*a*c^2*(1+1/a/x)^(3/2)
*x^2*(1-1/a/x)^(1/2)+7/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.608.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x\left(-136-15ax+112a^2x^2-90a^3x^3+24a^4x^4\right)+105\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

input
```
Integrate[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3
+ 24*a^4*x^4) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

3.608.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6745$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \; 27$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \; 2005$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \; 6745$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \; 27$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \; 2005$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \; 6745$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \; 27$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

3.608. $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow 27$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow 2005$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow 6745$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow 27$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow 2005$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow 6745$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow 27$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow 2005$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow 6745$$

$$a^4c^2\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow 27$$

$$c^2\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

3.608. $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$\Big\downarrow \color{blue}{2005}$$

$$c^2 \int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)^2 dx$$

input
```
Int[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

output
```
$Aborted
```

3.608.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.608.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.55

method	result
risch	$\frac{\left(24 a^{4} x^{4}-90 a^{3} x^{3}+112 a^{2} x^{2}-15 a x -136\right) \left(a x +1\right) c^{2} \sqrt{\frac{a x -1}{a x +1}}}{120 a}+\frac{7 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{2} \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}}{8 \sqrt{a^{2}}\, \left(a x -1\right)}$
default	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(a x +1\right)^{2} c^{2} \left(24 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a^{2} x^{2}-90 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}\, a x +16 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}-105 \sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}\, a x +120 \left(\left(a x -1\right) \left(a x +\right.\right.\right.}{120 a \left(a x -1\right) \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$

input
```
int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*a^4*x^4-90*a^3*x^3+112*a^2*x^2-15*a*x-136)*(a*x+1)/a*c^2*((a*x-1
)/(a*x+1))^(1/2)+7/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c
^2*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.608.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.54

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(24\,a^5c^2x^5-66\,a^4c^2x^4+22\,a^3c^2x^3+97\,a^2c^2x^2-15\right.}{120\,a}$$

input
```
integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (24*a^5*c^2*x^5 - 66*a^4*c^2*x^4 + 22*a^3*c^2*x^3 +
 97*a^2*c^2*x^2 - 151*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

3.608.6 Sympy [F]

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\Bigg(&\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\\ &+\int\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\\ &+\int\left(-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx\\ &+\int\left(-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx\\ &+\int\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\Bigg)\end{aligned}$$

input
```
integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral
(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(2*a**2*x**
2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2*a**3*x**3*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**4*x**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x**5*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.608.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=\frac{1}{120}a\Bigg(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\,\Big(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^2}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)}{(ax+1)}}$$

input
```
integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) +
790*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^2*((a*x - 1)/(a*x + 1))^(5/2)
+ 490*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))
/(5*(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1
)^3*a^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x
 + 1)^5 - a^2))
```

3.608.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=-\frac{7\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{8\,|a|}$$
$$-\frac{1}{120}\sqrt{a^2x^2-1}\Bigg(\big(15\,c^2\operatorname{sgn}(ax+1)-2\,\big(56\,ac^2\operatorname{sgn}(ax+1)+3\,\big(4\,a^3c^2x\operatorname{sgn}(ax+1)-15\,a^2c^2\operatorname{sgn}(ax+$$

input
```
integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
-7/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/1
20*sqrt(a^2*x^2 - 1)*((15*c^2*sgn(a*x + 1) - 2*(56*a*c^2*sgn(a*x + 1) + 3*
(4*a^3*c^2*x*sgn(a*x + 1) - 15*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 136*c^2*sgn
(a*x + 1)/a)
```

3.608.9 Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{\frac{49\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{79\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}+\frac{7\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}} + \frac{7\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((49*c^2*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^2*((a*x - 1)/(a*x + 1))^(1/
2))/4 - (224*c^2*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^2*((a*x - 1)/(a*x
 + 1))^(7/2))/6 + (7*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x -
1))/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x +
 1)^3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*
c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.609 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

3.609.1 Optimal result

Integrand size = 20, antiderivative size = 145

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x^3+\frac{5c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{2a}$$

output
```
5/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+5/6*a*c*(1-1/a/x)^(3/2)*x
^2*(1+1/a/x)^(1/2)-1/3*a^2*c*(1-1/a/x)^(5/2)*x^3*(1+1/a/x)^(1/2)-5/2*c*x*(
1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

3.609.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-22+9ax-2a^2x^2)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input
```
Integrate[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-22 + 9*a*x - 2*a^2*x^2) + 15*Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

3.609.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right) e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{ 2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{ 6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

3.609. $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

3.609. $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

$$-c\int e^{-3\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

input
```
Int[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
$Aborted
```

3.609.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.609.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(2a^2x^2-9ax+22)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{6a}+\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-24\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}+24a\ln(\right.}{6(ax-1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$

input
```
int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*a^2*x^2-9*a*x+22)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)+5/2*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)/(a*x+1))^(1/2)*((a*
x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.609.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = \frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3cx^3-7\,a^2cx^2+13\,acx+22\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) - (2*a^3*c*x^3 - 7*a^2*c*x^2 + 13*a*c*x + 22*c)*sqrt((a*x - 1
)/(a*x + 1)))/a
```

3.609.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
-c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a
*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**2*x**2*s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.609.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$$
$$=\frac{1}{6}\,a\left(\frac{2\left(33\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
1/6*a*(2*(33*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3
/2) + 15*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x
- 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) + 15*c*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2)
```

3.609.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$$
$$=-\frac{5\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}$$
$$-\frac{1}{6}\sqrt{a^2x^2-1}\left((2\,acx\operatorname{sgn}(ax+1)-9\,c\operatorname{sgn}(ax+1))x+\frac{22\,c\operatorname{sgn}(ax+1)}{a}\right)$$

input
```
integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
-5/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*s
qrt(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 9*c*sgn(a*x + 1))*x + 22*c*sgn(a
*x + 1)/a)
```

3.609.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{5\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{5\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+11\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}$$

input
```
int((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(5*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (5*c*((a*x - 1)/(a*x + 1))^(1
/2) - (40*c*((a*x - 1)/(a*x + 1))^(3/2))/3 + 11*c*((a*x - 1)/(a*x + 1))^(5
/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(
a*x - 1)^3)/(a*x + 1)^3)
```

3.610 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

3.610.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

output
```
-1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

3.610.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

output
```
-1/3*1/(a*c*E^(3*ArcCoth[a*x]))
```

3.610.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$$

$$\Big\downarrow \ 6737$$

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

output
```
-1/3*1/(a*c*E^(3*ArcCoth[a*x]))
```

3.610.3.1 Defintions of rubi rules used

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

3.610.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

method	result	size
gosper	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3ac}$	24
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3ac}$	24
trager	$-\frac{(ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3ac(ax+1)}$	38

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

---

output
```
-1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

3.610.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2cx+ac)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/3*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x + a*c)
```

3.610.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

output
```
-(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x**2 - a*x
 - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a*
*2*x**2 - a*x - 1), x))/c
```

3.610.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3\,ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/3*((a*x - 1)/(a*x + 1))^(3/2)/(a*c)
```

3.610.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^3ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x + 1)
^3*a*c)
```

3.610.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3\,a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2),x)
```

output
```
-((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c)
```

3.611 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.611.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{2e^{-3\coth^{-1}(ax)}}{15ac^2}-\frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2(1-a^2x^2)}$$

output
```
2/15/a/c^2*((a*x-1)/(a*x+1))^(3/2)+1/5*(-2*a*x-3)/a/c^2*((a*x-1)/(a*x+1))^
(3/2)/(-a^2*x^2+1)
```

3.611.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(7+6ax+2a^2x^2)}{15c^2(1+ax)^3}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(7 + 6*a*x + 2*a^2*x^2))/(15*c^2*(1 + a*x)^3)
```

3.611.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6739, 27, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \ 6739$$

$$-\frac{2\int \frac{e^{-3\coth^{-1}(ax)}}{c(1-a^2x^2)}dx}{5c}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}$$

$$\Big\downarrow \ 27$$

$$-\frac{2\int \frac{e^{-3\coth^{-1}(ax)}}{1-a^2x^2}dx}{5c^2}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}$$

$$\Big\downarrow \ 6737$$

$$\frac{2e^{-3\coth^{-1}(ax)}}{15ac^2}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

output
```
2/(15*a*c^2*E^(3*ArcCoth[a*x])) - (3 + 2*a*x)/(5*a*c^2*E^(3*ArcCoth[a*x])*
(1 - a^2*x^2))
```

3.611.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.611.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result	size
trager	$\frac{(2a^2x^2+6ax+7)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^2(ax+1)^2}$	47
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^2x^2+6ax+7)}{15(a^2x^2-1)a\,c^2}$	49
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^2x^2+6ax+7)}{15(ax-1)c^2a(ax+1)}$	52

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15/a/c^2*(2*a^2*x^2+6*a*x+7)/(a*x+1)^2*(-(-a*x+1)/(a*x+1))^(1/2)
```

3.611.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{(2\,a^2x^2+6\,ax+7)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/15*(2*a^2*x^2 + 6*a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 + 2*a^
2*c^2*x + a*c^2)
```

3.611.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

output
```
(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a*
*3*x**3 - 2*a**2*x**2 + a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1),
 x))/c**2
```

3.611.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 15\sqrt{\frac{ax-1}{ax+1}}}{60\,ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/60*(3*((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 15*
sqrt((a*x - 1)/(a*x + 1)))/(a*c^2)
```

3.611.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2 + 5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x + 1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^5ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 5*(a + sqrt(a^2 - 1/x^2))*x + 1)
/(((a + sqrt(a^2 - 1/x^2))*x + 1)^5*a*c^2)
```

3.611.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\frac{15\,\sqrt{\frac{a\,x-1}{a\,x+1}}-10\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{60\,a\,c^2}$$

input

```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^2,x)
```

output

```
(15*((a*x - 1)/(a*x + 1))^(1/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x
 - 1)/(a*x + 1))^(5/2))/(60*a*c^2)
```

3.612 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.612.1 Optimal result

Integrand size $= 22$, antiderivative size $= 91$

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{8e^{-3\coth^{-1}(ax)}}{35ac^3} + \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3(1-a^2x^2)^2} - \frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3(1-a^2x^2)}$$

output
```
8/35/a/c^3*((a*x-1)/(a*x+1))^(3/2)+1/7*(4*a*x+3)/a/c^3*((a*x-1)/(a*x+1))^(
3/2)/(-a^2*x^2+1)^2-12/35*(2*a*x+3)/a/c^3*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^
2+1)
```

3.612.2 Mathematica [A] (verified)

Time $= 0.41$ (sec) , antiderivative size $= 66$, normalized size of antiderivative $= 0.73$

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-13-4ax+20a^2x^2+24a^3x^3+8a^4x^4)}{35c^3(-1+ax)(1+ax)^4}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-13 - 4*a*x + 20*a^2*x^2 + 24*a^3*x^3 + 8*a^4*x^
4))/(35*c^3*(-1 + a*x)*(1 + a*x)^4)
```

3.612.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6739, 27, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{12\int \frac{e^{-3\coth^{-1}(ax)}}{c^2\left(1-a^2x^2\right)^2}dx}{7c}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{12\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-a^2x^2\right)^2}dx}{7c^3}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{6739}$$

$$\frac{12\left(-\frac{2}{5}\int \frac{e^{-3\coth^{-1}(ax)}}{1-a^2x^2}dx-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5a\left(1-a^2x^2\right)}\right)}{7c^3}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{6737}$$

$$\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12\left(\frac{2e^{-3\coth^{-1}(ax)}}{15a}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5a\left(1-a^2x^2\right)}\right)}{7c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

output
```
(3 + 4*a*x)/(7*a*c^3*E^(3*ArcCoth[a*x])*(1 - a^2*x^2)^2) + (12*(2/(15*a*E^
(3*ArcCoth[a*x])) - (3 + 2*a*x)/(5*a*E^(3*ArcCoth[a*x])*(1 - a^2*x^2))))/(
7*c^3)
```

3.612.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.612.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \left(8a^4x^4+24a^3x^3+20a^2x^2-4ax-13\right)}{35\left(a^2x^2-1\right)^2 c^3 a}$	65
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \left(8a^4x^4+24a^3x^3+20a^2x^2-4ax-13\right)}{35\left(ax-1\right)^2 c^3 a \left(ax+1\right)^2}$	68
trager	$\frac{\left(8a^4x^4+24a^3x^3+20a^2x^2-4ax-13\right)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^3\left(ax-1\right)\left(ax+1\right)^3}$	70

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/35*((a*x-1)/(a*x+1))^(3/2)*(8*a^4*x^4+24*a^3*x^3+20*a^2*x^2-4*a*x-13)/(a
^2*x^2-1)^2/c^3/a
```

3.612.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{(8\,a^4x^4+24\,a^3x^3+20\,a^2x^2-4\,ax-13)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/35*(8*a^4*x^4 + 24*a^3*x^3 + 20*a^2*x^2 - 4*a*x - 13)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

3.612.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

output
```
-(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 - 3*a
**5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x) + Integr
al(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 - 3*a**5*x
**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x))/c**3
```

3.612.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{560}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-28\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-140\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{35}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/560*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 28*((a*x - 1)/(a*x + 1))^(5/2)
+ 70*((a*x - 1)/(a*x + 1))^(3/2) - 140*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3
) - 35/(a^2*c^3*sqrt((a*x - 1)/(a*x + 1))))
```

3.612.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(a^2cx^2-c)^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^3, x)
```

3.612.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{1}{16\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^3} - \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8\,a\,c^3} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{20\,a\,c^3} - \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{112\,a\,c^3}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^3,x)
```

output
```
1/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2)) + ((a*x - 1)/(a*x + 1))^(1/2)/(4*
a*c^3) - ((a*x - 1)/(a*x + 1))^(3/2)/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/
2)/(20*a*c^3) - ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3)
```

3.613 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

3.613.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{16e^{-3\coth^{-1}(ax)}}{63ac^4} + \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2} - \frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4(1-a^2x^2)}$$

output
```
16/63/a/c^4*((a*x-1)/(a*x+1))^(3/2)+1/9*(2*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^
(3/2)/(-a^2*x^2+1)^3+10/63*(4*a*x+3)/a/c^4*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x
^2+1)^2-8/21*(2*a*x+3)/a/c^4*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2+1)
```

3.613.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(19-6ax-66a^2x^2-56a^3x^3+24a^4x^4+48a^5x^5+16a^6x^6)}{63c^4(-1+ax)^2(1+ax)^5}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 6*a*x - 66*a^2*x^2 - 56*a^3*x^3 + 24*a^4*x^
4 + 48*a^5*x^5 + 16*a^6*x^6))/(63*c^4*(-1 + a*x)^2*(1 + a*x)^5)
```

3.613.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6739, 27, 6739, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{10\int \frac{e^{-3\coth^{-1}(ax)}}{c^3(1-a^2x^2)^3}dx}{9c}+\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{10\int \frac{e^{-3\coth^{-1}(ax)}}{(1-a^2x^2)^3}dx}{9c^4}+\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6739}$$

$$\frac{10\left(\frac{12}{7}\int \frac{e^{-3\coth^{-1}(ax)}}{(1-a^2x^2)^2}dx+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7a(1-a^2x^2)^2}\right)}{9c^4}+\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6739}$$

$$\frac{10\left(\frac{12}{7}\left(-\frac{2}{5}\int \frac{e^{-3\coth^{-1}(ax)}}{1-a^2x^2}dx-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5a(1-a^2x^2)}\right)+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7a(1-a^2x^2)^2}\right)}{9c^4}+\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}$$

$$\Big\downarrow \text{6737}$$

$$\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}+\frac{10\left(\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7a(1-a^2x^2)^2}+\frac{12}{7}\left(\frac{2e^{-3\coth^{-1}(ax)}}{15a}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5a(1-a^2x^2)}\right)\right)}{9c^4}$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

output
```
(1 + 2*a*x)/(9*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x^2)^3) + (10*((3 + 4*a*x
)/(7*a*E^(3*ArcCoth[a*x])*(1 - a^2*x^2)^2) + (12*(2/(15*a*E^(3*ArcCoth[a*x
])) - (3 + 2*a*x)/(5*a*E^(3*ArcCoth[a*x])*(1 - a^2*x^2))))/7))/(9*c^4)
```

3.613.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.613.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19\right)}{63\left(a^2x^2-1\right)^3c^4a}$	81
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19\right)}{63\left(ax-1\right)^3c^4\left(ax+1\right)^3a}$	84
trager	$\frac{\left(16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19\right)\sqrt{-\frac{-ax+1}{ax+1}}}{63a\,c^4\left(ax-1\right)^2\left(ax+1\right)^4}$	86

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/63*((a*x-1)/(a*x+1))^(3/2)*(16*a^6*x^6+48*a^5*x^5+24*a^4*x^4-56*a^3*x^3-
66*a^2*x^2-6*a*x+19)/(a^2*x^2-1)^3/c^4/a
```

3.613. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

3.613.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\left(16\,a^6x^6+48\,a^5x^5+24\,a^4x^4-56\,a^3x^3-66\,a^2x^2-6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/63*(16*a^6*x^6 + 48*a^5*x^5 + 24*a^4*x^4 - 56*a^3*x^3 - 66*a^2*x^2 - 6*a
*x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*
x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

3.613.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$$

$$= \frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^9x^9+a^8x^8-4a^7x^7-4a^6x^6+6a^5x^5+6a^4x^4-4a^3x^3-4a^2x^2+ax+1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^9x^9+a^8x^8-4a^7x^7-4a^6x^6+6a^5x^5+6a^4x^4-4a^3x^3-4a}}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

output
```
(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**9*x**9 + a**8*x**8 - 4*a*
*7*x**7 - 4*a**6*x**6 + 6*a**5*x**5 + 6*a**4*x**4 - 4*a**3*x**3 - 4*a**2*x
**2 + a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**9*
x**9 + a**8*x**8 - 4*a**7*x**7 - 4*a**6*x**6 + 6*a**5*x**5 + 6*a**4*x**4 -
 4*a**3*x**3 - 4*a**2*x**2 + a*x + 1), x))/c**4
```

3.613.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$$
$$=\frac{1}{4032}a\left(\frac{7\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-54\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+189\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-420\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+945\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{21\left(\frac{18\,(ax-1)}{ax+1}-1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/4032*a*((7*((a*x - 1)/(a*x + 1))^(9/2) - 54*((a*x - 1)/(a*x + 1))^(7/2)
+ 189*((a*x - 1)/(a*x + 1))^(5/2) - 420*((a*x - 1)/(a*x + 1))^(3/2) + 945*
sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + 21*(18*(a*x - 1)/(a*x + 1) - 1)/(a^
2*c^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

3.613.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^4, x)
```

3.613.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx=\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^4}+\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{64\,a\,c^4}$$
$$-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{576\,a\,c^4}+\frac{\frac{6\,(a\,x-1)}{a\,x+1}-\frac{1}{3}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^4,x)
```

output
```
(15*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - (5*((a*x - 1)/(a*x + 1))^(3/
2))/(48*a*c^4) + (3*((a*x - 1)/(a*x + 1))^(5/2))/(64*a*c^4) - (3*((a*x - 1
)/(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*c^4)
+ ((6*(a*x - 1))/(a*x + 1) - 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(3/2))
```

3.613. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

3.614 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

3.614.1 Optimal result

Integrand size = 22, antiderivative size = 229

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{8(1+ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

output
```
8/3*(a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(a*x+
1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(a*x+1)^8*(-a^2*c
*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(a*x+1)^9*(-a^2*c*x^2+c)^(9
/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c)^(9/2)/a^10
/(1-1/a^2/x^2)^(9/2)/x^9
```

3.614.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(1+ax)^6\sqrt{c-a^2cx^2}\left(193-528ax+588a^2x^2-308a^3x^3+63a^4x^4\right)}{630a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

output
```
(c^4*(1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 - 528*a*x + 588*a^2*x^2 - 308*a^
3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.614.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9dx}{x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int (1-ax)^4(ax+1)^5dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int \left((ax+1)^9-8(ax+1)^8+24(ax+1)^7-32(ax+1)^6+16(ax+1)^5\right)dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(ax+1)^{10}}{10a}-\frac{8(ax+1)^9}{9a}+\frac{3(ax+1)^8}{a}-\frac{32(ax+1)^7}{7a}+\frac{8(ax+1)^6}{3a}\right)\left(c-a^2cx^2\right)^{9/2}}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

output
```
((c - a^2*c*x^2)^(9/2)*((8*(1 + a*x)^6)/(3*a) - (32*(1 + a*x)^7)/(7*a) + (
3*(1 + a*x)^8)/a - (8*(1 + a*x)^9)/(9*a) + (1 + a*x)^10/(10*a)))/(a^9*(1 -
 1/(a^2*x^2))^(9/2)*x^9)
```

3.614.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.614.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\left(63 a^{9} x^{9}+70 a^{8} x^{8}-315 a^{7} x^{7}-360 a^{6} x^{6}+630 a^{5} x^{5}+756 a^{4} x^{4}-630 a^{3} x^{3}-840 a^{2} x^{2}+315 a x +630\right) x \,c^{4} \sqrt{-c \left(a^{2} x^{2}-1\right)}}{630 \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	113
gosper	$\frac{x \left(63 a^{9} x^{9}+70 a^{8} x^{8}-315 a^{7} x^{7}-360 a^{6} x^{6}+630 a^{5} x^{5}+756 a^{4} x^{4}-630 a^{3} x^{3}-840 a^{2} x^{2}+315 a x +630\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{9}{2}}}{630 \left(a x +1\right)^{5} \left(a x -1\right)^{4} \sqrt{\frac{a x -1}{a x +1}}}$	116

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE
)
```

output
```
1/630*(63*a^9*x^9+70*a^8*x^8-315*a^7*x^7-360*a^6*x^6+630*a^5*x^5+756*a^4*x
^4-630*a^3*x^3-840*a^2*x^2+315*a*x+630)*x*c^4*(-c*(a^2*x^2-1))^(1/2)/(a*x+
1)/((a*x-1)/(a*x+1))^(1/2)
```

3.614.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.51

$$\int e^{\coth^{-1}(ax)}(c -a^2cx^2)^{9/2}\,dx = \frac{(63\,a^9c^4x^{10} + 70\,a^8c^4x^9 - 315\,a^7c^4x^8 - 360\,a^6c^4x^7 + 630\,a^5c^4x^6 + 756\,a^4c^4x^5 - 630\,a^3c^4x^4}{630\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fri
cas")
```

output
```
1/630*(63*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 - 360*a^6*c^4*x^
7 + 630*a^5*c^4*x^6 + 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 - 840*a^2*c^4*x^3
+ 315*a*c^4*x^2 + 630*c^4*x)*sqrt(-a^2*c)/a
```

3.614.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c - a^2cx^2\right)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Timed out
```

3.614.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c - a^2cx^2\right)^{9/2}\,dx = \int \frac{(-a^2cx^2 + c)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="max
ima")
```

output
```
integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.614.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \frac{(-a^2cx^2+c)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="gia
c")
```

output
```
integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.614.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \frac{(c-a^2\,c\,x^2)^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.615 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

3.615.1 Optimal result

Integrand size = 22, antiderivative size = 183

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{8(1+ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

output
```
-8/5*(a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(a*x+1)^
6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(a*x+1)^7*(-a^2*c*x
^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*x^2+c)^(7/2)
/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

3.615.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(1+ax)^5\sqrt{c-a^2cx^2}\left(-93+185ax-135a^2x^2+35a^3x^3\right)}{280a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/280*(c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(-93 + 185*a*x - 135*a^2*x^2 +
 35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.615.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int e^{\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7dx}{x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int -(1-ax)^3(ax+1)^4dx}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{7/2} \int (1-ax)^3(ax+1)^4dx}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{7/2} \int \left(-(ax+1)^7+6(ax+1)^6-12(ax+1)^5+8(ax+1)^4\right) dx}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(ax+1)^8}{8a}+\frac{6(ax+1)^7}{7a}-\frac{2(ax+1)^6}{a}+\frac{8(ax+1)^5}{5a}\right) \left(c-a^2cx^2\right)^{7/2}}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

output
```
-(((c - a^2*c*x^2)^(7/2)*((8*(1 + a*x)^5)/(5*a) - (2*(1 + a*x)^6)/a + (6*(
1 + a*x)^7)/(7*a) - (1 + a*x)^8/(8*a)))/(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7))
```

3.615.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.615.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{\left(35 a^{7} x^{7}+40 a^{6} x^{6}-140 a^{5} x^{5}-168 a^{4} x^{4}+210 a^{3} x^{3}+280 a^{2} x^{2}-140 a x -280\right) x \,c^{3} \sqrt{-c \left(a^{2} x^{2}-1\right)}}{280 \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	97
gosper	$\frac{x \left(35 a^{7} x^{7}+40 a^{6} x^{6}-140 a^{5} x^{5}-168 a^{4} x^{4}+210 a^{3} x^{3}+280 a^{2} x^{2}-140 a x -280\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}{280 \left(a x -1\right)^{3} \left(a x +1\right)^{4} \sqrt{\frac{a x -1}{a x +1}}}$	100

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/280*(35*a^7*x^7+40*a^6*x^6-140*a^5*x^5-168*a^4*x^4+210*a^3*x^3+280*a^2*
x^2-140*a*x-280)*x*c^3*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)
```

3.615.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx =$$

$$-\frac{(35\,a^7c^3x^8+40\,a^6c^3x^7-140\,a^5c^3x^6-168\,a^4c^3x^5+210\,a^3c^3x^4+280\,a^2c^3x^3-140\,ac^3x^2-280\,c^3x)\sqrt{-a^2}}{280\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

output
```
-1/280*(35*a^7*c^3*x^8 + 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 - 168*a^4*c^3*x^
5 + 210*a^3*c^3*x^4 + 280*a^2*c^3*x^3 - 140*a*c^3*x^2 - 280*c^3*x)*sqrt(-a
^2*c)/a
```

3.615.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

3.615.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \frac{(-a^2cx^2+c)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

output
```
integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.615.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \frac{(-a^2cx^2+c)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

output
```
integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.615.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.616 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

3.616.1 Optimal result

Integrand size = 22, antiderivative size = 136

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

output

```
(a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(a*x+1)^5*(
-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1)^6*(-a^2*c*x^2+
c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

3.616.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(1+ax)^4\left(11-14ax+5a^2x^2\right)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input

```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

output

```
(c^2*(1 + a*x)^4*(11 - 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sq
rt[1 - 1/(a^2*x^2)]*x)
```

3.616.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5dx}{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int (1-ax)^2(ax+1)^3dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int \left((ax+1)^5-4(ax+1)^4+4(ax+1)^3\right)dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(ax+1)^6}{6a}-\frac{4(ax+1)^5}{5a}+\frac{(ax+1)^4}{a}\right)\left(c-a^2cx^2\right)^{5/2}}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

output
```
((c - a^2*c*x^2)^(5/2)*((1 + a*x)^4/a - (4*(1 + a*x)^5)/(5*a) + (1 + a*x)^
6/(6*a)))/(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

3.616.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.616.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(5 a^{5} x^{5}+6 a^{4} x^{4}-15 a^{3} x^{3}-20 a^{2} x^{2}+15 a x +30\right) x \,c^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}{30 \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	81
gosper	$\frac{x \left(5 a^{5} x^{5}+6 a^{4} x^{4}-15 a^{3} x^{3}-20 a^{2} x^{2}+15 a x +30\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{30 \left(a x -1\right)^{2} \left(a x +1\right)^{3} \sqrt{\frac{a x -1}{a x +1}}}$	84

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/30*(5*a^5*x^5+6*a^4*x^4-15*a^3*x^3-20*a^2*x^2+15*a*x+30)*x*c^2*(-c*(a^2*
x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

3.616.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.54

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(5\,a^5c^2x^6+6\,a^4c^2x^5-15\,a^3c^2x^4-20\,a^2c^2x^3+15\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
1/30*(5*a^5*c^2*x^6 + 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 + 15
*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

3.616.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.616.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.616.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.616.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.617 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

3.617.1 Optimal result

Integrand size = 22, antiderivative size = 93

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{2(1+ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
-2/3*(a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(a*x+1
)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

3.617.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c(1+ax)^3(-5+3ax)\sqrt{c-a^2cx^2}}{12a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/12*(c*(1 + a*x)^3*(-5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^
2*x^2)]*x)
```

3.617.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2} \int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3dx}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2} \int -\left((1-ax)(ax+1)^2\right)dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{3/2} \int (1-ax)(ax+1)^2dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{3/2} \int \left(2(ax+1)^2-(ax+1)^3\right)dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{2(ax+1)^3}{3a}-\frac{(ax+1)^4}{4a}\right)\left(c-a^2cx^2\right)^{3/2}}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(((c - a^2*c*x^2)^(3/2)*((2*(1 + a*x)^3)/(3*a) - (1 + a*x)^4/(4*a)))/(a^3
*(1 - 1/(a^2*x^2))^(3/2)*x^3))
```

3.617. $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2} dx$

3.617.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.617.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{(3a^{3}x^{3}+4a^{2}x^{2}-6ax-12)xc\sqrt{-c(a^{2}x^{2}-1)}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	63
gosper	$\frac{x(3a^{3}x^{3}+4a^{2}x^{2}-6ax-12)\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{12(ax-1)(ax+1)^{2}\sqrt{\frac{ax-1}{ax+1}}}$	68

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/12*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*x*c*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((
a*x-1)/(a*x+1))^(1/2)
```

3.617.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(3\,a^3cx^4+4\,a^2cx^3-6\,acx^2-12\,cx\right)\sqrt{-a^2c}}{12\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/12*(3*a^3*c*x^4 + 4*a^2*c*x^3 - 6*a*c*x^2 - 12*c*x)*sqrt(-a^2*c)/a
```

3.617.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.617.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.617.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.617.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.618 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.618.1 Optimal result

Integrand size = 22, antiderivative size = 68

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

3.618.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

output
```
((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.618.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int (ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(ax+1)^2\sqrt{c-a^2cx^2}}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

output
```
((1 + a*x)^2*Sqrt[c - a^2*c*x^2])/(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.618.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.618.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
default	$\frac{(ax+2)x\sqrt{-c(a^2x^2-1)}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	45

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

3.618.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2+2\,x)}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

3.618.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.618.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.618.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.618.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.619 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

3.619.1 Optimal result

Integrand size = 22, antiderivative size = 38

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

output
```
x*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

3.619.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

3.619.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6746, 6747, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \; 6746$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \; 6747$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \frac{1}{ax-1}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \; 16$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

input

```
Int[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
```

output

```
(Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

3.619.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 6746

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.619.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\ln(ax-1)\sqrt{-c(a^2x^2-1)}}{ca(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	51

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-ln(a*x-1)*(-c*(a^2*x^2-1))^(1/2)/c/a/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

3.619.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{-a^2c}\log{(ax-1)}}{a^2c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-sqrt(-a^2*c)*log(a*x - 1)/(a^2*c)
```

3.619.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

3.619.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.619.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.619.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.620 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

3.620.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

output
```
1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1
/a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

3.620.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3(-1+(-1+ax)\text{arctanh}(ax))}{(-2+2ax)\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1 + (-1 + a*x)*ArcTanh[a*x]))/((-2 + 2*
a*x)*(c - a^2*c*x^2)^(3/2))
```

3.620.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{2(ax-1)^2}-\frac{1}{2(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\text{arctanh}(ax)}{2a}+\frac{1}{2a(1-ax)}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)))
/(c - a^2*c*x^2)^(3/2)
```

3.620.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.620.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\sqrt{-c(a^{2}x^{2}-1)}\,(a\ln(ax+1)x-a\ln(ax-1)x-\ln(ax+1)+\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^{2}x^{2}-1)c^{2}a}$	84

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-a*ln(a*x
-1)*x-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

3.620.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\,(a^3c^2x-a^2c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)
/(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

3.620.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.620.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.620.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.620.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.621 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

3.621.1 Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(
1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)
^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*ar
ctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

3.621.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(2+3ax-3a^2x^2+3(-1+ax)^2(1+ax)\text{arctanh}(ax))}{8c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 +
 a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

3.621.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{4(ax-1)^2}+\frac{1}{8(ax+1)^2}-\frac{1}{4(ax-1)^3}-\frac{3}{8(a^2x^2-1)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\text{arctanh}(ax)}{8a}+\frac{1}{4a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{1}{8a(1-ax)^2}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

3.621. $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)
) - 1/(8*a*(1 + a*x)) + (3*ArcTanh[a*x])/(8*a)))/(c - a^2*c*x^2)^(5/2))
```

3.621.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.621.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(3 a^{3} \ln \left(a x+1\right) x^{3}-3 a^{3} \ln \left(a x-1\right) x^{3}-3 a^{2} \ln \left(a x+1\right) x^{2}+3 a^{2} \ln \left(a x-1\right) x^{2}-6 a^{2} x^{2}-3 a \ln \left(a x+1\right) x+3 a \ln \left(a x-1\right) x+6 a x+}{16 \sqrt{\frac{a x-1}{a x+1}}\, \left(a x-1\right) \left(a^{2} x^{2}-1\right) c^{3} a \left(a x+1\right)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*ln(a*x+
1)*x^3-3*a^3*ln(a*x-1)*x^3-3*a^2*ln(a*x+1)*x^2+3*a^2*ln(a*x-1)*x^2-6*a^2*x
^2-3*a*ln(a*x+1)*x+3*a*ln(a*x-1)*x+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x
^2-1)/c^3/a/(a*x+1)
```

3.621.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\,(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\,(3\,a^2x^2-3\,ax-2)\sqrt{-a^2c}}{16\,(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
-1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(
-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a
^2*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

3.621.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.621.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.621.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.622 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

3.622.1 Optimal result

Integrand size = 22, antiderivative size = 277

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}$$

output
```
1/24*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*
(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/
x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/32*a^6*(1-1/a^2/x^2)^(7/2)*
x^7/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)
/(-a^2*c*x^2+c)^(7/2)+5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*
c*x^2+c)^(7/2)
```

3.622.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-8-25ax+25a^2x^2+15a^3x^3-15a^4x^4+15(-1+ax)^3(1+ax)^2\operatorname{arctanh}(ax))}{48c^3(-1+ax)^3(1+ax)^2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/48*(Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 25*a*x + 25*a^2*x^2 + 15*a^3*x^3 - 15
*a^4*x^4 + 15*(-1 + a*x)^3*(1 + a*x)^2*ArcTanh[a*x]))/(c^3*(-1 + a*x)^3*(1
 + a*x)^2*Sqrt[c - a^2*c*x^2])
```

3.622.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{1}{(1-ax)^4(ax+1)^3}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \left(\frac{3}{16(ax-1)^2}+\frac{1}{8(ax+1)^2}-\frac{3}{16(ax-1)^3}+\frac{1}{16(ax+1)^3}+\frac{1}{8(ax-1)^4}-\frac{5}{16(a^2x^2-1)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{5\text{arctanh}(ax)}{16a}+\frac{3}{16a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{3}{32a(1-ax)^2}-\frac{1}{32a(ax+1)^2}+\frac{1}{24a(1-ax)^3}\right)}{\left(c-a^2cx^2\right)^{7/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

output
```
(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7*(1/(24*a*(1 - a*x)^3) + 3/(32*a*(1 - a*x)
^2) + 3/(16*a*(1 - a*x)) - 1/(32*a*(1 + a*x)^2) - 1/(8*a*(1 + a*x)) + (5*A
rcTanh[a*x])/(16*a)))/(c - a^2*c*x^2)^(7/2)
```

3.622.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.622. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

3.622.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-15\ln(ax+1)x^4a^4+15\ln(ax-1)x^4a^4-30a^4x^4-30a^3\ln(ax+1)x^3+30a^3\ln(ax-}{96\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)^2}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
1/96/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1
)*x^5*a^5-15*ln(a*x-1)*x^5*a^5-15*ln(a*x+1)*x^4*a^4+15*ln(a*x-1)*x^4*a^4-3
0*a^4*x^4-30*a^3*ln(a*x+1)*x^3+30*a^3*ln(a*x-1)*x^3+30*a^3*x^3+30*a^2*ln(a
*x+1)*x^2-30*a^2*ln(a*x-1)*x^2+50*a^2*x^2+15*a*ln(a*x+1)*x-15*a*ln(a*x-1)*
x-50*a*x-15*ln(a*x+1)+15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x+1)^2
```

3.622.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{15\,(a^6x^5-a^5x^4-2\,a^4x^3+2\,a^3x^2+a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\,(15\,a^4x^4-15\,a^3x^3-25\,a}{96\,(a^7c^4x^5-a^6c^4x^4-2\,a^5c^4x^3+2\,a^4c^4x^2+a^3c^4x-a^2c^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

output
```
-1/96*(15*(a^6*x^5 - a^5*x^4 - 2*a^4*x^3 + 2*a^3*x^2 + a^2*x - a)*sqrt(-c)
*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15*a^
4*x^4 - 15*a^3*x^3 - 25*a^2*x^2 + 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 -
 a^6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 + a^3*c^4*x - a^2*c^4)
```

3.622.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

3.622.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\int \frac{1}{(-a^2cx^2+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.622.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\int \frac{1}{(-a^2cx^2+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.622.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.623 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

3.623.1 Optimal result

Integrand size = 24, antiderivative size = 176

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}$$
$$-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}$$
$$-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{11\left(c-a^2cx^2\right)^{9/2}}{90a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a}-\frac{77c^{9/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}$$

output
```
-77/384*c^3*x*(-a^2*c*x^2+c)^(3/2)-77/480*c^2*x*(-a^2*c*x^2+c)^(5/2)-11/80
*c*x*(-a^2*c*x^2+c)^(7/2)+11/90*(-a^2*c*x^2+c)^(9/2)/a+1/10*(a*x+1)*(-a^2*
c*x^2+c)^(9/2)/a-77/256*c^(9/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a
-77/256*c^4*x*(-a^2*c*x^2+c)^(1/2)
```

3.623.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\bigl(c$$
$$-a^2cx^2\bigr)^{9/2}\,dx=\frac{c^4\sqrt{c-a^2cx^2}\Bigl(\sqrt{1+ax}(2560-10615ax-2185a^2x^2+16390a^3x^3+9210a^4x^4-15048a^5}{11520a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

output
```
(c^4*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(2560 - 10615*a*x - 2185*a^2*x^2 +
 16390*a^3*x^3 + 9210*a^4*x^4 - 15048*a^5*x^5 - 10552*a^6*x^6 + 7216*a^7*x
^7 + 5584*a^8*x^8 - 1408*a^9*x^9 - 1152*a^10*x^10) + 6930*Sqrt[1 - a*x]*Ar
cSin[Sqrt[1 - a*x]/Sqrt[2]]))/(11520*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

3.623.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6691, 469, 455, 211, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{9/2}dx$$

$$\Big\downarrow \text{ 6691}$$

$$-c\int (ax+1)^2\left(c-a^2cx^2\right)^{7/2}dx$$

$$\Big\downarrow \text{ 469}$$

$$-c\left(\frac{11}{10}\int (ax+1)\left(c-a^2cx^2\right)^{7/2}dx-\frac{(ax+1)\left(c-a^2cx^2\right)^{9/2}}{10ac}\right)$$

$$\Big\downarrow \text{ 455}$$

$$-c\left(\frac{11}{10}\left(\int \left(c-a^2cx^2\right)^{7/2}dx-\frac{\left(c-a^2cx^2\right)^{9/2}}{9ac}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{9/2}}{10ac}\right)$$

$$\Big\downarrow \text{ 211}$$

$$-c\left(\frac{11}{10}\left(\frac{7}{8}c\int \left(c-a^2cx^2\right)^{5/2}dx-\frac{\left(c-a^2cx^2\right)^{9/2}}{9ac}+\frac{1}{8}x\left(c-a^2cx^2\right)^{7/2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{9/2}}{10ac}\right)$$

$$\Big\downarrow \text{ 211}$$

$$-c\Bigg(\frac{11}{10}\Bigg(\frac{7}{8}c\bigg(\frac{5}{6}c\int\left(c-a^2cx^2\right)^{3/2}dx+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\bigg)-\frac{\left(c-a^2cx^2\right)^{9/2}}{9ac}+\frac{1}{8}x\left(c-a^2cx^2\right)^{7/2}\Bigg)-\frac{(ax+1)\,(c}{1}$$

$$\Big\downarrow \; 211$$

$$-c\Bigg(\frac{11}{10}\Bigg(\frac{7}{8}c\bigg(\frac{5}{6}c\bigg(\frac{3}{4}c\int\sqrt{c-a^2cx^2}dx+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\bigg)+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\bigg)-\frac{\left(c-a^2cx^2\right)^{9/2}}{9ac}+\frac{1}{8}x\left(c-a^2\right.$$

$$\Big\downarrow \; 211$$

$$-c\Bigg(\frac{11}{10}\Bigg(\frac{7}{8}c\bigg(\frac{5}{6}c\bigg(\frac{3}{4}c\bigg(\frac{1}{2}c\int\frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\sqrt{c-a^2cx^2}\bigg)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\bigg)+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\bigg)-\frac{(c-}{}$$

$$\Big\downarrow \; 224$$

$$-c\Bigg(\frac{11}{10}\Bigg(\frac{7}{8}c\bigg(\frac{5}{6}c\bigg(\frac{3}{4}c\bigg(\frac{1}{2}c\int\frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\bigg)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\bigg)+\frac{1}{6}x\left(c-a^2cx^2\right)$$

$$\Big\downarrow \; 216$$

$$-c\Bigg(\frac{11}{10}\Bigg(\frac{7}{8}c\Bigg(\frac{5}{6}c\Bigg(\frac{3}{4}c\Bigg(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\Bigg)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\Bigg)+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\Bigg)-$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

output
```
-(c*(-1/10*((1 + a*x)*(c - a^2*c*x^2)^(9/2))/(a*c) + (11*((x*(c - a^2*c*x^
2)^(7/2))/8 - (c - a^2*c*x^2)^(9/2)/(9*a*c) + (7*c*((x*(c - a^2*c*x^2)^(5/
2))/6 + (5*c*((x*(c - a^2*c*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c - a^2*c*x^2])/
2 + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)))/4))/6))/8)
)/10))
```

3.623. $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

3.623.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.623. $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

3.623.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(1152a^9x^9+2560a^8x^8-3024a^7x^7-10240a^6x^6+312a^5x^5+15360a^4x^4+6150a^3x^3-10240a^2x^2-8055ax+2560)(a^2x^2-1)c^5}{11520a\sqrt{-c(a^2x^2-1)}}-\frac{77}{}$
default	$\frac{x\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}{10}+\frac{9c\left(\frac{x\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{8}+\frac{7c\left(\frac{x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}\right)}{6}\right)}{8}\right.}{10}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/11520*(1152*a^9*x^9+2560*a^8*x^8-3024*a^7*x^7-10240*a^6*x^6+312*a^5*x^5
+15360*a^4*x^4+6150*a^3*x^3-10240*a^2*x^2-8055*a*x+2560)*(a^2*x^2-1)/a/(-c
*(a^2*x^2-1))^(1/2)*c^5-77/256/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*
c*x^2+c)^(1/2))*c^5
```

3.623.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.87

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \left[\frac{3465\sqrt{-c}c^4\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(1152a^9c^4x^9+2560a^8c^4x^8-\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/23040*(3465*sqrt(-c)*c^4*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqr
t(-c)*x - c) + 2*(1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 -
 10240*a^6*c^4*x^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^
3 - 10240*a^2*c^4*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a,
1/11520*(3465*c^(9/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
 c)) + (1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 10240*a^6
*c^4*x^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 - 10240*
a^2*c^4*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a]
```

3.623.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(162) = 324$.

Time = 3.36 (sec) , antiderivative size = 714, normalized size of antiderivative = 4.06

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \begin{cases} \frac{-2c^4\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)+6c^4\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \text{f}\\ \frac{a^4\sqrt{c}x^4}{4} & \text{o}\end{cases}\right.}{} \\ -c^{\frac{9}{2}}x \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Piecewise(((-2*c**4*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
 Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 6*c**4*Piecewise((sqrt(-a**2*c*
x**2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x*
*4/4, True)) - 6*c**4*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**6*x**6/7 - a**
4*x**4/35 - 4*a**2*x**2/105 - 8/105), Ne(c, 0)), (a**6*sqrt(c)*x**6/6, Tru
e)) + 2*c**4*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**8*x**8/9 - a**6*x**6/63
 - 2*a**4*x**4/105 - 8*a**2*x**2/315 - 16/315), Ne(c, 0)), (a**8*sqrt(c)*x
**8/8, True)) + c**4*Piecewise((7*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*s
qrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**
2), True))/256 + sqrt(-a**2*c*x**2 + c)*(a**9*x**9/10 - a**7*x**7/80 - 7*a
**5*x**5/480 - 7*a**3*x**3/384 - 7*a*x/256), Ne(c, 0)), (a**9*sqrt(c)*x**9
/9, True)) - 2*c**4*Piecewise((5*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sq
rt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2
), True))/128 + sqrt(-a**2*c*x**2 + c)*(a**7*x**7/8 - a**5*x**5/48 - 5*a**
3*x**3/192 - 5*a*x/128), Ne(c, 0)), (a**7*sqrt(c)*x**7/7, True)) + 2*c**4*
Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/
sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/8 + (a**3*x*
*3/4 - a*x/8)*sqrt(-a**2*c*x**2 + c), Ne(c, 0)), (a**3*sqrt(c)*x**3/3, Tru
e)) - c**4*Piecewise((a*x*sqrt(-a**2*c*x**2 + c)/2 + c*Piecewise((log(-2*a
*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log...
```

3.623.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{1}{10}\left(-a^2cx^2+c\right)^{\frac{9}{2}}x$$
$$-\frac{11}{80}\left(-a^2cx^2+c\right)^{\frac{7}{2}}cx-\frac{77}{480}\left(-a^2cx^2+c\right)^{\frac{5}{2}}c^2x-\frac{77}{384}\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^3x$$
$$-\frac{35}{64}\sqrt{a^2cx^2-4\,acx+3\,c}c^4x+\frac{63}{256}\sqrt{-a^2cx^2+c}c^4x-\frac{35\,c^6\arcsin\left(ax-2\right)}{64\,a\left(-c\right)^{\frac{3}{2}}}$$
$$+\frac{63\,c^{\frac{9}{2}}\arcsin\left(ax\right)}{256\,a}+\frac{2\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{9\,a}+\frac{35\sqrt{a^2cx^2-4\,acx+3\,c}c^4}{32\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
1/10*(-a^2*c*x^2 + c)^(9/2)*x - 11/80*(-a^2*c*x^2 + c)^(7/2)*c*x - 77/480*
(-a^2*c*x^2 + c)^(5/2)*c^2*x - 77/384*(-a^2*c*x^2 + c)^(3/2)*c^3*x - 35/64
*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^4*x + 63/256*sqrt(-a^2*c*x^2 + c)*c^4*x
 - 35/64*c^6*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 63/256*c^(9/2)*arcsin(a*x)/a
 + 2/9*(-a^2*c*x^2 + c)^(9/2)/a + 35/32*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^
4/a
```

3.623.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{77\,c^5\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{256\sqrt{-c}|a|}$$
$$+\frac{1}{11520}\sqrt{-a^2cx^2+c}\Bigg(\frac{2560\,c^4}{a}-\big(8055\,c^4+2\left(5120\,ac^4-\left(3075\,a^2c^4+4\left(1920\,a^3c^4+\left(39\,a^4c^4-2\left(640\,a^5\right.\right.\right.\right.\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
77/256*c^5*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(
a)) + 1/11520*sqrt(-a^2*c*x^2 + c)*(2560*c^4/a - (8055*c^4 + 2*(5120*a*c^4
 - (3075*a^2*c^4 + 4*(1920*a^3*c^4 + (39*a^4*c^4 - 2*(640*a^5*c^4 + (189*a
^6*c^4 - 8*(9*a^8*c^4*x + 20*a^7*c^4)*x)*x)*x)*x)*x)*x)*x)*x)
```

3.623.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \frac{{\left(c-a^2\,c\,x^2\right)}^{9/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input

```
int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1),x)
```

output

```
int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1), x)
```

3.624 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

3.624.1 Optimal result

Integrand size = 24, antiderivative size = 153

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx &= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}\\ &-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}\\ &+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{45c^{7/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}\end{aligned}$$

output
```
-15/64*c^2*x*(-a^2*c*x^2+c)^(3/2)-3/16*c*x*(-a^2*c*x^2+c)^(5/2)+9/56*(-a^2
*c*x^2+c)^(7/2)/a+1/8*(a*x+1)*(-a^2*c*x^2+c)^(7/2)/a-45/128*c^(7/2)*arctan
(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-45/128*c^3*x*(-a^2*c*x^2+c)^(1/2)
```

3.624.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}(c -a^2cx^2)^{7/2}\,dx = \frac{c^3\sqrt{c-a^2cx^2}\Big(\sqrt{1+ax}(256-837ax-187a^2x^2+978a^3x^3+558a^4x^4-600a^5x^5-424a^6}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

output
```
(c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(256 - 837*a*x - 187*a^2*x^2 + 978
*a^3*x^3 + 558*a^4*x^4 - 600*a^5*x^5 - 424*a^6*x^6 + 144*a^7*x^7 + 112*a^8
*x^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(896*a*Sqrt[1 -
a*x]*Sqrt[1 - a^2*x^2])
```

3.624.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6717, 6691, 469, 455, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{7/2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int (ax+1)^2\left(c-a^2cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{9}{8}\int (ax+1)\left(c-a^2cx^2\right)^{5/2}dx-\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{9}{8}\left(\int \left(c-a^2cx^2\right)^{5/2}dx-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\int \left(c-a^2cx^2\right)^{3/2}dx-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{c-a^2cx^2}dx+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\right)-\frac{(ax+1)\,(c-a}{8ac}\right.$$

$$\Big\downarrow \; 211$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}x(c-a^2cx^2\right.\right.$$

$$\Big\downarrow \; 224$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}\right.\right.$$

$$\Big\downarrow \; 216$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}x(c-a\right.\right.$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

output
```
-(c*(-1/8*((1 + a*x)*(c - a^2*c*x^2)^(7/2))/(a*c) + (9*((x*(c - a^2*c*x^2)
^(5/2))/6 - (c - a^2*c*x^2)^(7/2)/(7*a*c) + (5*c*((x*(c - a^2*c*x^2)^(3/2)
)/4 + (3*c*((x*Sqrt[c - a^2*c*x^2])/2 + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(2*a)))/4))/6))/8))
```

3.624.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.624.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

method	result
risch	$\frac{(112a^{7}x^{7}+256a^{6}x^{6}-168a^{5}x^{5}-768a^{4}x^{4}-210a^{3}x^{3}+768a^{2}x^{2}+581ax-256)(a^{2}x^{2}-1)c^{4}}{896a\sqrt{-c(a^{2}x^{2}-1)}}-\frac{45\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}cx^{2}+c}}\right)c^{4}}{128\sqrt{a^{2}c}}$
default	$\frac{x(-a^{2}cx^{2}+c)^{\frac{7}{2}}}{8}+\frac{7c\left(\frac{x\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^{2}cx^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}cx^{2}+c}}\right)}{2\sqrt{a^{2}c}}\right)}{4}\right)}{6}\right)}{8}+\frac{2\left(-a^{2}c\left(x-\frac{1}{a}\right)\right.}{}$

input
```
int((-a^2*c*x^2+c)^(7/2)*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/896*(112*a^7*x^7+256*a^6*x^6-168*a^5*x^5-768*a^4*x^4-210*a^3*x^3+768*a^2
*x^2+581*a*x-256)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c^4-45/128/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^4
```

3.624.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.87

$$\int e^{2\coth^{-1}(ax)}(c-a^{2}cx^{2})^{7/2}\,dx=\left[\frac{315\sqrt{-c}c^{3}\log\left(2a^{2}cx^{2}-2\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right)-2\left(112a^{7}c^{3}x^{7}+256a^{6}c^{3}x^{6}-168\right.}{1792a}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) - 2*(112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 - 768*
a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256*c^3)*s
qrt(-a^2*c*x^2 + c))/a, 1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*s
qrt(c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c
^3*x^5 - 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x
 - 256*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

3.624.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(139) = 278$.

Time = 2.97 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.01

$$\int e^{2\coth^{-1}(ax)}\left(c - a^2cx^2\right)^{7/2}\,dx = \begin{cases} \frac{-2c^3\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)+4c^3\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \text{f}\\ \frac{a^4\sqrt{c}x^4}{4} & \text{o}\end{cases}\right)}{} \\ -c^{\frac{7}{2}}x \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Piecewise(((-2*c**3*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
 Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 4*c**3*Piecewise((sqrt(-a**2*c*
x**2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x*
*4/4, True)) - 2*c**3*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**6*x**6/7 - a**
4*x**4/35 - 4*a**2*x**2/105 - 8/105), Ne(c, 0)), (a**6*sqrt(c)*x**6/6, Tru
e)) - c**3*Piecewise((5*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*
c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))
/128 + sqrt(-a**2*c*x**2 + c)*(a**7*x**7/8 - a**5*x**5/48 - 5*a**3*x**3/19
2 - 5*a*x/128), Ne(c, 0)), (a**7*sqrt(c)*x**7/7, True)) + c**3*Piecewise((
c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), N
e(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/16 + sqrt(-a**2*c*x**2
+ c)*(a**5*x**5/6 - a**3*x**3/24 - a*x/16), Ne(c, 0)), (a**5*sqrt(c)*x**5/
5, True)) + c**3*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a
**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), Tr
ue))/8 + (a**3*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2 + c), Ne(c, 0)), (a**3*sq
rt(c)*x**3/3, True)) - c**3*Piecewise((a*x*sqrt(-a**2*c*x**2 + c)/2 + c*Pi
ecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c,
 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c, 0)), (a*sqrt(c)*x,
 True)))/a, Ne(a, 0)), (-c**(7/2)*x, True))
```

3.624.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.13

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{1}{8}\left(-a^2cx^2+c\right)^{\frac{7}{2}}x-\frac{3}{16}\left(-a^2cx^2+c\right)^{\frac{5}{2}}cx-\frac{15}{64}\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^2x-\frac{5}{8}\sqrt{a^2cx^2-4\,acx+3\,c}c^3x+\frac{35}{128}\sqrt{-a^2cx^2+c}c^3x-\frac{5\,c^5\arcsin\left(ax-2\right)}{8\,a\left(-c\right)^{\frac{3}{2}}}+\frac{35\,c^{\frac{7}{2}}\arcsin\left(ax\right)}{128\,a}+\frac{2\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{7\,a}+\frac{5\sqrt{a^2cx^2-4\,acx+3\,c}c^3}{4\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
1/8*(-a^2*c*x^2 + c)^(7/2)*x - 3/16*(-a^2*c*x^2 + c)^(5/2)*c*x - 15/64*(-a
^2*c*x^2 + c)^(3/2)*c^2*x - 5/8*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^3*x + 35
/128*sqrt(-a^2*c*x^2 + c)*c^3*x - 5/8*c^5*arcsin(a*x - 2)/(a*(-c)^(3/2)) +
 35/128*c^(7/2)*arcsin(a*x)/a + 2/7*(-a^2*c*x^2 + c)^(7/2)/a + 5/4*sqrt(a^
2*c*x^2 - 4*a*c*x + 3*c)*c^3/a
```

3.624.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{45\,c^4\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{128\,\sqrt{-c}|a|}$$

$$+\frac{1}{896}\sqrt{-a^2cx^2+c}\Bigg(\frac{256\,c^3}{a}-(581\,c^3+2\,(384\,ac^3-(105\,a^2c^3+4\,(96\,a^3c^3+(21\,a^4c^3-2\,(7\,a^6c^3x+16\,a^5c$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
45/128*c^4*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(
a)) + 1/896*sqrt(-a^2*c*x^2 + c)*(256*c^3/a - (581*c^3 + 2*(384*a*c^3 - (1
05*a^2*c^3 + 4*(96*a^3*c^3 + (21*a^4*c^3 - 2*(7*a^6*c^3*x + 16*a^5*c^3)*x)
*x)*x)*x)*x)*x)
```

3.624.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{7/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.625 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

3.625.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7(c-a^2cx^2)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}$$

output
```
-7/24*c*x*(-a^2*c*x^2+c)^(3/2)+7/30*(-a^2*c*x^2+c)^(5/2)/a+1/6*(a*x+1)*(-a
^2*c*x^2+c)^(5/2)/a-7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/
a-7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

3.625.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx=\frac{c^2\sqrt{c-a^2cx^2}\Big(\sqrt{1+ax}(96-231ax-57a^2x^2+182a^3x^3+106a^4x^4-56a^5x^5-40a^6x^6)+}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

output
```
(c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(96 - 231*a*x - 57*a^2*x^2 + 182*a
^3*x^3 + 106*a^4*x^4 - 56*a^5*x^5 - 40*a^6*x^6) + 210*Sqrt[1 - a*x]*ArcSin
[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

3.625.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6691, 469, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int (ax+1)^2\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{7}{6}\int (ax+1)\left(c-a^2cx^2\right)^{3/2}dx-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{7}{6}\left(\int \left(c-a^2cx^2\right)^{3/2}dx-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\int \sqrt{c-a^2cx^2}dx-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\Bigg(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{(ax+1)\,(c-a^2c}{6ac}$$

$$\Big\downarrow 224$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{(ax+}{}\right.$$

$$\Big\downarrow 216$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{(ax+1)\left(c-\right.}{6ac}\right.$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

output
```
-(c*(-1/6*((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(a*c) + (7*((x*(c - a^2*c*x^2)
^(3/2))/4 - (c - a^2*c*x^2)^(5/2)/(5*a*c) + (3*c*((x*Sqrt[c - a^2*c*x^2])/
2 + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)))/4))/6))
```

3.625.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.625. $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.625.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\left(40 a^{5} x^{5}+96 a^{4} x^{4}-10 a^{3} x^{3}-192 a^{2} x^{2}-135 a x +96\right) \left(a^{2} x^{2}-1\right) c^{3}}{240 a \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{7 \arctan\left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c^{3}}{16 \sqrt{a^{2} c}}$
default	$\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{6}+\frac{5 c \left(\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4}+\frac{3 c \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan\left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)}{4}\right)}{6}+\frac{\frac{2 \left(-a^{2} c \left(x -\frac{1}{a}\right)^{2}-2 \left(x -\frac{1}{a}\right) a c \right)^{\frac{5}{2}}}{5}-2 a c \left(\right.}{}$

input
```
int((-a^2*c*x^2+c)^(5/2)*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*a^5*x^5+96*a^4*x^4-10*a^3*x^3-192*a^2*x^2-135*a*x+96)*(a^2*x^2-
1)/a/(-c*(a^2*x^2-1))^(1/2)*c^3-7/16/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/
(-a^2*c*x^2+c)^(1/2))*c^3
```

3.625. $\int e^{2 \coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

3.625.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.85

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\left[\frac{105\sqrt{-c}c^2\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(40a^5c^2x^5+96a^4c^2x^4-10a^3\right.}{480a}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 2*(40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*
c^2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x
^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*
c^2)*sqrt(-a^2*c*x^2 + c))/a]
```

3.625.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(117) = 234.

Time = 2.53 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.42

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\begin{cases}\dfrac{-2c^2\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)+2c^2\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \\ \frac{a^4\sqrt{c}x^4}{4} & \end{cases}\right.}{} \\ -c^{\frac{5}{2}}x\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Piecewise(((-2*c**2*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
 Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 2*c**2*Piecewise((sqrt(-a**2*c*
x**2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x*
*4/4, True)) + c**2*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt
(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2),
 True))/16 + sqrt(-a**2*c*x**2 + c)*(a**5*x**5/6 - a**3*x**3/24 - a*x/16),
 Ne(c, 0)), (a**5*sqrt(c)*x**5/5, True)) - c**2*Piecewise((a*x*sqrt(-a**2*
c*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 +
 c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c
, 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

3.625.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \frac{1}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}x - \frac{7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx - \frac{3}{4}\sqrt{a^2cx^2-4\,acx+3\,c}c^2x + \frac{5}{16}\sqrt{-a^2cx^2+c}c^2x - \frac{3\,c^4\arcsin\left(ax-2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}} + \frac{5\,c^{\frac{5}{2}}\arcsin\left(ax\right)}{16\,a} + \frac{2\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{5\,a} + \frac{3\sqrt{a^2cx^2-4\,acx+3\,c}c^2}{2\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(
a^2*c*x^2 - 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x - 3/4*c
^4*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a + 2/5*(-a^2
*c*x^2 + c)^(5/2)/a + 3/2*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2/a
```

3.625.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{16\sqrt{-c}|a|} - \frac{1}{240}\sqrt{-a^2cx^2+c}\left(\left(135\,c^2+2\left(96\,ac^2+\left(5\,a^2c^2-4\left(5\,a^4c^2x+12\,a^3c^2\right)x\right)x\right)x\right)x - \frac{96\,c^2}{a}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a)
) - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 + 2*(96*a*c^2 + (5*a^2*c^2 - 4*(5
*a^4*c^2*x + 12*a^3*c^2)*x)*x)*x)*x - 96*c^2/a)
```

3.625.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.626 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

3.626.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

output
```
5/12*(-a^2*c*x^2+c)^(3/2)/a+1/4*(a*x+1)*(-a^2*c*x^2+c)^(3/2)/a-5/8*c^(3/2)
*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

3.626.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}\left(16-25ax-7a^2x^2+10a^3x^3+6a^4x^4\right)+30\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right.\right.}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
(c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(16 - 25*a*x - 7*a^2*x^2 + 10*a^3*x^
3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sq
rt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

3.626.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6717, 6691, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int (ax+1)^2\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{5}{4}\int (ax+1)\sqrt{c-a^2cx^2}dx-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{5}{4}\left(\int \sqrt{c-a^2cx^2}dx-\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{5}{4}\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{5}{4}\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{5}{4}\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}-\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(c*(-1/4*((1 + a*x)*(c - a^2*c*x^2)^(3/2))/(a*c) + (5*((x*Sqrt[c - a^2*c*
x^2])/2 - (c - a^2*c*x^2)^(3/2)/(3*a*c) + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sq
rt[c - a^2*c*x^2]])/(2*a)))/4))
```

3.626.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

3.626. $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.626.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(6a^3x^3+16a^2x^2+9ax-16)(a^2x^2-1)c^2}{24a\sqrt{-c(a^2x^2-1)}}-\frac{5\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c^2}{8\sqrt{a^2c}}$
default	$\frac{x(-a^2cx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2cx^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}+\frac{\frac{2\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}{3}-2ac\left(-\frac{\left(-2a^2c\left(x-\frac{1}{a}\right)-2ac\right)\sqrt{}}{4}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3+16*a^2*x^2+9*a*x-16)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*
c^2-5/8/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^2
```

3.626.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.68

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=\left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(6a^3cx^3+16a^2cx^2+9acx-16\right.}{48a}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) - 2*(6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 +
c))/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
 - c)) - (6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c
))/a]
```

3.626.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.

Time = 2.24 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.34

$$\int e^{2\coth^{-1}(ax)}\big(c - a^2cx^2\big)^{3/2}\,dx = \begin{cases} \frac{-2c\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(-2acx+2\sqrt{-c}\sqrt{-a^2cx^2+c}\right)}{\sqrt{-c}} & \text{for } c\neq\\ \frac{ax\log(ax)}{\sqrt{-a^2cx^2}} & \text{otherw}\end{cases}\right)}{8}\\ \frac{a^3\sqrt{c}x^3}{3}\end{cases}\right.}{} \\ -c^{\frac{3}{2}}x \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Piecewise(((-2*c*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c), Ne
(c, 0)), (a**2*sqrt(c)*x**2/2, True)) - c*Piecewise((c*Piecewise((log(-2*a
*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*
x)/sqrt(-a**2*c*x**2), True))/8 + (a**3*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2
+ c), Ne(c, 0)), (a**3*sqrt(c)*x**3/3, True)) - c*Piecewise((a*x*sqrt(-a**
2*c*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2
 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne
(c, 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(3/2)*x, True))
```

3.626.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\big(c - a^2cx^2\big)^{3/2}\,dx &= \frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x \\ &\quad - \sqrt{a^2cx^2-4\,acx+3\,c}cx + \frac{3}{8}\sqrt{-a^2cx^2+c}cx - \frac{c^3\arcsin(ax-2)}{a(-c)^{\frac{3}{2}}} \\ &\quad + \frac{3\,c^{\frac{3}{2}}\arcsin(ax)}{8\,a} + \frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a} + \frac{2\sqrt{a^2cx^2-4\,acx+3\,c}c}{a}\end{aligned}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c*x + 3/8*s
qrt(-a^2*c*x^2 + c)*c*x - c^3*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)
*arcsin(a*x)/a + 2/3*(-a^2*c*x^2 + c)^(3/2)/a + 2*sqrt(a^2*c*x^2 - 4*a*c*x
 + 3*c)*c/a
```

3.626.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = -\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx+8\,ac\right)x+9\,c\right)x-\frac{16\,c}{a}\right) + \frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x + 8*a*c)*x + 9*c)*x - 16*c/a) +
5/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.626.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1), x)
```

3.627 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.627.1 Optimal result

Integrand size = 24, antiderivative size = 86

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

output
```
-3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)
^(1/2)/a+1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

3.627.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]
/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

3.627.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6691, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{3}{2}\int \frac{ax+1}{\sqrt{c-a^2cx^2}}dx-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{3}{2}\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/2*((1 + a*x)*Sqrt[c - a^2*c*x^2])/(a*c) + (3*(-(Sqrt[c - a^2*c*x^2
]/(a*c)) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a*Sqrt[c])))/2))
```

3.627. $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.627.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.627. $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.627.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{(ax+4)\left(a^{2}x^{2}-1\right)c}{2a\sqrt{-c\left(a^{2}x^{2}-1\right)}}-\frac{3c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}$	69
default	$\frac{x\sqrt{-a^{2}c\,x^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}+\frac{2\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}}\right)}{\sqrt{a^{2}c}}}{a}$	136

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

3.627.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^{2}cx^{2}}\,dx$$
$$=\left[\frac{2\sqrt{-a^{2}cx^{2}+c}(ax+4)+3\sqrt{-c}\log\left(2\,a^{2}cx^{2}-2\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right)}{4\,a},\frac{\sqrt{-a^{2}cx^{2}+c}(ax+4)+3}{2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x +
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a
]
```

3.627.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.627.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}x-\frac{3\sqrt{c}\arcsin(ax)}{2a}+\frac{2\sqrt{-a^2cx^2+c}}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
 + c)/a
```

3.627.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.627.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.628 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

3.628.1 Optimal result

Integrand size = 24, antiderivative size = 59

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=-\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

output
```
arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)-2*(a*x+1)/a/(-a^2*c*x^2
+c)^(1/2)
```

3.628.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=-\frac{2\sqrt{1-a^2x^2}\left(\sqrt{1+ax}+\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*(Sqrt[1 + a*x] + Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[c - a^2*c*x^2])
```

3.628.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6717, 6691, 457, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \ 6691$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \ 457$$

$$-c\left(\frac{2(ax+1)}{ac\sqrt{c-a^2cx^2}}-\frac{\int \frac{1}{\sqrt{c-a^2cx^2}}dx}{c}\right)$$

$$\Big\downarrow \ 224$$

$$-c\left(\frac{2(ax+1)}{ac\sqrt{c-a^2cx^2}}-\frac{\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}}{c}\right)$$

$$\Big\downarrow \ 216$$

$$-c\left(\frac{2(ax+1)}{ac\sqrt{c-a^2cx^2}}-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{ac^{3/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*((2*(1 + a*x))/(a*c*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[c
 - a^2*c*x^2]]/(a*c^(3/2))))
```

3.628. $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$

3.628.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.628.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{\sqrt{a^{2}c}}+\frac{2\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}}{a^{2}c\left(x-\frac{1}{a}\right)}$	79

input
```
int((a*x+1)/(-a^2*c*x^2+c)^(1/2)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x-1/
a)*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)
```

3.628.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.59

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \left[-\frac{(ax-1)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\,(a^2cx-ac)}, -\frac{(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a^2cx^2+c}}{a^2cx-ac}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x - a*c), -((a*x - 1)*sqrt(c)*
arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x
^2 + c))/(a^2*c*x - a*c)]
```

3.628.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{ax+1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral((a*x + 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)), x)
```

3.628.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{-a^2cx^2+c}}{a^2cx-ac}+\frac{\arcsin(ax)}{a\sqrt{c}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x - a*c) + arcsin(a*x)/(a*sqrt(c))
```

3.628.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.628.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{a\,x+1}{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}\,dx$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)), x)
```

3.629 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

3.629.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{2(1+ax)}{3a\,(c-a^2cx^2)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

output
```
-2/3*(a*x+1)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

3.629.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{(2-ax)\sqrt{1+ax}\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/3*((2 - a*x)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)^(3/2)*Sqrt
[c - a^2*c*x^2])
```

3.629.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6717, 6691, 457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{\int \frac{1}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{2(ax+1)}{3ac\left(c-a^2cx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{x}{3c^2\sqrt{c-a^2cx^2}}+\frac{2(ax+1)}{3ac\left(c-a^2cx^2\right)^{3/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(c*((2*(1 + a*x))/(3*a*c*(c - a^2*c*x^2)^(3/2)) + x/(3*c^2*Sqrt[c - a^2*c
*x^2])))
```

3.629.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.629.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(ax-2)(ax+1)^{2}}{3(-a^{2}c\,x^{2}+c)^{\frac{3}{2}}a}$	31
trager	$\frac{(ax-2)\sqrt{-a^{2}c\,x^{2}+c}}{3c^{2}(ax-1)^{2}a}$	34
default	$\frac{x}{c\sqrt{-a^{2}c\,x^{2}+c}}+\frac{\frac{2}{3ac\left(x-\frac{1}{a}\right)\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}}+\frac{2\left(-2a^{2}c\left(x-\frac{1}{a}\right)-2ac\right)}{3a\,c^{2}\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}}}{a}$	127

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x-2)*(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)/a
```

3.629.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(ax-2)}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(-a^2*c*x^2 + c)*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

3.629.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{ax+1}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x - 1)), x)
```

3.629.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{x}{3\sqrt{-a^2cx^2+c}c} + \frac{2}{3\left(\sqrt{-a^2cx^2+c}a^2cx-\sqrt{-a^2cx^2+c}ac\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x - sqrt
(-a^2*c*x^2 + c)*a*c)
```

3.629.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(43) = 86.

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(ac-3\sqrt{-a^2c}\sqrt{c}\right)\operatorname{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}-\sqrt{-a^2c}ac^2\right)} - \frac{2\left(2\,a^2c+3\,a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}+\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^3c\operatorname{sgn}(x)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/3*(a*c - 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) - sqrt(-a^2*c)*a*c^
2) - 2/3*(2*a^2*c + 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sq
rt(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) + sqrt(-a^2*c + c/x^2) - sq
rt(c)/x)^3*c*sgn(x))
```

3.629.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-2)}{3\,a\,c^2\,(a\,x-1)^2}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(3/2)*(a*x - 1)),x)
```

output
```
((c - a^2*c*x^2)^(1/2)*(a*x - 2))/(3*a*c^2*(a*x - 1)^2)
```

3.630 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

3.630.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

output
```
-2/5*(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c^2
/(-a^2*c*x^2+c)^(1/2)
```

3.630.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{2+ax-4a^2x^2+2a^3x^3}{5ac^2(-1+ax)^2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/5*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(a*c^2*(-1 + a*x)^2*Sqrt[c - a^2*c*
x^2])
```

3.630.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6717, 6691, 457, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{3\int \frac{1}{(c-a^2cx^2)^{5/2}}dx}{5c}+\frac{2(ax+1)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{3\left(\frac{2\int \frac{1}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{2(ax+1)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{3\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{2(ax+1)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-(c*((2*(1 + a*x))/(5*a*c*(c - a^2*c*x^2)^(5/2)) + (3*(x/(3*c*(c - a^2*c*x
^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2*c*x^2])))/(5*c)))
```

3.630.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.630.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result
gosper	$-\frac{(ax+1)^{2}(2a^{3}x^{3}-4a^{2}x^{2}+ax+2)}{5a(-a^{2}c\,x^{2}+c)^{\frac{5}{2}}}$
trager	$\frac{(2a^{3}x^{3}-4a^{2}x^{2}+ax+2)\sqrt{-a^{2}c\,x^{2}+c}}{5c^{3}(ax-1)^{3}a(ax+1)}$
default	$\frac{x}{3c(-a^{2}c\,x^{2}+c)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{-a^{2}c\,x^{2}+c}}+\frac{\frac{2}{5ac\left(x-\frac{1}{a}\right)\left(-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\frac{8a\left(-\frac{-2a^{2}c\left(x-\frac{1}{a}\right)-2ac}{6a^{2}c^{2}\left(-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\frac{}{3a^{2}c^{3}}\right.}{5}}{a}$

input
```
int((a*x+1)/(-a^2*c*x^2+c)^(5/2)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a*x+1)^2*(2*a^3*x^3-4*a^2*x^2+a*x+2)/a/(-a^2*c*x^2+c)^(5/2)
```

3.630.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3x^3-4\,a^2x^2+ax+2)\sqrt{-a^2cx^2+c}}{5\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 -
2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

3.630.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x - 1)), x)
```

3.630.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{2}{5\left((-a^2cx^2+c)^{\frac{3}{2}}a^2cx-(-a^2cx^2+c)^{\frac{3}{2}}ac\right)} - \frac{2\,x}{5\sqrt{-a^2cx^2+c}c^2} - \frac{x}{5\,(-a^2cx^2+c)^{\frac{3}{2}}c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x - (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/
(sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

3.630.8 Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax+1}{(-a^2cx^2+c)^{\frac{5}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x - 1)), x)
```

3.630.9 Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2\right)}{5\,a\,c^3\,{\left(a\,x-1\right)}^3\,\left(a\,x+1\right)}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(5/2)*(a*x - 1)),x)
```

output
```
((c - a^2*c*x^2)^(1/2)*(a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2))/(5*a*c^3*(a*x -
1)^3*(a*x + 1))
```

3.631 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

3.631.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}} - \frac{x}{7c\,(c-a^2cx^2)^{5/2}} - \frac{4x}{21c^2\,(c-a^2cx^2)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

output

```
-2/7*(a*x+1)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/c^
2/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

3.631.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\sqrt{1-a^2x^2}(6+9ax-24a^2x^2+4a^3x^3+16a^4x^4-8a^5x^5)}{21ac^3(1-ax)^{7/2}(1+ax)^{3/2}\sqrt{c-a^2cx^2}}$$

input

```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output

```
-1/21*(Sqrt[1 - a^2*x^2]*(6 + 9*a*x - 24*a^2*x^2 + 4*a^3*x^3 + 16*a^4*x^4
- 8*a^5*x^5))/(a*c^3*(1 - a*x)^(7/2)*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])
```

3.631.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6691, 457, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{5\int \frac{1}{(c-a^2cx^2)^{7/2}}dx}{7c}+\frac{2(ax+1)}{7ac\,(c-a^2cx^2)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{5\left(\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}dx}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}+\frac{2(ax+1)}{7ac\,(c-a^2cx^2)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{5\left(\frac{4\left(\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}+\frac{2(ax+1)}{7ac\,(c-a^2cx^2)^{7/2}}\right)$$

$$\Big\downarrow 208$$

$$-c\left(\frac{5\left(\frac{4\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}+\frac{2(ax+1)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-(c*((2*(1 + a*x))/(7*a*c*(c - a^2*c*x^2)^(7/2)) + (5*(x/(5*c*(c - a^2*c*x
^2)^(5/2)) + (4*(x/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2
*c*x^2])))/(5*c)))/(7*c)))
```

3.631.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

3.631. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.631.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
gosper	$\frac{(ax+1)^2(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6)}{21a(-a^2c\,x^2+c)^{\frac{7}{2}}}$
trager	$\frac{(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6)\sqrt{-a^2c\,x^2+c}}{21c^4(ax-1)^4(ax+1)^2a}$
default	$\frac{x}{5c(-a^2c\,x^2+c)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2c\,x^2+c}}}{c}+\frac{\frac{2}{7ac\left(x-\frac{1}{a}\right)\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}-\frac{12a\left(-\frac{-2a^2c\left(x-\frac{1}{a}\right)}{10a^2c^2\left(-a^2c\left(x-\frac{1}{a}\right)^2\right.}\right.}{}}{}$

input
```
int((a*x+1)/(-a^2*c*x^2+c)^(7/2)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/21*(a*x+1)^2*(8*a^5*x^5-16*a^4*x^4-4*a^3*x^3+24*a^2*x^2-9*a*x-6)/a/(-a^2
*c*x^2+c)^(7/2)
```

3.631.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{(8\,a^5x^5-16\,a^4x^4-4\,a^3x^3+24\,a^2x^2-9\,ax-6)\sqrt{-a^2cx^2+c}}{21\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/21*(8*a^5*x^5 - 16*a^4*x^4 - 4*a^3*x^3 + 24*a^2*x^2 - 9*a*x - 6)*sqrt(-a
^2*c*x^2 + c)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 -
 a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

3.631.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{7}{2}}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x - 1)), x)
```

3.631.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{2}{7\left((-a^2cx^2+c)^{\frac{5}{2}}a^2cx-(-a^2cx^2+c)^{\frac{5}{2}}ac\right)} - \frac{8\,x}{21\,\sqrt{-a^2cx^2+c}c^3} - \frac{4\,x}{21\,(-a^2cx^2+c)^{\frac{3}{2}}c^2} - \frac{x}{7\,(-a^2cx^2+c)^{\frac{5}{2}}c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x - (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x
/(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/
((-a^2*c*x^2 + c)^(5/2)*c)
```

3.631.8 Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax+1}{(-a^2cx^2+c)^{\frac{7}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x - 1)), x)
```

3.631.9 Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,(a\,x-1)^3} - \frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,(a\,x-1)^4} - \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}+\frac{5}{28\,a\,c^4}\right)}{(a\,x-1)^2\,(a\,x+1)^2} + \frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,(a\,x-1)\,(a\,x+1)}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(7/2)*(a*x - 1)),x)
```

output
```
(c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x - 1)^3) - (c - a^2*c*x^2)^(1/2)/(28*a
*c^4*(a*x - 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) + 5/(28*a*c^4)
))/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x -
1)*(a*x + 1))
```

3.632 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

3.632.1 Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}} - \frac{8x}{45c^3\,(c-a^2cx^2)^{3/2}} - \frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

output
```
-2/9*(a*x+1)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/c^
2/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2*c
*x^2+c)^(1/2)
```

3.632.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\sqrt{1-a^2x^2}(-10-25ax+60a^2x^2+10a^3x^3-80a^4x^4+24a^5x^5+32a^6x^6-16a^7x^7)}{45ac^4(1-ax)^{9/2}(1+ax)^{5/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

output
```
(Sqrt[1 - a^2*x^2]*(-10 - 25*a*x + 60*a^2*x^2 + 10*a^3*x^3 - 80*a^4*x^4 +
24*a^5*x^5 + 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(9/2)*(1 + a*x)
^(5/2)*Sqrt[c - a^2*c*x^2])
```

3.632.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6717, 6691, 457, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{11/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{7\int \frac{1}{(c-a^2cx^2)^{9/2}}dx}{9c}+\frac{2(ax+1)}{9ac\,(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}dx}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}+\frac{2(ax+1)}{9ac\,(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}dx}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}+\frac{2(ax+1)}{9ac\,(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow 209$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}+\frac{2(ax+1)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

$$\Big\downarrow 208$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}+\frac{2(ax+1)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

3.632. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx$

output
```
-(c*((2*(1 + a*x))/(9*a*c*(c - a^2*c*x^2)^(9/2)) + (7*(x/(7*c*(c - a^2*c*x
^2)^(7/2)) + (6*(x/(5*c*(c - a^2*c*x^2)^(5/2)) + (4*(x/(3*c*(c - a^2*c*x^2
)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2*c*x^2])))/(5*c)))/(7*c)))/(9*c)))
```

3.632.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.632. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx$

3.632.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

method	result
gosper	$-\frac{(ax+1)^{2}\left(16a^{7}x^{7}-32a^{6}x^{6}-24a^{5}x^{5}+80a^{4}x^{4}-10a^{3}x^{3}-60a^{2}x^{2}+25ax+10\right)}{45a\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{2}}}$
trager	$\frac{\left(16a^{7}x^{7}-32a^{6}x^{6}-24a^{5}x^{5}+80a^{4}x^{4}-10a^{3}x^{3}-60a^{2}x^{2}+25ax+10\right)\sqrt{-a^{2}c\,x^{2}+c}}{45c^{5}(ax-1)^{5}(ax+1)^{3}a}$
default	$16a$ $\frac{x}{7c\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^{2}\sqrt{-a^{2}c\,x^{2}+c}}\right)}{7c}}{c}+\frac{2}{9ac\left(x-\frac{1}{a}\right)\left(-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{7}{2}}}-$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(a*x+1)^2*(16*a^7*x^7-32*a^6*x^6-24*a^5*x^5+80*a^4*x^4-10*a^3*x^3-60
*a^2*x^2+25*a*x+10)/a/(-a^2*c*x^2+c)^(9/2)
```

3.632.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^{2}cx^{2}\right)^{9/2}}\,dx=\frac{\left(16\,a^{7}x^{7}-32\,a^{6}x^{6}-24\,a^{5}x^{5}+80\,a^{4}x^{4}-10\,a^{3}x^{3}-60\,a^{2}x^{2}+25\,ax+10\right)\sqrt{-a^{2}cx^{2}+}}{45\left(a^{9}c^{5}x^{8}-2\,a^{8}c^{5}x^{7}-2\,a^{7}c^{5}x^{6}+6\,a^{6}c^{5}x^{5}-6\,a^{4}c^{5}x^{3}+2\,a^{3}c^{5}x^{2}+2\,a^{2}c^{5}x-ac^{5}\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
1/45*(16*a^7*x^7 - 32*a^6*x^6 - 24*a^5*x^5 + 80*a^4*x^4 - 10*a^3*x^3 - 60*
a^2*x^2 + 25*a*x + 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 -
 2*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5
*x - a*c^5)
```

3.632.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax+1}{(-c\,(ax-1)\,(ax+1))^{\frac{9}{2}}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x - 1)), x)
```

3.632.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{2}{9\left((-a^2cx^2+c)^{\frac{7}{2}}a^2cx-(-a^2cx^2+c)^{\frac{7}{2}}ac\right)} - \frac{16\,x}{45\sqrt{-a^2cx^2+c}\,c^4} - \frac{8\,x}{45\,(-a^2cx^2+c)^{\frac{3}{2}}c^3} - \frac{2\,x}{15\,(-a^2cx^2+c)^{\frac{5}{2}}c^2} - \frac{x}{9\,(-a^2cx^2+c)^{\frac{7}{2}}c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x - (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*
x/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*
x/((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

3.632.8 Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax+1}{(-a^2cx^2+c)^{\frac{9}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*x + 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x - 1)), x)
```

3.632.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,(a\,x-1)^5}-\frac{5\,\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,(a\,x-1)^4}+\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}+\frac{5}{24\,a\,c^5}\right)}{(a\,x-1)^3\,(a\,x+1)^3}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}-\frac{5}{144\,a\,c^5}\right)}{(a\,x-1)^2\,(a\,x+1)^2}+\frac{16\,x\,\sqrt{c-a^2\,c\,x^2}}{45\,c^5\,(a\,x-1)\,(a\,x+1)}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(9/2)*(a*x - 1)),x)
```

output
```
(c - a^2*c*x^2)^(1/2)/(72*a*c^5*(a*x - 1)^5) - (5*(c - a^2*c*x^2)^(1/2))/(
144*a*c^5*(a*x - 1)^4) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) + 5/(24*
a*c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5
) - 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2
))/(45*c^5*(a*x - 1)*(a*x + 1))
```

3.633 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

3.633.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{8(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

output
```
-8/7*(a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(a*x+
1)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(a*x+1)^9*(-a^2
*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c
)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

3.633.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(1+ax)^7\sqrt{c-a^2cx^2}(-44+98ax-77a^2x^2+21a^3x^3)}{210a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

output
```
(c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(-44 + 98*a*x - 77*a^2*x^2 + 21*a^3*x
^3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.633.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2} e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2} \int e^{3\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{9/2} x^9 dx}{x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2} \int -(1-ax)^3(ax+1)^6 dx}{a^9x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{9/2} \int (1-ax)^3(ax+1)^6 dx}{a^9x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{9/2} \int \left(-(ax+1)^9+6(ax+1)^8-12(ax+1)^7+8(ax+1)^6\right) dx}{a^9x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(ax+1)^{10}}{10a}+\frac{2(ax+1)^9}{3a}-\frac{3(ax+1)^8}{2a}+\frac{8(ax+1)^7}{7a}\right) \left(c-a^2cx^2\right)^{9/2}}{a^9x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

output
```
-(((c - a^2*c*x^2)^(9/2)*((8*(1 + a*x)^7)/(7*a) - (3*(1 + a*x)^8)/(2*a) +
(2*(1 + a*x)^9)/(3*a) - (1 + a*x)^10/(10*a)))/(a^9*(1 - 1/(a^2*x^2))^(9/2)
*x^9))
```

3.633.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.633.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{x\left(21a^{9}x^{9}+70a^{8}x^{8}-240a^{6}x^{6}-210a^{5}x^{5}+252a^{4}x^{4}+420a^{3}x^{3}-315ax-210\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{2}}}{210\left(ax+1\right)^{6}\left(ax-1\right)^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
default	$\frac{\left(21a^{9}x^{9}+70a^{8}x^{8}-240a^{6}x^{6}-210a^{5}x^{5}+252a^{4}x^{4}+420a^{3}x^{3}-315ax-210\right)x\,c^{4}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{210\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	102

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE
)
```

output
```
1/210*x*(21*a^9*x^9+70*a^8*x^8-240*a^6*x^6-210*a^5*x^5+252*a^4*x^4+420*a^3
*x^3-315*a*x-210)*(-a^2*c*x^2+c)^(9/2)/(a*x+1)^6/(a*x-1)^3/((a*x-1)/(a*x+1
))^(3/2)
```

3.633.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int e^{3\coth^{-1}(ax)}\left(c -a^2cx^2\right)^{9/2}\,dx = \frac{(21\,a^9c^4x^{10} + 70\,a^8c^4x^9 - 240\,a^6c^4x^7 - 210\,a^5c^4x^6 + 252\,a^4c^4x^5 + 420\,a^3c^4x^4 - 315\,ac^4x^2}{210\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fri
cas")
```

output
```
1/210*(21*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 240*a^6*c^4*x^7 - 210*a^5*c^4*x^
6 + 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 - 210*c^4*x)*sqrt(-a
^2*c)/a
```

3.633.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Timed out
```

3.633.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(21\,a^{11}\sqrt{-c}c^4x^{11}+49\,a^{10}\sqrt{-c}c^4x^{10}-70\,a^9\sqrt{-c}c^4x^9-240\,a^8\sqrt{-c}c^4x^8+30\,a^7\sqrt{-c}c^4x^7+\right.}{210}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="max
ima")
```

output
```
1/210*(21*a^11*sqrt(-c)*c^4*x^11 + 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt
(-c)*c^4*x^9 - 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 + 462*a^
6*sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 - 420*a^4*sqrt(-c)*c^4*x^4 -
 315*a^3*sqrt(-c)*c^4*x^3 + 105*a^2*sqrt(-c)*c^4*x^2 + 210*sqrt(-c)*c^4)*(
a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

3.633.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="gia
c")
```

output
```
integrate((-a^2*c*x^2 + c)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.633.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.634 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

3.634.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

output
```
2/3*(a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(a*x+1)
^7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*
x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

3.634.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.45

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(1+ax)^6\left(37-54ax+21a^2x^2\right)\sqrt{c-a^2cx^2}}{168a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/168*(c^3*(1 + a*x)^6*(37 - 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a
^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.634.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2} e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7dx}{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int (1-ax)^2(ax+1)^5dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int \left((ax+1)^7-4(ax+1)^6+4(ax+1)^5\right)dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(ax+1)^8}{8a}-\frac{4(ax+1)^7}{7a}+\frac{2(ax+1)^6}{3a}\right)\left(c-a^2cx^2\right)^{7/2}}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

output
```
((c - a^2*c*x^2)^(7/2)*((2*(1 + a*x)^6)/(3*a) - (4*(1 + a*x)^7)/(7*a) + (1
 + a*x)^8/(8*a)))/(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

3.634.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.634.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x\left(21a^{7}x^{7}+72a^{6}x^{6}+28a^{5}x^{5}-168a^{4}x^{4}-210a^{3}x^{3}+56a^{2}x^{2}+252ax+168\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}{168\left(ax-1\right)^{2}\left(ax+1\right)^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
default	$-\frac{\left(21a^{7}x^{7}+72a^{6}x^{6}+28a^{5}x^{5}-168a^{4}x^{4}-210a^{3}x^{3}+56a^{2}x^{2}+252ax+168\right)x\,c^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{168\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	102

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
1/168*x*(21*a^7*x^7+72*a^6*x^6+28*a^5*x^5-168*a^4*x^4-210*a^3*x^3+56*a^2*x
^2+252*a*x+168)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^2/(a*x+1)^5/((a*x-1)/(a*x+1))
^(3/2)
```

3.634.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^7c^3x^8+72\,a^6c^3x^7+28\,a^5c^3x^6-168\,a^4c^3x^5-210\,a^3c^3x^4+56\,a^2c^3x^3+252\,ac^3x^2+168\,c^3x\right)\sqrt{-a^2c}}{168\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

output
```
-1/168*(21*a^7*c^3*x^8 + 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 - 168*a^4*c^3*x^5
 - 210*a^3*c^3*x^4 + 56*a^2*c^3*x^3 + 252*a*c^3*x^2 + 168*c^3*x)*sqrt(-a^2
*c)/a
```

3.634.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

3.634.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^9\sqrt{-c}c^3x^9+51\,a^8\sqrt{-c}c^3x^8-44\,a^7\sqrt{-c}c^3x^7-196\,a^6\sqrt{-c}c^3x^6-42\,a^5\sqrt{-c}c^3x^5+266\,a^4\sqrt{-c}c^3x^4+\right.}{168\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

output
```
-1/168*(21*a^9*sqrt(-c)*c^3*x^9 + 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c
)*c^3*x^7 - 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 + 266*a^4*s
qrt(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 - 84*a^2*sqrt(-c)*c^3*x^2 - 168
*sqrt(-c)*c^3)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

3.634.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

output
```
integrate((-a^2*c*x^2 + c)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.634.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.635 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

3.635.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{2(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

output
```
-2/5*(a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1
)^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

3.635.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(1+ax)^5(-7+5ax)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

output
```
(c^2*(1 + a*x)^5*(-7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

3.635.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2} e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5dx}{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int -\left((1-ax)(ax+1)^4\right)dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{5/2}\int (1-ax)(ax+1)^4dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{5/2}\int \left(2(ax+1)^4-(ax+1)^5\right)dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{2(ax+1)^5}{5a}-\frac{(ax+1)^6}{6a}\right)\left(c-a^2cx^2\right)^{5/2}}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

output
```
-(((c - a^2*c*x^2)^(5/2)*((2*(1 + a*x)^5)/(5*a) - (1 + a*x)^6/(6*a)))/(a^5
*(1 - 1/(a^2*x^2))^(5/2)*x^5))
```

3.635.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.635.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result	size
gosper	$\frac{x\left(5a^{5}x^{5}+18a^{4}x^{4}+15a^{3}x^{3}-20a^{2}x^{2}-45ax-30\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}{30\left(ax+1\right)^{4}\left(ax-1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	84
default	$\frac{\left(5a^{5}x^{5}+18a^{4}x^{4}+15a^{3}x^{3}-20a^{2}x^{2}-45ax-30\right)x\,c^{2}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{30\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	86

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/30*x*(5*a^5*x^5+18*a^4*x^4+15*a^3*x^3-20*a^2*x^2-45*a*x-30)*(-a^2*c*x^2+
c)^(5/2)/(a*x+1)^4/(a*x-1)/((a*x-1)/(a*x+1))^(3/2)
```

3.635.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(5\,a^5c^2x^6+18\,a^4c^2x^5+15\,a^3c^2x^4-20\,a^2c^2x^3-45\,ac^2x^2-30\,c^2x)\sqrt{-a^2c}}{30\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
1/30*(5*a^5*c^2*x^6 + 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 - 4
5*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

3.635.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.635.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(5\,a^7\sqrt{-c}c^2x^7+13\,a^6\sqrt{-c}c^2x^6-3\,a^5\sqrt{-c}c^2x^5-35\,a^4\sqrt{-c}c^2x^4-25\,a^3\sqrt{-c}c^2x^3+15\,a^2}{30\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
1/30*(5*a^7*sqrt(-c)*c^2*x^7 + 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^
2*x^5 - 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 + 15*a^2*sqrt(-c
)*c^2*x^2 + 30*sqrt(-c)*c^2)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1
))
```

3.635.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.635.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.636 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

3.636.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
1/4*(a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

3.636.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c\sqrt{c-a^2cx^2}(4+6ax+4a^2x^2+a^3x^3)}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/4*(c*Sqrt[c - a^2*c*x^2]*(4 + 6*a*x + 4*a^2*x^2 + a^3*x^3))/(a*Sqrt[1 -
 1/(a^2*x^2)])
```

3.636.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3dx}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int (ax+1)^3dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(ax+1)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

3.636.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.636.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{(ax-1)(ax+1)^{2}\sqrt{-c(a^{2}x^{2}-1)}\,c}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	48
gosper	$\frac{x(a^{3}x^{3}+4a^{2}x^{2}+6ax+4)\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{4(ax+1)^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	60

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*c/a
```

3.636.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int e^{3\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{3/2}\,dx=-\frac{\left(a^{3}cx^{4}+4\,a^{2}cx^{3}+6\,acx^{2}+4\,cx\right)\sqrt{-a^{2}c}}{4\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/4*(a^3*c*x^4 + 4*a^2*c*x^3 + 6*a*c*x^2 + 4*c*x)*sqrt(-a^2*c)/a
```

3.636.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int \frac{\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.636.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(40) = 80$.

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = -\frac{\left(a^5\sqrt{-c}cx^5+3\,a^4\sqrt{-c}cx^4+2\,a^3\sqrt{-c}cx^3-2\,a^2\sqrt{-c}cx^2-4\,\sqrt{-c}c\right)(ax+1)^2}{4\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
-1/4*(a^5*sqrt(-c)*c*x^5 + 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 - 2
*a^2*sqrt(-c)*c*x^2 - 4*sqrt(-c)*c)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(
a*x - 1))
```

3.636.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.637 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.637.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1
/2)
```

3.637.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/(Sqrt[1 - 1
/(a^2*x^2)]*x)
```

3.637.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax+\frac{4}{1-ax}-3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/(a*Sqrt[1
- 1/(a^2*x^2)]*x))
```

---

3. 637. $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.637.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.637.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\left(a^{2} x^{2}+6 a x+8 \ln \left(a x-1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x-1\right)}{2 a \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	67

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

3.637.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(a^2x^2+6\,ax+8\,\log(ax-1))\sqrt{-a^2c}}{2\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

3.637.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.637.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.637.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.637.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.638 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

3.638.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

output
```
2*x*(1-1/a^2/x^2)^(1/2)/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+x*ln(-a*x+1)*(1-1/a^
2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

3.638.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x\left(\frac{2}{1-ax}+\log(1-ax)\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2/(1 - a*x) + Log[1 - a*x]))/Sqrt[c - a^2*c*x^2]
```

3.638.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \frac{ax+1}{(1-ax)^2}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{1}{ax-1}+\frac{2}{(ax-1)^2}\right)dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2/(a*(1 - a*x)) + Log[1 - a*x]/a))/Sqrt[c - a^
2*c*x^2]
```

3.638.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.638.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{-c(a^{2}x^{2}-1)}\,(a\ln(ax-1)x-\ln(ax-1)-2)}{ac(ax+1)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x-1)*x-ln(a*x-1)-2)/a/c/(a*x+1)^2/((a*x-1)
/(a*x+1))^(3/2)
```

3.638.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=-\frac{\sqrt{-a^2c}((ax-1)\log(ax-1)-2)}{a^3cx-a^2c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-sqrt(-a^2*c)*((a*x - 1)*log(a*x - 1) - 2)/(a^3*c*x - a^2*c)
```

3.638.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

3.638.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.638.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.638.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.639 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

3.639.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)^2\,(c-a^2cx^2)^{3/2}}$$

output
```
-1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

3.639.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}{2c^2(-1+ax)^3(1+ax)}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)^3*(1 +
a*x))
```

3.639.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(ax-1)^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)^2(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/2*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/((1 - a*x)^2*(c - a^2*c*x^2)^(3/2))
```

3.639.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.639.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{a x -1}{2 a \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}$	39
default	$-\frac{\sqrt{-c \left(a^{2} x^{2}-1\right)}}{2 \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(a x +1\right) \left(a^{2} x^{2}-1\right) c^{2} a}$	56

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/2*(a*x-1)/a/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

3.639.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{\sqrt{-a^{2} c}}{2 \,{\left(a^{4} c^{2} x^{2} - 2 \, a^{3} c^{2} x + a^{2} c^{2}\right)}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

3.639.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.639.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.639.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.639.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\frac{1}{2\,a^3\,c}+\frac{x}{2\,a^2\,c}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}+x^2\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x\,\sqrt{c-a^2\,c\,x^2}}{a}}$$

input
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
((1/(2*a^3*c) + x/(2*a^2*c))*((a*x - 1)/(a*x + 1))^(1/2))/((c - a^2*c*x^2)
^(1/2)/a^2 + x^2*(c - a^2*c*x^2)^(1/2) - (2*x*(c - a^2*c*x^2)^(1/2))/a)
```

3.640 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

3.640.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2
)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*
arctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

3.640.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-10+9ax-3a^2x^2+3(-1+ax)^3\operatorname{arctanh}(ax)\right)}{24c^2(-1+ax)^3\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh
[a*x]))/(24*c^2*(-1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

3.640.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)^4(ax+1)}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{8(ax-1)^2}-\frac{1}{4(ax-1)^3}+\frac{1}{2(ax-1)^4}-\frac{1}{8(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\text{arctanh}(ax)}{8a}+\frac{1}{8a(1-ax)}+\frac{1}{8a(1-ax)^2}+\frac{1}{6a(1-ax)^3}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(6*a*(1 - a*x)^3) + 1/(8*a*(1 - a*x)^2
) + 1/(8*a*(1 - a*x)) + ArcTanh[a*x]/(8*a)))/(c - a^2*c*x^2)^(5/2)
```

3.640. $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$

3.640.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.640.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-9a^2\ln(ax+1)x^2+9a^2\ln(ax-1)x^2-6a^2x^2+9a\ln(ax+1)x-9a\ln(ax-1)x+18a}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)(a^2x^2-1)c^3a}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/48/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^
3*ln(a*x+1)*x^3-3*a^3*ln(a*x-1)*x^3-9*a^2*ln(a*x+1)*x^2+9*a^2*ln(a*x-1)*x^
2-6*a^2*x^2+9*a*ln(a*x+1)*x-9*a*ln(a*x-1)*x+18*a*x-3*ln(a*x+1)+3*ln(a*x-1)
-20)/(a^2*x^2-1)/c^3/a
```

3.640.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{3\left(a^4x^3-3\,a^3x^2+3\,a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(3\,a^2x^2-9\,ax+10\right)\sqrt{-a^2c}}{48\left(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-a^2c^3\right)}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output

```
-1/48*(3*(a^4*x^3 - 3*a^3*x^2 + 3*a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*s
qrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 - 9*a*x + 10)*sq
rt(-a^2*c))/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^3)
```

3.640.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output

```
Timed out
```

3.640.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output

```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.640.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.640.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{1}{(c-a^2\,c\,x^2)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.641 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

3.641.1 Optimal result

Integrand size = 24, antiderivative size = 278

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = & -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}} \\ & -\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}} \\ & +\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}\end{aligned}$$

output
```
-1/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6
*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2
/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2
)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+
1)/(-a^2*c*x^2+c)^(7/2)-5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^
2*c*x^2+c)^(7/2)
```

3.641.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(32-15ax-35a^2x^2+45a^3x^3-15a^4x^4+15(-1+ax)^4(1+ax)\operatorname{arctanh}}{96c^3(-1+ax)^4(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(32 - 15*a*x - 35*a^2*x^2 + 45*a^3*x^3 - 15*a^4*x
^4 + 15*(-1 + a*x)^4*(1 + a*x)*ArcTanh[a*x]))/(96*c^3*(-1 + a*x)^4*(1 + a*
x)*Sqrt[c - a^2*c*x^2])
```

3.641.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int -\frac{1}{(1-ax)^5(ax+1)^2}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{1}{(1-ax)^5(ax+1)^2}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \left(\frac{1}{8(ax-1)^2}+\frac{1}{32(ax+1)^2}-\frac{3}{16(ax-1)^3}+\frac{1}{4(ax-1)^4}-\frac{1}{4(ax-1)^5}-\frac{5}{32(a^2x^2-1)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{5\text{arctanh}(ax)}{32a}+\frac{1}{8a(1-ax)}-\frac{1}{32a(ax+1)}+\frac{3}{32a(1-ax)^2}+\frac{1}{12a(1-ax)^3}+\frac{1}{16a(1-ax)^4}\right)}{\left(c-a^2cx^2\right)^{7/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-((a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7*(1/(16*a*(1 - a*x)^4) + 1/(12*a*(1 - a*
x)^3) + 3/(32*a*(1 - a*x)^2) + 1/(8*a*(1 - a*x)) - 1/(32*a*(1 + a*x)) + (5
*ArcTanh[a*x])/(32*a)))/(c - a^2*c*x^2)^(7/2))
```

3.641.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.641. $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

3.641.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-45\ln(ax+1)x^4a^4+45\ln(ax-1)x^4a^4-30a^4x^4+30a^3\ln(ax+1)x^3-30a^3\ln(a}{192\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*
(15*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5-45*ln(a*x+1)*x^4*a^4+45*ln(a*x-
1)*x^4*a^4-30*a^4*x^4+30*a^3*ln(a*x+1)*x^3-30*a^3*ln(a*x-1)*x^3+90*a^3*x^3
+30*a^2*ln(a*x+1)*x^2-30*a^2*ln(a*x-1)*x^2-70*a^2*x^2-45*a*ln(a*x+1)*x+45*
a*ln(a*x-1)*x-30*a*x+15*ln(a*x+1)-15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a
```

3.641.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{15\,(a^6x^5-3\,a^5x^4+2\,a^4x^3+2\,a^3x^2-3\,a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(15\,a^4x^4-45\,a^3x^3+3}{192\,(a^7c^4x^5-3\,a^6c^4x^4+2\,a^5c^4x^3+2\,a^4c^4x^2-3\,a^3c^4x+a^2c^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

output
```
-1/192*(15*(a^6*x^5 - 3*a^5*x^4 + 2*a^4*x^3 + 2*a^3*x^2 - 3*a^2*x + a)*sqr
t(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(
15*a^4*x^4 - 45*a^3*x^3 + 35*a^2*x^2 + 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4
*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c
^4)
```

3.641.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

3.641.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.641.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.641.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\int \frac{1}{(c-a^2\,c\,x^2)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input

```
int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output

```
int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.642 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

3.642.1 Optimal result

Integrand size = 24, antiderivative size = 234

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{8(1-ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

output
```
8/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(-a*
x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(-a*x+1)^8*(-a^
2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(-a*x+1)^9*(-a^2*c*x^2+c
)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x^2+c)^(9/2)
/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

3.642.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(-1+ax)^6\sqrt{c-a^2cx^2}\left(193+528ax+588a^2x^2+308a^3x^3+63a^4x^4\right)}{630a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

output
```
(c^4*(-1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 + 528*a*x + 588*a^2*x^2 + 308*a
^3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.642.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.47, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \ 6746$$

$$\frac{\left(c-a^2cx^2\right)^{9/2} \int e^{-\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{9/2} x^9 dx}{x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\left(c-a^2cx^2\right)^{9/2} \int -(1-ax)^5(ax+1)^4 dx}{a^9x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\left(c-a^2cx^2\right)^{9/2} \int (1-ax)^5(ax+1)^4 dx}{a^9x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \ 49$$

$$-\frac{\left(c-a^2cx^2\right)^{9/2} \int \left((1-ax)^9-8(1-ax)^8+24(1-ax)^7-32(1-ax)^6+16(1-ax)^5\right) dx}{a^9x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\left(-\frac{(1-ax)^{10}}{10a}+\frac{8(1-ax)^9}{9a}-\frac{3(1-ax)^8}{a}+\frac{32(1-ax)^7}{7a}-\frac{8(1-ax)^6}{3a}\right) \left(c-a^2cx^2\right)^{9/2}}{a^9x^9 \left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Int[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

output
```
-(((c - a^2*c*x^2)^(9/2)*((-8*(1 - a*x)^6)/(3*a) + (32*(1 - a*x)^7)/(7*a)
- (3*(1 - a*x)^8)/a + (8*(1 - a*x)^9)/(9*a) - (1 - a*x)^10/(10*a)))/(a^9*(
1 - 1/(a^2*x^2))^(9/2)*x^9))
```

3.642.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.642.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(63 a^{9} x^{9}-70 a^{8} x^{8}-315 a^{7} x^{7}+360 a^{6} x^{6}+630 a^{5} x^{5}-756 a^{4} x^{4}-630 a^{3} x^{3}+840 a^{2} x^{2}+315 a x -630\right) x \,c^{4} \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{a x -1}{a x +1}}}{630 a x -630}$	113
gosper	$\frac{x \left(63 a^{9} x^{9}-70 a^{8} x^{8}-315 a^{7} x^{7}+360 a^{6} x^{6}+630 a^{5} x^{5}-756 a^{4} x^{4}-630 a^{3} x^{3}+840 a^{2} x^{2}+315 a x -630\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{9}{2}} \sqrt{\frac{a x -1}{a x +1}}}{630 \left(a x +1\right)^{4} \left(a x -1\right)^{5}}$	116

input
```
int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/630*(63*a^9*x^9-70*a^8*x^8-315*a^7*x^7+360*a^6*x^6+630*a^5*x^5-756*a^4*x
^4-630*a^3*x^3+840*a^2*x^2+315*a*x-630)*x*c^4*(-c*(a^2*x^2-1))^(1/2)*((a*x
-1)/(a*x+1))^(1/2)/(a*x-1)
```

3.642.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.50

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{(63\,a^9c^4x^{10}-70\,a^8c^4x^9-315\,a^7c^4x^8+360\,a^6c^4x^7+630\,a^5c^4x^6-756\,a^4c^4x^5-630\,a^3c^4x^4}{630\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
1/630*(63*a^9*c^4*x^10 - 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 + 360*a^6*c^4*x^
7 + 630*a^5*c^4*x^6 - 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 + 840*a^2*c^4*x^3
+ 315*a*c^4*x^2 - 630*c^4*x)*sqrt(-a^2*c)/a
```

3.642.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.642.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.642.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.642.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{9/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.643 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

3.643.1 Optimal result

Integrand size $=24$, antiderivative size $=187$

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{8(1-ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

output
```
-8/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(-a*x+1
)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(-a*x+1)^7*(-a^2*
c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2*c*x^2+c)^(
7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

3.643.2 Mathematica [A] (verified)

Time $=0.06$ (sec) , antiderivative size $=71$, normalized size of antiderivative $=0.38$

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(-1+ax)^5\sqrt{c-a^2cx^2}\left(93+185ax+135a^2x^2+35a^3x^3\right)}{280a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

output
```
-1/280*(c^3*(-1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(93 + 185*a*x + 135*a^2*x^2 +
 35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.643.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int e^{-\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7dx}{x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int (1-ax)^4(ax+1)^3dx}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int \left(-(1-ax)^7+6(1-ax)^6-12(1-ax)^5+8(1-ax)^4\right) dx}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(1-ax)^8}{8a}-\frac{6(1-ax)^7}{7a}+\frac{2(1-ax)^6}{a}-\frac{8(1-ax)^5}{5a}\right) \left(c-a^2cx^2\right)^{7/2}}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Int[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

output
```
((c - a^2*c*x^2)^(7/2)*((-8*(1 - a*x)^5)/(5*a) + (2*(1 - a*x)^6)/a - (6*(1
 - a*x)^7)/(7*a) + (1 - a*x)^8/(8*a)))/(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

3.643.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.643.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{\left(35 a^{7} x^{7}-40 a^{6} x^{6}-140 a^{5} x^{5}+168 a^{4} x^{4}+210 a^{3} x^{3}-280 a^{2} x^{2}-140 a x+280\right) x\, c^{3} \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{a x-1}{a x+1}}}{280 \left(a x-1\right)}$	97
gosper	$\frac{x \left(35 a^{7} x^{7}-40 a^{6} x^{6}-140 a^{5} x^{5}+168 a^{4} x^{4}+210 a^{3} x^{3}-280 a^{2} x^{2}-140 a x+280\right) \left(-a^{2} c\, x^{2}+c\right)^{\frac{7}{2}} \sqrt{\frac{a x-1}{a x+1}}}{280 \left(a x-1\right)^{4} \left(a x+1\right)^{3}}$	100

input
```
int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/280*(35*a^7*x^7-40*a^6*x^6-140*a^5*x^5+168*a^4*x^4+210*a^3*x^3-280*a^2*
x^2-140*a*x+280)*x*c^3*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x
-1)
```

3.643.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx =$$
$$-\frac{\left(35\,a^{7}c^{3}x^{8}-40\,a^{6}c^{3}x^{7}-140\,a^{5}c^{3}x^{6}+168\,a^{4}c^{3}x^{5}+210\,a^{3}c^{3}x^{4}-280\,a^{2}c^{3}x^{3}-140\,ac^{3}x^{2}+280\,c^{3}x\right)\sqrt{-a^{2}}}{280\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
-1/280*(35*a^7*c^3*x^8 - 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 + 168*a^4*c^3*x^
5 + 210*a^3*c^3*x^4 - 280*a^2*c^3*x^3 - 140*a*c^3*x^2 + 280*c^3*x)*sqrt(-a
^2*c)/a
```

3.643.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.643.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \int \left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.643.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.643.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{7/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.644 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

3.644.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

output
```
(-a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(-a*x+1)^5
*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x+1)^6*(-a^2*c*x
^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

3.644.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.45

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(-1+ax)^4\left(11+14ax+5a^2x^2\right)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

output
```
(c^2*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*S
qrt[1 - 1/(a^2*x^2)]*x)
```

3.644.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2} \int e^{-\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5dx}{x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2} \int -(1-ax)^3(ax+1)^2dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{5/2} \int (1-ax)^3(ax+1)^2dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{5/2} \int \left((1-ax)^5-4(1-ax)^4+4(1-ax)^3\right)dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(1-ax)^6}{6a}+\frac{4(1-ax)^5}{5a}-\frac{(1-ax)^4}{a}\right) \left(c-a^2cx^2\right)^{5/2}}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Int[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

output
```
-(((c - a^2*c*x^2)^(5/2)*(-((1 - a*x)^4/a) + (4*(1 - a*x)^5)/(5*a) - (1 -
a*x)^6/(6*a)))/(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5))
```

3.644.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.644.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\left(5 a^{5} x^{5}-6 a^{4} x^{4}-15 a^{3} x^{3}+20 a^{2} x^{2}+15 a x -30\right) x \,c^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{a x -1}{a x +1}}}{30 a x -30}$	81
gosper	$\frac{x \left(5 a^{5} x^{5}-6 a^{4} x^{4}-15 a^{3} x^{3}+20 a^{2} x^{2}+15 a x -30\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\frac{a x -1}{a x +1}}}{30 \left(a x -1\right)^{3} \left(a x +1\right)^{2}}$	84

input
```
int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*x*c^2*(-c*(a^2*
x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

3.644.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int e^{-\coth^{-1}(ax)}(c - a^2cx^2)^{5/2}\,dx = \frac{(5\,a^5c^2x^6 - 6\,a^4c^2x^5 - 15\,a^3c^2x^4 + 20\,a^2c^2x^3 + 15\,ac^2x^2 - 30\,c^2x)\sqrt{-a^2c}}{30\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
1/30*(5*a^5*c^2*x^6 - 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 + 15
*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

3.644.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c - a^2cx^2)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.644.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}(c - a^2cx^2)^{5/2}\,dx = \int (-a^2cx^2 + c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.644.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.644.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.645 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

3.645.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
-2/3*(-a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(-a*x
+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

3.645.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c(-1+ax)^3(5+3ax)\sqrt{c-a^2cx^2}}{12a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

output
```
-1/12*(c*(-1 + a*x)^3*(5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^
2*x^2)]*x)
```

3.645.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3dx}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int (1-ax)^2(ax+1)dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int \left(2(1-ax)^2-(1-ax)^3\right)dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(1-ax)^4}{4a}-\frac{2(1-ax)^3}{3a}\right)\left(c-a^2cx^2\right)^{3/2}}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

output
```
((c - a^2*c*x^2)^(3/2)*((-2*(1 - a*x)^3)/(3*a) + (1 - a*x)^4/(4*a)))/(a^3*
(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

3.645.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.645.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\left(3 a^{3} x^{3}-4 a^{2} x^{2}-6 a x +12\right) x c \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{a x -1}{a x +1}}}{12 \left(a x -1\right)}$	63
gosper	$\frac{x \left(3 a^{3} x^{3}-4 a^{2} x^{2}-6 a x +12\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\frac{a x -1}{a x +1}}}{12 \left(a x +1\right) \left(a x -1\right)^{2}}$	68

input
```
int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*x*c*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(
a*x+1))^(1/2)/(a*x-1)
```

3.645.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.45

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{(3\,a^3cx^4-4\,a^2cx^3-6\,acx^2+12\,cx)\sqrt{-a^2c}}{12\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
-1/12*(3*a^3*c*x^4 - 4*a^2*c*x^3 - 6*a*c*x^2 + 12*c*x)*sqrt(-a^2*c)/a
```

3.645.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.645.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.645.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.645.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.646 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.646.1 Optimal result

Integrand size = 24, antiderivative size = 69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-
1/a^2/x^2)^(1/2)
```

3.646.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(-2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

output
```
((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.646.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int (ax-1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(1-ax)^2\sqrt{c-a^2cx^2}}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

output
```
((1 - a*x)^2*Sqrt[c - a^2*c*x^2])/(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.646.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.646.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(ax-2)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
default	$\frac{(ax-2)x\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	45

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

3.646.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2-2\,x)}{2\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

3.646.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.646.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.646.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.646.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.647 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

3.647.1 Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

output
```
x*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

3.647.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

3.647.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6746, 6747, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \; 6746$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \; 6747$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \frac{1}{ax+1}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \; 16$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

3.647.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.647.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\ln(ax+1)\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{c(ax-1)a}$	51

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/c/(a*x-1)/a
```

3.647.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=-\frac{\sqrt{-a^2c}\log{(ax+1)}}{a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

output
```
-sqrt(-a^2*c)*log(a*x + 1)/(a^2*c)
```

3.647.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.647.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

3.647.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

3.647.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input

```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2),x)
```

output

```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.648 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

3.648.1 Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\,(c-a^2cx^2)^{3/2}} - \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}(ax)}{2\,(c-a^2cx^2)^{3/2}}$$

output
```
1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a^2*(1-1/
a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

3.648.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+(1+ax)\operatorname{arctanh}(ax))}{2(c+acx)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (1 + a*x)*ArcTanh[a*x]))/(2*(c + a*c*x)*Sqr
t[c - a^2*c*x^2])
```

3.648.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 6746$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 6747$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int -\frac{1}{(1-ax)(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(1-ax)(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 54$$

$$-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{2(ax+1)^2}-\frac{1}{2(a^2x^2-1)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\text{arctanh}(ax)}{2a}-\frac{1}{2a(ax+1)}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-((a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1/2*1/(a*(1 + a*x)) + ArcTanh[a*x]/(2
*a)))/(c - a^2*c*x^2)^(3/2))
```

3.648.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.648.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(a\ln\left(ax+1\right)x-a\ln\left(ax-1\right)x+\ln\left(ax+1\right)-\ln\left(ax-1\right)-2\right)}{4\left(a^{2}x^{2}-1\right)c^{2}a}$	84

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-a*ln(a*
x-1)*x+ln(a*x+1)-ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

3.648.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{(a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\sqrt{-a^2c}}{4\left(a^3c^2x+a^2c^2\right)}$$

input

```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

output

```
-1/4*((a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)
/(a^2*x^2 - 1)) - 2*sqrt(-a^2*c))/(a^3*c^2*x + a^2*c^2)
```

3.648.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output

```
Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.648.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

output

```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

3.648.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

3.648.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2), x)
```

3.649 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

3.649.1 Optimal result

Integrand size = 24, antiderivative size = 183

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1
/a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arct
anh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

3.649.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(2-3ax-3a^2x^2+3(-1+ax)(1+ax)^2\operatorname{arctanh}(ax)\right)}{8(-1+ax)(c+acx)^2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)*(1 + a*x)^2
*ArcTanh[a*x]))/(8*(-1 + a*x)*(c + a*c*x)^2*Sqrt[c - a^2*c*x^2])
```

3.649.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)^2(ax+1)^3}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{8(ax-1)^2}+\frac{1}{4(ax+1)^2}+\frac{1}{4(ax+1)^3}-\frac{3}{8(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\text{arctanh}(ax)}{8a}+\frac{1}{8a(1-ax)}-\frac{1}{4a(ax+1)}-\frac{1}{8a(ax+1)^2}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a*(1 - a*x)) - 1/(8*a*(1 + a*x)^2)
- 1/(4*a*(1 + a*x)) + (3*ArcTanh[a*x])/(8*a)))/(c - a^2*c*x^2)^(5/2)
```

3.649.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.649.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3+3a^2\ln(ax+1)x^2-3a^2\ln(ax-1)x^2-6a^2x^2-3a\ln(ax+1)x+3a\ln(ax-}{16(ax+1)(a^2x^2-1)c^3a(ax-1)}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*ln(a*x
+1)*x^3-3*a^3*ln(a*x-1)*x^3+3*a^2*ln(a*x+1)*x^2-3*a^2*ln(a*x-1)*x^2-6*a^2*
x^2-3*a*ln(a*x+1)*x+3*a*ln(a*x-1)*x-6*a*x-3*ln(a*x+1)+3*ln(a*x-1)+4)/(a^2*
x^2-1)/c^3/a/(a*x-1)
```

3.649.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\,(a^4x^3+a^3x^2-a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(3\,a^2x^2+3\,ax-2)\sqrt{-a^2c}}{16\,(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
-1/16*(3*(a^4*x^3 + a^3*x^2 - a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(
-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 + 3*a*x - 2)*sqrt(-a
^2*c))/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

3.649.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.649.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

3.649.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

3.649.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2), x)
```

3.650 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

3.650.1 Optimal result

Integrand size = 24, antiderivative size = 276

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}$$

output

```
-1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*
(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/24*a^6*(1-1/a^2/x^
2)^(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*(1-1/a^2/x^2)^(7/2)*x
^7/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)
/(-a^2*c*x^2+c)^(7/2)-5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*
c*x^2+c)^(7/2)
```

3.650.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-8+25ax+25a^2x^2-15a^3x^3-15a^4x^4+15(-1+ax)^2(1+ax)^3\text{arcta}}{48(-1+ax)^2(c+acx)^3\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-8 + 25*a*x + 25*a^2*x^2 - 15*a^3*x^3 - 15*a^4*x
^4 + 15*(-1 + a*x)^2*(1 + a*x)^3*ArcTanh[a*x]))/(48*(-1 + a*x)^2*(c + a*c*
x)^3*Sqrt[c - a^2*c*x^2])
```

3.650.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int -\frac{1}{(1-ax)^3(ax+1)^4}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{1}{(1-ax)^3(ax+1)^4}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int\left(\frac{1}{8(ax-1)^2}+\frac{3}{16(ax+1)^2}-\frac{1}{16(ax-1)^3}+\frac{3}{16(ax+1)^3}+\frac{1}{8(ax+1)^4}-\frac{5}{16(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{5\text{arctanh}(ax)}{16a}+\frac{1}{8a(1-ax)}-\frac{3}{16a(ax+1)}+\frac{1}{32a(1-ax)^2}-\frac{3}{32a(ax+1)^2}-\frac{1}{24a(ax+1)^3}\right)}{(c-a^2cx^2)^{7/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
-((a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7*(1/(32*a*(1 - a*x)^2) + 1/(8*a*(1 - a*x
)) - 1/(24*a*(1 + a*x)^3) - 3/(32*a*(1 + a*x)^2) - 3/(16*a*(1 + a*x)) + (5
*ArcTanh[a*x])/(16*a)))/(c - a^2*c*x^2)^(7/2))
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.650.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5+15\ln(ax+1)x^4a^4-15\ln(ax-1)x^4a^4-30a^4x^4-30a^3\ln(ax+1)x^3+3}{96(ax+1}$

input

```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-1/96*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+
1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5+15*ln(a*x+1)*x^4*a^4-15*ln(a*x-1)*x^4*a^4-
30*a^4*x^4-30*a^3*ln(a*x+1)*x^3+30*a^3*ln(a*x-1)*x^3-30*a^3*x^3-30*a^2*ln(
a*x+1)*x^2+30*a^2*ln(a*x-1)*x^2+50*a^2*x^2+15*a*ln(a*x+1)*x-15*a*ln(a*x-1)
*x+50*a*x+15*ln(a*x+1)-15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x-1)^2
```

3.650.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx =$$

$$-\frac{15\left(a^6x^5+a^5x^4-2\,a^4x^3-2\,a^3x^2+a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(15\,a^4x^4+15\,a^3x^3-25\,a\right.}{96\left(a^7c^4x^5+a^6c^4x^4-2\,a^5c^4x^3-2\,a^4c^4x^2+a^3c^4x+a^2c^4\right)}$$

input

```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="frica
s")
```

output

```
-1/96*(15*(a^6*x^5 + a^5*x^4 - 2*a^4*x^3 - 2*a^3*x^2 + a^2*x + a)*sqrt(-c)
*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15*a^
4*x^4 + 15*a^3*x^3 - 25*a^2*x^2 - 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 +
 a^6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 + a^3*c^4*x + a^2*c^4)
```

3.650.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

3.650.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

3.650.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-a^2\,c\,x^2\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2), x)
```

3.651 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

3.651.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7(c-a^2cx^2)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}$$

output
```
-7/24*c*x*(-a^2*c*x^2+c)^(3/2)-7/30*(-a^2*c*x^2+c)^(5/2)/a-1/6*(-a*x+1)*(-
a^2*c*x^2+c)^(5/2)/a-7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))
/a-7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

3.651.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}(96+39ax-327a^2x^2+202a^3x^3+86a^4x^4-136a^5x^5+40a^6x^6)\right.}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(96 + 39*a*x - 327*a^2*x^2 + 202
*a^3*x^3 + 86*a^4*x^4 - 136*a^5*x^5 + 40*a^6*x^6)) + 210*Sqrt[1 - a*x]*Arc
Sin[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

3.651.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6692, 469, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2} e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int (1-ax)^2\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{7}{6}\int (1-ax)\left(c-a^2cx^2\right)^{3/2}dx+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{7}{6}\left(\int \left(c-a^2cx^2\right)^{3/2}dx+\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\int \sqrt{c-a^2cx^2}dx+\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\Bigg(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{(1-ax)\,(c-a^2c}{6ac}$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{(1-a}{}$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{(1-ax)\left(c-\right.}{6ac}$$

input
```
Int[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a*c) + (7*((x*(c - a^2*c*x^2)^(3
/2))/4 + (c - a^2*c*x^2)^(5/2)/(5*a*c) + (3*c*((x*Sqrt[c - a^2*c*x^2])/2 +
 (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)))/4))/6))
```

3.651.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.651. $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.651.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(40 a^{5} x^{5}-96 a^{4} x^{4}-10 a^{3} x^{3}+192 a^{2} x^{2}-135 a x-96\right) \left(a^{2} x^{2}-1\right) c^{3}}{240 a \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{7 \arctan\left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c^{3}}{16 \sqrt{a^{2} c}}$
default	$\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{6}+\frac{5 c \left(\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4}+\frac{3 c \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan\left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)}{4}\right)}{6}-\frac{2 \Bigg(\frac{\left(-a^{2} c \left(x+\frac{1}{a}\right)^{2}+2 \left(x+\frac{1}{a}\right) a c \right)^{\frac{5}{2}}}{5}+a c \Bigg(}{}$

input
```
int((a*x-1)*(-a^2*c*x^2+c)^(5/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*a^5*x^5-96*a^4*x^4-10*a^3*x^3+192*a^2*x^2-135*a*x-96)*(a^2*x^2-
1)/a/(-c*(a^2*x^2-1))^(1/2)*c^3-7/16/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/
(-a^2*c*x^2+c)^(1/2))*c^3
```

3.651.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.84

$$\int e^{-2\coth^{-1}(ax)}(c -a^2cx^2)^{5/2}\,dx = \left[\frac{105\sqrt{-c}c^2\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(40a^5c^2x^5-96a^4c^2x^4-10a^3\right.}{480a}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 2*(40*a^5*c^2*x^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*
c^2*x^2 - 135*a*c^2*x - 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x
^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^2*x^2 - 135*a*c^2*x - 96*
c^2)*sqrt(-a^2*c*x^2 + c))/a]
```

3.651.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(119) = 238.

Time = 2.74 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.40

$$\int e^{-2\coth^{-1}(ax)}(c -a^2cx^2)^{5/2}\,dx = \begin{cases} \dfrac{2c^2\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-2c^2\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \text{fo}\\ \frac{a^4\sqrt{c}x^4}{4} & \text{ot}\end{cases}\right.}{} \\ -c^{\frac{5}{2}}x \end{cases}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise(((2*c**2*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) - 2*c**2*Piecewise((sqrt(-a**2*c*x
**2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x**
4/4, True)) + c**2*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(
-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2),
True))/16 + sqrt(-a**2*c*x**2 + c)*(a**5*x**5/6 - a**3*x**3/24 - a*x/16),
Ne(c, 0)), (a**5*sqrt(c)*x**5/5, True)) - c**2*Piecewise((a*x*sqrt(-a**2*c
*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 +
c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c,
 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

3.651.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \frac{1}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}x - \frac{7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx - \frac{3}{4}\sqrt{a^2cx^2+4\,acx+3\,c}c^2x + \frac{5}{16}\sqrt{-a^2cx^2+c}c^2x + \frac{3\,c^4\arcsin\left(ax+2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}} + \frac{5\,c^{\frac{5}{2}}\arcsin\left(ax\right)}{16\,a} - \frac{2\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{5\,a} - \frac{3\sqrt{a^2cx^2+4\,acx+3\,c}c^2}{2\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(
a^2*c*x^2 + 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x + 3/4*c
^4*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a - 2/5*(-a^2
*c*x^2 + c)^(5/2)/a - 3/2*sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c^2/a
```

3.651.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{16\sqrt{-c}|a|} - \frac{1}{240}\sqrt{-a^2cx^2+c}\left(\left(135\,c^2-2\left(96\,ac^2-\left(5\,a^2c^2-4\left(5\,a^4c^2x-12\,a^3c^2\right)x\right)x\right)x+\frac{96\,c^2}{a}\right)$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a)
) - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 - 2*(96*a*c^2 - (5*a^2*c^2 - 4*(5
*a^4*c^2*x - 12*a^3*c^2)*x)*x)*x)*x + 96*c^2/a)
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{5/2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1), x)
```

3.652 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

3.652.1 Optimal result

Integrand size = 24, antiderivative size = 108

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

output
```
-5/12*(-a^2*c*x^2+c)^(3/2)/a-1/4*(-a*x+1)*(-a^2*c*x^2+c)^(3/2)/a-5/8*c^(3/
2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

3.652.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(-16+7ax+25a^2x^2-22a^3x^3+6a^4x^4)+30\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-16 + 7*a*x + 25*a^2*x^2 - 22*a^3*x
^3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*S
qrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

3.652.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6717, 6692, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2} e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int (1-ax)^2\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{5}{4}\int (1-ax)\sqrt{c-a^2cx^2}dx+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{5}{4}\left(\int \sqrt{c-a^2cx^2}dx+\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{5}{4}\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{5}{4}\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{5}{4}\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

input
```
Int[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(((1 - a*x)*(c - a^2*c*x^2)^(3/2))/(4*a*c) + (5*((x*Sqrt[c - a^2*c*x^2
])/2 + (c - a^2*c*x^2)^(3/2)/(3*a*c) + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(2*a)))/4))
```

3.652.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.652.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\left(6 a^{3} x^{3}-16 a^{2} x^{2}+9 a x +16\right) \left(a^{2} x^{2}-1\right) c^{2}}{24 a \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{5 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c^{2}}{8 \sqrt{a^{2} c}}$
default	$\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4}+\frac{3 c \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)}{4}-\frac{2 \left(\frac{\left(-a^{2} c \left(x +\frac{1}{a}\right)^{2}+2 \left(x +\frac{1}{a}\right) a c \right)^{\frac{3}{2}}}{3}+a c \left(-\frac{\left(-2 a^{2} c \left(x +\frac{1}{a}\right)+2 a c \right)}{}\right.\right.}{}$

input
```
int((a*x-1)*(-a^2*c*x^2+c)^(3/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3-16*a^2*x^2+9*a*x+16)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*
c^2-5/8/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^2
```

3.652.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.67

$$\int e^{-2 \coth^{-1}(ax)} \left(c - a^2 c x^2\right)^{3/2} \, dx = \left[\frac{15 \sqrt{-c} c \log\left(2 a^2 c x^2 - 2 \sqrt{-a^2 c x^2 + c} a \sqrt{-c} x - c\right) - 2 \left(6 a^3 c x^3 - 16 a^2 c x^2 + 9 a c x + 16\right.}{48 a}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) - 2*(6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 +
c))/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
 - c)) - (6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c
))/a]
```

3.652.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(97) = 194$.

Time = 2.41 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.30

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \begin{cases} \frac{2c\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(-2acx+2\sqrt{-c}\sqrt{-a^2cx^2+c}\right)}{\sqrt{-c}} & \text{for } c\neq 0\\ \frac{ax\log(ax)}{\sqrt{-a^2cx^2}} & \text{otherwis}\end{cases}\right)}{8} \\ \frac{a^3\sqrt{c}x^3}{3}\end{cases}\right.}{} \\ -c^{\frac{3}{2}}x\end{cases}$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise(((2*c*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c), Ne(
c, 0)), (a**2*sqrt(c)*x**2/2, True)) - c*Piecewise((c*Piecewise((log(-2*a*
c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x
)/sqrt(-a**2*c*x**2), True))/8 + (a**3*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2 +
 c), Ne(c, 0)), (a**3*sqrt(c)*x**3/3, True)) - c*Piecewise((a*x*sqrt(-a**2
*c*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2
+ c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(
c, 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(3/2)*x, True))
```

3.652.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = {} & \frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x \\ & -\sqrt{a^2cx^2+4\,acx+3\,c}cx+\frac{3}{8}\sqrt{-a^2cx^2+c}cx+\frac{c^3\arcsin\left(ax+2\right)}{a\left(-c\right)^{\frac{3}{2}}} \\ & +\frac{3\,c^{\frac{3}{2}}\arcsin\left(ax\right)}{8\,a}-\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a}-\frac{2\sqrt{a^2cx^2+4\,acx+3\,c}c}{a}\end{aligned}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c*x + 3/8*s
qrt(-a^2*c*x^2 + c)*c*x + c^3*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)
*arcsin(a*x)/a - 2/3*(-a^2*c*x^2 + c)^(3/2)/a - 2*sqrt(a^2*c*x^2 + 4*a*c*x
 + 3*c)*c/a
```

3.652.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = -\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx-8\,ac\right)x+9\,c\right)x+\frac{16\,c}{a}\right) + \frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\sqrt{-c}|a|}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x - 8*a*c)*x + 9*c)*x + 16*c/a) +
5/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.653 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.653.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

output
```
-3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)
^(1/2)/a-1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

3.653.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}(4-5ax+a^2x^2)+6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 -
a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

3.653.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6692, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{3}{2}\int \frac{1-ax}{\sqrt{c-a^2cx^2}}dx+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{3}{2}\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a*c) + (3*(Sqrt[c - a^2*c*x^2]/(a*
c) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a*Sqrt[c])))/2))
```

3.653. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.653.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.653. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.653.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(ax-4)\left(a^{2}x^{2}-1\right)c}{2a\sqrt{-c\left(a^{2}x^{2}-1\right)}}-\frac{3c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}$	69
default	$\frac{x\sqrt{-a^{2}c\,x^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}-\frac{2\left(\sqrt{-a^{2}c\left(x+\frac{1}{a}\right)^{2}+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\left(x+\frac{1}{a}\right)^{2}+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^{2}c}}\right)}{a}$	127

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

3.653.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^{2}cx^{2}}\,dx$$
$$=\left[\frac{2\sqrt{-a^{2}cx^{2}+c}(ax-4)+3\sqrt{-c}\log\left(2\,a^{2}cx^{2}-2\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right)}{4\,a},\frac{\sqrt{-a^{2}cx^{2}+c}(ax-4)+3}{2}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x -
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a
]
```

3.653.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.653.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}x-\frac{3\sqrt{c}\arcsin{(ax)}}{2a}-\frac{2\sqrt{-a^2cx^2+c}}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2
 + c)/a
```

3.653.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.654 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

3.654.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

output
```
arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)+2*(-a*x+1)/a/(-a^2*c*x^
2+c)^(1/2)
```

3.654.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=-\frac{2\sqrt{1-a^2x^2}\left((-1+ax)\sqrt{1+ax}+\sqrt{1-ax}(1+ax)\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*((-1 + a*x)*Sqrt[1 + a*x] + Sqrt[1 - a*x]*(1 + a*x)*
ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*(1 + a*x)*Sqrt[c - a^2*c*
x^2])
```

3.654.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6717, 6692, 457, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(-\frac{\int \frac{1}{\sqrt{c-a^2cx^2}}dx}{c}-\frac{2(1-ax)}{ac\sqrt{c-a^2cx^2}}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(-\frac{\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}}{c}-\frac{2(1-ax)}{ac\sqrt{c-a^2cx^2}}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{ac^{3/2}}-\frac{2(1-ax)}{ac\sqrt{c-a^2cx^2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

output
```
-(c*((-2*(1 - a*x))/(a*c*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]]/(a*c^(3/2))))
```

3.654.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.654.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c\, x^{2}+c}}\right)}{\sqrt{a^{2} c}}+\frac{2 \sqrt{-a^{2} c \left(x+\frac{1}{a}\right)^{2}+2 \left(x+\frac{1}{a}\right) a c}}{a^{2} c \left(x+\frac{1}{a}\right)}$	73

input
```
int((a*x-1)/(-a^2*c*x^2+c)^(1/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x+1/
a)*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)
```

3.654.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.52

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \left[-\frac{(ax+1)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\,(a^2cx+ac)}, -\frac{(ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a^2cx^2+c}}{a^2cx+ac}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x + a*c), -((a*x + 1)*sqrt(c)*
arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x
^2 + c))/(a^2*c*x + a*c)]
```

3.654.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{ax-1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral((a*x - 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.654.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{-a^2cx^2+c}}{a^2cx+ac} + \frac{\arcsin(ax)}{a\sqrt{c}}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x + a*c) + arcsin(a*x)/(a*sqrt(c))
```

3.654.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{a\,x-1}{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)), x)
```

3.655 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

3.655.1 Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

output
```
2/3*(-a*x+1)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

3.655.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{1-ax}(2+ax)\sqrt{1-a^2x^2}}{3ac(1+ax)^{3/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 - a*x]*(2 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 + a*x)^(3/2)*Sqrt[c
- a^2*c*x^2])
```

3.655.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6717, 6692, 457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{\int \frac{1}{(c-a^2cx^2)^{3/2}}dx}{3c}-\frac{2(1-ax)}{3ac\,(c-a^2cx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{x}{3c^2\sqrt{c-a^2cx^2}}-\frac{2(1-ax)}{3ac\,(c-a^2cx^2)^{3/2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-(c*((-2*(1 - a*x))/(3*a*c*(c - a^2*c*x^2)^(3/2)) + x/(3*c^2*Sqrt[c - a^2*
c*x^2])))
```

3.655.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.655.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{(ax-1)^2(ax+2)}{3a(-a^2cx^2+c)^{\frac{3}{2}}}$	31
trager	$\frac{(ax+2)\sqrt{-a^2cx^2+c}}{3c^2(ax+1)^2a}$	34
default	$\frac{x}{c\sqrt{-a^2cx^2+c}}-\frac{2\left(-\frac{1}{3ac\left(x+\frac{1}{a}\right)\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}-\frac{-2a^2c\left(x+\frac{1}{a}\right)+2ac}{3a\,c^2\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}\right)}{a}$	115

input
```
int((a*x-1)/(-a^2*c*x^2+c)^(3/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x-1)^2*(a*x+2)/a/(-a^2*c*x^2+c)^(3/2)
```

3.655.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(ax+2)}{3\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(-a^2*c*x^2 + c)*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

3.655.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{ax-1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)), x)
```

3.655.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{x}{3\sqrt{-a^2cx^2+c}c} + \frac{2}{3\left(\sqrt{-a^2cx^2+c}a^2cx+\sqrt{-a^2cx^2+c}ac\right)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x + sqrt
(-a^2*c*x^2 + c)*a*c)
```

3.655.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(43) = 86$.

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(ac+3\sqrt{-a^2c}\sqrt{c}\right)\mathrm{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}+\sqrt{-a^2c}ac^2\right)} + \frac{2\left(2a^2c-3a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}-\sqrt{-a^2c+\frac{c}{x^2}}+\frac{\sqrt{c}}{x}\right)^3 c\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(a*c + 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) + sqrt(-a^2*c)*a*c
^2) + 2/3*(2*a^2*c - 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(s
qrt(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) - sqrt(-a^2*c + c/x^2) + s
qrt(c)/x)^3*c*sgn(x))
```

3.655.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+2)}{3\,a\,c^2\,(a\,x+1)^2}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)),x)
```

output
```
((c - a^2*c*x^2)^(1/2)*(a*x + 2))/(3*a*c^2*(a*x + 1)^2)
```

3.656 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

3.656.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

output
```
2/5*(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c^2
/(-a^2*c*x^2+c)^(1/2)
```

3.656.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=-\frac{\sqrt{1-a^2x^2}(-2+ax+4a^2x^2+2a^3x^3)}{5ac^2\sqrt{1-ax}(1+ax)^{5/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-1/5*(Sqrt[1 - a^2*x^2]*(-2 + a*x + 4*a^2*x^2 + 2*a^3*x^3))/(a*c^2*Sqrt[1
- a*x]*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])
```

3.656.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6717, 6692, 457, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{3\int \frac{1}{(c-a^2cx^2)^{5/2}}dx}{5c}-\frac{2(1-ax)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{3\left(\frac{2\int \frac{1}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}-\frac{2(1-ax)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{3\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}-\frac{2(1-ax)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-(c*((-2*(1 - a*x))/(5*a*c*(c - a^2*c*x^2)^(5/2)) + (3*(x/(3*c*(c - a^2*c*
x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2*c*x^2])))/(5*c)))
```

3.656. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$

3.656.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.656.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.63

method	result
gosper	$-\frac{(ax-1)^{2} (2a^{3}x^{3}+4a^{2}x^{2}+ax-2)}{5a(-a^{2}c\,x^{2}+c)^{\frac{5}{2}}}$
trager	$\frac{(2a^{3}x^{3}+4a^{2}x^{2}+ax-2)\sqrt{-a^{2}c\,x^{2}+c}}{5c^{3}(ax+1)^{3}a(ax-1)}$
default	$\frac{x}{3c(-a^{2}c\,x^{2}+c)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{-a^{2}c\,x^{2}+c}}-\frac{2\Bigg(-\frac{1}{5ac(x+\frac{1}{a})(-a^{2}c(x+\frac{1}{a})^{2}+2(x+\frac{1}{a})ac)^{\frac{3}{2}}}+\frac{4a\Bigg(-\frac{-2a^{2}c(x+\frac{1}{a})+2ac}{6a^{2}c^{2}(-a^{2}c(x+\frac{1}{a})^{2}+2(x+\frac{1}{a})ac)^{\frac{3}{2}}}-}{5}}{a}$

3.656. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^{2}cx^{2})^{5/2}}\,dx$

input
```
int((a*x-1)/(-a^2*c*x^2+c)^(5/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a*x-1)^2*(2*a^3*x^3+4*a^2*x^2+a*x-2)/a/(-a^2*c*x^2+c)^(5/2)
```

3.656.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3x^3+4\,a^2x^2+ax-2)\sqrt{-a^2cx^2+c}}{5\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*a^3*x^3 + 4*a^2*x^2 + a*x - 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 +
2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

3.656.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax-1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)), x)
```

3.656.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{2}{5\left((-a^2cx^2+c)^{\frac{3}{2}}a^2cx+(-a^2cx^2+c)^{\frac{3}{2}}ac\right)} - \frac{2\,x}{5\,\sqrt{-a^2cx^2+c}c^2} - \frac{x}{5\,(-a^2cx^2+c)^{\frac{3}{2}}c}$$

---

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x + (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/
(sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

3.656.8 Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax-1}{(-a^2cx^2+c)^{\frac{5}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*x - 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)), x)
```

3.656.9 Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3+4\,a^2\,x^2+a\,x-2\right)}{5\,a\,c^3\,\left(a\,x-1\right)\,{\left(a\,x+1\right)}^3}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)),x)
```

output
```
((c - a^2*c*x^2)^(1/2)*(a*x + 4*a^2*x^2 + 2*a^3*x^3 - 2))/(5*a*c^3*(a*x -
1)*(a*x + 1)^3)
```

3.657 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

3.657.1 Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

output
```
2/7*(-a*x+1)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/c^
2/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

3.657.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=-\frac{\sqrt{1-a^2x^2}(-6+9ax+24a^2x^2+4a^3x^3-16a^4x^4-8a^5x^5)}{21ac^3(1-ax)^{3/2}(1+ax)^{7/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
-1/21*(Sqrt[1 - a^2*x^2]*(-6 + 9*a*x + 24*a^2*x^2 + 4*a^3*x^3 - 16*a^4*x^4
 - 8*a^5*x^5))/(a*c^3*(1 - a*x)^(3/2)*(1 + a*x)^(7/2)*Sqrt[c - a^2*c*x^2])
```

3.657.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6692, 457, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{5\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}dx}{7c}-\frac{2(1-ax)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{5\left(\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}dx}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}-\frac{2(1-ax)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{5\left(\frac{4\left(\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}-\frac{2(1-ax)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{5\left(\frac{4\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}-\frac{2(1-ax)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
-(c*((-2*(1 - a*x))/(7*a*c*(c - a^2*c*x^2)^(7/2)) + (5*(x/(5*c*(c - a^2*c*
x^2)^(5/2)) + (4*(x/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^
2*c*x^2])))/(5*c)))/(7*c)))
```

3.657.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

3.657. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.657.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result
gosper	$\frac{(ax-1)^2(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)}{21(-a^2cx^2+c)^{\frac{7}{2}}a}$
trager	$\frac{(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)\sqrt{-a^2cx^2+c}}{21c^4(ax+1)^4(ax-1)^2a}$
default	$\frac{x}{5c(-a^2cx^2+c)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(-a^2cx^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2cx^2+c}}}{c}-\frac{2\left(-\frac{1}{7ac\left(x+\frac{1}{a}\right)\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}+\frac{6a\left(-\frac{-2a^2c\left(x\right.}{10a^2c^2\left(-a^2c\left(x+\right.\right.}\right.}{}\right.}{}$

input
```
int((a*x-1)/(-a^2*c*x^2+c)^(7/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/21*(a*x-1)^2*(8*a^5*x^5+16*a^4*x^4-4*a^3*x^3-24*a^2*x^2-9*a*x+6)/(-a^2*c
*x^2+c)^(7/2)/a
```

3.657.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\frac{(8\,a^5x^5+16\,a^4x^4-4\,a^3x^3-24\,a^2x^2-9\,ax+6)\sqrt{-a^2cx^2+c}}{21\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/21*(8*a^5*x^5 + 16*a^4*x^4 - 4*a^3*x^3 - 24*a^2*x^2 - 9*a*x + 6)*sqrt(-a
^2*c*x^2 + c)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 -
 a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

3.657.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax-1}{(-c(ax-1)(ax+1))^{\frac{7}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)), x)
```

3.657.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{2}{7\left((-a^2cx^2+c)^{\frac{5}{2}}a^2cx+(-a^2cx^2+c)^{\frac{5}{2}}ac\right)} - \frac{8\,x}{21\sqrt{-a^2cx^2+c}c^3} - \frac{4\,x}{21\,(-a^2cx^2+c)^{\frac{3}{2}}c^2} - \frac{x}{7\,(-a^2cx^2+c)^{\frac{5}{2}}c}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x + (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x
/(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/
((-a^2*c*x^2 + c)^(5/2)*c)
```

3.657.8 Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{ax-1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*x - 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)), x)
```

3.657.9 Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,{\left(a\,x+1\right)}^3}+\frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,{\left(a\,x+1\right)}^4}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}-\frac{5}{28\,a\,c^4}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}+\frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)),x)
```

output
```
(c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x + 1)^3) + (c - a^2*c*x^2)^(1/2)/(28*a
*c^4*(a*x + 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) - 5/(28*a*c^4)
))/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x -
1)*(a*x + 1))
```

3.658 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx$

3.658.1 Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

output
```
2/9*(-a*x+1)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/c^
2/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2*c
*x^2+c)^(1/2)
```

3.658.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = \frac{\sqrt{1-a^2x^2}\left(10-25ax-60a^2x^2+10a^3x^3+80a^4x^4+24a^5x^5-32a^6x^6-16a^7x^7\right)}{45ac^4(1-ax)^{5/2}(1+ax)^{9/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

output
```
(Sqrt[1 - a^2*x^2]*(10 - 25*a*x - 60*a^2*x^2 + 10*a^3*x^3 + 80*a^4*x^4 + 2
4*a^5*x^5 - 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(5/2)*(1 + a*x)^
(9/2)*Sqrt[c - a^2*c*x^2])
```

3.658.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6717, 6692, 457, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{11/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{7\int \frac{1}{(c-a^2cx^2)^{9/2}}dx}{9c}-\frac{2(1-ax)}{9ac\,(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}dx}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}-\frac{2(1-ax)}{9ac\,(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}dx}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}-\frac{2(1-ax)}{9ac\,(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}-\frac{2(1-ax)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}-\frac{2(1-ax)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

3.658. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

output
```
-(c*((-2*(1 - a*x))/(9*a*c*(c - a^2*c*x^2)^(9/2)) + (7*(x/(7*c*(c - a^2*c*
x^2)^(7/2)) + (6*(x/(5*c*(c - a^2*c*x^2)^(5/2)) + (4*(x/(3*c*(c - a^2*c*x^
2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2*c*x^2])))/(5*c)))/(7*c)))/(9*c)))
```

3.658.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.658. $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

3.658.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

method	result
gosper	$-\frac{(ax-1)^2(16a^7x^7+32a^6x^6-24a^5x^5-80a^4x^4-10a^3x^3+60a^2x^2+25ax-10)}{45a(-a^2c\,x^2+c)^{\frac{9}{2}}}$
trager	$\frac{(16a^7x^7+32a^6x^6-24a^5x^5-80a^4x^4-10a^3x^3+60a^2x^2+25ax-10)\sqrt{-a^2c\,x^2+c}}{45c^5(ax+1)^5(ax-1)^3a}$
default	$\frac{x}{7c(-a^2c\,x^2+c)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2c\,x^2+c}}\right)}{7c}}{c}-\frac{2\left(-\frac{1}{9ac\left(x+\frac{1}{a}\right)\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{7}{2}}}-\right.}{}$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(a*x-1)^2*(16*a^7*x^7+32*a^6*x^6-24*a^5*x^5-80*a^4*x^4-10*a^3*x^3+60
*a^2*x^2+25*a*x-10)/a/(-a^2*c*x^2+c)^(9/2)
```

3.658.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{(16\,a^7x^7+32\,a^6x^6-24\,a^5x^5-80\,a^4x^4-10\,a^3x^3+60\,a^2x^2+25\,ax-10)\sqrt{-a^2cx^2+}}{45\,(a^9c^5x^8+2\,a^8c^5x^7-2\,a^7c^5x^6-6\,a^6c^5x^5+6\,a^4c^5x^3+2\,a^3c^5x^2-2\,a^2c^5x-ac^5)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
1/45*(16*a^7*x^7 + 32*a^6*x^6 - 24*a^5*x^5 - 80*a^4*x^4 - 10*a^3*x^3 + 60*
a^2*x^2 + 25*a*x - 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 + 2*a^8*c^5*x^7 -
 2*a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 - 2*a^2*c^5
*x - a*c^5)
```

3.658.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx=\int \frac{ax-1}{(-c\,(ax-1)\,(ax+1))^{\frac{9}{2}}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x + 1)), x)
```

3.658.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx=\frac{2}{9\left((-a^2cx^2+c)^{\frac{7}{2}}a^2cx+(-a^2cx^2+c)^{\frac{7}{2}}ac\right)}-\frac{16\,x}{45\,\sqrt{-a^2cx^2+c}c^4}-\frac{8\,x}{45\,(-a^2cx^2+c)^{\frac{3}{2}}c^3}-\frac{2\,x}{15\,(-a^2cx^2+c)^{\frac{5}{2}}c^2}-\frac{x}{9\,(-a^2cx^2+c)^{\frac{7}{2}}c}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x + (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*
x/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*
x/((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

3.658.8 Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax-1}{(-a^2cx^2+c)^{\frac{9}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*x - 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x + 1)), x)
```

3.658.9 Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx &= \frac{5\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,(a\,x+1)^4} + \frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,(a\,x+1)^5} \\ &\quad + \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}-\frac{5}{24\,a\,c^5}\right)}{(a\,x-1)^3\,(a\,x+1)^3} - \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}+\frac{5}{144\,a\,c^5}\right)}{(a\,x-1)^2\,(a\,x+1)^2} + \frac{16\,x\,\sqrt{c-a^2\,c\,x^2}}{45\,c^5\,(a\,x-1)\,(a\,x+1)}\end{aligned}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(9/2)*(a*x + 1)),x)
```

output
```
(5*(c - a^2*c*x^2)^(1/2))/(144*a*c^5*(a*x + 1)^4) + (c - a^2*c*x^2)^(1/2)/
(72*a*c^5*(a*x + 1)^5) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) - 5/(24*
a*c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5
) + 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2
))/(45*c^5*(a*x - 1)*(a*x + 1))
```

3.659 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

3.659.1 Optimal result

Integrand size = 24, antiderivative size = 189

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

output
```
-8/7*(-a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(-a*
x+1)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(-a*x+1)^9*(-
a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x
^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

3.659.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(-1+ax)^7\sqrt{c-a^2cx^2}(44+98ax+77a^2x^2+21a^3x^3)}{210a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*(-1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(44 + 98*a*x + 77*a^2*x^2 + 21*a^3*x
^3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.659.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9dx}{x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int (1-ax)^6(ax+1)^3dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int \left(-(1-ax)^9+6(1-ax)^8-12(1-ax)^7+8(1-ax)^6\right)dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(1-ax)^{10}}{10a}-\frac{2(1-ax)^9}{3a}+\frac{3(1-ax)^8}{2a}-\frac{8(1-ax)^7}{7a}\right)\left(c-a^2cx^2\right)^{9/2}}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Int[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - a^2*c*x^2)^(9/2)*((-8*(1 - a*x)^7)/(7*a) + (3*(1 - a*x)^8)/(2*a) - (
2*(1 - a*x)^9)/(3*a) + (1 - a*x)^10/(10*a)))/(a^9*(1 - 1/(a^2*x^2))^(9/2)*
x^9)
```

3.659.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.659.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{x\left(21a^{9}x^{9}-70a^{8}x^{8}+240a^{6}x^{6}-210a^{5}x^{5}-252a^{4}x^{4}+420a^{3}x^{3}-315ax+210\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210\left(ax+1\right)^{3}\left(ax-1\right)^{6}}$	100
default	$\frac{\left(21a^{9}x^{9}-70a^{8}x^{8}+240a^{6}x^{6}-210a^{5}x^{5}-252a^{4}x^{4}+420a^{3}x^{3}-315ax+210\right)x\,c^{4}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210\left(ax-1\right)^{2}}$	102

input
```
int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/210*x*(21*a^9*x^9-70*a^8*x^8+240*a^6*x^6-210*a^5*x^5-252*a^4*x^4+420*a^3
*x^3-315*a*x+210)*(-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^3/(
a*x-1)^6
```

3.659.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = \frac{(21\,a^9c^4x^{10} - 70\,a^8c^4x^9 + 240\,a^6c^4x^7 - 210\,a^5c^4x^6 - 252\,a^4c^4x^5 + 420\,a^3c^4x^4 - 315\,ac^4x^2}{210\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
1/210*(21*a^9*c^4*x^10 - 70*a^8*c^4*x^9 + 240*a^6*c^4*x^7 - 210*a^5*c^4*x^
6 - 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 + 210*c^4*x)*sqrt(-a
^2*c)/a
```

3.659.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.659.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.08

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = \frac{(21\,a^{11}\sqrt{-c}c^4x^{11} - 49\,a^{10}\sqrt{-c}c^4x^{10} - 70\,a^9\sqrt{-c}c^4x^9 + 240\,a^8\sqrt{-c}c^4x^8 + 30\,a^7\sqrt{-c}c^4x^7}{210}$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
1/210*(21*a^11*sqrt(-c)*c^4*x^11 - 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt
(-c)*c^4*x^9 + 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 - 462*a^
6*sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 + 420*a^4*sqrt(-c)*c^4*x^4 -
 315*a^3*sqrt(-c)*c^4*x^3 - 105*a^2*sqrt(-c)*c^4*x^2 - 210*sqrt(-c)*c^4)*(
a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

3.659.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((-a^2*c*x^2 + c)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.659.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{9/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.660 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

3.660.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

output
```
2/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(-a*x+
1)^7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2
*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

3.660.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(-1+ax)^6\left(37+54ax+21a^2x^2\right)\sqrt{c-a^2cx^2}}{168a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-1/168*(c^3*(-1 + a*x)^6*(37 + 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(
a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.660.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7dx}{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int -(1-ax)^5(ax+1)^2dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{7/2}\int (1-ax)^5(ax+1)^2dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{7/2}\int \left((1-ax)^7-4(1-ax)^6+4(1-ax)^5\right)dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(1-ax)^8}{8a}+\frac{4(1-ax)^7}{7a}-\frac{2(1-ax)^6}{3a}\right)\left(c-a^2cx^2\right)^{7/2}}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Int[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((c - a^2*c*x^2)^(7/2)*((-2*(1 - a*x)^6)/(3*a) + (4*(1 - a*x)^7)/(7*a) -
 (1 - a*x)^8/(8*a)))/(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7))
```

3.660.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.660.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x\left(21a^{7}x^{7}-72a^{6}x^{6}+28a^{5}x^{5}+168a^{4}x^{4}-210a^{3}x^{3}-56a^{2}x^{2}+252ax-168\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168\left(ax+1\right)^{2}\left(ax-1\right)^{5}}$	100
default	$-\frac{\left(21a^{7}x^{7}-72a^{6}x^{6}+28a^{5}x^{5}+168a^{4}x^{4}-210a^{3}x^{3}-56a^{2}x^{2}+252ax-168\right)x\,c^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168\left(ax-1\right)^{2}}$	102

input
```
int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/168*x*(21*a^7*x^7-72*a^6*x^6+28*a^5*x^5+168*a^4*x^4-210*a^3*x^3-56*a^2*x
^2+252*a*x-168)*(-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*
x-1)^5
```

3.660.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^7c^3x^8-72\,a^6c^3x^7+28\,a^5c^3x^6+168\,a^4c^3x^5-210\,a^3c^3x^4-56\,a^2c^3x^3+252\,ac^3x^2-168\,c^3x\right)\sqrt{-a^2c}}{168\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
-1/168*(21*a^7*c^3*x^8 - 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 + 168*a^4*c^3*x^5
 - 210*a^3*c^3*x^4 - 56*a^2*c^3*x^3 + 252*a*c^3*x^2 - 168*c^3*x)*sqrt(-a^2
*c)/a
```

3.660.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.660.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.21

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^9\sqrt{-c}c^3x^9-51\,a^8\sqrt{-c}c^3x^8-44\,a^7\sqrt{-c}c^3x^7+196\,a^6\sqrt{-c}c^3x^6-42\,a^5\sqrt{-c}c^3x^5-266\,a^4\sqrt{-c}c^3x^4+\right.}{168\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

---

output
```
-1/168*(21*a^9*sqrt(-c)*c^3*x^9 - 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c
)*c^3*x^7 + 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 - 266*a^4*s
qrt(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 + 84*a^2*sqrt(-c)*c^3*x^2 + 168
*sqrt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

3.660.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.660.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.661 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

3.661.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

output
```
-2/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x
+1)^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

3.661.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(-1+ax)^5(7+5ax)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*(-1 + a*x)^5*(7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

3.661.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5dx}{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int (1-ax)^4(ax+1)dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int \left(2(1-ax)^4-(1-ax)^5\right)dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(1-ax)^6}{6a}-\frac{2(1-ax)^5}{5a}\right)\left(c-a^2cx^2\right)^{5/2}}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input

```
Int[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output

```
((c - a^2*c*x^2)^(5/2)*((-2*(1 - a*x)^5)/(5*a) + (1 - a*x)^6/(6*a)))/(a^5*
(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

3.661.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.661.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{x\left(5a^{5}x^{5}-18a^{4}x^{4}+15a^{3}x^{3}+20a^{2}x^{2}-45ax+30\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30\left(ax-1\right)^{4}\left(ax+1\right)}$	84
default	$\frac{\left(5a^{5}x^{5}-18a^{4}x^{4}+15a^{3}x^{3}+20a^{2}x^{2}-45ax+30\right)x\,c^{2}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30\left(ax-1\right)^{2}}$	86

input
```
int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*(-a^2*c*x^2+
c)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^4/(a*x+1)
```

3.661.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(5\,a^5c^2x^6-18\,a^4c^2x^5+15\,a^3c^2x^4+20\,a^2c^2x^3-45\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
1/30*(5*a^5*c^2*x^6 - 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 - 4
5*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

3.661.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.661.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.47

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(5\,a^7\sqrt{-c}c^2x^7-13\,a^6\sqrt{-c}c^2x^6-3\,a^5\sqrt{-c}c^2x^5+35\,a^4\sqrt{-c}c^2x^4-25\,a^3\sqrt{-c}c^2x^3-15\,a^2}{30\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
1/30*(5*a^7*sqrt(-c)*c^2*x^7 - 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^
2*x^5 + 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 - 15*a^2*sqrt(-c
)*c^2*x^2 - 30*sqrt(-c)*c^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1
))
```

3.661.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\left(c-a^2\,c\,x^2\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.662 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

3.662.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
1/4*(-a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

3.662.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c\sqrt{c-a^2cx^2}(-4+6ax-4a^2x^2+a^3x^3)}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-1/4*(c*Sqrt[c - a^2*c*x^2]*(-4 + 6*a*x - 4*a^2*x^2 + a^3*x^3))/(a*Sqrt[1
- 1/(a^2*x^2)])
```

3.662.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3dx}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int (ax-1)^3dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

3.662.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.662.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1) (ax-1)^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}\, c}{4 a}$	48
gosper	$\frac{x \left(a^{3} x^{3}-4 a^{2} x^{2}+6 a x -4\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{4 \left(a x -1\right)^{3}}$	60

input
```
int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*c/a
```

3.662.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int e^{-3\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{3/2} \, dx = -\frac{\left(a^{3} c x^{4} - 4 \, a^{2} c x^{3} + 6 \, a c x^{2} - 4 \, c x\right) \sqrt{-a^{2} c}}{4 \, a}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
-1/4*(a^3*c*x^4 - 4*a^2*c*x^3 + 6*a*c*x^2 - 4*c*x)*sqrt(-a^2*c)/a
```

3.662.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.662.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(40)=80$.

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^5\sqrt{-c}cx^5-3\,a^4\sqrt{-c}cx^4+2\,a^3\sqrt{-c}cx^3+2\,a^2\sqrt{-c}cx^2+4\,\sqrt{-c}c\right)(ax-1)^2}{4\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
-1/4*(a^5*sqrt(-c)*c*x^5 - 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 + 2
*a^2*sqrt(-c)*c*x^2 + 4*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(
a*x + 1))
```

3.662.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.663 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.663.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(
1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1
/2)
```

3.663.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/(Sqrt[1 -
1/(a^2*x^2)]*x)
```

3.663.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax+\frac{4}{ax+1}-3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/(a*Sqrt[1 -
1/(a^2*x^2)]*x)
```

3.663.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.663.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(a^2x^2-6ax+8\ln(ax+1))\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a(ax-1)^2}$	67

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/a/(a*x-1)^2
```

3.663.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(a^2x^2-6\,ax+8\,\log{(ax+1)})\sqrt{-a^2c}}{2\,a^2}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

3.663.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.663.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.663.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.663.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.664 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

3.664.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

output
```
2*x*(1-1/a^2/x^2)^(1/2)/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+x*ln(a*x+1)*(1-1/a^2/
x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

3.664.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(\frac{2}{1+ax}+\log(1+ax)\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2/(1 + a*x) + Log[1 + a*x]))/Sqrt[c - a^2*c*x^2]
```

3.664.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{1-ax}{(ax+1)^2}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \frac{1-ax}{(ax+1)^2}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{2}{(ax+1)^2}+\frac{1}{-ax-1}\right)dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{2}{a(ax+1)}-\frac{\log(ax+1)}{a}\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

output
```
-((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2/(a*(1 + a*x)) - Log[1 + a*x]/a))/Sqrt[c -
 a^2*c*x^2])
```

3.664.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.664.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(a \ln \left(a x +1\right) x +\ln \left(a x +1\right)+2\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{a c \left(a x -1\right)^{2}}$	62

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x+ln(a*x+1)+2)*((a*x-1)/(a*x+1))^(3/2
)/a/c/(a*x-1)^2
```

3.664.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{-a^2c}((ax+1)\log(ax+1)+2)}{a^3cx+a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

output
```
-sqrt(-a^2*c)*((a*x + 1)*log(a*x + 1) + 2)/(a^3*c*x + a^2*c)
```

3.664.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.664.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

3.664.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.665 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

3.665.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

output
```
-1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

3.665.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}{2c^2(-1+ax)(1+ax)^3}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)*(1 + a*
x)^3)
```

3.665.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(ax+1)^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)^2(c-a^2cx^2)^{3/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-1/2*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/((1 + a*x)^2*(c - a^2*c*x^2)^(3/2))
```

3.665.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.665.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a(-a^2c\,x^2+c)^{\frac{3}{2}}}$	39
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c(a^2x^2-1)}}{2(ax-1)(a^2x^2-1)a\,c^2}$	56

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

3.665.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2c}}{2\,(a^4c^2x^2+2\,a^3c^2x+a^2c^2)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

output
```
-1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

3.665.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.665.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.665.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.665.9 Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,c\,\left(x\,\sqrt{c-a^2\,c\,x^2}+\frac{\sqrt{c-a^2\,c\,x^2}}{a}\right)}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(3/2),x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(2*a^2*c*(x*(c - a^2*c*x^2)^(1/2) + (c - a^2*c
*x^2)^(1/2)/a))
```

3.666 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

3.666.1 Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output

```
1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-
1/a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^
(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arc
tanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

3.666.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.39

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-10-9ax-3a^2x^2+3(1+ax)^3\operatorname{arctanh}(ax)\right)}{24c^2(1+ax)^3\sqrt{c-a^2cx^2}}$$

input

```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-1/24*(Sqrt[1 - 1/(a^2*x^2)]*x*(-10 - 9*a*x - 3*a^2*x^2 + 3*(1 + a*x)^3*Ar
cTanh[a*x]))/(c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

3.666.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{(1-ax)(ax+1)^4}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)(ax+1)^4}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{8(ax+1)^2}+\frac{1}{4(ax+1)^3}+\frac{1}{2(ax+1)^4}-\frac{1}{8(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\text{arctanh}(ax)}{8a}-\frac{1}{8a(ax+1)}-\frac{1}{8a(ax+1)^2}-\frac{1}{6a(ax+1)^3}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-1/6*1/(a*(1 + a*x)^3) - 1/(8*a*(1 + a
*x)^2) - 1/(8*a*(1 + a*x)) + ArcTanh[a*x]/(8*a)))/(c - a^2*c*x^2)^(5/2))
```

3.666.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.666.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3+9a^2\ln(ax+1)x^2-9a^2\ln(ax-1)x^2-6a^2x^2+9a\ln(ax+1)x-9a\ln(ax-1}{48(ax+1)(ax-1)(a^2x^2-1)c^3a}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3
*ln(a*x+1)*x^3-3*a^3*ln(a*x-1)*x^3+9*a^2*ln(a*x+1)*x^2-9*a^2*ln(a*x-1)*x^2
-6*a^2*x^2+9*a*ln(a*x+1)*x-9*a*ln(a*x-1)*x-18*a*x+3*ln(a*x+1)-3*ln(a*x-1)-
20)/(a^2*x^2-1)/c^3/a
```

3.666.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{3\left(a^4x^3+3\,a^3x^2+3\,a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(3\,a^2x^2+9\,ax+10\right)\sqrt{-a^2c}}{48\left(a^5c^3x^3+3\,a^4c^3x^2+3\,a^3c^3x+a^2c^3\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
-1/48*(3*(a^4*x^3 + 3*a^3*x^2 + 3*a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*s
qrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 + 9*a*x + 10)*sq
rt(-a^2*c))/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2*c^3)
```

3.666.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.666.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.666.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.666.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2), x)
```

3.667 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

3.667.1 Optimal result

Integrand size = 24, antiderivative size = 275

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx &= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}}\\
&\quad-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}\\
&\quad-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}\end{aligned}$$

output
```
1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/16*a^6*(1
-1/a^2/x^2)^(7/2)*x^7/(a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6*(1-1/a^2/x^2
)^(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2/x^2)^(7/2)*x^
7/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(
-a^2*c*x^2+c)^(7/2)+5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*
x^2+c)^(7/2)
```

3.667.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(32+15ax-35a^2x^2-45a^3x^3-15a^4x^4+15(-1+ax)(1+ax)^4\operatorname{arctanh}(ax)\right)}{96c^3(-1+ax)(1+ax)^4\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
-1/96*(Sqrt[1 - 1/(a^2*x^2)]*x*(32 + 15*a*x - 35*a^2*x^2 - 45*a^3*x^3 - 15
*a^4*x^4 + 15*(-1 + a*x)*(1 + a*x)^4*ArcTanh[a*x]))/(c^3*(-1 + a*x)*(1 + a
*x)^4*Sqrt[c - a^2*c*x^2])
```

3.667.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$$

$$\Big\downarrow \ 6746$$

$$\frac{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$\Big\downarrow \ 6747$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{1}{(1-ax)^2(ax+1)^5}dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$\Big\downarrow \ 54$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int\left(\frac{1}{32(ax-1)^2}+\frac{1}{8(ax+1)^2}+\frac{3}{16(ax+1)^3}+\frac{1}{4(ax+1)^4}+\frac{1}{4(ax+1)^5}-\frac{5}{32(a^2x^2-1)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$\Big\downarrow 2009$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{5\text{arctanh}(ax)}{32a}+\frac{1}{32a(1-ax)}-\frac{1}{8a(ax+1)}-\frac{3}{32a(ax+1)^2}-\frac{1}{12a(ax+1)^3}-\frac{1}{16a(ax+1)^4}\right)}{\left(c-a^2cx^2\right)^{7/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7*(1/(32*a*(1 - a*x)) - 1/(16*a*(1 + a*x)^4
) - 1/(12*a*(1 + a*x)^3) - 3/(32*a*(1 + a*x)^2) - 1/(8*a*(1 + a*x)) + (5*A
rcTanh[a*x])/(32*a)))/(c - a^2*c*x^2)^(7/2)
```

3.667.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.667. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

3.667.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5+45\ln(ax+1)x^4a^4-45\ln(ax-1)x^4a^4-30a^4x^4+30a^3\ln(ax+1)x^3-30}{192(ax+1)}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(
15*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5+45*ln(a*x+1)*x^4*a^4-45*ln(a*x-1
)*x^4*a^4-30*a^4*x^4+30*a^3*ln(a*x+1)*x^3-30*a^3*ln(a*x-1)*x^3-90*a^3*x^3-
30*a^2*ln(a*x+1)*x^2+30*a^2*ln(a*x-1)*x^2-70*a^2*x^2-45*a*ln(a*x+1)*x+45*a
*ln(a*x-1)*x+30*a*x-15*ln(a*x+1)+15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a
```

3.667.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{15\left(a^6x^5+3\,a^5x^4+2\,a^4x^3-2\,a^3x^2-3\,a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(15\,a^4x^4+45\,a^3x^3+\right.}{192\left(a^7c^4x^5+3\,a^6c^4x^4+2\,a^5c^4x^3-2\,a^4c^4x^2-3\,a^3c^4x-a^2c^4\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="frica
s")
```

output
```
-1/192*(15*(a^6*x^5 + 3*a^5*x^4 + 2*a^4*x^3 - 2*a^3*x^2 - 3*a^2*x - a)*sqr
t(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(
15*a^4*x^4 + 45*a^3*x^3 + 35*a^2*x^2 - 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4
*x^5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c
^4)
```

3.667.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

3.667.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxim
a")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

3.667.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac"
)
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

3.667.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{(c-a^2\,c\,x^2)^{7/2}}\,dx$$

input

```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2),x)
```

output

```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2), x)
```

3.668 $\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

3.668.1 Optimal result

Integrand size = 25, antiderivative size = 76

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)^
(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.668.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^2(4+3ax)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.668.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int x^2(ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax^3+x^2\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\frac{ax^4}{4}+\frac{x^3}{3}\right)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

output

```
(Sqrt[c - a^2*c*x^2]*(x^3/3 + (a*x^4)/4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.668.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.668.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x^{3}\left(3ax+4\right)\sqrt{-a^{2}c\,x^{2}+c}}{12\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{\left(3ax+4\right)x^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}}{12\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}$	48

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/12*x^3*(3*a*x+4)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

3.668.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.33

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{(3\,ax^4+4\,x^3)\sqrt{-a^2c}}{12\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/12*(3*a*x^4 + 4*x^3)*sqrt(-a^2*c)/a
```

3.668.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.668.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.668.8 Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.668.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.669 $\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

3.669.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

3.669.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x(3+2ax)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x*(3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.669.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int x(ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax^2+x\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{ax^3}{3}+\frac{x^2}{2}\right)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

output

```
(Sqrt[c - a^2*c*x^2]*(x^2/2 + (a*x^3)/3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.669.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.669.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x^{2} \left(2 a x +3\right) \sqrt{-a^{2} c \,x^{2}+c}}{6 \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	47
default	$\frac{\left(2 a x +3\right) x^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}{6 \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	48

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/6*x^2*(2*a*x+3)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

3.669.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(2\,ax^3+3\,x^2)\sqrt{-a^2c}}{6\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
1/6*(2*a*x^3 + 3*x^2)*sqrt(-a^2*c)/a
```

3.669.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.669.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.669.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+cx}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.669.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int \frac{x\,\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.670 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.670.1 Optimal result

Integrand size = 22, antiderivative size = 68

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

3.670.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

output
```
((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.670.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int (ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(ax+1)^2\sqrt{c-a^2cx^2}}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

output

```
((1 + a*x)^2*Sqrt[c - a^2*c*x^2])/(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.670.3.1 Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.670.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
default	$\frac{(ax+2)x\sqrt{-c(a^2x^2-1)}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	45

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

3.670.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{-a^2c}(ax^2+2\,x)}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

3.670.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.670.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.670.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.670.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.671 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

3.671.1 Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

output

```
(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1
-1/a^2/x^2)^(1/2)
```

3.671.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input

```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

output

```
(Sqrt[c - a^2*c*x^2]*(x + Log[x]/a))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.671.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{ax+1}{x}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}(ax+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.671.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.671.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{(ax+\ln(x))\sqrt{-c(a^2x^2-1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBO
SE)
```

output
```
(a*x+ln(x))*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

3.671.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.26

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{-a^2c}(ax+\log{(x)})}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="f
ricas")
```

output
```
sqrt(-a^2*c)*(a*x + log(x))/a
```

3.671.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.671.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.671.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.671.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.672 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.672.1 Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)
/x/(1-1/a^2/x^2)^(1/2)
```

3.672.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(1/(a*x)) + Log[x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.672.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \textcolor{blue}{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textcolor{blue}{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{ax+1}{x^2}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textcolor{blue}{49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(\frac{a}{x}+\frac{1}{x^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(a\log(x)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output

```
(Sqrt[c - a^2*c*x^2]*(-x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.672.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.672.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{(a\ln(x)x-1)\sqrt{-c(a^2x^2-1)}}{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}}$	48

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVER
BOSE)
```

output
```
(a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/x/((a*x-1)/(a*x+1))^(1/2)
```

3.672.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.30

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(ax\log(x)-1)}{ax}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"fricas")
```

output
```
sqrt(-a^2*c)*(a*x*log(x) - 1)/(a*x)
```

3.672.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-c(ax-1)(ax+1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.672.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.672.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"giac")
```

output

```
integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.672.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input

```
int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output

```
int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.673 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

3.673.1 Optimal result

Integrand size = 27, antiderivative size = 137

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

output
```
-3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+3/5*x^2*(-a^2*c*
x^2+c)^(1/2)/a^2+1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(1/
2)+3/20*(5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

3.673.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(24+15ax+12a^2x^2+10a^3x^3+4a^4x^4)+15\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{20a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 10*a^3*x^3 + 4*a^4*x^4) +
 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(2
0*a^4)
```

3.673.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.40, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6717, 6701, 541, 25, 27, 533, 27, 533, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^3\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{x^3(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx^3(10ax+9)}{\sqrt{c-a^2cx^2}}dx}{5a^2c}-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^3(10ax+9)}{\sqrt{c-a^2cx^2}}dx}{5a^2c}-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\int \frac{x^3(10ax+9)}{\sqrt{c-a^2cx^2}}dx-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{5}\left(\frac{\int \frac{6acx^2(6ax+5)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{1}{5}\left(\frac{3\int \frac{x^2(6ax+5)}{\sqrt{c-a^2cx^2}}dx}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 533$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\int \frac{3acx(5ax+4)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\int \frac{x(5ax+4)}{\sqrt{c-a^2cx^2}}dx}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 533$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\int \frac{ac(8ax+5)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\int \frac{8ax+5}{\sqrt{c-a^2cx^2}}dx}{2a}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 455$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{5\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

3.673. $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

$$\Big\downarrow 224$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{5\int\frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 216$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\frac{5\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/5*(x^4*Sqrt[c - a^2*c*x^2])/c + ((-5*x^3*Sqrt[c - a^2*c*x^2])/(2*a
*c) + (3*((-2*x^2*Sqrt[c - a^2*c*x^2])/(a*c) + ((-5*x*Sqrt[c - a^2*c*x^2])
/(2*a*c) + ((-8*Sqrt[c - a^2*c*x^2])/(a*c) + (5*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(a*Sqrt[c]))/(2*a))/a))/(2*a))/5))
```

3.673.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.673. $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

3.673.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24)(a^2x^2-1)c}{20a^4\sqrt{-c(a^2x^2-1)}}-\frac{3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{4a^3\sqrt{a^2c}}$
default	$-\frac{x^2(-a^2c\,x^2+c)^{\frac{3}{2}}}{5a^2c}-\frac{4(-a^2c\,x^2+c)^{\frac{3}{2}}}{5c\,a^4}+\frac{x\sqrt{-a^2c\,x^2+c}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}}{a^3}+\frac{-\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{2a^2c}+\frac{\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan}{}}{2a^2}}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(4*a^4*x^4+10*a^3*x^3+12*a^2*x^2+15*a*x+24)*(a^2*x^2-1)/a^4/(-c*(a^2
*x^2-1))^(1/2)*c-3/4/a^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))*c
```

3.673.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.34

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$

$$=\left[\frac{2\left(4\,a^4x^4+10\,a^3x^3+12\,a^2x^2+15\,ax+24\right)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}\right.}{40\,a^4}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/40*(2*(4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-a^2*c*x
^2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*
x - c))/a^4, 1/20*((4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqr
t(-a^2*c*x^2 + c) + 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^
2*c*x^2 - c)))/a^4]
```

3.673.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.673.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = -\frac{(-a^2cx^2+c)^{\frac{3}{2}}x^2}{5\,a^2c} + \frac{5\,\sqrt{-a^2cx^2+c}x}{4\,a^3} - \frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{2\,a^3c} - \frac{3\,\sqrt{c}\arcsin{(ax)}}{4\,a^4} + \frac{2\,\sqrt{-a^2cx^2+c}}{a^4} - \frac{4\,(-a^2cx^2+c)^{\frac{3}{2}}}{5\,a^4c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) + 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 -
 1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) - 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sq
rt(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

3.673.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.673.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.674 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

3.674.1 Optimal result

Integrand size = 27, antiderivative size = 112

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

output
```
-7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3+2/3*x^2*(-a^2*c*
x^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)+1/24*(21*a*x+32)*(-a^2*c*x^2+c
)^(1/2)/a^3
```

3.674.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(32+21ax+16a^2x^2+6a^3x^3)+21\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{24a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*A
rcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

3.674.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.44, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6717, 6701, 541, 25, 27, 533, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6717$$

$$-\int e^{2\text{arctanh}(ax)}x^2\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \ 6701$$

$$-c\int \frac{x^2(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \ 541$$

$$-c\left(-\frac{\int -\frac{a^2cx^2(8ax+7)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \ 25$$

$$-c\left(\frac{\int \frac{a^2cx^2(8ax+7)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \ 27$$

$$-c\left(\frac{1}{4}\int \frac{x^2(8ax+7)}{\sqrt{c-a^2cx^2}}dx-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \ 533$$

$$-c\left(\frac{1}{4}\left(\frac{\int \frac{acx(21ax+16)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \ 27$$

$$-c\left(\frac{1}{4}\left(\frac{\int \frac{x(21ax+16)}{\sqrt{c-a^2cx^2}}dx}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \ 533$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{\int \frac{ac(32ax+21)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 27$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{\int \frac{32ax+21}{\sqrt{c-a^2cx^2}}dx}{2a}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 455$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{21\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 224$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{21\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 216$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{\frac{21\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

input

```
Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

output

```
-(c*(-1/4*(x^3*Sqrt[c - a^2*c*x^2])/c + ((-8*x^2*Sqrt[c - a^2*c*x^2])/(3*a
*c) + ((-21*x*Sqrt[c - a^2*c*x^2])/(2*a*c) + ((-32*Sqrt[c - a^2*c*x^2])/(a
*c) + (21*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a*Sqrt[c]))/(2*a))/(
3*a))/4))
```

3.674. $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

3.674.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

3.674. $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.674.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\left(6 a^{3} x^{3}+16 a^{2} x^{2}+21 a x +32\right) \left(a^{2} x^{2}-1\right) c}{24 a^{3} \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{7 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{8 a^{2} \sqrt{a^{2} c}}$
default	$-\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4 a^{2} c}+\frac{\frac{9 x \sqrt{-a^{2} c \,x^{2}+c}}{8}+\frac{9 c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{8 \sqrt{a^{2} c}}}{a^{2}}-\frac{2 \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{3 a^{3} c}+\frac{2 \sqrt{-a^{2} c \left(x -\frac{1}{a}\right)^{2}-2 \left(x -\frac{1}{a}\right) a c}-\frac{2 a c \arctan \left(\right.}{}}{a^{3}}$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x+1)*x^2/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3+16*a^2*x^2+21*a*x+32)*(a^2*x^2-1)/a^3/(-c*(a^2*x^2-1))^(1
/2)*c-7/8/a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

3.674.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.50

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\,(6\,a^3x^3+16\,a^2x^2+21\,ax+32)\sqrt{-a^2cx^2+c}+21\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48\,a^3},\frac{(6}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/48*(2*(6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*
sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3,
1/24*((6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqr
t(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

3.674.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.674.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{9\sqrt{-a^2cx^2+c}x}{8a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{4a^2c}-\frac{7\sqrt{c}\arcsin(ax)}{8a^3}+\frac{2\sqrt{-a^2cx^2+c}}{a^3}-\frac{2(-a^2cx^2+c)^{\frac{3}{2}}}{3a^3c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/
8*sqrt(c)*arcsin(a*x)/a^3 + 2*sqrt(-a^2*c*x^2 + c)/a^3 - 2/3*(-a^2*c*x^2 +
 c)^(3/2)/(a^3*c)
```

3.674.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3x+\frac{8}{a}\right)x+\frac{21}{a^2}\right)x+\frac{32}{a^3}\right)+\frac{7c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8a^2\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x + 8/a)*x + 21/a^2)*x + 32/a^3) + 7/8*c*
log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

3.674.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.675 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

3.675.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

output
```
-arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2+1/3*x^2*(-a^2*c*x^2+
c)^(1/2)+1/3*(3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

3.675.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(5+3ax+a^2x^2)\sqrt{c-a^2cx^2}+3\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{3a^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
((5 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c
- a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

3.675.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6701, 541, 25, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{x(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx(6ax+5)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx(6ax+5)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}\int \frac{x(6ax+5)}{\sqrt{c-a^2cx^2}}dx-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{3}\left(\frac{\int \frac{2ac(5ax+3)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}\left(\frac{\int \frac{5ax+3}{\sqrt{c-a^2cx^2}}dx}{a}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{1}{3}\left(\frac{3\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow 224$$

$$-c\left(\frac{1}{3}\left(\frac{3\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow 216$$

$$-c\left(\frac{1}{3}\left(\frac{\frac{3\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/3*(x^2*Sqrt[c - a^2*c*x^2])/c + ((-3*x*Sqrt[c - a^2*c*x^2])/(a*c)
+ ((-5*Sqrt[c - a^2*c*x^2])/(a*c) + (3*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]])/(a*Sqrt[c]))/a)/3))
```

3.675.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.675. $\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.675.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\left(a^{2} x^{2}+3 a x +5\right) \left(a^{2} x^{2}-1\right) c}{3 a^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{\arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{a \sqrt{a^{2} c}}$
default	$-\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{3 a^{2} c}+\frac{x \sqrt{-a^{2} c \,x^{2}+c}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{\sqrt{a^{2} c}}}{a}+\frac{2 \sqrt{-a^{2} c \left(x -\frac{1}{a}\right)^{2}-2 \left(x -\frac{1}{a}\right) a c}-\frac{2 a c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \left(x -\frac{1}{a}\right)^{2}-2 \left(x -\frac{1}{a}\right.}}\right.}{\sqrt{a^{2} c}}}{a^{2}}$

input
```
int(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a^2*x^2+3*a*x+5)*(a^2*x^2-1)/a^2/(-c*(a^2*x^2-1))^(1/2)*c-1/a/(a^2*c
)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

3.675.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\sqrt{-a^2cx^2+c}(a^2x^2+3\,ax+5)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2},\ \frac{\sqrt{-a^2cx^2+c}(a^2x}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*
c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^
2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqr
t(c)*x/(a^2*c*x^2 - c)))/a^2]
```

3.675.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.675.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{-a^2cx^2+c}x}{a}-\frac{\sqrt{c}\arcsin{(ax)}}{a^2}+\frac{2\sqrt{-a^2cx^2+c}}{a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^2c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-a^2*c*x^2 + c)*x/a - sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 + c
)/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

3.675.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x+\frac{3}{a}\right)x+\frac{5}{a^2}\right)+\frac{c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(-a^2*c*x^2 + c)*((x + 3/a)*x + 5/a^2) + c*log(abs(-sqrt(-a^2*c)*x
 + sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

3.675.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.676 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.676.1 Optimal result

Integrand size = 24, antiderivative size = 86

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

output
```
-3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)
^(1/2)/a+1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

3.676.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]
/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

3.676.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6691, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{3}{2}\int \frac{ax+1}{\sqrt{c-a^2cx^2}}dx-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{3}{2}\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/2*((1 + a*x)*Sqrt[c - a^2*c*x^2])/(a*c) + (3*(-(Sqrt[c - a^2*c*x^2
]/(a*c)) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a*Sqrt[c])))/2))
```

3.676.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.676. $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.676.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{(ax+4)\left(a^{2}x^{2}-1\right)c}{2a\sqrt{-c\left(a^{2}x^{2}-1\right)}}-\frac{3c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}$	69
default	$\frac{x\sqrt{-a^{2}c\,x^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}+\frac{2\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac}}\right)}{\sqrt{a^{2}c}}}{a}$	136

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

3.676.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^{2}cx^{2}}\,dx$$

$$=\left[\frac{2\sqrt{-a^{2}cx^{2}+c}(ax+4)+3\sqrt{-c}\log\left(2\,a^{2}cx^{2}-2\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}x-c\right)}{4\,a},\frac{\sqrt{-a^{2}cx^{2}+c}(ax+4)+3}{2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x +
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a
]
```

3.676.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.676.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}x-\frac{3\sqrt{c}\arcsin{(ax)}}{2a}+\frac{2\sqrt{-a^2cx^2+c}}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
 + c)/a
```

3.676.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.676.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.677 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

3.677.1 Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c-a^2cx^2}-2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
-2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+arctanh((-a^2*c*x^2+c)
^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)
```

3.677.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c-a^2cx^2}+2\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)-\sqrt{c}\log(x)+\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*
(-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c
*x^2]]
```

3.677.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6717, 6701, 541, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{(ax+1)^2}{x\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2c(2ax+1)}{x\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2c(2ax+1)}{x\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\int \frac{2ax+1}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{538}$$

$$-c\left(2a\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\int \frac{1}{x\sqrt{c-a^2cx^2}}dx+2a\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\int \frac{1}{x\sqrt{c-a^2cx^2}}dx+\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \textbf{243}$$

$$-c\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2+\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \textbf{73}$$

$$-c\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{a^2c}+\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \textbf{221}$$

$$-c\left(\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
-(c*(-(Sqrt[c - a^2*c*x^2]/c) + (2*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2
]])/Sqrt[c] - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/Sqrt[c]))
```

3.677.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.677. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.677. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

3.677.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(61) = 122$.

Time = 0.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.72

method	result
default	$-\sqrt{-a^{2} c \,x^{2}+c}+\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)+2 \sqrt{-a^{2} c \left(x -\frac{1}{a}\right)^{2}-2 \left(x -\frac{1}{a}\right) a c}-\frac{2 a c \arctan \left(\frac{}{\sqrt{-a^{2}}}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2
*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)
^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2))
```

3.677.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \Bigg[& 2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right) \\
&+ \frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right) \\
&+ \sqrt{-a^2cx^2+c}, \sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right) \\
&+ \sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right) \\
&+ \sqrt{-a^2cx^2+c}\Bigg]
\end{aligned}
$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
[2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*
sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqr
t(-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^
2 - c)) + sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x -
 c) + sqrt(-a^2*c*x^2 + c)]
```

3.677.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x*(a*x - 1)), x)
```

3.677.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -a^2\left(\frac{\sqrt{c}\arcsin(ax)}{a^2} - \frac{\sqrt{-a^2cx^2+c}}{a^2}\right) - a\left(\frac{\sqrt{c}\arcsin(ax)}{a} - \frac{\sqrt{c}\log\left(\frac{2\sqrt{-a^2cx^2+c}\sqrt{c}}{|x|} + \frac{2c}{|x|}\right)}{a}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
-a^2*(sqrt(c)*arcsin(a*x)/a^2 - sqrt(-a^2*c*x^2 + c)/a^2) - a*(sqrt(c)*arc
sin(a*x)/a - sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x
))/a)
```

3.677.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} + \sqrt{-a^2cx^2+c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
-2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
2*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqr
t(-a^2*c*x^2 + c)
```

3.677.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

3.678 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.678.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) + 2a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
-a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+2*a*arctanh((-a^2*c*x^
2+c)^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
```

3.678.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) - 2a\sqrt{c}\log(x) + 2a\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c
]*(-1 + a^2*x^2))] - 2*a*Sqrt[c]*Log[x] + 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt
[c - a^2*c*x^2]]
```

3.678.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6701, 540, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^2}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{(ax+1)^2}{x^2\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int -\frac{ac(ax+2)}{x\sqrt{c-a^2cx^2}}dx}{c}-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{ac(ax+2)}{x\sqrt{c-a^2cx^2}}dx}{c}-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(a\int \frac{ax+2}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{538}$$

$$-c\left(a\left(a\int \frac{1}{\sqrt{c-a^2cx^2}}dx+2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx+a\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{243}$$

$$-c\left(a\left(\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(a\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{2\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{a^2c}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(a\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
-(c*(-(Sqrt[c - a^2*c*x^2]/(c*x)) + a*(ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/Sqrt[c] - (2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c])))
```

3.678.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.678. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.678.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{(a^2x^2-1)c}{x\sqrt{-c(a^2x^2-1)}}-\left(\frac{a^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}-\frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}\right)c$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}+2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)-2a\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c}}{x}\right.\right.$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/x^2/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2-1)/x/(-c*(a^2*x^2-1))^(1/2)*c-(a^2/(a^2*c)^(1/2)*arctan((a^2*c)^
(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*a/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)
^(1/2))/x))*c
```

3.678.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.55

$$\int\frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$=\left[\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a\sqrt{c}x\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},\ 4a\sqrt{-c}x\arctan\left(\right.\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas"
)
```

output
```
[(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a
*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) +
sqrt(-a^2*c*x^2 + c))/x, 1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*s
qrt(-c)/(a^2*c*x^2 - c)) + a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^
2 + c)*a*sqrt(-c)*x - c) + 2*sqrt(-a^2*c*x^2 + c))/x]
```

3.678.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^2\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

3.678.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^2}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^2), x)
```

3.678.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=-\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2
*a^2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

3.678.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^2\,(a\,x-1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

3.679 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

3.679.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output

```
3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a^2*c*x^2+c)^(
1/2)/x^2+2*a*(-a^2*c*x^2+c)^(1/2)/x
```

3.679.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\frac{1}{2}\left(\frac{(1+4ax)\sqrt{c-a^2cx^2}}{x^2}-3a^2\sqrt{c}\log(x)+3a^2\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)\right)$$

input

```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

output

```
(((1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt
[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

3.679.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6701, 540, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^3}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{(ax+1)^2}{x^3\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int -\frac{ac(3ax+4)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{ac(3ax+4)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{2}a\int \frac{3ax+4}{x^2\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{534}$$

$$-c\left(\frac{1}{2}a\left(3a\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{243}$$

$$-c\left(\frac{1}{2}a\left(\frac{3}{2}a\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(\frac{1}{2}a\left(-\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow 221$$

$$-c\left(\frac{1}{2}a\left(-\frac{3a\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

output
```
-(c*(-1/2*Sqrt[c - a^2*c*x^2]/(c*x^2) + (a*((-4*Sqrt[c - a^2*c*x^2])/(c*x)
 - (3*a*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/2))
```

3.679.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.679. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.679.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\left(4 a^{3} x^{3}+a^{2} x^{2}-4 a x -1\right) c}{2 x^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}+\frac{3 a^{2} \sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)}{2}$
default	$\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{2 c \,x^{2}}-\frac{3 a^{2} \left(\sqrt{-a^{2} c \,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)\right)}{2}-2 a \left(-\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{c x}-2 a^{2} \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\right.\right.$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(4*a^3*x^3+a^2*x^2-4*a*x-1)/x^2/(-c*(a^2*x^2-1))^(1/2)*c+3/2*a^2*c^(1
/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

3.679.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= \left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\sqrt{-a^2cx^2+c}(4\,ax+1)}{4\,x^2}, \frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)}{2\,x^2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas"
)
```

output
```
[1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
 2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2, 1/2*(3*a^2*sqrt(-c)*
x^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^
2 + c)*(4*a*x + 1))/x^2]
```

3.679.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^3\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

3.679.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^3}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^3), x)
```

3.679.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.56

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$

$$+\frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^3a^2c-4\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}c|a|+\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

output

```
-3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c
) + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c - 4*(sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2
*c*x^2 + c))*a^2*c^2 + 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))^2 - c)^2
```

3.679.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^3\,(a\,x-1)}\,dx$$

input

```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

output

```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

3.680 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

3.680.1 Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output

```
a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2)
/x^3+a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

3.680.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\frac{(1+3ax+5a^2x^2)\sqrt{c-a^2cx^2}}{3x^3}-a^3\sqrt{c}\log(x)+a^3\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input

```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

output

```
((1 + 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) - a^3*Sqrt[c]*Log[x]
 + a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

3.680.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6717, 6701, 540, 25, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^4}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{(ax+1)^2}{x^4\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int -\frac{ac(5ax+6)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{ac(5ax+6)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}a\int \frac{5ax+6}{x^3\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{539}$$

$$-c\left(\frac{1}{3}a\left(-\frac{\int -\frac{2ac(3ax+5)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}a\left(a\int \frac{3ax+5}{x^2\sqrt{c-a^2cx^2}}dx-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{534}$$

$$-c\left(\frac{1}{3}a\left(a\left(3a\int\frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow 243$$

$$-c\left(\frac{1}{3}a\left(a\left(\frac{3}{2}a\int\frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow 73$$

$$-c\left(\frac{1}{3}a\left(a\left(-\frac{3\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow 221$$

$$-c\left(\frac{1}{3}a\left(a\left(-\frac{3a\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

output
```
-(c*(-1/3*Sqrt[c - a^2*c*x^2]/(c*x^3) + (a*((-3*Sqrt[c - a^2*c*x^2])/(c*x^
2) + a*((-5*Sqrt[c - a^2*c*x^2])/(c*x) - (3*a*ArcTanh[Sqrt[c - a^2*c*x^2]/
Sqrt[c]])/Sqrt[c])))/3))
```

3.680.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.680. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

3.680.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\left(5a^{4}x^{4}+3a^{3}x^{3}-4a^{2}x^{2}-3ax-1\right)c}{3x^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}}+a^{3}\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)$
default	$\frac{\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{3c\,x^{3}}-2a^{3}\left(\sqrt{-a^{2}c\,x^{2}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)\right)-2a\left(-\frac{\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{2c\,x^{2}}-\right.$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/x^4/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(5*a^4*x^4+3*a^3*x^3-4*a^2*x^2-3*a*x-1)/x^3/(-c*(a^2*x^2-1))^(1/2)*c+
a^3*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

3.680.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.66

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$$
$$=\left[\frac{3\,a^{3}\sqrt{c}x^{3}\log\left(-\frac{a^{2}cx^{2}-2\,\sqrt{-a^{2}cx^{2}+c}\sqrt{c}-2\,c}{x^{2}}\right)+2\,\sqrt{-a^{2}cx^{2}+c}(5\,a^{2}x^{2}+3\,ax+1)}{6\,x^{3}},\;3\,a^{3}\sqrt{-c}x^{3}\arctan\left(\right.\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas"
)
```

output
```
[1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
 2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3, 1/3*(3*a
^3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sq
rt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3]
```

3.680.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}(ax+1)}{x^4\left(ax-1\right)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

3.680.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^4}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^4), x)
```

3.680.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(83)=166$.

Time $=0.29$ (sec) , antiderivative size $=250$, normalized size of antiderivative $=2.53$

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=-\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$
$$+\frac{2\left(3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^3c-3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}c|a|+12\left(\sqrt{-a^2c}x-\sqrt{-a^2}\right.\right.}{3\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-\right.}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c
) + 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c - 3*(sqrt(-a^2*
c)*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) + 12*(sqrt(-a^2*c)*x
- sqrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))*a^3*c^3 - 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x
- sqrt(-a^2*c*x^2 + c))^2 - c)^3
```

3.680.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^4\,\left(a\,x-1\right)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

3.681 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

3.681.1 Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a^2*c*x^2+c)^(
1/2)/x^4+2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2+4
/3*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

3.681.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}(6+16ax+21a^2x^2+32a^3x^3)}{24x^4}-\frac{7}{8}a^4\sqrt{c}\log(x)+\frac{7}{8}a^4\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) - (7
*a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2
]])/8
```

3.681.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6717, 6701, 540, 25, 27, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^5}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{(ax+1)^2}{x^5\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int -\frac{ac(7ax+8)}{x^4\sqrt{c-a^2cx^2}}dx}{4c}-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{ac(7ax+8)}{x^4\sqrt{c-a^2cx^2}}dx}{4c}-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{4}a\int \frac{7ax+8}{x^4\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{539}$$

$$-c\left(\frac{1}{4}a\left(-\frac{\int -\frac{ac(16ax+21)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 25$$

$$-c\left(\frac{1}{4}a\left(\frac{\int\frac{ac(16ax+21)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\int\frac{16ax+21}{x^3\sqrt{c-a^2cx^2}}dx-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 539$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{\int-\frac{ac(21ax+32)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 25$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{\int\frac{ac(21ax+32)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\int\frac{21ax+32}{x^2\sqrt{c-a^2cx^2}}dx-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 534$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(21a\int\frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 243$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(\frac{21}{2}a\int\frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2ca}}{4cx^4}$$

$$\Big\downarrow 73$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{21\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a}}{4cx}$$

$$\Big\downarrow 221$$

3.681. $\int\frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{21a\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2}}{4cx^4}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

output
```
-(c*(-1/4*Sqrt[c - a^2*c*x^2]/(c*x^4) + (a*((-8*Sqrt[c - a^2*c*x^2])/(3*c*
x^3) + (a*((-21*Sqrt[c - a^2*c*x^2])/(2*c*x^2) + (a*((-32*Sqrt[c - a^2*c*x
^2])/(c*x) - (21*a*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/2))/3))
/4))
```

3.681.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.681. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.681.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\left(32 a^{5} x^{5}+21 a^{4} x^{4}-16 a^{3} x^{3}-15 a^{2} x^{2}-16 a x -6\right) c}{24 x^{4} \sqrt{-c \left(a^{2} x^{2}-1\right)}}+\frac{7 a^{4} \sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)}{8}$
default	$\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4 c \,x^{4}}-\frac{9 a^{2} \left(-\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{2 c \,x^{2}}-\frac{a^{2} \left(\sqrt{-a^{2} c \,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)\right)}{2}\right)}{4}-2 a^{4} \Bigg(\sqrt{-a^{2} c \,x^{2}+c}-\sqrt{c}$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x+1)/x^5/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(32*a^5*x^5+21*a^4*x^4-16*a^3*x^3-15*a^2*x^2-16*a*x-6)/x^4/(-c*(a^2*
x^2-1))^(1/2)*c+7/8*a^4*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

3.681.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.38

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= \left[\frac{21\,a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{-a^2cx^2+c}}{48\,x^4},\ 21\,a^4\sqrt{-c}x^4\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="fricas"
)
```

output
```
[1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
 - 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 +
c))/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a
^2*c*x^2 - c)) + (32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 +
c))/x^4]
```

3.681.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^5\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

3.681.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^5}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^5), x)
```

3.681.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(106) = 212$.

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.49

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}$$

$$+\,21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2+96\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(
-c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(
-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 + 96*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^3*a^4*c^3 - 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*
sqrt(-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 +
 32*a^3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 -
c)^4
```

3.681.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^5\,(a\,x-1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

3.682 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

3.682.1 Optimal result

Integrand size = 27, antiderivative size = 228

$$
\begin{aligned}
\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}} \\
&\quad+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}} \\
&\quad+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}
$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^
3/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
+3/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)
^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^5/x/(1-1/a^
2/x^2)^(1/2)
```

3.682.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^4}+\frac{2x^2}{a^3}+\frac{4x^3}{3a^2}+\frac{3x^4}{4a}+\frac{x^5}{5}+\frac{4\log(1-ax)}{a^5}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a^4 + (2*x^2)/a^3 + (4*x^3)/(3*a^2) + (3*x^4)/
(4*a) + x^5/5 + (4*Log[1 - a*x])/a^5))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.682.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.39, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \textcolor{blue}{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textcolor{blue}{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{x^3(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{x^3(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textcolor{blue}{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int\left(-ax^4-3x^3-\frac{4x^2}{a}-\frac{4x}{a^2}-\frac{4}{a^3(ax-1)}-\frac{4}{a^3}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow 2009$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^4}-\frac{4x}{a^3}-\frac{2x^2}{a^2}-\frac{ax^5}{5}-\frac{4x^3}{3a}-\frac{3x^4}{4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*((-4*x)/a^3 - (2*x^2)/a^2 - (4*x^3)/(3*a) - (3*x^4)
/4 - (a*x^5)/5 - (4*Log[1 - a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.682.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.682.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\left(12 a^{5} x^{5}+45 a^{4} x^{4}+80 a^{3} x^{3}+120 a^{2} x^{2}+240 a x+240 \ln \left(a x-1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x-1\right)}{60 a^{4} \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	92

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/60*(12*a^5*x^5+45*a^4*x^4+80*a^3*x^3+120*a^2*x^2+240*a*x+240*ln(a*x-1))*
(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.682.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\left(12\,a^{5}x^{5}+45\,a^{4}x^{4}+80\,a^{3}x^{3}+120\,a^{2}x^{2}+240\,ax+240\,\log\left(ax-1\right)\right)\sqrt{-a^{2}c}}{60\,a^{5}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/60*(12*a^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x + 240*l
og(a*x - 1))*sqrt(-a^2*c)/a^5
```

3.682.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \int \frac{x^{3}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x
)
```

3.682.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.682.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.682.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\,\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.683 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

3.683.1 Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^
2/(1-1/a^2/x^2)^(1/2)+x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x
^3*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1
/2)/a^4/x/(1-1/a^2/x^2)^(1/2)
```

3.683.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^3}+\frac{2x^2}{a^2}+\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1-ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a^3 + (2*x^2)/a^2 + x^3/a + x^4/4 + (4*Log[1 -
 a*x])/a^4))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.683.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{x^2(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{x^2(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax^3-3x^2-\frac{4x}{a}-\frac{4}{a^2(ax-1)}-\frac{4}{a^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^3}-\frac{4x}{a^2}-\frac{ax^4}{4}-\frac{2x^2}{a}-x^3\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

3.683. $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

output
```
-((Sqrt[c - a^2*c*x^2]*((-4*x)/a^2 - (2*x^2)/a - x^3 - (a*x^4)/4 - (4*Log[
1 - a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.683.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.683.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\left(a^{4} x^{4}+4 a^{3} x^{3}+8 a^{2} x^{2}+16 a x +16 \ln \left(a x -1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x -1\right)}{4 a^{3} \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	83

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4*(a^4*x^4+4*a^3*x^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/
2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.683.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log{(ax-1)})\sqrt{-a^2c}}{4\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(-a^2
*c)/a^4
```

3.683.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x
)
```

3.683.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.683.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.683.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.684 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

3.684.1 Optimal result

Integrand size = 25, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x*(-a^2*c*x^2+c)^(1/2)/
a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*l
n(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^3/x/(1-1/a^2/x^2)^(1/2)
```

3.684.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(ax(24+9ax+2a^2x^2)+24\log(1-ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6*
a^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.684.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6746, 6747, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{x(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{x(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax^2-3x-\frac{4}{a}-\frac{4}{a(ax-1)}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^2}-\frac{ax^3}{3}-\frac{4x}{a}-\frac{3x^2}{2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

output

```
-((Sqrt[c - a^2*c*x^2]*((-4*x)/a - (3*x^2)/2 - (a*x^3)/3 - (4*Log[1 - a*x]
)/a^2))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.684.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.684.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{(2a^3x^3+9a^2x^2+24ax+24\ln(ax-1))\sqrt{-c(a^2x^2-1)}\,(ax-1)}{6a^2(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	76

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/6*(2*a^3*x^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-
1)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.684.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log{(ax-1)})\sqrt{-a^2c}}{6\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(-a^2*c)/a^3
```

3.684.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.684.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+cx}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.684.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+cx}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.684.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.685 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.685.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1
/2)
```

3.685.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/(Sqrt[1 - 1
/(a^2*x^2)]*x)
```

3.685.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax+\frac{4}{1-ax}-3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/(a*Sqrt[1
- 1/(a^2*x^2)]*x))
```

3.685. $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.685.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.685.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\left(a^{2} x^{2}+6 a x+8 \ln \left(a x-1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x-1\right)}{2 a \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	67

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

3.685.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(a^2x^2+6\,ax+8\,\log{(ax-1)})\sqrt{-a^2c}}{2\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

3.685.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.685.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.685.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.685.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.686 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

3.686.1 Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1
-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2
)
```

3.686.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/(Sqrt[1 - 1/(a^2
*x^2)]*x)
```

3.686.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a}{ax-1}-a+\frac{1}{x}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}(-ax-4\log(1-ax)+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(
a^2*x^2)]*x))
```

3.686. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}dx$

3.686.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.686.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(-a x+\ln\left(x\right)-4 \ln\left(a x-1\right)\right) \left(a x-1\right)}{\left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	59

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBO
SE)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x)-4*ln(a*x-1))*(a*x-1)/(a*x+1)^2/((a*x-1
)/(a*x+1))^(3/2)
```

3.686.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.25

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax+4\log(ax-1)-\log(x))}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="f
ricas")
```

output
```
sqrt(-a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a
```

3.686.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c(ax-1)(ax+1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.686.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.686.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.686.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.687 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.687.1 Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(-a^2*c*x^2+c)^(1/2
)/x/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^
(1/2)
```

3.687.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{ax}-3\log(x)+4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(a*x) - 3*Log[x] + 4*Log[1 - a*x]))/(Sqrt[1 - 1/(a
^2*x^2)]*x)
```

3.687.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x^2(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x^2(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^2}{ax-1}+\frac{3a}{x}+\frac{1}{x^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(3a\log(x)-4a\log(1-ax)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-x^(-1) + 3*a*Log[x] - 4*a*Log[1 - a*x]))/(a*Sqrt[
1 - 1/(a^2*x^2)]*x))
```

3.687.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.687.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(3a\ln(x)x-4a\ln(ax-1)x-1)(ax-1)}{(ax+1)^2x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVER
BOSE)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(3*a*ln(x)*x-4*a*ln(a*x-1)*x-1)*(a*x-1)/(a*x+1)^2/
x/((a*x-1)/(a*x+1))^(3/2)
```

3.687.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(4\,ax\log{(ax-1)}-3\,ax\log{(x)}+1)}{ax}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"fricas")
```

output
```
sqrt(-a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a*x)
```

3.687.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Timed out
```

3.687.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.687.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"giac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.687.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.688 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

3.688.1 Optimal result

Integrand size = 27, antiderivative size = 153

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
1/2*(-a^2*c*x^2+c)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3*(-a^2*c*x^2+c)^(1/2)/
x^2/(1-1/a^2/x^2)^(1/2)-4*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/
2)+4*a*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

3.688.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{2ax^2}+\frac{3}{x}-4a\log(x)+4a\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(2*a*x^2) + 3/x - 4*a*Log[x] + 4*a*Log[1 - a*x]))/
(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.688.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \textbf{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x^3(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x^3(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^3}{ax-1}+\frac{4a^2}{x}+\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(1-ax)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-1/2*1/x^2 - (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1
- a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.688.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.688.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\left(8a^{2}\ln\left(x\right)x^{2}-8a^{2}\ln\left(ax-1\right)x^{2}-6ax-1\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{2\left(ax+1\right)^{2}x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	77

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVER
BOSE)
```

output
```
-1/2*(8*a^2*ln(x)*x^2-8*a^2*ln(a*x-1)*x^2-6*a*x-1)*(-c*(a^2*x^2-1))^(1/2)*
(a*x-1)/(a*x+1)^2/x^2/((a*x-1)/(a*x+1))^(3/2)
```

3.688.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{8\,a^3\sqrt{-c}x^2\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+\sqrt{-a^2c}(6\,ax+1)}{2\,ax^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm=
"fricas")
```

output
```
1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + sqrt(-a^2*c)*(6*a*x + 1))/(a*x^2)
```

3.688.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

output
```
Timed out
```

3.688.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.688.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm=
"giac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.688.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.689 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

3.689.1 Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output

```
1/3*(-a^2*c*x^2+c)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+3/2*(-a^2*c*x^2+c)^(1/2
)/x^3/(1-1/a^2/x^2)^(1/2)+4*a*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)
-4*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(-a*x+1)*(
-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

3.689.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.39

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{3ax^3}+\frac{3}{2x^2}+\frac{4a}{x}-4a^2\log(x)+4a^2\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input

```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(3*a*x^3) + 3/(2*x^2) + (4*a)/x - 4*a^2*Log[x] + 4
*a^2*Log[1 - a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.689.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x^4(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x^4(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^4}{ax-1}+\frac{4a^3}{x}+\frac{4a^2}{x^2}+\frac{3a}{x^3}+\frac{1}{x^4}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(4a^3\log(x)-4a^3\log(1-ax)-\frac{4a^2}{x}-\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

3.689. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}dx$

output

```
-((Sqrt[c - a^2*c*x^2]*(-1/3*1/x^3 - (3*a)/(2*x^2) - (4*a^2)/x + 4*a^3*Log
[x] - 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.689.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.689.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{\left(24 a^{3} \ln \left(x \right) x^{3}-24 a^{3} \ln \left(a x -1\right) x^{3}-24 a^{2} x^{2}-9 a x -2\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x -1\right)}{6 x^{3} \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	85

input

```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVER
BOSE)
```

output
```
-1/6*(24*a^3*ln(x)*x^3-24*a^3*ln(a*x-1)*x^3-24*a^2*x^2-9*a*x-2)*(-c*(a^2*x
^2-1))^(1/2)*(a*x-1)/x^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.689.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.51

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{24\,a^4\sqrt{-c}x^3\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+\left(24\,a^2x^2+9\,ax+2\right)\sqrt{-a^2c}}{6\,ax^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm=
"fricas")
```

output
```
1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*
x - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(-a^2*c
))/(a*x^3)
```

3.689.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

output
```
Timed out
```

3.689.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.689.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm=
"giac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.689.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.690 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

3.690.1 Optimal result

Integrand size = 27, antiderivative size = 228

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5} + \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4} + \frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3} + \frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2} - \frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output

```
1/4*(-a^2*c*x^2+c)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)/x^
4/(1-1/a^2/x^2)^(1/2)+2*a*(-a^2*c*x^2+c)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+4*a
^2*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(x)*(-a^2*c*x^2+c)
^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/
a^2/x^2)^(1/2)
```

3.690.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.35

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{4ax^4}+\frac{1}{x^3}+\frac{2a}{x^2}+\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(4*a*x^4) + x^(-3) + (2*a)/x^2 + (4*a^2)/x - 4*a^3
*Log[x] + 4*a^3*Log[1 - a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.690.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.38, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x^5}\,dx$$

$$\Big\downarrow \; 6746$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 6747$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x^5(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x^5(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^5}{ax-1}+\frac{4a^4}{x}+\frac{4a^3}{x^2}+\frac{4a^2}{x^3}+\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(4a^4\log(x)-4a^4\log(1-ax)-\frac{4a^3}{x}-\frac{2a^2}{x^2}-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-1/4*1/x^4 - a/x^3 - (2*a^2)/x^2 - (4*a^3)/x + 4*a
^4*Log[x] - 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.690.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.690. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

3.690.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(16\ln\left(x\right)x^{4}a^{4}-16\ln\left(ax-1\right)x^{4}a^{4}-16a^{3}x^{3}-8a^{2}x^{2}-4ax-1\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{4x^{4}\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	93

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x,method=_RETURNVER
BOSE)
```

output
```
-1/4*(16*ln(x)*x^4*a^4-16*ln(a*x-1)*x^4*a^4-16*a^3*x^3-8*a^2*x^2-4*a*x-1)*
(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/x^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.690.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{16\,a^{5}\sqrt{-c}x^{4}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx+\sqrt{-a^{2}c}(2\,ax-1)\sqrt{-c}+ac}{ax^{2}-x}\right)+\left(16\,a^{3}x^{3}+8\,a^{2}x^{2}+4\,ax+1\right)\sqrt{-a^{2}c}}{4\,ax^{4}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm=
"fricas")
```

output
```
1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*
x - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)
*sqrt(-a^2*c))/(a*x^4)
```

3.690.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

output
```
Timed out
```

3.690.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.690.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm=
"giac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.690.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.691 $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx$

3.691.1 Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a\,(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{2\,(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a^2(1-ax)\,(c-a^2cx^2)^{3/2}} + \frac{7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a^2\,(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a^2\,(c-a^2cx^2)^{3/2}}$$

output
```
(1-1/a^2/x^2)^(3/2)*x^4/a/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^5
/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a^2/(-a*x+1)/(-a^2*c*x^2
+c)^(3/2)+7/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)+
1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)
```

3.691.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(2\left(\frac{2x}{a}+x^2+\frac{1}{a^2-a^3x}\right)+\frac{7\log(1-ax)}{a^2}+\frac{\log(1+ax)}{a^2}\right)}{4\,(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*x^3*(2*((2*x)/a + x^2 + (a^2 - a^3*x)^(-1)) + (7*
Log[1 - a*x])/a^2 + Log[1 + a*x]/a^2))/(4*(c - a^2*c*x^2)^(3/2))
```

3.691.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{x^4}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{x}{a^3}+\frac{7}{4a^4(ax-1)}+\frac{1}{4a^4(ax+1)}+\frac{1}{2a^4(ax-1)^2}+\frac{1}{a^4}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^5(1-ax)}+\frac{7\log(1-ax)}{4a^5}+\frac{\log(ax+1)}{4a^5}+\frac{x}{a^4}+\frac{x^2}{2a^3}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(x/a^4 + x^2/(2*a^3) + 1/(2*a^5*(1 - a*x)
) + (7*Log[1 - a*x])/(4*a^5) + Log[1 + a*x]/(4*a^5)))/(c - a^2*c*x^2)^(3/2
)
```

3.691. $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}dx$

3.691.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.691.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(2 a^{3} x^{3}+2 a^{2} x^{2}+a \ln \left(a x +1\right) x +7 a \ln \left(a x -1\right) x -4 a x -\ln \left(a x +1\right)-7 \ln \left(a x -1\right)-2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2} a^{5}}$	106

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3+2*a^2*x^2+a*
ln(a*x+1)*x+7*a*ln(a*x-1)*x-4*a*x-ln(a*x+1)-7*ln(a*x-1)-2)/(a^2*x^2-1)/c^2
/a^5
```

3.691.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{(2\,a^3x^3+2\,a^2x^2-4\,ax+(ax-1)\log{(ax+1)}+7\,(ax-1)\log{(ax-1)}-2)\sqrt{-a^2c}}{4\,(a^7c^2x-a^6c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/4*(2*a^3*x^3 + 2*a^2*x^2 - 4*a*x + (a*x - 1)*log(a*x + 1) + 7*(a*x - 1)*
log(a*x - 1) - 2)*sqrt(-a^2*c)/(a^7*c^2*x - a^6*c^2)
```

3.691.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**4/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
 x)
```

3.691.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.691.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^4}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.691.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^4}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.692 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

3.692.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a\,(c-a^2cx^2)^{3/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a\,(c-a^2cx^2)^{3/2}}$$

output
```
(1-1/a^2/x^2)^(3/2)*x^4/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a
/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+5/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a/(-
a^2*c*x^2+c)^(3/2)-1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a/(-a^2*c*x^2+c)^
(3/2)
```

3.692.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(x+\frac{1}{2a-2a^2x}+\frac{5\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*x^3*(x + (2*a - 2*a^2*x)^(-1) + (5*Log[1 - a*x])/
(4*a) - Log[1 + a*x]/(4*a)))/(c - a^2*c*x^2)^(3/2)
```

3.692.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{x^3}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(-\frac{1}{4a^3(ax+1)}+\frac{1}{a^3}+\frac{5}{4a^3(ax-1)}+\frac{1}{2a^3(ax-1)^2}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}+\frac{x}{a^3}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 -
 a*x])/(4*a^4) - Log[1 + a*x]/(4*a^4)))/(c - a^2*c*x^2)^(3/2)
```

3.692. $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}dx$

3.692.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.692.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{\sqrt{-c(a^{2}x^{2}-1)}\,(-4a^{2}x^{2}+a\ln(ax+1)x-5a\ln(ax-1)x+4ax-\ln(ax+1)+5\ln(ax-1)+2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^{2}x^{2}-1)c^{2}a^{4}}$	98

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(-4*a^2*x^2+a*ln(a*x+1
)*x-5*a*ln(a*x-1)*x+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^4
```

3.692.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.40

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{(4\,a^2x^2-4\,ax-(ax-1)\log{(ax+1)}+5\,(ax-1)\log{(ax-1)}-2)\sqrt{-a^2c}}{4\,(a^6c^2x-a^5c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)*sqrt(-a^2*c)/(a^6*c^2*x - a^5*c^2)
```

3.692.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
 x)
```

3.692.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(x^3/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.692.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.692.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.693 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

3.693.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

output
```
1/2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+3/4*(1-1/a^2/x^2
)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln
(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

3.693.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.48

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{2}{1-ax}+3\log(1-ax)+\log(1+ax)\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*x^3*(2/(1 - a*x) + 3*Log[1 - a*x] + Log[1 + a*x])
)/(4*(c - a^2*c*x^2)^(3/2))
```

3.693.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{x^2}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{4a^2(ax+1)}+\frac{3}{4a^2(ax-1)}+\frac{1}{2a^2(ax-1)^2}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(
4*a^3) + Log[1 + a*x]/(4*a^3)))/(c - a^2*c*x^2)^(3/2)
```

3.693. $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}dx$

3.693.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.693.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(a \ln \left(a x +1\right) x +3 a \ln \left(a x -1\right) x -\ln \left(a x +1\right)-3 \ln \left(a x -1\right)-2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\,\left(a^{2} x^{2}-1\right) c^{2} a^{3}}$	86

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x+3*a*ln(a
*x-1)*x-ln(a*x+1)-3*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a^3
```

3.693.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{-a^2c}((ax-1)\log{(ax+1)}+3\,(ax-1)\log{(ax-1)}-2)}{4\,(a^5c^2x-a^4c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 2)/(
a^5*c^2*x - a^4*c^2)
```

3.693.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
 x)
```

3.693.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \int \frac{x^2}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.693.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.693.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.694 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

3.694.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\,(c-a^2cx^2)^{3/2}}$$

output
```
1/2*a*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a*(1-1/a^2
/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

3.694.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{1}{1-ax}-\text{arctanh}(ax)\right)}{2\,(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a*(1 - 1/(a^2*x^2))^(3/2)*x^3*((1 - a*x)^(-1) - ArcTanh[a*x]))/(2*(c - a^
2*c*x^2)^(3/2))
```

3.694.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6746, 6747, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \ 6746$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \ 6747$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{x}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \ 86$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{2(a^2x^2-1)a}+\frac{1}{2(ax-1)^2a}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^2(1-ax)}-\frac{\text{arctanh}(ax)}{2a^2}\right)}{(c-a^2cx^2)^{3/2}}$$

input

```
Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

output

```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*a^2*(1 - a*x)) - ArcTanh[a*x]/(2*a^
2)))/(c - a^2*c*x^2)^(3/2)
```

3.694.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.694.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(a \ln \left(a x +1\right) x -a \ln \left(a x -1\right) x -\ln \left(a x +1\right)+\ln \left(a x -1\right)+2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2} a^{2}}$	84

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-a*ln(a*
x-1)*x-ln(a*x+1)+ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^2
```

3.694.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\left(a^4c^2x-a^3c^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)
/(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^4*c^2*x - a^3*c^2)
```

3.694.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.694.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

output
```
integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.694.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

output
```
integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.694.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.695 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

3.695.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

output

```
1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1
/a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

3.695.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3(-1+(-1+ax)\text{arctanh}(ax))}{(-2+2ax)\left(c-a^2cx^2\right)^{3/2}}$$

input

```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output

```
(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1 + (-1 + a*x)*ArcTanh[a*x]))/((-2 + 2*
a*x)*(c - a^2*c*x^2)^(3/2))
```

3.695.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{2(ax-1)^2}-\frac{1}{2(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\operatorname{arctanh}(ax)}{2a}+\frac{1}{2a(1-ax)}\right)}{(c-a^2cx^2)^{3/2}}$$

input

```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output

```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)))
/(c - a^2*c*x^2)^(3/2)
```

3.695.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.695.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(a\ln(ax+1)x-a\ln(ax-1)x-\ln(ax+1)+\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a}$	84

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-a*ln(a*x
-1)*x-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

3.695.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)
/(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

3.695.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.695.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.695.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.695.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{1}{\left(c-a^2\,c\,x^2\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.696 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

3.696.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}} - \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

output

```
1/2*a^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2
/x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-3/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3
*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)
/(-a^2*c*x^2+c)^(3/2)
```

3.696.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.38

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{1}{2-2ax}+\log(x)-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(1+ax)\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

input

```
Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

output

```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*((2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a
*x])/4 - Log[1 + a*x]/4))/(c - a^2*c*x^2)^(3/2)
```

3.696.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{x(1-ax)^2(ax+1)}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{93}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(-\frac{3a}{4(ax-1)}-\frac{a}{4(ax+1)}+\frac{a}{2(ax-1)^2}+\frac{1}{x}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2(1-ax)}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*(1 - a*x)) + Log[x] - (3*Log[1 - a*
x])/4 - Log[1 + a*x]/4))/(c - a^2*c*x^2)^(3/2)
```

3.696.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.696.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(a \ln \left(a x +1\right) x -4 a \ln \left(x \right) x +3 a \ln \left(a x -1\right) x -\ln \left(a x +1\right)+4 \ln \left(x \right)-3 \ln \left(a x -1\right)+2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2}}$	93

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x+1)*x-4*a*ln(
x)*x+3*a*ln(a*x-1)*x-ln(a*x+1)+4*ln(x)-3*ln(a*x-1)+2)/(a^2*x^2-1)/c^2
```

3.696.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2c}((ax-1)\log(ax+1)+3\,(ax-1)\log(ax-1)-4\,(ax-1)\log(x)+2)}{4\,(a^2c^2x-ac^2)}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

output

```
-1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 4*(
a*x - 1)*log(x) + 2)/(a^2*c^2*x - a*c^2)
```

3.696.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(3/2),x)
```

output

```
Timed out
```

3.696.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

output

```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.696.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.696.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.697 $\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx$

3.697.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}\\ &\quad+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}\\ &\quad-\frac{5a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}\end{aligned}$$

output
```
-a^3*(1-1/a^2/x^2)^(3/2)*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^4*(1-1/a^2/x^2)^(3
/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-
a^2*c*x^2+c)^(3/2)-5/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+
c)^(3/2)+1/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

3.697.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(-\frac{4}{x}+\frac{2a}{1-ax}+4a\log(x)-5a\log(1-ax)+a\log(1+ax)\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

input

```
Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

output

```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-4/x + (2*a)/(1 - a*x) + 4*a*Log[x] - 5*
a*Log[1 - a*x] + a*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/2))
```

3.697.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{x^2(1-ax)^2(ax+1)}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(-\frac{5a^2}{4(ax-1)}+\frac{a^2}{4(ax+1)}+\frac{a^2}{2(ax-1)^2}+\frac{a}{x}+\frac{1}{x^2}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a}{2(1-ax)}+a\log(x)-\frac{5}{4}a\log(1-ax)+\frac{1}{4}a\log(ax+1)-\frac{1}{x}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-x^(-1) + a/(2*(1 - a*x)) + a*Log[x] - (
5*a*Log[1 - a*x])/4 + (a*Log[1 + a*x])/4))/(c - a^2*c*x^2)^(3/2)
```

3.697.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.697. $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

3.697.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(a^{2} \ln \left(a x +1\right) x^{2}+4 a^{2} \ln \left(x \right) x^{2}-5 a^{2} \ln \left(a x -1\right) x^{2}-a \ln \left(a x +1\right) x -4 a \ln \left(x \right) x +5 a \ln \left(a x -1\right) x -6 a x +4\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2} x}$	118

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^2*ln(a*x+1)*x^2+4*a^
2*ln(x)*x^2-5*a^2*ln(a*x-1)*x^2-a*ln(a*x+1)*x-4*a*ln(x)*x+5*a*ln(a*x-1)*x-
6*a*x+4)/(a^2*x^2-1)/c^2/x
```

3.697.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{-a^2c}\left(6\,ax-\left(a^2x^2-ax\right)\log\left(ax+1\right)+5\left(a^2x^2-ax\right)\log\left(ax-1\right)-4\left(a^2x^2-ax\right)\log\left(x\right)-4\right)}{4\left(a^2c^2x^2-ac^2x\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
-1/4*sqrt(-a^2*c)*(6*a*x - (a^2*x^2 - a*x)*log(a*x + 1) + 5*(a^2*x^2 - a*x
)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 4)/(a^2*c^2*x^2 - a*c^2*x)
```

3.697.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.697.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.697.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.697.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.698 $\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx$

3.698.1 Optimal result

Integrand size = 25, antiderivative size = 252

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{2\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}\\ &\quad+\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}\\ &\quad-\frac{7a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}\end{aligned}$$

output
```
-1/2*a^3*(1-1/a^2/x^2)^(3/2)*x/(-a^2*c*x^2+c)^(3/2)-a^4*(1-1/a^2/x^2)^(3/2
)*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^5*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*
c*x^2+c)^(3/2)+2*a^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-7/
4*a^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^5*(1-1
/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

3.698.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c-a^2cx^2)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3\left(-\frac{2}{x^2}-\frac{4a}{x}+\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(1+\right.}{4\,(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8
*a^2*Log[x] - 7*a^2*Log[1 - a*x] - a^2*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(
3/2))
```

3.698.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^6}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{x^3(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(-\frac{7a^3}{4(ax-1)}-\frac{a^3}{4(ax+1)}+\frac{a^3}{2(ax-1)^2}+\frac{2a^2}{x}+\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^2}{2(1-ax)}+2a^2\log(x)-\frac{7}{4}a^2\log(1-ax)-\frac{1}{4}a^2\log(ax+1)-\frac{a}{x}-\frac{1}{2x^2}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1/2*1/x^2 - a/x + a^2/(2*(1 - a*x)) + 2
*a^2*Log[x] - (7*a^2*Log[1 - a*x])/4 - (a^2*Log[1 + a*x])/4))/(c - a^2*c*x
^2)^(3/2)
```

3.698.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.698. $\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

3.698.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.55

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(a^3\ln(ax+1)x^3-8a^3\ln(x)x^3+7a^3\ln(ax-1)x^3-a^2\ln(ax+1)x^2+8a^2\ln(x)x^2-7a^2\ln(ax-1)x^2+6a^2x^2-2ax-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2x^2}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^3*ln(a*x+1)*x^3-8*a
^3*ln(x)*x^3+7*a^3*ln(a*x-1)*x^3-a^2*ln(a*x+1)*x^2+8*a^2*ln(x)*x^2-7*a^2*l
n(a*x-1)*x^2+6*a^2*x^2-2*a*x-2)/(a^2*x^2-1)/c^2/x^2
```

3.698.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx =$$

$$-\frac{\left(6\,a^2x^2-2\,ax+\left(a^3x^3-a^2x^2\right)\log\left(ax+1\right)+7\left(a^3x^3-a^2x^2\right)\log\left(ax-1\right)-8\left(a^3x^3-a^2x^2\right)\log\left(x\right)-2\right)}{4\left(a^2c^2x^3-ac^2x^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
-1/4*(6*a^2*x^2 - 2*a*x + (a^3*x^3 - a^2*x^2)*log(a*x + 1) + 7*(a^3*x^3 -
a^2*x^2)*log(a*x - 1) - 8*(a^3*x^3 - a^2*x^2)*log(x) - 2)*sqrt(-a^2*c)/(a^
2*c^2*x^3 - a*c^2*x^2)
```

3.698.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.698.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.698.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.698.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{1}{x^3\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input

```
int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output

```
int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.699 $\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx$

3.699.1 Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}{(c-a^2cx^2)^{5/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1-ax)^2\,(c-a^2cx^2)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{a(1-ax)\,(c-a^2cx^2)^{5/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1+ax)\,(c-a^2cx^2)^{5/2}}+\frac{23\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16a\,(c-a^2cx^2)^{5/2}}-\frac{7\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16a\,(c-a^2cx^2)^{5/2}}$$

output
```
(1-1/a^2/x^2)^(5/2)*x^6/(-a^2*c*x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a
/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+(1-1/a^2/x^2)^(5/2)*x^5/a/(-a*x+1)/(-a^2*
c*x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+
23/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-7/16*(1-1/
a^2/x^2)^(5/2)*x^5*ln(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)
```

3.699.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.38

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(x-\frac{1}{8a(-1+ax)^2}+\frac{1}{a-a^2x}-\frac{1}{8a+8a^2x}+\frac{23\log(1-ax)}{16a}-\frac{7\log(1+ax)}{16a}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*x^5*(x - 1/(8*a*(-1 + a*x)^2) + (a - a^2*x)^(-1)
- (8*a + 8*a^2*x)^(-1) + (23*Log[1 - a*x])/(16*a) - (7*Log[1 + a*x])/(16*a
)))/(c - a^2*c*x^2)^(5/2)
```

3.699.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.43, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x^5}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{x^5}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{7}{16a^5(ax+1)}-\frac{1}{8a^5(ax+1)^2}-\frac{1}{a^5}-\frac{23}{16a^5(ax-1)}-\frac{1}{a^5(ax-1)^2}-\frac{1}{4a^5(ax-1)^3}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow 2009$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(ax+1)}+\frac{1}{8a^6(1-ax)^2}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(ax+1)}{16a^6}-\frac{x}{a^5}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-(x/a^5) + 1/(8*a^6*(1 - a*x)^2) - 1/(
a^6*(1 - a*x)) + 1/(8*a^6*(1 + a*x)) - (23*Log[1 - a*x])/(16*a^6) + (7*Log
[1 + a*x])/(16*a^6)))/(c - a^2*c*x^2)^(5/2))
```

3.699.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.699.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(-16a^4x^4+7a^3\ln(ax+1)x^3-23a^3\ln(ax-1)x^3+16a^3x^3-7a^2\ln(ax+1)x^2+23a^2\ln(ax-1)x^2+34a^2x^2-7a\ln(ax+1)x}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^6(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(-16*a^4*x^4+7
*a^3*ln(a*x+1)*x^3-23*a^3*ln(a*x-1)*x^3+16*a^3*x^3-7*a^2*ln(a*x+1)*x^2+23*
a^2*ln(a*x-1)*x^2+34*a^2*x^2-7*a*ln(a*x+1)*x+23*a*ln(a*x-1)*x-18*a*x+7*ln(
a*x+1)-23*ln(a*x-1)-12)/(a^2*x^2-1)/c^3/a^6/(a*x+1)
```

3.699.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^{5/2}}\,dx =$$

$$-\frac{\left(16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\left(a^3x^3-a^2x^2-ax+1\right.}{16\left(a^{10}c^3x^3-a^9c^3x^2-a^8c^3x+a^7c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^
2 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1)
+ 12)*sqrt(-a^2*c)/(a^10*c^3*x^3 - a^9*c^3*x^2 - a^8*c^3*x + a^7*c^3)
```

3.699.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**5/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.699.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^5}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(x^5/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.699.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.699.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^5}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.700 $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx$

3.700.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx &= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} \\ &\quad+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} \\ &\quad+\frac{11\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\left(c-a^2cx^2\right)^{5/2}}\end{aligned}$$

output

```
-1/8*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+3/4*(1-1/a^2/
x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*(1-1/a^2/x^2)^(5/2)*x^5/(
a*x+1)/(-a^2*c*x^2+c)^(5/2)+11/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2
*c*x^2+c)^(5/2)+5/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2
)
```

3.700.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{2(-6+3ax+5a^2x^2)}{(-1+ax)^2(1+ax)}+11\log(1-ax)+5\log(1+ax)\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*x^5*((-2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*
(1 + a*x)) + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5/2))
```

3.700.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x^4}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{x^4}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-\frac{5}{16a^4(ax+1)}+\frac{1}{8a^4(ax+1)^2}-\frac{11}{16a^4(ax-1)}-\frac{3}{4a^4(ax-1)^2}-\frac{1}{4a^4(ax-1)^3}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{3}{4a^5(1-ax)}-\frac{1}{8a^5(ax+1)}+\frac{1}{8a^5(1-ax)^2}-\frac{11\log(1-ax)}{16a^5}-\frac{5\log(ax+1)}{16a^5}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a^5*(1 - a*x)^2) - 3/(4*a^5*(1 -
a*x)) - 1/(8*a^5*(1 + a*x)) - (11*Log[1 - a*x])/(16*a^5) - (5*Log[1 + a*x]
)/(16*a^5)))/(c - a^2*c*x^2)^(5/2))
```

3.700.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.700. $\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

3.700.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(12+11a^3\ln(ax-1)x^3+5a^3\ln(ax+1)x^3-11a^2\ln(ax-1)x^2-5a^2\ln(ax+1)x^2-10a^2x^2-11a\ln(ax-1)x-5a\ln(ax+}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^5(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(12+11*a^3*ln
(a*x-1)*x^3+5*a^3*ln(a*x+1)*x^3-11*a^2*ln(a*x-1)*x^2-5*a^2*ln(a*x+1)*x^2-1
0*a^2*x^2-11*a*ln(a*x-1)*x-5*a*ln(a*x+1)*x-6*a*x+11*ln(a*x-1)+5*ln(a*x+1))
/(a^2*x^2-1)/c^3/a^5/(a*x+1)
```

3.700.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(10\,a^2x^2+6\,ax-5\,(a^3x^3-a^2x^2-ax+1)\log{(ax+1)}-11\,(a^3x^3-a^2x^2-ax+1)}{16\,(a^9c^3x^3-a^8c^3x^2-a^7c^3x+a^6c^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
1/16*(10*a^2*x^2 + 6*a*x - 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) -
11*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 12)*sqrt(-a^2*c)/(a^9*c^3*
x^3 - a^8*c^3*x^2 - a^7*c^3*x + a^6*c^3)
```

3.700.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.700.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.700.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.700.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.701 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

3.701.1 Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
-1/8*a*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/2*a*(1-1/
a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a*(1-1/a^2/x^2)^(5/2)
*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a*(1-1/a^2/x^2)^(5/2)*x^5*arctanh(a*
x)/(-a^2*c*x^2+c)^(5/2)
```

3.701.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.41

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(\frac{2+ax-5a^2x^2}{(-1+ax)^2(1+ax)}-3\operatorname{arctanh}(ax)\right)}{8\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(a*(1 - 1/(a^2*x^2))^(5/2)*x^5*((2 + a*x - 5*a^2*x^2)/((-1 + a*x)^2*(1 + a
*x)) - 3*ArcTanh[a*x]))/(8*(c - a^2*c*x^2)^(5/2))
```

3.701.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{6746, 6747, 25, 99, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x^3}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{x^3}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-\frac{1}{2a^3(ax-1)^2}-\frac{1}{8a^3(ax+1)^2}-\frac{1}{4a^3(ax-1)^3}-\frac{3}{8a^3(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\text{arctanh}(ax)}{8a^4}-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(ax+1)}+\frac{1}{8a^4(1-ax)^2}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

3.701. $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}dx$

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a^4*(1 - a*x)^2) - 1/(2*a^4*(1 -
a*x)) + 1/(8*a^4*(1 + a*x)) + (3*ArcTanh[a*x])/(8*a^4)))/(c - a^2*c*x^2)^(
5/2))
```

3.701.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.701.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-3a^2\ln(ax+1)x^2+3a^2\ln(ax-1)x^2+10a^2x^2-3a\ln(ax+1)x+3a\ln(ax-1)x-2ax+}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^4(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*ln(a*x+
1)*x^3-3*a^3*ln(a*x-1)*x^3-3*a^2*ln(a*x+1)*x^2+3*a^2*ln(a*x-1)*x^2+10*a^2*
x^2-3*a*ln(a*x+1)*x+3*a*ln(a*x-1)*x-2*a*x+3*ln(a*x+1)-3*ln(a*x-1)-4)/(a^2*
x^2-1)/c^3/a^4/(a*x+1)
```

3.701.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(5a^2x^2-ax-2\right)\sqrt{-a^2c}}{16\left(a^8c^3x^3-a^7c^3x^2-a^6c^3x+a^5c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(
-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(5*a^2*x^2 - a*x - 2)*sqrt(-a^2
*c))/(a^8*c^3*x^3 - a^7*c^3*x^2 - a^6*c^3*x + a^5*c^3)
```

3.701.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.701.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(x^3/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.701.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.701.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.702 $\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

3.702.1 Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
-1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/4*a^2*(
1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)
^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5*ar
ctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

3.702.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(2-3ax-a^2x^2+(-1+ax)^2(1+ax)\operatorname{arctanh}(ax)\right)}{8a^2c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - a^2*x^2 + (-1 + a*x)^2*(1 + a*x)*Arc
Tanh[a*x]))/(8*a^2*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

3.702.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^3}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x^2}{(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{x^2}{(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-\frac{1}{4a^2(ax-1)^2}+\frac{1}{8a^2(ax+1)^2}-\frac{1}{4a^2(ax-1)^3}+\frac{1}{8a^2\left(a^2x^2-1\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{\text{arctanh}(ax)}{8a^3}-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(ax+1)}+\frac{1}{8a^3(1-ax)^2}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a^3*(1 - a*x)^2) - 1/(4*a^3*(1 -
a*x)) - 1/(8*a^3*(1 + a*x)) - ArcTanh[a*x]/(8*a^3)))/(c - a^2*c*x^2)^(5/2)
)
```

3.702.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.702.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(a^3\ln(ax+1)x^3-a^3\ln(ax-1)x^3-a^2\ln(ax+1)x^2+a^2\ln(ax-1)x^2-2a^2x^2-a\ln(ax+1)x+a\ln(ax-1)x-6ax+\ln(ax}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^3(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a^3*ln(a*x+1
)*x^3-a^3*ln(a*x-1)*x^3-a^2*ln(a*x+1)*x^2+a^2*ln(a*x-1)*x^2-2*a^2*x^2-a*ln
(a*x+1)*x+a*ln(a*x-1)*x-6*a*x+ln(a*x+1)-ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a^3/(
a*x+1)
```

3.702.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.73

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(a^2x^2+3\,ax-2\right)\sqrt{-a^2c}}{16\left(a^7c^3x^3-a^6c^3x^2-a^5c^3x+a^4c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a
^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 + 3*a*x - 2)*sqrt(-a^2*c
))/(a^7*c^3*x^3 - a^6*c^3*x^2 - a^5*c^3*x + a^4*c^3)
```

3.702.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.702.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.702.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.702.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.703 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

3.703.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
-1/8*a^3*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/8*a^3*(
1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^3*(1-1/a^2/x^2)^
(5/2)*x^5*arctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

3.703.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{1}{(-1+ax)^2}-\frac{1}{1+ax}+\operatorname{arctanh}(ax)\right)}{8\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-(-1 + a*x)^(-2) - (1 + a*x)^(-1) + ArcT
anh[a*x]))/(8*(c - a^2*c*x^2)^(5/2))
```

3.703.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6746, 6747, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{x}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{8(a^2x^2-1)a}-\frac{1}{8(ax+1)^2a}-\frac{1}{4(ax-1)^3a}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{\operatorname{arctanh}(ax)}{8a^2}+\frac{1}{8a^2(ax+1)}+\frac{1}{8a^2(1-ax)^2}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a^2*(1 - a*x)^2) + 1/(8*a^2*(1 +
a*x)) - ArcTanh[a*x]/(8*a^2)))/(c - a^2*c*x^2)^(5/2))
```

3.703.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.703.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, (a^{3} \ln \left(a x +1\right) x^{3}-a^{3} \ln \left(a x -1\right) x^{3}-a^{2} \ln \left(a x +1\right) x^{2}+a^{2} \ln \left(a x -1\right) x^{2}-2 a^{2} x^{2}-a \ln \left(a x +1\right) x +a \ln \left(a x -1\right) x +2 a x +\ln (a x}{16 \sqrt{\frac{a x -1}{a x +1}}\, \left(a x -1\right) \left(a^{2} x^{2}-1\right) c^{3} a^{2} \left(a x +1\right)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a^3*ln(a*x+1
)*x^3-a^3*ln(a*x-1)*x^3-a^2*ln(a*x+1)*x^2+a^2*ln(a*x-1)*x^2-2*a^2*x^2-a*ln
(a*x+1)*x+a*ln(a*x-1)*x+2*a*x+ln(a*x+1)-ln(a*x-1)-4)/(a^2*x^2-1)/c^3/a^2/(
a*x+1)
```

3.703.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(a^2x^2-ax+2\right)\sqrt{-a^2c}}{16\left(a^6c^3x^3-a^5c^3x^2-a^4c^3x+a^3c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

output
```
-1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a
^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 - a*x + 2)*sqrt(-a^2*c))
/(a^6*c^3*x^3 - a^5*c^3*x^2 - a^4*c^3*x + a^3*c^3)
```

3.703.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.703.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

output
```
integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.703.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

output
```
integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.703.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.704 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

3.704.1 Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(
1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)
^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*ar
ctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

3.704.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(2+3ax-3a^2x^2+3(-1+ax)^2(1+ax)\text{arctanh}(ax))}{8c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 +
 a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

3.704.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{4(ax-1)^2}+\frac{1}{8(ax+1)^2}-\frac{1}{4(ax-1)^3}-\frac{3}{8\left(a^2x^2-1\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\text{arctanh}(ax)}{8a}+\frac{1}{4a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{1}{8a(1-ax)^2}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)
) - 1/(8*a*(1 + a*x)) + (3*ArcTanh[a*x])/(8*a)))/(c - a^2*c*x^2)^(5/2))
```

3.704.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.704.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(3a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-3a^2\ln(ax+1)x^2+3a^2\ln(ax-1)x^2-6a^2x^2-3a\ln(ax+1)x+3a\ln(ax-1)x+6ax+}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*a^3*ln(a*x+
1)*x^3-3*a^3*ln(a*x-1)*x^3-3*a^2*ln(a*x+1)*x^2+3*a^2*ln(a*x-1)*x^2-6*a^2*x
^2-3*a*ln(a*x+1)*x+3*a*ln(a*x-1)*x+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x
^2-1)/c^3/a/(a*x+1)
```

3.704.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx =$$

$$-\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\left(3\,a^2x^2-3\,ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
-1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(
-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a
^2*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

3.704.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.704.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.704.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.704.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.705 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

3.705.1 Optimal result

Integrand size = 25, antiderivative size = 271

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}\\ &\quad-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{\left(c-a^2cx^2\right)^{5/2}}\\ &\quad+\frac{11a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\left(c-a^2cx^2\right)^{5/2}}\end{aligned}$$

output
```
-1/8*a^5*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/2*a^5*(
1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^5*(1-1/a^2/x^2)
^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/
(-a^2*c*x^2+c)^(5/2)+11/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*
x^2+c)^(5/2)+5/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/
2)
```

3.705.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.32

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{2}{(-1+ax)^2}+\frac{8}{-1+ax}-\frac{2}{1+ax}-16\log(x)+11\log(1-ax)+5\log(\right.}{16\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-2/(-1 + a*x)^2 + 8/(-1 + a*x) - 2/(1 +
a*x) - 16*Log[x] + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^
(5/2))
```

3.705.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.35, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 6746$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 6747$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{x(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{x(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 99$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int\left(-\frac{11a}{16(ax-1)}-\frac{5a}{16(ax+1)}+\frac{a}{2(ax-1)^2}-\frac{a}{8(ax+1)^2}-\frac{a}{4(ax-1)^3}+\frac{1}{x}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{2(1-ax)}+\frac{1}{8(ax+1)}+\frac{1}{8(1-ax)^2}-\frac{11}{16}\log(1-ax)-\frac{5}{16}\log(ax+1)+\log(x)\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*(1 - a*x)^2) + 1/(2*(1 - a*x)) +
1/(8*(1 + a*x)) + Log[x] - (11*Log[1 - a*x])/16 - (5*Log[1 + a*x])/16))/(c
 - a^2*c*x^2)^(5/2))
```

3.705.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.705. $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

3.705.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(5a^3\ln(ax+1)x^3-16a^3\ln(x)x^3+11a^3\ln(ax-1)x^3-5a^2\ln(ax+1)x^2+16a^2\ln(x)x^2-11a^2\ln(ax-1)x^2+6a^2x^2-5a}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(5*a^3*ln(a*x
+1)*x^3-16*a^3*ln(x)*x^3+11*a^3*ln(a*x-1)*x^3-5*a^2*ln(a*x+1)*x^2+16*a^2*l
n(x)*x^2-11*a^2*ln(a*x-1)*x^2+6*a^2*x^2-5*a*ln(a*x+1)*x+16*a*ln(x)*x-11*a*
ln(a*x-1)*x+2*a*x+5*ln(a*x+1)-16*ln(x)+11*ln(a*x-1)-12)/(a^2*x^2-1)/c^3/(a
*x+1)
```

3.705.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx =$$
$$-\frac{\left(6\,a^2x^2+2\,ax+5\,\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+11\,\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)-16\,(a^3}{16\,\left(a^4c^3x^3-a^3c^3x^2-a^2c^3x+ac^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

output
```
-1/16*(6*a^2*x^2 + 2*a*x + 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) +
11*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 16*(a^3*x^3 - a^2*x^2 - a*
x + 1)*log(x) - 12)*sqrt(-a^2*c)/(a^4*c^3*x^3 - a^3*c^3*x^2 - a^2*c^3*x +
a*c^3)
```

3.705.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.705.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.705.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.705.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{1}{x\,\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input

```
int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output

```
int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.706 $\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx$

3.706.1 Optimal result

Integrand size = 25, antiderivative size = 307

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}{\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}\\&\quad-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{\left(c-a^2cx^2\right)^{5/2}}\\&\quad+\frac{23a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}-\frac{7a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\left(c-a^2cx^2\right)^{5/2}}\end{aligned}$$

output

```
a^5*(1-1/a^2/x^2)^(5/2)*x^4/(-a^2*c*x^2+c)^(5/2)-1/8*a^6*(1-1/a^2/x^2)^(5/
2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-3/4*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(-a
*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c
*x^2+c)^(5/2)-a^6*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*c*x^2+c)^(5/2)+23/16
*a^6*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-7/16*a^6*(1-1
/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

3.706.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.32

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(\frac{16}{x}-\frac{2a}{(-1+ax)^2}+\frac{12a}{-1+ax}+\frac{2a}{1+ax}-16a\log(x)+23a\log(1-ax)-\right.}{16\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(16/x - (2*a)/(-1 + a*x)^2 + (12*a)/(-1 +
 a*x) + (2*a)/(1 + a*x) - 16*a*Log[x] + 23*a*Log[1 - a*x] - 7*a*Log[1 + a*
x]))/(16*(c - a^2*c*x^2)^(5/2))
```

3.706.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.35, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^7}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{x^2(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{x^2(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-\frac{23a^2}{16(ax-1)}+\frac{7a^2}{16(ax+1)}+\frac{3a^2}{4(ax-1)^2}+\frac{a^2}{8(ax+1)^2}-\frac{a^2}{4(ax-1)^3}+\frac{a}{x}+\frac{1}{x^2}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3a}{4(1-ax)}-\frac{a}{8(ax+1)}+\frac{a}{8(1-ax)^2}+a\log(x)-\frac{23}{16}a\log(1-ax)+\frac{7}{16}a\log(ax+1)-\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-x^(-1) + a/(8*(1 - a*x)^2) + (3*a)/(4
*(1 - a*x)) - a/(8*(1 + a*x)) + a*Log[x] - (23*a*Log[1 - a*x])/16 + (7*a*L
og[1 + a*x])/16))/(c - a^2*c*x^2)^(5/2))
```

3.706.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.706. $\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx$

3.706.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(7\ln(ax+1)x^4a^4+16\ln(x)x^4a^4-23\ln(ax-1)x^4a^4-7a^3\ln(ax+1)x^3-16a^3\ln(x)x^3+23a^3\ln(ax-1)x^3-30a^3x^3-7a^2}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(7*ln(a*x+1)*x
^4*a^4+16*ln(x)*x^4*a^4-23*ln(a*x-1)*x^4*a^4-7*a^3*ln(a*x+1)*x^3-16*a^3*ln
(x)*x^3+23*a^3*ln(a*x-1)*x^3-30*a^3*x^3-7*a^2*ln(a*x+1)*x^2-16*a^2*ln(x)*x
^2+23*a^2*ln(a*x-1)*x^2+22*a^2*x^2+7*a*ln(a*x+1)*x+16*a*ln(x)*x-23*a*ln(a*
x-1)*x+28*a*x-16)/(a^2*x^2-1)/c^3/x/(a*x+1)
```

3.706.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx =$$

$$-\frac{\left(30\,a^3x^3-22\,a^2x^2-28\,ax-7\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax+1\right)+23\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log}{16\left(a^4c^3x^4-a^3c^3x^3-a^2c^3x^2+ac^3x\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/16*(30*a^3*x^3 - 22*a^2*x^2 - 28*a*x - 7*(a^4*x^4 - a^3*x^3 - a^2*x^2 +
 a*x)*log(a*x + 1) + 23*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(a*x - 1) -
 16*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(x) + 16)*sqrt(-a^2*c)/(a^4*c^3
*x^4 - a^3*c^3*x^3 - a^2*c^3*x^2 + a*c^3*x)
```

3.706.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.706.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.706.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.706.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{1}{x^2\,\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.707 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

3.707.1 Optimal result

Integrand size = 27, antiderivative size = 76

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)
^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.707.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^2(-4+3ax)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
(x^2*(-4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.707.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -x^2(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int x^2(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(x^2-ax^3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{x^3}{3}-\frac{ax^4}{4}\right)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(x^3/3 - (a*x^4)/4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.707.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.707.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x^{3}(3ax-4)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{12ax-12}$	47
default	$\frac{(3ax-4)x^{3}\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{12ax-12}$	48

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/12*x^3*(3*a*x-4)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

3.707.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.33

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{(3\,ax^4-4\,x^3)\sqrt{-a^2c}}{12\,a}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="f
ricas")
```

output
```
1/12*(3*a*x^4 - 4*x^3)*sqrt(-a^2*c)/a
```

3.707.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int x^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

input
```
integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.707.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.707.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int \sqrt{-a^2cx^2+c}\,x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.707.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int x^2\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.708 $\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

3.708.1 Optimal result

Integrand size = 25, antiderivative size = 74

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(
1/2)/(1-1/a^2/x^2)^(1/2)
```

3.708.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x(-3+2ax)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
(x*(-3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.708.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -x(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int x(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(x-ax^2\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{x^2}{2}-\frac{ax^3}{3}\right)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(x^2/2 - (a*x^3)/3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.708.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.708.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x^{2}\left(2ax-3\right)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	47
default	$\frac{\left(2ax-3\right)x^{2}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	48

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/6*x^2*(2*a*x-3)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

3.708.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{(2\,ax^3-3\,x^2)\sqrt{-a^2c}}{6\,a}$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

output
```
1/6*(2*a*x^3 - 3*x^2)*sqrt(-a^2*c)/a
```

3.708.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

input
```
integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.708.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+cx}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.708.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+cx}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.708.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.709 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.709.1 Optimal result

Integrand size = 24, antiderivative size = 69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-
1/a^2/x^2)^(1/2)
```

3.709.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(-2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

output
```
((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.709.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int (ax-1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(1-ax)^2\sqrt{c-a^2cx^2}}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

output
```
((1 - a*x)^2*Sqrt[c - a^2*c*x^2])/(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.709.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.709.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(ax-2)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
default	$\frac{(ax-2)x\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	45

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

3.709.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^{2}cx^{2}}\,dx=\frac{\sqrt{-a^{2}c}(ax^{2}-2\,x)}{2\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

3.709.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.709.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.709.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.709.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.710 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

3.710.1 Optimal result

Integrand size = 27, antiderivative size = 70

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1
-1/a^2/x^2)^(1/2)
```

3.710.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x - Log[x]/a))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.710.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{1-ax}{x}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{1-ax}{x}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x}-a\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}(\log(x)-ax)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-(a*x) + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.710. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}dx$

3.710.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.710.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(-a x +\ln \left(x \right)\right) \sqrt{\frac{a x -1}{a x +1}}}{a x -1}$	46

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE
)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x))*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

3.710.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax-\log(x))}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fri
cas")
```

output
```
sqrt(-a^2*c)*(a*x - log(x))/a
```

3.710.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)(ax+1)}}{x}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

3.710.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.710.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.711 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.711.1 Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/
x/(1-1/a^2/x^2)^(1/2)
```

3.711.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(a*x) + Log[x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.711.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{1-ax}{x^2}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{1-ax}{x^2}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{a}{x}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-a\log(x)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-x^(-1) - a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

3.711. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.711.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.711.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{(a\ln(x)x+1)\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)x}$	48

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBO
SE)
```

output
```
(a*ln(x)*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/x
```

3.711.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.31

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(ax\log{(x)}+1)}{ax}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="f
ricas")
```

output
```
sqrt(-a^2*c)*(a*x*log(x) + 1)/(a*x)
```

3.711.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x^2}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x**2, x)
```

3.711.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.711.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.711.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

3.712 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

3.712.1 Optimal result

Integrand size = 27, antiderivative size = 137

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

output
```
3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+3/5*x^2*(-a^2*c*x
^2+c)^(1/2)/a^2-1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(1/2
)+3/20*(-5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

3.712.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(24-15ax+12a^2x^2-10a^3x^3+4a^4x^4)-15\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{20a^4}$$

input
```
Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(24 - 15*a*x + 12*a^2*x^2 - 10*a^3*x^3 + 4*a^4*x^4) -
 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(2
0*a^4)
```

3.712.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6717, 6702, 541, 25, 27, 533, 27, 533, 27, 533, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^3\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{x^3(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx^3(9-10ax)}{\sqrt{c-a^2cx^2}}dx}{5a^2c}-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^3(9-10ax)}{\sqrt{c-a^2cx^2}}dx}{5a^2c}-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\int \frac{x^3(9-10ax)}{\sqrt{c-a^2cx^2}}dx-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{5}\left(\frac{\int -\frac{6acx^2(5-6ax)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}+\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\int\frac{x^2(5-6ax)}{\sqrt{c-a^2cx^2}}dx}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 533$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{\int-\frac{3acx(4-5ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}+\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\int\frac{x(4-5ax)}{\sqrt{c-a^2cx^2}}dx}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 533$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{\int-\frac{ac(5-8ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}+\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 25$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{\int\frac{ac(5-8ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{\int\frac{5-8ax}{\sqrt{c-a^2cx^2}}dx}{2a}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow 455$$

3.712. $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{5\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \textbf{224}$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{5\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \textbf{216}$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{\frac{5\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

input
```
Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(-1/5*(x^4*Sqrt[c - a^2*c*x^2])/c + ((5*x^3*Sqrt[c - a^2*c*x^2])/(2*a*
c) - (3*((2*x^2*Sqrt[c - a^2*c*x^2])/(a*c) - ((5*x*Sqrt[c - a^2*c*x^2])/(2
*a*c) - ((8*Sqrt[c - a^2*c*x^2])/(a*c) + (5*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
a^2*c*x^2]])/(a*Sqrt[c]))/(2*a))/a))/(2*a))/5))
```

3.712. $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

3.712.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

3.712. $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.712.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\left(4 a^{4} x^{4}-10 a^{3} x^{3}+12 a^{2} x^{2}-15 a x +24\right) \left(a^{2} x^{2}-1\right) c}{20 a^{4} \sqrt{-c \left(a^{2} x^{2}-1\right)}}+\frac{3 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{4 a^{3} \sqrt{a^{2} c}}$
default	$-\frac{x^{2} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{5 a^{2} c}-\frac{4 \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{5 c \,a^{4}}-\frac{2 \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)}{a^{3}}-\frac{2 \left(-\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4 a^{2} c}+\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\right.}{a}$

input
```
int(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(4*a^4*x^4-10*a^3*x^3+12*a^2*x^2-15*a*x+24)*(a^2*x^2-1)/a^4/(-c*(a^2
*x^2-1))^(1/2)*c+3/4/a^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))*c
```

3.712.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.34

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$

$$=\left[\frac{2\left(4\,a^{4}x^{4}-10\,a^{3}x^{3}+12\,a^{2}x^{2}-15\,ax+24\right)\sqrt{-a^{2}cx^{2}+c}+15\,\sqrt{-c}\log\left(2\,a^{2}cx^{2}+2\,\sqrt{-a^{2}cx^{2}+c}a\sqrt{-c}\right.}{40\,a^{4}}\right.$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/40*(2*(4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x
^2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*
x - c))/a^4, 1/20*((4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqr
t(-a^2*c*x^2 + c) - 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^
2*c*x^2 - c)))/a^4]
```

3.712.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x**3*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.712.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x^2}{5\,a^2c}-\frac{5\sqrt{-a^2cx^2+c}x}{4\,a^3}+\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{2\,a^3c}+\frac{3\sqrt{c}\arcsin(ax)}{4\,a^4}+\frac{2\sqrt{-a^2cx^2+c}}{a^4}-\frac{4\,(-a^2cx^2+c)^{\frac{3}{2}}}{5\,a^4c}$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) - 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 +
 1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) + 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sq
rt(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

3.712.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.712.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

input
```
int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.713 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

3.713.1 Optimal result

Integrand size = 27, antiderivative size = 112

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

output
```
-7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3-2/3*x^2*(-a^2*c*
x^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)-1/24*(-21*a*x+32)*(-a^2*c*x^2+
c)^(1/2)/a^3
```

3.713.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(-32+21ax-16a^2x^2+6a^3x^3)+21\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{24a^3}$$

input
```
Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*
ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

3.713.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {6717, 6702, 541, 25, 27, 533, 25, 27, 533, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^2\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{x^2(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx^2(7-8ax)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^2(7-8ax)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{4}\int \frac{x^2(7-8ax)}{\sqrt{c-a^2cx^2}}dx-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{4}\left(\frac{\int -\frac{acx(16-21ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}+\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\int \frac{acx(16-21ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\int \frac{x(16-21ax)}{\sqrt{c-a^2cx^2}}dx}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 533$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{\int -\frac{ac(21-32ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}+\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 25$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{\int \frac{ac(21-32ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 27$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{\int \frac{21-32ax}{\sqrt{c-a^2cx^2}}dx}{2a}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 455$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{21\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 224$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{21\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \; 216$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{\frac{21\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

input
```
Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

3.713. $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

output
```
-(c*(-1/4*(x^3*Sqrt[c - a^2*c*x^2])/c + ((8*x^2*Sqrt[c - a^2*c*x^2])/(3*a*
c) - ((21*x*Sqrt[c - a^2*c*x^2])/(2*a*c) - ((32*Sqrt[c - a^2*c*x^2])/(a*c)
 + (21*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a*Sqrt[c]))/(2*a))/(3*a
))/4))
```

3.713.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

3.713. $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.713.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\left(6 a^{3} x^{3}-16 a^{2} x^{2}+21 a x -32\right) \left(a^{2} x^{2}-1\right) c}{24 a^{3} \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{7 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{8 a^{2} \sqrt{a^{2} c}}$
default	$-\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4 a^{2} c}+\frac{\frac{9 x \sqrt{-a^{2} c \,x^{2}+c}}{8}+\frac{9 c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{8 \sqrt{a^{2} c}}}{a^{2}}+\frac{2 \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{3 a^{3} c}-\frac{2 \left(\sqrt{-a^{2} c \left(x +\frac{1}{a}\right)^{2}+2 \left(x +\frac{1}{a}\right) a c}+\frac{a c \arctan}{} \right.}{a^{3}}$

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3-16*a^2*x^2+21*a*x-32)*(a^2*x^2-1)/a^3/(-c*(a^2*x^2-1))^(1
/2)*c-7/8/a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

3.713.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.50

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$

$$=\left[\frac{2\left(6\,a^3x^3-16\,a^2x^2+21\,ax-32\right)\sqrt{-a^2cx^2+c}+21\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48\,a^3},\ (6\right.$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(2*(6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*
sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3,
1/24*((6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*sqr
t(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

3.713.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x**2*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.713.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{9\sqrt{-a^2cx^2+c}x}{8a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{4a^2c}-\frac{7\sqrt{c}\arcsin(ax)}{8a^3}-\frac{2\sqrt{-a^2cx^2+c}}{a^3}+\frac{2(-a^2cx^2+c)^{\frac{3}{2}}}{3a^3c}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/
8*sqrt(c)*arcsin(a*x)/a^3 - 2*sqrt(-a^2*c*x^2 + c)/a^3 + 2/3*(-a^2*c*x^2 +
 c)^(3/2)/(a^3*c)
```

3.713.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,x-\frac{8}{a}\right)x+\frac{21}{a^2}\right)x-\frac{32}{a^3}\right) + \frac{7\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,a^2\sqrt{-c}|a|}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x - 8/a)*x + 21/a^2)*x - 32/a^3) + 7/8*c*
log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

3.713.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.714 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

3.714.1 Optimal result

Integrand size = 25, antiderivative size = 84

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

output
```
arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2+1/3*x^2*(-a^2*c*x^2+c
)^(1/2)+1/3*(-3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

3.714.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(5-3ax+a^2x^2)\sqrt{c-a^2cx^2}-3\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{3a^2}$$

input
```
Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
((5 - 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] - 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c
- a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

3.714.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6702, 541, 25, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{x(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx(5-6ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx(5-6ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}\int \frac{x(5-6ax)}{\sqrt{c-a^2cx^2}}dx-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{3}\left(\frac{\int -\frac{2ac(3-5ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}+\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}\left(\frac{3x\sqrt{c-a^2cx^2}}{ac}-\frac{\int \frac{3-5ax}{\sqrt{c-a^2cx^2}}dx}{a}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{1}{3}\left(\frac{3x\sqrt{c-a^2cx^2}}{ac}-\frac{3\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow 224$$

$$-c\left(\frac{1}{3}\left(\frac{3x\sqrt{c-a^2cx^2}}{ac}-\frac{3\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow 216$$

$$-c\left(\frac{1}{3}\left(\frac{3x\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{3\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

input

```
Int[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output

```
-(c*(-1/3*(x^2*Sqrt[c - a^2*c*x^2])/c + ((3*x*Sqrt[c - a^2*c*x^2])/(a*c) -
 ((5*Sqrt[c - a^2*c*x^2])/(a*c) + (3*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x
^2]])/(a*Sqrt[c]))/a)/3))
```

3.714.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.714.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(a^2x^2-3ax+5)(a^2x^2-1)c}{3a^2\sqrt{-c(a^2x^2-1)}}+\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c}{a\sqrt{a^2c}}$
default	$-\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c}-\frac{2\left(\frac{x\sqrt{-a^2cx^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{2\sqrt{a^2c}}\right)}{a}+\frac{2\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{2ac\arctan\Bigg(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\big(}}}{\sqrt{a^2c}}}{a^2}$

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a^2*x^2-3*a*x+5)*(a^2*x^2-1)/a^2/(-c*(a^2*x^2-1))^(1/2)*c+1/a/(a^2*c
)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

3.714.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.79

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$$

$$= \left[\frac{2\sqrt{-a^2cx^2+c}(a^2x^2-3\,ax+5)+3\sqrt{-c}\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2}, \sqrt{-a^2cx^2+c}(a^2x\right.$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*
c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^
2 + c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqr
t(c)*x/(a^2*c*x^2 - c)))/a^2]
```

3.714.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.714.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = -\frac{\sqrt{-a^2cx^2+cx}}{a}+\frac{\sqrt{c}\arcsin{(ax)}}{a^2}+\frac{2\sqrt{-a^2cx^2+c}}{a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^2c}$$

input

```
integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output

```
-sqrt(-a^2*c*x^2 + c)*x/a + sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 +
c)/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

3.714.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x-\frac{3}{a}\right)x+\frac{5}{a^2}\right)-\frac{c\log\left(\left|-\sqrt{-a^2cx}+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

input

```
integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output

```
1/3*sqrt(-a^2*c*x^2 + c)*((x - 3/a)*x + 5/a^2) - c*log(abs(-sqrt(-a^2*c)*x
 + sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

3.714.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\,\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input

```
int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output

```
int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.715 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.715.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

output
```
-3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)
^(1/2)/a-1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

3.715.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}(4-5ax+a^2x^2)+6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 -
a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

3.715.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6692, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{3}{2}\int \frac{1-ax}{\sqrt{c-a^2cx^2}}dx+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{3}{2}\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a*c) + (3*(Sqrt[c - a^2*c*x^2]/(a*
c) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a*Sqrt[c])))/2))
```

3.715. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.715.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.715. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.715.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(ax-4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}-\frac{2\left(\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}\right)}{a}$	127

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

3.715.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$

$$= \left[\frac{2\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4\,a}, \frac{\sqrt{-a^2cx^2+c}(ax-4)+3}{2}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x -
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a
]
```

3.715.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.715.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}x-\frac{3\sqrt{c}\arcsin{(ax)}}{2\,a}-\frac{2\sqrt{-a^2cx^2+c}}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2
 + c)/a
```

3.715.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

3.715.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.716 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

3.716.1 Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\sqrt{c-a^2cx^2}+2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+arctanh((-a^2*c*x^2+c)^
(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)
```

3.716.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\sqrt{c-a^2cx^2}-2\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(-1+a^2x^2\right)}\right)-\sqrt{c}\log(x)+\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*
(-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c
*x^2]]
```

3.716.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6702, 541, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{(1-ax)^2}{x\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2c(1-2ax)}{x\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2c(1-2ax)}{x\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\int \frac{1-2ax}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{538}$$

$$-c\left(-2a\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-2a\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \color{blue}{243}$$

$$-c\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \color{blue}{73}$$

$$-c\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{a^2c}-\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \color{blue}{221}$$

$$-c\left(-\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

input

```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

output

```
-(c*(-(Sqrt[c - a^2*c*x^2]/c) - (2*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2
]])/Sqrt[c] - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/Sqrt[c]))
```

3.716.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.716. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.716. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

3.716.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

method	result
default	$-\sqrt{-a^{2}c\,x^{2}+c}+\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)+2\sqrt{-a^{2}c\left(x+\frac{1}{a}\right)^{2}+2\left(x+\frac{1}{a}\right)ac}+\frac{2ac\arctan\left(\frac{}{\sqrt{-a^{2}}}\right.}{}$

input
```
int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2
*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)
^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))
```

3.716.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \Bigg[&-2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)\\ &+\frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)\\ &+\sqrt{-a^2cx^2+c},\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)\\ &+\sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)\\ &+\sqrt{-a^2cx^2+c}\Bigg]\end{aligned}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
[-2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2
*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x
^2 - c)) + sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) + sqrt(-a^2*c*x^2 + c)]
```

3.716.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x*(a*x + 1)), x)
```

3.716.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = a^2\left(\frac{\sqrt{c}\arcsin{(ax)}}{a^2}+\frac{\sqrt{-a^2cx^2+c}}{a^2}\right) + a\left(\frac{\sqrt{c}\arcsin{(ax)}}{a}+\frac{\sqrt{c}\log\left(\frac{2\sqrt{-a^2cx^2+c}\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
a^2*(sqrt(c)*arcsin(a*x)/a^2 + sqrt(-a^2*c*x^2 + c)/a^2) + a*(sqrt(c)*arcs
in(a*x)/a + sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x)
)/a)
```

3.716.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} + \sqrt{-a^2cx^2+c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
-2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) +
2*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqr
t(-a^2*c*x^2 + c)
```

3.716.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.717 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.717.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
-a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)-2*a*arctanh((-a^2*c*x^
2+c)^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
```

3.717.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) + 2a\sqrt{c}\log(x) - 2a\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c
]*(-1 + a^2*x^2))] + 2*a*Sqrt[c]*Log[x] - 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt
[c - a^2*c*x^2]]
```

3.717.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6702, 540, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^2}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{(1-ax)^2}{x^2\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int \frac{ac(2-ax)}{x\sqrt{c-a^2cx^2}}dx}{c}-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-a\int \frac{2-ax}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{538}$$

$$-c\left(-a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-a\int \frac{1}{\sqrt{c-a^2cx^2}}dx\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(-a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-a\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(-a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{243}$$

$$-c\left(-a\left(\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow 73$$

$$-c\left(-a\left(-\frac{2\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{a^2c}-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow 221$$

$$-c\left(-a\left(-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
-(c*(-(Sqrt[c - a^2*c*x^2]/(c*x)) - a*(-(ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2
*c*x^2]]/Sqrt[c]) - (2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c])))
```

3.717.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.717. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.717. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.717.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{(a^2x^2-1)c}{x\sqrt{-c(a^2x^2-1)}}-\left(\frac{a^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}\right)c$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}+2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)+2a\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c}}{x}\right.\right.$

input
```
int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2-1)/x/(-c*(a^2*x^2-1))^(1/2)*c-(a^2/(a^2*c)^(1/2)*arctan((a^2*c)^
(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*a/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)
^(1/2))/x))*c
```

3.717.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$=\left[\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a\sqrt{c}x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},\right.$$
$$-\frac{4\,a\sqrt{-c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-a\sqrt{-c}x\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\sqrt{-a^2cx^2+}}{2\,x}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
[(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a
*sqrt(c)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) +
sqrt(-a^2*c*x^2 + c))/x, -1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*
sqrt(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x
^2 + c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c))/x]
```

3.717.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^2\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

3.717.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^2), x)
```

3.717.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
 a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*
a^2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

3.717.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x^2\,\left(a\,x+1\right)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.718 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

3.718.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a^2*c*x^2+c)^(
1/2)/x^2-2*a*(-a^2*c*x^2+c)^(1/2)/x
```

3.718.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\frac{1}{2}\left(\frac{(1-4ax)\sqrt{c-a^2cx^2}}{x^2}-3a^2\sqrt{c}\log(x)+3a^2\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
(((1 - 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt
[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

3.718.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6717, 6702, 540, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^3}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{(1-ax)^2}{x^3\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int \frac{ac(4-3ax)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{2}a\int \frac{4-3ax}{x^2\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{534}$$

$$-c\left(-\frac{1}{2}a\left(-3a\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{243}$$

$$-c\left(-\frac{1}{2}a\left(-\frac{3}{2}a\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(-\frac{1}{2}a\left(\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(-\frac{1}{2}a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
-(c*(-1/2*Sqrt[c - a^2*c*x^2]/(c*x^2) - (a*((-4*Sqrt[c - a^2*c*x^2])/(c*x)
 + (3*a*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/2))
```

3.718.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.718. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.718.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result
risch	$\frac{\left(4 a^{3} x^{3}-a^{2} x^{2}-4 a x +1\right) c}{2 x^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}+\frac{3 a^{2} \sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)}{2}$
default	$\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{2 c \,x^{2}}-\frac{3 a^{2} \left(\sqrt{-a^{2} c \,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)\right)}{2}+2 a \left(-\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{c x}-2 a^{2} \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\right.\right.$

input
```
int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(4*a^3*x^3-a^2*x^2-4*a*x+1)/x^2/(-c*(a^2*x^2-1))^(1/2)*c+3/2*a^2*c^(1/
2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

3.718.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= \left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\,\sqrt{-a^2cx^2+c}(4\,ax-1)}{4\,x^2},\frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)}{2\,x^2}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
 2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 1))/x^2, 1/2*(3*a^2*sqrt(-c)*
x^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^
2 + c)*(4*a*x - 1))/x^2]
```

3.718.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^3\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

3.718.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^3}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^3), x)
```

3.718.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^3a^2c+4\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}c|a|+\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
-3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c
) + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c + 4*(sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2
*c*x^2 + c))*a^2*c^2 - 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))^2 - c)^2
```

3.718.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^3\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.719 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

3.719.1 Optimal result

Integrand size = 27, antiderivative size = 101

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
-a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2
)/x^3-a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

3.719.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\frac{(1-3ax+5a^2x^2)\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\log(x)-a^3\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
((1 - 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) + a^3*Sqrt[c]*Log[x]
 - a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

3.719.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6717, 6702, 540, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^4}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{(1-ax)^2}{x^4\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int \frac{ac(6-5ax)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{3}a\int \frac{6-5ax}{x^3\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{539}$$

$$-c\left(-\frac{1}{3}a\left(-\frac{\int \frac{2ac(5-3ax)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{3}a\left(-a\int \frac{5-3ax}{x^2\sqrt{c-a^2cx^2}}dx-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{534}$$

$$-c\left(-\frac{1}{3}a\left(-a\left(-3a\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{243}$$

$$-c\left(-\frac{1}{3}a\left(-a\left(-\frac{3}{2}a\int\frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow 73$$

$$-c\left(-\frac{1}{3}a\left(-a\left(\frac{3\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow 221$$

$$-c\left(-\frac{1}{3}a\left(-a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
-(c*(-1/3*Sqrt[c - a^2*c*x^2]/(c*x^3) - (a*((-3*Sqrt[c - a^2*c*x^2])/(c*x^
2) - a*((-5*Sqrt[c - a^2*c*x^2])/(c*x) + (3*a*ArcTanh[Sqrt[c - a^2*c*x^2]/
Sqrt[c]])/Sqrt[c])))/3))
```

3.719.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.719. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.719.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(5a^{4}x^{4}-3a^{3}x^{3}-4a^{2}x^{2}+3ax-1\right)c}{3x^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}}-a^{3}\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)$
default	$\frac{\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{3c\,x^{3}}+2a^{3}\left(\sqrt{-a^{2}c\,x^{2}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)\right)+2a\Bigg(-\frac{\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{2c\,x^{2}}-\frac{a^{2}\Big(\sqrt{-a^{2}c\,x^{2}}}{}$

input
```
int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(5*a^4*x^4-3*a^3*x^3-4*a^2*x^2+3*a*x-1)/x^3/(-c*(a^2*x^2-1))^(1/2)*c-
a^3*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

3.719.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.63

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \left[\frac{3\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\sqrt{-a^2cx^2+c}(5\,a^2x^2-3\,ax+1)}{6\,x^3}, -\frac{3\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a^2cx^2+c}(5\,a^2x^2-3\,ax+1)}{3\,x^3}\right]$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
 2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3, -1/3*(3*
a^3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - s
qrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3]
```

3.719.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^4\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

3.719.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^4}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^4), x)
```

3.719.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(85) = 170$.

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.48

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$
$$-\frac{2\Big(3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^3c+3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}c|a|-12\big(\sqrt{-a^2c}x-\sqrt{-a^2}}{3\Big(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 - 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c + 3*(sqrt(-a^2*c
)*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) - 12*(sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqr
t(-a^2*c*x^2 + c))*a^3*c^3 + 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^2 - c)^3
```

3.719.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^4\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.720 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

3.720.1 Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} -\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a^2*c*x^2+c)^(
1/2)/x^4-2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2-4
/3*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

3.720.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}(6-16ax+21a^2x^2-32a^3x^3)}{24x^4} -\frac{7}{8}a^4\sqrt{c}\log(x)+\frac{7}{8}a^4\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(6 - 16*a*x + 21*a^2*x^2 - 32*a^3*x^3))/(24*x^4) - (7
*a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2
]])/8
```

3.720.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6717, 6702, 540, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^5}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{(1-ax)^2}{x^5\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int \frac{ac(8-7ax)}{x^4\sqrt{c-a^2cx^2}}dx}{4c}-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{4}a\int \frac{8-7ax}{x^4\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{539}$$

$$-c\left(-\frac{1}{4}a\left(-\frac{\int \frac{ac(21-16ax)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\int \frac{21-16ax}{x^3\sqrt{c-a^2cx^2}}dx-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 539$$

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{\int \frac{ac(32-21ax)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 27$$

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\int\frac{32-21ax}{x^2\sqrt{c-a^2cx^2}}dx-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow 534$$

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(-21a\int\frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-}}{4c}\right.\right.$$

$$\Big\downarrow 243$$

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(-\frac{21}{2}a\int\frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\sqrt{c}\right.\right.$$

$$\Big\downarrow 73$$

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(\frac{21\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\sqrt{c}\right.\right.$$

$$\Big\downarrow 221$$

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(\frac{21a\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-}}{4}\right.\right.$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
-(c*(-1/4*Sqrt[c - a^2*c*x^2]/(c*x^4) - (a*((-8*Sqrt[c - a^2*c*x^2])/(3*c*
x^3) - (a*((-21*Sqrt[c - a^2*c*x^2])/(2*c*x^2) - (a*((-32*Sqrt[c - a^2*c*x
^2])/(c*x) + (21*a*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/2))/3))
/4))
```

3.720. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

3.720.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.720. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.720.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(32a^5x^5-21a^4x^4-16a^3x^3+15a^2x^2-16ax+6)c}{24x^4\sqrt{-c(a^2x^2-1)}}+\frac{7a^4\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{8}$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{4c\,x^4}-\frac{9a^2\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2}\right)}{4}-2a^4\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\right.$

input
```
int((a*x-1)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(32*a^5*x^5-21*a^4*x^4-16*a^3*x^3+15*a^2*x^2-16*a*x+6)/x^4/(-c*(a^2*x
^2-1))^(1/2)*c+7/8*a^4*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

3.720.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$=\left[\frac{21\,a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\,(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{-a^2cx^2+c}}{48\,x^4},\ 21\,a^4\sqrt{-c}x^4\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
 - 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 +
c))/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a
^2*c*x^2 - c)) - (32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 +
c))/x^4]
```

3.720.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^5\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

3.720.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^5}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^5), x)
```

3.720.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(106) = 212.

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.49

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2-96\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+}\right.}{}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

output
```
-7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(
-c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(
-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 - 96*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^3*a^4*c^3 + 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*
sqrt(-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 -
 32*a^3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 -
c)^4
```

3.720.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.721 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

3.721.1 Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\\ &\quad+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}\\ &\quad+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^5\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)-2*x*(-a^2*c*x^2+c)^(1/2)/a^
3/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
-3/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)
^(1/2)/(1-1/a^2/x^2)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^5/x/(1-1/a^2
/x^2)^(1/2)
```

3.721.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^4}-\frac{2x^2}{a^3}+\frac{4x^3}{3a^2}-\frac{3x^4}{4a}+\frac{x^5}{5}-\frac{4\log(1+ax)}{a^5}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a^4 - (2*x^2)/a^3 + (4*x^3)/(3*a^2) - (3*x^4)/
(4*a) + x^5/5 - (4*Log[1 + a*x])/a^5))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.721.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{x^3(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax^4-3x^3+\frac{4x^2}{a}-\frac{4x}{a^2}-\frac{4}{a^3(ax+1)}+\frac{4}{a^3}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(ax+1)}{a^4}+\frac{4x}{a^3}-\frac{2x^2}{a^2}+\frac{ax^5}{5}+\frac{4x^3}{3a}-\frac{3x^4}{4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a^3 - (2*x^2)/a^2 + (4*x^3)/(3*a) - (3*x^4)/4
+ (a*x^5)/5 - (4*Log[1 + a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.721.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.721. $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

3.721.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(-12 a^{5} x^{5}+45 a^{4} x^{4}-80 a^{3} x^{3}+120 a^{2} x^{2}-240 a x+240 \ln \left(a x+1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{60 \left(a x-1\right)^{2} a^{4}}$	92

input
```
int(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/60*(-12*a^5*x^5+45*a^4*x^4-80*a^3*x^3+120*a^2*x^2-240*a*x+240*ln(a*x+1)
)*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/a^4
```

3.721.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\left(12\,a^{5}x^{5}-45\,a^{4}x^{4}+80\,a^{3}x^{3}-120\,a^{2}x^{2}+240\,ax-240\,\log\left(ax+1\right)\right)\sqrt{-a^{2}c}}{60\,a^{5}}$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="f
ricas")
```

output
```
1/60*(12*a^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x - 240*l
og(a*x + 1))*sqrt(-a^2*c)/a^5
```

3.721.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.721.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int \sqrt{-a^2cx^2+c}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.721.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int \sqrt{-a^2cx^2+c}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.721.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

input
```
int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.722 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

3.722.1 Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = -\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
-4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a
^2/(1-1/a^2/x^2)^(1/2)-x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*
x^3*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1
/2)/a^4/x/(1-1/a^2/x^2)^(1/2)
```

3.722.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{4x}{a^3}+\frac{2x^2}{a^2}-\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1+ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-4*x)/a^3 + (2*x^2)/a^2 - x^3/a + x^4/4 + (4*Log[1
+ a*x])/a^4))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.722.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax^3-3x^2+\frac{4x}{a}+\frac{4}{a^2(ax+1)}-\frac{4}{a^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4\log(ax+1)}{a^3}-\frac{4x}{a^2}+\frac{ax^4}{4}+\frac{2x^2}{a}-x^3\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-4*x)/a^2 + (2*x^2)/a - x^3 + (a*x^4)/4 + (4*Log[1
+ a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.722.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.722.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\left(a^{4} x^{4}-4 a^{3} x^{3}+8 a^{2} x^{2}-16 a x +16 \ln \left(a x +1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{4 a^{3} \left(a x -1\right)^{2}}$	83

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
1/4*(a^4*x^4-4*a^3*x^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/
2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

3.722.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log\left(ax+1\right))\sqrt{-a^2c}}{4\,a^4}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="f
ricas")
```

output
```
1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(-a^2
*c)/a^4
```

3.722.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.722.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.722.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+cx^2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.722.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.723 $\int e^{-3\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

3.723.1 Optimal result

Integrand size = 25, antiderivative size = 151

$$\int e^{-3\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx = \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

output

```
4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x*(-a^2*c*x^2+c)^(1/2)/
a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*l
n(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^3/x/(1-1/a^2/x^2)^(1/2)
```

3.723.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.43

$$\int e^{-3\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}(ax(24-9ax+2a^2x^2)-24\log(1+ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

input

```
Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output

```
(Sqrt[c - a^2*c*x^2]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6*
a^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.723.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{x(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax^2-3x+\frac{4}{a}-\frac{4}{a(ax+1)}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(ax+1)}{a^2}+\frac{ax^3}{3}+\frac{4x}{a}-\frac{3x^2}{2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a - (3*x^2)/2 + (a*x^3)/3 - (4*Log[1 + a*x])/a
^2))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.723.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.723.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\left(-2 a^{3} x^{3}+9 a^{2} x^{2}-24 a x +24 \ln \left(a x +1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{6 a^{2} \left(a x -1\right)^{2}}$	76

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/6*(-2*a^3*x^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*
x+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

3.723.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\left(2\,a^3x^3-9\,a^2x^2+24\,ax-24\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{6\,a^3}$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

output
```
1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(-a^2*c)/a^3
```

3.723.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.723.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+cx}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.723.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+cx}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.723.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int x\sqrt{c-a^2\,c\,x^2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.724 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

3.724.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(
1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1
/2)
```

3.724.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/(Sqrt[1 -
1/(a^2*x^2)]*x)
```

3.724.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax+\frac{4}{ax+1}-3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/(a*Sqrt[1 -
1/(a^2*x^2)]*x)
```

3.724.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.724.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(a^2x^2-6ax+8\ln\left(ax+1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2\left(ax-1\right)^2a}$	67

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/(a*x-1)^2/a
```

3.724.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(a^2x^2-6\,ax+8\,\log{(ax+1)})\sqrt{-a^2c}}{2\,a^2}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

3.724.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.724.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.724.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.724.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.725 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

3.725.1 Optimal result

Integrand size = 27, antiderivative size = 112

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1
-1/a^2/x^2)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)
```

3.725.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x
^2)]*x)
```

3.725.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{93}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a}{ax+1}+a+\frac{1}{x}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}(ax-4\log(ax+1)+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x
^2)]*x)
```

3.725.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.725.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(-a x+4 \ln \left(a x+1\right)-\ln \left(x\right)\right) \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{\left(a x-1\right)^{2}}$	61

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE
)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(-a*x+4*ln(a*x+1)-ln(x))*(a*x+1)*((a*x-1)/(a*x+1))
^(3/2)/(a*x-1)^2
```

3.725.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.23

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax-4\log(ax+1)+\log(x))}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fri
cas")
```

output
```
sqrt(-a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a
```

3.725.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

3.725.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.725.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.725.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.726 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

3.726.1 Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(-a^2*c*x^2+c)^(1/
2)/x/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^
(1/2)
```

3.726.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{ax}-3\log(x)+4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(1/(a*x)) - 3*Log[x] + 4*Log[1 + a*x]))/(Sqrt[1 - 1
/(a^2*x^2)]*x)
```

3.726.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^2(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(\frac{4a^2}{ax+1}-\frac{3a}{x}+\frac{1}{x^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

3.726. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}dx$

3.726.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.726.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{(4a\ln(ax+1)x-3a\ln(x)x-1)\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2x}$	64

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBO
SE)
```

output
```
(4*a*ln(a*x+1)*x-3*a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/(a*x-1)^2/x
```

3.726.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(4\,ax\log{(ax+1)}-3\,ax\log{(x)}-1)}{ax}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="f
ricas")
```

output
```
sqrt(-a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a*x)
```

3.726.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Timed out
```

3.726.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.726.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.726.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.727 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

3.727.1 Optimal result

Integrand size = 27, antiderivative size = 152

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2} + \frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-1/2*(-a^2*c*x^2+c)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3*(-a^2*c*x^2+c)^(1/2)
/x^2/(1-1/a^2/x^2)^(1/2)+4*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1
/2)-4*a*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

3.727.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{2ax^2}+\frac{3}{x}+4a\log(x)-4a\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/2*1/(a*x^2) + 3/x + 4*a*Log[x] - 4*a*Log[1 + a*x]
))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.727.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^3(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^3}{ax+1}+\frac{4a^2}{x}-\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 +
a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.727.3.1 Defintions of rubi rules used

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.727.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\left(8 a^{2} \ln \left(a x +1\right) x^{2}-8 a^{2} \ln \left(x \right) x^{2}-6 a x +1\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 \left(a x -1\right)^{2} x^{2}}$	77

input

```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBO
SE)
```

output

```
-1/2*(8*a^2*ln(a*x+1)*x^2-8*a^2*ln(x)*x^2-6*a*x+1)*(-c*(a^2*x^2-1))^(1/2)*
(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

3.727.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{8\,a^3\sqrt{-c}x^2\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+\sqrt{-a^2c}(6\,ax-1)}{2\,ax^2}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="f
ricas")
```

output
```
1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + sqrt(-a^2*c)*(6*a*x - 1))/(a*x^2)
```

3.727.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Timed out
```

3.727.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.727.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.727.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^3}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.728 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

3.728.1 Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
-1/3*(-a^2*c*x^2+c)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+3/2*(-a^2*c*x^2+c)^(1/
2)/x^3/(1-1/a^2/x^2)^(1/2)-4*a*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2
)-4*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(a*x+1)*(
-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

3.728.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.39

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{3ax^3}+\frac{3}{2x^2}-\frac{4a}{x}-4a^2\log(x)+4a^2\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/3*1/(a*x^3) + 3/(2*x^2) - (4*a)/x - 4*a^2*Log[x]
+ 4*a^2*Log[1 + a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.728.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^4(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(\frac{4a^4}{ax+1}-\frac{4a^3}{x}+\frac{4a^2}{x^2}-\frac{3a}{x^3}+\frac{1}{x^4}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^3\log(x)+4a^3\log(ax+1)-\frac{4a^2}{x}+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[x
] + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.728.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.728.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\left(24a^{3}\ln(ax+1)x^{3}-24a^{3}\ln(x)x^{3}-24a^{2}x^{2}+9ax-2\right)\sqrt{-c(a^{2}x^{2}-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6x^{3}(ax-1)^{2}}$	85

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBO
SE)
```

output
```
1/6*(24*a^3*ln(a*x+1)*x^3-24*a^3*ln(x)*x^3-24*a^2*x^2+9*a*x-2)*(-c*(a^2*x^
2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

3.728.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.51

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{24\,a^4\sqrt{-c}x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)-(24\,a^2x^2-9\,ax+2)\sqrt{-a^2c}}{6\,ax^3}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="f
ricas")
```

output
```
1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(-a^2*c)*(2*a*
x + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(-a^2*c
))/(a*x^3)
```

3.728.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Timed out
```

3.728.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.728.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.728.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.729 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

3.729.1 Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = & -\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4} \\ & -\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2} \\ & +\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^3\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\end{aligned}$$

output
```
-1/4*(-a^2*c*x^2+c)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
^4/(1-1/a^2/x^2)^(1/2)-2*a*(-a^2*c*x^2+c)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+4*
a^2*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(x)*(-a^2*c*x^2+c
)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/
a^2/x^2)^(1/2)
```

3.729.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.35

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{4ax^4}+\frac{1}{x^3}-\frac{2a}{x^2}+\frac{4a^2}{x}+4a^3\log(x)-4a^3\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/4*1/(a*x^4) + x^(-3) - (2*a)/x^2 + (4*a^2)/x + 4*
a^3*Log[x] - 4*a^3*Log[1 + a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.729.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^5(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int\left(-\frac{4a^5}{ax+1}+\frac{4a^4}{x}-\frac{4a^3}{x^2}+\frac{4a^2}{x^3}-\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(4a^4\log(x)-4a^4\log(ax+1)+\frac{4a^3}{x}-\frac{2a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^4
*Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.729.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.729. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

3.729.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(16 \ln \left(a x+1\right) x^{4} a^{4}-16 \ln \left(x\right) x^{4} a^{4}-16 a^{3} x^{3}+8 a^{2} x^{2}-4 a x+1\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{4 x^{4} \left(a x-1\right)^{2}}$	93

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBO
SE)
```

output
```
-1/4*(16*ln(a*x+1)*x^4*a^4-16*ln(x)*x^4*a^4-16*a^3*x^3+8*a^2*x^2-4*a*x+1)*
(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

3.729.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.46

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{16\,a^5\sqrt{-c}x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+(16\,a^3x^3-8\,a^2x^2+4\,ax-1)\sqrt{-a^2c}}{4\,ax^4}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="f
ricas")
```

output
```
1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*
x + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)
*sqrt(-a^2*c))/(a*x^4)
```

3.729.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Timed out
```

3.729.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.729.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.729.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.730 $\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

3.730.1 Optimal result

Integrand size = 27, antiderivative size = 136

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}\\ &\quad-\frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2
+c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)-4*x^m*hypergeom([1, 1+m],[2+m],a*x)*(-
a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

3.730.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\begin{aligned}&\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx\\ &=\frac{x^m\sqrt{c-a^2cx^2}(6+ax+m(3+ax)-4(2+m)\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x^m*Sqrt[c - a^2*c*x^2]*(6 + a*x + m*(3 + a*x) - 4*(2 + m)*Hypergeometric
2F1[1, 1 + m, 2 + m, a*x]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

3.730.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{m+1}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{x^m(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{x^m(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(\frac{4x^m}{1-ax}-3x^m-ax^{m+1}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(\frac{4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{m+1}-\frac{ax^{m+2}}{m+2}-\frac{3x^{m+1}}{m+1}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*((-3*x^(1 + m))/(1 + m) - (a*x^(2 + m))/(2 + m) + (
4*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(1 + m)))/(a*Sqrt[1 -
 1/(a^2*x^2)]*x))
```

3.730.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.730.4 Maple [F]

$$\int \frac{x^m\sqrt{-a^2c\,x^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

3.730.5 Fricas [F]

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*
x + 1))/(a^2*x^2 - 2*a*x + 1), x)
```

3.730.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.730.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}\,x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.730.8 Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.730.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.731 $\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

3.731.1 Optimal result

Integrand size = 27, antiderivative size = 172

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}} - \frac{2acx^{2+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{(2+m)\sqrt{c-a^2cx^2}}$$

output
```
-c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x
^2+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)-2*a*c*x^(2+m)*hypergeom([1/2,
 1+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)
+x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

3.731.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\left(\frac{2\sqrt{1-ax}\sqrt{-c(1+ax)}\,\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,ax,-ax\right)}{\sqrt{-1+ax}\sqrt{1+ax}}+\frac{\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{1+m}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x^(1 + m)*((2*Sqrt[1 - a*x]*Sqrt[-(c*(1 + a*x))]*AppellF1[1 + m, 1/2, -1/
2, 2 + m, a*x, -(a*x)])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[c - a^2*c*x
^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x
^2]))/(1 + m)
```

3.731.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6717, 6701, 559, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^m\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{x^m(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{559}$$

$$-c\left(-\frac{\int -\frac{a^2cx^m(2m+2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{a^2c(m+2)}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^m(2m+2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{a^2c(m+2)}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{\int \frac{x^m(2m+2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{557}$$

$$-c\left(\frac{2a(m+2)\int \frac{x^{m+1}}{\sqrt{c-a^2cx^2}}dx+(2m+3)\int \frac{x^m}{\sqrt{c-a^2cx^2}}dx}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow 279$$

$$-c\left(\frac{\frac{2a(m+2)\sqrt{1-a^2x^2}\int \frac{x^{m+1}}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}+\frac{(2m+3)\sqrt{1-a^2x^2}\int \frac{x^m}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow 278$$

$$-c\left(\frac{\frac{(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)\sqrt{c-a^2cx^2}}+\frac{2a\sqrt{1-a^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{\sqrt{c-a^2cx^2}}}{m+2}-\frac{x^{m+}}{c}\right.$$

input
```
Int[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(c*(2 + m))) + (((3 + 2*m)*x^(1 + m
)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])
/((1 + m)*Sqrt[c - a^2*c*x^2]) + (2*a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/Sqrt[c - a^2*c*x^2])/(2 + m
)))
```

3.731.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.731.4 Maple [F]

$$\int \frac{(ax+1)\, x^{m} \sqrt{-a^{2} c\, x^{2}+c}}{ax-1}dx$$

input
```
int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

3.731.5 Fricas [F]

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

3.731.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.731.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

3.731.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.731.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.732 $\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

3.732.1 Optimal result

Integrand size = 25, antiderivative size = 82

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+c
)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

3.732.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{x^m(2+m+ax+amx)\sqrt{c-a^2cx^2}}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x^m*(2 + m + a*x + a*m*x)*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*(2 + m)*Sqrt[1
- 1/(a^2*x^2)])
```

3.732.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{m+1}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int x^m(ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(x^m+ax^{m+1}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m)))/(a*Sqrt[
1 - 1/(a^2*x^2)]*x)
```

3.732.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.732.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{1+m} \sqrt{-a^{2} c\, x^{2}+c}\, \left(a m x +a x +m +2\right)}{\left(1+m \right) \left(2+m \right) \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	62
risch	$-\frac{\sqrt{-\frac{c \left(a^{2} x^{2}-1\right)}{\left(a x -1\right) \left(a x +1\right)}}\, \left(a x -1\right) c \left(a m x +a x +m +2\right) x \,x^{m}}{\sqrt{\frac{a x -1}{a x +1}}\, \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{-c}\, \left(2+m \right) \left(1+m \right)}$	95

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
x^(1+m)/(1+m)/(2+m)/(a*x+1)*(-a^2*c*x^2+c)^(1/2)*(a*m*x+a*x+m+2)/((a*x-1)/
(a*x+1))^(1/2)
```

3.732.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\frac{\sqrt{-a^2cx^2+c}((am+a)x^2+(m+2)x)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-(am^2+3\,am+2\,a)x+3\,m+2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
-sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 + (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x
+ 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

3.732.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.732.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{(a\sqrt{-c}(m+1)x^2+\sqrt{-c}(m+2)x)(ax+1)x^m}{(m^2+3\,m+2)ax+m^2+3\,m+2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
(a*sqrt(-c)*(m + 1)*x^2 + sqrt(-c)*(m + 2)*x)*(a*x + 1)*x^m/((m^2 + 3*m +
2)*a*x + m^2 + 3*m + 2)
```

3.732.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}\,x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.732.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(m+1\right)}{m^2+3\,m+2}+\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(m+2\right)}{a\,\left(m^2+3\,m+2\right)}\right)}{x-\frac{1}{a}}$$

input
```
int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
 + m^2 + 2) + (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/
(x - 1/a)
```

3.733 $\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

3.733.1 Optimal result

Integrand size = 27, antiderivative size = 83

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=-\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+
c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

3.733.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{x^{1+m}}{a(1+m)}+\frac{x^{2+m}}{2+m}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(x^(1 + m)/(a*(1 + m))) + x^(2 + m)/(2 + m)))/(Sqrt
[1 - 1/(a^2*x^2)]*x)
```

3.733.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{m+1}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -x^m(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int x^m(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{53}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(x^m-ax^{m+1}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(\frac{x^{m+1}}{m+1}-\frac{ax^{m+2}}{m+2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(x^(1 + m)/(1 + m) - (a*x^(2 + m))/(2 + m)))/(a*Sqr
t[1 - 1/(a^2*x^2)]*x))
```

3.733.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.733.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{x^{1+m}\sqrt{-a^{2} c\,x^{2}+c}\,\left(a m x+a x-m-2\right) \sqrt{\frac{a x-1}{a x+1}}}{\left(1+m\right) \left(2+m\right) \left(a x-1\right)}$	64
risch	$-\frac{\sqrt{\frac{a x-1}{a x+1}}\,\sqrt{-\frac{c\left(a^{2} x^{2}-1\right)}{\left(a x-1\right) \left(a x+1\right)}}\,\left(a x+1\right) c \left(a m x+a x-m-2\right) x\,x^{m}}{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\sqrt{-c}\,\left(2+m\right) \left(1+m\right)}$	97

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
x^(1+m)/(1+m)/(2+m)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)*(a*m*x+a*x-m-2)*((a*x-1)/
(a*x+1))^(1/2)
```

3.733.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\frac{\sqrt{-a^2cx^2+c}((am+a)x^2-(m+2)x)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-(am^2+3\,am+2\,a)x+3\,m+2}$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="f
ricas")
```

output
```
-sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 - (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x
+ 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

3.733.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.733.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\left(a\sqrt{-c}(m+1)x^2-\sqrt{-c}(m+2)x\right)(ax-1)x^m}{(m^2+3\,m+2)ax-m^2-3\,m-2}$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="m
axima")
```

output
```
(a*sqrt(-c)*(m + 1)*x^2 - sqrt(-c)*(m + 2)*x)*(a*x - 1)*x^m/((m^2 + 3*m +
2)*a*x - m^2 - 3*m - 2)
```

3.733.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.733.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,(m+1)}{m^2+3\,m+2}-\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,(m+2)}{a\,(m^2+3\,m+2)}\right)}{x-\frac{1}{a}}$$

input
```
int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
 + m^2 + 2) - (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/
(x - 1/a)
```

3.734 $\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

3.734.1 Optimal result

Integrand size = 27, antiderivative size = 172

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}\\&\quad+\frac{2acx^{2+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{(2+m)\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
-c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x
^2+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)+2*a*c*x^(2+m)*hypergeom([1/2,
 1+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)
+x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

3.734.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\left(-\frac{2\sqrt{c-acx}\,\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,-ax,ax\right)}{\sqrt{1-ax}}+\frac{\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{1+m}$$

input

```
Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output

```
(x^(1 + m)*((-2*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/2, -1/2, 2 + m, -(a*x),
a*x])/Sqrt[1 - a*x] + (Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)
/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]))/(1 + m)
```

3.734.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6717, 6702, 559, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^m\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{x^m(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{559}$$

$$-c\left(-\frac{\int -\frac{a^2cx^m(2m-2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{a^2c(m+2)}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^m(2m-2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{a^2c(m+2)}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{\int \frac{x^m(2m-2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{557}$$

$$-c\left(\frac{(2m+3)\int \frac{x^m}{\sqrt{c-a^2cx^2}}dx-2a(m+2)\int \frac{x^{m+1}}{\sqrt{c-a^2cx^2}}dx}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow 279$$

$$-c\left(\frac{\frac{(2m+3)\sqrt{1-a^2x^2}\int \frac{x^m}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}-\frac{2a(m+2)\sqrt{1-a^2x^2}\int \frac{x^{m+1}}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow 278$$

$$-c\Bigg(\frac{\frac{(2m+3)\sqrt{1-a^2x^2}x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)\sqrt{c-a^2cx^2}}-\frac{2a\sqrt{1-a^2x^2}x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{\sqrt{c-a^2cx^2}}}{m+2}-\frac{x^{m+}}{c}$$

input
```
Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(-((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(c*(2 + m))) + (((3 + 2*m)*x^(1 + m
)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])
/((1 + m)*Sqrt[c - a^2*c*x^2]) - (2*a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/Sqrt[c - a^2*c*x^2])/(2 + m
)))
```

3.734.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.734. $\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.734.4 Maple [F]

$$\int \frac{x^m \sqrt{-a^2 c\, x^2 + c}\, (a x - 1)}{a x + 1} dx$$

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

3.734.5 Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}(ax-1)x^m}{ax+1}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

3.734.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x**m*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.734.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}(ax-1)x^m}{ax+1}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

3.734.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.734.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.735 $\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

3.735.1 Optimal result

Integrand size $=27$, antiderivative size $=137$

$$\begin{aligned}
&\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx\\
&=-\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}\\
&\quad+\frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

output
```
-3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^
2+c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)+4*x^m*hypergeom([1, 1+m],[2+m],-a*x)*
(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

3.735.2 Mathematica [A] (verified)

Time $=0.06$ (sec) , antiderivative size $=75$, normalized size of antiderivative $=0.55$

$$\begin{aligned}
&\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx\\
&=\frac{x^m\sqrt{c-a^2cx^2}(-6+ax+m(-3+ax)+4(2+m)\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

input
```
Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(x^m*Sqrt[c - a^2*c*x^2]*(-6 + a*x + m*(-3 + a*x) + 4*(2 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, -(a*x)]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

3.735.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{m+1}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{x^m(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(\frac{4x^m}{ax+1}-3x^m+ax^{m+1}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{m+1}+\frac{ax^{m+2}}{m+2}-\frac{3x^{m+1}}{m+1}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

3.735. $\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

output
```
(Sqrt[c - a^2*c*x^2]*((-3*x^(1 + m))/(1 + m) + (a*x^(2 + m))/(2 + m) + (4*
x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(1 + m)))/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

3.735.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.735.4 Maple [F]

$$\int x^m\sqrt{-a^2c\,x^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

3.735.5 Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\,x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x
 + 1), x)
```

3.735.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.735.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\,x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.735.8 Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.735.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int x^m\,\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.736 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

3.736.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left(8,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

output
```
-256*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([8, 4-1/2*n],[
5-1/2*n],(a-1/x)/(a+1/x))/a/(8-n)
```

3.736.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 267 vs. 2(81) = 162.

Time = 2.39 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.30

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\frac{c^3e^{n\coth^{-1}(ax)}\left(-912n+58n^3-n^5-5040ax+912an^2x-58an^4x+an^6x+1368a^2nx^2-64a^2n^3x^2+\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
-1/5040*(c^3*E^(n*ArcCoth[a*x])*(-912*n + 58*n^3 - n^5 - 5040*a*x + 912*a*
n^2*x - 58*a*n^4*x + a*n^6*x + 1368*a^2*n*x^2 - 64*a^2*n^3*x^2 + a^2*n^5*x
^2 + 5040*a^3*x^3 - 152*a^3*n^2*x^3 + 2*a^3*n^4*x^3 - 576*a^4*n*x^4 + 6*a^
4*n^3*x^4 - 3024*a^5*x^5 + 24*a^5*n^2*x^5 + 120*a^6*n*x^6 + 720*a^7*x^7 +
E^(2*ArcCoth[a*x])*n*(-1152 + 576*n + 104*n^2 - 52*n^3 - 2*n^4 + n^5)*Hype
rgeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (-2304 + 784*n^2
- 56*n^4 + n^6)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/a
```

3.736.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \text{6745}$$

$$-a^6c^3\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

$$\Big\downarrow \ 6745$$

$$-a^6c^3\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

3.736. $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

$$\Big\downarrow \ 2005$$

$$-c^3 \int e^{n \coth^{-1}(ax)} \left(a^2 x^2-1\right)^3 dx$$

$$\Big\downarrow \ 6745$$

$$-a^6 c^3 \int \frac{e^{n \coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3 \int e^{n \coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3 \int e^{n \coth^{-1}(ax)} \left(a^2 x^2-1\right)^3 dx$$

$$\Big\downarrow \ 6745$$

$$-a^6 c^3 \int \frac{e^{n \coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3 \int e^{n \coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3 \int e^{n \coth^{-1}(ax)} \left(a^2 x^2-1\right)^3 dx$$

$$\Big\downarrow \ 6745$$

$$-a^6 c^3 \int \frac{e^{n \coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6}dx$$

$$\Big\downarrow \ 27$$

$$-c^3 \int e^{n \coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6dx$$

$$\Big\downarrow \ 2005$$

$$-c^3 \int e^{n \coth^{-1}(ax)} \left(a^2 x^2-1\right)^3 dx$$

$$\Big\downarrow \ 6745$$

$$-a^6 c^3 \int \frac{e^{n \coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 x^6}{a^6}dx$$

3.736. $\int e^{n \coth^{-1}(ax)} \left(c-a^2 c x^2\right)^3 \, dx$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^3x^6dx$$

$$\Big\downarrow \text{2005}$$

$$-c^3\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^3dx$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.736.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.736.4 Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^3dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

3.736.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\int -\left(a^2cx^2-c\right)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*((a*x + 1)/(
a*x - 1))^(1/2*n), x)
```

3.736.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-c^3\left(\int 3a^2x^2e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-3a^4x^4e^{n\operatorname{acoth}(ax)}\right)dx\right.$$
$$\left.+\int a^6x^6e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-e^{n\operatorname{acoth}(ax)}\right)dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**3,x)
```

output
```
-c**3*(Integral(3*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-3*a**4*x**4*
exp(n*acoth(a*x)), x) + Integral(a**6*x**6*exp(n*acoth(a*x)), x) + Integra
l(-exp(n*acoth(a*x)), x))
```

3.736.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\int -\left(a^2cx^2-c\right)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-integrate((a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.736.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\int -\left(a^2cx^2-c\right)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input

```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output

```
integrate(-(a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.736.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^3\,dx$$

input

```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^3,x)
```

output

```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^3, x)
```

3.737 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

3.737.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\text{Hypergeometric2F1}\left(6,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

output
```
64*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([6, 3-1/2*n],[4-
1/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

3.737.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(81) = 162.

Time = 1.40 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.21

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{c^2e^{n\coth^{-1}(ax)}\left(22n-n^3+120ax-22an^2x+an^4x-28a^2nx^2+a^2n^3x^2-80a^3x^3+2a^3n^2x^3+6a^4nx^4+\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
(c^2*E^(n*ArcCoth[a*x])*(22*n - n^3 + 120*a*x - 22*a*n^2*x + a*n^4*x - 28*
a^2*n*x^2 + a^2*n^3*x^2 - 80*a^3*x^3 + 2*a^3*n^2*x^3 + 6*a^4*n*x^4 + 24*a^
5*x^5 + E^(2*ArcCoth[a*x])*n*(32 - 16*n - 2*n^2 + n^3)*Hypergeometric2F1[1
, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (64 - 20*n^2 + n^4)*Hypergeometr
ic2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/(120*a)
```

3.737.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2 e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{ 2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{ 6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

3.737. $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4dx$$

$$\Big\downarrow \text{2005}$$

$$c^2\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)^2dx$$

$$\Big\downarrow \text{6745}$$

$$a^4c^2\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)^2x^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

3.737. $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

$$c^2 \int e^{n \coth^{-1}(ax)} \left(a^2-\frac{1}{x^2}\right)^2 x^4 dx$$
$$\Big\downarrow \text{2005}$$
$$c^2 \int e^{n \coth^{-1}(ax)} \left(a^2x^2-1\right)^2 dx$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.737.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.737.4 Maple [F]

$$\int e^{n \,\mathrm{arccoth}(ax)} \left(-a^2 c \,x^2+c\right)^2 dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
```

3.737.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\int\left(a^2cx^2-c\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n)
, x)
```

3.737.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=c^2\left(\int\left(-2a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx+\int a^4x^4e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**2,x)
```

output
```
c**2*(Integral(-2*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(a**4*x**4*exp
(n*acoth(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

3.737.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\int\left(a^2cx^2-c\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.737.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\int \left(a^2cx^2-c\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input

```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output

```
integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.737.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^2\,dx$$

input

```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^2,x)
```

output

```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^2, x)
```

3.738 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

3.738.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)\,dx = -\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(4,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

output
```
-16*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([4, 2-1/2*n],[3-1
/2*n],(a-1/x)/(a+1/x))/a/(4-n)
```

3.738.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.41

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)\,dx = -\frac{ce^{n\coth^{-1}(ax)}\left(-n-6ax+an^2x+a^2nx^2+2a^3x^3+e^{2\coth^{-1}(ax)}(-2+n)n\,\text{Hypergeometric2F1}\left(1,1+\right.\right.}{6a}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
-1/6*(c*E^(n*ArcCoth[a*x])*(-n - 6*a*x + a*n^2*x + a^2*n*x^2 + 2*a^3*x^3 +
 E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2
*ArcCoth[a*x])] + (-4 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCo
th[a*x])]))/a
```

3.738.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \text{2005}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \text{6745}$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \ \textcolor{blue}{2005}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \ \textcolor{blue}{6745}$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \ \textcolor{blue}{27}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \ \textcolor{blue}{2005}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \ \textcolor{blue}{6745}$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \ \textcolor{blue}{27}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \ \textcolor{blue}{2005}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \ \textcolor{blue}{6745}$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \ \textcolor{blue}{27}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \ \textcolor{blue}{2005}$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \ \textcolor{blue}{6745}$$

3.738. $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)\ dx$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \ 27$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \ 2005$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \ 6745$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \ 27$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \ 2005$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \ 6745$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \ 27$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \ 2005$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

$$\Big\downarrow \ 6745$$

$$-a^2c\int \frac{e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2}{a^2}dx$$

$$\Big\downarrow \ 27$$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2-\frac{1}{x^2}\right)x^2dx$$

$$\Big\downarrow \ 2005$$

3.738. $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

$$-c\int e^{n\coth^{-1}(ax)}\left(a^2x^2-1\right)dx$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
$Aborted
```

3.738.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 6745
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && In
tegerQ[p]
```

3.738.4 Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
```

3.738.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int -\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.738.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-c\left(\int a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c),x)
```

output
```
-c*(Integral(a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))
, x))
```

3.738.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int -\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-integrate((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.738.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int -\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.738.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2), x)
```

3.739 $\int e^{n \coth^{-1}(ax)} \, dx$

3.739.1 Optimal result

Integrand size = 8, antiderivative size = 78

$$\int e^{n \coth^{-1}(ax)} \, dx = \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left(2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

output
```
4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n
],(a-1/x)/(a+1/x))/a/(2-n)
```

3.739.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int e^{n \coth^{-1}(ax)} \, dx$$
$$= \frac{e^{n \coth^{-1}(ax)} \left(e^{2\coth^{-1}(ax)} n \, \text{Hypergeometric2F1}\left(1, 1+\frac{n}{2}, 2+\frac{n}{2}, e^{2\coth^{-1}(ax)}\right) + (2+n) \left(ax + \text{Hypergeome}\right.\right.}{a(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x]),x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
+ n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 +
n/2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

3.739.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6720, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6720}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 141}$$

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

input
```
Int[E^(n*ArcCoth[a*x]),x]
```

output
```
(4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
 1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

3.739.3.1 Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

3.739.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}dx$$

input
```
int(exp(n*arccoth(a*x)),x)
```

output
```
int(exp(n*arccoth(a*x)),x)
```

3.739.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.739.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x)),x)
```

output
```
Integral(exp(n*acoth(a*x)), x)
```

3.739.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.739.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\,dx = \int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.739.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\,dx = \int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

input
```
int(exp(n*acoth(a*x)),x)
```

output
```
int(exp(n*acoth(a*x)), x)
```

3.740 $\int \frac{e^{n \coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

3.740.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{n \coth^{-1}(ax)}}{acn}$$

output
```
exp(n*arccoth(a*x))/a/c/n
```

3.740.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{n \coth^{-1}(ax)}}{acn}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
E^(n*ArcCoth[a*x])/(a*c*n)
```

3.740.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$$
$$\Big\downarrow \text{6737}$$
$$\frac{e^{n\coth^{-1}(ax)}}{acn}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
E^(n*ArcCoth[a*x])/(a*c*n)
```

3.740.3.1 Defintions of rubi rules used

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

3.740.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)}}{a c n}$	18
parallelrisch	$\frac{{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)}}{a c n}$	18
risch	$\frac{\left(a x -1\right)^{-\frac{n}{2}} \left(a x +1\right)^{\frac{n}{2}}}{c a n}$	29

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
exp(n*arccoth(a*x))/a/c/n
```

3.740.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acn}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*n)
```

3.740.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \begin{cases} \frac{x}{c} & \text{for}\ a=0 \wedge n=0 \\ \frac{xe^{\frac{i\pi n}{2}}}{c} & \text{for}\ a=0 \\ -\frac{\log\left(x-\frac{1}{a}\right)}{2ac}+\frac{\log\left(x+\frac{1}{a}\right)}{2ac} & \text{for}\ n=0 \\ \frac{e^{n\,\operatorname{acoth}(ax)}}{acn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c),x)
```

output
```
Piecewise((x/c, Eq(a, 0) & Eq(n, 0)), (x*exp(I*pi*n/2)/c, Eq(a, 0)), (-log
(x - 1/a)/(2*a*c) + log(x + 1/a)/(2*a*c), Eq(n, 0)), (exp(n*acoth(a*x))/(a
*c*n), True))
```

3.740.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{\left(\frac{1}{2}\,n\log(ax+1)-\frac{1}{2}\,n\log(ax-1)\right)}}{acn}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
e^(1/2*n*log(a*x + 1) - 1/2*n*log(a*x - 1))/(a*c*n)
```

3.740.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2cx^2-c}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.740.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{{\left(\frac{1}{a\,x}+1\right)}^{n/2}}{a\,c\,n\,{\left(1-\frac{1}{a\,x}\right)}^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2),x)
```

output
```
(1/(a*x) + 1)^(n/2)/(a*c*n*(1 - 1/(a*x))^(n/2))
```

3.741 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

3.741.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=\frac{2e^{n\coth^{-1}(ax)}}{ac^2n(4-n^2)}-\frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2(4-n^2)(1-a^2x^2)}$$

output
```
2*exp(n*arccoth(a*x))/a/c^2/n/(-n^2+4)-exp(n*arccoth(a*x))*(-2*a*x+n)/a/c^
2/(-n^2+4)/(-a^2*x^2+1)
```

3.741.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx=-\frac{e^{n\coth^{-1}(ax)}(-2+n^2-2anx+2a^2x^2)}{ac^2n(-4+n^2)(-1+a^2x^2)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
-((E^(n*ArcCoth[a*x])*(-2 + n^2 - 2*a*n*x + 2*a^2*x^2))/(a*c^2*n*(-4 + n^2
)*(-1 + a^2*x^2)))
```

3.741.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6739, 27, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{2\int \frac{e^{n\coth^{-1}(ax)}}{c(1-a^2x^2)}dx}{c(4-n^2)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{ac^2(4-n^2)(1-a^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{n\coth^{-1}(ax)}}{1-a^2x^2}dx}{c^2(4-n^2)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{ac^2(4-n^2)(1-a^2x^2)}$$

$$\Big\downarrow \text{6737}$$

$$\frac{2e^{n\coth^{-1}(ax)}}{ac^2n(4-n^2)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{ac^2(4-n^2)(1-a^2x^2)}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
(2*E^(n*ArcCoth[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcCoth[a*x])*(n - 2*a*
x))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

3.741.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.741.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} \left(2 a^{2} x^{2}-2 a n x +n^{2}-2\right)}{\left(a^{2} x^{2}-1\right) c^{2} a n \left(n^{2}-4\right)}$	55
risch	$-\frac{\left(2 a^{2} x^{2}-2 a n x +n^{2}-2\right) \left(a x -1\right)^{-\frac{n}{2}} \left(a x +1\right)^{\frac{n}{2}}}{\left(a^{2} x^{2}-1\right) c^{2} a n \left(n^{2}-4\right)}$	66
parallelrisch	$\frac{-2 x^{2} {\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} a^{2}+2 x \,{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} a n -{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} n^{2}+2 \,{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)}}{c^{2} \left(a^{2} x^{2}-1\right) a n \left(n^{2}-4\right)}$	78

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(n*arccoth(a*x))*(2*a^2*x^2-2*a*n*x+n^2-2)/(a^2*x^2-1)/c^2/a/n/(n^2-4)
```

3.741.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{(2\,a^2x^2 - 2\,anx + n^2 - 2)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^2n^3 - 4\,ac^2n - (a^3c^2n^3 - 4\,a^3c^2n)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
(2*a^2*x^2 - 2*a*n*x + n^2 - 2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^3 -
 4*a*c^2*n - (a^3*c^2*n^3 - 4*a^3*c^2*n)*x^2)
```

3.741.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$$

$$= \begin{cases} \frac{xe^{\frac{i\pi n}{2}}}{c^2} \\ -\frac{a^2x^2\operatorname{acoth}(ax)}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)}-4ac^2e^{2\operatorname{acoth}(ax)}} - \frac{2ax\operatorname{acoth}(ax)}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)}-4ac^2e^{2\operatorname{acoth}(ax)}} + \frac{ax}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)}-4ac^2e^{2\operatorname{acoth}(ax)}} - \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} + \frac{a^2x^2\log\left(x+\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} - \frac{2ax}{4a^3c^2x^2-4ac^2} + \frac{\log\left(x-\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} - \frac{\log\left(x+\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} \\ \frac{\int\frac{e^{2\operatorname{acoth}(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2} \\ -\frac{2a^2x^2e^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} + \frac{2anxe^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} - \frac{n^2e^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} + \end{cases}$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((x*exp(I*pi*n/2)/c**2, Eq(a, 0)), (-a**2*x**2*acoth(a*x)/(4*a**3
*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) - 2*a*x*acoth(a
*x)/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) + a*
x/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) - acot
h(a*x)/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) +
 2/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))), Eq(n
, -2)), (-a**2*x**2*log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) + a**2*x**2
*log(x + 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - 2*a*x/(4*a**3*c**2*x**2 - 4*
a*c**2) + log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - log(x + 1/a)/(4*a**
3*c**2*x**2 - 4*a*c**2), Eq(n, 0)), (Integral(exp(2*acoth(a*x))/(a**4*x**4
 - 2*a**2*x**2 + 1), x)/c**2, Eq(n, 2)), (-2*a**2*x**2*exp(n*acoth(a*x))/(
a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n) + 2*a
*n*x*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*
n**3 + 4*a*c**2*n) - n**2*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*
c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n) + 2*exp(n*acoth(a*x))/(a**3*c**2*n
**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n), True))
```

3.741.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.741.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

3.741.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,x^2}{a\,c^2\,n\,\left(n^2-4\right)}-\frac{2\,x}{a^2\,c^2\,\left(n^2-4\right)}+\frac{n^2-2}{a^3\,c^2\,n\,\left(n^2-4\right)}\right)}{\left(\frac{1}{a^2}-x^2\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^2,x)
```

output
```
(((a*x + 1)/(a*x))^(n/2)*((2*x^2)/(a*c^2*n*(n^2 - 4)) - (2*x)/(a^2*c^2*(n^
2 - 4)) + (n^2 - 2)/(a^3*c^2*n*(n^2 - 4))))/((1/a^2 - x^2)*((a*x - 1)/(a*x
))^(n/2))
```

3.742 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

3.742.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{24e^{n\coth^{-1}(ax)}}{ac^3n\,(64-20n^2+n^4)} - \frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3\,(16-n^2)\,(1-a^2x^2)^2} - \frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3\,(4-n^2)\,(16-n^2)\,(1-a^2x^2)}$$

output
```
24*exp(n*arccoth(a*x))/a/c^3/n/(n^4-20*n^2+64)-exp(n*arccoth(a*x))*(-4*a*x
+n)/a/c^3/(-n^2+16)/(-a^2*x^2+1)^2-12*exp(n*arccoth(a*x))*(-2*a*x+n)/a/c^3
/(n^4-20*n^2+64)/(-a^2*x^2+1)
```

3.742.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(n^4-4an^3x+24(-1+a^2x^2)^2-8anx(-5+3a^2x^2)+4n^2(-4+3a^2x^2)\right)}{ac^3n\,(-16+n^2)\,(-4+n^2)\,(-1+a^2x^2)^2}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
(E^(n*ArcCoth[a*x])*(n^4 - 4*a*n^3*x + 24*(-1 + a^2*x^2)^2 - 8*a*n*x*(-5 +
 3*a^2*x^2) + 4*n^2*(-4 + 3*a^2*x^2)))/(a*c^3*n*(-16 + n^2)*(-4 + n^2)*(-1
 + a^2*x^2)^2)
```

3.742.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6739, 27, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{12\int \frac{e^{n\coth^{-1}(ax)}}{c^2(1-a^2x^2)^2}dx}{c(16-n^2)}-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3(16-n^2)(1-a^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{12\int \frac{e^{n\coth^{-1}(ax)}}{(1-a^2x^2)^2}dx}{c^3(16-n^2)}-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3(16-n^2)(1-a^2x^2)^2}$$

$$\Big\downarrow \text{6739}$$

$$\frac{12\left(\frac{2\int \frac{e^{n\coth^{-1}(ax)}}{1-a^2x^2}dx}{4-n^2}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{a(4-n^2)(1-a^2x^2)}\right)}{c^3(16-n^2)}-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3(16-n^2)(1-a^2x^2)^2}$$

$$\Big\downarrow \text{6737}$$

$$\frac{12\left(\frac{2e^{n\coth^{-1}(ax)}}{an(4-n^2)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{a(4-n^2)(1-a^2x^2)}\right)}{c^3(16-n^2)}-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3(16-n^2)(1-a^2x^2)^2}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

---

output

```
-((E^(n*ArcCoth[a*x])*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2)) + (
12*((2*E^(n*ArcCoth[a*x]))/(a*n*(4 - n^2)) - (E^(n*ArcCoth[a*x])*(n - 2*a*
x))/(a*(4 - n^2)*(1 - a^2*x^2))))/(c^3*(16 - n^2))
```

3.742.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.742.4 Maple [A] (verified)

Time = 17.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

method	result
gosper	$\frac{\left(24 a^{4} x^{4}-24 a^{3} x^{3} n+12 a^{2} n^{2} x^{2}-4 a\,n^{3} x-48 a^{2} x^{2}+n^{4}+40 a n x-16 n^{2}+24\right) \mathrm{e}^{n\,\operatorname{arccoth}\left(a x\right)}}{\left(a^{2} x^{2}-1\right)^{2} c^{3} a \left(n^{2}-16\right) \left(n^{2}-4\right) n}$
risch	$\frac{\left(24 a^{4} x^{4}-24 a^{3} x^{3} n+12 a^{2} n^{2} x^{2}-4 a\,n^{3} x-48 a^{2} x^{2}+n^{4}+40 a n x-16 n^{2}+24\right) \left(a x-1\right)^{-\frac{n}{2}} \left(a x+1\right)^{\frac{n}{2}}}{\left(a^{2} x^{2}-1\right)^{2} c^{3} a \left(n^{2}-16\right) \left(n^{2}-4\right) n}$
parallelrisch	$\frac{24 x^{4} \mathrm{e}^{n\,\operatorname{arccoth}\left(a x\right)} a^{4}+40 x\,\mathrm{e}^{n\,\operatorname{arccoth}\left(a x\right)} a n-48 x^{2} \mathrm{e}^{n\,\operatorname{arccoth}\left(a x\right)} a^{2}-24 a^{3} x^{3} \mathrm{e}^{n\,\operatorname{arccoth}\left(a x\right)} n+12 x^{2} \mathrm{e}^{n\,\operatorname{arccoth}\left(a x\right)} a^{2} n^{2}-4 x\,\mathrm{e}^{n\ \mathrm{arc}}}{c^{3} \left(a^{2} x^{2}-1\right)^{2} a \left(n^{2}-16\right) \left(n^{2}-4\right) n}$

input

```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output

```
(24*a^4*x^4-24*a^3*n*x^3+12*a^2*n^2*x^2-4*a*n^3*x-48*a^2*x^2+n^4+40*a*n*x-
16*n^2+24)*exp(n*arccoth(a*x))/(a^2*x^2-1)^2/c^3/a/(n^2-16)/(n^2-4)/n
```

3.742.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.37

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$$
$$=\frac{(24\,a^4x^4-24\,a^3nx^3+n^4+12\,(a^2n^2-4\,a^2)x^2-16\,n^2-4\,(an^3-10\,an)x+24)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^5-20\,ac^3n^3+64\,ac^3n+(a^5c^3n^5-20\,a^5c^3n^3+64\,a^5c^3n)x^4-2\,(a^3c^3n^5-20\,a^3c^3n^3+64\,a^3c^3n)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
(24*a^4*x^4 - 24*a^3*n*x^3 + n^4 + 12*(a^2*n^2 - 4*a^2)*x^2 - 16*n^2 - 4*(
a*n^3 - 10*a*n)*x + 24)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^5 - 20*a*c^
3*n^3 + 64*a*c^3*n + (a^5*c^3*n^5 - 20*a^5*c^3*n^3 + 64*a^5*c^3*n)*x^4 - 2
*(a^3*c^3*n^5 - 20*a^3*c^3*n^3 + 64*a^3*c^3*n)*x^2)
```

3.742.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**3,x)
```

output
```
Timed out
```

3.742.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx=\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(a^2cx^2-c)^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

3.742.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

3.742.9 Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.51

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$$
$$=\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{24\,x^4}{a\,c^3\,n\,\left(n^4-20\,n^2+64\right)}-\frac{4\,x\,\left(n^2-10\right)}{a^4\,c^3\,\left(n^4-20\,n^2+64\right)}-\frac{24\,x^3}{a^2\,c^3\,\left(n^4-20\,n^2+64\right)}+\frac{n^4-16\,n^2+24}{a^5\,c^3\,n\,\left(n^4-20\,n^2+64\right)}+\frac{x^2\,\left(12\,n^2-48\right)}{a^3\,c^3\,n\,\left(n^4-20\,n^2+64\right.}\right.}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^4}+x^4-\frac{2\,x^2}{a^2}\right)}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^3,x)
```

output
```
(((a*x + 1)/(a*x))^(n/2)*((24*x^4)/(a*c^3*n*(n^4 - 20*n^2 + 64)) - (4*x*(n
^2 - 10))/(a^4*c^3*(n^4 - 20*n^2 + 64)) - (24*x^3)/(a^2*c^3*(n^4 - 20*n^2
+ 64)) + (n^4 - 16*n^2 + 24)/(a^5*c^3*n*(n^4 - 20*n^2 + 64)) + (x^2*(12*n^
2 - 48))/(a^3*c^3*n*(n^4 - 20*n^2 + 64))))/(((a*x - 1)/(a*x))^(n/2)*(1/a^4
 + x^4 - (2*x^2)/a^2))
```

3.743 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

3.743.1 Optimal result

Integrand size = 22, antiderivative size = 197

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx &= \frac{720e^{n\coth^{-1}(ax)}}{ac^4n\,(36-n^2)\,(64-20n^2+n^4)} - \frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4\,(36-n^2)\,(1-a^2x^2)^3} \\ &\quad - \frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4\,(16-n^2)\,(36-n^2)\,(1-a^2x^2)^2} \\ &\quad - \frac{360e^{n\coth^{-1}(ax)}(n-2ax)}{ac^4\,(4-n^2)\,(16-n^2)\,(36-n^2)\,(1-a^2x^2)}\end{aligned}$$

output

```
720*exp(n*arccoth(a*x))/a/c^4/n/(-n^2+36)/(n^4-20*n^2+64)-exp(n*arccoth(a*
x))*(-6*a*x*x+n)/a/c^4/(-n^2+36)/(-a^2*x^2+1)^3-30*exp(n*arccoth(a*x))*(-4*a
*x+n)/a/c^4/(n^4-52*n^2+576)/(-a^2*x^2+1)^2-360*exp(n*arccoth(a*x))*(-2*a*
x+n)/a/c^4/(-n^2+36)/(n^4-20*n^2+64)/(-a^2*x^2+1)
```

3.743.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.77

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \\ -\frac{e^{n\coth^{-1}(ax)}\left(n^6-6an^5x-120an^3x(-2+a^2x^2)+720(-1+a^2x^2)^3+10n^4(-5+3a^2x^2)-48anx(33-\right.}{ac^4n\,(-36+n^2)\,(-16+n^2)\,(-4+n^2)\,(-1+a^2x^2)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n^6 - 6*a*n^5*x - 120*a*n^3*x*(-2 + a^2*x^2) + 720*
(-1 + a^2*x^2)^3 + 10*n^4*(-5 + 3*a^2*x^2) - 48*a*n*x*(33 - 40*a^2*x^2 + 1
5*a^4*x^4) + 8*n^2*(68 - 105*a^2*x^2 + 45*a^4*x^4)))/(a*c^4*n*(-36 + n^2)*
(-16 + n^2)*(-4 + n^2)*(-1 + a^2*x^2)^3))
```

3.743.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6739, 27, 6739, 6739, 6737}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{30\int \frac{e^{n\coth^{-1}(ax)}}{c^3\left(1-a^2x^2\right)^3}dx}{c\left(36-n^2\right)}-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{30\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-a^2x^2\right)^3}dx}{c^4\left(36-n^2\right)}-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}$$

$$\Big\downarrow \text{6739}$$

$$\frac{30\left(\frac{12\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-a^2x^2\right)^2}dx}{16-n^2}-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{a\left(16-n^2\right)\left(1-a^2x^2\right)^2}\right)}{c^4\left(36-n^2\right)}-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}$$

$$\Big\downarrow \text{6739}$$

$$\frac{30\left(\frac{12\left(\frac{2\int \frac{e^{n\coth^{-1}(ax)}}{1-a^2x^2}dx}{4-n^2}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{a\left(4-n^2\right)\left(1-a^2x^2\right)}\right)}{16-n^2}-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{a\left(16-n^2\right)\left(1-a^2x^2\right)^2}\right)}{c^4\left(36-n^2\right)}-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}$$

$$\Big\downarrow \text{6737}$$

$$\frac{30\left(\frac{12\left(\frac{2e^{n\coth^{-1}(ax)}}{an\left(4-n^2\right)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{a\left(4-n^2\right)\left(1-a^2x^2\right)}\right)}{16-n^2}-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{a(16-n^2)(1-a^2x^2)^2}\right)}{c^4\left(36-n^2\right)}-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3)) + (
30*(-((E^(n*ArcCoth[a*x])*(n - 4*a*x))/(a*(16 - n^2)*(1 - a^2*x^2)^2)) + (
12*((2*E^(n*ArcCoth[a*x]))/(a*n*(4 - n^2)) - (E^(n*ArcCoth[a*x])*(n - 2*a*
x))/(a*(4 - n^2)*(1 - a^2*x^2))))/(16 - n^2)))/(c^4*(36 - n^2))
```

3.743.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6737
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.743. $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

3.743.4 Maple [A] (verified)

Time = 59.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{(720a^6x^6-720a^5x^5n+360a^4n^2x^4-120a^3n^3x^3-2160a^4x^4+30a^2n^4x^2+1920a^3x^3n-6a\,n^5x-840a^2n^2x^2+n^6+240a\,n^3x+2}{(a^2x^2-1)^3c^4an(n^6-56n^4+784n^2-2304)}$
risch	$-\frac{(720a^6x^6-720a^5x^5n+360a^4n^2x^4-120a^3n^3x^3-2160a^4x^4+30a^2n^4x^2+1920a^3x^3n-6a\,n^5x-840a^2n^2x^2+n^6+240a\,n^3x+2}{c^4(n^2-36)(n^2-16)(n^2-4)an(a^2x^2-1)^3}$
parallelrisch	$\frac{-720x^6\mathrm{e}^{n\ \mathrm{arccoth}(ax)}a^6+2160x^4\mathrm{e}^{n\ \mathrm{arccoth}(ax)}a^4+720a^5\mathrm{e}^{n\ \mathrm{arccoth}(ax)}x^5n-360x^4\mathrm{e}^{n\ \mathrm{arccoth}(ax)}a^4n^2+120x^3\mathrm{e}^{n\ \mathrm{arccoth}(ax)}a^3}{}$

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-(720*a^6*x^6-720*a^5*n*x^5+360*a^4*n^2*x^4-120*a^3*n^3*x^3-2160*a^4*x^4+3
0*a^2*n^4*x^2+1920*a^3*n*x^3-6*a*n^5*x-840*a^2*n^2*x^2+n^6+240*a*n^3*x+216
0*a^2*x^2-50*n^4-1584*a*n*x+544*n^2-720)*exp(n*arccoth(a*x))/(a^2*x^2-1)^3
/c^4/a/n/(n^6-56*n^4+784*n^2-2304)
```

3.743.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.57

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$=\frac{(720\,a^6x^6-720\,a^5nx^5+n^6+360\,(a^4n^2-6\,a^4)x^4-50\,n^4-120\,(a^3n^3-16\,a^3n)x^3+30}{ac^4n^7-56\,ac^4n^5+784\,ac^4n^3-(a^7c^4n^7-56\,a^7c^4n^5+784\,a^7c^4n^3-2304\,a^7c^4n)x^6-2304\,ac^4n+3\,(a^5c^4}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
(720*a^6*x^6 - 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 - 6*a^4)*x^4 - 50*n^4 -
120*(a^3*n^3 - 16*a^3*n)*x^3 + 30*(a^2*n^4 - 28*a^2*n^2 + 72*a^2)*x^2 + 54
4*n^2 - 6*(a*n^5 - 40*a*n^3 + 264*a*n)*x - 720)*((a*x + 1)/(a*x - 1))^(1/2
*n)/(a*c^4*n^7 - 56*a*c^4*n^5 + 784*a*c^4*n^3 - (a^7*c^4*n^7 - 56*a^7*c^4*
n^5 + 784*a^7*c^4*n^3 - 2304*a^7*c^4*n)*x^6 - 2304*a*c^4*n + 3*(a^5*c^4*n^
7 - 56*a^5*c^4*n^5 + 784*a^5*c^4*n^3 - 2304*a^5*c^4*n)*x^4 - 3*(a^3*c^4*n^
7 - 56*a^3*c^4*n^5 + 784*a^3*c^4*n^3 - 2304*a^3*c^4*n)*x^2)
```

3.743.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**4,x)
```

output
```
Timed out
```

3.743.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(a^2cx^2-c)^4}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

3.743.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(a^2cx^2-c)^4}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

3.743.9 Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.59

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$

$$= \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n^6-50\,n^4+544\,n^2-720}{a^7\,c^4\,n\,(n^6-56\,n^4+784\,n^2-2304)}-\frac{720\,x^5}{a^2\,c^4\,(n^6-56\,n^4+784\,n^2-2304)}-\frac{x^3\,(120\,n^2-1920)}{a^4\,c^4\,(n^6-56\,n^4+784\,n^2-2304)}+\frac{720}{a\,c^4\,n\,(n^6-56\,n^4}\right.}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\left(\frac{1}{a^6}-x^6+\right.}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^4,x)
```

output
```
(((a*x + 1)/(a*x))^(n/2)*((544*n^2 - 50*n^4 + n^6 - 720)/(a^7*c^4*n*(784*n
^2 - 56*n^4 + n^6 - 2304)) - (720*x^5)/(a^2*c^4*(784*n^2 - 56*n^4 + n^6 -
2304)) - (x^3*(120*n^2 - 1920))/(a^4*c^4*(784*n^2 - 56*n^4 + n^6 - 2304))
+ (720*x^6)/(a*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) - (6*x*(n^4 - 40*n^2
 + 264))/(a^6*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^2*(30*n^4 - 840*n^
2 + 2160))/(a^5*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^4*(360*n^2 - 2
160))/(a^3*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304))))/(((a*x - 1)/(a*x))^(n/
2)*(1/a^6 - x^6 + (3*x^4)/a^2 - (3*x^2)/a^4))
```

3.744 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

3.744.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{32\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-5+n)}\left(c-a^2cx^2\right)^{3/2}\text{Hypergeometric2F1}\left(5,\frac{5-n}{2},\frac{7-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
32*(1-1/a/x)^(5/2-1/2*n)*(1+1/a/x)^(-5/2+1/2*n)*(-a^2*c*x^2+c)^(3/2)*hyper
geom([5, 5/2-1/2*n],[7/2-1/2*n],(a-1/x)/(a+1/x))/a^4/(5-n)/(1-1/a^2/x^2)^(
3/2)/x^3
```

3.744.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(116) = 232.

Time = 2.93 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.41

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{c^2\left(96a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(ae^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x(n+ax)+2e^{(1+n)\coth^{-1}(ax)}(-1+n)\,\mathrm{Hy}\right.\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
(c^2*(96*a^3*c*(1 - 1/(a^2*x^2))^(3/2)*x^3*(a*E^(n*ArcCoth[a*x])*Sqrt[1 -
1/(a^2*x^2)]*x*(n + a*x) + 2*E^((1 + n)*ArcCoth[a*x])*(-1 + n)*Hypergeomet
ric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]) - c*(-1 + a^2*x^2)*(2
*E^(n*ArcCoth[a*x])*(-1 + a^2*x^2)^2*(-(a*(-21 + n^2)*x) + 2*n*(1 - n^2 +
(3 + n^2)*Cosh[2*ArcCoth[a*x]]) + a*(3 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh
[3*ArcCoth[a*x]]) + 16*a*E^((1 + n)*ArcCoth[a*x])*(-3 + 3*n - n^2 + n^3)*S
qrt[1 - 1/(a^2*x^2)]*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*Arc
Coth[a*x])])))/(192*a*(c - a^2*c*x^2)^(3/2))
```

3.744.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6749, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3dx}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 6749}$$

$$-\frac{\left(c-a^2cx^2\right)^{3/2}\int \left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n+3}{2}}x^5d\frac{1}{x}}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 141}$$

$$\frac{32\left(c-a^2cx^2\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-5}{2}}\text{Hypergeometric2F1}\left(5,\frac{5-n}{2},\frac{7-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
(32*(1 - 1/(a*x))^((5 - n)/2)*(1 + 1/(a*x))^((-5 + n)/2)*(c - a^2*c*x^2)^(
3/2)*Hypergeometric2F1[5, (5 - n)/2, (7 - n)/2, (a - x^(-1))/(a + x^(-1))]
)/(a^4*(5 - n)*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

3.744.3.1 Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

3.744.4 Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

3.744.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*
n), x)
```

3.744.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}e^{n\,\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*exp(n*acoth(a*x)), x)
```

3.744.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.744.8 Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.744.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2), x)
```

3.745 $\int e^{n \coth^{-1}(ax)} \sqrt{c-a^2 c x^2} \, dx$

3.745.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{n \coth^{-1}(ax)} \sqrt{c-a^2 c x^2} \, dx = \frac{8\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)} \sqrt{c-a^2 c x^2}\, \text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
8*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*hypergeom([3, 3/2-1/2*n],[5
/2-1/2*n],(a-1/x)/(a+1/x))*(-a^2*c*x^2+c)^(1/2)/a^2/(3-n)/x/(1-1/a^2/x^2)^
(1/2)
```

3.745.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int e^{n \coth^{-1}(ax)} \sqrt{c-a^2 c x^2} \, dx = -\frac{c e^{n \coth^{-1}(ax)} \sqrt{1-\frac{1}{a^2x^2}}\,x \Bigg(a\sqrt{1-\frac{1}{a^2x^2}}\,x(n+ax)+2e^{\coth^{-1}(ax)}(-1+n)\,\text{Hypergeometric2F1}\Bigg(1,\frac{1+n}{2},\frac{3+n}{2}}{2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-1/2*(c*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*(a*Sqrt[1 - 1/(a^2*x^2)
]*x*(n + a*x) + 2*E^ArcCoth[a*x]*(-1 + n)*Hypergeometric2F1[1, (1 + n)/2,
(3 + n)/2, E^(2*ArcCoth[a*x])]))/Sqrt[c - a^2*c*x^2]
```

3.745.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6749, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6749}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n+1}{2}}x^3d\frac{1}{x}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{141}$$

$$\frac{8\sqrt{c-a^2cx^2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(8*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*Sqrt[c - a^2*c*x^2
]*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(a + x^(-1))])/(
a^2*(3 - n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

3.745.3.1 Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

3.745.4 Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} \sqrt{-a^2 c\, x^2 + c}dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

3.745.5 Fricas [F]

$$\int e^{n \coth^{-1}(ax)} \sqrt{c - a^2 c x^2}\, dx = \int \sqrt{-a^2 c x^2 + c} \left(\frac{ax + 1}{ax - 1}\right)^{\frac{1}{2}n} dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.745.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-c\left(ax-1\right)\left(ax+1\right)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*acoth(a*x)), x)
```

3.745.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.745.8 Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.745.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

input

```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2),x)
```

output

```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

3.746 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

3.746.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

output
```
2*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1, 1/2-1/2*n],
[3/2-1/2*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/(1-n)/(-a^2*c*x^2+c)^(1/2
)
```

3.746.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2e^{(1+n)\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e^{2\coth^{-1}(ax)}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,(a^2cx+a^2cnx)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*E^((1 + n)*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[1, (1 +
 n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(Sqrt[1 - 1/(a^2*x^2)]*(a^2*c*x + a
^2*c*n*x))
```

3.746.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6749, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{6749}$$

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}xd\frac{1}{x}}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{141}$$

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)
/2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))
])/((1 - n)*Sqrt[c - a^2*c*x^2])
```

3.746.3.1 Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

3.746.4 Maple [F]

$$\int \frac{e^{n \,\mathrm{arccoth}\left(a x \right)}}{\sqrt{-a^{2} c \,x^{2}+c}}d x$$

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

3.746.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 -
c), x)
```

3.746.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.746.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

3.746.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

3.746.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2), x)
```

3.747 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

3.747.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

3.747.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac(-1+n^2)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

3.747.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6738}$$

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\,(1-n^2)\sqrt{c-a^2cx^2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

3.747.3.1 Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

3.747.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{(ax-1)(ax+1)(ax-n)\,\mathrm{e}^{n\,\operatorname{arccoth}(ax)}}{(n^2-1)a(-a^2cx^2+c)^{\frac{3}{2}}}$	49

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

3.747.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-(a^3c^2n^2-a^3c^2)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
 a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

3.747.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.747.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.747.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.747.9 Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{\left(\frac{x}{c\,\left(n^2-1\right)}-\frac{n}{a\,c\,\left(n^2-1\right)}\right)\,{\left(\frac{a\,x+1}{a\,x}\right)}^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x}\right)}^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
-((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2
*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.748 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

3.748.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*
arccoth(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

3.748.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{e^{n\coth^{-1}(ax)}\left(-26n+2n^3+27ax-3an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-\right.}{4ac^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*C
osh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCot
h[a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

3.748.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6739, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{c(9-n^2)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6738}$$

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
- (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2
*c*x^2])
```

3.748.3.1 Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.748.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{(ax-1)(ax+1)(6a^{3}x^{3}-6n\,x^{2}a^{2}+3n^{2}xa-n^{3}-9ax+7n)\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}}{a(n^{4}-10n^{2}+9)(-a^{2}c\,x^{2}+c)^{\frac{5}{2}}}$	84

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcc
oth(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

3.748.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{(6\,a^3x^3-6\,a^2nx^2-n^3+3\,(an^2-3\,a)x+7\,n)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^4-10\,ac^3n^2+(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^4+9\,ac^3-2\,(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c
^3*n^2 + 9*a^3*c^3)*x^2)
```

3.748.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n\,\operatorname{acoth}(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.748.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.748.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.748.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6\,x^3}{c^2\,(n^4-10\,n^2+9)}+\frac{7\,n-n^3}{a^3\,c^2\,(n^4-10\,n^2+9)}+\frac{3\,x\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x^2}{a\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/
(a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)
) - (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x
^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.749 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

3.749.1 Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{e^{n \coth^{-1}(ax)}(n-5ax)}{ac(25-n^2)(c-a^2cx^2)^{5/2}} - \frac{20e^{n \coth^{-1}(ax)}(n-3ax)}{ac^2(9-n^2)(25-n^2)(c-a^2cx^2)^{3/2}} - \frac{120e^{n \coth^{-1}(ax)}(n-ax)}{ac^3(1-n^2)(9-n^2)(25-n^2)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-5*a*x+n)/a/c/(-n^2+25)/(-a^2*c*x^2+c)^(5/2)-20*exp(
n*arccoth(a*x))*(-3*a*x+n)/a/c^2/(n^4-34*n^2+225)/(-a^2*c*x^2+c)^(3/2)-120
*exp(n*arccoth(a*x))*(-a*x+n)/a/c^3/(-n^2+25)/(n^4-10*n^2+9)/(-a^2*c*x^2+c
)^(1/2)
```

3.749.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.80

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{a^2 e^{n \coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3 \left(-\frac{10(225-34n^2+n^4)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{2250n}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{340n^3}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{10n^5}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} + 15(25-26n^2+\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/16*(a^2*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))^(3/2)*x^3*((-10*(225 - 34*
n^2 + n^4))/Sqrt[1 - 1/(a^2*x^2)] + (2250*n)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) -
 (340*n^3)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (10*n^5)/(a*Sqrt[1 - 1/(a^2*x^2)]
*x) + 15*(25 - 26*n^2 + n^4)*Cosh[3*ArcCoth[a*x]] - 45*Cosh[5*ArcCoth[a*x]
] + 50*n^2*Cosh[5*ArcCoth[a*x]] - 5*n^4*Cosh[5*ArcCoth[a*x]] - 125*n*Sinh[
3*ArcCoth[a*x]] + 130*n^3*Sinh[3*ArcCoth[a*x]] - 5*n^5*Sinh[3*ArcCoth[a*x]
] + 9*n*Sinh[5*ArcCoth[a*x]] - 10*n^3*Sinh[5*ArcCoth[a*x]] + n^5*Sinh[5*Ar
cCoth[a*x]]))/(c^2*(-5 + n)*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(5 + n)*(c -
 a^2*c*x^2)^(3/2))
```

3.749.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6739, 6739, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{20\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6739}$$

$$\frac{20\left(\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}dx}{c(9-n^2)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}\right)}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6738}$$

$$\frac{20\left(-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}\right)}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
 + (20*(-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^
(3/2))) - (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt
[c - a^2*c*x^2])))/(c*(25 - n^2))
```

3.749.3.1 Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.749.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{(ax-1)(ax+1)\left(120a^5x^5-120n\,a^4x^4+60a^3n^2x^3-20a^2n^3x^2-300a^3x^3+5a\,n^4x+260n\,x^2a^2-n^5-110n^2xa+30n^3+225ax-149n\right)\mathrm{e}^{n\,a}}{a(n^6-35n^4+259n^2-225)(-a^2c\,x^2+c)^{\frac{7}{2}}}$

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(120*a^5*x^5-120*a^4*n*x^4+60*a^3*n^2*x^3-20*a^2*n^3*x^2-3
00*a^3*x^3+5*a*n^4*x+260*a^2*n*x^2-n^5-110*a*n^2*x+30*n^3+225*a*x-149*n)*e
xp(n*arccoth(a*x))/a/(n^6-35*n^4+259*n^2-225)/(-a^2*c*x^2+c)^(7/2)
```

3.749.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.75

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx =$$

$$-\frac{(120\,a^5x^5-120\,a^4nx^4-n^5+60\,(a^3n^2-5\,a^3)x^3+30\,n^3-20\,(a^2n^3-13\,a^2n)}{ac^4n^6-35\,ac^4n^4+259\,ac^4n^2-(a^7c^4n^6-35\,a^7c^4n^4+259\,a^7c^4n^2-225\,a^7c^4)x^6-225\,ac^4+3\,(a^5c^4n^6-}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
-(120*a^5*x^5 - 120*a^4*n*x^4 - n^5 + 60*(a^3*n^2 - 5*a^3)*x^3 + 30*n^3 -
20*(a^2*n^3 - 13*a^2*n)*x^2 + 5*(a*n^4 - 22*a*n^2 + 45*a)*x - 149*n)*sqrt(
-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^6 - 35*a*c^4*n^4 +
259*a*c^4*n^2 - (a^7*c^4*n^6 - 35*a^7*c^4*n^4 + 259*a^7*c^4*n^2 - 225*a^7*
c^4)*x^6 - 225*a*c^4 + 3*(a^5*c^4*n^6 - 35*a^5*c^4*n^4 + 259*a^5*c^4*n^2 -
 225*a^5*c^4)*x^4 - 3*(a^3*c^4*n^6 - 35*a^3*c^4*n^4 + 259*a^3*c^4*n^2 - 22
5*a^3*c^4)*x^2)
```

3.749.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

3.749.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

3.749.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

3.749.9 Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.74

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=$$
$$-\frac{{\left(\frac{a\,x+1}{a\,x}\right)}^{n/2}\,\left(\frac{120\,x^5}{c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{120\,n\,x^4}{a\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}+\frac{x^3\,\left(60\,n^2-300\right)}{a^2\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{n\,\left(n^4-30\,n^2+149\right)}{a^5\,c^3\,\left(n^6-35\,n^4+259\,n^2-\right.}\right.}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^4}+x^4\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(7/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((120*x^5)/(c^3*(259*n^2 - 35*n^4 + n^6 - 225))
- (120*n*x^4)/(a*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) + (x^3*(60*n^2 - 300)
)/(a^2*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) - (n*(n^4 - 30*n^2 + 149))/(a^5
*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) + (5*x*(n^4 - 22*n^2 + 45))/(a^4*c^3*
(259*n^2 - 35*n^4 + n^6 - 225)) - (20*n*x^2*(n^2 - 13))/(a^3*c^3*(259*n^2
- 35*n^4 + n^6 - 225))))/(((c - a^2*c*x^2)^(1/2)/a^4 + x^4*(c - a^2*c*x^2)
^(1/2) - (2*x^2*(c - a^2*c*x^2)^(1/2))/a^2)*((a*x - 1)/(a*x))^(n/2))
```

3.749. $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

3.750 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

3.750.1 Optimal result

Integrand size = 24, antiderivative size = 239

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\,(49-n^2)\,(c-a^2cx^2)^{7/2}}\\ &\quad-\frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{5/2}}\\ &\quad-\frac{840e^{n\coth^{-1}(ax)}(n-3ax)}{ac^3\,(9-n^2)\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{3/2}}\\ &\quad-\frac{5040e^{n\coth^{-1}(ax)}(n-ax)}{ac^4\,(1-n^2)\,(9-n^2)\,(25-n^2)\,(49-n^2)\,\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
-exp(n*arccoth(a*x))*(-7*a*x+n)/a/c/(-n^2+49)/(-a^2*c*x^2+c)^(7/2)-42*exp(
n*arccoth(a*x))*(-5*a*x+n)/a/c^2/(n^4-74*n^2+1225)/(-a^2*c*x^2+c)^(5/2)-84
0*exp(n*arccoth(a*x))*(-3*a*x+n)/a/c^3/(-n^2+49)/(n^4-34*n^2+225)/(-a^2*c*
x^2+c)^(3/2)-5040*exp(n*arccoth(a*x))*(-a*x+n)/a/c^4/(n^4-74*n^2+1225)/(n^
4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

3.750.2 Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.09

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = \frac{ae^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\left(-\frac{35n}{-1+n^2}+\frac{35ax}{-1+n^2}-\frac{63a\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(3\coth^{-1}(ax)\right)}{-9+n^2}+\frac{35a\sqrt{1-\frac{}{a^2}}}{}\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

output
```
(a*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))*x^2*((-35*n)/(-1 + n^2) + (35*a*x)
/(-1 + n^2) - (63*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]])/(-9 + n^
2) + (35*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[5*ArcCoth[a*x]])/(-25 + n^2) - (7*
a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[7*ArcCoth[a*x]])/(-49 + n^2) + (21*a*n*Sqrt
[1 - 1/(a^2*x^2)]*x*Sinh[3*ArcCoth[a*x]])/(-9 + n^2) - (7*a*n*Sqrt[1 - 1/(
a^2*x^2)]*x*Sinh[5*ArcCoth[a*x]])/(-25 + n^2) + (a*n*Sqrt[1 - 1/(a^2*x^2)]
*x*Sinh[7*ArcCoth[a*x]])/(-49 + n^2)))/(64*c^3*(c - a^2*c*x^2)^(3/2))
```

3.750.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6739, 6739, 6739, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{42\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}dx}{c\left(49-n^2\right)}-\frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{6739}$$

$$\frac{42\left(\frac{20\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}dx}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}\right)}{c\left(49-n^2\right)}-\frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 6739}$$

$$\frac{42\left(\frac{20\left(\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}dx}{c\left(9-n^2\right)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}\right)}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}\right)}{c\left(49-n^2\right)}-\frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 6738}$$

$$\frac{42\left(\frac{20\left(-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}\right)}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}\right)}{c\left(49-n^2\right)}-\frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}$$

input

```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

output

```
-((E^(n*ArcCoth[a*x])*(n - 7*a*x))/(a*c*(49 - n^2)*(c - a^2*c*x^2)^(7/2)))
 + (42*(-((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)
^(5/2))) + (20*(-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2
*c*x^2)^(3/2))) - (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n
^2)*Sqrt[c - a^2*c*x^2])))/(c*(25 - n^2))))/(c*(49 - n^2))
```

3.750. $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

3.750.3.1 Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.750.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.91

method	result
gosper	$\frac{(ax-1)(ax+1)(5040a^7x^7-5040n\,a^6x^6+2520a^5n^2x^5-840a^4n^3x^4-17640a^5x^5+210a^3n^4x^3+15960n\,a^4x^4-42a^2n^5x^2-7140a^3n^2x^3-}{a(n^8-84n^6+1974n^4-12916n^2}$

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(5040*a^7*x^7-5040*a^6*n*x^6+2520*a^5*n^2*x^5-840*a^4*n^3*
x^4-17640*a^5*x^5+210*a^3*n^4*x^3+15960*a^4*n*x^4-42*a^2*n^5*x^2-7140*a^3*
n^2*x^3+7*a*n^6*x+2100*a^2*n^3*x^2-n^7+22050*a^3*x^3-455*a*n^4*x-17178*a^2
*n*x^2+77*n^5+6433*a*n^2*x-1519*n^3-11025*a*x+6483*n)*exp(n*arccoth(a*x))/
a/(n^8-84*n^6+1974*n^4-12916*n^2+11025)/(-a^2*c*x^2+c)^(9/2)
```

3.750.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.90

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx =$$

$$-\frac{(5040\,a^7x^7-5040\,a^6nx^6-n^7+2520\,(a^5n^2-7\,a^5)x^5+}{ac^5n^8-84\,ac^5n^6+1974\,ac^5n^4+(a^9c^5n^8-84\,a^9c^5n^6+1974\,a^9c^5n^4-12916\,a^9c^5n^2+11025\,a^9c^5)x^8-12}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
-(5040*a^7*x^7 - 5040*a^6*n*x^6 - n^7 + 2520*(a^5*n^2 - 7*a^5)*x^5 + 77*n^
5 - 840*(a^4*n^3 - 19*a^4*n)*x^4 + 210*(a^3*n^4 - 34*a^3*n^2 + 105*a^3)*x^
3 - 1519*n^3 - 42*(a^2*n^5 - 50*a^2*n^3 + 409*a^2*n)*x^2 + 7*(a*n^6 - 65*a
*n^4 + 919*a*n^2 - 1575*a)*x + 6483*n)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*
x - 1))^(1/2*n)/(a*c^5*n^8 - 84*a*c^5*n^6 + 1974*a*c^5*n^4 + (a^9*c^5*n^8
- 84*a^9*c^5*n^6 + 1974*a^9*c^5*n^4 - 12916*a^9*c^5*n^2 + 11025*a^9*c^5)*x
^8 - 12916*a*c^5*n^2 - 4*(a^7*c^5*n^8 - 84*a^7*c^5*n^6 + 1974*a^7*c^5*n^4
- 12916*a^7*c^5*n^2 + 11025*a^7*c^5)*x^6 + 11025*a*c^5 + 6*(a^5*c^5*n^8 -
84*a^5*c^5*n^6 + 1974*a^5*c^5*n^4 - 12916*a^5*c^5*n^2 + 11025*a^5*c^5)*x^4
 - 4*(a^3*c^5*n^8 - 84*a^3*c^5*n^6 + 1974*a^3*c^5*n^4 - 12916*a^3*c^5*n^2
+ 11025*a^3*c^5)*x^2)
```

3.750.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Timed out
```

3.750.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

3.750.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

3.750.9 Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.85

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{5040\,x^7}{c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}+\frac{-n^7+77\,n^5-1519\,n^3+6483\,n}{a^7\,c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}-\frac{5040\,n\,x^6}{a\,c^4\,(n^8-84\,n^6+1974\,n^4-12916}\right.}{}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(9/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((5040*x^7)/(c^4*(1974*n^4 - 12916*n^2 - 84*n^6
+ n^8 + 11025)) + (6483*n - 1519*n^3 + 77*n^5 - n^7)/(a^7*c^4*(1974*n^4 -
12916*n^2 - 84*n^6 + n^8 + 11025)) - (5040*n*x^6)/(a*c^4*(1974*n^4 - 12916
*n^2 - 84*n^6 + n^8 + 11025)) + (x^5*(2520*n^2 - 17640))/(a^2*c^4*(1974*n^
4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (x^3*(210*n^4 - 7140*n^2 + 22050)
)/(a^4*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (7*x*(919*n^2
- 65*n^4 + n^6 - 1575))/(a^6*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11
025)) - (840*n*x^4*(n^2 - 19))/(a^3*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n
^8 + 11025)) - (42*n*x^2*(n^4 - 50*n^2 + 409))/(a^5*c^4*(1974*n^4 - 12916*
n^2 - 84*n^6 + n^8 + 11025))))/(((a*x - 1)/(a*x))^(n/2)*((c - a^2*c*x^2)^(
1/2)/a^6 - x^6*(c - a^2*c*x^2)^(1/2) + (3*x^4*(c - a^2*c*x^2)^(1/2))/a^2 -
 (3*x^2*(c - a^2*c*x^2)^(1/2))/a^4))
```

3.751 $\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

3.751.1 Optimal result

Integrand size = 27, antiderivative size = 359

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx &= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\,(c-a^2cx^2)^{3/2}}\\
&+\frac{(2+2n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1-n)(1+n)\,(c-a^2cx^2)^{3/2}}\\
&+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{(c-a^2cx^2)^{3/2}}\\
&-\frac{2n\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a(1-n)\,(c-a^2cx^2)^{3/2}}\end{aligned}$$

output
```
-(2+n)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x
^3/a/(1+n)/(-a^2*c*x^2+c)^(3/2)+(n^2+2*n+2)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^
(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3/a/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)+(1-
1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^4/(-a^2*c
*x^2+c)^(3/2)-2*n*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/
2+1/2*n)*x^3*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/a/(1-n
)/(-a^2*c*x^2+c)^(3/2)
```

3.751.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.37

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx=\frac{\frac{ce^{n\coth^{-1}(ax)}(-1+anx)}{-1+n^2}-\frac{c(-1+a^2x^2)\left(e^{n\coth^{-1}(ax)}(1+n)+\frac{2e^{(1+n)\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2}\right.}{a\sqrt{1-\frac{1}{a^2x^2}}x}\right.}{1+n}}{a^4c^2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
((c*E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(-1 + n^2) - (c*(-1 + a^2*x^2)*(E^(n*
ArcCoth[a*x])*(1 + n) + (2*E^((1 + n)*ArcCoth[a*x])*n*Hypergeometric2F1[1,
 (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(a*Sqrt[1 - 1/(a^2*x^2)]*x)))/
(1 + n))/(a^4*c^2*Sqrt[c - a^2*c*x^2])
```

3.751.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.77, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6746, 6748, 144, 25, 27, 172, 25, 27, 172, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6748}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^2d\frac{1}{x}}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{144}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x\left(-\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}-\int -\frac{\left(an+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x}{a^2}d\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\int \frac{\left(an+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\int \left(an+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 172$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\Bigg(\frac{\frac{a(n+2)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a\int -\frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an(n+1)+\frac{n+2}{x}\right)x}{a}d\frac{1}{x}}{n+1}}{a^2}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1}}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\Bigg(\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an(n+1)+\frac{n+2}{x}\right)x}{a}d\frac{1}{x}}{n+1}+\frac{a(n+2)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}}{a^2}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\Bigg(\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an(n+1)+\frac{n+2}{x}\right)xd\frac{1}{x}}{n+1}+\frac{a(n+2)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}}{a^2}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\Big(}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 172$$

3.751. $\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}dx$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{\frac{a\int n\left(1-n^2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{1-n}-\frac{a\left(n^2+2n+2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{1-n}}{n+1}}{a^2}+\frac{a(n+2)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{\frac{an\left(1-n^2\right)\int\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{1-n}-\frac{a\left(n^2+2n+2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{1-n}}{n+1}}{a^2}+\frac{a(n+2)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 141$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{\frac{2an\left(1-n^2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)^2}-\frac{a\left(n^2+2n+2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{1-n}}{n+1}}{a^2}+a(n\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(((1 - 1/(a^2*x^2))^(3/2)*x^3*(-((1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x) + ((a*(2 + n)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((
-1 + n)/2))/(1 + n) + (-((a*(2 + 2*n + n^2)*(1 - 1/(a*x))^((1 - n)/2)*(1 +
 1/(a*x))^((-1 + n)/2))/(1 - n)) + (2*a*n*(1 - n^2)*(1 - 1/(a*x))^((1 - n)
/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2,
 (a + x^(-1))/(a - x^(-1))])/(1 - n)^2)/(1 + n))/a^2))/(c - a^2*c*x^2)^(3/
2))
```

3.751. $\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

3.751.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

3.751. $\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.751.4 Maple [F]

$$\int \frac{e^{n\ \mathrm{arccoth}(ax)} x^3}{\left(-a^2 c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

3.751.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x
^4 - 2*a^2*c^2*x^2 + c^2), x)
```

3.751.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^3e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.751.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.751.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.751.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

3.752 $\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

3.752.1 Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c(1-n^2)\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-a*x+n)/a^3/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)-2*(1-1/a
/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1, 1/2-1/2*n],[3/2-1/2
*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/a^2/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

3.752.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx =$$

$$-\frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-n+ax)+2e^{\coth^{-1}(ax)}(-1+n)\left(-1+a^2x^2\right)\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\right.\right.}{a^4c(-1+n)(1+n)\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-n + a*x) + 2*E^ArcCoth[
a*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^
(2*ArcCoth[a*x])]))/(a^4*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c
 - a^2*c*x^2]))
```

3.752.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6743, 6746, 6749, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6743}$$

$$-\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{6746}$$

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{a^2c\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{6749}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}xd\frac{1}{x}}{a^2c\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{141}$$

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) -
(2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)
/2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))
])/(a^2*c*(1 - n)*Sqrt[c - a^2*c*x^2])
```

3.752.3.1 Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6743
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^
3*c*(n^2 - 4*(p + 1)^2))), x] - Simp[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2))   Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c
, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^
2 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !Integer
Q[n])
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

3.752. $\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

3.752.4 Maple [F]

$$\int \frac{\mathrm{e}^{n \,\mathrm{arccoth}(a x)} x^{2}}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

3.752.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^{2} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n}}{{\left(-a^{2} c x^{2} + c\right)}^{\frac{3}{2}}}\,{d x}$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x
^4 - 2*a^2*c^2*x^2 + c^2), x)
```

3.752.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^{2} e^{n \operatorname{acoth}{\left(a x \right)}}}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.752.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.752.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.752.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

3.753 $\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

3.753.1 Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(1-anx)}{a^2c(1-n^2)\sqrt{c-a^2cx^2}}$$

output
```
exp(n*arccoth(a*x))*(-a*n*x+1)/a^2/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

3.753.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(-1+anx)}{a^2c(-1+n^2)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(a^2*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

3.753.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {6740}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6740$$

$$\frac{(1-anx)e^{n\coth^{-1}(ax)}}{a^2c\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

3.753.3.1 Defintions of rubi rules used

rule 6740
```
Int[(E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(E^(n*ArcCoth[a*x])/(a^2*c*(n^2 - 1)*Sqrt[c + d*
x^2])), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

3.753.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{(ax-1)(ax+1)(anx-1)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a^{2}(n^{2}-1)(-a^{2}c\,x^{2}+c)^{\frac{3}{2}}}$	49

input
```
int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arccoth(a*x))/a^2/(n^2-1)/(-a^2*c*x^2+c)^
(3/2)
```

3.753.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(anx-1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2n^2-a^2c^2-(a^4c^2n^2-a^4c^2)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

output
```
sqrt(-a^2*c*x^2 + c)*(a*n*x - 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*n^
2 - a^2*c^2 - (a^4*c^2*n^2 - a^4*c^2)*x^2)
```

3.753.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{xe^{n\operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.753.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.753.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.753.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(\frac{1}{a^2\,c\,(n^2-1)}-\frac{n\,x}{a\,c\,(n^2-1)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
-((1/(a^2*c*(n^2 - 1)) - (n*x)/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/(
(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.754 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

3.754.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

3.754.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac\,(-1+n^2)\,\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

3.754.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 6738}$$
$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

3.754.3.1 Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

3.754.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{(ax-1)(ax+1)(ax-n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(n^2-1)a(-a^2cx^2+c)^{\frac{3}{2}}}$	49

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

3.754.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-(a^3c^2n^2-a^3c^2)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
 a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

3.754.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \int \frac{e^{n\operatorname{acoth}(ax)}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.754.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.754.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

3.754.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{\left(\frac{x}{c\,\left(n^2-1\right)}-\frac{n}{a\,c\,\left(n^2-1\right)}\right)\,{\left(\frac{a\,x+1}{a\,x}\right)}^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x}\right)}^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
-((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2
*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.755 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

3.755.1 Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1-n^2)\left(c-a^2cx^2\right)^{3/2}} - \frac{2^{\frac{1+n}{2}}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)\left(c-a^2cx^2\right)^{3/2}}$$

output

```
-a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3
/(1+n)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*
(1+1/a/x)^(-1/2+1/2*n)*x^3/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)-2^(1/2+1/2*n)*a^3
*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*x^3*hypergeom([1/2-1/2*n, 1/2-1
/2*n],[3/2-1/2*n],1/2*(a-1/x)/a)/(1-n)/(-a^2*c*x^2+c)^(3/2)
```

3.755.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.46

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-1+anx)-2e^{\coth^{-1}(ax)}(-1+n)\left(-1+a^2x^2\right)\text{Hypergeo}\right.}{ac(-1+n)(1+n)\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*n*x) - 2*E^ArcCoth[
a*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E
^(2*ArcCoth[a*x])]))/(a*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c
- a^2*c*x^2])
```

3.755.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6746, 6749, 100, 27, 88, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6749}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}}{x^2}d\frac{1}{x}}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^3\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a^3\int\frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an+\frac{n+1}{x}\right)}{a^2}d\frac{1}{x}}{n+1}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^3\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a\int\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an+\frac{n+1}{x}\right)d\frac{1}{x}}{n+1}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 88$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^3\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a\left(a(n+1)\int\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}d\frac{1}{x}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{1-n}\right)}{n+1}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 79$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^3\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a\left(\frac{a^2\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{1-n}-\frac{a^2 2^{\frac{n+1}{2}}(n+1)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2}\right.}{1-n}\right.}{n+1}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-(((1 - 1/(a^2*x^2))^(3/2)*x^3*((a^3*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*
x))^((-1 + n)/2))/(1 + n) - (a*((a^2*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x
))^((-1 + n)/2))/(1 - n) - (2^((1 + n)/2)*a^2*(1 + n)*(1 - 1/(a*x))^((1 -
n)/2)*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(2*a
)])/(1 - n)))/(1 + n)))/(c - a^2*c*x^2)^(3/2))
```

3.755. $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

3.755.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

3.755. $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

3.755.4 Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

3.755.5 Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x (c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^{2}cx^{2}+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^5 -
 2*a^2*c^2*x^3 + c^2*x), x)
```

3.755.6 Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x (c-a^2cx^2)^{3/2}}\,dx = \int \frac{e^{n \operatorname{acoth}{(ax)}}}{x \left(-c \left(ax-1\right) \left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.755.7 Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x \left(c-a^2 c x^2\right)^{3/2}} \, dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} x} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

3.755.8 Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x \left(c-a^2 c x^2\right)^{3/2}} \, dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} x} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

3.755.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x \left(c-a^2 c x^2\right)^{3/2}} \, dx=\int \frac{{\mathrm{e}}^{n\,\mathrm{acoth}\left(a\,x\right)}}{x\,{\left(c-a^2\,c\,x^2\right)}^{3/2}} \,d x$$

input
```
int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)),x)
```

output
```
int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)), x)
```

3.756 $\int \frac{e^{n\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

3.756.1 Optimal result

Integrand size = 27, antiderivative size = 463

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx = &-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}\\
&-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}\\
&+\frac{\left(15+6n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1-n)(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}\\
&-\frac{\left(18+7n-2n^2-n^3\right)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{\left(9-10n^2+n^4\right)\left(c-a^2cx^2\right)^{5/2}}\\
&-\frac{2\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)\left(c-a^2cx^2\right)^{5/2}}\end{aligned}$$

output
```
-(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+
n)/(-a^2*c*x^2+c)^(5/2)-(6+n)*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(
1+1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+(n^2+6*n+15)*(1
-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3
*n^2+n+3)/(-a^2*c*x^2+c)^(5/2)-(-n^3-2*n^2+7*n+18)*(1-1/a^2/x^2)^(5/2)*(1-
1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c
)^(5/2)-2*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)
*x^5*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/(1-n)/(-a^2*c*
x^2+c)^(5/2)
```

3.756.2 Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.43

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(-1+a^2x^2)\Bigg(\frac{8e^{n\coth^{-1}(ax)}(n-ax)}{-1+n^2}+\frac{e^{n\coth^{-1}(ax)}\big(26n-2n^3-27ax+3an^2x+2n(-1+n^2)\cosh(2\coth^{-1}(}{9-10n^2+n^4}}{4}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
((-1 + a^2*x^2)*((8*E^(n*ArcCoth[a*x])*(n - a*x))/(-1 + n^2) + (E^(n*ArcCo
th[a*x])*(26*n - 2*n^3 - 27*a*x + 3*a*n^2*x + 2*n*(-1 + n^2)*Cosh[2*ArcCot
h[a*x]] - 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))/(9
 - 10*n^2 + n^4) - (8*a*E^((1 + n)*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*H
ypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(1 + n)))/(
4*a^5*c*(c - a^2*c*x^2)^(3/2))
```

3.756.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.78, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {6746, 6749, 144, 25, 27, 172, 25, 27, 172, 27, 172, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 6749}$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}xd\frac{1}{x}}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 144}$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{n+3}-\frac{a\int -\frac{\left(a(n+3)+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}x}{a^2}d\frac{1}{x}}{n+3}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{a\int \frac{\left(a(n+3)+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}x}{a^2}d\frac{1}{x}}{n+3}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\int \left(a(n+3)+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}xd\frac{1}{x}}{a(n+3)}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 172$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{a(n+6)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{n+1}-\frac{a\int -\frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(n+1)(n+3)+\frac{2(n+6)}{x}\right)x}{a}d\frac{1}{x}}{n+1}}{a(n+3)}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{n}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(n+1)(n+3)+\frac{2(n+6)}{x}\right)x}{a}d\frac{1}{x}}{n+1}+\frac{a(n+6)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}}{a(n+3)}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\right.}{n+}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(n+1)(n+3)+\frac{2(n+6)}{x}\right)xd\frac{1}{x}}{n+1}+\frac{a(n+6)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}}{a(n+3)}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\right.}{n+3}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 172$$

3.756. $\int \frac{e^{n\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}dx$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{a\int\frac{\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(1-n)(n+1)(n+3)-\frac{n^2+6n+15}{x}\right)x}{a}d\frac{1}{x}}{1-n}-\frac{a\left(n^2+6n+15\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{1-n}}{n+1}+\frac{a(n+6)\left(\frac{1}{ax}\right.}{}}{a(n+3)}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow 27$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{\int\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(1-n)(n+1)(n+3)-\frac{n^2+6n+15}{x}\right)xd\frac{1}{x}}{1-n}-\frac{a\left(n^2+6n+15\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{1-n}}{n+1}+\frac{a(n+6)\left(\frac{1}{ax}+1\right.}{}}{a(n+3)}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow 172$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{\frac{a\int\left(n^4-10n^2+9\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{3-n}+\frac{a\left(-n^3-2n^2+7n+18\right)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{3-n}}{1-n}-\frac{a\left(n^2+6n+15\right)\left(1-\frac{1}{ax}\right)^{\frac{1-}{2}}}{1-n}}{n+1}}{a(n+3)}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow 27$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{\frac{a\left(n^4-10n^2+9\right)\int\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{3-n}+\frac{a\left(-n^3-2n^2+7n+18\right)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{3-n}}{1-n}-\frac{a\left(n^2+6n+15\right)\left(1-\frac{1}{ax}\right)^{\frac{1-}{2}}}{1-n}}{n+1}}{a(n+3)}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow 141$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{\frac{2a\left(n^4-10n^2+9\right)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)(3-n)}+\frac{a\left(-n^3-2n^2+7n+18\right)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right.}{3-n}}{1-n}}{n+1}}{a(n+3)}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

3.756. $\int\frac{e^{n\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

input

```
Int[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

output

```
-(((1 - 1/(a^2*x^2))^(5/2)*x^5*(((1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^
((-3 + n)/2))/(3 + n) + ((a*(6 + n)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x
))^((-3 + n)/2))/(1 + n) + (-((a*(15 + 6*n + n^2)*(1 - 1/(a*x))^((1 - n)/2
)*(1 + 1/(a*x))^((-3 + n)/2))/(1 - n)) + ((a*(18 + 7*n - 2*n^2 - n^3)*(1 -
 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2))/(3 - n) + (2*a*(9 - 10*n
^2 + n^4)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeomet
ric2F1[1, (-1 + n)/2, (1 + n)/2, (a + x^(-1))/(a - x^(-1))])/((1 - n)*(3 -
 n)))/(1 - n))/(1 + n))/(a*(3 + n))))/(c - a^2*c*x^2)^(5/2))
```

3.756.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 144

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

3.756. $\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx$

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

3.756.4 Maple [F]

$$\int \frac{\mathrm{e}^{n \,\mathrm{arccoth}\left(a x \right)} x^{4}}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

3.756.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*
x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3), x)
```

3.756.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^4e^{n\operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**4/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.756.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.756.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.756.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{x^4\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

input
```
int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2), x)
```

3.757 $\int \frac{e^{n \coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

3.757.1 Optimal result

Integrand size = 27, antiderivative size = 330

$$\begin{aligned}\int \frac{e^{n \coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx &= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} \\ &- \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}} \\ &+ \frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)(1-n^2)\left(c-a^2cx^2\right)^{5/2}} \\ &- \frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\left(c-a^2cx^2\right)^{5/2}}\end{aligned}$$

output
```
-a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(
3+n)/(-a^2*c*x^2+c)^(5/2)-3*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(
1+1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a*(1-1/a^2/x^
2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)
/(-a^2*c*x^2+c)^(5/2)-6*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*(1+1/a
/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

3.757.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.33

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx =$$

$$-\frac{e^{n\coth^{-1}(ax)}\Big(3(10-2n^2-9anx+an^3x)-6(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+an(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\ \mathrm{co}}{4a^4c^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/4*(E^(n*ArcCoth[a*x])*(3*(10 - 2*n^2 - 9*a*n*x + a*n^3*x) - 6*(-1 + n^2
)*Cosh[2*ArcCoth[a*x]] + a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*Arc
Coth[a*x]]))/(a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

3.757.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6746, 6749, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 6746$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \ 6749$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}d\frac{1}{x}}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \ 55$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\int\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}d\frac{1}{x}}{n+3}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\left(\frac{2\int\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}d\frac{1}{x}}{n+1}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}\right)}{n+3}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\left(\frac{2\left(-\frac{\int\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}d\frac{1}{x}}{1-n}-\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{1-n}\right)}{n+1}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}\right)}{n+3}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-}{2}}}{}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}+\frac{3\left(\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}+\frac{2\left(\frac{a\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{(1-n)(3-n)}-\frac{a\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{a\right.}{1-n}\right.}{n+1}\right.}{n+3}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

input

```
Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

3.757. $\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

output

```
-((((3*((2*(-((a*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2))/(1
- n)) + (a*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2))/((1 - n)*
(3 - n))))/(1 + n) + (a*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)
/2))/(1 + n)))/(3 + n) + (a*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3
+ n)/2))/(3 + n))*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(c - a^2*c*x^2)^(5/2))
```

3.757.3.1 Defintions of rubi rules used

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6746

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

3.757. $\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

3.757.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

method	result	size
gosper	$-\frac{(ax-1)(ax+1)\left(a^{3} n^{3} x^{3}-7 a^{3} x^{3} n-3 a^{2} n^{2} x^{2}+9 a^{2} x^{2}+6 a n x-6\right) {\mathrm e}^{n \,\operatorname{arccoth}(a x)}}{a^{4} \left(n^{4}-10 n^{2}+9\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}$	93

input
```
int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)*(a*x+1)*(a^3*n^3*x^3-7*a^3*n*x^3-3*a^2*n^2*x^2+9*a^2*x^2+6*a*n*x-
6)*exp(n*arccoth(a*x))/a^4/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

3.757.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{\left(c-a^2 c x^2\right)^{5/2}}\, dx = \frac{\sqrt{-a^{2} c x^{2} + c}{\left({\left(a^{3} n^{3} - 7 \, a^{3} n\right)} x^{3} + 6 \, a n x - 3 \, {\left(a^{2} n^{2} - 3 \, a^{2}\right)} x^{2} - 6\right)} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2}}}{a^{4} c^{3} n^{4} - 10 \, a^{4} c^{3} n^{2} + 9 \, a^{4} c^{3} + {\left(a^{8} c^{3} n^{4} - 10 \, a^{8} c^{3} n^{2} + 9 \, a^{8} c^{3}\right)} x^{4} - 2 \, {\left(a^{6} c^{3} n^{4} - 10 \, a^{6} c^{3}}$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
sqrt(-a^2*c*x^2 + c)*((a^3*n^3 - 7*a^3*n)*x^3 + 6*a*n*x - 3*(a^2*n^2 - 3*a
^2)*x^2 - 6)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^3*n^4 - 10*a^4*c^3*n^2 +
 9*a^4*c^3 + (a^8*c^3*n^4 - 10*a^8*c^3*n^2 + 9*a^8*c^3)*x^4 - 2*(a^6*c^3*n
^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^2)
```

3.757.6 Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{\left(c-a^2 c x^2\right)^{5/2}}\, dx = \int \frac{x^{3} e^{n \operatorname{acoth}{\left(a x \right)}}}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.757.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.757.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.757.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6}{a^6\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{6\,n\,x}{a^5\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{x^2\,\left(3\,n^2-9\right)}{a^4\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{n\,x^3\,\left(n^2-7\right)}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*(6/(a^6*c^2*(n^4 - 10*n^2 + 9)) - (6*n*x)/(a^5*c
^2*(n^4 - 10*n^2 + 9)) + (x^2*(3*n^2 - 9))/(a^4*c^2*(n^4 - 10*n^2 + 9)) -
(n*x^3*(n^2 - 7))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a
^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.757. $\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

3.758 $\int \frac{e^{n \coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

3.758.1 Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{e^{n \coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{e^{n \coth^{-1}(ax)}(n-3ax)}{a^3c(9-n^2)\left(c-a^2cx^2\right)^{3/2}}+\frac{e^{n \coth^{-1}(ax)}(3-n^2)(n-ax)}{a^3c^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-3*a*x+n)/a^3/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+exp(n*
arccoth(a*x))*(-n^2+3)*(-a*x+n)/a^3/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2
)
```

3.758.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int \frac{e^{n \coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n \coth^{-1}(ax)}\left(10n-2n^3-9ax+an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-1+\right.}{4a^3c^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(10*n - 2*n^3 - 9*a*x + a*n^2*x - 2*n*(-1 + n^2)*Cosh[
2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*
x]]))/(4*a^3*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

3.758.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6743, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6743}$$

$$-\frac{(3-n^2)\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{a^2c(9-n^2)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6738}$$

$$\frac{(3-n^2)(n-ax)e^{n\coth^{-1}(ax)}}{a^3c^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))
) + (E^(n*ArcCoth[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(1 - n^2)*(9 - n^2)*
Sqrt[c - a^2*c*x^2])
```

3.758.3.1 Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6743
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^
3*c*(n^2 - 4*(p + 1)^2))), x] - Simp[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2))   Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c
, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^
2 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !Integer
Q[n])
```

3.758.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{(ax-1)(ax+1)\left(a^3n^2x^3-a^2n^3x^2-3a^3x^3+3n\,x^2a^2+2n^2xa-2n\right)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{\left(n^4-10n^2+9\right)a^3\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$	96

input
```
int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(a^3*n^2*x^3-a^2*n^3*x^2-3*a^3*x^3+3*a^2*n*x^2+2*a*n^2*x-2
*n)*exp(n*arccoth(a*x))/(n^4-10*n^2+9)/a^3/(-a^2*c*x^2+c)^(5/2)
```

3.758.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx =$$

$$-\frac{\sqrt{-a^2cx^2+c}\left(2\,an^2x+(a^3n^2-3\,a^3)x^3-(a^2n^3-3\,a^2n)x^2-2\,n\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3+(a^7c^3n^4-10\,a^7c^3n^2+9\,a^7c^3)x^4-2\,(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
-sqrt(-a^2*c*x^2 + c)*(2*a*n^2*x + (a^3*n^2 - 3*a^3)*x^3 - (a^2*n^3 - 3*a^
2*n)*x^2 - 2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*n^4 - 10*a^3*c^3*n^
2 + 9*a^3*c^3 + (a^7*c^3*n^4 - 10*a^7*c^3*n^2 + 9*a^7*c^3)*x^4 - 2*(a^5*c^
3*n^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^2)
```

3.758.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.758.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.758.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.758.9 Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.72

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,n}{a^5\,c^2\,(n^4-10\,n^2+9)} - \frac{x^3\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)} - \frac{2\,n^2\,x}{a^4\,c^2\,(n^4-10\,n^2+9)} + \frac{n\,x^2\,(n^2-3)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2} - x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input

```
int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

output

```
(((a*x + 1)/(a*x))^(n/2)*((2*n)/(a^5*c^2*(n^4 - 10*n^2 + 9)) - (x^3*(n^2 -
 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n^2*x)/(a^4*c^2*(n^4 - 10*n^2 + 9))
 + (n*x^2*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2
)/a^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.759 $\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

3.759.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} + \frac{2e^{n\coth^{-1}(ax)}n(n-ax)}{a^2c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

output

```
exp(n*arccoth(a*x))*(-a*n*x+3)/a^2/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+2*exp(n
*arccoth(a*x))*n*(-a*x+n)/a^2/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

3.759.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(6+2n^2-9anx+an^3x+6(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)-an(-1+n\right.}{4a^2c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

input

```
Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

output

```
(E^(n*ArcCoth[a*x])*(6 + 2*n^2 - 9*a*n*x + a*n^3*x + 6*(-1 + n^2)*Cosh[2*A
rcCoth[a*x]] - a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]
))/(4*a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

3.759.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6741, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6741}$$

$$\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c(9-n^2)(c-a^2cx^2)^{3/2}}-\frac{2n\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{ac(9-n^2)}$$

$$\Big\downarrow \text{6738}$$

$$\frac{2n(n-ax)e^{n\coth^{-1}(ax)}}{a^2c^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}+\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c(9-n^2)(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(3 - a*n*x))/(a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) +
 (2*E^(n*ArcCoth[a*x])*n*(n - a*x))/(a^2*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c -
a^2*c*x^2])
```

3.759.3.1 Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6741
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(2*(p + 1) + a*n*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^2*
c*(n^2 - 4*(p + 1)^2))), x] - Simp[n*((2*p + 3)/(a*c*(n^2 - 4*(p + 1)^2)))
  Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
 x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[p, -3/2] &
& NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.759.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{(ax-1)(ax+1)(2a^3x^3n-2a^2n^2x^2+a\,n^3x-3anx-n^2+3)\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}}{a^2(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	86

input
```
int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)*(a*x+1)*(2*a^3*n*x^3-2*a^2*n^2*x^2+a*n^3*x-3*a*n*x-n^2+3)*exp(n*a
rccoth(a*x))/a^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

3.759.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3nx^3-2\,a^2n^2x^2-n^2+(an^3-3\,an)x+3)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2c^3n^4-10\,a^2c^3n^2+9\,a^2c^3+(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3)x^4-2\,(a^4c^3n^4-10\,a^4c^3}$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

output
```
(2*a^3*n*x^3 - 2*a^2*n^2*x^2 - n^2 + (a*n^3 - 3*a*n)*x + 3)*sqrt(-a^2*c*x^
2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^3*n^4 - 10*a^2*c^3*n^2 + 9*a^2
*c^3 + (a^6*c^3*n^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^4 - 2*(a^4*c^3*n^4 - 1
0*a^4*c^3*n^2 + 9*a^4*c^3)*x^2)
```

3.759.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{xe^{n\,\operatorname{acoth}(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.759.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.759.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.759.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.81

$$\int \frac{e^{n\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx=$$
$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n^2-3}{a^4\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{2\,n^2\,x^2}{a^2\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{2\,n\,x^3}{a\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{n\,x\,\left(n^2-3\right)}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((n^2 - 3)/(a^4*c^2*(n^4 - 10*n^2 + 9)) + (2*n^2
*x^2)/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n*x^3)/(a*c^2*(n^4 - 10*n^2 + 9))
- (n*x*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a
^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.760 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}} \, dx$

3.760.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}} \, dx=-\frac{e^{n \coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{6e^{n \coth^{-1}(ax)}(n-ax)}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*
arccoth(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

3.760.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}} \, dx=\frac{e^{n \coth^{-1}(ax)}\left(-26n+2n^3+27ax-3an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-\right.}{4ac^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*C
osh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCot
h[a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

3.760.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6739, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}dx}{c\left(9-n^2\right)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6738}$$

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
- (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2
*c*x^2])
```

3.760.3.1 Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

3.760.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{(ax-1)(ax+1)(6a^3x^3-6n\,x^2a^2+3n^2xa-n^3-9ax+7n)\,\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	84

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcc
oth(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

3.760.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx =$$

$$-\frac{(6\,a^3x^3-6\,a^2nx^2-n^3+3\,(an^2-3\,a)x+7\,n)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^4-10\,ac^3n^2+(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^4+9\,ac^3-2\,(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c
^3*n^2 + 9*a^3*c^3)*x^2)
```

3.760.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.760.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.760.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

3.760.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=$$
$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6\,x^3}{c^2\,(n^4-10\,n^2+9)}+\frac{7\,n-n^3}{a^3\,c^2\,(n^4-10\,n^2+9)}+\frac{3\,x\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x^2}{a\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/
(a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)
) - (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x
^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.761 $\int \frac{e^{n \coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

3.761.1 Optimal result

Integrand size = 27, antiderivative size = 944

$$
\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = &-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&-\frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}} \\
&+\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(1-n^2\right)\left(c-a^2cx^2\right)^{5/2}} \\
&-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{\left(9-10n^2+n^4\right)\left(c-a^2cx^2\right)^{5/2}} \\
&+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&+\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}} \\
&-\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5}{(3+n)\left(1-n^2\right)\left(c-a^2cx^2\right)^{5/2}} \\
&-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x^5}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}} \\
&+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} \\
&-\frac{2^{\frac{5+n}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-n),\frac{1}{2}(-3-n),\frac{1}{2}(-1-n),\frac{a-\frac{1}{x}}{2a}\right)}{(3+n)\left(c-a^2cx^2\right)^{5/2}}
\end{aligned}
$$

output
```
-a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5
/(3+n)/(-a^2*c*x^2+c)^(5/2)-3*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*
n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a^5*(1-1/
a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^
2+n+3)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n
)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)+4*a^5*(1-
1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(3+n)/(
-a^2*c*x^2+c)^(5/2)+8*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/
a/x)^(-1/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)-8*a^5*(1-1/a^2/x^2)
^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)/(
-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/
a/x)^(1/2+1/2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*
(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c
)^(5/2)+4*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(3/2+1/
2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-2^(5/2+1/2*n)*a^5*(1-1/a^2/x^2)^(5/2)*
(1-1/a/x)^(-3/2-1/2*n)*x^5*hypergeom([-3/2-1/2*n, -3/2-1/2*n],[-1/2-1/2*n]
,1/2*(a-1/x)/a)/(3+n)/(-a^2*c*x^2+c)^(5/2)
```

3.761.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.23

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(42-2n^2-45anx+5an^3x)+6a(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}x\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(42 - 2*n^2 - 45*a*n*x + 5*
a*n^3*x) + 6*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[2*ArcCoth[a*x]] - n
*(-1 + n^2)*(-1 + a^2*x^2)*Cosh[3*ArcCoth[a*x]]) - 8*E^((1 + n)*ArcCoth[a*
x])*(9 - 9*n - n^2 + n^3)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (
3 + n)/2, -E^(2*ArcCoth[a*x])])/(4*a*c^2*(-1 + n)*(1 + n)*(-9 + n^2)*Sqrt[
1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])
```

3.761.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 624, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6749, 137, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{6749}$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}}{x^4}d\frac{1}{x}}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{137}$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-4a^4\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}+1}+6a^4\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}+2}-4a^4\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{a^5 2^{\frac{n+5}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-3),\frac{1}{2}(-n-3),\frac{1}{2}(-n-1),\frac{a-\frac{1}{x}}{2a}\right)}{n+3}+\frac{3a^5\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n}}{n^2+4n+3}\right.}{}$$

input
```
Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

3.761. $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

output

```
-(((1 - 1/(a^2*x^2))^(5/2)*x^5*((a^5*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*
x))^((-3 + n)/2))/(3 + n) + (3*a^5*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x)
)^((-3 + n)/2))/(3 + 4*n + n^2) - (6*a^5*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/
(a*x))^((-3 + n)/2))/((3 + n)*(1 - n^2)) + (6*a^5*(1 - 1/(a*x))^((3 - n)/2
)*(1 + 1/(a*x))^((-3 + n)/2))/(9 - 10*n^2 + n^4) - (4*a^5*(1 - 1/(a*x))^((
-3 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2))/(3 + n) - (8*a^5*(1 - 1/(a*x))^((-1
 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2))/(3 + 4*n + n^2) + (8*a^5*(1 - 1/(a*x)
)^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2))/((3 + n)*(1 - n^2)) + (6*a^5*(1
- 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((1 + n)/2))/(3 + n) + (6*a^5*(1 - 1
/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2))/(3 + 4*n + n^2) - (4*a^5*(
1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((3 + n)/2))/(3 + n) + (2^((5 + n)
/2)*a^5*(1 - 1/(a*x))^((-3 - n)/2)*Hypergeometric2F1[(-3 - n)/2, (-3 - n)/
2, (-1 - n)/2, (a - x^(-1))/(2*a)])/(3 + n)))/(c - a^2*c*x^2)^(5/2))
```

3.761.3.1 Defintions of rubi rules used

rule 137

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[m, 0] &&
ILtQ[n, 0]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

3.761. $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

3.761.4 Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

3.761.5 Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x (c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^7
- 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 - c^3*x), x)
```

3.761.6 Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x (c-a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n \operatorname{acoth}{(ax)}}}{x \left(-c \left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(5/2)), x)
```

3.761.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

3.761.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

3.761.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x\,\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)),x)
```

output
```
int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)), x)
```

3.762 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.762.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}x\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-1-2p,\frac{1}{2}(n\right.}{1+2p}$$

output
```
((a-1/x)/(a+1/x))^(1/2*n-p)*(1-1/a/x)^(-1/2*n+p)*(1+1/a/x)^(1+1/2*n+p)*x*(
-a^2*c*x^2+c)^p*hypergeom([-1-2*p, 1/2*n-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((
1-1/a^2/x^2)^p)
```

3.762.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{e^{(-2+n)\coth^{-1}(ax)}\left(-1+e^{2\coth^{-1}(ax)}\right)\left(-1+a^2x^2\right)\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(1,-\frac{n}{2}-p,2-\frac{n}{2}+\right.}{a(n-2(1+p))}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
-((E^((-2 + n)*ArcCoth[a*x])*(-1 + E^(2*ArcCoth[a*x]))*(-1 + a^2*x^2)*(c -
 a^2*c*x^2)^p*Hypergeometric2F1[1, -1/2*n - p, 2 - n/2 + p, E^(-2*ArcCoth[
a*x])])/(a*(n - 2*(1 + p))))
```

3.762.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^p e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n}{2}+p}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}\text{Hypergeometric2F1}\left(-2p-1,\frac{1}{2}(n-2p),\right.}{2p+1}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 - 1/(a*x))^(-1/2*n + p)*(1 +
 1/(a*x))^(1 + n/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, (n
 - 2*p)/2, -2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

3.762.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

3.762.4 Maple [F]

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)} \left(-a^{2} c\,x^{2}+c\right)^{p} dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

3.762.5 Fricas [F]

$$\int e^{n \coth^{-1}(ax)} \left(c-a^2 c x^2\right)^p \, dx = \int \left(-a^{2} c x^{2} + c\right)^{p} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n} \,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.762.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-c(ax-1)\,(ax+1)\right)^pe^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p*exp(n*acoth(a*x)), x)
```

3.762.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.762.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.762.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

output
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p, x)
```

3.763 $\int e^{2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.763.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+2p}x(c-a^2cx^2)^p}{1+2p}$$

output

```
(1+1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

3.763.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{e^{2p\coth^{-1}(ax)}(1+ax)\left(c-a^2cx^2\right)^p}{a+2ap}$$

input

```
Integrate[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output

```
(E^(2*p*ArcCoth[a*x])*(1 + a*x)*(c - a^2*c*x^2)^p)/(a + 2*a*p)
```

3.763.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6746, 6750, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^p e^{2p\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{ 6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int \left(1+\frac{1}{ax}\right)^{2p}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

input

```
Int[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output

```
((1 + 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))
^p)
```

3.763.3.1 Defintions of rubi rules used

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6746

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

3.763.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

method	result
gosper	$\frac{(ax+1)\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{p}}{a(1+2p)}$
parallelrisch	$-\frac{-\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}x\left(-a^{2}c\,x^{2}+c\right)^{p}a-\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{p}}{a(1+2p)}$
risch	$\frac{(ax+1)(ax+1)^{2p}(ax-1)^{-p}c^{p}(ax-1)^{p}\mathrm{e}^{-\frac{ip\pi\left(\mathrm{csgn}(i(ax-1)(ax+1))^{3}-\mathrm{csgn}(i(ax-1))\,\mathrm{csgn}(i(ax-1)(ax+1))^{2}-\mathrm{csgn}(i(ax-1)(ax+1))^{2}\,\mathrm{csg}\right.}{}}}{}$

input
```
int(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x,method=_RETURNVERBOSE)
```

output
```
(a*x+1)/a/(1+2*p)*exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p
```

3.763.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{p}\,dx=\frac{(ax+1)(-a^{2}cx^{2}+c)^{p}\left(\frac{ax+1}{ax-1}\right)^{p}}{2\,ap+a}$$

input
```
integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
(a*x + 1)*(-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p/(2*a*p + a)
```

3.763.6 Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \begin{cases} -\frac{ix}{\sqrt{c}} & \text{for } a=0\wedge p=-\frac{1}{2} \\ c^p x e^{i\pi p} & \text{for } a=0 \\ \int \frac{e^{-\operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx & \text{for } p=-\frac{1}{2} \\ \frac{ax(-a^2cx^2+c)^p e^{2p\operatorname{acoth}(ax)}}{2ap+a}+\frac{(-a^2cx^2+c)^p e^{2p\operatorname{acoth}(ax)}}{2ap+a} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*p*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

output
```
Piecewise((-I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(I*pi*p), Eq(
a, 0)), (Integral(exp(-acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p,
 -1/2)), (a*x*(-a**2*c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a) + (-a*
*2*c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a), True))
```

3.763.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{(a(-c)^p x+(-c)^p)(ax+1)^{2p}}{a(2p+1)}$$

input
```
integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
(a*(-c)^p*x + (-c)^p)*(a*x + 1)^(2*p)/(a*(2*p + 1))
```

3.763.8 Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.763.9 Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)\,\left(\frac{a\,x+1}{a\,x}\right)^p}{a\,\left(2\,p+1\right)\,\left(\frac{a\,x-1}{a\,x}\right)^p}$$

input
```
int(exp(2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

output
```
((c - a^2*c*x^2)^p*(a*x + 1)*((a*x + 1)/(a*x))^p)/(a*(2*p + 1)*((a*x - 1)/
(a*x))^p)
```

3.764 $\int e^{-2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.764.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{1+2p}x\left(c-a^2cx^2\right)^p}{1+2p}$$

output
```
(1-1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

3.764.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{e^{-2p\coth^{-1}(ax)}(-1+ax)\left(c-a^2cx^2\right)^p}{a+2ap}$$

input
```
Integrate[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
((-1 + a*x)*(c - a^2*c*x^2)^p)/(E^(2*p*ArcCoth[a*x])*(a + 2*a*p))
```

3.764.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6746, 6750, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^p e^{-2p\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \color{blue}{6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}(c-a^2cx^2)^p\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \color{blue}{6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)(c-a^2cx^2)^p\int \left(1-\frac{1}{ax}\right)^{2p}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \color{blue}{48}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}(c-a^2cx^2)^p}{2p+1}$$

input
```
Int[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
((1 - 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))
^p)
```

3.764.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

3.764.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{(ax-1)\left(-a^{2}c\,x^{2}+c\right)^{p}\mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}}{a(1+2p)}$
parallelrisch	$\frac{\left(x\left(-a^{2}c\,x^{2}+c\right)^{p}a-\left(-a^{2}c\,x^{2}+c\right)^{p}\right)\mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}}{a(1+2p)}$
risch	$\frac{(ax-1)\left((ax-1)^{p}\right)^{2}c^{p}\mathrm{e}^{-\frac{ip\pi\left(\mathrm{csgn}(i(ax-1)(ax+1))^{3}-\mathrm{csgn}(i(ax-1))\,\mathrm{csgn}(i(ax-1)(ax+1))^{2}-\mathrm{csgn}(i(ax-1)(ax+1))^{2}\,\mathrm{csgn}(i(ax+1))+\mathrm{csgn}(i\right.}{}}}{}$

input
```
int((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/a/(1+2*p)*(-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x))
```

3.764.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{p}\,dx=\frac{(ax-1)(-a^{2}cx^{2}+c)^{p}}{(2\,ap+a)\left(\frac{ax+1}{ax-1}\right)^{p}}$$

input
```
integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

output
```
(a*x - 1)*(-a^2*c*x^2 + c)^p/((2*a*p + a)*((a*x + 1)/(a*x - 1))^p)
```

3.764.6 Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \begin{cases} \frac{ix}{\sqrt{c}} & \text{for } a=0 \wedge p=-\frac{1}{2} \\ c^p x e^{-i\pi p} & \text{for } a=0 \\ \int \frac{e^{\operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx & \text{for } p=-\frac{1}{2} \\ \frac{ax(-a^2cx^2+c)^p}{2ape^{2p\operatorname{acoth}(ax)}+ae^{2p\operatorname{acoth}(ax)}} - \frac{(-a^2cx^2+c)^p}{2ape^{2p\operatorname{acoth}(ax)}+ae^{2p\operatorname{acoth}(ax)}} & \text{otherwise} \end{cases}$$

input
```
integrate((-a**2*c*x**2+c)**p/exp(2*p*acoth(a*x)),x)
```

output
```
Piecewise((I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(-I*pi*p), Eq(
a, 0)), (Integral(exp(acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p,
-1/2)), (a*x*(-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*p*
acoth(a*x))) - (-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*
p*acoth(a*x))), True))
```

3.764.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{(a(-c)^p x - (-c)^p)(ax-1)^{2p}}{a(2p+1)}$$

input
```
integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

output
```
(a*(-c)^p*x - (-c)^p)*(a*x - 1)^(2*p)/(a*(2*p + 1))
```

3.764.8 Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.764.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x-1\right)\,\left(\frac{a\,x-1}{a\,x}\right)^p}{a\,\left(2\,p+1\right)\,\left(\frac{a\,x+1}{a\,x}\right)^p}$$

input
```
int(exp(-2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

output
```
((c - a^2*c*x^2)^p*(a*x - 1)*((a*x - 1)/(a*x))^p)/(a*(2*p + 1)*((a*x + 1)/
(a*x))^p)
```

3.765 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

3.765.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\text{Hypergeometric2F1}\left(-2-p,-1+p,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

output
```
2^(2+p)*c*(a*x+1)^(1-p)*(-a^2*c*x^2+c)^(-1+p)*hypergeom([-1+p, -2-p],[p],-
1/2*a*x+1/2)/a/(1-p)
```

3.765.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{2^{2+p}(1-ax)^{-1+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2-p,-1+p,p,\frac{1}{2}(1-ax)\right)}{a(-1+p)}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
-((2^(2 + p)*(1 - a*x)^(-1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-2 - p
, -1 + p, p, (1 - a*x)/2])/(a*(-1 + p)*(1 - a^2*x^2)^p))
```

3.765.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 6691, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int e^{4\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6691}$$

$$c^2\int (ax+1)^4\left(c-a^2cx^2\right)^{p-2}dx$$

$$\Big\downarrow \text{473}$$

$$c^2(ax+1)^{1-p}(c-acx)^{1-p}\left(c-a^2cx^2\right)^{p-1}\int (ax+1)^{p+2}(c-acx)^{p-2}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\text{Hypergeometric2F1}\left(-p-2,p-1,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[
-2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

3.765.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.765.4 Maple [F]

$$\int \frac{\left(a x +1\right)^{2} \left(-a^{2} c \,x^{2}+c \right)^{p}}{\left(a x -1\right)^{2}}d x$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

output
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

3.765.5 Fricas [F]

$$\int e^{4 \coth^{-1}(ax)} \left(c-a^2 c x^2\right)^p \, dx = \int \frac{(ax+1)^2 (-a^2 c x^2 + c)^p}{(ax-1)^2} \, dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x + 1), x
)
```

3.765.6 Sympy [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(-c(ax-1)\left(ax+1\right)\right)^p\left(ax+1\right)^2}{\left(ax-1\right)^2}\,dx$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**p,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

3.765.7 Maxima [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(ax+1\right)^2\left(-a^2cx^2+c\right)^p}{\left(ax-1\right)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

3.765.8 Giac [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(ax+1\right)^2\left(-a^2cx^2+c\right)^p}{\left(ax-1\right)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

3.765.9 Mupad [F(-1)]

Timed out.

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)^2}{\left(a\,x-1\right)^2}\,dx$$

input
```
int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2, x)
```

3.766 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.766.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{5}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\Big(-1-2p,\frac{3}{2}-p,-}{1+2p}$$

output

```
((a-1/x)/(a+1/x))^(3/2-p)*(1-1/a/x)^(-3/2+p)*(1+1/a/x)^(5/2+p)*x*(-a^2*c*x
^2+c)^p*hypergeom([-1-2*p, 3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^
2)^p)
```

3.766.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=$$

$$-\frac{4^{1+p}e^{5\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)^{-2p}(c-a^2cx^2)^p\,\text{Hypergeometric}}{5a+2ap}$$

input

```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
-((4^(1 + p)*E^(5*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[
a*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[
5/2 + p, 2 + 2*p, 7/2 + p, E^(2*ArcCoth[a*x])])/((5*a + 2*a*p)*(a*Sqrt[1 -
 1/(a^2*x^2)]*x)^(2*p)))
```

3.766.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(1+\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{5}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,\frac{3}{2}-p,-2p,\frac{2}{\left(a+\frac{1}{x}\right)}\right.}{2p+1}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/(a*x))^(-3/2 + p)*(1 + 1/(a*
x))^(5/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 3/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

3.766.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

3.766.4 Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

3.766.5 Fricas [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas"
)
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)
)/(a^2*x^2 - 2*a*x + 1), x)
```

3.766.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(-c(ax-1)\,(ax+1)\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**p,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.766.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima"
)
```

output
```
integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.766.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(-a^2cx^2+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.766.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^p}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.767 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.767.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1-ax)\right)}{ap}$$

output
```
2^(p+1)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[p+1],-1/2*a*x+1/2)/a/p/((a*x
+1)^p)
```

3.767.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{1+p}(1-ax)^p\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1-ax)\right)}{ap}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(2^(1 + p)*(1 - a*x)^p*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 +
p, (1 - a*x)/2])/(a*p*(1 - a^2*x^2)^p)
```

3.767.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 6691, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int (ax+1)^2\left(c-a^2cx^2\right)^{p-1}dx$$

$$\Big\downarrow \text{473}$$

$$-c(ax+1)^{-p}(c-acx)^{-p}\left(c-a^2cx^2\right)^p\int (ax+1)^{p+1}(c-acx)^{p-1}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(1-ax)\right)}{ap}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x)
/2])/(a*p*(1 + a*x)^p)
```

3.767.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.767.4 Maple [F]

$$\int \frac{(ax+1)\left(-a^{2}c\,x^{2}+c\right)^{p}}{ax-1}dx$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
```

output
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
```

3.767.5 Fricas [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

3.767.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.07 (sec) , antiderivative size = 648, normalized size of antiderivative = 12.00

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\text{Too large to display}$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**p,x)
```

output

```
a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x**
*(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (
1/2 - p,), 1/(a**2*x**2))/(2*gamma(1/2 - p)*gamma(p + 1)) + c**p*x**2*gamm
a(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi)
)/(2*gamma(-p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a - 0**p*log(
1/(a**2*x**2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atan
h(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*ga
mma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*gamma
(1/2 - p)*gamma(p + 1)) + c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 -
 p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)), True
)) + Piecewise((0**p*log(a**2*x**2 - 1)/(2*a) - 0**p*acoth(a*x)/a + a*c**p
*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_pol
ar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p - 2)*c**p*p*x**(2*p - 1)*
exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/
(a**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*l
og(-a**2*x**2 + 1)/(2*a) - 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(
1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p - 2)*c**p*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*
gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*ga...
```

3.767.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output

```
integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

3.767.8 Giac [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

3.767.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^p\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1), x)
```

3.768 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.768.1 Optimal result

Integrand size = 20, antiderivative size = 118

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{3}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,\frac{1}{2}-p,-\right.}{1+2p}$$

output
```
((a-1/x)/(a+1/x))^(1/2-p)*(1-1/a/x)^(-1/2+p)*(1+1/a/x)^(3/2+p)*x*(-a^2*c*x
^2+c)^p*hypergeom([-1-2*p, 1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^
2)^p)
```

3.768.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx =$$
$$-\frac{4^{1+p}e^{3\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)^{-2p}(c-a^2cx^2)^p\,\text{Hypergeometric}}{3a+2ap}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

output
```
-((4^(1 + p)*E^(3*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[
a*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[
3/2 + p, 2 + 2*p, 5/2 + p, E^(2*ArcCoth[a*x])])/((3*a + 2*a*p)*(a*Sqrt[1 -
 1/(a^2*x^2)]*x)^(2*p)))
```

3.768.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{ 6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(1+\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{3}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\Bigl(-2p-1,\frac{1}{2}-p,-2p,\frac{2}{(a+\frac{1}{x})}}{2p+1}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*
x))^(3/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 1/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

3.768.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

3.768.4 Maple [F]

$$\int \frac{\left(-a^{2} c \,x^{2}+c \right)^{p}}{\sqrt{\frac{a x -1}{a x +1}}}d x$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

3.768.5 Fricas [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(-a^2cx^2+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas"
)
```

output
```
integral((a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1),
 x)
```

3.768.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(-c(ax-1)\,(ax+1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**p,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.768.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(-a^2cx^2+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima"
)
```

output
```
integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.768.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(-a^2cx^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.768.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.769 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.769.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,-\frac{1}{2}-p,\right.}{1+2p}$$

output
```
((a-1/x)/(a+1/x))^(-1/2-p)*(1-1/a/x)^(1/2+p)*(1+1/a/x)^(1/2+p)*x*(-a^2*c*x
^2+c)^p*hypergeom([-1-2*p, -1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x
^2)^p)
```

3.769.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=$$
$$-\frac{4^{1+p}e^{\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)^{-2p}(c-a^2cx^2)^p\,\text{Hypergeometric2}}{a+2ap}$$

input
```
Integrate[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

output
```
-((4^(1 + p)*E^ArcCoth[a*x]*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]
/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2
+ p, 2 + 2*p, 3/2 + p, E^(2*ArcCoth[a*x])])/((a + 2*a*p)*(a*Sqrt[1 - 1/(a^
2*x^2)]*x)^(2*p)))
```

3.769.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(1+\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{1}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,-p-\frac{1}{2},-2p,\right.}{2p+1}$$

input
```
Int[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*(1 - 1/(a*x))^(1/2 + p)*(1 + 1/(a*
x))^(1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -1/2 - p, -2
*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

3.769.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

3.769.4 Maple [F]

$$\int \left(-a^{2} c \,x^{2}+c \right)^{p} \sqrt{\frac{a x -1}{a x +1}}d x$$

input
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

3.769.5 Fricas [F]

$$\int e^{-\coth^{-1}(ax)} \left(c-a^2 c x^2\right)^p \, dx = \int \left(-a^{2} c x^{2} + c\right)^{p} \sqrt{\frac{a x - 1}{a x + 1}} \, dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
integral((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.769.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\sqrt{\frac{ax-1}{ax+1}}(-c(ax-1)\,(ax+1))^p\,dx$$

input
```
integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p, x)
```

3.769.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.769.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.769.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \left(c-a^2\,c\,x^2\right)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.770 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.770.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1+ax)\right)}{ap}$$

output

```
-2^(p+1)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[p+1],1/2*a*x+1/2)/a/p/((-a*
x+1)^p)
```

3.770.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{-1+p}(1-ax)^{2+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(1-p,2+p,3+p,\frac{1}{2}(1-ax)\right)}{a(2+p)}$$

input

```
Integrate[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

output

```
(2^(-1 + p)*(1 - a*x)^(2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1 - p, 2
 + p, 3 + p, (1 - a*x)/2])/(a*(2 + p)*(1 - a^2*x^2)^p)
```

3.770.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 6692, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow 6717$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow 6692$$

$$-c\int (1-ax)^2\left(c-a^2cx^2\right)^{p-1}\,dx$$

$$\Big\downarrow 473$$

$$-c(1-ax)^{-p}(acx+c)^{-p}\left(c-a^2cx^2\right)^p\int (1-ax)^{p+1}(axc+c)^{p-1}dx$$

$$\Big\downarrow 79$$

$$-\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(ax+1)\right)}{ap}$$

input
```
Int[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

output
```
-((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*
x)/2])/(a*p*(1 - a*x)^p))
```

3.770.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.770.4 Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^p\,(ax-1)}{ax+1}dx$$

input
```
int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
```

output
```
int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
```

3.770.5 Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)(-a^2cx^2+c)^p}{ax+1}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

3.770.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.31 (sec) , antiderivative size = 648, normalized size of antiderivative = 11.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\text{Too large to display}$$

input

```
integrate((-a**2*c*x**2+c)**p*(a*x-1)/(a*x+1),x)
```

output

```
a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x*
*(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (
1/2 - p,), 1/(a**2*x**2))/(2*gamma(1/2 - p)*gamma(p + 1)) - c**p*x**2*gamm
a(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi)
)/(2*gamma(-p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(
1/(a**2*x**2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atan
h(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*ga
mma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*gamma
(1/2 - p)*gamma(p + 1)) - c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 -
 p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)), True
)) - Piecewise((0**p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a + a*c**p
*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_pol
ar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) + a**(2*p - 2)*c**p*p*x**(2*p - 1)*
exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/
(a**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*l
og(-a**2*x**2 + 1)/(2*a) + 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(
1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) + a**(2*p - 2)*c**p*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*
gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*ga...
```

3.770.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1}\,dx$$

input

```
integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output

```
integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

3.770.8 Giac [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)(-a^2cx^2+c)^p}{ax+1}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

3.770.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(c-a^2\,c\,x^2)^p\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1), x)
```

3.771 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

3.771.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{-\frac{1}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\Bigl(-1-2p,-\frac{3}{2}-p,}{1+2p}$$

output
```
((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2+p)*(1+1/a/x)^(-1/2+p)*x*(-a^2*c*
x^2+c)^p*hypergeom([-1-2*p, -3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/
x^2)^p)
```

3.771.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\frac{4^{1+p}e^{-\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}x\right)^{-2p}(c-a^2cx^2)^p\,\text{Hypergeometric2F}}{a-2ap}$$

input
```
Integrate[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

output
```
(4^(1 + p)*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/(-1 + E^(2*ArcCo
th[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2 + p, 2 + 2*p, 1/
2 + p, E^(2*ArcCoth[a*x])])/(E^ArcCoth[a*x]*(a - 2*a*p)*(a*Sqrt[1 - 1/(a^2
*x^2)]*x)^(2*p))
```

3.771.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^px^{2p}dx$$

$$\Big\downarrow \text{6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(1+\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p-\frac{1}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,-p-\frac{3}{2},-2p,\right.}{2p+1}$$

input
```
Int[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2 + p)*(1 + 1/(a*
x))^(-1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -3/2 - p, -
2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

3.771.3.1 Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

3.771.4 Maple [F]

$$\int \left(-a^{2} c \,x^{2}+c \right)^{p} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}d x$$

input
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

3.771.5 Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \left(-a^{2} c x^{2}+c\right)^{p} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1),
 x)
```

3.771.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.771.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.771.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.771.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int {\left(c-a^2\,c\,x^2\right)}^p\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.772 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.772.1 Optimal result

Integrand size = 20, antiderivative size = 342

$$
\begin{aligned}
&\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx \\
&= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a}-\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} \\
&\quad-\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a}+\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} \\
&\quad+\frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} \\
&\quad+c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x+\frac{35c^4\csc^{-1}(ax)}{16a}+\frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}
$$

output

```
47/42*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)/a+8/7*c^4*(1-1/a/x)^(5/2)*(1+1/a
/x)^(9/2)/a+c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(9/2)*x+35/16*c^4*arccsc(a*x)/a+
c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-67/48*c^4*(1+1/a/x)^(3/2)*(
1-1/a/x)^(1/2)/a-91/120*c^4*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-131/280*c^4*
(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a+61/70*c^4*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2
)/a-51/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.772.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.35

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(240+280ax-1056a^2x^2-1330a^3x^3+1952a^4x^4+3045a^5x^5-2816a^6x^6+1680a^7x^7)}{x^6}+3675a^6\arcsin\left(\frac{1}{ax}\right)+1680a^6\,\mathrm{lo}}{1680a^7}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

output
```
(c^4*((Sqrt[1 - 1/(a^2*x^2)]*(240 + 280*a*x - 1056*a^2*x^2 - 1330*a^3*x^3
+ 1952*a^4*x^4 + 3045*a^5*x^5 - 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a
^6*ArcSin[1/(a*x)] + 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a
^7)
```

3.772.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.98, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.150, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6748$$

$$-c^4\int \left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \; 108$$

$$-c^4\left(\int \frac{\left(a-\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\int \left(a-\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow 171$$

$$-c^4\left(\frac{\frac{1}{7}a\int \frac{\left(7a-\frac{47}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x}{a}d\frac{1}{x}-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow 27$$

$$-c^4\left(\frac{\frac{1}{7}\int \left(7a-\frac{47}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}xd\frac{1}{x}-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow 171$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{6}a\int \frac{3\left(14a-\frac{61}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x}{a}d\frac{1}{x}-\frac{47}{6}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{a^2}-x\left(1-\right.\right.$$

$$\Big\downarrow 27$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\int \left(14a-\frac{61}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}xd\frac{1}{x}-\frac{47}{6}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{a^2}-x\right.$$

$$\Big\downarrow 171$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}a\int \frac{\left(70a-\frac{131}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{61}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{47}{6}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow 27$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\int \frac{\left(70a-\frac{131}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{61}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{47}{6}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)}{a^2}\right.$$

$$\Big\downarrow 171$$

3.772. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{131}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{4}a\int -\frac{7\left(40a-\frac{91}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{61}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{47}{6}a(1-\right.\right.}{a^2}\right.$$

$\downarrow$ 27

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\int \frac{\left(40a-\frac{91}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{131}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{61}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{47}{6}a(1-\frac{1}{ax}\right.\right.}{a^2}\right.$$

$\downarrow$ 171

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{91}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int -\frac{5\left(24a-\frac{67}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{131}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{61}{5}a\right.\right.\right.}{a^2}\right.$$

$\downarrow$ 27

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\int \frac{\left(24a-\frac{67}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{91}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{131}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{61}{5}a\sqrt{1\right.\right.\right.}{a^2}\right.$$

$\downarrow$ 171

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int -\frac{3\left(16a-\frac{51}{x}\right)\sqrt{1+\frac{1}{ax}}\,x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{91}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{131}{4}a\right.\right.\right.\right.}{}\right.$$

$\downarrow$ 27

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\int \frac{\left(16a-\frac{51}{x}\right)\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{91}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{131}{4}a\sqrt{1\right.\right.\right.\right.}{}\right.$$

3.772. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx$

$$\Big\downarrow \text{171}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(16a-\frac{35}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{91}{3}a\sqrt{1$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(a\int \frac{\left(16a-\frac{35}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{91}{3}a\sqrt{1-$$

$$\Big\downarrow \text{27}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{\left(16a-\frac{35}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{91}{3}a\sqrt{1-$$

$$\Big\downarrow \text{175}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(16a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-35\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}$$

$$\Big\downarrow \text{39}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-35\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\right.$$

$$\Big\downarrow \text{103}$$

3.772. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

$$-c^4\Bigg(\frac{\frac{1}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\Big(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\Big)+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{67}{2}a\sqrt{}}{}$$

$$\Big\downarrow \color{blue}{221}$$

$$-c^4\Bigg(\frac{\frac{1}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a\text{arctanh}\Big(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Big)+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{67}{2}a\sqrt{}}{}$$

$$\Big\downarrow \color{blue}{223}$$

$$-c^4\Bigg(\frac{\frac{1}{7}\Big(\frac{1}{2}\Big(\frac{1}{5}\Big(\frac{7}{4}\Big(\frac{5}{3}\Big(\frac{3}{2}\Big(-35a\arcsin\left(\frac{1}{ax}\right)-16a\text{arctanh}\Big(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Big)+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Big)+\frac{67}{2}a\sqrt{1-}}{}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

output
```
-(c^4*(-((1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(9/2)*x) + ((-8*a*(1 - 1/(a*x))
^(5/2)*(1 + 1/(a*x))^(9/2))/7 + ((-47*a*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^
(9/2))/6 + ((-61*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/5 + ((131*a*Sqrt
[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/4 + (7*((91*a*Sqrt[1 - 1/(a*x)]*(1 + 1/
(a*x))^(5/2))/3 + (5*((67*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (3*
(51*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] - 35*a*ArcSin[1/(a*x)] - 16*a*Ar
cTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2))/3))/4)/5)/2)/7)/a^2))
```

3.772.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

---

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.772. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.772.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{(ax-1)(2816a^6x^6-3045a^5x^5-1952a^4x^4+1330a^3x^3+1056a^2x^2-280ax-240)c^4}{1680x^7a^8\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{35a^7\arctan\left(\right.}{16}\right.}{a^8\sqrt{}}$
default	$\frac{(ax-1)c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+3675a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}+3675a^7x^7\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(a*x-1)*(2816*a^6*x^6-3045*a^5*x^5-1952*a^4*x^4+1330*a^3*x^3+1056*
a^2*x^2-280*a*x-240)/x^7*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)+(a^8*ln(a^2*x/(a^
2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+35/16*a^7*arctan(1/(a^2*x^2-1)^(1/
2))+a^7*((a*x-1)*(a*x+1))^(1/2))*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*
(a*x+1))^(1/2)/(a*x+1)
```

3.772.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx =$$

$$-\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(1680}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
-1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4
*x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 - 1136*a^7*c^4*x^7 + 229*a^6*c^4*x
^6 + 4997*a^5*c^4*x^5 + 622*a^4*c^4*x^4 - 2386*a^3*c^4*x^3 - 776*a^2*c^4*x
^2 + 520*a*c^4*x + 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

3.772.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4\Bigg(\int\frac{a^8}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^2}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^4}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\Bigg(-\frac{4a}{x^2\sqrt{\frac{ax}{ax+}}}}{a^8}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**4,x)
```

output
```
c**4*(Integral(a**8/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x*
*8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a**2/(x**6*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(6*a**4/(x**4*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))), x) + Integral(-4*a**6/(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))), x))/a**8
```

3.772.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=$$
$$-\frac{1}{840}\Bigg(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{5355\,c^4\left(\frac{ax-1}{ax+1}\right)}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/a^2 - (5355*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 31465*c^4*((a*x - 1)/(a*x
+ 1))^(13/2) + 72051*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 71801*c^4*((a*x -
1)/(a*x + 1))^(9/2) + 4569*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 17619*c^4*((a
*x - 1)/(a*x + 1))^(5/2) + 10185*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 1995*c^
4*sqrt((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a
^2/(a*x + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x
+ 1)^5 - 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 -
(a*x - 1)^8*a^2/(a*x + 1)^8 + a^2))*a
```

3.772.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.35

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$

$$= -\frac{35\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{8\,a\operatorname{sgn}\left(ax+1\right)} - \frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\operatorname{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^4}{a\operatorname{sgn}\left(ax+1\right)}$$

$$-\frac{3045\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a|+6720\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4+6860\left(x|a|-\sqrt{a^2x^2-1}\right)^{11}c^4|a|+20}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
-35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^4*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2
- 1)*c^4/(a*sgn(a*x + 1)) - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*
c^4*abs(a) + 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4 + 6860*(x*abs(a)
 - sqrt(a^2*x^2 - 1))^11*c^4*abs(a) + 20160*(x*abs(a) - sqrt(a^2*x^2 - 1))
^10*a*c^4 + 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^4*abs(a) + 49280*(x*ab
s(a) - sqrt(a^2*x^2 - 1))^8*a*c^4 + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6
*a*c^4 - 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) + 38976*(x*abs(a
) - sqrt(a^2*x^2 - 1))^4*a*c^4 - 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4
*abs(a) + 12992*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4 - 3045*(x*abs(a) -
sqrt(a^2*x^2 - 1))*c^4*abs(a) + 2816*a*c^4)/(((x*abs(a) - sqrt(a^2*x^2 - 1
))^2 + 1)^7*a*abs(a)*sgn(a*x + 1))
```

3.772.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$

$$= \frac{\frac{19\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{97\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8}+\frac{839\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}+\frac{1523\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{280}+\frac{71801\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{840}+\frac{3431\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+899\,c^4}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\frac{a\,(a}{(a\,x}}$$

$$-\frac{35\,c^4\operatorname{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}+\frac{2\,c^4\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input

```
int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output

```
((19*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (97*c^4*((a*x - 1)/(a*x + 1))^(3
/2))/8 + (839*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (1523*c^4*((a*x - 1)/(
a*x + 1))^(7/2))/280 + (71801*c^4*((a*x - 1)/(a*x + 1))^(9/2))/840 + (3431
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (899*c^4*((a*x - 1)/(a*x + 1))^(13
/2))/24 + (51*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a
*x + 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3
- (14*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(
a*x - 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*
x - 1)/(a*x + 1))^(1/2)))/(8*a) + (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/a
```

3.773 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.773.1 Optimal result

Integrand size = 20, antiderivative size = 268

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx &= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}\\&\quad+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a}\\&\quad+c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{15c^3\csc^{-1}(ax)}{8a}+\frac{c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

output
```
6/5*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)/a+c^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(7
/2)*x+15/8*c^3*arccsc(a*x)/a+c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/
a-31/24*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-43/60*c^3*(1+1/a/x)^(5/2)*(1
-1/a/x)^(1/2)/a+23/20*c^3*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-23/8*c^3*(1-1/
a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.773.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{c^3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(-24-30ax+88a^2x^2+135a^3x^3-184a^4x^4+120a^5x^5\right)}{x^4}+225a^4\arcsin\left(\frac{1}{ax}\right)+120a^4\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right.}{120a^5}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

output
```
(c^3*((Sqrt[1 - 1/(a^2*x^2)]*(-24 - 30*a*x + 88*a^2*x^2 + 135*a^3*x^3 - 18
4*a^4*x^4 + 120*a^5*x^5))/x^4 + 225*a^4*ArcSin[1/(a*x)] + 120*a^4*Log[(1 +
 Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5)
```

3.773.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.98, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.950, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6748}$$

$$-c^3\int \left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^3\left(\int \frac{\left(a-\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^3\left(\frac{\int \left(a-\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^3\left(\frac{\frac{1}{5}a\int\frac{\left(5a-\frac{23}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x}{a}d\frac{1}{x}-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow 27$$

$$-c^3\left(\frac{\frac{1}{5}\int\left(5a-\frac{23}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow 171$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}a\int\frac{\left(20a-\frac{43}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\right.\right.$$

$$\Big\downarrow 27$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\int\frac{\left(20a-\frac{43}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\right.\right.$$

$$\Big\downarrow 171$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{43}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int-\frac{5\left(12a-\frac{31}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{6}{5}a\left(1-\frac{1}{ax}\right)}{a^2}\right.$$

$$\Big\downarrow 27$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\int\frac{\left(12a-\frac{31}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{43}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}}{a^2}\right.$$

$$\Big\downarrow 171$$

3.773. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx$

$$-c^3 \left(\frac{\frac{1}{5} \left(\frac{1}{4} \left(\frac{5}{3} \left(\frac{31}{2} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{3/2} - \frac{1}{2} a \int -\frac{3\left(8a-\frac{23}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}} d\frac{1}{x} \right) + \frac{43}{3} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{5/2} \right) - \frac{23}{4} a \sqrt{1-}}{a^2} \right.$$

$$\Big\downarrow 27$$

$$-c^3 \left(\frac{\frac{1}{5} \left(\frac{1}{4} \left(\frac{5}{3} \left(\frac{3}{2} \int \frac{\left(8a-\frac{23}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}} d\frac{1}{x} + \frac{31}{2} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{3/2} \right) + \frac{43}{3} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{5/2} \right) - \frac{23}{4} a \sqrt{1-\frac{1}{ax}} (}{a^2} \right.$$

$$\Big\downarrow 171$$

$$-c^3 \left(\frac{\frac{1}{5} \left(\frac{1}{4} \left(\frac{5}{3} \left(\frac{3}{2} \left(23a \sqrt{1-\frac{1}{ax}} \sqrt{\frac{1}{ax}+1} - a \int -\frac{\left(8a-\frac{15}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} d\frac{1}{x} \right) + \frac{31}{2} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{3/2} \right) + \frac{43}{3} a \sqrt{1-\frac{1}{ax}} (}{a^2} \right.$$

$$\Big\downarrow 25$$

$$-c^3 \left(\frac{\frac{1}{5} \left(\frac{1}{4} \left(\frac{5}{3} \left(\frac{3}{2} \left(a \int \frac{\left(8a-\frac{15}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} d\frac{1}{x} + 23a \sqrt{1-\frac{1}{ax}} \sqrt{\frac{1}{ax}+1} \right) + \frac{31}{2} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{3/2} \right) + \frac{43}{3} a \sqrt{1-\frac{1}{ax}} (}{a^2} \right.$$

$$\Big\downarrow 27$$

$$-c^3 \left(\frac{\frac{1}{5} \left(\frac{1}{4} \left(\frac{5}{3} \left(\frac{3}{2} \left(\int \frac{\left(8a-\frac{15}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} d\frac{1}{x} + 23a \sqrt{1-\frac{1}{ax}} \sqrt{\frac{1}{ax}+1} \right) + \frac{31}{2} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{3/2} \right) + \frac{43}{3} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+\right.}{a^2} \right.$$

$$\Big\downarrow 175$$

$$-c^3 \left(\frac{\frac{1}{5} \left(\frac{1}{4} \left(\frac{5}{3} \left(\frac{3}{2} \left(8a \int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} d\frac{1}{x} - 15 \int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} d\frac{1}{x} + 23a \sqrt{1-\frac{1}{ax}} \sqrt{\frac{1}{ax}+1} \right) + \frac{31}{2} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right.}{a^2} \right.$$

3.773. $\int e^{\coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^3 dx$

$$\Big\downarrow \text{39}$$

$$-c^3\Bigg(\frac{\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/}}{a^2}$$

$$\Big\downarrow \text{103}$$

$$-c^3\Bigg(\frac{\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\Bigg(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\Bigg)+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}}{a^2}$$

$$\Big\downarrow \text{221}$$

$$-c^3\Bigg(\frac{\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\text{arctanh}\Bigg(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\Big(}{a^2}$$

$$\Big\downarrow \text{223}$$

$$-c^3\Bigg(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-15a\arcsin\left(\frac{1}{ax}\right)-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\Big(\frac{1}{ax}\right.\right.\right.}{a^2}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

output
```
-(c^3*(-((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)*x) + ((-6*a*(1 - 1/(a*x))
^(3/2)*(1 + 1/(a*x))^(7/2))/5 + ((-23*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7
/2))/4 + ((43*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2))/3 + (5*((31*a*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (3*(23*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 +
1/(a*x)] - 15*a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1
/(a*x)]]))/2))/3)/4)/5)/a^2))
```

3.773. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.773.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.773. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.773.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{(ax-1)\left(184a^4x^4-135a^3x^3-88a^2x^2+30ax+24\right)c^3}{120x^5a^6\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{15a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}+a^5\sqrt{(ax-1)(ax+1)}\right)}{a^6\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+225a^5x^5\sqrt{a^2}\,\sqrt{a^2x^2-1}+225a^5x^5\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+120\ln\left(\right.\right.}{120\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/120*(a*x-1)*(184*a^4*x^4-135*a^3*x^3-88*a^2*x^2+30*a*x+24)/x^5*c^3/a^6/
((a*x-1)/(a*x+1))^(1/2)+(a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)
^(1/2)+15/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^5*((a*x-1)*(a*x+1))^(1/2))*c
^3/a^6/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.773.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(120\,a^6c^3x}{120\,a^6x^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
-1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^
5*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) - (120*a^6*c^3*x^6 - 64*a^5*c^3*x^5 - 49*a^4*c^3*x^4 + 223
*a^3*c^3*x^3 + 58*a^2*c^3*x^2 - 54*a*c^3*x - 24*c^3)*sqrt((a*x - 1)/(a*x +
 1)))/(a^6*x^5)
```

3.773.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$

$$=\frac{c^3\left(\int\frac{a^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a^2}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^6}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**3,x)
```

output
```
c**3*(Integral(a**6/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x
**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a**2/(x**4*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**4/(x**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))), x))/a**6
```

3.773.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx =$$
$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
 - (345*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1345*c^3*((a*x - 1)/(a*x + 1))^
(9/2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 86*c^3*((a*x - 1)/(a*x + 1)
)^(5/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 105*c^3*sqrt((a*x - 1)/(a*
x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a
*x - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^
2/(a*x + 1)^6 + a^2))*a
```

3.773.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.32

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=-\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,a\mathrm{sgn}\left(ax+1\right)}-\frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^3}{a\mathrm{sgn}\left(ax+1\right)}$$
$$-\frac{135\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|+360\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3+150\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a|+720\,(x|a}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
-15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^3*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2
 - 1)*c^3/(a*sgn(a*x + 1)) - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3
*abs(a) + 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3 + 150*(x*abs(a) - sqr
t(a^2*x^2 - 1))^7*c^3*abs(a) + 720*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^3
 + 1120*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^3 - 150*(x*abs(a) - sqrt(a^2*x
^2 - 1))^3*c^3*abs(a) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - 135*(
x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a) + 184*a*c^3)/(((x*abs(a) - sqrt(a
^2*x^2 - 1))^2 + 1)^5*a*abs(a)*sgn(a*x + 1))
```

3.773.9 Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{\frac{7\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{61\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{43\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{30}+\frac{827\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{30}+\frac{269\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{12}+\frac{23\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}$$
$$-\frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+\frac{2\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((7*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (61*c^3*((a*x - 1)/(a*x + 1))^(3/
2))/12 + (43*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (827*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/30 + (269*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (23*c^3*((a
*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x -
 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*
x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1
))^(1/2)))/(4*a) + (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.774 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$

3.774.1 Optimal result

Integrand size = 20, antiderivative size = 194

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x+\frac{3c^2\csc^{-1}(ax)}{2a}+\frac{c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

output
```
c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x+3/2*c^2*arccsc(a*x)/a+c^2*arctanh((1
-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/6*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a
+4/3*c^2*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2)/a
```

3.774.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(2+3ax-8a^2x^2+6a^3x^3\right)+9a^2x^2\arcsin\left(\frac{1}{ax}\right)+6a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^
2*ArcSin[1/(a*x)] + 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*
x^2)
```

3.774.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6748}$$

$$-c^2\int \left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^2\left(\int \frac{\left(a-\frac{4}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(\frac{\int \left(a-\frac{4}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(\frac{\frac{1}{3}a\int \frac{\left(3a-\frac{7}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(\frac{\frac{1}{3}\int \frac{\left(3a-\frac{7}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{ 171}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int -\frac{3\left(2a-\frac{5}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+\right.\right.$$

$$\Big\downarrow \text{ 27}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\int \frac{\left(2a-\frac{5}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)\right.$$

$$\Big\downarrow \text{ 171}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(2a-\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/}}{a^2}\right.$$

$$\Big\downarrow \text{ 25}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\int \frac{\left(2a-\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-\right.$$

$$\Big\downarrow \text{ 27}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{\left(2a-\frac{3}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-\right.$$

$$\Big\downarrow \text{ 175}$$

3.774. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(2a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-3\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}}{a^2}\right.$$

$$\Big\downarrow 39$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(-3\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a}{a^2}\right.$$

$$\Big\downarrow 103$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(-3\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3}}{a^2}\right.$$

$$\Big\downarrow 221$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(-3\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/}}{a^2}\right.$$

$$\Big\downarrow 223$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(-3a\arcsin\left(\frac{1}{ax}\right)-2a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2}\right.$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

output
```
-(c^2*(-((1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x) + ((-4*a*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(5/2))/3 + ((7*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))
/2 + (3*(5*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] - 3*a*ArcSin[1/(a*x)] - 2
*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2)/3)/a^2))
```

3.774. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.774.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.774. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.774.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{(ax-1)\left(8a^{2}x^{2}-3ax-2\right)c^{2}}{6x^{3}a^{4}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a^{4}\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{\sqrt{a^{2}}}+\frac{3a^{3}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}+a^{3}\sqrt{(ax-1)(ax+1)}\right)c^{2}\sqrt{(ax-1)(ax+1)}}{a^{4}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c^{2}\Bigg(-6\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{4}x^{4}+6\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{2}x^{2}+9\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{3}x^{3}+9a^{3}x^{3}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+6\ln\Bigg(}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^{4}x^{3}\sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x-1)*(8*a^2*x^2-3*a*x-2)/x^3*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)+(a^4*
ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+3/2*a^3*arctan(1/(a^2*
x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)
*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.774.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^2x^4-2\,a^3}{6\,a^4x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 6*a^3*c^2*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) + 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (6*a^4*c^2*x^4 - 2*a^3*c^2*x^3 - 5*a^2*c^2*x^2 + 5*a*c^2*x + 2*
c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

3.774.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{2a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**2,x)
```

output
```
c**2*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x*
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a**2/(x**2*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))), x))/a**4
```

3.774.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.15

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx =$$

$$-\frac{1}{3}\,a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+29\,c^2\left(\frac{ax}{ax}\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(a}{(a}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
(15*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2) +
 c^2*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*
x + 1)^4 + a^2))
```

3.774.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.28

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$

$$= -\frac{3\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^2}{a\mathrm{sgn}\left(ax+1\right)}$$

$$-\frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2+12\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-3\left(x|a|-\sqrt{a^2\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
-3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^2*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^2/(a*sgn(a*x + 1)) - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a
) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2 + 12*(x*abs(a) - sqrt(a^2*x^
2 - 1))^2*a*c^2 - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^2*abs(a) + 8*a*c^2)/(
((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

3.774.9 Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\sqrt{\frac{ax-1}{ax+1}}+\frac{c^2\left(\frac{ax-1}{ax+1}\right)^{3/2}}{3}+\frac{29\,c^2\left(\frac{ax-1}{ax+1}\right)^{5/2}}{3}+5\,c^2\left(\frac{ax-1}{ax+1}\right)^{7/2}}{a+\frac{2\,a\,(ax-1)}{ax+1}-\frac{2\,a\,(ax-1)^3}{(ax+1)^3}-\frac{a\,(ax-1)^4}{(ax+1)^4}}-\frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(c^2*((a*x - 1)/(a*x + 1))^(1/2) + (c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 + (
29*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(7/2))
/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x
- 1)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^
2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.775 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.775.1 Optimal result

Integrand size = 18, antiderivative size = 107

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\csc^{-1}(ax)}{a}+\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

output
```
c*arccsc(a*x)/a+c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1+1/a/x)^(
3/2)*x*(1-1/a/x)^(1/2)-2*c*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.775.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)+\arcsin\left(\frac{1}{ax}\right)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

output
```
(c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/a
```

3.775.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6748, 108, 27, 171, 25, 27, 35, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-c\int \sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 108}$$

$$-c\left(\int \frac{\left(a-\frac{2}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a^2\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(\frac{\int \frac{\left(a-\frac{2}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 171}$$

$$-c\left(\frac{2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c\left(\frac{a\int \frac{\left(a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(\frac{\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 35$$

$$-c\left(\frac{a\int \frac{\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 140$$

$$-c\left(\frac{a\left(\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 39$$

$$-c\left(\frac{a\left(\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 103$$

$$-c\left(\frac{a\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 221$$

$$-c\left(\frac{a\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 223$$

3.775. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

$$-c\left(\frac{a\left(-\arcsin\left(\frac{1}{ax}\right)-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

output
```
-(c*(-(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x) + (2*a*Sqrt[1 - 1/(a*x)]*S
qrt[1 + 1/(a*x)] + a*(-ArcSin[1/(a*x)] - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1
+ 1/(a*x)]]))/a^2))
```

3.775.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.775. $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.775.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{(ax-1)c}{x\,a^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{\sqrt{a^{2}}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\right)c\sqrt{(ax-1)(ax+1)}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c\left(-\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax+\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{2}x+ax\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x\sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)/x*c/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(a*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(1/
2)))*c/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.775.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^{2}x^{2}}\right)\,dx = -\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^{2}cx^{2}-c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^{2}x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
-(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) + a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 - c)*
sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

3.775.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2),x)
```

output
```
c*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x**2
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x))/a**2
```

3.775.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=-\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*
c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

3.775.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c}{a\mathrm{sgn}\left(ax+1\right)}-\frac{2\,c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
-2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c/
(a*sgn(a*x + 1)) - 2*c/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)*sgn(
a*x + 1))
```

3.775.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

input
```
int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x
 + 1)^2)
```

3.776 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.776.1 Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

output
```
arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c-2*(1+1/a/x)^(1/2)/a/c/(1-1/a/
x)^(1/2)+x*(1+1/a/x)^(1/2)/c/(1-1/a/x)^(1/2)
```

3.776.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+ax)}{-1+ax}+\frac{\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

output
```
((Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(-1 + a*x) + Log[(1 + Sqrt[1 - 1/(a^
2*x^2)])*x]/a)/c
```

3.776.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6748, 114, 25, 27, 35, 105, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$

$$\Big\downarrow \; 6748$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 114$$

$$-\frac{-\int -\frac{\left(a+\frac{1}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\left(a+\frac{1}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \; 35$$

$$-\frac{\frac{\int \frac{\sqrt{1+\frac{1}{ax}}\,x}{\left(1-\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \; 105$$

$$-\frac{\frac{\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \textcolor{blue}{103}$$

$$-\frac{\frac{\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$-\frac{\frac{\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

output
```
-((-((Sqrt[1 + 1/(a*x)]*x)/Sqrt[1 - 1/(a*x)]) + ((2*Sqrt[1 + 1/(a*x)])/Sqr
t[1 - 1/(a*x)] - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a)/c)
```

3.776.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

3.776. $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.776.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.38

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{a^{4}\left(x-\frac{1}{a}\right)}\right)a^{2}\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-3\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x^{2}-2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}+6\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,ax+4\ln\Big(}{2a\sqrt{a^{2}}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-1}{ax+1}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

3.776. $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^{2}x^{2}}}\,dx$

output
```
1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
-1)^(1/2))/(a^2)^(1/2)-1/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^
2/c/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.776.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c
*x - a*c)
```

3.776.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\int \frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

output
```
a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))), x)/c
```

3.776.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -a\left(\frac{\frac{3\,(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*c
*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

3.776.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
undef
```

3.776.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 4)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.777 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.777.1 Optimal result

Integrand size = 20, antiderivative size = 180

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=-\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

output
```
arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-4/3/a/c^2/(1-1/a/x)^(3/2)/(
1+1/a/x)^(1/2)+x/c^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)-11/3/a/c^2/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)+8/3*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

3.777.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x\left(8-5ax-7a^2x^2+3a^3x^3\right)}{3(-1+ax)^2(1+ax)}+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 +
a*x)^2*(1 + a*x)) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

3.777.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6748, 114, 25, 27, 169, 25, 27, 169, 25, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{-\int -\frac{\left(a+\frac{3}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\left(a+\frac{3}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\int \frac{\left(a+\frac{3}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \text{ 169}$$

$$-\frac{\frac{\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{1}{3}a\int -\frac{\left(3a+\frac{8}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\frac{1}{3}a\int \frac{\left(3a+\frac{8}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{3}\int \frac{\left(3a+\frac{8}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-a\int -\frac{\left(3a+\frac{11}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{1}{3}\left(a\int \frac{\left(3a+\frac{11}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{3}\left(\int \frac{\left(3a+\frac{11}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{3}\left(a\int \frac{3x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{3}\left(3a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \ 103$$

3.777. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

$$-\frac{\frac{\frac{1}{3}\left(-3\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{\frac{1}{3}\left(-3a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

output
```
-((-(x/((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])) + ((4*a)/(3*(1 - 1/(a*x))^
(3/2)*Sqrt[1 + 1/(a*x)]) + ((11*a)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) -
 (8*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 3*a*ArcTanh[Sqrt[1 - 1/(a*x)]
*Sqrt[1 + 1/(a*x)]])/3)/a^2)/c^2)
```

3.777.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.777.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.21

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{4}\sqrt{a^{2}}}+\frac{\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{4a^{6}\left(x+\frac{1}{a}\right)}-\frac{19\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{12a^{6}\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{6a^{7}\left(x-\frac{1}{a}\right)^{2}}\right)a^{4}\sqrt{(ax-1)(}}{c^{2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{5}x^{5}-24\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{6}x^{5}+21((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}\,a^{3}x^{3}+45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)+1/4/a^6/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)
-19/12/a^6/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1/6/a^7/(x-1/a)^2*((x
-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(
a*x+1))^(1/2)/(a*x+1)
```

3.777.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+\right.}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^
2 - 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c
^2)
```

3.777.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1)))
, x)/c**2
```

3.777.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.89

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2
*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 12*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c^2) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

3.777.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.777.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.71

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2}-\frac{\frac{17\,(a\,x-1)}{3\,(a\,x+1)}-\frac{14\,{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{1}{3}}{4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

input
```
int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^2) - ((17*(a*x - 1))/(3*(a*x + 1)) - (1
4*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 4
*a*c^2*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/(a*c^2)
```

3.778 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.778.1 Optimal result

Integrand size = 20, antiderivative size = 254

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = {} & -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} \\
& -\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}} \\
& +\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}
\end{aligned}
$$

output

```
-6/5/a/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)-29/15/a/c^3/(1-1/a/x)^(3/2)/(1+
1/a/x)^(3/2)+x/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)+arctanh((1-1/a/x)^(1/2)
*(1+1/a/x)^(1/2))/a/c^3-34/5/a/c^3/(1+1/a/x)^(3/2)/(1-1/a/x)^(1/2)+21/5*(1
-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+16/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(
1/2)
```

3.778.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-48+33ax+87a^2x^2-52a^3x^3-38a^4x^4+15a^5x^5)}{15(-1+ax)^3(1+ax)^2}+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-48 + 33*a*x + 87*a^2*x^2 - 52*a^3*x^3 - 38*a
^4*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^3*(1 + a*x)^2) + Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/(a*c^3)
```

3.778.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.91, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6748, 114, 25, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(a+\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(a+\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\int \frac{\left(a+\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{5}a\int -\frac{\left(5a+\frac{24}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\frac{1}{5}a\int \frac{\left(5a+\frac{24}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{5}\int \frac{\left(5a+\frac{24}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{5}\left(\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{3}a\int -\frac{3\left(5a+\frac{29}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{5}\left(\int \frac{\left(5a+\frac{29}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{5}\left(a\left(-\int -\frac{\left(5a+\frac{68}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 25$$

3.778. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

$$-\frac{\frac{\frac{1}{5}\left(a\int \frac{\left(5a+\frac{68}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{5}\left(\int \frac{\left(5a+\frac{68}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{5}\left(\frac{1}{3}a\int \frac{3\left(5a+\frac{21}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{5}\left(\int \frac{\left(5a+\frac{21}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{5}\left(a\int \frac{5x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}(}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{5}\left(5a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^3}$$

$$\Big\downarrow 103$$

3.778. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

$$-\frac{\frac{\frac{1}{5}\left(-5\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-}{c^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{1}{5}\left(-5a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-}{c^3}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

output
```
-((-(x/((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2))) + ((6*a)/(5*(1 - 1/(a*x)
)^(5/2)*(1 + 1/(a*x))^(3/2)) + ((29*a)/(3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x)
)^(3/2)) + (34*a)/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) - (21*a*Sqrt[1 -
 1/(a*x)])/(1 + 1/(a*x))^(3/2) - (16*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)
] - 5*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/5)/a^2)/c^3)
```

3.778.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

3.778. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.778.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.15

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{20a^{10}\left(x-\frac{1}{a}\right)^3}-\frac{23\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{60a^9\left(x-\frac{1}{a}\right)^2}-\frac{493\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{240a^8\left(x-\frac{1}{a}\right)}-\right.}{c^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7-240\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^8x^7+285((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+525\sqrt{(ax-1)(ax+1)}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(1/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/20/a^10/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(
1/2)-23/60/a^9/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-493/240/a^8/(x-
1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1/24/a^9/(x+1/a)^2*(a^2*(x+1/a)^2-2
*a*(x+1/a))^(1/2)+25/48/a^8/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^6
/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.778.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{15\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a}{15\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
 1) - 15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
 1) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x - 48)*sq
rt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^
3)
```

3.778.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{a^6\int \frac{x^6}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*Integral(x**6/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x
**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

3.778.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.76

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{1}{240}\,a\left(\frac{\frac{37\,(ax-1)}{ax+1}+\frac{410\,(ax-1)^2}{(ax+1)^2}-\frac{930\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+24\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{240}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
1/240*a*((37*(a*x - 1)/(a*x + 1) + 410*(a*x - 1)^2/(a*x + 1)^2 - 930*(a*x
- 1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a
*x - 1)/(a*x + 1))^(5/2)) + 5*(((a*x - 1)/(a*x + 1))^(3/2) + 24*sqrt((a*x
- 1)/(a*x + 1)))/(a^2*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c
^3) - 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

3.778.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.778.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.67

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3} - \frac{\frac{82\,(a\,x-1)^2}{3\,(a\,x+1)^2} - \frac{62\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{37\,(a\,x-1)}{15\,(a\,x+1)} + \frac{1}{5}}{16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2} - 16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^3} - \frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^3}$$

input
```
int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(2*a*c^3) - ((82*(a*x - 1)^2)/(3*(a*x + 1)^2)
- (62*(a*x - 1)^3)/(a*x + 1)^3 + (37*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(16*
a*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
+ ((a*x - 1)/(a*x + 1))^(3/2)/(48*a*c^3) - (atan(((a*x - 1)/(a*x + 1))^(1/
2)*1i)*2i)/(a*c^3)
```

3.779 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.779.1 Optimal result

Integrand size = 20, antiderivative size = 328

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx &= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}\\ &\quad-\frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}\\ &\quad+\frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}}\\ &\quad+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}\end{aligned}$$

output

```
-8/7/a/c^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(5/2)-11/7/a/c^4/(1-1/a/x)^(5/2)/(1+1
/a/x)^(5/2)-62/21/a/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^(5/2)+x/c^4/(1-1/a/x)^(7
/2)/(1+1/a/x)^(5/2)+arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-269/21/
a/c^4/(1+1/a/x)^(5/2)/(1-1/a/x)^(1/2)+262/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/
x)^(5/2)+163/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+128/35*(1-1/a/x)^(1/
2)/a/c^4/(1+1/a/x)^(1/2)
```

3.779.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.35

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(384-279ax-1065a^2x^2+715a^3x^3+965a^4x^4-559a^5x^5-281a^6x^6+105a^7x^7)}{105(-1+ax)^4(1+ax)^3}+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(384 - 279*a*x - 1065*a^2*x^2 + 715*a^3*x^3 +
965*a^4*x^4 - 559*a^5*x^5 - 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^4*
(1 + a*x)^3) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

3.779.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.94, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {6748, 114, 25, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(a+\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(a+\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{\left(a+\frac{7}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{1}{7}a\int -\frac{\left(7a+\frac{48}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{1}{7}a\int \frac{\left(7a+\frac{48}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\int \frac{\left(7a+\frac{48}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{1}{5}a\int -\frac{5\left(7a+\frac{55}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\left(\int \frac{\left(7a+\frac{55}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{7}\left(-\frac{1}{3}a\int -\frac{\left(21a+\frac{248}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/}}}{c^4}$$

3.779. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}a\int \frac{\left(21a+\frac{248}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\int \frac{\left(21a+\frac{248}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-a\int -\frac{3\left(7a+\frac{269}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{}{(1}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\int \frac{\left(7a+\frac{269}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{}{(1-}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}a\int \frac{\left(35a+\frac{524}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{}{7\left(1-\frac{1}{ax}\right)^{7}}}{a^2}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}\int \frac{\left(35a+\frac{524}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}}}{a^2}}{c^4}$$

$$\Big\downarrow \ 169$$

3.779. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

$$-\frac{\frac{\frac{1}{7}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{3\left(35a+\frac{163}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\Big(\frac{1}{ax}+}}{a^2}}{c^4}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{7}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{1}{5}\left(\int \frac{\left(35a+\frac{163}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/}}}{a^2}}{c^4}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{7}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{1}{5}\left(a\int \frac{35x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{}{\left(1-\frac{1}{ax}\right)^{5/}}}{a^2}}{c^4}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{7}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{1}{5}\left(35a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{}{\Big(1-\frac{1}{ax}}}{a^2}}{c^4}$$

$$\Big\downarrow 103$$

$$-\frac{\frac{\frac{1}{7}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{1}{5}\left(-35\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/}}}{a^2}}{c^4}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{\frac{1}{7}\Bigg(\frac{1}{3}\Bigg(3\left(\frac{1}{5}\left(-35a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/}}}{a^2}}{c^4}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

3.779. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

output

```
-((-(x/((1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2))) + ((8*a)/(7*(1 - 1/(a*x)
)^(7/2)*(1 + 1/(a*x))^(5/2)) + ((11*a)/((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(5/2)) + (62*a)/(3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)) + ((269*a)/(Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + 3*((-262*a*Sqrt[1 - 1/(a*x)])/(5*(1
 + 1/(a*x))^(5/2)) + ((-163*a*Sqrt[1 - 1/(a*x)])/(1 + 1/(a*x))^(3/2) - (12
8*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 35*a*ArcTanh[Sqrt[1 - 1/(a*x)]*
Sqrt[1 + 1/(a*x)]])/5))/3)/7)/a^2)/c^4)
```

3.779.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

3.779. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.779.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.12

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{56a^{13}\left(x-\frac{1}{a}\right)^4}-\frac{17\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{112a^{12}\left(x-\frac{1}{a}\right)^3}-\frac{211\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{336a^{11}\left(x-\frac{1}{a}\right)^2}-\frac{1657\sqrt{(x-}}{672}\right.}{c^4\sqrt{\frac{ax-}{ax-}}}$
default	Expression too large to display

input

```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output

```
1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(1/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/56/a^13/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(
1/2)-17/112/a^12/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-211/336/a^11/
(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1657/672/a^10/(x-1/a)*((x-1/a)
^2*a^2+2*(x-1/a)*a)^(1/2)-7/60/a^11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^
(1/2)+379/480/a^10/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+1/80/a^12/(x+
1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^8/c^4/((a*x-1)/(a*x+1))^(1/2)*
((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.779.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.84

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\right.}{105\left(a^7c^4x^6-2\,a^6c\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4
 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (
105*a^7*x^7 - 281*a^6*x^6 - 559*a^5*x^5 + 965*a^4*x^4 + 715*a^3*x^3 - 1065
*a^2*x^2 - 279*a*x + 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 2*a^6*
c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

3.779.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{x^8}{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*Integral(x**8/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x
**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1)) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x)/c**4
```

3.779.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{1}{6720}a\left(\frac{5\left(\frac{39(ax-1)}{ax+1}+\frac{287(ax-1)^2}{(ax+1)^2}+\frac{2611(ax-1)^3}{(ax+1)^3}-\frac{5628(ax-1)^4}{(ax+1)^4}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{7\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+50\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+705\sqrt{\frac{a}{a}}\right.}{a^2c^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
1/6720*a*(5*(39*(a*x - 1)/(a*x + 1) + 287*(a*x - 1)^2/(a*x + 1)^2 + 2611*(
a*x - 1)^3/(a*x + 1)^3 - 5628*(a*x - 1)^4/(a*x + 1)^4 + 3)/(a^2*c^4*((a*x
- 1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 7*(3*((a*x
- 1)/(a*x + 1))^(5/2) + 50*((a*x - 1)/(a*x + 1))^(3/2) + 705*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) + 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4
) - 6720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

3.779.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.779.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{47\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{41\,(a\,x-1)^2}{3\,(a\,x+1)^2}+\frac{373\,(a\,x-1)^3}{3\,(a\,x+1)^3}-\frac{268\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{13\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{7}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}+\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^4}$$

input
```
int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(47*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((41*(a*x - 1)^2)/(3*(a*x +
1)^2) + (373*(a*x - 1)^3)/(3*(a*x + 1)^3) - (268*(a*x - 1)^4)/(a*x + 1)^4
+ (13*(a*x - 1))/(7*(a*x + 1)) + 1/7)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2
) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2)) + (5*((a*x - 1)/(a*x + 1))^(3/2)
)/(96*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - (atan(((a*x - 1)/
(a*x + 1))^(1/2)*1i)*2i)/(a*c^4)
```

3.779. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.780 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

3.780.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\log(x)}{a}$$

output
```
-1/9*c^5/a^10/x^9-1/4*c^5/a^9/x^8+3/7*c^5/a^8/x^7+4/3*c^5/a^7/x^6-2/5*c^5/
a^6/x^5-3*c^5/a^5/x^4-2/3*c^5/a^4/x^3+4*c^5/a^3/x^2+3*c^5/a^2/x+c^5*x+2*c^
5*ln(x)/a
```

3.780.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

output
```
-1/9*c^5/(a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a
^7*x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) +
(4*c^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

3.780.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^5 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6717$$

$$-\int \frac{c^5 e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^5}{a^{10}}dx$$

$$\Big\downarrow \; 27$$

$$-\frac{c^5\int e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^5dx}{a^{10}}$$

$$\Big\downarrow \; 6707$$

$$\frac{c^5\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}dx}{a^{10}}$$

$$\Big\downarrow \; 6700$$

$$\frac{c^5\int \frac{(1-ax)^4(ax+1)^6}{x^{10}}dx}{a^{10}}$$

$$\Big\downarrow \; 99$$

$$\frac{c^5\int \left(a^{10}+\frac{2a^9}{x}-\frac{3a^8}{x^2}-\frac{8a^7}{x^3}+\frac{2a^6}{x^4}+\frac{12a^5}{x^5}+\frac{2a^4}{x^6}-\frac{8a^3}{x^7}-\frac{3a^2}{x^8}+\frac{2a}{x^9}+\frac{1}{x^{10}}\right)dx}{a^{10}}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^5\left(a^{10}x+2a^9\log(x)+\frac{3a^8}{x}+\frac{4a^7}{x^2}-\frac{2a^6}{3x^3}-\frac{3a^5}{x^4}-\frac{2a^4}{5x^5}+\frac{4a^3}{3x^6}+\frac{3a^2}{7x^7}-\frac{a}{4x^8}-\frac{1}{9x^9}\right)}{a^{10}}$$

input

```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

output

```
(c^5*(-1/9*1/x^9 - a/(4*x^8) + (3*a^2)/(7*x^7) + (4*a^3)/(3*x^6) - (2*a^4)
/(5*x^5) - (3*a^5)/x^4 - (2*a^6)/(3*x^3) + (4*a^7)/x^2 + (3*a^8)/x + a^10*
x + 2*a^9*Log[x]))/a^10
```

3.780.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.780. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

3.780.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
default	$\frac{c^{5}\left(a^{10}x-\frac{1}{9x^{9}}+2a^{9}\ln\left(x\right)-\frac{3a^{5}}{x^{4}}-\frac{2a^{6}}{3x^{3}}+\frac{4a^{3}}{3x^{6}}+\frac{4a^{7}}{x^{2}}+\frac{3a^{8}}{x}+\frac{3a^{2}}{7x^{7}}-\frac{a}{4x^{8}}-\frac{2a^{4}}{5x^{5}}\right)}{a^{10}}$
risch	$c^{5}x+\frac{3a^{8}c^{5}x^{8}+4a^{7}c^{5}x^{7}-\frac{2}{3}a^{6}c^{5}x^{6}-3a^{5}c^{5}x^{5}-\frac{2}{5}a^{4}c^{5}x^{4}+\frac{4}{3}a^{3}c^{5}x^{3}+\frac{3}{7}a^{2}c^{5}x^{2}-\frac{1}{4}a\,c^{5}x-\frac{1}{9}c^{5}}{a^{10}x^{9}}+\frac{2c^{5}\ln\left(x\right)}{a}$
norman	$\frac{a^{9}c^{5}x^{10}-\frac{c^{5}}{9a}-\frac{c^{5}x}{4}+\frac{3a\,c^{5}x^{2}}{7}-\frac{2a^{3}c^{5}x^{4}}{5}-3a^{4}c^{5}x^{5}-\frac{2a^{5}c^{5}x^{6}}{3}+4a^{6}c^{5}x^{7}+3a^{7}c^{5}x^{8}+\frac{4c^{5}a^{2}x^{3}}{3}}{a^{9}x^{9}}+\frac{2c^{5}\ln\left(x\right)}{a}$
parallelrisch	$\frac{1260a^{10}c^{5}x^{10}+2520c^{5}\ln\left(x\right)a^{9}x^{9}+3780a^{8}c^{5}x^{8}+5040a^{7}c^{5}x^{7}-840a^{6}c^{5}x^{6}-3780a^{5}c^{5}x^{5}-504a^{4}c^{5}x^{4}+1680a^{3}c^{5}x^{3}+540a^{2}c^{5}x^{2}-}{1260a^{10}x^{9}}$
meijerg	$-\frac{c^{5}\left(-ax-\ln\left(-ax+1\right)\right)}{a}+\frac{5c^{5}\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)\right)}{a}-\frac{10c^{5}\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)-\frac{1}{2a^{2}x^{2}}-\frac{1}{ax}\right)}{a}+ 10c^{5}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5/a^10*(a^10*x-1/9/x^9+2*a^9*ln(x)-3*a^5/x^4-2/3*a^6/x^3+4/3*a^3/x^6+4*a
^7/x^2+3*a^8/x+3/7*a^2/x^7-1/4*a/x^8-2/5*a^4/x^5)
```

3.780.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^{2}x^{2}}\right)^{5}\,dx$$
$$=\frac{1260\,a^{10}c^{5}x^{10}+2520\,a^{9}c^{5}x^{9}\log\left(x\right)+3780\,a^{8}c^{5}x^{8}+5040\,a^{7}c^{5}x^{7}-840\,a^{6}c^{5}x^{6}-3780\,a^{5}c^{5}x^{5}-504\,a^{4}c^{5}x^{4}}{1260\,a^{10}x^{9}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

output
```
1/1260*(1260*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) + 3780*a^8*c^5*x^8 +
5040*a^7*c^5*x^7 - 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 +
1680*a^3*c^5*x^3 + 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

3.780.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$$
$$=\frac{a^{10}c^5x+2a^9c^5\log{(x)}+\frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315ac^5x-140c^5}{1260x^9}}{a^{10}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**5,x)
```

output
```
(a**10*c**5*x + 2*a**9*c**5*log(x) + (3780*a**8*c**5*x**8 + 5040*a**7*c**5
*x**7 - 840*a**6*c**5*x**6 - 3780*a**5*c**5*x**5 - 504*a**4*c**5*x**4 + 16
80*a**3*c**5*x**3 + 540*a**2*c**5*x**2 - 315*a*c**5*x - 140*c**5)/(1260*x*
*9))/a**10
```

3.780.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=c^5x+\frac{2\,c^5\log\left(x\right)}{a}$$
$$+\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2-315}{1260\,a^{10}x^9}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

output
```
c^5*x + 2*c^5*log(x)/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 - 840
*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 + 540
*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

3.780.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=c^5x+\frac{2\,c^5\log\left(|x|\right)}{a}$$
$$+\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2-315}{1260\,a^{10}x^9}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

output
```
c^5*x + 2*c^5*log(abs(x))/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7
- 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3
+ 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

3.780.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$$
$$=\frac{c^5\left(\frac{3\,a^2\,x^2}{7}-\frac{a\,x}{4}+\frac{4\,a^3\,x^3}{3}-\frac{2\,a^4\,x^4}{5}-3\,a^5\,x^5-\frac{2\,a^6\,x^6}{3}+4\,a^7\,x^7+3\,a^8\,x^8+a^{10}\,x^{10}+2\,a^9\,x^9\,\ln\left(x\right)-\frac{1}{9}\right)}{a^{10}\,x^9}$$

input
```
int(((c - c/(a^2*x^2))^5*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^5*((3*a^2*x^2)/7 - (a*x)/4 + (4*a^3*x^3)/3 - (2*a^4*x^4)/5 - 3*a^5*x^5
- (2*a^6*x^6)/3 + 4*a^7*x^7 + 3*a^8*x^8 + a^10*x^10 + 2*a^9*x^9*log(x) - 1
/9))/(a^10*x^9)
```

3.781 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.781.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

output
```
1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/a^3/
x^2+2*c^4/a^2/x+c^4*x+2*c^4*ln(x)/a
```

3.781.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x
^4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

3.781.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^4e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\int e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4dx}{a^8}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{c^4\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{c^4\int \frac{(1-ax)^3(ax+1)^5}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^4\int \left(-a^8-\frac{2a^7}{x}+\frac{2a^6}{x^2}+\frac{6a^5}{x^3}-\frac{6a^3}{x^5}-\frac{2a^2}{x^6}+\frac{2a}{x^7}+\frac{1}{x^8}\right)dx}{a^8}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4\left(a^8(-x)-2a^7\log(x)-\frac{2a^6}{x}-\frac{3a^5}{x^2}+\frac{3a^3}{2x^4}+\frac{2a^2}{5x^5}-\frac{a}{3x^6}-\frac{1}{7x^7}\right)}{a^8}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
-((c^4*(-1/7*1/x^7 - a/(3*x^6) + (2*a^2)/(5*x^5) + (3*a^3)/(2*x^4) - (3*a^
5)/x^2 - (2*a^6)/x - a^8*x - 2*a^7*Log[x]))/a^8)
```

3.781. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx$

3.781.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.781. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.781.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^4\left(a^8x+2a^7\ln(x)-\frac{3a^3}{2x^4}+\frac{a}{3x^6}+\frac{3a^5}{x^2}+\frac{2a^6}{x}+\frac{1}{7x^7}-\frac{2a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{2a^6c^4x^6+3a^5c^4x^5-\frac{3}{2}a^3c^4x^3-\frac{2}{5}a^2c^4x^2+\frac{1}{3}a\,c^4x+\frac{1}{7}c^4}{a^8x^7}+\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^7c^4x^8+\frac{c^4}{7a}+\frac{c^4x}{3}-\frac{2a\,c^4x^2}{5}-\frac{3a^2c^4x^3}{2}+3a^4c^4x^5+2a^5c^4x^6}{a^7x^7}+\frac{2c^4\ln(x)}{a}$
parallelrisch	$\frac{210a^8c^4x^8+420c^4\ln(x)a^7x^7+420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70a\,c^4x+30c^4}{210a^8x^7}$
meijerg	$-\frac{c^4(-ax-\ln(-ax+1))}{a}+\frac{4c^4(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{6c^4\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4/a^8*(a^8*x+2*a^7*ln(x)-3/2*a^3/x^4+1/3*a/x^6+3*a^5/x^2+2*a^6/x+1/7/x^7
-2/5*a^2/x^5)
```

3.781.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{210\,a^8c^4x^8+420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/210*(210*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 + 630*a^
5*c^4*x^5 - 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x
^7)
```

3.781.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{a^8c^4x+2a^7c^4\log(x)+\frac{420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70ac^4x+30c^4}{210x^7}}{a^8}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**4,x)
```

output
```
(a**8*c**4*x + 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 + 630*a**5*c**4*x*
*5 - 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 + 70*a*c**4*x + 30*c**4)/(210*
x**7))/a**8
```

3.781.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = c^4x+\frac{2\,c^4\log(x)}{a}+\frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
c^4*x + 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^
3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

3.781.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = c^4x+\frac{2\,c^4\log(|x|)}{a}+\frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
c^4*x + 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 3
15*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

3.781.9 Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4\left(\frac{a\,x}{3}-\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5+2\,a^6\,x^6+a^8\,x^8+2\,a^7\,x^7\,\ln\left(x\right)+\frac{1}{7}\right)}{a^8\,x^7}$$

input
```
int(((c - c/(a^2*x^2))^4*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^4*((a*x)/3 - (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 + 2*a^6*x^6 + a^
8*x^8 + 2*a^7*x^7*log(x) + 1/7))/(a^8*x^7)
```

3.782 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.782.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

output
```
-1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3+2*c^3/a^3/x^2+c^3/a^2/x+c
^3*x+2*c^3*ln(x)/a
```

3.782.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
-1/5*c^3/(a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2)
 + c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

3.782.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^3e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^3}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^3dx}{a^6}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c^3\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c^3\int \frac{(1-ax)^2(ax+1)^4}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^3\int \left(a^6+\frac{2a^5}{x}-\frac{a^4}{x^2}-\frac{4a^3}{x^3}-\frac{a^2}{x^4}+\frac{2a}{x^5}+\frac{1}{x^6}\right)dx}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(a^6x+2a^5\log(x)+\frac{a^4}{x}+\frac{2a^3}{x^2}+\frac{a^2}{3x^3}-\frac{a}{2x^4}-\frac{1}{5x^5}\right)}{a^6}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
(c^3*(-1/5*1/x^5 - a/(2*x^4) + a^2/(3*x^3) + (2*a^3)/x^2 + a^4/x + a^6*x +
 2*a^5*Log[x]))/a^6
```

3.782.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.782. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.782.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

method	result
default	$\frac{c^{3}\left(a^{6}x+2a^{5}\ln(x)-\frac{a}{2x^{4}}+\frac{a^{2}}{3x^{3}}+\frac{2a^{3}}{x^{2}}+\frac{a^{4}}{x}-\frac{1}{5x^{5}}\right)}{a^{6}}$
risch	$c^{3}x+\frac{a^{4}c^{3}x^{4}+2a^{3}c^{3}x^{3}+\frac{1}{3}a^{2}c^{3}x^{2}-\frac{1}{2}a\,c^{3}x-\frac{1}{5}c^{3}}{a^{6}x^{5}}+\frac{2c^{3}\ln(x)}{a}$
norman	$\frac{a^{3}c^{3}x^{4}+a^{5}c^{3}x^{6}-\frac{c^{3}}{5a}-\frac{c^{3}x}{2}+\frac{a\,c^{3}x^{2}}{3}+2a^{2}c^{3}x^{3}}{a^{5}x^{5}}+\frac{2c^{3}\ln(x)}{a}$
parallelrisch	$\frac{30a^{6}c^{3}x^{6}+60c^{3}\ln(x)a^{5}x^{5}+30a^{4}c^{3}x^{4}+60a^{3}c^{3}x^{3}+10a^{2}c^{3}x^{2}-15a\,c^{3}x-6c^{3}}{30a^{6}x^{5}}$
meijerg	$-\frac{c^{3}(-ax-\ln(-ax+1))}{a}+\frac{3c^{3}(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{3c^{3}\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^{2}x^{2}}-\frac{1}{ax}\right)}{a}+$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/a^6*(a^6*x+2*a^5*ln(x)-1/2*a/x^4+1/3*a^2/x^3+2*a^3/x^2+a^4/x-1/5/x^5)
```

3.782.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^{2}x^{2}}\right)^{3}dx$$
$$=\frac{30\,a^{6}c^{3}x^{6}+60\,a^{5}c^{3}x^{5}\log\left(x\right)+30\,a^{4}c^{3}x^{4}+60\,a^{3}c^{3}x^{3}+10\,a^{2}c^{3}x^{2}-15\,ac^{3}x-6\,c^{3}}{30\,a^{6}x^{5}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/30*(30*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 + 60*a^3*c^3
*x^3 + 10*a^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.782.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{a^6c^3x+2a^5c^3\log(x)+\frac{30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15ac^3x-6c^3}{30x^5}}{a^6}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**3,x)
```

output
```
(a**6*c**3*x + 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 + 60*a**3*c**3*x**3
 + 10*a**2*c**3*x**2 - 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

3.782.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=c^3x+\frac{2\,c^3\log(x)}{a}+\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
c^3*x + 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^2*c^
3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.782.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=c^3x+\frac{2\,c^3\log(|x|)}{a}+\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
c^3*x + 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a
^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.782.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3\,\left(\frac{a^2\,x^2}{3}-\frac{a\,x}{2}+2\,a^3\,x^3+a^4\,x^4+a^6\,x^6+2\,a^5\,x^5\,\ln\left(x\right)-\frac{1}{5}\right)}{a^6\,x^5}$$

input
```
int(((c - c/(a^2*x^2))^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^3*((a^2*x^2)/3 - (a*x)/2 + 2*a^3*x^3 + a^4*x^4 + a^6*x^6 + 2*a^5*x^5*lo
g(x) - 1/5))/(a^6*x^5)
```

3.783 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.783.1 Optimal result

Integrand size = 22, antiderivative size = 39

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

output
```
1/3*c^2/a^4/x^3+c^2/a^3/x^2+c^2*x+2*c^2*ln(x)/a
```

3.783.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

3.783.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{c^2e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^2}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\int e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^2dx}{a^4}$$

$$\Big\downarrow \text{ 6707}$$

$$-\frac{c^2\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{c^2\int \frac{(1-ax)(ax+1)^3}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 84}$$

$$-\frac{c^2\int \left(-a^4-\frac{2a^3}{x}+\frac{2a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^2\left(a^4(-x)-2a^3\log(x)-\frac{a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
-((c^2*(-1/3*1/x^3 - a/x^2 - a^4*x - 2*a^3*Log[x]))/a^4)
```

3.783.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.783. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.783.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^2\left(a^4x+2a^3\ln(x)+\frac{1}{3x^3}+\frac{a}{x^2}\right)}{a^4}$
risch	$c^2x+\frac{a\,c^2x+\frac{1}{3}c^2}{a^4x^3}+\frac{2c^2\ln(x)}{a}$
norman	$\frac{c^2x+a^3c^2x^4+\frac{c^2}{3a}}{a^3x^3}+\frac{2c^2\ln(x)}{a}$
parallelrisch	$\frac{3a^4c^2x^4+6c^2\ln(x)a^3x^3+3a\,c^2x+c^2}{3a^4x^3}$
meijerg	$-\frac{c^2(-ax-\ln(-ax+1))}{a}+\frac{2c^2(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{c^2\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+ c^2\ln(-$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/a^4*(a^4*x+2*a^3*ln(x)+1/3/x^3+a/x^2)
```

3.783.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log{(x)}+3\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/(a^4*x^3)
```

3.783.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x+2a^3c^2\log{(x)}+\frac{3ac^2x+c^2}{3x^3}}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**2,x)
```

output
```
(a**4*c**2*x + 2*a**3*c**2*log(x) + (3*a*c**2*x + c**2)/(3*x**3))/a**4
```

3.783.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = c^2x+\frac{2\,c^2\log\left(x\right)}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
c^2*x + 2*c^2*log(x)/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

3.783.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = c^2x+\frac{2\,c^2\log\left(|x|\right)}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
c^2*x + 2*c^2*log(abs(x))/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

3.783.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\left(a\,x+a^4\,x^4+2\,a^3\,x^3\,\ln\left(x\right)+\frac{1}{3}\right)}{a^4\,x^3}$$

input
```
int(((c - c/(a^2*x^2))^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^2*(a*x + a^4*x^4 + 2*a^3*x^3*log(x) + 1/3))/(a^4*x^3)
```

3.784 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.784.1 Optimal result

Integrand size = 20, antiderivative size = 21

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

output
```
-c/a^2/x+c*x+2*c*ln(x)/a
```

3.784.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
-(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

3.784.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{ce^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\int e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c\int \frac{(ax+1)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\int \left(a^2+\frac{2a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(a^2x+2a\log(x)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
(c*(-x^(-1) + a^2*x + 2*a*Log[x]))/a^2
```

3.784. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.784.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.784.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{c\left(a^{2}x+2a\ln\left(x\right)-\frac{1}{x}\right)}{a^{2}}$	22
risch	$-\frac{c}{a^{2}x}+cx+\frac{2c\ln\left(x\right)}{a}$	22
parallelrisch	$\frac{a^{2}c\,x^{2}+2c\ln\left(x\right)ax-c}{a^{2}x}$	27
norman	$\frac{ac\,x^{2}-\frac{c}{a}}{ax}+\frac{2c\ln\left(x\right)}{a}$	30
meijerg	$-\frac{c\left(-ax-\ln\left(-ax+1\right)\right)}{a}+\frac{c\left(-\ln\left(-ax+1\right)+\ln\left(x\right)+\ln\left(-a\right)\right)}{a}+\frac{c\ln\left(-ax+1\right)}{a}-\frac{c\left(\ln\left(-ax+1\right)-\ln\left(x\right)-\ln\left(-a\right)+\frac{1}{ax}\right)}{a}$	86

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
c/a^2*(a^2*x+2*a*ln(x)-1/x)
```

3.784.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx^2+2\,acx\log{(x)}-c}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^2*c*x^2 + 2*a*c*x*log(x) - c)/(a^2*x)
```

3.784.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx+2ac\log{(x)}-\frac{c}{x}}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2),x)
```

output
```
(a**2*c*x + 2*a*c*log(x) - c/x)/a**2
```

3.784.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{2\,c\log{(x)}}{a}-\frac{c}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
c*x + 2*c*log(x)/a - c/(a^2*x)
```

3.784.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{2\,c\log\left(|x|\right)}{a}-\frac{c}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
c*x + 2*c*log(abs(x))/a - c/(a^2*x)
```

3.784.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\,(a^2\,x^2+2\,a\,x\,\ln\left(x\right)-1)}{a^2\,x}$$

input
```
int(((c - c/(a^2*x^2))*(a*x + 1))/(a*x - 1),x)
```

output
```
(c*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

3.785 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.785.1 Optimal result

Integrand size = 22, antiderivative size = 36

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}+\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}$$

output
```
x/c+1/a/c/(-a*x+1)+2*ln(-a*x+1)/a/c
```

3.785.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x+\frac{1}{a-a^2x}+\frac{2\log(1-ax)}{a}}{c}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
(x + (a - a^2*x)^(-1) + (2*Log[1 - a*x])/a)/c
```

3.785.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^2e^{2\text{arctanh}(ax)}}{c\left(a^2-\frac{1}{x^2}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{e^{2\text{arctanh}(ax)}}{a^2-\frac{1}{x^2}}dx}{c}$$

$$\Big\downarrow \text{6707}$$

$$\frac{a^2\int \frac{e^{2\text{arctanh}(ax)}x^2}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^2\int \frac{x^2}{(1-ax)^2}dx}{c}$$

$$\Big\downarrow \text{49}$$

$$\frac{a^2\int \left(\frac{1}{a^2}+\frac{2}{a^2(ax-1)}+\frac{1}{a^2(ax-1)^2}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(\frac{1}{a^3(1-ax)}+\frac{2\log(1-ax)}{a^3}+\frac{x}{a^2}\right)}{c}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
(a^2*(x/a^2 + 1/(a^3*(1 - a*x)) + (2*Log[1 - a*x])/a^3))/c
```

3.785. $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$

3.785.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.785.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{x}{c}-\frac{1}{ac\left(ax-1\right)}+\frac{2\ln\left(ax-1\right)}{ac}$	36
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{1}{a^{3}\left(ax-1\right)}+\frac{2\ln\left(ax-1\right)}{a^{3}}\right)}{c}$	37
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{2x}{c}}{ax-1}+\frac{2\ln\left(ax-1\right)}{ac}$	39
parallelrisch	$\frac{a^{2}x^{2}+2a\ln\left(ax-1\right)x-2ax-2\ln\left(ax-1\right)}{c\left(ax-1\right)a}$	45

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c-1/a/c/(a*x-1)+2/a/c*ln(a*x-1)
```

3.785.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2x^2-ax+2\,(ax-1)\log\,(ax-1)-1}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^2*c*x - a*c)
```

3.785.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=a^2\left(-\frac{1}{a^4cx-a^3c}+\frac{x}{a^2c}+\frac{2\log\,(ax-1)}{a^3c}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2),x)
```

output
```
a**2*(-1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c))
```

3.785.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{a^2cx-ac}+\frac{2\ \log\,(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
x/c - 1/(a^2*c*x - a*c) + 2*log(a*x - 1)/(a*c)
```

3.785.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}+\frac{2\log(|ax-1|)}{ac}-\frac{1}{(ax-1)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
x/c + 2*log(abs(a*x - 1))/(a*c) - 1/((a*x - 1)*a*c)
```

3.785.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}+\frac{1}{a\,(c-a\,c\,x)}+\frac{2\ln(a\,x-1)}{a\,c}$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))*(a*x - 1)),x)
```

output
```
x/c + 1/(a*(c - a*c*x)) + (2*log(a*x - 1))/(a*c)
```

3.786 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.786.1 Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

output
```
x/c^2-1/4/a/c^2/(-a*x+1)^2+7/4/a/c^2/(-a*x+1)+17/8*ln(-a*x+1)/a/c^2-1/8*ln
(a*x+1)/a/c^2
```

3.786.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x]
)/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

3.786.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{a^4e^{2\text{arctanh}(ax)}}{c^2\left(a^2-\frac{1}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 6707}$$

$$-\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{a^4\int \frac{x^4}{(1-ax)^3(ax+1)}dx}{c^2}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^4\int \left(\frac{1}{8a^4(ax+1)}-\frac{1}{a^4}-\frac{17}{8a^4(ax-1)}-\frac{7}{4a^4(ax-1)^2}-\frac{1}{2a^4(ax-1)^3}\right)dx}{c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4\left(-\frac{7}{4a^5(1-ax)}+\frac{1}{4a^5(1-ax)^2}-\frac{17\log(1-ax)}{8a^5}+\frac{\log(ax+1)}{8a^5}-\frac{x}{a^4}\right)}{c^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
-((a^4*(-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1
 - a*x])/(8*a^5) + Log[1 + a*x]/(8*a^5)))/c^2)
```

3.786. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$

3.786.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.786.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{a^{4} \left(-\frac{\ln \left(a x +1\right)}{8 a^{5}}+\frac{x}{a^{4}}+\frac{17 \ln \left(a x -1\right)}{8 a^{5}}-\frac{1}{4 a^{5} \left(a x -1\right)^{2}}-\frac{7}{4 a^{5} \left(a x -1\right)}\right)}{c^{2}}$	60
risch	$\frac{x}{c^{2}}+\frac{-\frac{7 c^{2} x}{4}+\frac{3 c^{2}}{2 a}}{c^{4} \left(a x -1\right)^{2}}-\frac{\ln \left(a x +1\right)}{8 a\, c^{2}}+\frac{17 \ln \left(-a x +1\right)}{8 a\, c^{2}}$	62
norman	$\frac{\frac{a^{3} x^{4}}{c}+\frac{9 x}{4 c}-\frac{5 a\, x^{2}}{4 c}-\frac{5 a^{2} x^{3}}{2 c}}{\left(a x +1\right) c \left(a x -1\right)^{2}}+\frac{17 \ln \left(a x -1\right)}{8 a\, c^{2}}-\frac{\ln \left(a x +1\right)}{8 a\, c^{2}}$	85
parallelrisch	$\frac{8 a^{3} x^{3}+17 a^{2} \ln \left(a x -1\right) x^{2}-a^{2} \ln \left(a x +1\right) x^{2}-28 a^{2} x^{2}-34 a \ln \left(a x -1\right) x +2 a \ln \left(a x +1\right) x +18 a x +17 \ln \left(a x -1\right)-\ln \left(a x +1\right)}{8 c^{2} \left(a x -1\right)^{2} a}$	101

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^4/c^2*(-1/8*ln(a*x+1)/a^5+x/a^4+17/8/a^5*ln(a*x-1)-1/4/a^5/(a*x-1)^2-7/4
/a^5/(a*x-1))
```

3.786.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{8\,a^3x^3-16\,a^2x^2-6\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+17\,(a^2x^2-2\,ax+1)\log(ax-1)+12}{8\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^2*x^2 - 2*a^2*c^2*x +
a*c^2)
```

3.786.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-7ax+6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*((-7*a*x + 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) + x/(a
**4*c**2) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2))
```

3.786.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{7\,ax-6}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}+\frac{x}{c^2}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{17\,\log\left(ax-1\right)}{8\,ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(7*a*x - 6)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 - 1/8*log(a*x
 + 1)/(a*c^2) + 17/8*log(a*x - 1)/(a*c^2)
```

3.786.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{\log\left(\left|ax+1\right|\right)}{8\,ac^2}+\frac{17\,\log\left(\left|ax-1\right|\right)}{8\,ac^2}-\frac{7\,ax-6}{4\left(ax-1\right)^2ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
x/c^2 - 1/8*log(abs(a*x + 1))/(a*c^2) + 17/8*log(abs(a*x - 1))/(a*c^2) - 1
/4*(7*a*x - 6)/((a*x - 1)^2*a*c^2)
```

3.786.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}+\frac{17\,\ln\left(a\,x-1\right)}{8\,a\,c^2}-\frac{\ln\left(a\,x+1\right)}{8\,a\,c^2}$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^2*(a*x - 1)),x)
```

output
```
x/c^2 - ((7*x)/4 - 3/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (17*log(a*x
- 1))/(8*a*c^2) - log(a*x + 1)/(8*a*c^2)
```

3.787 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.787.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}+\frac{1}{12ac^3(1-ax)^3}-\frac{5}{8ac^3(1-ax)^2}+\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

output
```
x/c^3+1/12/a/c^3/(-a*x+1)^3-5/8/a/c^3/(-a*x+1)^2+39/16/a/c^3/(-a*x+1)-1/16
/a/c^3/(a*x+1)+9/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

3.787.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{\frac{2(-11+7ax+24a^2x^2-15a^3x^3-12a^4x^4+6a^5x^5)}{(-1+ax)^3(1+ax)}+27\log(1-ax)-3\log(1+ax)}{12ac^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
((2*(-11 + 7*a*x + 24*a^2*x^2 - 15*a^3*x^3 - 12*a^4*x^4 + 6*a^5*x^5))/((-1
 + a*x)^3*(1 + a*x)) + 27*Log[1 - a*x] - 3*Log[1 + a*x])/(12*a*c^3)
```

3.787.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^6e^{2\text{arctanh}(ax)}}{c^3\left(a^2-\frac{1}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^6\int \frac{e^{2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6707}$$

$$\frac{a^6\int \frac{e^{2\text{arctanh}(ax)}x^6}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^6\int \frac{x^6}{(1-ax)^4(ax+1)^2}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^6\int \left(-\frac{1}{4a^6(ax+1)}+\frac{1}{16a^6(ax+1)^2}+\frac{1}{a^6}+\frac{9}{4a^6(ax-1)}+\frac{39}{16a^6(ax-1)^2}+\frac{5}{4a^6(ax-1)^3}+\frac{1}{4a^6(ax-1)^4}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6\left(\frac{39}{16a^7(1-ax)}-\frac{1}{16a^7(ax+1)}-\frac{5}{8a^7(1-ax)^2}+\frac{1}{12a^7(1-ax)^3}+\frac{9\log(1-ax)}{4a^7}-\frac{\log(ax+1)}{4a^7}+\frac{x}{a^6}\right)}{c^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
(a^6*(x/a^6 + 1/(12*a^7*(1 - a*x)^3) - 5/(8*a^7*(1 - a*x)^2) + 39/(16*a^7*
(1 - a*x)) - 1/(16*a^7*(1 + a*x)) + (9*Log[1 - a*x])/(4*a^7) - Log[1 + a*x
]/(4*a^7)))/c^3
```

3.787.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.787. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.787.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

method	result
default	$\frac{a^{6}\left(-\frac{\ln(ax+1)}{4a^{7}}-\frac{1}{16a^{7}(ax+1)}+\frac{x}{a^{6}}-\frac{39}{16a^{7}(ax-1)}+\frac{9\ln(ax-1)}{4a^{7}}-\frac{1}{12a^{7}(ax-1)^{3}}-\frac{5}{8a^{7}(ax-1)^{2}}\right)}{c^{3}}$
risch	$\frac{x}{c^{3}}+\frac{-\frac{5a^{2}c^{3}x^{3}}{2}+2a\,c^{3}x^{2}+\frac{13c^{3}x}{6}-\frac{11c^{3}}{6a}}{c^{6}(ax-1)^{2}(a^{2}x^{2}-1)}+\frac{9\ln(-ax+1)}{4a\,c^{3}}-\frac{\ln(ax+1)}{4a\,c^{3}}$
norman	$\frac{\frac{a^{5}x^{6}}{c}-\frac{5x}{2c}+\frac{3a\,x^{2}}{2c}+\frac{31a^{2}x^{3}}{6c}-\frac{8a^{3}x^{4}}{3c}-\frac{17a^{4}x^{5}}{6c}}{(ax-1)^{3}(ax+1)^{2}c^{2}}+\frac{9\ln(ax-1)}{4a\,c^{3}}-\frac{\ln(ax+1)}{4a\,c^{3}}$
parallelrisch	$\frac{12a^{5}x^{5}+27\ln(ax-1)x^{4}a^{4}-3\ln(ax+1)x^{4}a^{4}-46a^{4}x^{4}-54a^{3}\ln(ax-1)x^{3}+6a^{3}\ln(ax+1)x^{3}+14a^{3}x^{3}+48a^{2}x^{2}+54a\ln(ax-1)x-}{12c^{3}(ax-1)^{2}(a^{2}x^{2}-1)a}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
a^6/c^3*(-1/4*ln(a*x+1)/a^7-1/16/a^7/(a*x+1)+x/a^6-39/16/a^7/(a*x-1)+9/4/a
^7*ln(a*x-1)-1/12/a^7/(a*x-1)^3-5/8/a^7/(a*x-1)^2)
```

3.787.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{3}}\,dx$$
$$=\frac{12\,a^{5}x^{5}-24\,a^{4}x^{4}-30\,a^{3}x^{3}+48\,a^{2}x^{2}+14\,ax-3\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1\right)\log\left(ax+1\right)+27\left(a^{4}x^{4}-2\right.}{12\left(a^{5}c^{3}x^{4}-2\,a^{4}c^{3}x^{3}+2\,a^{2}c^{3}x-ac^{3}\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/12*(12*a^5*x^5 - 24*a^4*x^4 - 30*a^3*x^3 + 48*a^2*x^2 + 14*a*x - 3*(a^4*
x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x + 1) + 27*(a^4*x^4 - 2*a^3*x^3 + 2*a*
x - 1)*log(a*x - 1) - 22)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c
^3)
```

3.787.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-15a^3x^3+12a^2x^2+13ax-11}{6a^{11}c^3x^4-12a^{10}c^3x^3+12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{9\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*((-15*a**3*x**3 + 12*a**2*x**2 + 13*a*x - 11)/(6*a**11*c**3*x**4 - 12
*a**10*c**3*x**3 + 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (9*log(
x - 1/a)/4 - log(x + 1/a)/4)/(a**7*c**3))
```

3.787.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{\log(ax+1)}{4\,ac^3}+\frac{9\,\log(ax-1)}{4\,ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3
+ 2*a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 9/4*log(a*x -
1)/(a*c^3)
```

3.787.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}-\frac{\log(|ax+1|)}{4\,ac^3}+\frac{9\,\log(|ax-1|)}{4\,ac^3}-\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\,(ax+1)(ax-1)^3ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 9/4*log(abs(a*x - 1))/(a*c^3) - 1/
6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

3.787.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}-\frac{\frac{13\,x}{6}+2\,a\,x^2-\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}+\frac{9\,\ln(a\,x-1)}{4\,a\,c^3}-\frac{\ln(a\,x+1)}{4\,a\,c^3}$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^3*(a*x - 1)),x)
```

output
```
x/c^3 - ((13*x)/6 + 2*a*x^2 - 11/(6*a) - (5*a^2*x^3)/2)/(c^3 + 2*a^3*c^3*x
^3 - a^4*c^3*x^4 - 2*a*c^3*x) + (9*log(a*x - 1))/(4*a*c^3) - log(a*x + 1)/
(4*a*c^3)
```

3.788 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.788.1 Optimal result

Integrand size $=22$, antiderivative size $=145$

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{x}{c^4}-\frac{1}{32ac^4(1-ax)^4}+\frac{13}{48ac^4(1-ax)^3}-\frac{35}{32ac^4(1-ax)^2}+\frac{99}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}-\frac{11}{64ac^4(1+ax)}+\frac{303\log(1-ax)}{128ac^4}-\frac{47\log(1+ax)}{128ac^4}$$

output
```
x/c^4-1/32/a/c^4/(-a*x+1)^4+13/48/a/c^4/(-a*x+1)^3-35/32/a/c^4/(-a*x+1)^2+
99/32/a/c^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2-11/64/a/c^4/(a*x+1)+303/128*ln(-
a*x+1)/a/c^4-47/128*ln(a*x+1)/a/c^4
```

3.788.2 Mathematica [A] (verified)

Time $=0.12$ (sec) , antiderivative size $=98$, normalized size of antiderivative $=0.68$

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{\frac{2(400-275ax-1258a^2x^2+866a^3x^3+1254a^4x^4-819a^5x^5-384a^6x^6+192a^7x^7)}{(-1+ax)^4(1+ax)^2}+909\log(1-ax)-141\log(1+ax)}{384ac^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
((2*(400 - 275*a*x - 1258*a^2*x^2 + 866*a^3*x^3 + 1254*a^4*x^4 - 819*a^5*x
^5 - 384*a^6*x^6 + 192*a^7*x^7))/((-1 + a*x)^4*(1 + a*x)^2) + 909*Log[1 -
a*x] - 141*Log[1 + a*x])/(384*a*c^4)
```

3.788.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^8 e^{2\text{arctanh}(ax)}}{c^4\left(a^2-\frac{1}{x^2}\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^8\int \frac{e^{2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^8\int \frac{e^{2\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{a^8\int \frac{x^8}{(1-ax)^5(ax+1)^3}dx}{c^4}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^8\int \left(\frac{47}{128a^8(ax+1)}-\frac{11}{64a^8(ax+1)^2}+\frac{1}{32a^8(ax+1)^3}-\frac{1}{a^8}-\frac{303}{128a^8(ax-1)}-\frac{99}{32a^8(ax-1)^2}-\frac{35}{16a^8(ax-1)^3}-\frac{13}{16a^8(ax-1)^4}-\frac{}{8a^8(}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8\left(-\frac{99}{32a^9(1-ax)}+\frac{11}{64a^9(ax+1)}+\frac{35}{32a^9(1-ax)^2}-\frac{1}{64a^9(ax+1)^2}-\frac{13}{48a^9(1-ax)^3}+\frac{1}{32a^9(1-ax)^4}-\frac{303\log(1-ax)}{128a^9}+\frac{47\log(ax+1)}{128a^9}\right.}{c^4}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
-((a^8*(-(x/a^8) + 1/(32*a^9*(1 - a*x)^4) - 13/(48*a^9*(1 - a*x)^3) + 35/(
32*a^9*(1 - a*x)^2) - 99/(32*a^9*(1 - a*x)) - 1/(64*a^9*(1 + a*x)^2) + 11/
(64*a^9*(1 + a*x)) - (303*Log[1 - a*x])/(128*a^9) + (47*Log[1 + a*x])/(128
*a^9)))/c^4)
```

3.788.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.788. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.788.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
default	$\frac{a^{8}\left(\frac{1}{64a^{9}(ax+1)^{2}}-\frac{11}{64a^{9}(ax+1)}-\frac{47\ln(ax+1)}{128a^{9}}+\frac{x}{a^{8}}-\frac{1}{32a^{9}(ax-1)^{4}}-\frac{13}{48a^{9}(ax-1)^{3}}-\frac{35}{32a^{9}(ax-1)^{2}}-\frac{99}{32a^{9}(ax-1)}+\frac{303\ln(ax-1)}{128a^{9}}\right)}{c^{4}}$
risch	$\frac{x}{c^{4}}+\frac{-\frac{209a^{4}c^{4}x^{5}}{64}+\frac{81a^{3}c^{4}x^{4}}{32}+\frac{529a^{2}c^{4}x^{3}}{96}-\frac{437a\,c^{4}x^{2}}{96}-\frac{467c^{4}x}{192}+\frac{25c^{4}}{12a}}{c^{8}(ax-1)^{2}(a^{2}x^{2}-1)^{2}}+\frac{303\ln(-ax+1)}{128a\,c^{4}}-\frac{47\ln(ax+1)}{128a\,c^{4}}$
norman	$\frac{\frac{a^{7}x^{8}}{c}+\frac{175x}{64c}-\frac{111a\,x^{2}}{64c}-\frac{199a^{2}x^{3}}{24c}+\frac{115a^{3}x^{4}}{24c}+\frac{545a^{4}x^{5}}{64c}-\frac{803a^{5}x^{6}}{192c}-\frac{37a^{6}x^{7}}{12c}}{c^{3}(ax-1)^{4}(ax+1)^{3}}+\frac{303\ln(ax-1)}{128a\,c^{4}}-\frac{47\ln(ax+1)}{128a\,c^{4}}$
parallelrisch	$\underline{282a\ln(ax+1)x+141a^{2}\ln(ax+1)x^{2}-38a^{5}x^{5}-1468a^{3}x^{3}+282\ln(ax+1)x^{5}a^{5}-141\ln(ax+1)x^{6}a^{6}+141\ln(ax+1)x^{4}a^{4}+909\ln(}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
a^8/c^4*(1/64/a^9/(a*x+1)^2-11/64/a^9/(a*x+1)-47/128/a^9*ln(a*x+1)+1/a^8*x
-1/32/a^9/(a*x-1)^4-13/48/a^9/(a*x-1)^3-35/32/a^9/(a*x-1)^2-99/32/a^9/(a*x
-1)+303/128/a^9*ln(a*x-1))
```

3.788.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.61

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{4}}\,dx$$
$$=\frac{384\,a^{7}x^{7}-768\,a^{6}x^{6}-1638\,a^{5}x^{5}+2508\,a^{4}x^{4}+1732\,a^{3}x^{3}-2516\,a^{2}x^{2}-550\,ax-141\,(a^{6}x^{6}-2\,a^{5}x^{5}-a}{384\,(a^{7}c^{4}x^{6}-2\,a^{6}c^{4}x^{5}-a^{5}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/384*(384*a^7*x^7 - 768*a^6*x^6 - 1638*a^5*x^5 + 2508*a^4*x^4 + 1732*a^3*
x^3 - 2516*a^2*x^2 - 550*a*x - 141*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*
x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 909*(a^6*x^6 - 2*a^5*x^5 - a^4*x
^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 800)/(a^7*c^4*x^6 - 2
*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a
*c^4)
```

3.788.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$=a^8\left(\frac{-627a^5x^5+486a^4x^4+1058a^3x^3-874a^2x^2-467ax+400}{192a^{15}c^4x^6-384a^{14}c^4x^5-192a^{13}c^4x^4+768a^{12}c^4x^3-192a^{11}c^4x^2-384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{303\log\left(x-\frac{1}{a}\right)}{128}-\frac{47\log\left(x+\frac{1}{a}\right)}{128}}{a^9c^4}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*((-627*a**5*x**5 + 486*a**4*x**4 + 1058*a**3*x**3 - 874*a**2*x**2 - 4
67*a*x + 400)/(192*a**15*c**4*x**6 - 384*a**14*c**4*x**5 - 192*a**13*c**4*
x**4 + 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 - 384*a**10*c**4*x + 192*
a**9*c**4) + x/(a**8*c**4) + (303*log(x - 1/a)/128 - 47*log(x + 1/a)/128)/
(a**9*c**4))
```

3.788.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=-\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}+\frac{x}{c^4}-\frac{47\,\log(ax+1)}{128\,ac^4}+\frac{303\,\log(ax-1)}{128\,ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x -
 400)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4
*x^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 47/128*log(a*x + 1)/(a*c^4) + 303/12
8*log(a*x - 1)/(a*c^4)
```

3.788.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4} - \frac{47\log\left(|ax+1|\right)}{128\,ac^4} + \frac{303\log\left(|ax-1|\right)}{128\,ac^4} - \frac{627\,a^5x^5 - 486\,a^4x^4 - 1058\,a^3x^3 + 874\,a^2x^2 + 467\,ax - 400}{192\,(ax+1)^2(ax-1)^4ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
x/c^4 - 47/128*log(abs(a*x + 1))/(a*c^4) + 303/128*log(abs(a*x - 1))/(a*c^
4) - 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a
*x - 400)/((a*x + 1)^2*(a*x - 1)^4*a*c^4)
```

3.788.9 Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4} + \frac{\frac{467\,x}{192} + \frac{437\,a\,x^2}{96} - \frac{25}{12\,a} - \frac{529\,a^2\,x^3}{96} - \frac{81\,a^3\,x^4}{32} + \frac{209\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6 + 2\,a^5\,c^4\,x^5 + a^4\,c^4\,x^4 - 4\,a^3\,c^4\,x^3 + a^2\,c^4\,x^2 + 2\,a\,c^4\,x - c^4} + \frac{303\,\ln\left(a\,x-1\right)}{128\,a\,c^4} - \frac{47\,\ln\left(a\,x+1\right)}{128\,a\,c^4}$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^4*(a*x - 1)),x)
```

output
```
x/c^4 + ((467*x)/192 + (437*a*x^2)/96 - 25/(12*a) - (529*a^2*x^3)/96 - (81
*a^3*x^4)/32 + (209*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*
c^4*x^4 + 2*a^5*c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) + (303*log(a*x - 1))/(1
28*a*c^4) - (47*log(a*x + 1))/(128*a*c^4)
```

3.789 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.789.1 Optimal result

Integrand size = 22, antiderivative size = 343

$$\begin{aligned}
&\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx \\
&= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}-\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} \\
&\quad -\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} \\
&\quad +\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} \\
&\quad +c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x+\frac{15c^4\csc^{-1}(ax)}{16a}+\frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}$$

output
```
8/7*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)/a+c^4*(1-1/a/x)^(5/2)*(1+1/a/x)^(
11/2)*x+15/16*c^4*arccsc(a*x)/a+3*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1
/2))/a-37/16*c^4*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-61/40*c^4*(1+1/a/x)^(5/
2)*(1-1/a/x)^(1/2)/a-303/280*c^4*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-57/70*c
^4*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a+15/14*c^4*(1+1/a/x)^(11/2)*(1-1/a/x)^
(1/2)/a-63/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.789.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.37

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}(-80-280ax-96a^2x^2+770a^3x^3+992a^4x^4-525a^5x^5-2496a^6x^6+560a^7x^7)+525a^6x}{560a^7x^6}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
(c^4*(Sqrt[1 - 1/(a^2*x^2)]*(-80 - 280*a*x - 96*a^2*x^2 + 770*a^3*x^3 + 99
2*a^4*x^4 - 525*a^5*x^5 - 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin
[1/(a*x)] + 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6
)
```

3.789.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.98, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.091, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 25, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{3\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6748}$$
$$-c^4\int \left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{108}$$
$$-c^4\left(\int \frac{\left(3a-\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}\right)$$
$$\Big\downarrow \text{27}$$
$$-c^4\left(\frac{\int \left(3a-\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}\right)$$

$$\Big\downarrow 171$$

$$-c^4\left(\frac{\frac{1}{7}a\int \frac{3\left(7a-\frac{15}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x}{a}d\frac{1}{x}-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}\right)$$

$$\Big\downarrow 27$$

$$-c^4\left(\frac{\frac{3}{7}\int\left(7a-\frac{15}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}xd\frac{1}{x}-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}\right)$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{6}a\int\frac{3\left(14a-\frac{19}{x}\right)\left(1+\frac{1}{ax}\right)^{9/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\int\frac{\left(14a-\frac{19}{x}\right)\left(1+\frac{1}{ax}\right)^{9/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\Bigg($$

$$\Big\downarrow 171$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{5}a\int-\frac{\left(70a-\frac{101}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)}{a^2}$$

$$\Big\downarrow 25$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}a\int\frac{\left(70a-\frac{101}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)}{a^2}$$

$$\Big\downarrow 27$$

3.789. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\int\frac{\left(70a-\frac{101}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}}{a^2}$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{101}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{4}a\int-\frac{7\left(40a-\frac{61}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{5}{2}a\sqrt{1-}\right.}{a^2}$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\int\frac{\left(40a-\frac{61}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{101}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)+\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\right.}{a^2}$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{61}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int-\frac{5\left(24a-\frac{37}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{101}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)+\frac{19}{5}a\right.\right.}{a^2}$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\int\frac{\left(24a-\frac{37}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{61}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{101}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)+\frac{19}{5}a\sqrt{1-}\right.\right.}{a^2}$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int-\frac{3\left(16a-\frac{21}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{61}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{101}{4}a\right.\right.\right.}{}$$

3.789. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

$$\Big\downarrow 27$$

$$-c^4\Bigg(\frac{\frac{3}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\int\frac{\left(16a-\frac{21}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+\frac{61}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\Bigg)+\frac{101}{4}a\sqrt{1}{a}$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(\frac{\frac{3}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int-\frac{\left(16a-\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\Bigg)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+\frac{61}{3}a\sqrt{1}{}$$

$$\Big\downarrow 25$$

$$-c^4\Bigg(\frac{\frac{3}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(a\int\frac{\left(16a-\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+\frac{61}{3}a\sqrt{1-}}{}$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(\frac{\frac{3}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(\int\frac{\left(16a-\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+\frac{61}{3}a\sqrt{1-\frac{1}{a}}}{}$$

$$\Big\downarrow 175$$

$$-c^4\Bigg(\frac{\frac{3}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-5\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\Big(}{}$$

$$\Big\downarrow 39$$

3.789. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx$

$$-c^4\Bigg(\frac{3}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-5\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\Big(\frac{1}{ax}$$

$$\Big\downarrow \ 103$$

$$-c^4\Bigg(\frac{3}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-5\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{37}{2}a\sqrt{}$$

$$\Big\downarrow \ 221$$

$$-c^4\Bigg(\frac{3}{7}\Bigg(\frac{1}{2}\Bigg(\frac{1}{5}\Bigg(\frac{7}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(-5\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{37}{2}a\sqrt{1}$$

$$\Big\downarrow \ 223$$

$$-c^4\Bigg(\frac{3}{7}\bigg(\frac{1}{2}\bigg(\frac{1}{5}\bigg(\frac{7}{4}\bigg(\frac{5}{3}\bigg(\frac{3}{2}\bigg(-5a\arcsin\left(\frac{1}{ax}\right)-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\bigg)+\frac{37}{2}a\sqrt{1-}$$

input

```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output

```
-(c^4*(-((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(11/2)*x) + ((-8*a*(1 - 1/(a*x)
)^(3/2)*(1 + 1/(a*x))^(11/2))/7 + (3*((-5*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x)
)^(11/2))/2 + ((19*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/5 + ((101*a*Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/4 + (7*((61*a*Sqrt[1 - 1/(a*x)]*(1 +
1/(a*x))^(5/2))/3 + (5*((37*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (
3*(21*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] - 5*a*ArcSin[1/(a*x)] - 16*a*A
rcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2))/3))/4)/5)/2))/7)/a^2))
```

3.789. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.789.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.789. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.789.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(ax-1)\left(560a^7x^7-2496a^6x^6-525a^5x^5+992a^4x^4+770a^3x^3-96a^2x^2-280ax-80\right)c^4}{560x^7a^8\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{15a^7\arctan\left(\frac{}{\sqrt{a}}\right.}{16}\right.}{a^8(ax+1)\sqrt{\frac{ax-}{ax+}}}$
default	$\frac{(ax-1)^2c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+525a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}+525a^7x^7\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+168\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/560*(a*x-1)*(560*a^7*x^7-2496*a^6*x^6-525*a^5*x^5+992*a^4*x^4+770*a^3*x^
3-96*a^2*x^2-280*a*x-80)/x^7*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)+(3*a^8*ln(a^2
*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+15/16*a^7*arctan(1/(a^2*x^2-
1)^(1/2)))*c^4/a^8/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.789.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right) - (560\,a}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
-1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 - 1936*a^7*c^4*x^7 - 3021*a^6*c^4*x^
6 + 467*a^5*c^4*x^5 + 1762*a^4*c^4*x^4 + 674*a^3*c^4*x^3 - 376*a^2*c^4*x^2
 - 360*a*c^4*x - 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

3.789.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(\int\left(-\frac{4a^2}{\frac{ax^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^4}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}dx+\int\left(-\frac{4a^6}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2}{}}\right.\right.}{a^8}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**4,x)
```

output
```
c**4*(Integral(-4*a**2/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
 - x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4
/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6/(a*x**3*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a*x + 1)), x) + Integral(a**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*
*9*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**8*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x))/a**8
```

3.789.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.11

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx =$$

$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2205\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2 - (2205*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 13615*c^4*((a*x - 1)/(a*x +
 1))^(13/2) + 33621*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 39071*c^4*((a*x - 1
)/(a*x + 1))^(9/2) + 12799*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 20811*c^4*((a
*x - 1)/(a*x + 1))^(5/2) - 7665*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 1155*c^4
*sqrt((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^
2/(a*x + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x +
 1)^5 - 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (
a*x - 1)^8*a^2/(a*x + 1)^8 + a^2))*a
```

3.789.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.34

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$

$$=-\frac{15\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{8\,a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^4}{a\mathrm{sgn}\left(ax+1\right)}$$

$$+\frac{525\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a|-4480\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4-980\left(x|a|-\sqrt{a^2x^2-1}\right)^{11}c^4|a|-2016}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
-15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^4*l
og(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^
2 - 1)*c^4/(a*sgn(a*x + 1)) + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13
*c^4*abs(a) - 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4 - 980*(x*abs(a)
 - sqrt(a^2*x^2 - 1))^11*c^4*abs(a) - 20160*(x*abs(a) - sqrt(a^2*x^2 - 1))
^10*a*c^4 + 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^4*abs(a) - 38080*(x*abs
(a) - sqrt(a^2*x^2 - 1))^8*a*c^4 - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*
a*c^4 - 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) - 32256*(x*abs(a)
- sqrt(a^2*x^2 - 1))^4*a*c^4 + 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*ab
s(a) - 12992*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4 - 525*(x*abs(a) - sqrt
(a^2*x^2 - 1))*c^4*abs(a) - 2496*a*c^4)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2
 + 1)^7*a*abs(a)*sgn(a*x + 1))
```

3.789.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{\frac{12799\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{280}-\frac{219\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8}-\frac{2973\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}-\frac{33\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{39071\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}+\frac{4803\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+38}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-a(}$$
$$-\frac{15\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}+\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((12799*c^4*((a*x - 1)/(a*x + 1))^(7/2))/280 - (219*c^4*((a*x - 1)/(a*x +
1))^(3/2))/8 - (2973*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 - (33*c^4*((a*x -
 1)/(a*x + 1))^(1/2))/8 + (39071*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (4
803*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (389*c^4*((a*x - 1)/(a*x + 1))^
(13/2))/8 + (63*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/
(a*x + 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^
3 - (14*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a
*(a*x - 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((
a*x - 1)/(a*x + 1))^(1/2)))/(8*a) + (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/
2)))/a
```

3.790 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.790.1 Optimal result

Integrand size = 22, antiderivative size = 269

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx\\ &=-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}\\ &\quad-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}\\ &\quad+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x+\frac{3c^3\csc^{-1}(ax)}{8a}+\frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

output
```
c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x+3/8*c^3*arccsc(a*x)/a+3*c^3*arctanh(
(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-17/8*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2
)/a-29/20*c^3*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-21/20*c^3*(1+1/a/x)^(7/2)*
(1-1/a/x)^(1/2)/a+6/5*c^3*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a-27/8*c^3*(1-1/
a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.790.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(8+30ax+24a^2x^2-55a^3x^3-152a^4x^4+40a^5x^5)+15a^4x^4\arcsin\left(\frac{1}{ax}\right)+120a^4x^4\log\left(\left(1\right.\right.\right.}{40a^5x^4}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
(c^3*(Sqrt[1 - 1/(a^2*x^2)]*(8 + 30*a*x + 24*a^2*x^2 - 55*a^3*x^3 - 152*a^
4*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] + 120*a^4*x^4*Log[(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

3.790.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.98, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {6748, 108, 27, 171, 27, 171, 25, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-c^3\int \left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^3\left(\int \frac{3\left(a-\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^3\left(\frac{3\int \left(a-\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow 171$$

$$-c^3\left(\frac{3\left(\frac{1}{5}a\int\frac{\left(5a-\frac{7}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow 27$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\int\frac{\left(5a-\frac{7}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow 171$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{4}a\int-\frac{\left(20a-\frac{29}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\right.$$

$$\Big\downarrow 25$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}a\int\frac{\left(20a-\frac{29}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\right.$$

$$\Big\downarrow 27$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\int\frac{\left(20a-\frac{29}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\right.$$

$$\Big\downarrow 171$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int-\frac{5\left(12a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{2}{5}a\sqrt{1-\right.}{a^2}\right.$$

3.790. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

$$\Big\downarrow \text{27}$$

$$-c^3\Bigg(\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{5}{3}\int \frac{\left(12a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\Bigg)+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\Bigg)-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}(}{a^2}$$

$$\Big\downarrow \text{171}$$

$$-c^3\Bigg(\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{5}{3}\Bigg(\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int -\frac{3\left(8a-\frac{9}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\Bigg)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\Bigg)+\frac{7}{4}a\sqrt{1-}{a^2}$$

$$\Big\downarrow \text{27}$$

$$-c^3\Bigg(\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\int \frac{\left(8a-\frac{9}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\Bigg)+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}(}{a^2}$$

$$\Big\downarrow \text{171}$$

$$-c^3\Bigg(\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(8a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\Bigg)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}}{a^2}$$

$$\Big\downarrow \text{25}$$

$$-c^3\Bigg(\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{5}{3}\Bigg(\frac{3}{2}\Bigg(a\int \frac{\left(8a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}(}{a^2}$$

$$\Big\downarrow \text{27}$$

3.790. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{\left(8a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \text{175}$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(8a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \text{39}$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \text{103}$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \text{221}$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\right.\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \text{223}$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(a\left(-\arcsin\left(\frac{1}{ax}\right)\right)-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.\right.\right.\right.\right.}{a^2}\right.$$

3.790. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
-(c^3*(-((1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)*x) + (3*((-2*a*Sqrt[1 - 1
/(a*x)]*(1 + 1/(a*x))^(9/2))/5 + ((7*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/
2))/4 + ((29*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2))/3 + (5*((17*a*Sqrt[1
 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (3*(9*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)] - a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x
)]]))/2))/3)/4)/5))/a^2))
```

3.790.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.790.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{(ax-1)\left(152a^4x^4+55a^3x^3-24a^2x^2-30ax-8\right)c^3}{40x^5a^6\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}+a^5\sqrt{(ax-1)(ax+1)}\right)c^3}{a^6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+15a^5x^5\sqrt{a^2}\,\sqrt{a^2x^2-1}+15a^5x^5\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+120\ln(\right.}{40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(a*x-1)*(152*a^4*x^4+55*a^3*x^3-24*a^2*x^2-30*a*x-8)/x^5*c^3/a^6/((a
*x-1)/(a*x+1))^(1/2)+(3*a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^
(1/2)+3/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^5*((a*x-1)*(a*x+1))^(1/2))*c^3
/a^6/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.790.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx =$$

$$-\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(40\,a^6c^3x^6}{40\,a^6x^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
-1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5*
log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (40*a^6*c^3*x^6 - 112*a^5*c^3*x^5 - 207*a^4*c^3*x^4 - 31*a
^3*c^3*x^3 + 54*a^2*c^3*x^2 + 38*a*c^3*x + 8*c^3)*sqrt((a*x - 1)/(a*x + 1)
))/(a^6*x^5)
```

3.790.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$

$$=\frac{c^3\left(\int\frac{3a^2}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^4}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-}}{ax+1}}}{a^6}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**3,x)
```

output
```
c**3*(Integral(3*a**2/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4
/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**6/(a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x) + Integral(-1/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**
6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**6
```

3.790.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.12

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx =$$

$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{135\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+57}{4\,(}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
- (135*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 575*c^3*((a*x - 1)/(a*x + 1))^(9
/2) + 842*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 298*c^3*((a*x - 1)/(a*x + 1))^
(5/2) - 465*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x
+ 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x
 - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/
(a*x + 1)^6 + a^2))*a
```

3.790.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.32

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$

$$=-\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,a\text{sgn}\left(ax+1\right)}-\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^3}{a\text{sgn}\left(ax+1\right)}$$

$$+\frac{55\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|-200\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3-10\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a|-720\left(x|a|\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
-3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^3*lo
g(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2
 - 1)*c^3/(a*sgn(a*x + 1)) + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3
*abs(a) - 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3 - 10*(x*abs(a) - sqrt
(a^2*x^2 - 1))^7*c^3*abs(a) - 720*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^3 -
 800*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^3 + 10*(x*abs(a) - sqrt(a^2*x^2
- 1))^3*c^3*abs(a) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - 55*(x*ab
s(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a) - 152*a*c^3)/(((x*abs(a) - sqrt(a^2*x
^2 - 1))^2 + 1)^5*a*abs(a)*sgn(a*x + 1))
```

3.790.9 Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{\frac{149\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{10}-\frac{93\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4}-\frac{21\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{421\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{10}+\frac{115\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}+\frac{27\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{4}}{a+\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{5\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{a\,(a\,x-1)^6}{(a\,x+1)^6}}$$
$$-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+\frac{6\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((149*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^
(3/2))/4 - (21*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (421*c^3*((a*x - 1)/(a
*x + 1))^(7/2))/10 + (115*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 + (27*c^3*((a
*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x -
 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*
x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1)
)^(1/2)))/(4*a) + (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.791 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.791.1 Optimal result

Integrand size = 22, antiderivative size = 195

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = &-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}\\ &-\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}+c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x\\ &-\frac{c^2\csc^{-1}(ax)}{2a}+\frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

output
```
-1/2*c^2*arccsc(a*x)/a+3*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-11
/6*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-4/3*c^2*(1+1/a/x)^(5/2)*(1-1/a/x)
^(1/2)/a+c^2*(1+1/a/x)^(7/2)*x*(1-1/a/x)^(1/2)-5/2*c^2*(1-1/a/x)^(1/2)*(1+
1/a/x)^(1/2)/a
```

3.791.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\begin{aligned}&\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx\\ &=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(-2-9ax-16a^2x^2+6a^3x^3\right)-3a^2x^2\arcsin\left(\frac{1}{ax}\right)+18a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}\end{aligned}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 - 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*
x^2*ArcSin[1/(a*x)] + 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a
^3*x^2)
```

3.791.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {6748, 108, 27, 171, 25, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6748}$$

$$-c^2\int \sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^2\left(\int \frac{\left(3a-\frac{4}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a^2\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(\frac{\int \frac{\left(3a-\frac{4}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(\frac{\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int -\frac{\left(9a-\frac{11}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c^2\left(\frac{\frac{1}{3}a\int\frac{\left(9a-\frac{11}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^2\left(\frac{\frac{1}{3}\int\frac{\left(9a-\frac{11}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \; 171$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int-\frac{3\left(6a-\frac{5}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)\right.$$

$$\Big\downarrow \; 27$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(6a-\frac{5}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \; 171$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int-\frac{\left(6a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/}}{a^2}\right.$$

$$\Big\downarrow \; 25$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\int\frac{\left(6a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}\right.$$

$$\Big\downarrow \; 27$$

3.791. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{\left(6a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-\right.$$

$$\Big\downarrow \text{175}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(6a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}}{a^2}\right.$$

$$\Big\downarrow \text{39}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+6a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1}}{a^2}\right.$$

$$\Big\downarrow \text{103}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-6\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{a^2}\right.$$

$$\Big\downarrow \text{221}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-6a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2}\right.$$

$$\Big\downarrow \text{223}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\arcsin\left(\frac{1}{ax}\right)-6a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+}{a^2}\right.$$

3.791. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
-(c^2*(-(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x) + ((4*a*Sqrt[1 - 1/(a*x)
]*(1 + 1/(a*x))^(5/2))/3 + ((11*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2
 + (3*(5*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] + a*ArcSin[1/(a*x)] - 6*a*A
rcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2)/3)/a^2))
```

3.791.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.791.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(ax-1)\left(16a^{2}x^{2}+9ax+2\right)c^{2}}{6x^{3}a^{4}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^{4}\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{\sqrt{a^{2}}}-\frac{a^{3}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}+a^{3}\sqrt{(ax-1)(ax+1)}\right)c^{2}\sqrt{(ax-1)(ax+1)}}{a^{4}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^{2}c^{2}\left(-18\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{4}x^{4}+18\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,a^{2}x^{2}-3\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{3}x^{3}-3a^{3}x^{3}\sqrt{a^{2}}\,\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+18\ln\left(a^{2}x+\right.\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^{4}x^{3}\sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x-1)*(16*a^2*x^2+9*a*x+2)/x^3*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)+(3*a
^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-1/2*a^3*arctan(1/(a
^2*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4/(a*x+1)/((a*x-1)/(a*
x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.791.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^2x^4-10\,a^3}{6\,a^4x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 18*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 25*a^2*c^2*x^2 - 11*a*c^2*x -
 2*c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

3.791.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{c^2\left(\int\left(-\frac{2a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-}}{ax+1}}\right.}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**2,x)
```

output
```
c**2*(Integral(-2*a**2/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
 - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4/(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a
**4
```

3.791.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.14

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{1}{3}a\left(\frac{3\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{9\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{9\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+37\,c^2\left(\frac{ax-1}{ax+1}\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax}{(ax}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 9*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 - 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (
15*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 37*c^2*((a*x - 1)/(a*x + 1))^(5/2) +
17*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 21*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/
(a*x + 1)^4 + a^2))
```

3.791.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.27

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^2}{a\mathrm{sgn}\left(ax+1\right)}$$
$$+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2-36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2}\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)
*c^2/(a*sgn(a*x + 1)) + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)
 - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2 - 36*(x*abs(a) - sqrt(a^2*x^2
 - 1))^2*a*c^2 - 9*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^2*abs(a) - 16*a*c^2)/(
((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

3.791.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{\frac{17\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+5\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((17*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(1/2
) + (37*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(
7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6
*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.792 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.792.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

output

```
-3*c*arccsc(a*x)/a+3*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1+1/a
/x)^(3/2)*x*(1-1/a/x)^(1/2)
```

3.792.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(1+ax)-3\arcsin\left(\frac{1}{ax}\right)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

input

```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output

```
(c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) - 3*ArcSin[1/(a*x)] + 3*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/a
```

3.792.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6748, 109, 27, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \textcolor{blue}{6748}$$

$$-c\int \frac{\left(1+\frac{1}{ax}\right)^{5/2} x^2}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \textcolor{blue}{109}$$

$$-c\left(x\left(-\sqrt{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/2}-\int -\frac{3\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-c\left(\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \textcolor{blue}{140}$$

$$-c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}+\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \textcolor{blue}{39}$$

$$-c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \textcolor{blue}{103}$$

$$-c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{223}$$

$$-c\left(\frac{3\left(\arcsin\left(\frac{1}{ax}\right)-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
-(c*(-(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x) + (3*(ArcSin[1/(a*x)] - Ar
cTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/a))
```

3.792.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.792. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.792.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.70

method	result
risch	$\frac{(ax-1)c}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{(ax-1)(ax+1)}}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+3ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-}}\right.\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/x*c/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(3*a*ln(a^2*x/(a^2)^(1/2)+(a^2
*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)-3*arctan(1/(a^2*x^2-1)^
(1/2))*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

3.792.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2+2\,acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 3*a*c*x*log(sqrt((a*x - 1)/(a
*x + 1)) + 1) - 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 +
2*a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

3.792.6 Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\int \frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2),x)
```

output
```
c*(Integral(a**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**3*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/(a*x + 1)), x))/a**2
```

3.792.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*
c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.792.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c}{a\mathrm{sgn}\left(ax+1\right)}+\frac{2\,c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c
/(a*sgn(a*x + 1)) + 2*c/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)*sgn
(a*x + 1))
```

3.792.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

input
```
int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6*c*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x
 + 1)^2)
```

3.793 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.793.1 Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=-\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

output
```
3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c-5/3*(1+1/a/x)^(1/2)/a/c/(1-
1/a/x)^(3/2)+x*(1+1/a/x)^(1/2)/c/(1-1/a/x)^(3/2)-14/3*(1+1/a/x)^(1/2)/a/c/
(1-1/a/x)^(1/2)
```

3.793.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(14-19ax+3a^2x^2)}{(-1+ax)^2}+\frac{9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
((Sqrt[1 - 1/(a^2*x^2)]*x*(14 - 19*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + (9*Log
[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

3.793.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6748, 110, 27, 169, 25, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{\sqrt{1+\frac{1}{ax}}x^2}{\left(1-\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{110}$$

$$-\frac{\int \frac{\left(3a+\frac{2}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\left(3a+\frac{2}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{169}$$

$$-\frac{\frac{\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{1}{3}a\int -\frac{\left(9a+\frac{5}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{1}{3}a\int \frac{\left(9a+\frac{5}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{3}\int \frac{\left(9a+\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{14a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-a\int -\frac{9x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{3}\left(9a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}\right)+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \ 103$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{14a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-9\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \ 221$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{14a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-9a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
-((-((Sqrt[1 + 1/(a*x)]*x)/(1 - 1/(a*x))^(3/2)) + ((5*a*Sqrt[1 + 1/(a*x)])
/(3*(1 - 1/(a*x))^(3/2)) + ((14*a*Sqrt[1 + 1/(a*x)])/Sqrt[1 - 1/(a*x)] - 9
*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3)/a^2)/c)
```

3.793. $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.793.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.793. $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.793.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^5\left(x-\frac{1}{a}\right)^2}-\frac{13\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{3a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-9\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+6\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}ax+27\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^2x^2}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
-1)^(1/2))/(a^2)^(1/2)-2/3/a^5/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)
-13/3/a^4/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^2/c/((a*x-1)/(a*x+1
))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.793.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int\frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-16\,a^2x^2-5\,ax\right.}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x
^2 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

3.793.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\int \frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

output
```
a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x)/c
```

3.793.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((
a*x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/(a^2*c))
```

3.793.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.793.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}-\frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)}-\frac{6\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((11*(a*x - 1))/(3*(a*x + 1
)) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) -
 a*c*((a*x - 1)/(a*x + 1))^(5/2))
```

3.794 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.794.1 Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

output

```
3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-6/5*(1+1/a/x)^(1/2)/a/c^2
/(1-1/a/x)^(5/2)-9/5*(1+1/a/x)^(1/2)/a/c^2/(1-1/a/x)^(3/2)+x*(1+1/a/x)^(1/
2)/c^2/(1-1/a/x)^(5/2)-24/5*(1+1/a/x)^(1/2)/a/c^2/(1-1/a/x)^(1/2)
```

3.794.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-24+57ax-39a^2x^2+5a^3x^3)}{5(-1+ax)^3}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

input

```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-24 + 57*a*x - 39*a^2*x^2 + 5*a^3*x^3))/(5*(-
1 + a*x)^3) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

3.794.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6748, 114, 27, 35, 110, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{-\int -\frac{3\left(a+\frac{1}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{3\int \frac{\left(a+\frac{1}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}x}{\left(1-\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 110}$$

$$-\frac{\frac{3\left(\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2}{5}\int -\frac{\left(5a+\frac{4}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{3\left(\frac{\int \frac{\left(5a+\frac{4}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{3\left(\frac{\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{1}{3}a\int -\frac{3\left(5a+\frac{3}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{3\left(\frac{\int \frac{\left(5a+\frac{3}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{3\left(\frac{a\left(-\int -\frac{5x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{8a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{3\left(\frac{5a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 103$$

$$-\frac{\frac{3\left(\frac{-5\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+\frac{8a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

3.794. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{3\left(\frac{-5a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{8a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
-((-((Sqrt[1 + 1/(a*x)]*x)/(1 - 1/(a*x))^(5/2)) + (3*((2*Sqrt[1 + 1/(a*x)]
)/(5*(1 - 1/(a*x))^(5/2)) + ((3*a*Sqrt[1 + 1/(a*x)])/(1 - 1/(a*x))^(3/2) +
 (8*a*Sqrt[1 + 1/(a*x)])/Sqrt[1 - 1/(a*x)] - 5*a*ArcTanh[Sqrt[1 - 1/(a*x)]
*Sqrt[1 + 1/(a*x)]])/(5*a)))/a)/c^2)
```

3.794.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

3.794. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.794.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.24

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^8\left(x-\frac{1}{a}\right)^3}-\frac{6\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^7\left(x-\frac{1}{a}\right)^2}-\frac{24\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{5a^6\left(x-\frac{1}{a}\right)}\right)a^4\sqrt{(ax-1}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{120\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^5x^4+125\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4-480\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3-85((ax-1)(ax+1}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(3/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/5/a^8/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/
2)-6/5/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-24/5/a^6/(x-1/a)*((
x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*
(a*x+1))^(1/2)/(a*x+1)
```

3.794.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{15\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(5\,a^4}{5\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
1) + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a
*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

3.794.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

3.794.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{1}{20}\,a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1
)^3/(a*x + 1)^3 + 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x
- 1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) -
 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

3.794.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^2|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac^2\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
-3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^2*abs(a)*sgn(a*x + 1)) + sqr
t(a^2*x^2 - 1)/(a*c^2*sgn(a*x + 1))
```

3.794.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.67

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2} - \frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{25\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{9\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((15*(a*x - 1)^2)/(a*x +
1)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(
4*a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^2*((a*x - 1)/(a*x + 1))^(7/2))
```

3.795 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.795.1 Optimal result

Integrand size = 22, antiderivative size = 255

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

output
```
3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-8/7/a/c^3/(1-1/a/x)^(7/2)
/(1+1/a/x)^(1/2)-53/35/a/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)-88/35/a/c^3/(
1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+x/c^3/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)-281/3
5/a/c^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+176/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/
a/x)^(1/2)
```

3.795.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(176-423ax+125a^2x^2+368a^3x^3-286a^4x^4+35a^5x^5\right)}{35(-1+ax)^4(1+ax)}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(176 - 423*a*x + 125*a^2*x^2 + 368*a^3*x^3 - 2
86*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)^4*(1 + a*x)) + 3*Log[(1 + Sqrt[1
- 1/(a^2*x^2)])*x])/(a*c^3)
```

3.795.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {6748, 114, 25, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 25, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

$$\Big\downarrow \; 6748$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \; 114$$

$$-\frac{-\int -\frac{\left(3a+\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\left(3a+\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\left(3a+\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \text{169}$$

$$-\frac{\frac{\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}-\frac{1}{7}a\int -\frac{\left(21a+\frac{32}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{1}{7}a\int \frac{\left(21a+\frac{32}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{1}{7}\int \frac{\left(21a+\frac{32}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \text{169}$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{1}{5}a\int -\frac{3\left(35a+\frac{53}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\int \frac{\left(35a+\frac{53}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \text{169}$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{1}{3}a\int -\frac{\left(105a+\frac{176}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \text{25}$$

3.795. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}a\int \frac{\left(105a+\frac{176}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{\left(105a+\frac{176}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-a\int -\frac{\left(105a+\frac{281}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)^{7/}}}{c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(a\int \frac{\left(105a+\frac{281}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\int \frac{\left(105a+\frac{281}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(a\int \frac{105x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{}{(1}}{c^3}$$

$$\Big\downarrow \; 27$$

3.795. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(105a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-}{c^3}$$

$$\Big\downarrow \text{103}$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(-105\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}}}{a^2}}{c^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(-105a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}}}{a^2}}{c^3}$$

input

```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output

```
-((-(x/((1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])) + ((8*a)/(7*(1 - 1/(a*x))^
(7/2)*Sqrt[1 + 1/(a*x)]) + ((53*a)/(5*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)
]) + (3*((88*a)/(3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + ((281*a)/(Sqrt
[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (176*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a
*x)] - 105*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3))/5)/7)/a^2)/
c^3)
```

3.795.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.795. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.795. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.795.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.17

method	result
risch	$\frac{ax-1}{a\,c^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{6}\sqrt{a^{2}}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{14a^{11}\left(x-\frac{1}{a}\right)^{4}}-\frac{71\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{140a^{10}\left(x-\frac{1}{a}\right)^{3}}-\frac{477\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2\left(x-\frac{1}{a}\right)a}}{280a^{9}\left(x-\frac{1}{a}\right)^{2}}-\frac{2931\sqrt{(x}}{56}\right.}{c^{3}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-3675\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{7}x^{7}-3360\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{8}x^{7}+2555((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}\,a^{5}x^{5}+11025\sqrt{(ax-1)(}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(3/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/14/a^11/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(
1/2)-71/140/a^10/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-477/280/a^9/(
x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-2931/560/a^8/(x-1/a)*((x-1/a)^2
*a^2+2*(x-1/a)*a)^(1/2)+1/16/a^8/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)
)*a^6/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.795.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.80

$$\int\frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{3}}\,dx$$
$$=\frac{105\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\log\left(\right.}{35\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/35*(105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) - 105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(s
qrt((a*x - 1)/(a*x + 1)) - 1) + (35*a^5*x^5 - 286*a^4*x^4 + 368*a^3*x^3 +
125*a^2*x^2 - 423*a*x + 176)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a
^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

3.795.6 Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{a^6\int\frac{x^6}{\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-ax\sqrt{}}}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*Integral(x**6/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a
*x + 1) - 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/(a*x + 1)), x)/c**3
```

3.795.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{1}{560}\,a\left(\frac{\frac{51\,(ax-1)}{ax+1}+\frac{294\,(ax-1)^2}{(ax+1)^2}+\frac{2170\,(ax-1)^3}{(ax+1)^3}-\frac{3640\,(ax-1)^4}{(ax+1)^4}+5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\,\log\left(\sqrt{\frac{ax-}{ax+}}\right.}{a^2c^3}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
1/560*a*((51*(a*x - 1)/(a*x + 1) + 294*(a*x - 1)^2/(a*x + 1)^2 + 2170*(a*x
 - 1)^3/(a*x + 1)^3 - 3640*(a*x - 1)^4/(a*x + 1)^4 + 5)/(a^2*c^3*((a*x - 1
)/(a*x + 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 1680*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c^3) + 35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

3.795.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.795.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.63

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^3}-\frac{\frac{42\,(a\,x-1)^2}{5\,(a\,x+1)^2}+\frac{62\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{104\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{51\,(a\,x-1)}{35\,(a\,x+1)}+\frac{1}{7}}{16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}+\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}$$

input
```
int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(16*a*c^3) - ((42*(a*x - 1)^2)/(5*(a*x + 1)^2)
 + (62*(a*x - 1)^3)/(a*x + 1)^3 - (104*(a*x - 1)^4)/(a*x + 1)^4 + (51*(a*x
 - 1))/(35*(a*x + 1)) + 1/7)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 16*a*
c^3*((a*x - 1)/(a*x + 1))^(9/2)) + (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/
(a*c^3)
```

3.796 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.796.1 Optimal result

Integrand size = 22, antiderivative size = 329

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx &= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}\\ &\quad-\frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}\\ &\quad-\frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}}\\ &\quad+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}\end{aligned}$$

output
```
-10/9/a/c^4/(1-1/a/x)^(9/2)/(1+1/a/x)^(3/2)-29/21/a/c^4/(1-1/a/x)^(7/2)/(1
+1/a/x)^(3/2)-208/105/a/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)-1147/315/a/c^4
/(1-1/a/x)^(3/2)/(1+1/a/x)^(3/2)+x/c^4/(1-1/a/x)^(9/2)/(1+1/a/x)^(3/2)+3*a
rctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-1462/105/a/c^4/(1+1/a/x)^(3/
2)/(1-1/a/x)^(1/2)+2609/315*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+1664/315
*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(1/2)
```

3.796.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1664+4047ax+339a^2x^2-7399a^3x^3+4029a^4x^4+2967a^5x^5-2669a^6x^6+315a^7x^7)}{315(-1+ax)^5(1+ax)^2}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

input

```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output

```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1664 + 4047*a*x + 339*a^2*x^2 - 7399*a^3*x^3
 + 4029*a^4*x^4 + 2967*a^5*x^5 - 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a
*x)^5*(1 + a*x)^2) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

3.796.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.95, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.045, Rules used = {6748, 114, 25, 27, 169, 27, 169, 25, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(3a+\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(3a+\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{\left(3a+\frac{7}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{9}a\int -\frac{3\left(9a+\frac{20}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{3}\int \frac{\left(9a+\frac{20}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{7}a\int -\frac{\left(63a+\frac{145}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}a\int \frac{\left(63a+\frac{145}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\int \frac{\left(63a+\frac{145}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{5}a\int -\frac{\left(315a+\frac{832}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+\right.}}{c^4}$$

3.796. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

$$\Big\downarrow 25$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}a\int\frac{\left(315a+\frac{832}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}(\frac{1}{ax}+1}}{c^4}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\int\frac{\left(315a+\frac{832}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3}}}{c^4}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{3}a\int-\frac{3\left(315a+\frac{1147}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}(}}{a^2}}{c^4}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\int\frac{\left(315a+\frac{1147}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}(\frac{1}{ax}+1}}{a^2}}{c^4}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(a\left(-\int-\frac{\left(315a+\frac{2924}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}(\frac{1}{ax}+1}}{a^2}}{c^4}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(a\int\frac{\left(315a+\frac{2924}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+}{a^2}}{c^4}$$

$$\Big\downarrow 27$$

3.796. $\int\frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}dx$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{\left(315a+\frac{2924}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+}{a^2}}{c^4}$$

$\downarrow$ 169

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{\left(945a+\frac{2609}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{}{7\left(1-\frac{1}{ax}\right)^{7}}\right.}{a^2}}{c^4}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(945a+\frac{2609}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}}\right.}{a^2}}{c^4}$$

$\downarrow$ 169

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\int \frac{945x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right.\right.}{a^2}}{c^4}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(945a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)}\right.\right.}{a^2}}{c^4}$$

$\downarrow$ 103

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-945\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{}{5\left(1-\frac{1}{ax}\right)}\right.\right.}{a^2}}{c^4}$$

$\downarrow$ 221

3.796. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-945a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{}{5\left(1-\frac{1}{ax}\right)^{5}}\right)\right)}{a^2}}{c^4}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
-((-(x/((1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2))) + ((10*a)/(9*(1 - 1/(a*x
))^(9/2)*(1 + 1/(a*x))^(3/2)) + ((29*a)/(7*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x
))^(3/2)) + ((208*a)/(5*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)) + ((1147*
a)/(3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)) + (1462*a)/(Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)) - (2609*a*Sqrt[1 - 1/(a*x)])/(3*(1 + 1/(a*x))^(3/2
)) + ((-1664*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 945*a*ArcTanh[Sqrt[1
 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3)/5)/7)/3)/a^2)/c^4)
```

3.796.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.796. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.796.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.13

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{36a^{14}\left(x-\frac{1}{a}\right)^5}-\frac{59\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{252a^{13}\left(x-\frac{1}{a}\right)^4}-\frac{1507\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{1680a^{12}\left(x-\frac{1}{a}\right)^3}-\right.}{c^4\sqrt{}}$
default	$-\frac{-138915\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^9x^9-120960\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^{10}x^9+98595((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^7x^7+416745}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(3/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/36/a^14/(x-1/a)^5*((x-1/a)^2*a^2+2*(x-1/a)*a)^(
1/2)-59/252/a^13/(x-1/a)^4*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1507/1680/a^1
2/(x-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-691/315/a^11/(x-1/a)^2*((x-1
/a)^2*a^2+2*(x-1/a)*a)^(1/2)-113591/20160/a^10/(x-1/a)*((x-1/a)^2*a^2+2*(x
-1/a)*a)^(1/2)-1/96/a^11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+31/19
2/a^10/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^8/c^4/((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

3.796.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{945\left(a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6-4a^5x^5+5a^4x^4-5a^2x\right.}{315\left(a^7c^4x^6-4a^6\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/315*(945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a
^2*x^2 + 4*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^7*x^7 - 26
69*a^6*x^6 + 2967*a^5*x^5 + 4029*a^4*x^4 - 7399*a^3*x^3 + 339*a^2*x^2 + 40
47*a*x - 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5
*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

3.796.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

output
```
Timed out
```

3.796.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.69

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$=\frac{1}{20160}a\left(\frac{\frac{415\,(ax-1)}{ax+1}+\frac{2511\,(ax-1)^2}{(ax+1)^2}+\frac{11739\,(ax-1)^3}{(ax+1)^3}+\frac{80745\,(ax-1)^4}{(ax+1)^4}-\frac{135765\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{105\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+30\sqrt{}\right.}{a^2c^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
1/20160*a*((415*(a*x - 1)/(a*x + 1) + 2511*(a*x - 1)^2/(a*x + 1)^2 + 11739
*(a*x - 1)^3/(a*x + 1)^3 + 80745*(a*x - 1)^4/(a*x + 1)^4 - 135765*(a*x - 1
)^5/(a*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*
x - 1)/(a*x + 1))^(9/2)) + 105*(((a*x - 1)/(a*x + 1))^(3/2) + 30*sqrt((a*x
 - 1)/(a*x + 1)))/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^
2*c^4) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

3.796.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.796.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.62

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{5\sqrt{\frac{a\,x-1}{a\,x+1}}}{32\,a\,c^4} - \frac{\frac{279\,{\left(a\,x-1\right)}^2}{35\,{\left(a\,x+1\right)}^2}+\frac{559\,{\left(a\,x-1\right)}^3}{15\,{\left(a\,x+1\right)}^3}+\frac{769\,{\left(a\,x-1\right)}^4}{3\,{\left(a\,x+1\right)}^4}-\frac{431\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{83\,\left(a\,x-1\right)}{63\,\left(a\,x+1\right)}+\frac{1}{9}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}-64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}} + \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{192\,a\,c^4} - \frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^4}$$

input
```
int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(5*((a*x - 1)/(a*x + 1))^(1/2))/(32*a*c^4) - ((279*(a*x - 1)^2)/(35*(a*x +
 1)^2) + (559*(a*x - 1)^3)/(15*(a*x + 1)^3) + (769*(a*x - 1)^4)/(3*(a*x +
1)^4) - (431*(a*x - 1)^5)/(a*x + 1)^5 + (83*(a*x - 1))/(63*(a*x + 1)) + 1/
9)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^
(11/2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(192*a*c^4) - (atan(((a*x - 1)/(a*x
+ 1))^(1/2)*1i)*6i)/(a*c^4)
```

3.797 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

3.797.1 Optimal result

Integrand size = 22, antiderivative size = 116

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=&\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}\\&+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}\end{aligned}$$

output
```
1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/
a^6/x^5+14/3*c^5/a^4/x^3+4*c^5/a^3/x^2-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

3.797.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=&\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}\\&+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}\end{aligned}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

output
```
c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*
x^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (
3*c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

3.797.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^5 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \textbf{6717}$$

$$\int \frac{c^5\left(a^2-\frac{1}{x^2}\right)^5 e^{4\text{arctanh}(ax)}}{a^{10}}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{c^5\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^5dx}{a^{10}}$$

$$\Big\downarrow \textbf{6707}$$

$$-\frac{c^5\int \frac{e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}dx}{a^{10}}$$

$$\Big\downarrow \textbf{6700}$$

$$-\frac{c^5\int \frac{(1-ax)^3(ax+1)^7}{x^{10}}dx}{a^{10}}$$

$$\Big\downarrow \textbf{99}$$

$$-\frac{c^5\int \left(-a^{10}-\frac{4a^9}{x}-\frac{3a^8}{x^2}+\frac{8a^7}{x^3}+\frac{14a^6}{x^4}-\frac{14a^4}{x^6}-\frac{8a^3}{x^7}+\frac{3a^2}{x^8}+\frac{4a}{x^9}+\frac{1}{x^{10}}\right)dx}{a^{10}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{c^5\left(a^{10}(-x)-4a^9\log(x)+\frac{3a^8}{x}-\frac{4a^7}{x^2}-\frac{14a^6}{3x^3}+\frac{14a^4}{5x^5}+\frac{4a^3}{3x^6}-\frac{3a^2}{7x^7}-\frac{a}{2x^8}-\frac{1}{9x^9}\right)}{a^{10}}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

output
```
-((c^5*(-1/9*1/x^9 - a/(2*x^8) - (3*a^2)/(7*x^7) + (4*a^3)/(3*x^6) + (14*a
^4)/(5*x^5) - (14*a^6)/(3*x^3) - (4*a^7)/x^2 + (3*a^8)/x - a^10*x - 4*a^9*
Log[x]))/a^10)
```

3.797. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5dx$

3.797.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.797. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

3.797.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

method	result
default	$\frac{c^5\left(a^{10}x+\frac{1}{9x^9}+4a^9\ln(x)+\frac{14a^6}{3x^3}-\frac{4a^3}{3x^6}+\frac{4a^7}{x^2}-\frac{3a^8}{x}+\frac{3a^2}{7x^7}+\frac{a}{2x^8}-\frac{14a^4}{5x^5}\right)}{a^{10}}$
risch	$c^5x+\frac{-3a^8c^5x^8+4a^7c^5x^7+\frac{14}{3}a^6c^5x^6-\frac{14}{5}a^4c^5x^4-\frac{4}{3}a^3c^5x^3+\frac{3}{7}a^2c^5x^2+\frac{1}{2}a\,c^5x+\frac{1}{9}c^5}{a^{10}x^9}+\frac{4c^5\ln(x)}{a}$
parallelrisch	$\frac{630a^{10}c^5x^{10}+2520c^5\ln(x)a^9x^9-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315a\,c^5x+70}{630a^{10}x^9}$
norman	$\frac{-4a^9c^5x^{10}+a^{10}c^5x^{11}-\frac{c^5}{9a}-\frac{7c^5x}{18}+\frac{a\,c^5x^2}{14}+\frac{22a^3c^5x^4}{15}-\frac{14a^4c^5x^5}{5}-\frac{14a^5c^5x^6}{3}+\frac{2a^6c^5x^7}{3}+7a^7c^5x^8+\frac{37c^5a^2x^3}{21}}{(ax-1)a^9x^9}+\frac{4c^5\ln(x)}{a}$
meijerg	$-\frac{c^5\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{4c^5x}{-ax+1}-\frac{5c^5\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}+\frac{5c^5\left(-\frac{7c}{-7a}\right.}{}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5/a^10*(a^10*x+1/9/x^9+4*a^9*ln(x)+14/3*a^6/x^3-4/3*a^3/x^6+4*a^7/x^2-3*
a^8/x+3/7*a^2/x^7+1/2*a/x^8-14/5*a^4/x^5)
```

3.797.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = \frac{630\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log(x)-1890\,a^8c^5x^8+2520\,a^7c^5x^7+2940\,a^6c^5x^6-1764\,a^4c^5x^4-840\,a^3c^5x^3}{630\,a^{10}x^9}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

output
```
1/630*(630*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) - 1890*a^8*c^5*x^8 + 25
20*a^7*c^5*x^7 + 2940*a^6*c^5*x^6 - 1764*a^4*c^5*x^4 - 840*a^3*c^5*x^3 + 2
70*a^2*c^5*x^2 + 315*a*c^5*x + 70*c^5)/(a^10*x^9)
```

3.797.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx$$
$$=\frac{a^{10}c^5x+4a^9c^5\log(x)+\frac{-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315ac^5x+70c^5}{630x^9}}{a^{10}}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**5,x)
```

output
```
(a**10*c**5*x + 4*a**9*c**5*log(x) + (-1890*a**8*c**5*x**8 + 2520*a**7*c**
5*x**7 + 2940*a**6*c**5*x**6 - 1764*a**4*c**5*x**4 - 840*a**3*c**5*x**3 +
270*a**2*c**5*x**2 + 315*a*c**5*x + 70*c**5)/(630*x**9))/a**10
```

3.797.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx=c^5x+\frac{4\,c^5\log(x)}{a}$$
$$-\frac{1890\,a^8c^5x^8-2520\,a^7c^5x^7-2940\,a^6c^5x^6+1764\,a^4c^5x^4+840\,a^3c^5x^3-270\,a^2c^5x^2-315\,ac^5x-70\,c^5}{630\,a^{10}x^9}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

output
```
c^5*x + 4*c^5*log(x)/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 - 2940
*a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 315*
a*c^5*x - 70*c^5)/(a^10*x^9)
```

3.797.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx=-\frac{4\,c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}$$
$$+\frac{\left(630\,c^5+\frac{4049\,c^5}{ax-1}+\frac{6201\,c^5}{(ax-1)^2}-\frac{18036\,c^5}{(ax-1)^3}-\frac{89124\,c^5}{(ax-1)^4}-\frac{160146\,c^5}{(ax-1)^5}-\frac{153090\,c^5}{(ax-1)^6}-\frac{80220\,c^5}{(ax-1)^7}-\frac{21420\,c^5}{(ax-1)^8}-\frac{2520\,c^5}{(ax-1)^9}\right)(ax-}{630\,a\left(\frac{1}{ax-1}+1\right)^9}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

output
```
-4*c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^5*log(abs(-1/(a*x -
1) - 1))/a + 1/630*(630*c^5 + 4049*c^5/(a*x - 1) + 6201*c^5/(a*x - 1)^2 -
18036*c^5/(a*x - 1)^3 - 89124*c^5/(a*x - 1)^4 - 160146*c^5/(a*x - 1)^5 - 1
53090*c^5/(a*x - 1)^6 - 80220*c^5/(a*x - 1)^7 - 21420*c^5/(a*x - 1)^8 - 25
20*c^5/(a*x - 1)^9)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^9)
```

3.797.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$$
$$=\frac{c^5\left(\frac{a\,x}{2}+\frac{3\,a^2\,x^2}{7}-\frac{4\,a^3\,x^3}{3}-\frac{14\,a^4\,x^4}{5}+\frac{14\,a^6\,x^6}{3}+4\,a^7\,x^7-3\,a^8\,x^8+a^{10}\,x^{10}+4\,a^9\,x^9\,\ln\left(x\right)+\frac{1}{9}\right)}{a^{10}\,x^9}$$

input
```
int(((c - c/(a^2*x^2))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^5*((a*x)/2 + (3*a^2*x^2)/7 - (4*a^3*x^3)/3 - (14*a^4*x^4)/5 + (14*a^6*x
^6)/3 + 4*a^7*x^7 - 3*a^8*x^8 + a^10*x^10 + 4*a^9*x^9*log(x) + 1/9))/(a^10
*x^9)
```

3.798 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.798.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

output
```
-1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/
x^3+2*c^4/a^3/x^2-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

3.798.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
-1/7*c^4/(a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*
x^4) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x
+ (4*c^4*Log[x])/a
```

3.798.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c^4\left(a^2-\frac{1}{x^2}\right)^4 e^{4\text{arctanh}(ax)}}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^4\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4dx}{a^8}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c^4\int \frac{e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c^4\int \frac{(1-ax)^2(ax+1)^6}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^4\int \left(a^8+\frac{4a^7}{x}+\frac{4a^6}{x^2}-\frac{4a^5}{x^3}-\frac{10a^4}{x^4}-\frac{4a^3}{x^5}+\frac{4a^2}{x^6}+\frac{4a}{x^7}+\frac{1}{x^8}\right)dx}{a^8}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^4\left(a^8x+4a^7\log(x)-\frac{4a^6}{x}+\frac{2a^5}{x^2}+\frac{10a^4}{3x^3}+\frac{a^3}{x^4}-\frac{4a^2}{5x^5}-\frac{2a}{3x^6}-\frac{1}{7x^7}\right)}{a^8}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
(c^4*(-1/7*1/x^7 - (2*a)/(3*x^6) - (4*a^2)/(5*x^5) + a^3/x^4 + (10*a^4)/(3
*x^3) + (2*a^5)/x^2 - (4*a^6)/x + a^8*x + 4*a^7*Log[x]))/a^8
```

3.798.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.798. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.798.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^4\left(a^8x+4a^7\ln(x)+\frac{a^3}{x^4}+\frac{10a^4}{3x^3}-\frac{2a}{3x^6}+\frac{2a^5}{x^2}-\frac{4a^6}{x}-\frac{1}{7x^7}-\frac{4a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{-4a^6c^4x^6+2a^5c^4x^5+\frac{10}{3}a^4c^4x^4+a^3c^4x^3-\frac{4}{5}a^2c^4x^2-\frac{2}{3}a\,c^4x-\frac{1}{7}c^4}{a^8x^7}+\frac{4c^4\ln(x)}{a}$
parallelrisch	$\frac{105a^8c^4x^8+420c^4\ln(x)a^7x^7-420a^6c^4x^6+210a^5c^4x^5+350a^4c^4x^4+105a^3c^4x^3-84a^2c^4x^2-70a\,c^4x-15c^4}{105a^8x^7}$
norman	$\frac{-5a^7c^4x^8+a^8c^4x^9+\frac{c^4}{7a}+\frac{11c^4x}{21}+\frac{2a\,c^4x^2}{15}-\frac{9a^2c^4x^3}{5}-\frac{7a^3c^4x^4}{3}+\frac{4a^4c^4x^5}{3}+6a^5c^4x^6}{(ax-1)a^7x^7}+\frac{4c^4\ln(x)}{a}$
meijerg	$-\frac{c^4\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{3c^4x}{-ax+1}-\frac{2c^4\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}-2c^4\left(-\frac{5}{-5a}\right.$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4/a^8*(a^8*x+4*a^7*ln(x)+a^3/x^4+10/3*a^4/x^3-2/3*a/x^6+2*a^5/x^2-4*a^6/
x-1/7/x^7-4/5*a^2/x^5)
```

3.798.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{105\,a^8c^4x^8+420\,a^7c^4x^7\log{(x)}-420\,a^6c^4x^6+210\,a^5c^4x^5+350\,a^4c^4x^4+105\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4}{105\,a^8x^7}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/105*(105*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) - 420*a^6*c^4*x^6 + 210*a^
5*c^4*x^5 + 350*a^4*c^4*x^4 + 105*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*
x - 15*c^4)/(a^8*x^7)
```

3.798.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= \frac{a^8c^4x+4a^7c^4\log(x)+\frac{-420a^6c^4x^6+210a^5c^4x^5+350a^4c^4x^4+105a^3c^4x^3-84a^2c^4x^2-70ac^4x-15c^4}{105x^7}}{a^8}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**4,x)
```

output
```
(a**8*c**4*x + 4*a**7*c**4*log(x) + (-420*a**6*c**4*x**6 + 210*a**5*c**4*x
**5 + 350*a**4*c**4*x**4 + 105*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c
**4*x - 15*c**4)/(105*x**7))/a**8
```

3.798.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= c^4x+\frac{4\,c^4\log(x)}{a}$$
$$-\frac{420\,a^6c^4x^6-210\,a^5c^4x^5-350\,a^4c^4x^4-105\,a^3c^4x^3+84\,a^2c^4x^2+70\,ac^4x+15\,c^4}{105\,a^8x^7}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
c^4*x + 4*c^4*log(x)/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 350*a^
4*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^8*x
^7)
```

3.798.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

$$\begin{aligned}
&\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx\\
&=-\frac{4\,c^4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^4\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}\\
&\quad+\frac{\left(105\,c^4+\frac{659\,c^4}{ax-1}+\frac{1253\,c^4}{(ax-1)^2}-\frac{231\,c^4}{(ax-1)^3}-\frac{3885\,c^4}{(ax-1)^4}-\frac{5250\,c^4}{(ax-1)^5}-\frac{2730\,c^4}{(ax-1)^6}-\frac{420\,c^4}{(ax-1)^7}\right)(ax-1)}{105\,a\left(\frac{1}{ax-1}+1\right)^7}
\end{aligned}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
-4*c^4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^4*log(abs(-1/(a*x -
1) - 1))/a + 1/105*(105*c^4 + 659*c^4/(a*x - 1) + 1253*c^4/(a*x - 1)^2 - 2
31*c^4/(a*x - 1)^3 - 3885*c^4/(a*x - 1)^4 - 5250*c^4/(a*x - 1)^5 - 2730*c^
4/(a*x - 1)^6 - 420*c^4/(a*x - 1)^7)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^7)
```

3.798.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\begin{aligned}
&\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx\\
&=\frac{c^4\left(a^3\,x^3-\frac{4\,a^2\,x^2}{5}-\frac{2\,a\,x}{3}+\frac{10\,a^4\,x^4}{3}+2\,a^5\,x^5-4\,a^6\,x^6+a^8\,x^8+4\,a^7\,x^7\,\ln\left(x\right)-\frac{1}{7}\right)}{a^8\,x^7}
\end{aligned}$$

input
```
int(((c - c/(a^2*x^2))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^4*(a^3*x^3 - (4*a^2*x^2)/5 - (2*a*x)/3 + (10*a^4*x^4)/3 + 2*a^5*x^5 - 4
*a^6*x^6 + a^8*x^8 + 4*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

3.799 $\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^3 \, dx$

3.799.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^3 \, dx=\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

output
```
1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/
a
```

3.799.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^3 \, dx=\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) +
c^3*x + (4*c^3*Log[x])/a
```

3.799.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c^3\left(a^2-\frac{1}{x^2}\right)^3 e^{4\text{arctanh}(ax)}}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^3dx}{a^6}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{c^3\int \frac{e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{c^3\int \frac{(1-ax)(ax+1)^5}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{84}$$

$$-\frac{c^3\int \left(-a^6-\frac{4a^5}{x}-\frac{5a^4}{x^2}+\frac{5a^2}{x^4}+\frac{4a}{x^5}+\frac{1}{x^6}\right)dx}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3\left(a^6(-x)-4a^5\log(x)+\frac{5a^4}{x}-\frac{5a^2}{3x^3}-\frac{a}{x^4}-\frac{1}{5x^5}\right)}{a^6}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
-((c^3*(-1/5*1/x^5 - a/x^4 - (5*a^2)/(3*x^3) + (5*a^4)/x - a^6*x - 4*a^5*L
og[x]))/a^6)
```

3.799. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx$

3.799.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.799. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.799.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

method	result
default	$\frac{c^{3}\left(a^{6}x+4a^{5}\ln\left(x\right)+\frac{a}{x^{4}}+\frac{5a^{2}}{3x^{3}}-\frac{5a^{4}}{x}+\frac{1}{5x^{5}}\right)}{a^{6}}$
risch	$c^{3}x+\frac{-5a^{4}c^{3}x^{4}+\frac{5}{3}a^{2}c^{3}x^{2}+a\,c^{3}x+\frac{1}{5}c^{3}}{a^{6}x^{5}}+\frac{4c^{3}\ln\left(x\right)}{a}$
parallelrisch	$\frac{15a^{6}c^{3}x^{6}+60c^{3}\ln\left(x\right)a^{5}x^{5}-75a^{4}c^{3}x^{4}+25a^{2}c^{3}x^{2}+15a\,c^{3}x+3c^{3}}{15a^{6}x^{5}}$
norman	$\frac{-6a^{5}c^{3}x^{6}+a^{6}c^{3}x^{7}-\frac{c^{3}}{5a}-\frac{4c^{3}x}{5}-\frac{2a\,c^{3}x^{2}}{3}+\frac{5a^{2}c^{3}x^{3}}{3}+5a^{3}c^{3}x^{4}}{\left(ax-1\right)a^{5}x^{5}}+\frac{4c^{3}\ln\left(x\right)}{a}$
meijerg	$-\frac{c^{3}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{3}x}{-ax+1}-\frac{2c^{3}\left(-\frac{5ax}{-5ax+5}+4\ln\left(-ax+1\right)-1-4\ln\left(x\right)-4\ln\left(-a\right)+\frac{1}{3x^{3}a^{3}}+\frac{1}{a^{2}x^{2}}+\frac{3}{ax}\right)}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/a^6*(a^6*x+4*a^5*ln(x)+a/x^4+5/3*a^2/x^3-5*a^4/x+1/5/x^5)
```

3.799.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{15\,a^{6}c^{3}x^{6}+60\,a^{5}c^{3}x^{5}\log\left(x\right)-75\,a^{4}c^{3}x^{4}+25\,a^{2}c^{3}x^{2}+15\,ac^{3}x+3\,c^{3}}{15\,a^{6}x^{5}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/15*(15*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) - 75*a^4*c^3*x^4 + 25*a^2*c^3
*x^2 + 15*a*c^3*x + 3*c^3)/(a^6*x^5)
```

3.799.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{a^6c^3x+4a^5c^3\log(x)+\frac{-75a^4c^3x^4+25a^2c^3x^2+15ac^3x+3c^3}{15x^5}}{a^6}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**3,x)
```

output
```
(a**6*c**3*x + 4*a**5*c**3*log(x) + (-75*a**4*c**3*x**4 + 25*a**2*c**3*x**
2 + 15*a*c**3*x + 3*c**3)/(15*x**5))/a**6
```

3.799.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = c^3x+\frac{4c^3\log(x)}{a}-\frac{75a^4c^3x^4-25a^2c^3x^2-15ac^3x-3c^3}{15a^6x^5}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
c^3*x + 4*c^3*log(x)/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*c^3*
x - 3*c^3)/(a^6*x^5)
```

3.799.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(59) = 118.

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.16

$$\begin{aligned}&\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx\\&=-\frac{4c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}\\&\quad+\frac{\left(15c^3+\frac{107c^3}{ax-1}+\frac{235c^3}{(ax-1)^2}+\frac{170c^3}{(ax-1)^3}-\frac{30c^3}{(ax-1)^4}-\frac{60c^3}{(ax-1)^5}\right)(ax-1)}{15a\left(\frac{1}{ax-1}+1\right)^5}\end{aligned}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
-4*c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^3*log(abs(-1/(a*x -
1) - 1))/a + 1/15*(15*c^3 + 107*c^3/(a*x - 1) + 235*c^3/(a*x - 1)^2 + 170*
c^3/(a*x - 1)^3 - 30*c^3/(a*x - 1)^4 - 60*c^3/(a*x - 1)^5)*(a*x - 1)/(a*(1
/(a*x - 1) + 1)^5)
```

3.799.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3\left(a\,x+\frac{5\,a^2\,x^2}{3}-5\,a^4\,x^4+a^6\,x^6+4\,a^5\,x^5\,\ln\left(x\right)+\frac{1}{5}\right)}{a^6\,x^5}$$

input
```
int(((c - c/(a^2*x^2))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^3*(a*x + (5*a^2*x^2)/3 - 5*a^4*x^4 + a^6*x^6 + 4*a^5*x^5*log(x) + 1/5))
/(a^6*x^5)
```

3.800 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.800.1 Optimal result

Integrand size = 22, antiderivative size = 51

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

output
```
-1/3*c^2/a^4/x^3-2*c^2/a^3/x^2-6*c^2/a^2/x+c^2*x+4*c^2*ln(x)/a
```

3.800.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
-1/3*c^2/(a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*
Log[x])/a
```

3.800.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c^2\left(a^2-\frac{1}{x^2}\right)^2 e^{4\text{arctanh}(ax)}}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^2dx}{a^4}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c^2\int \frac{e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c^2\int \frac{(ax+1)^4}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{49}$$

$$\frac{c^2\int \left(a^4+\frac{4a^3}{x}+\frac{6a^2}{x^2}+\frac{4a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\left(a^4x+4a^3\log(x)-\frac{6a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
(c^2*(-1/3*1/x^3 - (2*a)/x^2 - (6*a^2)/x + a^4*x + 4*a^3*Log[x]))/a^4
```

3.800. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx$

3.800.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.800.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result
default	$\frac{c^{2}\left(a^{4}x+4a^{3}\ln\left(x\right)-\frac{1}{3x^{3}}-\frac{2a}{x^{2}}-\frac{6a^{2}}{x}\right)}{a^{4}}$
risch	$c^{2}x+\frac{-6a^{2}c^{2}x^{2}-2a\,c^{2}x-\frac{1}{3}c^{2}}{a^{4}x^{3}}+\frac{4c^{2}\ln\left(x\right)}{a}$
parallelrisch	$\frac{3a^{4}c^{2}x^{4}+12c^{2}\ln\left(x\right)a^{3}x^{3}-18a^{2}c^{2}x^{2}-6a\,c^{2}x-c^{2}}{3a^{4}x^{3}}$
norman	$\frac{-7a^{3}c^{2}x^{4}+a^{4}c^{2}x^{5}+\frac{c^{2}}{3a}+\frac{5c^{2}x}{3}+4a\,c^{2}x^{2}}{\left(ax-1\right)a^{3}x^{3}}+\frac{4c^{2}\ln\left(x\right)}{a}$
meijerg	$-\frac{c^{2}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{c^{2}x}{-ax+1}+\frac{c^{2}\left(-\frac{3ax}{-3ax+3}+2\ln\left(-ax+1\right)-1-2\ln\left(x\right)-2\ln\left(-a\right)+\frac{1}{ax}\right)}{a}+ 2c^{2}\left(\frac{ax}{-ax+1}\right.$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/a^4*(a^4*x+4*a^3*ln(x)-1/3/x^3-2*a/x^2-6*a^2/x)
```

3.800.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{3\,a^4c^2x^4+12\,a^3c^2x^3\log{(x)}-18\,a^2c^2x^2-6\,ac^2x-c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^2*x^4 + 12*a^3*c^2*x^3*log(x) - 18*a^2*c^2*x^2 - 6*a*c^2*x -
c^2)/(a^4*x^3)
```

3.800.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x+4a^3c^2\log{(x)}+\frac{-18a^2c^2x^2-6ac^2x-c^2}{3x^3}}{a^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**2,x)
```

output
```
(a**4*c**2*x + 4*a**3*c**2*log(x) + (-18*a**2*c**2*x**2 - 6*a*c**2*x - c**
2)/(3*x**3))/a**4
```

3.800.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x+\frac{4\,c^2\log{(x)}}{a}-\frac{18\,a^2c^2x^2+6\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
c^2*x + 4*c^2*log(x)/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x^3)
```

3.800.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{4\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(3\,c^2+\frac{34\,c^2}{ax-1}+\frac{66\,c^2}{(ax-1)^2}+\frac{36\,c^2}{(ax-1)^3}\right)(ax-1)}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
-4*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^2*log(abs(-1/(a*x -
1) - 1))/a + 1/3*(3*c^2 + 34*c^2/(a*x - 1) + 66*c^2/(a*x - 1)^2 + 36*c^2/(
a*x - 1)^3)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^3)
```

3.800.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{c^2\,\left(6\,a\,x+18\,a^2\,x^2-3\,a^4\,x^4-12\,a^3\,x^3\,\ln\left(x\right)+1\right)}{3\,a^4\,x^3}$$

input
```
int(((c - c/(a^2*x^2))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
-(c^2*(6*a*x + 18*a^2*x^2 - 3*a^4*x^4 - 12*a^3*x^3*log(x) + 1))/(3*a^4*x^3
)
```

3.801 $\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx$

3.801.1 Optimal result

Integrand size = 20, antiderivative size = 33

$$\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx=\frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}$$

output
```
c/a^2/x+c*x-4*c*ln(x)/a+8*c*ln(-a*x+1)/a
```

3.801.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx=\frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

3.801.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c\left(a^2-\frac{1}{x^2}\right) e^{4\text{arctanh}(ax)}}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{c\int \frac{e^{4\text{arctanh}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{c\int \frac{(ax+1)^3}{x^2(1-ax)}dx}{a^2}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c\int \left(-\frac{8a^2}{ax-1}-a^2+\frac{4a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(a^2(-x)+4a\log(x)-8a\log(1-ax)-\frac{1}{x}\right)}{a^2}$$

input

```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output

```
-((c*(-x^(-1) - a^2*x + 4*a*Log[x] - 8*a*Log[1 - a*x]))/a^2)
```

3.801.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.801. $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.801.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result
default	$\frac{c\left(a^{2} x+\frac{1}{x}-4 a \ln \left(x\right)+8 a \ln \left(a x-1\right)\right)}{a^{2}}$
risch	$\frac{c}{a^{2} x}+c x-\frac{4 c \ln \left(x\right)}{a}+\frac{8 c \ln \left(-a x+1\right)}{a}$
parallelrisch	$-\frac{-a^{2} c \,x^{2}+4 c \ln \left(x\right) a x-8 c \ln \left(a x-1\right) a x-c}{a^{2} x}$
norman	$\frac{a^{2} c \,x^{3}-\frac{c}{a}}{x \left(a x-1\right) a}-\frac{4 c \ln \left(x\right)}{a}+\frac{8 c \ln \left(a x-1\right)}{a}$
meijerg	$-\frac{c \left(-\frac{a x \left(-3 a x+6\right)}{3 \left(-a x+1\right)}-2 \ln \left(-a x+1\right)\right)}{a}+\frac{2 c \left(\frac{a x}{-a x+1}+\ln \left(-a x+1\right)\right)}{a}-\frac{2 c \left(\frac{2 a x}{-2 a x+2}-\ln \left(-a x+1\right)+1+\ln \left(x\right)+\ln \left(-a\right)\right)}{a}+$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
c/a^2*(a^2*x+1/x-4*a*ln(x)+8*a*ln(a*x-1))
```

3.801.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx^2+8\,acx\log\left(ax-1\right)-4\,acx\log\left(x\right)+c}{a^2x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^2*c*x^2 + 8*a*c*x*log(a*x - 1) - 4*a*c*x*log(x) + c)/(a^2*x)
```

3.801.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{4c\left(-\log\left(x\right)+2\log\left(x-\frac{1}{a}\right)\right)}{a}+\frac{c}{a^2x}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2),x)
```

output
```
c*x + 4*c*(-log(x) + 2*log(x - 1/a))/a + c/(a**2*x)
```

3.801.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{8\,c\log\left(ax-1\right)}{a}-\frac{4\,c\log\left(x\right)}{a}+\frac{c}{a^2x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
c*x + 8*c*log(a*x - 1)/a - 4*c*log(x)/a + c/(a^2*x)
```

3.801.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{4\,c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{(ax-1)c}{a\left(\frac{1}{ax-1}+1\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
-4*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4*c*log(abs(-1/(a*x - 1) -
 1))/a + (a*x - 1)*c/(a*(1/(a*x - 1) + 1))
```

3.801.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=c\,x+\frac{c}{a^2\,x}-\frac{4\,c\,\ln\left(x\right)}{a}+\frac{8\,c\,\ln\left(a\,x-1\right)}{a}$$

input
```
int(((c - c/(a^2*x^2))*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c*x + c/(a^2*x) - (4*c*log(x))/a + (8*c*log(a*x - 1))/a
```

3.802 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.802.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

output
```
x/c-1/a/c/(-a*x+1)^2+5/a/c/(-a*x+1)+4*ln(-a*x+1)/a/c
```

3.802.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

3.802.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$

$$\Big\downarrow \; 6717$$

$$\int \frac{a^2 e^{4\text{arctanh}(ax)}}{c\left(a^2-\frac{1}{x^2}\right)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}}{a^2-\frac{1}{x^2}}dx}{c}$$

$$\Big\downarrow \; 6707$$

$$-\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}x^2}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \; 6700$$

$$-\frac{a^2\int \frac{x^2(ax+1)}{(1-ax)^3}dx}{c}$$

$$\Big\downarrow \; 86$$

$$-\frac{a^2\int \left(-\frac{1}{a^2}-\frac{4}{a^2(ax-1)}-\frac{5}{a^2(ax-1)^2}-\frac{2}{a^2(ax-1)^3}\right)dx}{c}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2\left(-\frac{5}{a^3(1-ax)}+\frac{1}{a^3(1-ax)^2}-\frac{4\log(1-ax)}{a^3}-\frac{x}{a^2}\right)}{c}$$

input

```
Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output

```
-((a^2*(-(x/a^2) + 1/(a^3*(1 - a*x)^2) - 5/(a^3*(1 - a*x)) - (4*Log[1 - a*
x])/a^3))/c)
```

3.802.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.802. $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.802.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{c}+\frac{-5cx+\frac{4c}{a}}{c^2(ax-1)^2}+\frac{4\ln(ax-1)}{ac}$	43
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{1}{a^3(ax-1)^2}-\frac{5}{a^3(ax-1)}+\frac{4\ln(ax-1)}{a^3}\right)}{c}$	49
norman	$\frac{\frac{a^2x^3}{c}-\frac{6a\,x^2}{c}+\frac{4x}{c}}{(ax-1)^2}+\frac{4\ln(ax-1)}{ac}$	50
parallelrisch	$\frac{a^3x^3+4a^2\ln(ax-1)x^2-6a^2x^2-8a\ln(ax-1)x+4ax+4\ln(ax-1)}{(ax-1)^2ca}$	67

input

```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output

```
x/c+(-5*c*x+4*c/a)/c^2/(a*x-1)^2+4/a/c*ln(a*x-1)
```

3.802.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^3x^3-2\,a^2x^2-4\,ax+4\,(a^2x^2-2\,ax+1)\log{(ax-1)}+4}{a^3cx^2-2\,a^2cx+ac}$$

input

```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="fricas")
```

output

```
(a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(
a^3*c*x^2 - 2*a^2*c*x + a*c)
```

3.802.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{-5ax+4}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{4\log{(ax-1)}}{ac}$$

input

```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2),x)
```

output
```
(-5*a*x + 4)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

3.802.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{5\,ax-4}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{4\,\log\left(ax-1\right)}{ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-(5*a*x - 4)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

3.802.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{ax-1}{ac}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{\frac{5\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}}{a^4c^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - (5*a^3*
c/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

3.802.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{5\,x-\frac{4}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c}$$

input
```
int((a*x + 1)^2/((c - c/(a^2*x^2))*(a*x - 1)^2),x)
```

output
```
x/c - (5*x - 4/a)/(c + a^2*c*x^2 - 2*a*c*x) + (4*log(a*x - 1))/(a*c)
```

3.803 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.803.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

output
```
x/c^2+1/3/a/c^2/(-a*x+1)^3-2/a/c^2/(-a*x+1)^2+6/a/c^2/(-a*x+1)+4*ln(-a*x+1
)/a/c^2
```

3.803.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{-13+27ax-9a^2x^2-9a^3x^3+3a^4x^4+12(-1+ax)^3\log(1-ax)}{3ac^2(-1+ax)^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
(-13 + 27*a*x - 9*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 12*(-1 + a*x)^3*Log[1
- a*x])/(3*a*c^2*(-1 + a*x)^3)
```

3.803.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \; 6717$$

$$\int \frac{a^4e^{4\text{arctanh}(ax)}}{c^2\left(a^2-\frac{1}{x^2}\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\frac{a^4\int \frac{e^{4\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^2}dx}{c^2}$$

$$\Big\downarrow \; 6707$$

$$\frac{a^4\int \frac{e^{4\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \; 6700$$

$$\frac{a^4\int \frac{x^4}{(1-ax)^4}dx}{c^2}$$

$$\Big\downarrow \; 49$$

$$\frac{a^4\int \left(\frac{1}{a^4}+\frac{4}{a^4(ax-1)}+\frac{6}{a^4(ax-1)^2}+\frac{4}{a^4(ax-1)^3}+\frac{1}{a^4(ax-1)^4}\right)dx}{c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^4\left(\frac{6}{a^5(1-ax)}-\frac{2}{a^5(1-ax)^2}+\frac{1}{3a^5(1-ax)^3}+\frac{4\log(1-ax)}{a^5}+\frac{x}{a^4}\right)}{c^2}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
(a^4*(x/a^4 + 1/(3*a^5*(1 - a*x)^3) - 2/(a^5*(1 - a*x)^2) + 6/(a^5*(1 - a*
x)) + (4*Log[1 - a*x])/a^5))/c^2
```

3.803. $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$

3.803.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.803.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{c^{2}}+\frac{-6 a \,c^{2} x^{2}+10 c^{2} x -\frac{13 c^{2}}{3 a}}{c^{4} \left(a x -1\right)^{3}}+\frac{4 \ln \left(a x -1\right)}{a \,c^{2}}$	56
default	$\frac{a^{4} \left(\frac{x}{a^{4}}-\frac{1}{3 a^{5} \left(a x -1\right)^{3}}-\frac{2}{a^{5} \left(a x -1\right)^{2}}-\frac{6}{a^{5} \left(a x -1\right)}+\frac{4 \ln \left(a x -1\right)}{a^{5}}\right)}{c^{2}}$	61
norman	$\frac{\frac{a^{4} x^{5}}{c}+\frac{6 a \,x^{2}}{c}-\frac{4 x}{c}+\frac{8 a^{2} x^{3}}{3 c}-\frac{19 a^{3} x^{4}}{3 c}}{\left(a x -1\right)^{3} c \left(a x +1\right)}+\frac{4 \ln \left(a x -1\right)}{a \,c^{2}}$	82
parallelrisch	$\frac{3 a^{4} x^{4}+12 a^{3} \ln \left(a x -1\right) x^{3}-22 a^{3} x^{3}-36 a^{2} \ln \left(a x -1\right) x^{2}+30 a^{2} x^{2}+36 a \ln \left(a x -1\right) x -12 a x -12 \ln \left(a x -1\right)}{3 \left(a x -1\right)^{3} c^{2} a}$	91

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+(-6*a*c^2*x^2+10*c^2*x-13/3*c^2/a)/c^4/(a*x-1)^3+4/a/c^2*ln(a*x-1)
```

3.803.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\,a^4x^4-9\,a^3x^3-9\,a^2x^2+27\,ax+12\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(ax-1\right)-13}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*x^4 - 9*a^3*x^3 - 9*a^2*x^2 + 27*a*x + 12*(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)*log(a*x - 1) - 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
 - a*c^2)
```

3.803.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-18a^2x^2+30ax-13}{3a^8c^2x^3-9a^7c^2x^2+9a^6c^2x-3a^5c^2}+\frac{x}{a^4c^2}+\frac{4\log\left(ax-1\right)}{a^5c^2}\right)$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*((-18*a**2*x**2 + 30*a*x - 13)/(3*a**8*c**2*x**3 - 9*a**7*c**2*x**2 +
 9*a**6*c**2*x - 3*a**5*c**2) + x/(a**4*c**2) + 4*log(a*x - 1)/(a**5*c**2)
)
```

3.803.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{18\,a^2x^2-30\,ax+13}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}+\frac{x}{c^2}+\frac{4\,\log{(ax-1)}}{ac^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(18*a^2*x^2 - 30*a*x + 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
 - a*c^2) + x/c^2 + 4*log(a*x - 1)/(a*c^2)
```

3.803.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{ax-1}{ac^2}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{18\,a^5c^4}{ax-1}+\frac{6\,a^5c^4}{(ax-1)^2}+\frac{a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^2) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3
*(18*a^5*c^4/(a*x - 1) + 6*a^5*c^4/(a*x - 1)^2 + a^5*c^4/(a*x - 1)^3)/(a^6
*c^6)
```

3.803.9 Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{6\,a\,x^2-10\,x+\frac{13}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{4\,\ln{(a\,x-1)}}{a\,c^2}$$

input
```
int((a*x + 1)^2/((c - c/(a^2*x^2))^2*(a*x - 1)^2),x)
```

output
```
(6*a*x^2 - 10*x + 13/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x
) + x/c^2 + (4*log(a*x - 1))/(a*c^2)
```

3.804 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.804.1 Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(1+ax)}{32ac^3}$$

output
```
x/c^3-1/8/a/c^3/(-a*x+1)^4+11/12/a/c^3/(-a*x+1)^3-49/16/a/c^3/(-a*x+1)^2+1
11/16/a/c^3/(-a*x+1)+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

3.804.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{2(224-701ax+660a^2x^2-45a^3x^3-192a^4x^4+48a^5x^5)+387(-1+ax)^4\log(1-ax)-3(-1+ax)^4\log}{96ac^3(-1+ax)^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
(2*(224 - 701*a*x + 660*a^2*x^2 - 45*a^3*x^3 - 192*a^4*x^4 + 48*a^5*x^5) +
 387*(-1 + a*x)^4*Log[1 - a*x] - 3*(-1 + a*x)^4*Log[1 + a*x])/(96*a*c^3*(-
1 + a*x)^4)
```

3.804.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^6 e^{4\text{arctanh}(ax)}}{c^3\left(a^2-\frac{1}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^6\int \frac{e^{4\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^6\int \frac{e^{4\text{arctanh}(ax)}x^6}{\left(1-a^2x^2\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{a^6\int \frac{x^6}{(1-ax)^5(ax+1)}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^6\int \left(\frac{1}{32a^6(ax+1)}-\frac{1}{a^6}-\frac{129}{32a^6(ax-1)}-\frac{111}{16a^6(ax-1)^2}-\frac{49}{8a^6(ax-1)^3}-\frac{11}{4a^6(ax-1)^4}-\frac{1}{2a^6(ax-1)^5}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6\left(-\frac{111}{16a^7(1-ax)}+\frac{49}{16a^7(1-ax)^2}-\frac{11}{12a^7(1-ax)^3}+\frac{1}{8a^7(1-ax)^4}-\frac{129\log(1-ax)}{32a^7}+\frac{\log(ax+1)}{32a^7}-\frac{x}{a^6}\right)}{c^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
-((a^6*(-(x/a^6) + 1/(8*a^7*(1 - a*x)^4) - 11/(12*a^7*(1 - a*x)^3) + 49/(1
6*a^7*(1 - a*x)^2) - 111/(16*a^7*(1 - a*x)) - (129*Log[1 - a*x])/(32*a^7)
+ Log[1 + a*x]/(32*a^7)))/c^3)
```

3.804.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.804.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x}{c^{3}}+\frac{-\frac{111 a^{2} c^{3} x^{3}}{16}+\frac{71 a\,c^{3} x^{2}}{4}-\frac{749 c^{3} x}{48}+\frac{14 c^{3}}{3 a}}{c^{6} \left(a x -1\right)^{4}}+\frac{129 \ln \left(-a x +1\right)}{32 a\,c^{3}}-\frac{\ln \left(a x +1\right)}{32 a\,c^{3}}$
default	$\frac{a^{6} \left(-\frac{\ln \left(a x +1\right)}{32 a^{7}}+\frac{x}{a^{6}}-\frac{1}{8 a^{7} \left(a x -1\right)^{4}}-\frac{11}{12 a^{7} \left(a x -1\right)^{3}}-\frac{49}{16 a^{7} \left(a x -1\right)^{2}}-\frac{111}{16 a^{7} \left(a x -1\right)}+\frac{129 \ln \left(a x -1\right)}{32 a^{7}}\right)}{c^{3}}$
norman	$\frac{\frac{a^{6} x^{7}}{c}+\frac{65 x}{16 c}-\frac{49 a\,x^{2}}{8 c}-\frac{161 a^{2} x^{3}}{24 c}+\frac{301 a^{3} x^{4}}{24 c}+\frac{67 a^{4} x^{5}}{48 c}-\frac{20 a^{5} x^{6}}{3 c}}{\left(a x +1\right)^{2} \left(a x -1\right)^{4} c^{2}}+\frac{129 \ln \left(a x -1\right)}{32 a\,c^{3}}-\frac{\ln \left(a x +1\right)}{32 a\,c^{3}}$
parallelrisch	$\frac{12 a \ln \left(a x +1\right) x -18 a^{2} \ln \left(a x +1\right) x^{2}+96 a^{5} x^{5}+1702 a^{3} x^{3}-3 \ln \left(a x +1\right) x^{4} a^{4}+387 \ln \left(a x -1\right) x^{4} a^{4}+12 a^{3} \ln \left(a x +1\right) x^{3}+390 a x -1548}{96 \left(a x -1\right)^{4} c^{3} a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3+(-111/16*a^2*c^3*x^3+71/4*a*c^3*x^2-749/48*c^3*x+14/3*c^3/a)/c^6/(a*
x-1)^4+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

3.804.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{96\,a^{5}x^{5} - 384\,a^{4}x^{4} - 90\,a^{3}x^{3} + 1320\,a^{2}x^{2} - 1402\,ax - 3\,\left(a^{4}x^{4} - 4\,a^{3}x^{3} + 6\,a^{2}x^{2} - 4\,ax + 1\right)\log\left(ax + 1\right)}{96\,\left(a^{5}c^{3}x^{4} - 4\,a^{4}c^{3}x^{3} + 6\,a^{3}c^{3}x^{2} - 4\,a^{2}c^{3}x + a\right.}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/96*(96*a^5*x^5 - 384*a^4*x^4 - 90*a^3*x^3 + 1320*a^2*x^2 - 1402*a*x - 3*
(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x + 1) + 387*(a^4*x^4
 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 448)/(a^5*c^3*x^4 - 4*
a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

3.804.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-333a^3x^3+852a^2x^2-749ax+224}{48a^{11}c^3x^4-192a^{10}c^3x^3+288a^9c^3x^2-192a^8c^3x+48a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{129\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{a^7c^3}\right)$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*((-333*a**3*x**3 + 852*a**2*x**2 - 749*a*x + 224)/(48*a**11*c**3*x**4
 - 192*a**10*c**3*x**3 + 288*a**9*c**3*x**2 - 192*a**8*c**3*x + 48*a**7*c*
*3) + x/(a**6*c**3) + (129*log(x - 1/a)/32 - log(x + 1/a)/32)/(a**7*c**3))
```

3.804.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{333\,a^3x^3-852\,a^2x^2+749\,ax-224}{48\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}+\frac{x}{c^3}-\frac{\log(ax+1)}{32\,ac^3}+\frac{129\,\log(ax-1)}{32\,ac^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/(a^5*c^3*x^4 - 4*a^4*c^3
*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 - 1/32*log(a*x + 1)/(a
*c^3) + 129/32*log(a*x - 1)/(a*c^3)
```

3.804.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{ax-1}{ac^3} - \frac{4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3} - \frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3} - \frac{\frac{333\,a^{11}c^9}{ax-1}+\frac{147\,a^{11}c^9}{(ax-1)^2}+\frac{44\,a^{11}c^9}{(ax-1)^3}+\frac{6\,a^{11}c^9}{(ax-1)^4}}{48\,a^{12}c^{12}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^3) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/3
2*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(333*a^11*c^9/(a*x - 1) + 147*
a^11*c^9/(a*x - 1)^2 + 44*a^11*c^9/(a*x - 1)^3 + 6*a^11*c^9/(a*x - 1)^4)/(
a^12*c^12)
```

3.804.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3} - \frac{\frac{749\,x}{48}-\frac{71\,a\,x^2}{4}-\frac{14}{3\,a}+\frac{111\,a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3} + \frac{129\,\ln\left(a\,x-1\right)}{32\,a\,c^3} - \frac{\ln\left(a\,x+1\right)}{32\,a\,c^3}$$

input
```
int((a*x + 1)^2/((c - c/(a^2*x^2))^3*(a*x - 1)^2),x)
```

output
```
x/c^3 - ((749*x)/48 - (71*a*x^2)/4 - 14/(3*a) + (111*a^2*x^3)/16)/(c^3 + 6
*a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (129*log(a*x - 1
))/(32*a*c^3) - log(a*x + 1)/(32*a*c^3)
```

3.805 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.805.1 Optimal result

Integrand size = 22, antiderivative size = 146

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=&\ \frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}\\&+\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{261\log(1-ax)}{64ac^4}-\frac{5\log(1+ax)}{64ac^4}\end{aligned}$$

output
```
x/c^4+1/20/a/c^4/(-a*x+1)^5-7/16/a/c^4/(-a*x+1)^4+83/48/a/c^4/(-a*x+1)^3-6
7/16/a/c^4/(-a*x+1)^2+501/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+261/64*ln(-
a*x+1)/a/c^4-5/64*ln(a*x+1)/a/c^4
```

3.805.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.67

$$\begin{aligned}&\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx\\&=\frac{\frac{2(-2384+7541ax-4900a^2x^2-6800a^3x^3+9300a^4x^4-1365a^5x^5-1920a^6x^6+480a^7x^7)}{(-1+ax)^5(1+ax)}+3915\log(1-ax)-75\log(1+ax)}{960ac^4}\end{aligned}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
((2*(-2384 + 7541*a*x - 4900*a^2*x^2 - 6800*a^3*x^3 + 9300*a^4*x^4 - 1365*
a^5*x^5 - 1920*a^6*x^6 + 480*a^7*x^7))/((-1 + a*x)^5*(1 + a*x)) + 3915*Log
[1 - a*x] - 75*Log[1 + a*x])/(960*a*c^4)
```

3.805.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^8 e^{4\text{arctanh}(ax)}}{c^4\left(a^2-\frac{1}{x^2}\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^8\int \frac{e^{4\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6707}$$

$$\frac{a^8\int \frac{e^{4\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^8\int \frac{x^8}{(1-ax)^6(ax+1)^2}dx}{c^4}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^8\int \Big(-\frac{5}{64a^8(ax+1)}+\frac{1}{64a^8(ax+1)^2}+\frac{1}{a^8}+\frac{261}{64a^8(ax-1)}+\frac{501}{64a^8(ax-1)^2}+\frac{67}{8a^8(ax-1)^3}+\frac{83}{16a^8(ax-1)^4}+\frac{7}{4a^8(ax-1)^5}+\frac{1}{4a^8(ax-1}}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8\left(\frac{501}{64a^9(1-ax)}-\frac{1}{64a^9(ax+1)}-\frac{67}{16a^9(1-ax)^2}+\frac{83}{48a^9(1-ax)^3}-\frac{7}{16a^9(1-ax)^4}+\frac{1}{20a^9(1-ax)^5}+\frac{261\log(1-ax)}{64a^9}-\frac{5\log(ax+1)}{64a^9}+\frac{x}{a^8}\right.}{c^4}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
(a^8*(x/a^8 + 1/(20*a^9*(1 - a*x)^5) - 7/(16*a^9*(1 - a*x)^4) + 83/(48*a^9
*(1 - a*x)^3) - 67/(16*a^9*(1 - a*x)^2) + 501/(64*a^9*(1 - a*x)) - 1/(64*a
^9*(1 + a*x)) + (261*Log[1 - a*x])/(64*a^9) - (5*Log[1 + a*x])/(64*a^9)))/
c^4
```

3.805.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.805. $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.805.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
default	$\frac{a^{8}\left(-\frac{1}{64a^{9}(ax+1)}-\frac{5\ln(ax+1)}{64a^{9}}+\frac{x}{a^{8}}-\frac{1}{20a^{9}(ax-1)^{5}}-\frac{7}{16a^{9}(ax-1)^{4}}-\frac{83}{48a^{9}(ax-1)^{3}}-\frac{67}{16a^{9}(ax-1)^{2}}-\frac{501}{64a^{9}(ax-1)}+\frac{261\ln(ax-1)}{64a^{9}}\right)}{c^{4}}$
risch	$\frac{x}{c^{4}}+\frac{-\frac{251a^{4}c^{4}x^{5}}{32}+\frac{155a^{3}c^{4}x^{4}}{8}-\frac{55a^{2}c^{4}x^{3}}{6}-\frac{341a\,c^{4}x^{2}}{24}+\frac{8021c^{4}x}{480}-\frac{149c^{4}}{30a}}{c^{8}(ax-1)^{4}(a^{2}x^{2}-1)}-\frac{5\ln(ax+1)}{64a\,c^{4}}+\frac{261\ln(-ax+1)}{64a\,c^{4}}$
norman	$\frac{\frac{a^{8}x^{9}}{c}-\frac{115a^{3}x^{4}}{6c}-\frac{133x}{32c}+\frac{101a\,x^{2}}{16c}+\frac{1049a^{2}x^{3}}{96c}-\frac{3869a^{4}x^{5}}{480c}+\frac{4709a^{5}x^{6}}{240c}+\frac{43a^{6}x^{7}}{480c}-\frac{209a^{7}x^{8}}{30c}}{(ax+1)^{3}(ax-1)^{5}c^{3}}+\frac{261\ln(ax-1)}{64a\,c^{4}}-\frac{5\ln(ax+1)}{64a\,c^{4}}$
parallelrisch	$\frac{-300a\ln(ax+1)x+375a^{2}\ln(ax+1)x^{2}+16342a^{5}x^{5}-13600a^{3}x^{3}+300\ln(ax+1)x^{5}a^{5}-75\ln(ax+1)x^{6}a^{6}-375\ln(ax+1)x^{4}a^{4}+39}{}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
a^8/c^4*(-1/64/a^9/(a*x+1)-5/64/a^9*ln(a*x+1)+1/a^8*x-1/20/a^9/(a*x-1)^5-7
/16/a^9/(a*x-1)^4-83/48/a^9/(a*x-1)^3-67/16/a^9/(a*x-1)^2-501/64/a^9/(a*x-
1)+261/64/a^9*ln(a*x-1))
```

3.805.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.42

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{4}}\,dx = \frac{960\,a^{7}x^{7}-3840\,a^{6}x^{6}-2730\,a^{5}x^{5}+18600\,a^{4}x^{4}-13600\,a^{3}x^{3}-9800\,a^{2}x^{2}+15082\,ax-75\left(a^{6}x^{6}-4\,a^{5}x^{5}\right.}{960\left(a^{7}c^{4}x^{6}-4\,a^{6}c^{4}x^{5}+\right.}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/960*(960*a^7*x^7 - 3840*a^6*x^6 - 2730*a^5*x^5 + 18600*a^4*x^4 - 13600*a
^3*x^3 - 9800*a^2*x^2 + 15082*a*x - 75*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 -
5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 3915*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^
4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) - 4768)/(a^7*c^4*x^6 - 4*a^6*c^4*x
^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

3.805.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= a^8\left(\frac{-3765a^5x^5+9300a^4x^4-4400a^3x^3-6820a^2x^2+8021ax-2384}{480a^{15}c^4x^6-1920a^{14}c^4x^5+2400a^{13}c^4x^4-2400a^{11}c^4x^2+1920a^{10}c^4x-480a^9c^4}\right.$$
$$\left.+\frac{x}{a^8c^4}+\frac{\frac{261\log\left(x-\frac{1}{a}\right)}{64}-\frac{5\log\left(x+\frac{1}{a}\right)}{64}}{a^9c^4}\right)$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*((-3765*a**5*x**5 + 9300*a**4*x**4 - 4400*a**3*x**3 - 6820*a**2*x**2
+ 8021*a*x - 2384)/(480*a**15*c**4*x**6 - 1920*a**14*c**4*x**5 + 2400*a**1
3*c**4*x**4 - 2400*a**11*c**4*x**2 + 1920*a**10*c**4*x - 480*a**9*c**4) +
x/(a**8*c**4) + (261*log(x - 1/a)/64 - 5*log(x + 1/a)/64)/(a**9*c**4))
```

3.805.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{3765\,a^5x^5-9300\,a^4x^4+4400\,a^3x^3+6820\,a^2x^2-8021\,ax+2384}{480\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)}$$
$$+\frac{x}{c^4}-\frac{5\,\log(ax+1)}{64\,ac^4}+\frac{261\,\log(ax-1)}{64\,ac^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a
*x + 2384)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 +
4*a^2*c^4*x - a*c^4) + x/c^4 - 5/64*log(a*x + 1)/(a*c^4) + 261/64*log(a*x
- 1)/(a*c^4)
```

3.805.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{(ax-1)\left(\frac{257}{ax-1}+128\right)}{128\,ac^4\left(\frac{2}{ax-1}+1\right)} - \frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4} - \frac{5\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4} - \frac{\frac{7515\,a^{19}c^{16}}{ax-1}+\frac{4020\,a^{19}c^{16}}{(ax-1)^2}+\frac{1660\,a^{19}c^{16}}{(ax-1)^3}+\frac{420\,a^{19}c^{16}}{(ax-1)^4}+\frac{48\,a^{19}c^{16}}{(ax-1)^5}}{960\,a^{20}c^{20}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
1/128*(a*x - 1)*(257/(a*x - 1) + 128)/(a*c^4*(2/(a*x - 1) + 1)) - 4*log(ab
s(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 5/64*log(abs(-2/(a*x - 1) - 1))
/(a*c^4) - 1/960*(7515*a^19*c^16/(a*x - 1) + 4020*a^19*c^16/(a*x - 1)^2 +
1660*a^19*c^16/(a*x - 1)^3 + 420*a^19*c^16/(a*x - 1)^4 + 48*a^19*c^16/(a*x
 - 1)^5)/(a^20*c^20)
```

3.805.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{341\,a\,x^2}{24}-\frac{8021\,x}{480}+\frac{149}{30\,a}+\frac{55\,a^2\,x^3}{6}-\frac{155\,a^3\,x^4}{8}+\frac{251\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4} + \frac{x}{c^4} + \frac{261\,\ln\left(a\,x-1\right)}{64\,a\,c^4} - \frac{5\,\ln\left(a\,x+1\right)}{64\,a\,c^4}$$

input
```
int((a*x + 1)^2/((c - c/(a^2*x^2))^4*(a*x - 1)^2),x)
```

output
```
((341*a*x^2)/24 - (8021*x)/480 + 149/(30*a) + (55*a^2*x^3)/6 - (155*a^3*x^
4)/8 + (251*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 + 4*a^5*c^4*
x^5 - a^6*c^4*x^6 - 4*a*c^4*x) + x/c^4 + (261*log(a*x - 1))/(64*a*c^4) - (
5*log(a*x + 1))/(64*a*c^4)
```

3.806 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.806.1 Optimal result

Integrand size = 22, antiderivative size = 343

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx &= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}+\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}\\
&\quad+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a}+\frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}\\
&\quad+\frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a}\\
&\quad+c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x+\frac{35c^4\csc^{-1}(ax)}{16a}-\frac{c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

output
```
29/30*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)/a+7/6*c^4*(1-1/a/x)^(5/2)*(1+1/a
/x)^(7/2)/a+8/7*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)/a+c^4*(1-1/a/x)^(9/2)*
(1+1/a/x)^(7/2)*x+35/16*c^4*arccsc(a*x)/a-c^4*arctanh((1-1/a/x)^(1/2)*(1+1
/a/x)^(1/2))/a-1/16*c^4*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+7/40*c^4*(1+1/a/
x)^(5/2)*(1-1/a/x)^(1/2)/a+19/40*c^4*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-19/
16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.806.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.35

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=\frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(-240+280ax+1056a^2x^2-1330a^3x^3-1952a^4x^4+3045a^5x^5+2816a^6x^6+1680a^7x^7)}{x^6}+3675a^6\arcsin\left(\frac{1}{ax}\right)-1680a^6}{1680a^7}$$

input
```
Integrate[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

output
```
(c^4*((Sqrt[1 - 1/(a^2*x^2)]*(-240 + 280*a*x + 1056*a^2*x^2 - 1330*a^3*x^3
 - 1952*a^4*x^4 + 3045*a^5*x^5 + 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*
a^6*ArcSin[1/(a*x)] - 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*
a^7)
```

3.806.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.136, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6748}$$

$$-c^4\int \left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^4\left(\int -\frac{\left(a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\int\frac{\left(a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a^2}d\frac{1}{x}\right)$$

$$\Big\downarrow \; 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\int\left(a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \; 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{7}a\int\frac{7\left(a+\frac{7}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a}d\frac{1}{x}+\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\int\left(a+\frac{7}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}+\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}\right)$$

$$\Big\downarrow \; 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}a\int\frac{\left(6a+\frac{29}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a}d\frac{1}{x}+\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{7}{6}a(}{a^2}\right.$$

$$\Big\downarrow \; 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\int\left(6a+\frac{29}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}+\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/}}{a^2}\right.$$

$$\Big\downarrow \; 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{1}{5}a\int\frac{3\left(10a+\frac{19}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x}{a}d\frac{1}{x}+\frac{29}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)+}{a^2}\right.$$

$$\Big\downarrow \; 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\int\left(10a+\frac{19}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}+\frac{29}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)\right.}{a^2}\right.$$

3.806. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$

$$\Big\downarrow 171$$

$$-c^4\Bigg(x\bigg(-\Big(\frac{1}{ax}+1\Big)^{7/2}\bigg)\Big(1-\frac{1}{ax}\Big)^{9/2}-\frac{\frac{1}{6}\bigg(\frac{3}{5}\bigg(\frac{1}{4}a\int\frac{(40a-\frac{21}{x})(1+\frac{1}{ax})^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{19}{4}a\sqrt{1-\frac{1}{ax}}(\frac{1}{ax}+1)^{7/2}\bigg)+\frac{29}{5}a(1}{}$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(x\bigg(-\Big(\frac{1}{ax}+1\Big)^{7/2}\bigg)\Big(1-\frac{1}{ax}\Big)^{9/2}-\frac{\frac{1}{6}\bigg(\frac{3}{5}\bigg(\frac{1}{4}\int\frac{(40a-\frac{21}{x})(1+\frac{1}{ax})^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{19}{4}a\sqrt{1-\frac{1}{ax}}(\frac{1}{ax}+1)^{7/2}\bigg)+\frac{29}{5}a(1-}{}$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(x\bigg(-\Big(\frac{1}{ax}+1\Big)^{7/2}\bigg)\Big(1-\frac{1}{ax}\Big)^{9/2}-\frac{\frac{1}{6}\bigg(\frac{3}{5}\bigg(\frac{1}{4}\bigg(7a\sqrt{1-\frac{1}{ax}}(\frac{1}{ax}+1)^{5/2}-\frac{1}{3}a\int-\frac{15(8a+\frac{1}{x})(1+\frac{1}{ax})^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\bigg)+\frac{19}{4}}{}$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(x\bigg(-\Big(\frac{1}{ax}+1\Big)^{7/2}\bigg)\Big(1-\frac{1}{ax}\Big)^{9/2}-\frac{\frac{1}{6}\bigg(\frac{3}{5}\bigg(\frac{1}{4}\bigg(5\int\frac{(8a+\frac{1}{x})(1+\frac{1}{ax})^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}(\frac{1}{ax}+1)^{5/2}\bigg)+\frac{19}{4}a\sqrt{1}{}$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(x\bigg(-\Big(\frac{1}{ax}+1\Big)^{7/2}\bigg)\Big(1-\frac{1}{ax}\Big)^{9/2}-\frac{\frac{1}{6}\bigg(\frac{3}{5}\bigg(\frac{1}{4}\bigg(5\bigg(-\frac{1}{2}a\int-\frac{(16a+\frac{19}{x})\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\sqrt{1-\frac{1}{ax}}(\frac{1}{ax}+1)^{3/2}\bigg)+}{}$$

$$\Big\downarrow 25$$

3.806. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

$$-c^4 \Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2} - \frac{\frac{1}{6}\Bigg(\frac{3}{5}\Bigg(\frac{1}{4}\Bigg(5\Bigg(\frac{1}{2}a\int \frac{\left(16a+\frac{19}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+7a}{}$$

$$\Big\downarrow \mathbf{27}$$

$$-c^4 \Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2} - \frac{\frac{1}{6}\Bigg(\frac{3}{5}\Bigg(\frac{1}{4}\Bigg(5\Bigg(\frac{1}{2}\int \frac{\left(16a+\frac{19}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+7a\sqrt{}}{}$$

$$\Big\downarrow \mathbf{171}$$

$$-c^4 \Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2} - \frac{\frac{1}{6}\Bigg(\frac{3}{5}\Bigg(\frac{1}{4}\Bigg(5\Bigg(\frac{1}{2}\Bigg(a\Bigg(-\int -\frac{\left(16a+\frac{35}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\Bigg)-19a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)}{}$$

$$\Big\downarrow \mathbf{25}$$

$$-c^4 \Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2} - \frac{\frac{1}{6}\Bigg(\frac{3}{5}\Bigg(\frac{1}{4}\Bigg(5\Bigg(\frac{1}{2}\Bigg(a\int \frac{\left(16a+\frac{35}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-19a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{1}{2}a\sqrt{}}{}$$

$$\Big\downarrow \mathbf{27}$$

$$-c^4 \Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2} - \frac{\frac{1}{6}\Bigg(\frac{3}{5}\Bigg(\frac{1}{4}\Bigg(5\Bigg(\frac{1}{2}\Bigg(\int \frac{\left(16a+\frac{35}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-19a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)-\frac{1}{2}a\sqrt{1}}{}$$

$$\Big\downarrow \mathbf{175}$$

$$-c^4 \Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2} - \frac{\frac{1}{6}\Bigg(\frac{3}{5}\Bigg(\frac{1}{4}\Bigg(5\Bigg(\frac{1}{2}\Bigg(16a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+35\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-19a\sqrt{}}{}$$

3.806. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

$$\Big\downarrow \text{ 39}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-19a\sqrt{1-}\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 103}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 221}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 223}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(35a\arcsin\left(\frac{1}{ax}\right)-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-19a\right.\right.\right.\right.\right.}{}\right.$$

input
```
Int[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

output
```
-(c^4*(-((1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(7/2)*x) - ((7*a*(1 - 1/(a*x))^
(5/2)*(1 + 1/(a*x))^(7/2))/6 + (8*a*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(7/2
))/7 + ((29*a*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/5 + (3*((19*a*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/4 + (7*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2) + 5*(-1/2*(a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (-19*a*Sqrt[1
 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] + 35*a*ArcSin[1/(a*x)] - 16*a*ArcTanh[Sqrt[1
 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/2))/4))/5)/6)/a^2))
```

3.806. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.806.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.806. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 \, dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.806.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(ax+1)\left(2816a^6x^6+3045a^5x^5-1952a^4x^4-1330a^3x^3+1056a^2x^2+280ax-240\right)c^4\sqrt{\frac{ax-1}{ax+1}}}{1680x^7a^8}+\frac{\left(-\frac{a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{35a^7\ \mathrm{arctan}}{}\right.}{}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-3675a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}-3675a^7x^7\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{a^2}}\right.\right.}{}$

input
```
int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1680*(a*x+1)*(2816*a^6*x^6+3045*a^5*x^5-1952*a^4*x^4-1330*a^3*x^3+1056*a
^2*x^2+280*a*x-240)/x^7*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)+(-a^8*ln(a^2*x/(a^
2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+35/16*a^7*arctan(1/(a^2*x^2-1)^(1/
2))+a^7*((a*x-1)*(a*x+1))^(1/2))*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*
(a*x+1))^(1/2)/(a*x-1)
```

3.806.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx =$$

$$-\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(1680}{}$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4
*x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 + 4496*a^7*c^4*x^7 + 5861*a^6*c^4*
x^6 + 1093*a^5*c^4*x^5 - 3282*a^4*c^4*x^4 - 274*a^3*c^4*x^3 + 1336*a^2*c^4
*x^2 + 40*a*c^4*x - 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

3.806.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$

$$=\frac{c^4\Bigl(\int a^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^8}\,dx+\int\Bigl(-\frac{4a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\Bigr)\,dx+\int\frac{6a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\Bigl(-\frac{4a}{}}{a^8}$$

input
```
integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**4*(Integral(a**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**8, x) + Integral(-4*a**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/x**6, x) + Integral(6*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/x**4, x) + Integral(-4*a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x**2, x)
)/a**8
```

3.806.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.11

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx =$$
$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{1995\,c^4\left(\frac{ax-1}{ax+1}\right)}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/a^2 - (1995*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10185*c^4*((a*x - 1)/(a*x
+ 1))^(13/2) + 17619*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 4569*c^4*((a*x - 1
)/(a*x + 1))^(9/2) + 71801*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 72051*c^4*((a
*x - 1)/(a*x + 1))^(5/2) + 31465*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 5355*c^
4*sqrt((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a
^2/(a*x + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x
+ 1)^5 - 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 -
(a*x - 1)^8*a^2/(a*x + 1)^8 + a^2))*a
```

3.806.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.53

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{35\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{8\,a}$$
$$+\frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^4\operatorname{sgn}(ax+1)}{a}$$
$$-\frac{3045\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a|\operatorname{sgn}(ax+1)-6720\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4\operatorname{sgn}(ax+1)+6860\,(x|a|-}{}$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^4*log(a
bs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)
*c^4*sgn(a*x + 1)/a - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*ab
s(a)*sgn(a*x + 1) - 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x +
 1) + 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) - 201
60*(x*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 9065*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2
*x^2 - 1))^8*a*c^4*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a
*c^4*sgn(a*x + 1) - 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a
*x + 1) - 38976*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) - 6860
*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) - 12992*(x*abs(a
) - sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 3045*(x*abs(a) - sqrt(a^2*x^
2 - 1))*c^4*abs(a)*sgn(a*x + 1) - 2816*a*c^4*sgn(a*x + 1))/(((x*abs(a) - s
qrt(a^2*x^2 - 1))^2 + 1)^7*a*abs(a))
```

3.806.9 Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$

$$= \frac{\frac{51\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{899\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{24}+\frac{3431\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}+\frac{71801\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{840}+\frac{1523\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}+\frac{839\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+97\,c^4}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\frac{a\,(a}{(a}}$$

$$-\frac{35\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((51*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (899*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/24 + (3431*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (71801*c^4*((a*x -
1)/(a*x + 1))^(7/2))/840 + (1523*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (8
39*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (97*c^4*((a*x - 1)/(a*x + 1))^(1
3/2))/8 + (19*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a
*x + 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3
- (14*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(
a*x - 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*
x - 1)/(a*x + 1))^(1/2)))/(8*a) - (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/a
```

3.807 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.807.1 Optimal result

Integrand size = 22, antiderivative size = 269

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx\\ &= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}+\frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}+\frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a}\\ &\quad+\frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a}\\ &\quad+c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x+\frac{15c^3\csc^{-1}(ax)}{8a}-\frac{c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

output
```
5/4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)/a+6/5*c^3*(1-1/a/x)^(5/2)*(1+1/a/x
)^(5/2)/a+c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x+15/8*c^3*arccsc(a*x)/a-c^3
*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+1/24*c^3*(1+1/a/x)^(3/2)*(1-1/
a/x)^(1/2)/a+11/12*c^3*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-7/8*c^3*(1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2)/a
```

3.807.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.41

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(24-30ax-88a^2x^2+135a^3x^3+184a^4x^4+120a^5x^5)+225a^4x^4\arcsin\left(\frac{1}{ax}\right)-120a^4x^4\log\right.}{120a^5x^4}$$

input
```
Integrate[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

output
```
(c^3*(Sqrt[1 - 1/(a^2*x^2)]*(24 - 30*a*x - 88*a^2*x^2 + 135*a^3*x^3 + 184*
a^4*x^4 + 120*a^5*x^5) + 225*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5*x^4)
```

3.807.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-c^3\int \left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^3\left(\int -\frac{\left(a+\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\int \frac{\left(a+\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a^2}d\frac{1}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\int\left(a+\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \ 171$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{5}a\int\frac{5\left(a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}\right)$$

$$\Big\downarrow \ 27$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\int\left(a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}\right)$$

$$\Big\downarrow \ 171$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}a\int\frac{\left(4a+\frac{11}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{5}{4}a\left(1-\right.}{a^2}\right.$$

$$\Big\downarrow \ 27$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\int\left(4a+\frac{11}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+}{a^2}\right.$$

$$\Big\downarrow \ 171$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}a\int\frac{\left(12a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{6}{5}a\left(1-\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \ 27$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\int\frac{\left(12a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{6}{5}a\left(1-\frac{1}{ax}\right)}{a^2}\right.$$

$$\Big\downarrow \ 171$$

3.807. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int-\frac{3\left(8a+\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\Bigg)+\frac{11}{3}a\sqrt{1}}{}$$

$$\Big\downarrow 27$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{3}{2}\int\frac{\left(8a+\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)+\frac{11}{3}a\sqrt{1-\frac{1}{ax}}}{}$$

$$\Big\downarrow 171$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{3}{2}\Bigg(a\Bigg(-\int-\frac{\left(8a+\frac{15}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\Bigg)-7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{1}{2}a\sqrt{}}{}$$

$$\Big\downarrow 25$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{3}{2}\Bigg(a\int\frac{\left(8a+\frac{15}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{1}{2}a\sqrt{1-\frac{1}{ax}}}{}$$

$$\Big\downarrow 27$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{3}{2}\Bigg(\int\frac{\left(8a+\frac{15}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\big(}{}$$

$$\Big\downarrow 175$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{3}{2}\Bigg(8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+15\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-7a\sqrt{1-\frac{1}{ax}}\sqrt{}}{}$$

3.807. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

$$\Big\downarrow \; 39$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{3}{2}\Bigg(15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}-}}{}$$

$$\Big\downarrow \; 103$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{3}{2}\Bigg(15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-7a\sqrt{1}}{}$$

$$\Big\downarrow \; 221$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{3}{2}\Bigg(15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-7a\sqrt{1-}}{}$$

$$\Big\downarrow \; 223$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(15a\arcsin\left(\frac{1}{ax}\right)-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-7a\sqrt{1-\frac{1}{a}}\right.\right.\right.}{}$$

input
```
Int[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

output
```
-(c^3*(-((1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x) - ((5*a*(1 - 1/(a*x))^
(3/2)*(1 + 1/(a*x))^(5/2))/4 + (6*a*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2
))/5 + ((11*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2))/3 + ((a*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/2 + (3*(-7*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]
 + 15*a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
))/2)/3)/4)/a^2))
```

3.807. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.807.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.807. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.807.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.64

method	result
risch	$\frac{(ax+1)\left(184a^4x^4+135a^3x^3-88a^2x^2-30ax+24\right)c^3\sqrt{\frac{ax-1}{ax+1}}}{120x^5a^6}+\frac{\Big(-\frac{a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{15a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}+a^5\sqrt{(ax-1)(a}}{a^6(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\Big(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-225a^5x^5\sqrt{a^2}\,\sqrt{a^2x^2-1}-225a^5x^5\sqrt{a^2}\,\arctan\Big(\frac{1}{\sqrt{a^2x^2-1}}}{120\sqrt{(ax-1}}$

input
```
int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*(a*x+1)*(184*a^4*x^4+135*a^3*x^3-88*a^2*x^2-30*a*x+24)/x^5*c^3/a^6*(
(a*x-1)/(a*x+1))^(1/2)+(-a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)
^(1/2)+15/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^5*((a*x-1)*(a*x+1))^(1/2))*c
^3/a^6*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.807.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(120\,a^6c^3x}{120\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^
5*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) - (120*a^6*c^3*x^6 + 304*a^5*c^3*x^5 + 319*a^4*c^3*x^4 + 4
7*a^3*c^3*x^3 - 118*a^2*c^3*x^2 - 6*a*c^3*x + 24*c^3)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^6*x^5)
```

3.807.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\int a^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\right)dx+\int\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{3a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^6}$$

input
```
integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**3*(Integral(a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/x**6, x) + Integral(3*a**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/x**4, x) + Integral(-3*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/x**2, x))/a**6
```

3.807.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.12

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx =$$
$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+3\right.$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
 - (105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(
9/2) + 86*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 1654*c^3*((a*x - 1)/(a*x + 1))
^(5/2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 345*c^3*sqrt((a*x - 1)/(a*
x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a
*x - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^
2/(a*x + 1)^6 + a^2))*a
```

3.807.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.46

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{4\,a}$$
$$+\frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^3\operatorname{sgn}(ax+1)}{a}$$
$$-\frac{135\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|\operatorname{sgn}(ax+1)-360\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3\operatorname{sgn}(ax+1)+150\left(x|a|-\sqrt{a^2\right.}{}$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^3*log(a
bs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)
*c^3*sgn(a*x + 1)/a - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a
)*sgn(a*x + 1) - 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) +
 150*(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) - 720*(x*abs
(a) - sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) - 1120*(x*abs(a) - sqrt(a^2*
x^2 - 1))^4*a*c^3*sgn(a*x + 1) - 150*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*
abs(a)*sgn(a*x + 1) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x +
 1) - 135*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) - 184*a*c
^3*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

3.807.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{\frac{23\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{269\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{827\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{30}+\frac{43\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{30}+\frac{61\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{12}+\frac{7\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}$$
$$-\frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{2\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((23*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (269*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/12 + (827*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (43*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/30 + (61*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (7*c^3*((a
*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x -
 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*
x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1
))^(1/2)))/(4*a) - (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.808 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.808.1 Optimal result

Integrand size = 22, antiderivative size = 195

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx\\ &=-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}\\ &\quad+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{3c^2\csc^{-1}(ax)}{2a}-\frac{c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

output
```
4/3*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)/a+c^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(3
/2)*x+3/2*c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a
+3/2*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-1/2*c^2*(1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2)/a
```

3.808.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx\\ &=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(-2+3ax+8a^2x^2+6a^3x^3)+9a^2x^2\arcsin\left(\frac{1}{ax}\right)-6a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}\end{aligned}$$

input
```
Integrate[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 + 3*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x
^2*ArcSin[1/(a*x)] - 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3
*x^2)
```

3.808.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6748}$$

$$-c^2\int \left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^2\left(\int -\frac{\left(a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\int \frac{\left(a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{a^2}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\int \left(a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{3}a\int \frac{3\left(a+\frac{3}{x}\right)\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{a}d\frac{1}{x}+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{a^2}\right)$$

$$\Big\downarrow 27$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\int \left(a+\frac{3}{x}\right)\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{a^2}\right)$$

$$\Big\downarrow 171$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}a\int \frac{\left(2a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{3}{2}a\sqrt{1-\frac{1}{ax}}\big(\frac{1}{ax}}{a^2}$$

$$\Big\downarrow 27$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\int \frac{\left(2a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{3}{2}a\sqrt{1-\frac{1}{ax}}\big(\frac{1}{ax}+}{a^2}$$

$$\Big\downarrow 171$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(a\left(-\int -\frac{\left(2a+\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3}}{a^2}$$

$$\Big\downarrow 25$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(a\int \frac{\left(2a+\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\big(\frac{1}{ax}}{a^2}$$

$$\Big\downarrow 27$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(\int \frac{\left(2a+\frac{3}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\big(\frac{1}{ax}+1}{a^2}$$

3.808. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

$$\Big\downarrow \text{175}$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\Bigg(2a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+3\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)}{a^2}$$

$$\Big\downarrow \text{39}$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\Bigg(3\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{4}{3}}{a^2}$$

$$\Big\downarrow \text{103}$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\Bigg(3\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+a\Big(-\sqrt{1-\frac{1}{ax}}\Big)}{a^2}$$

$$\Big\downarrow \text{221}$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\Bigg(3\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}}}{a^2}$$

$$\Big\downarrow \text{223}$$

$$-c^2\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\Big(3a\arcsin\left(\frac{1}{ax}\right)-2a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+}}{a^2}$$

input
```
Int[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

3.808. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

output
```
-(c^2*(-((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x) - ((3*a*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2))/2 + (4*a*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2))
/3 + (-(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + 3*a*ArcSin[1/(a*x)] - 2*a
*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/2)/a^2))
```

3.808.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.808.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

method	result
risch	$\frac{(ax+1)(8a^2x^2+3ax-2)c^2\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-}}{a^4(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)c^2\left(-6\sqrt{a^2x^2-1}\sqrt{a^2}a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^2x^2-9\sqrt{a^2x^2-1}\sqrt{a^2}a^3x^3-9a^3x^3\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln\left(\right.\right.}{6\sqrt{(ax-1)(ax+1)}a^4x^3\sqrt{a^2}}$

input
```
int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)*(8*a^2*x^2+3*a*x-2)/x^3*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)+(-a^4*
ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+3/2*a^3*arctan(1/(a^2*
x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)
*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.808.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^2x^4+14\,a}{6\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^2*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (6*a^4*c^2*x^4 + 14*a^3*c^2*x^3 + 11*a^2*c^2*x^2 + a*c^2*x - 2*
c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

3.808.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^4}$$

input
```
integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**2*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-2*a**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/x**2, x))/a**4
```

3.808.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx =$$

$$-\frac{1}{3}\,a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^2\left(\frac{ax-1}{ax+1}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(a}{(a}}\right.$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
(3*c^2*((a*x - 1)/(a*x + 1))^(7/2) + c^2*((a*x - 1)/(a*x + 1))^(5/2) + 29*
c^2*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*
x + 1)^4 + a^2))
```

3.808.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.35

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{3\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}$$

$$+\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^2\operatorname{sgn}(ax+1)}{a}$$

$$-\frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|\operatorname{sgn}(ax+1)-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2\operatorname{sgn}(ax+1)-12\left(x|a|-\sqrt{a^2x^2-\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)}$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^2*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
2*sgn(a*x + 1)/a - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(
a*x + 1) - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) - 12*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 3*(x*abs(a) - sqrt(a^2*
x^2 - 1))*c^2*abs(a)*sgn(a*x + 1) - 8*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sq
rt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

3.808.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^2*((a*x - 1)/(a*x + 1))^(3/2))/
3 + (c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^2*((a*x - 1)/(a*x + 1))^(7/2))
/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x
- 1)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^
2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.809 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.809.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x+\frac{c\csc^{-1}(ax)}{a}-\frac{c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

output
```
c*arccsc(a*x)/a-c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1-1/a/x)^(
3/2)*x*(1+1/a/x)^(1/2)+2*c*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.809.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(1+ax)+\arcsin\left(\frac{1}{ax}\right)-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

output
```
(c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) + ArcSin[1/(a*x)] - Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x]))/a
```

3.809.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6748, 108, 25, 27, 171, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \, 6748$$

$$-c\int \left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \, 108$$

$$-c\left(\int -\frac{\left(a+\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}x}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \, 25$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\int \frac{\left(a+\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}x}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \, 27$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{\int \frac{\left(a+\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \, 171$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\int \frac{\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \, 140$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\frac{\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}+\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \color{blue}{39}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \color{blue}{103}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\frac{\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \color{blue}{221}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \color{blue}{223}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\arcsin\left(\frac{1}{ax}\right)-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

input
```
Int[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

3.809. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

output

```
-(c*(-((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) - (2*a*Sqrt[1 - 1/(a*x)]*S
qrt[1 + 1/(a*x)] + a*(ArcSin[1/(a*x)] - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 +
 1/(a*x)]]))/a^2))
```

3.809.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.809.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.18

method	result
risch	$\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(-\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,ax-ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+1)/x*c/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(-a*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(1/
2)))*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.809.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2cx^2+2\,acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 + 2*a
*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

3.809.6 Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^2}$$

input
```
integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a**2
```

3.809.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$= -a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*
c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

3.809.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a}+\frac{2\,c\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(
a*x + 1)/a + 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(
a))
```

3.809.9 Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*c
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1))^(
1/2)))/a
```

3.810 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.810.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

output
```
-arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c+2*(1-1/a/x)^(1/2)/a/c/(1+1/a
/x)^(1/2)+x*(1-1/a/x)^(1/2)/c/(1+1/a/x)^(1/2)
```

3.810.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+ax)}{1+ax}-\frac{\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

output
```
((Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x))/(1 + a*x) - Log[(1 + Sqrt[1 - 1/(a^2*
x^2)])*x]/a)/c
```

3.810.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6748, 114, 27, 35, 105, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{-\int \frac{\left(a-\frac{1}{x}\right)x}{a^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{-\frac{\int \frac{\sqrt{1-\frac{1}{ax}}x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{-\frac{\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{ 103}$$

$$-\frac{-\frac{\frac{2\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$-\frac{-\frac{\frac{2\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

input

```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

output

```
-((-((Sqrt[1 - 1/(a*x)]*x)/Sqrt[1 + 1/(a*x)]) - ((2*Sqrt[1 - 1/(a*x)])/Sqr
t[1 + 1/(a*x)] - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a)/c)
```

3.810.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 103

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.810.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}+\frac{\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{a^{4}\left(x+\frac{1}{a}\right)}\right)a^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{\left(-3\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x^{2}+2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}-6\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,ax+4\ln\left(\right.}{2a\sqrt{a^{2}}\,(ax+1)c\sqrt{(ax-1)(ax+}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)+1/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a
^2/c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.810.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)
+ log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

3.810.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2\int\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

output
```
a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

3.810.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$=-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

3.810.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{c-\frac{c}{a^2x^2}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
undef
```

3.810.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + ((a*x
- 1)/(a*x + 1))^(1/2)/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.811 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.811.1 Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=-\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

output
```
-arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-2/a/c^2/(1+1/a/x)^(3/2)/(1
-1/a/x)^(1/2)+x/c^2/(1+1/a/x)^(3/2)/(1-1/a/x)^(1/2)+5/3*(1-1/a/x)^(1/2)/a/
c^2/(1+1/a/x)^(3/2)+8/3*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

3.811.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.47

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-8-5ax+7a^2x^2+3a^3x^3\right)}{3(-1+ax)(1+ax)^2}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

output

```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 5*a*x + 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 +
 a*x)*(1 + a*x)^2) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

3.811.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6748, 114, 27, 169, 25, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \ 6748$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \ 114$$

$$-\frac{-\int \frac{\left(a-\frac{3}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{3}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{a\left(-\int -\frac{\left(a-\frac{4}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{a\int \frac{\left(a-\frac{4}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{4}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow 169$$

$$-\frac{-\frac{\frac{1}{3}a\int \frac{\left(3a-\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\frac{1}{3}\int \frac{\left(3a-\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow 169$$

$$-\frac{-\frac{\frac{1}{3}\left(a\int \frac{3x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\frac{1}{3}\left(3a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow 103$$

$$-\frac{-\frac{\frac{1}{3}\left(\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-3\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow 221$$

$$-\frac{-\frac{\frac{1}{3}\left(\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-3a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

3.811. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

output
```
-((-(x/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))) - ((-2*a)/(Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)) + (5*a*Sqrt[1 - 1/(a*x)])/(3*(1 + 1/(a*x))^(3/2))
+ ((8*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 3*a*ArcTanh[Sqrt[1 - 1/(a*x
)]*Sqrt[1 + 1/(a*x)]])/3)/a^2)/c^2)
```

3.811.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.811.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.19

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{6a^7\left(x+\frac{1}{a}\right)^2}+\frac{19\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{12a^6\left(x+\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2\left(x-\frac{1}{a}\right)a}}{4a^6\left(x-\frac{1}{a}\right)}\right)a^4}{c^2(ax-1)}$
default	$-\left(-45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5+24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^6x^5+21((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-45\sqrt{(ax-1)(ax+1)}\right.$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-1/6/a^7/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1
/2)+19/12/a^6/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1/4/a^6/(x-1/a)*((
x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^4/c^2*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*
(a*x+1))^(1/2)/(a*x-1)
```

3.811.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int\frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=$$
$$-\frac{3\left(a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(3\,a^3x^3+7\,a^2x^2-5\,ax-8\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-ac^2\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*(a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^2 - 1)
*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (3*a^3*x^3 + 7*a^2*x^2 - 5*a*x - 8)*
sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - a*c^2)
```

3.811.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{a^4\int \frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x
**2 + 1), x)/c**2
```

3.811.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=$$
$$-\frac{1}{12}a\Bigg(\frac{3\left(\frac{9(ax-1)}{ax+1}-1\right)}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+18\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{12\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\log\Bigg(\sqrt{\frac{ax-1}{ax+1}}-}{a^2c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/12*a*(3*(9*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2
) - a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) - (((a*x - 1)/(a*x + 1))^(3/2) + 18
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 12*log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

3.811.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^2, x)
```

3.811.9 Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\frac{9\,(a\,x-1)}{a\,x+1}-1}{4\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}+\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^2}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12\,a\,c^2}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^2,x)
```

output
```
((9*(a*x - 1))/(a*x + 1) - 1)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(1/2) - 4*a*c
^2*((a*x - 1)/(a*x + 1))^(3/2)) + (3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*c^2
) + ((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^2) + (atan(((a*x - 1)/(a*x + 1))^(
1/2)*1i)*2i)/(a*c^2)
```

3.812 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.812.1 Optimal result

Integrand size = 22, antiderivative size = 255

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}$$

output
```
-4/3/a/c^3/(1-1/a/x)^(3/2)/(1+1/a/x)^(5/2)+x/c^3/(1-1/a/x)^(3/2)/(1+1/a/x)
^(5/2)-arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-13/3/a/c^3/(1+1/a/x)
^(5/2)/(1-1/a/x)^(1/2)+14/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(5/2)+11/5*(1-
1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+16/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1
/2)
```

3.812.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(48+33ax-87a^2x^2-52a^3x^3+38a^4x^4+15a^5x^5)}{15(-1+ax)^2(1+ax)^3}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 33*a*x - 87*a^2*x^2 - 52*a^3*x^3 + 38*a^
4*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^2*(1 + a*x)^3) - Log[(1 + Sqrt[1 - 1/(
a^2*x^2)])*x])/(a*c^3)
```

3.812.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {6748, 114, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{\left(a-\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{3}a\int -\frac{\left(3a-\frac{16}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{\frac{1}{3}a\int \frac{\left(3a-\frac{16}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{3}\int \frac{\left(3a-\frac{16}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{3}\left(a\left(-\int -\frac{3\left(a-\frac{13}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{3}\left(3\int \frac{\left(a-\frac{13}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}a\int \frac{\left(5a-\frac{28}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\int \frac{\left(5a-\frac{28}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 169$$

3.812. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(\frac{1}{3}a\int\frac{3\left(5a-\frac{11}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+\right.}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(\int\frac{\left(5a-\frac{11}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(a\int\frac{5x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)^{3/}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(5a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)^{3}}}{c^3}$$

$$\Big\downarrow \; 103$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(-5\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-}{c^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(-5a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-}{c^3}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

3.812. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

output
```
-((-(x/((1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))) - ((-4*a)/(3*(1 - 1/(a*x
))^(3/2)*(1 + 1/(a*x))^(5/2)) + ((-13*a)/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2)) + 3*((14*a*Sqrt[1 - 1/(a*x)])/(5*(1 + 1/(a*x))^(5/2)) + ((11*a*Sqrt
[1 - 1/(a*x)])/(1 + 1/(a*x))^(3/2) + (16*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(
a*x)] - 5*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/5))/3)/a^2)/c^3)
```

3.812.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.812.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{3}}+\frac{\left(-\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{6}\sqrt{a^{2}}}+\frac{\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{20a^{10}\left(x+\frac{1}{a}\right)^{3}}-\frac{23\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{60a^{9}\left(x+\frac{1}{a}\right)^{2}}+\frac{493\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{240a^{8}\left(x+\frac{1}{a}\right)}-\right.}{c^{3}(ax-1)}$
default	$-\frac{\left(-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{7}x^{7}+240\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{8}x^{7}+285((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}\,a^{5}x^{5}-525\sqrt{(ax-1)(ax+}\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(-1/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+1/20/a^10/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^
(1/2)-23/60/a^9/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+493/240/a^8/(x
+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1/24/a^9/(x-1/a)^2*((x-1/a)^2*a^2+
2*(x-1/a)*a)^(1/2)-25/48/a^8/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^
6/c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.812.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.63

$$\int\frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{3}}\,dx=$$

$$-\frac{15\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(15\,a^{5}x^{5}+38\,a^{4}x\right.}{15\left(a^{5}c^{3}x^{4}-2\,a^{3}c^{3}x^{2}+ac^{3}\right)}$$

---

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
-1/15*(15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1
5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (15*a^5*x
^5 + 38*a^4*x^4 - 52*a^3*x^3 - 87*a^2*x^2 + 33*a*x + 48)*sqrt((a*x - 1)/(a
*x + 1)))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

3.812.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{a^6\int \frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*Integral(x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x
**4 + 3*a**2*x**2 - 1), x)/c**3
```

3.812.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{1}{240}a\left(\frac{5\left(\frac{23\,(ax-1)}{ax+1}-\frac{120\,(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+450\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}+\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
1/240*a*(5*(23*(a*x - 1)/(a*x + 1) - 120*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2
*c^3*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) +
(3*((a*x - 1)/(a*x + 1))^(5/2) + 40*((a*x - 1)/(a*x + 1))^(3/2) + 450*sqrt
((a*x - 1)/(a*x + 1)))/(a^2*c^3) - 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/
(a^2*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

3.812.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^3, x)
```

3.812.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3}-\frac{\frac{23\,(a\,x-1)}{3\,(a\,x+1)}-\frac{40\,{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{1}{3}}{16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{80\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^3}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^3,x)
```

output
```
(15*((a*x - 1)/(a*x + 1))^(1/2))/(8*a*c^3) - ((23*(a*x - 1))/(3*(a*x + 1))
 - (40*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/
2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(5/2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(
6*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) + (atan(((a*x - 1)/(a*x
+ 1))^(1/2)*1i)*2i)/(a*c^3)
```

3.813 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.813.1 Optimal result

Integrand size = 22, antiderivative size = 329

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx &= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}-\frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}\\ &\quad-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)^{7/2}}\\ &\quad+\frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{1+\frac{1}{ax}}}\\ &\quad+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}\end{aligned}$$

output
```
-6/5/a/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(7/2)-31/15/a/c^4/(1-1/a/x)^(3/2)/(1+
1/a/x)^(7/2)+x/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(7/2)-arctanh((1-1/a/x)^(1/2)
*(1+1/a/x)^(1/2))/a/c^4-28/3/a/c^4/(1+1/a/x)^(7/2)/(1-1/a/x)^(1/2)+115/21*
(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(7/2)+122/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/
x)^(5/2)+93/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+128/35*(1-1/a/x)^(1/2
)/a/c^4/(1+1/a/x)^(1/2)
```

3.813.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-384-279ax+1065a^2x^2+715a^3x^3-965a^4x^4-559a^5x^5+281a^6x^6+105a^7x^7)}{105(-1+ax)^3(1+ax)^4}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-384 - 279*a*x + 1065*a^2*x^2 + 715*a^3*x^3 -
 965*a^4*x^4 - 559*a^5*x^5 + 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^3
*(1 + a*x)^4) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

3.813.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.955, Rules used = {6748, 114, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{\left(a-\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{7}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{-\frac{1}{5}a\int -\frac{\left(5a-\frac{36}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{\frac{1}{5}a\int \frac{\left(5a-\frac{36}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{5}\int \frac{\left(5a-\frac{36}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{5}\left(-\frac{1}{3}a\int -\frac{5\left(3a-\frac{31}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\int \frac{\left(3a-\frac{31}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(a\left(-\int -\frac{\left(3a-\frac{112}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}\right)-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\big(\frac{1}{ax}+1}}{c^4}$$

$$\Big\downarrow \ 25$$

3.813. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(a\int \frac{\left(3a-\frac{112}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\int \frac{\left(3a-\frac{112}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \text{169}$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{1}{7}a\int \frac{3\left(7a-\frac{115}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)^{5/}}}{c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\int \frac{\left(7a-\frac{115}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}(}}{c^4}$$

$$\Big\downarrow \text{169}$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}a\int \frac{\left(35a-\frac{244}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+\right.}}{a^2}}{c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\int \frac{\left(35a-\frac{244}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right.}}{a^2}}{c^4}$$

$$\Big\downarrow \text{169}$$

3.813. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{3\left(35a-\frac{93}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)}{a^2}}{c^4}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(\int \frac{\left(35a-\frac{93}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-}{a^2}}{c^4}$$

$$\Big\downarrow 169$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(a\int \frac{35x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+\right.}}{a^2}}{c^4}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(35a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}\right.}}{a^2}}{c^4}$$

$$\Big\downarrow 103$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(-35\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{}{3\left(1-\right.}}{a^2}}{c^4}$$

$$\Big\downarrow 221$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(-35a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{}{3\left(1-\right.}}{a^2}}{c^4}$$

input

```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

3.813. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

output
```
-((-(x/((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2))) - ((-6*a)/(5*(1 - 1/(a*x
))^(5/2)*(1 + 1/(a*x))^(7/2)) + ((-31*a)/(3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*
x))^(7/2)) + (5*((-28*a)/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)) + (115*a*
Sqrt[1 - 1/(a*x)])/(7*(1 + 1/(a*x))^(7/2)) + (3*((122*a*Sqrt[1 - 1/(a*x)])
/(5*(1 + 1/(a*x))^(5/2)) + ((93*a*Sqrt[1 - 1/(a*x)])/(1 + 1/(a*x))^(3/2) +
 (128*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 35*a*ArcTanh[Sqrt[1 - 1/(a*
x)]*Sqrt[1 + 1/(a*x)]])/5))/7))/3)/5)/a^2)/c^4)
```

3.813.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

3.813. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.813.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.10

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{56a^{13}\left(x+\frac{1}{a}\right)^4}+\frac{17\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{112a^{12}\left(x+\frac{1}{a}\right)^3}-\frac{211\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{336a^{11}\left(x+\frac{1}{a}\right)^2}+1657\right.}{}$
default	Expression too large to display

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(-1/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-1/56/a^13/(x+1/a)^4*(a^2*(x+1/a)^2-2*a*(x+1/a))^
(1/2)+17/112/a^12/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-211/336/a^11
/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+1657/672/a^10/(x+1/a)*(a^2*(x
+1/a)^2-2*a*(x+1/a))^(1/2)-7/60/a^11/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)
^(1/2)-379/480/a^10/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-1/80/a^12/(x
-1/a)^3*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^8/c^4*((a*x-1)/(a*x+1))^(1/2)
*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.813.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.62

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx =$$

$$-\frac{105\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right.}{105\,(a^7c^4x^6-3\,a^5c^4x^4+}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
-1/105*(105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1) - 105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) - (105*a^7*x^7 + 281*a^6*x^6 - 559*a^5*x^5 - 965*a^4*x^4 +
 715*a^3*x^3 + 1065*a^2*x^2 - 279*a*x - 384)*sqrt((a*x - 1)/(a*x + 1)))/(a
^7*c^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

3.813.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{a^8\int \frac{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}\,dx}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*Integral(x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**8*x**8 - 4*a**6*x
**6 + 6*a**4*x**4 - 4*a**2*x**2 + 1), x)/c**4
```

3.813.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$=\frac{1}{6720}a\left(\frac{7\left(\frac{47\,(ax-1)}{ax+1}+\frac{655\,(ax-1)^2}{(ax+1)^2}-\frac{2625\,(ax-1)^3}{(ax+1)^3}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+329\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2940\right.}{a^2c^4}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
1/6720*a*(7*(47*(a*x - 1)/(a*x + 1) + 655*(a*x - 1)^2/(a*x + 1)^2 - 2625*(
a*x - 1)^3/(a*x + 1)^3 + 3)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4
*((a*x - 1)/(a*x + 1))^(5/2)) + 5*(3*((a*x - 1)/(a*x + 1))^(7/2) + 42*((a*
x - 1)/(a*x + 1))^(5/2) + 329*((a*x - 1)/(a*x + 1))^(3/2) + 2940*sqrt((a*x
 - 1)/(a*x + 1)))/(a^2*c^4) - 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2
*c^4) + 6720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

3.813.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^4, x)
```

3.813.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{35\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{\frac{131\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{175\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{47\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}+\frac{47\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{192\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{448\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^4,x)
```

output
```
(35*((a*x - 1)/(a*x + 1))^(1/2))/(16*a*c^4) - ((131*(a*x - 1)^2)/(3*(a*x +
 1)^2) - (175*(a*x - 1)^3)/(a*x + 1)^3 + (47*(a*x - 1))/(15*(a*x + 1)) + 1
/5)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))
^(7/2)) + (47*((a*x - 1)/(a*x + 1))^(3/2))/(192*a*c^4) + ((a*x - 1)/(a*x +
 1))^(5/2)/(32*a*c^4) + ((a*x - 1)/(a*x + 1))^(7/2)/(448*a*c^4) + (atan(((
a*x - 1)/(a*x + 1))^(1/2)*1i)*2i)/(a*c^4)
```

3.814 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.814.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

output

```
1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/a^3/
x^2+2*c^4/a^2/x+c^4*x-2*c^4*ln(x)/a
```

3.814.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

input

```
Integrate[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

output

```
c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x
^4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

3.814.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \textbf{6717}$$

$$-\int \frac{c^4e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4}{a^8}dx$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{c^4\int e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4dx}{a^8}$$

$$\Big\downarrow \textbf{6707}$$

$$-\frac{c^4\int \frac{e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$\Big\downarrow \textbf{6700}$$

$$-\frac{c^4\int \frac{(1-ax)^5(ax+1)^3}{x^8}dx}{a^8}$$

$$\Big\downarrow \textbf{99}$$

$$-\frac{c^4\int \left(-a^8+\frac{2a^7}{x}+\frac{2a^6}{x^2}-\frac{6a^5}{x^3}+\frac{6a^3}{x^5}-\frac{2a^2}{x^6}-\frac{2a}{x^7}+\frac{1}{x^8}\right)dx}{a^8}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{c^4\left(a^8(-x)+2a^7\log(x)-\frac{2a^6}{x}+\frac{3a^5}{x^2}-\frac{3a^3}{2x^4}+\frac{2a^2}{5x^5}+\frac{a}{3x^6}-\frac{1}{7x^7}\right)}{a^8}$$

input
```
Int[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^4*(-1/7*1/x^7 + a/(3*x^6) + (2*a^2)/(5*x^5) - (3*a^3)/(2*x^4) + (3*a^
5)/x^2 - (2*a^6)/x - a^8*x + 2*a^7*Log[x]))/a^8)
```

3.814.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.814. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.814.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^4\left(a^8x-2a^7\ln(x)+\frac{3a^3}{2x^4}-\frac{a}{3x^6}-\frac{3a^5}{x^2}+\frac{2a^6}{x}+\frac{1}{7x^7}-\frac{2a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{2a^6c^4x^6-3a^5c^4x^5+\frac{3}{2}a^3c^4x^3-\frac{2}{5}a^2c^4x^2-\frac{1}{3}a\,c^4x+\frac{1}{7}c^4}{a^8x^7}-\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^7c^4x^8+\frac{c^4}{7a}-\frac{c^4x}{3}-\frac{2a\,c^4x^2}{5}+\frac{3a^2c^4x^3}{2}-3a^4c^4x^5+2a^5c^4x^6}{a^7x^7}-\frac{2c^4\ln(x)}{a}$
parallelrisch	$-\frac{-210a^8c^4x^8+420c^4\ln(x)a^7x^7-420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3+84a^2c^4x^2+70a\,c^4x-30c^4}{210a^8x^7}$
meijerg	$\frac{c^4(ax-\ln(ax+1))}{a}-\frac{c^4\ln(ax+1)}{a}-\frac{4c^4(-\ln(ax+1)+\ln(x)+\ln(a))}{a}+\frac{4c^4\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+ 6c^4\Big(-\ln(ax$

input
```
int((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^4/a^8*(a^8*x-2*a^7*ln(x)+3/2*a^3/x^4-1/3*a/x^6-3*a^5/x^2+2*a^6/x+1/7/x^7
-2/5*a^2/x^5)
```

3.814.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{210\,a^8c^4x^8-420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/210*(210*a^8*c^4*x^8 - 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 - 630*a^
5*c^4*x^5 + 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x
^7)
```

3.814.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{a^8c^4x-2a^7c^4\log{(x)}+\frac{420a^6c^4x^6-630a^5c^4x^5+315a^3c^4x^3-84a^2c^4x^2-70ac^4x+30c^4}{210x^7}}{a^8}$$

input
```
integrate((c-c/a**2/x**2)**4*(a*x-1)/(a*x+1),x)
```

output
```
(a**8*c**4*x - 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 - 630*a**5*c**4*x*
*5 + 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**4*x + 30*c**4)/(210*
x**7))/a**8
```

3.814.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=c^4x-\frac{2\,c^4\log{(x)}}{a}+\frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^4*x - 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 315*a^
3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

3.814.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=c^4x-\frac{2\,c^4\log{(|x|)}}{a}+\frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^4*x - 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 3
15*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

3.814.9 Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=-\frac{c^4\left(\frac{a\,x}{3}+\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5-2\,a^6\,x^6-a^8\,x^8+2\,a^7\,x^7\,\ln\left(x\right)-\frac{1}{7}\right)}{a^8\,x^7}$$

input
```
int(((c - c/(a^2*x^2))^4*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c^4*((a*x)/3 + (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 - 2*a^6*x^6 - a
^8*x^8 + 2*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

3.815 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.815.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

output
```
-1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3-2*c^3/a^3/x^2+c^3/a^2/x+c
^3*x-2*c^3*ln(x)/a
```

3.815.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-1/5*c^3/(a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2)
 + c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

3.815.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^3e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^3}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^3dx}{a^6}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c^3\int \frac{e^{-2\text{arctanh}(ax)}(1-a^2x^2)^3}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c^3\int \frac{(1-ax)^4(ax+1)^2}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^3\int \left(a^6-\frac{2a^5}{x}-\frac{a^4}{x^2}+\frac{4a^3}{x^3}-\frac{a^2}{x^4}-\frac{2a}{x^5}+\frac{1}{x^6}\right)dx}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(a^6x-2a^5\log(x)+\frac{a^4}{x}-\frac{2a^3}{x^2}+\frac{a^2}{3x^3}+\frac{a}{2x^4}-\frac{1}{5x^5}\right)}{a^6}$$

input
```
Int[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^3*(-1/5*1/x^5 + a/(2*x^4) + a^2/(3*x^3) - (2*a^3)/x^2 + a^4/x + a^6*x -
 2*a^5*Log[x]))/a^6
```

3.815.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.815. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.815.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

method	result
default	$\frac{c^3\left(a^6x-2a^5\ln(x)+\frac{a}{2x^4}+\frac{a^2}{3x^3}-\frac{2a^3}{x^2}+\frac{a^4}{x}-\frac{1}{5x^5}\right)}{a^6}$
risch	$c^3x+\frac{a^4c^3x^4-2a^3c^3x^3+\frac{1}{3}a^2c^3x^2+\frac{1}{2}a\,c^3x-\frac{1}{5}c^3}{a^6x^5}-\frac{2c^3\ln(x)}{a}$
norman	$\frac{a^3c^3x^4+a^5c^3x^6-\frac{c^3}{5a}+\frac{c^3x}{2}+\frac{a\,c^3x^2}{3}-2a^2c^3x^3}{a^5x^5}-\frac{2c^3\ln(x)}{a}$
parallelrisch	$-\frac{-30a^6c^3x^6+60c^3\ln(x)a^5x^5-30a^4c^3x^4+60a^3c^3x^3-10a^2c^3x^2-15a\,c^3x+6c^3}{30a^6x^5}$
meijerg	$\frac{c^3(ax-\ln(ax+1))}{a}-\frac{c^3\ln(ax+1)}{a}-\frac{3c^3(-\ln(ax+1)+\ln(x)+\ln(a))}{a}+\frac{3c^3\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+3c^3\left(-\ln(ax\right.$

input
```
int((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^3/a^6*(a^6*x-2*a^5*ln(x)+1/2*a/x^4+1/3*a^2/x^3-2*a^3/x^2+a^4/x-1/5/x^5)
```

3.815.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{30\,a^6c^3x^6-60\,a^5c^3x^5\log\left(x\right)+30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/30*(30*a^6*c^3*x^6 - 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 - 60*a^3*c^3
*x^3 + 10*a^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.815.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{a^6c^3x-2a^5c^3\log(x)+\frac{30a^4c^3x^4-60a^3c^3x^3+10a^2c^3x^2+15ac^3x-6c^3}{30x^5}}{a^6}$$

input
```
integrate((c-c/a**2/x**2)**3*(a*x-1)/(a*x+1),x)
```

output
```
(a**6*c**3*x - 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 - 60*a**3*c**3*x**3
 + 10*a**2*c**3*x**2 + 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

3.815.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=c^3x-\frac{2\,c^3\log(x)}{a}+\frac{30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^3*x - 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^2*c^
3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.815.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=c^3x-\frac{2\,c^3\log(|x|)}{a}+\frac{30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^3*x - 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a
^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

3.815.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3\left(\frac{a\,x}{2}+\frac{a^2\,x^2}{3}-2\,a^3\,x^3+a^4\,x^4+a^6\,x^6-2\,a^5\,x^5\,\ln\left(x\right)-\frac{1}{5}\right)}{a^6\,x^5}$$

input
```
int(((c - c/(a^2*x^2))^3*(a*x - 1))/(a*x + 1),x)
```

output
```
(c^3*((a*x)/2 + (a^2*x^2)/3 - 2*a^3*x^3 + a^4*x^4 + a^6*x^6 - 2*a^5*x^5*lo
g(x) - 1/5))/(a^6*x^5)
```

3.816 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.816.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

output
```
1/3*c^2/a^4/x^3-c^2/a^3/x^2+c^2*x-2*c^2*ln(x)/a
```

3.816.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
```

output
```
c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

3.816.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = $\{6717, 27, 6707, 6700, 84, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^2e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^2}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^2dx}{a^4}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{c^2\int \frac{e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{c^2\int \frac{(1-ax)^3(ax+1)}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{84}$$

$$-\frac{c^2\int \left(-a^4+\frac{2a^3}{x}-\frac{2a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\left(a^4(-x)+2a^3\log(x)+\frac{a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^2*(-1/3*1/x^3 + a/x^2 - a^4*x + 2*a^3*Log[x]))/a^4)
```

3.816.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.816.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^2\left(a^4x-2a^3\ln(x)+\frac{1}{3x^3}-\frac{a}{x^2}\right)}{a^4}$
risch	$c^2x+\frac{-a\,c^2x+\frac{1}{3}c^2}{a^4x^3}-\frac{2c^2\ln(x)}{a}$
norman	$\frac{a^3c^2x^4+\frac{c^2}{3a}-c^2x}{a^3x^3}-\frac{2c^2\ln(x)}{a}$
parallelrisch	$-\frac{-3a^4c^2x^4+6c^2\ln(x)a^3x^3+3a\,c^2x-c^2}{3a^4x^3}$
meijerg	$\frac{c^2(ax-\ln(ax+1))}{a}-\frac{c^2\ln(ax+1)}{a}-\frac{2c^2(-\ln(ax+1)+\ln(x)+\ln(a))}{a}+\frac{2c^2\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+\ c^2\big(-\ln(ax+$

input
```
int((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^2/a^4*(a^4*x-2*a^3*ln(x)+1/3/x^3-a/x^2)
```

3.816.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{3\,a^4c^2x^4-6\,a^3c^2x^3\log{(x)}-3\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^2*x^4 - 6*a^3*c^2*x^3*log(x) - 3*a*c^2*x + c^2)/(a^4*x^3)
```

3.816.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x-2a^3c^2\log{(x)}+\frac{-3ac^2x+c^2}{3x^3}}{a^4}$$

input
```
integrate((c-c/a**2/x**2)**2*(a*x-1)/(a*x+1),x)
```

output
```
(a**4*c**2*x - 2*a**3*c**2*log(x) + (-3*a*c**2*x + c**2)/(3*x**3))/a**4
```

3.816.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x-\frac{2\,c^2\log\left(x\right)}{a}-\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^2*x - 2*c^2*log(x)/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

3.816.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x-\frac{2\,c^2\log\left(|x|\right)}{a}-\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^2*x - 2*c^2*log(abs(x))/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

3.816.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln\left(x\right)-1\right)}{3\,a^4\,x^3}$$

input
```
int(((c - c/(a^2*x^2))^2*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c^2*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

3.817 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.817.1 Optimal result

Integrand size = 20, antiderivative size = 21

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

output
```
-c/a^2/x+c*x-2*c*ln(x)/a
```

3.817.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

3.817.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{ce^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c\int e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 6707}$$

$$\frac{c\int \frac{e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6700}$$

$$\frac{c\int \frac{(1-ax)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\int \left(a^2-\frac{2a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\left(a^2x-2a\log(x)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*(-x^(-1) + a^2*x - 2*a*Log[x]))/a^2
```

3.817.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.817.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{c\left(a^{2} x-2 a \ln\left(x\right)-\frac{1}{x}\right)}{a^{2}}$	22
risch	$-\frac{c}{a^{2} x}+c x-\frac{2 c \ln\left(x\right)}{a}$	22
parallelrisch	$-\frac{-a^{2} c \,x^{2}+2 c \ln\left(x\right) a x+c}{a^{2} x}$	27
norman	$\frac{a c \,x^{2}-\frac{c}{a}}{a x}-\frac{2 c \ln\left(x\right)}{a}$	30
meijerg	$\frac{c\left(a x-\ln\left(a x+1\right)\right)}{a}-\frac{c \ln\left(a x+1\right)}{a}-\frac{c\left(-\ln\left(a x+1\right)+\ln\left(x\right)+\ln\left(a\right)\right)}{a}+\frac{c\left(\ln\left(a x+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{a x}\right)}{a}$	78

input
```
int((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c/a^2*(a^2*x-2*a*ln(x)-1/x)
```

3.817.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx^2-2\,acx\log\left(x\right)-c}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
(a^2*c*x^2 - 2*a*c*x*log(x) - c)/(a^2*x)
```

3.817.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx-2ac\log\left(x\right)-\frac{c}{x}}{a^2}$$

input
```
integrate((c-c/a**2/x**2)*(a*x-1)/(a*x+1),x)
```

output
```
(a**2*c*x - 2*a*c*log(x) - c/x)/a**2
```

3.817.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx-\frac{2\,c\log\left(x\right)}{a}-\frac{c}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c*x - 2*c*log(x)/a - c/(a^2*x)
```

3.817.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx-\frac{2\,c\log\left(|x|\right)}{a}-\frac{c}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c*x - 2*c*log(abs(x))/a - c/(a^2*x)
```

3.817.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c\,\left(2\,a\,x\,\ln\left(x\right)-a^2\,x^2+1\right)}{a^2\,x}$$

input
```
int(((c - c/(a^2*x^2))*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c*(2*a*x*log(x) - a^2*x^2 + 1))/(a^2*x)
```

3.818 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.818.1 Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{ac(1+ax)}-\frac{2\log(1+ax)}{ac}$$

output
```
x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

3.818.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x-\frac{1}{a+a^2x}-\frac{2\log(1+ax)}{a}}{c}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

output
```
(x - (a + a^2*x)^(-1) - (2*Log[1 + a*x])/a)/c
```

3.818.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^2e^{-2\text{arctanh}(ax)}}{c\left(a^2-\frac{1}{x^2}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}}{a^2-\frac{1}{x^2}}dx}{c}$$

$$\Big\downarrow \text{6707}$$

$$\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}x^2}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^2\int \frac{x^2}{(ax+1)^2}dx}{c}$$

$$\Big\downarrow \text{49}$$

$$\frac{a^2\int \left(\frac{1}{a^2}-\frac{2}{a^2(ax+1)}+\frac{1}{a^2(ax+1)^2}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(-\frac{1}{a^3(ax+1)}-\frac{2\log(ax+1)}{a^3}+\frac{x}{a^2}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

output
```
(a^2*(x/a^2 - 1/(a^3*(1 + a*x)) - (2*Log[1 + a*x])/a^3))/c
```

3.818.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.818.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{x}{c}-\frac{1}{ac\left(ax+1\right)}-\frac{2\ln\left(ax+1\right)}{ac}$	36
default	$\frac{a^{2}\left(-\frac{2\ln\left(ax+1\right)}{a^{3}}-\frac{1}{a^{3}\left(ax+1\right)}+\frac{x}{a^{2}}\right)}{c}$	37
norman	$\frac{\frac{a\,x^{2}}{c}+\frac{2x}{c}}{ax+1}-\frac{2\ln\left(ax+1\right)}{ac}$	39
parallelrisch	$\frac{a^{2}x^{2}-2a\ln\left(ax+1\right)x+2ax-2\ln\left(ax+1\right)}{c\left(ax+1\right)a}$	45

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

3.818.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2x^2+ax-2\,(ax+1)\log{(ax+1)}-1}{a^2cx+ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^2*x^2 + a*x - 2*(a*x + 1)*log(a*x + 1) - 1)/(a^2*c*x + a*c)
```

3.818.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = a^2\left(-\frac{1}{a^4cx+a^3c}+\frac{x}{a^2c}-\frac{2\log{(ax+1)}}{a^3c}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2),x)
```

output
```
a**2*(-1/(a**4*c*x + a**3*c) + x/(a**2*c) - 2*log(a*x + 1)/(a**3*c))
```

3.818.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{1}{a^2cx+ac}-\frac{2\,\log{(ax+1)}}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
x/c - 1/(a^2*c*x + a*c) - 2*log(a*x + 1)/(a*c)
```

3.818.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{2\log\left(|ax+1|\right)}{ac}-\frac{1}{(ax+1)ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
x/c - 2*log(abs(a*x + 1))/(a*c) - 1/((a*x + 1)*a*c)
```

3.818.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{a\,\left(c+a\,c\,x\right)}-\frac{2\,\ln\left(a\,x+1\right)}{a\,c}$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))*(a*x + 1)),x)
```

output
```
x/c - 1/(a*(c + a*c*x)) - (2*log(a*x + 1))/(a*c)
```

3.819 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.819.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}+\frac{1}{4ac^2(1+ax)^2}-\frac{7}{4ac^2(1+ax)}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(1+ax)}{8ac^2}$$

output
```
x/c^2+1/4/a/c^2/(a*x+1)^2-7/4/a/c^2/(a*x+1)+1/8*ln(-a*x+1)/a/c^2-17/8*ln(a
*x+1)/a/c^2
```

3.819.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx =\frac{2(-6-3ax+8a^2x^2+4a^3x^3)+(1+ax)^2\log(1-ax)-17(1+ax)^2\log(1+ax)}{8a(c+acx)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

output
```
(2*(-6 - 3*a*x + 8*a^2*x^2 + 4*a^3*x^3) + (1 + a*x)^2*Log[1 - a*x] - 17*(1
 + a*x)^2*Log[1 + a*x])/(8*a*(c + a*c*x)^2)
```

3.819.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^4e^{-2\text{arctanh}(ax)}}{c^2\left(a^2-\frac{1}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{a^4\int \frac{x^4}{(1-ax)(ax+1)^3}dx}{c^2}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^4\int \left(\frac{17}{8a^4(ax+1)}-\frac{7}{4a^4(ax+1)^2}+\frac{1}{2a^4(ax+1)^3}-\frac{1}{a^4}-\frac{1}{8a^4(ax-1)}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\left(\frac{7}{4a^5(ax+1)}-\frac{1}{4a^5(ax+1)^2}-\frac{\log(1-ax)}{8a^5}+\frac{17\log(ax+1)}{8a^5}-\frac{x}{a^4}\right)}{c^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

output
```
-((a^4*(-(x/a^4) - 1/(4*a^5*(1 + a*x)^2) + 7/(4*a^5*(1 + a*x)) - Log[1 - a
*x]/(8*a^5) + (17*Log[1 + a*x])/(8*a^5)))/c^2)
```

3.819. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$

3.819.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.819.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{a^{4} \left(-\frac{17 \ln \left(a x +1\right)}{8 a^{5}}+\frac{1}{4 a^{5} \left(a x +1\right)^{2}}-\frac{7}{4 a^{5} \left(a x +1\right)}+\frac{x}{a^{4}}+\frac{\ln \left(a x -1\right)}{8 a^{5}}\right)}{c^{2}}$	60
risch	$\frac{x}{c^{2}}+\frac{-\frac{7 c^{2} x}{4}-\frac{3 c^{2}}{2 a}}{c^{4} \left(a x +1\right)^{2}}-\frac{17 \ln \left(a x +1\right)}{8 a \,c^{2}}+\frac{\ln \left(-a x +1\right)}{8 a \,c^{2}}$	62
norman	$\frac{\frac{a^{3} x^{4}}{c}-\frac{9 x}{4 c}-\frac{5 a \,x^{2}}{4 c}+\frac{5 a^{2} x^{3}}{2 c}}{\left(a x +1\right)^{2} c \left(a x -1\right)}+\frac{\ln \left(a x -1\right)}{8 a \,c^{2}}-\frac{17 \ln \left(a x +1\right)}{8 a \,c^{2}}$	85
parallelrisch	$\frac{8 a^{3} x^{3}+a^{2} \ln \left(a x -1\right) x^{2}-17 a^{2} \ln \left(a x +1\right) x^{2}+28 a^{2} x^{2}+2 a \ln \left(a x -1\right) x -34 a \ln \left(a x +1\right) x +18 a x +\ln \left(a x -1\right)-17 \ln \left(a x +1\right)}{8 c^{2} \left(a x +1\right)^{2} a}$	98

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^4/c^2*(-17/8*ln(a*x+1)/a^5+1/4/a^5/(a*x+1)^2-7/4/a^5/(a*x+1)+x/a^4+1/8/a
^5*ln(a*x-1))
```

3.819.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{8\,a^3x^3+16\,a^2x^2-6\,ax-17\,(a^2x^2+2\,ax+1)\log{(ax+1)}+(a^2x^2+2\,ax+1)\log{(ax-1)}-12}{8\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/8*(8*a^3*x^3 + 16*a^2*x^2 - 6*a*x - 17*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1
) + (a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 12)/(a^3*c^2*x^2 + 2*a^2*c^2*x +
a*c^2)
```

3.819.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-7ax-6}{4a^7c^2x^2+8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{\log{\left(x-\frac{1}{a}\right)}}{8}-\frac{17\log{\left(x+\frac{1}{a}\right)}}{8}}{a^5c^2}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*((-7*a*x - 6)/(4*a**7*c**2*x**2 + 8*a**6*c**2*x + 4*a**5*c**2) + x/(a
**4*c**2) + (log(x - 1/a)/8 - 17*log(x + 1/a)/8)/(a**5*c**2))
```

3.819.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{7\,ax+6}{4\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}+\frac{x}{c^2}-\frac{17\,\log{(ax+1)}}{8\,ac^2}+\frac{\log{(ax-1)}}{8\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(7*a*x + 6)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) + x/c^2 - 17/8*log(a*
x + 1)/(a*c^2) + 1/8*log(a*x - 1)/(a*c^2)
```

3.819.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{17\,\log{(|ax+1|)}}{8\,ac^2}+\frac{\log{(|ax-1|)}}{8\,ac^2}-\frac{7\,ax+6}{4\,(ax+1)^2ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
x/c^2 - 17/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) - 1
/4*(7*a*x + 6)/((a*x + 1)^2*a*c^2)
```

3.819.9 Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{\frac{7\,x}{4}+\frac{3}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2}+\frac{\ln{(a\,x-1)}}{8\,a\,c^2}-\frac{17\,\ln{(a\,x+1)}}{8\,a\,c^2}$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^2*(a*x + 1)),x)
```

output
```
x/c^2 - ((7*x)/4 + 3/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) + log(a*x - 1)
/(8*a*c^2) - (17*log(a*x + 1))/(8*a*c^2)
```

3.820 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.820.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}+\frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}+\frac{5}{8ac^3(1+ax)^2} -\frac{39}{16ac^3(1+ax)}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(1+ax)}{4ac^3}$$

output
```
x/c^3+1/16/a/c^3/(-a*x+1)-1/12/a/c^3/(a*x+1)^3+5/8/a/c^3/(a*x+1)^2-39/16/a
/c^3/(a*x+1)+1/4*ln(-a*x+1)/a/c^3-9/4*ln(a*x+1)/a/c^3
```

3.820.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{2(11+7ax-24a^2x^2-15a^3x^3+12a^4x^4+6a^5x^5)+3(-1+ax)(1+ax)^3\log(1-ax)-27(-1+ax)(1-}{12a(-1+ax)(c+acx)^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

output
```
(2*(11 + 7*a*x - 24*a^2*x^2 - 15*a^3*x^3 + 12*a^4*x^4 + 6*a^5*x^5) + 3*(-1
 + a*x)*(1 + a*x)^3*Log[1 - a*x] - 27*(-1 + a*x)*(1 + a*x)^3*Log[1 + a*x])
/(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

3.820.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^6 e^{-2\text{arctanh}(ax)}}{c^3\left(a^2-\frac{1}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^6\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6707}$$

$$\frac{a^6\int \frac{e^{-2\text{arctanh}(ax)}x^6}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^6\int \frac{x^6}{(1-ax)^2(ax+1)^4}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^6\int \left(-\frac{9}{4a^6(ax+1)}+\frac{39}{16a^6(ax+1)^2}-\frac{5}{4a^6(ax+1)^3}+\frac{1}{4a^6(ax+1)^4}+\frac{1}{a^6}+\frac{1}{4a^6(ax-1)}+\frac{1}{16a^6(ax-1)^2}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6\left(\frac{1}{16a^7(1-ax)}-\frac{39}{16a^7(ax+1)}+\frac{5}{8a^7(ax+1)^2}-\frac{1}{12a^7(ax+1)^3}+\frac{\log(1-ax)}{4a^7}-\frac{9\log(ax+1)}{4a^7}+\frac{x}{a^6}\right)}{c^3}$$

3.820. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

output
```
(a^6*(x/a^6 + 1/(16*a^7*(1 - a*x)) - 1/(12*a^7*(1 + a*x)^3) + 5/(8*a^7*(1
+ a*x)^2) - 39/(16*a^7*(1 + a*x)) + Log[1 - a*x]/(4*a^7) - (9*Log[1 + a*x]
)/(4*a^7)))/c^3
```

3.820.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.820.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

method	result
default	$\frac{a^{6} \left(-\frac{9 \ln \left(a x +1\right)}{4 a^{7}}-\frac{1}{12 a^{7} \left(a x +1\right)^{3}}+\frac{5}{8 a^{7} \left(a x +1\right)^{2}}-\frac{39}{16 a^{7} \left(a x +1\right)}+\frac{x}{a^{6}}-\frac{1}{16 a^{7} \left(a x -1\right)}+\frac{\ln \left(a x -1\right)}{4 a^{7}}\right)}{c^{3}}$
risch	$\frac{x}{c^{3}}+\frac{-\frac{5 a^{2} c^{3} x^{3}}{2}-2 a \,c^{3} x^{2}+\frac{13 c^{3} x}{6}+\frac{11 c^{3}}{6 a}}{c^{6} \left(a x +1\right)^{2} \left(a^{2} x^{2}-1\right)}+\frac{\ln \left(-a x +1\right)}{4 a \,c^{3}}-\frac{9 \ln \left(a x +1\right)}{4 a \,c^{3}}$
norman	$\frac{\frac{a^{5} x^{6}}{c}+\frac{5 x}{2 c}+\frac{3 a \,x^{2}}{2 c}-\frac{31 a^{2} x^{3}}{6 c}-\frac{8 a^{3} x^{4}}{3 c}+\frac{17 a^{4} x^{5}}{6 c}}{c^{2} \left(a x +1\right)^{3} \left(a x -1\right)^{2}}+\frac{\ln \left(a x -1\right)}{4 a \,c^{3}}-\frac{9 \ln \left(a x +1\right)}{4 a \,c^{3}}$
parallelrisch	$\frac{12 a^{5} x^{5}+3 \ln \left(a x -1\right) x^{4} a^{4}-27 \ln \left(a x +1\right) x^{4} a^{4}+46 a^{4} x^{4}+6 a^{3} \ln \left(a x -1\right) x^{3}-54 a^{3} \ln \left(a x +1\right) x^{3}+14 a^{3} x^{3}-48 a^{2} x^{2}-6 a \ln \left(a x -1\right) x +5}{12 c^{3} \left(a x +1\right)^{2} \left(a^{2} x^{2}-1\right) a}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
a^6/c^3*(-9/4*ln(a*x+1)/a^7-1/12/a^7/(a*x+1)^3+5/8/a^7/(a*x+1)^2-39/16/a^7
/(a*x+1)+x/a^6-1/16/a^7/(a*x-1)+1/4/a^7*ln(a*x-1))
```

3.820.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{12\,a^5x^5+24\,a^4x^4-30\,a^3x^3-48\,a^2x^2+14\,ax-27\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log{(ax+1)}+3\,(a^4x^4+2}{12\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/12*(12*a^5*x^5 + 24*a^4*x^4 - 30*a^3*x^3 - 48*a^2*x^2 + 14*a*x - 27*(a^4
*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*
x - 1)*log(a*x - 1) + 22)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c
^3)
```

3.820.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-15a^3x^3-12a^2x^2+13ax+11}{6a^{11}c^3x^4+12a^{10}c^3x^3-12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{4}-\frac{9\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*((-15*a**3*x**3 - 12*a**2*x**2 + 13*a*x + 11)/(6*a**11*c**3*x**4 + 12
*a**10*c**3*x**3 - 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (log(x
- 1/a)/4 - 9*log(x + 1/a)/4)/(a**7*c**3))
```

3.820.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{9\,\log\left(ax+1\right)}{4\,ac^3}+\frac{\log\left(ax-1\right)}{4\,ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3
- 2*a^2*c^3*x - a*c^3) + x/c^3 - 9/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x -
1)/(a*c^3)
```

3.820.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}-\frac{9\log\left(|ax+1|\right)}{4\,ac^3}+\frac{\log\left(|ax-1|\right)}{4\,ac^3}-\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(ax+1)^3(ax-1)ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
x/c^3 - 9/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) - 1/
6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

3.820.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}-\frac{\frac{13\,x}{6}-2\,a\,x^2+\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}+\frac{\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{9\,\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^3*(a*x + 1)),x)
```

output
```
x/c^3 - ((13*x)/6 - 2*a*x^2 + 11/(6*a) - (5*a^2*x^3)/2)/(c^3 - 2*a^3*c^3*x
^3 - a^4*c^3*x^4 + 2*a*c^3*x) + log(a*x - 1)/(4*a*c^3) - (9*log(a*x + 1))/
(4*a*c^3)
```

3.821 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.821.1 Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{x}{c^4}-\frac{1}{64ac^4(1-ax)^2}+\frac{11}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}-\frac{13}{48ac^4(1+ax)^3}+\frac{35}{32ac^4(1+ax)^2}-\frac{99}{32ac^4(1+ax)}+\frac{47\log(1-ax)}{128ac^4}-\frac{303\log(1+ax)}{128ac^4}$$

output
```
x/c^4-1/64/a/c^4/(-a*x+1)^2+11/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4-13/4
8/a/c^4/(a*x+1)^3+35/32/a/c^4/(a*x+1)^2-99/32/a/c^4/(a*x+1)+47/128*ln(-a*x
+1)/a/c^4-303/128*ln(a*x+1)/a/c^4
```

3.821.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx =\frac{2(-400-275ax+1258a^2x^2+866a^3x^3-1254a^4x^4-819a^5x^5+384a^6x^6+192a^7x^7)+141(-1+ax)^2(1}{384a(-1+ax)^2(c+acx)^4}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

output
```
(2*(-400 - 275*a*x + 1258*a^2*x^2 + 866*a^3*x^3 - 1254*a^4*x^4 - 819*a^5*x
^5 + 384*a^6*x^6 + 192*a^7*x^7) + 141*(-1 + a*x)^2*(1 + a*x)^4*Log[1 - a*x
] - 909*(-1 + a*x)^2*(1 + a*x)^4*Log[1 + a*x])/(384*a*(-1 + a*x)^2*(c + a*
c*x)^4)
```

3.821.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^8e^{-2\text{arctanh}(ax)}}{c^4\left(a^2-\frac{1}{x^2}\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^8\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^8\int \frac{e^{-2\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{a^8\int \frac{x^8}{(1-ax)^3(ax+1)^5}dx}{c^4}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^8\int \left(\frac{303}{128a^8(ax+1)}-\frac{99}{32a^8(ax+1)^2}+\frac{35}{16a^8(ax+1)^3}-\frac{13}{16a^8(ax+1)^4}+\frac{1}{8a^8(ax+1)^5}-\frac{1}{a^8}-\frac{47}{128a^8(ax-1)}-\frac{11}{64a^8(ax-1)^2}-\right.}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8\left(-\frac{11}{64a^9(1-ax)}+\frac{99}{32a^9(ax+1)}+\frac{1}{64a^9(1-ax)^2}-\frac{35}{32a^9(ax+1)^2}+\frac{13}{48a^9(ax+1)^3}-\frac{1}{32a^9(ax+1)^4}-\frac{47\log(1-ax)}{128a^9}+\frac{303\log(ax+1)}{128a^9}\right.}{c^4}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

output
```
-((a^8*(-(x/a^8) + 1/(64*a^9*(1 - a*x)^2) - 11/(64*a^9*(1 - a*x)) - 1/(32*
a^9*(1 + a*x)^4) + 13/(48*a^9*(1 + a*x)^3) - 35/(32*a^9*(1 + a*x)^2) + 99/
(32*a^9*(1 + a*x)) - (47*Log[1 - a*x])/(128*a^9) + (303*Log[1 + a*x])/(128
*a^9)))/c^4)
```

3.821.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.821. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.821.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

method	result
default	$\frac{a^{8}\left(\frac{1}{32a^{9}(ax+1)^{4}}-\frac{13}{48a^{9}(ax+1)^{3}}+\frac{35}{32a^{9}(ax+1)^{2}}-\frac{99}{32a^{9}(ax+1)}-\frac{303\ln(ax+1)}{128a^{9}}+\frac{x}{a^{8}}-\frac{1}{64a^{9}(ax-1)^{2}}-\frac{11}{64a^{9}(ax-1)}+\frac{47\ln(ax-1)}{128a^{9}}\right)}{c^{4}}$
risch	$\frac{x}{c^{4}}+\frac{-\frac{209a^{4}c^{4}x^{5}}{64}-\frac{81a^{3}c^{4}x^{4}}{32}+\frac{529a^{2}c^{4}x^{3}}{96}+\frac{437a\,c^{4}x^{2}}{96}-\frac{467c^{4}x}{192}-\frac{25c^{4}}{12a}}{c^{8}(ax+1)^{2}(a^{2}x^{2}-1)^{2}}+\frac{47\ln(-ax+1)}{128a\,c^{4}}-\frac{303\ln(ax+1)}{128a\,c^{4}}$
norman	$\frac{\frac{a^{7}x^{8}}{c}-\frac{175x}{64c}-\frac{111a\,x^{2}}{64c}+\frac{199a^{2}x^{3}}{24c}+\frac{115a^{3}x^{4}}{24c}-\frac{545a^{4}x^{5}}{64c}-\frac{803a^{5}x^{6}}{192c}+\frac{37a^{6}x^{7}}{12c}}{(ax-1)^{3}c^{3}(ax+1)^{4}}+\frac{47\ln(ax-1)}{128a\,c^{4}}-\frac{303\ln(ax+1)}{128a\,c^{4}}$
parallelrisch	$-1818a\ln(ax+1)x+909a^{2}\ln(ax+1)x^{2}-38a^{5}x^{5}-1468a^{3}x^{3}-1818\ln(ax+1)x^{5}a^{5}-909\ln(ax+1)x^{6}a^{6}+909\ln(ax+1)x^{4}a^{4}+141$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
a^8/c^4*(1/32/a^9/(a*x+1)^4-13/48/a^9/(a*x+1)^3+35/32/a^9/(a*x+1)^2-99/32/
a^9/(a*x+1)-303/128/a^9*ln(a*x+1)+1/a^8*x-1/64/a^9/(a*x-1)^2-11/64/a^9/(a*
x-1)+47/128/a^9*ln(a*x-1))
```

3.821.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.63

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{4}}\,dx$$
$$=\frac{384\,a^{7}x^{7}+768\,a^{6}x^{6}-1638\,a^{5}x^{5}-2508\,a^{4}x^{4}+1732\,a^{3}x^{3}+2516\,a^{2}x^{2}-550\,ax-909\,(a^{6}x^{6}+2\,a^{5}x^{5}-a}{384\,(a^{7}c^{4}x^{6}+2\,a^{6}c^{4}x^{5}-a^{5}}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/384*(384*a^7*x^7 + 768*a^6*x^6 - 1638*a^5*x^5 - 2508*a^4*x^4 + 1732*a^3*
x^3 + 2516*a^2*x^2 - 550*a*x - 909*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*
x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + 141*(a^6*x^6 + 2*a^5*x^5 - a^4*x
^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 800)/(a^7*c^4*x^6 + 2
*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a
*c^4)
```

3.821.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$=a^8\left(\frac{-627a^5x^5-486a^4x^4+1058a^3x^3+874a^2x^2-467ax-400}{192a^{15}c^4x^6+384a^{14}c^4x^5-192a^{13}c^4x^4-768a^{12}c^4x^3-192a^{11}c^4x^2+384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{47\log\left(x-\frac{1}{a}\right)}{128}-\frac{303\log\left(x+\frac{1}{a}\right)}{128}}{a^9c^4}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*((-627*a**5*x**5 - 486*a**4*x**4 + 1058*a**3*x**3 + 874*a**2*x**2 - 4
67*a*x - 400)/(192*a**15*c**4*x**6 + 384*a**14*c**4*x**5 - 192*a**13*c**4*
x**4 - 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 + 384*a**10*c**4*x + 192*
a**9*c**4) + x/(a**8*c**4) + (47*log(x - 1/a)/128 - 303*log(x + 1/a)/128)/
(a**9*c**4))
```

3.821.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=-\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}+\frac{x}{c^4}-\frac{303\,\log\left(ax+1\right)}{128\,ac^4}+\frac{47\,\log\left(ax-1\right)}{128\,ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x +
 400)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4
*x^2 + 2*a^2*c^4*x + a*c^4) + x/c^4 - 303/128*log(a*x + 1)/(a*c^4) + 47/12
8*log(a*x - 1)/(a*c^4)
```

3.821.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4} - \frac{303\log\left(|ax+1|\right)}{128\,ac^4} + \frac{47\log\left(|ax-1|\right)}{128\,ac^4} - \frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\,(ax+1)^4(ax-1)^2ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
x/c^4 - 303/128*log(abs(a*x + 1))/(a*c^4) + 47/128*log(abs(a*x - 1))/(a*c^
4) - 1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a
*x + 400)/((a*x + 1)^4*(a*x - 1)^2*a*c^4)
```

3.821.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4} - \frac{\frac{467\,x}{192}-\frac{437\,a\,x^2}{96}+\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}+\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4} + \frac{47\,\ln\left(a\,x-1\right)}{128\,a\,c^4} - \frac{303\,\ln\left(a\,x+1\right)}{128\,a\,c^4}$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^4*(a*x + 1)),x)
```

output
```
x/c^4 - ((467*x)/192 - (437*a*x^2)/96 + 25/(12*a) - (529*a^2*x^3)/96 + (81
*a^3*x^4)/32 + (209*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*
c^4*x^4 + 2*a^5*c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) + (47*log(a*x - 1))/(12
8*a*c^4) - (303*log(a*x + 1))/(128*a*c^4)
```

3.822 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

3.822.1 Optimal result

Integrand size = 22, antiderivative size = 343

$$
\begin{aligned}
&\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx\\
&=\frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}+\frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}-\frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}\\
&\quad+\frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}+\frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a}\\
&\quad+\frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a}+\frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a}\\
&\quad+c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x+\frac{15c^4\csc^{-1}(ax)}{16a}-\frac{3c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}
$$

output
```
5/8*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)/a+11/10*c^4*(1-1/a/x)^(5/2)*(1+1/a
/x)^(5/2)/a+17/14*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)/a+8/7*c^4*(1-1/a/x)^
(9/2)*(1+1/a/x)^(5/2)/a+c^4*(1-1/a/x)^(11/2)*(1+1/a/x)^(5/2)*x+15/16*c^4*a
rccsc(a*x)/a-3*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+27/16*c^4*(1
+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-3/8*c^4*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a+
33/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.822.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.37

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=\frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}(80-280ax+96a^2x^2+770a^3x^3-992a^4x^4-525a^5x^5+2496a^6x^6+560a^7x^7)+525a^6x^6\right.}{560a^7x^6}$$

input
```
Integrate[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*(Sqrt[1 - 1/(a^2*x^2)]*(80 - 280*a*x + 96*a^2*x^2 + 770*a^3*x^3 - 992
*a^4*x^4 - 525*a^5*x^5 + 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[
1/(a*x)] - 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)
```

3.822.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.95, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.136, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6748}$$

$$-c^4\int \left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^4\left(\int -\frac{\left(3a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{11/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\int \frac{\left(3a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a^2}d\frac{1}{x}\right)$$

$$\Big\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\int\left(3a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{1}{7}a\int\frac{3\left(7a+\frac{17}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{8}{7}a\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/2}}{a^2}\right)$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\int\left(7a+\frac{17}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{8}{7}a\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/}}{a^2}$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{1}{6}a\int\frac{7\left(6a+\frac{11}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{17}{6}a\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\right)}{a^2}$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\int\left(6a+\frac{11}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{17}{6}a\left(\frac{1}{ax}+1\right)^{5/2}\Big(1-}{a^2}$$

$$\Big\downarrow 171$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{1}{5}a\int\frac{5\left(6a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{11}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/}}{a^2}$$

$$\Big\downarrow 27$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\int\left(6a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{11}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\Big(\frac{1}{ax}+}{a^2}$$

$$\Big\downarrow 171$$

3.822. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{1}{4}a\int\frac{3\left(8a-\frac{3}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{11}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{}$$

$$\Big\downarrow \; 27$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Big(\frac{7}{6}\Big(\frac{3}{4}\int\left(8a-\frac{3}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{11}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\Big(\frac{1}{ax}+1}{}$$

$$\Big\downarrow \; 171$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\left(\frac{1}{3}a\int\frac{3\left(8a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{11}{5}a\Big(}{}$$

$$\Big\downarrow \; 27$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\left(\int\frac{\left(8a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{11}{5}a\Big(1-}{}$$

$$\Big\downarrow \; 171$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(-\frac{1}{2}a\int-\frac{\left(16a-\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{9}{2}a\sqrt{\;}}{}$$

$$\Big\downarrow \; 25$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}a\int\frac{\left(16a-\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{9}{2}a\sqrt{1-}}{}$$

$$\Big\downarrow \; 27$$

3.822. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\int\frac{\left(16a-\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{9}{2}a\sqrt{1-$$

$$\Big\downarrow \color{blue}{171}$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(11a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int-\frac{\left(16a+\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\Bigg)-a\sqrt{1$$

$$\Big\downarrow \color{blue}{25}$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(a\int\frac{\left(16a+\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+11a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)-a\sqrt{1-$$

$$\Big\downarrow \color{blue}{27}$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(\int\frac{\left(16a+\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+11a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)-a\sqrt{1-$$

$$\Big\downarrow \color{blue}{175}$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+11a\sqrt{1-$$

$$\Big\downarrow \color{blue}{39}$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(5\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+11a\sqrt{1-\frac{1}{ax}}$$

3.822. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx$

$$\Big\downarrow \text{103}$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(5\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\Big(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\Big)+11}{}$$

$$\Big\downarrow \text{221}$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Bigg(\frac{7}{6}\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(5\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a\text{arctanh}\Big(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Big)+11a}{}$$

$$\Big\downarrow \text{223}$$

$$-c^4\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\Big(\frac{7}{6}\Big(\frac{3}{4}\Big(\frac{1}{2}\Big(5a\arcsin\left(\frac{1}{ax}\right)-16a\text{arctanh}\Big(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Big)+11a\sqrt{}}{}$$

input
```
Int[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

output
```
-(c^4*(-((1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^(5/2)*x) - ((8*a*(1 - 1/(a*x))
^(9/2)*(1 + 1/(a*x))^(5/2))/7 + (3*((17*a*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x)
)^(5/2))/6 + (7*((5*a*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/4 + (11*a*(
1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/5 + (3*((9*a*Sqrt[1 - 1/(a*x)]*(1
+ 1/(a*x))^(3/2))/2 - a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2) + (11*a*Sqrt
[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] + 5*a*ArcSin[1/(a*x)] - 16*a*ArcTanh[Sqrt[
1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/2))/4))/6))/7)/a^2))
```

3.822.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.822. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

rule 39

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.822. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.822.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(ax+1)(560a^7x^7+2496a^6x^6-525a^5x^5-992a^4x^4+770a^3x^3+96a^2x^2-280ax+80)c^4\sqrt{\frac{ax-1}{ax+1}}}{560x^7a^8} + \frac{\left(-\frac{3a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}} + \frac{15a^7}{}\right.}{}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^4\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-525a^7x^7\sqrt{a^2}\sqrt{a^2x^2-1}-525a^7x^7\sqrt{a^2}\arctan\left(\right.\right.}{}$

input
```
int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/560*(a*x+1)*(560*a^7*x^7+2496*a^6*x^6-525*a^5*x^5-992*a^4*x^4+770*a^3*x^
3+96*a^2*x^2-280*a*x+80)/x^7*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)+(-3*a^8*ln(a^
2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+15/16*a^7*arctan(1/(a^2*x^2
-1)^(1/2)))*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2
)
```

3.822.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx =$$
$$-\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(560\,a}{}$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 + 3056*a^7*c^4*x^7 + 1971*a^6*c^4*x^
6 - 1517*a^5*c^4*x^5 - 222*a^4*c^4*x^4 + 866*a^3*c^4*x^3 - 184*a^2*c^4*x^2
 - 200*a*c^4*x + 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

3.822.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^9+x^8}\right)dx+\int\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^8+x^7}\,dx+\int\frac{4a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^7+x^6}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^6+x^5}\right)dx+\int\left(-\right.\right.}{}$$

input
```
integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**9 + x**8), x) + In
tegral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**8 + x**7), x) + Integral(
4*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**7 + x**6), x) + Integral(-4
*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**6 + x**5), x) + Integral(-6*
a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Integral(6*a*
*5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(4*a**6
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-4*a**7*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**8*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**9*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x))/a**8
```

3.822.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.10

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx =$$
$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{1155\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2 + (1155*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 7665*c^4*((a*x - 1)/(a*x +
1))^(13/2) + 20811*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 12799*c^4*((a*x - 1)
/(a*x + 1))^(9/2) - 39071*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 33621*c^4*((a*
x - 1)/(a*x + 1))^(5/2) - 13615*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 2205*c^4
*sqrt((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^
2/(a*x + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x +
 1)^5 - 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (
a*x - 1)^8*a^2/(a*x + 1)^8 + a^2))*a
```

3.822.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.53

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{15\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{8\,a}$$

$$+\frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^4\operatorname{sgn}(ax+1)}{a}$$

$$+\,525\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a|\operatorname{sgn}(ax+1)+4480\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4\operatorname{sgn}(ax+1)-980\left(x|a|-\right.$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
-15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^4*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 -
1)*c^4*sgn(a*x + 1)/a + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*a
bs(a)*sgn(a*x + 1) + 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x
+ 1) - 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) + 201
60*(x*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 945*(x*abs(a) -
sqrt(a^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) + 38080*(x*abs(a) - sqrt(a^2*
x^2 - 1))^8*a*c^4*sgn(a*x + 1) + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*
c^4*sgn(a*x + 1) - 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x
 + 1) + 32256*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 980*(x
*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) + 12992*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 525*(x*abs(a) - sqrt(a^2*x^2 -
1))*c^4*abs(a)*sgn(a*x + 1) + 2496*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(
a^2*x^2 - 1))^2 + 1)^7*a*abs(a))
```

3.822.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{\frac{63\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{389\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8}+\frac{4803\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{39071\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{280}+\frac{12799\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{280}-\frac{2973\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}-21}{a+\frac{6\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{14\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{14\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{14\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{14\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{6\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}-a\,(}$$
$$-\frac{15\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((63*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (389*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/8 + (4803*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (39071*c^4*((a*x - 1
)/(a*x + 1))^(7/2))/280 + (12799*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 - (2
973*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 - (219*c^4*((a*x - 1)/(a*x + 1))^
(13/2))/8 - (33*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/
(a*x + 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^
3 - (14*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a
*(a*x - 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((
a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/
2)))/a
```

3.823 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.823.1 Optimal result

Integrand size = 22, antiderivative size = 269

$$\begin{aligned}
&\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx\\
&=\frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}+\frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}+\frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a}\\
&\quad+\frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a}+\frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a}\\
&\quad+c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{3c^3\csc^{-1}(ax)}{8a}-\frac{3c^3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}
\end{aligned}$$

output
```
5/4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)/a+27/20*c^3*(1-1/a/x)^(5/2)*(1+1/a
/x)^(3/2)/a+6/5*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)/a+c^3*(1-1/a/x)^(9/2)*
(1+1/a/x)^(3/2)*x+3/8*c^3*arccsc(a*x)/a-3*c^3*arctanh((1-1/a/x)^(1/2)*(1+1
/a/x)^(1/2))/a+3/8*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+21/8*c^3*(1-1/a/x
)^(1/2)*(1+1/a/x)^(1/2)/a
```

3.823.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.41

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{c^3\Bigg(\sqrt{1-\frac{1}{a^2x^2}}(-8+30ax-24a^2x^2-55a^3x^3+152a^4x^4+40a^5x^5)+15a^4x^4\arcsin\left(\frac{1}{ax}\right)-120a^4x^4\log\Bigg(}{40a^5x^4}$$

input
```
Integrate[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^3*(Sqrt[1 - 1/(a^2*x^2)]*(-8 + 30*a*x - 24*a^2*x^2 - 55*a^3*x^3 + 152*a
^4*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

3.823.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.864, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{-3\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{ 6748}$$
$$-c^3\int \left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{ 108}$$
$$-c^3\left(\int -\frac{3\left(a+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$
$$\Big\downarrow \text{ 27}$$
$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\int\left(a+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}a\int \frac{\left(5a+\frac{9}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{a}d\frac{1}{x}+\frac{2}{5}a\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Big(\frac{1}{5}\int\left(5a+\frac{9}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{2}{5}a\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}}{a^2}$$

$$\Big\downarrow \text{171}$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{1}{4}a\int\frac{5\left(4a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{a}d\frac{1}{x}+\frac{9}{4}a\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\Bigg)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Big(\frac{1}{5}\Big(\frac{5}{4}\int\left(4a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{9}{4}a\left(\frac{1}{ax}+1\right)^{3/2}\Big(1-\frac{1}{ax}\Big)}{a^2}$$

$$\Big\downarrow \text{171}$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{5}{4}\Bigg(\frac{1}{3}a\int\frac{3\left(4a+\frac{1}{x}\right)\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{a}d\frac{1}{x}+\frac{5}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\Bigg)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Big(\frac{1}{5}\Big(\frac{5}{4}\Big(\int\left(4a+\frac{1}{x}\right)\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{5}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{a^2}$$

$$\Big\downarrow \text{171}$$

3.823. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{5}{4}\Bigg(\frac{1}{2}a\int\frac{\left(8a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{5}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}a\sqrt{1}}{}$$

$$\Big\downarrow 27$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{5}{4}\Bigg(\frac{1}{2}\int\frac{\left(8a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{5}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}a\sqrt{1-}}{}$$

$$\Big\downarrow 171$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{5}{4}\Bigg(\frac{1}{2}\Bigg(7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int-\frac{\left(8a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\Bigg)+\frac{5}{3}a(1-}{}$$

$$\Big\downarrow 25$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{5}{4}\Bigg(\frac{1}{2}\Bigg(a\int\frac{\left(8a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{5}{3}a(1-}{}$$

$$\Big\downarrow 27$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{5}{4}\Bigg(\frac{1}{2}\Bigg(\int\frac{\left(8a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\Bigg)+\frac{5}{3}a\left(1-\frac{1}{ax}\right)}{}$$

$$\Big\downarrow 175$$

$$-c^3\Bigg(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\Bigg(\frac{1}{5}\Bigg(\frac{5}{4}\Bigg(\frac{1}{2}\Bigg(8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}}{}$$

3.823. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$

$$\Big\downarrow \; 39$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 103$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+7a\sqrt{1\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 221$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+7a\sqrt{1-\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 223$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(a\arcsin\left(\frac{1}{ax}\right)-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+7a\sqrt{1-\right.\right.\right.\right.}{}\right.$$

input
```
Int[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

output
```
-(c^3*(-((1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2)*x) - (3*((2*a*(1 - 1/(a*x
))^(7/2)*(1 + 1/(a*x))^(3/2))/5 + ((9*a*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(3/2))/4 + (5*((a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (5*a*(1 - 1/(
a*x))^(3/2)*(1 + 1/(a*x))^(3/2))/3 + (7*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*
x)] + a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
)/2))/4)/5))/a^2))
```

3.823. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

3.823.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.823. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.823.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.64

method	result
risch	$\frac{(ax+1)\left(152a^4x^4-55a^3x^3-24a^2x^2+30ax-8\right)c^3\sqrt{\frac{ax-1}{ax+1}}}{40x^5a^6}+\frac{\left(-\frac{3a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}+a^5\sqrt{(ax-1)(ax+}\right.}{a^6(ax-1)}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-15a^5x^5\sqrt{a^2}\,\sqrt{a^2x^2-1}-15a^5x^5\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-}}\right.\right.}{40(ax-1)\sqrt{(ax}}$

input
```
int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*(a*x+1)*(152*a^4*x^4-55*a^3*x^3-24*a^2*x^2+30*a*x-8)/x^5*c^3/a^6*((a*
x-1)/(a*x+1))^(1/2)+(-3*a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^
(1/2)+3/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^5*((a*x-1)*(a*x+1))^(1/2))*c^3
/a^6*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

3.823.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(40\,a^6c^3x^6}{40\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5*
log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (40*a^6*c^3*x^6 + 192*a^5*c^3*x^5 + 97*a^4*c^3*x^4 - 79*a^
3*c^3*x^3 + 6*a^2*c^3*x^2 + 22*a*c^3*x - 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))
/(a^6*x^5)
```

3.823.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$

$$=\frac{c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^7+x^6}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^6+x^5}\right)dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{3a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{3a^4}{}\right.}{a^6}$$

input
```
integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**7 + x**6), x) + Int
egral(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**6 + x**5), x) + Integral(
-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Integral(3
*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(3*a
**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-3*a*
*5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**6*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**7*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**6
```

3.823.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.12

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx =$$
$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+46}{4(}\right.$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
+ (105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 465*c^3*((a*x - 1)/(a*x + 1))^(9
/2) - 298*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 842*c^3*((a*x - 1)/(a*x + 1))^
(5/2) - 575*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 135*c^3*sqrt((a*x - 1)/(a*x
+ 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x
 - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/
(a*x + 1)^6 + a^2))*a
```

3.823.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.47

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{4\,a}$$
$$+\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^3\mathrm{sgn}(ax+1)}{a}$$
$$+\frac{55\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|\mathrm{sgn}(ax+1)+200\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3\mathrm{sgn}(ax+1)-10\left(x|a|-\sqrt{a^2x^2}\right.}{}$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output

```
-3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^3*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1
)*c^3*sgn(a*x + 1)/a + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a
)*sgn(a*x + 1) + 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) -
 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) + 720*(x*abs(
a) - sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) + 800*(x*abs(a) - sqrt(a^2*x^
2 - 1))^4*a*c^3*sgn(a*x + 1) + 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs
(a)*sgn(a*x + 1) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1)
 - 55*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 152*a*c^3*s
gn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

3.823.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{\frac{27\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{115\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{421\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10}+\frac{149\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{10}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}-\frac{21\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}$$
$$-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{6\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input

```
int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output

```
((27*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (115*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/4 + (421*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 + (149*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 - (21*c^3*((a
*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x -
 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*
x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1)
)^(1/2)))/(4*a) - (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.824 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.824.1 Optimal result

Integrand size = 22, antiderivative size = 195

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=&\ \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a}\\&+\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x\\&-\frac{c^2\csc^{-1}(ax)}{2a}-\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\end{aligned}$$

output
```
-1/2*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+11
/6*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/a+4/3*c^2*(1-1/a/x)^(5/2)*(1+1/a/x)
^(1/2)/a+c^2*(1-1/a/x)^(7/2)*x*(1+1/a/x)^(1/2)+5/2*c^2*(1-1/a/x)^(1/2)*(1+
1/a/x)^(1/2)/a
```

3.824.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\begin{aligned}&\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx\\&=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(2-9ax+16a^2x^2+6a^3x^3\right)-3a^2x^2\arcsin\left(\frac{1}{ax}\right)-18a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}\end{aligned}$$

input
```
Integrate[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 - 9*a*x + 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x
^2*ArcSin[1/(a*x)] - 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^
3*x^2)
```

3.824.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6748}$$

$$-c^2\int \left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 108}$$

$$-c^2\left(\int -\frac{\left(3a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}x}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\int \frac{\left(3a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}x}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\int \frac{\left(3a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{ 171}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}a\int\frac{\left(9a+\frac{11}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{a^2}\right)$$

$$\Big\downarrow 27$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\int\frac{\left(9a+\frac{11}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{a^2}\right)$$

$$\Big\downarrow 171$$

$$-c^2\Bigg(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{1}{2}a\int\frac{3\left(6a+\frac{5}{x}\right)\sqrt{1-\frac{1}{ax}}x}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{2}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}\right)+\frac{4}{3}a\sqrt{\frac{1}{ax}+1}(1-}{a^2}$$

$$\Big\downarrow 27$$

$$-c^2\Bigg(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(6a+\frac{5}{x}\right)\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{2}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}\right)+\frac{4}{3}a\sqrt{\frac{1}{ax}+1}(1-}{a^2}$$

$$\Big\downarrow 171$$

$$-c^2\Bigg(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\Bigg(\frac{3}{2}\left(a\int\frac{\left(6a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{\frac{1}{ax}+1}(1-}{a^2}$$

$$\Big\downarrow 27$$

$$-c^2\Bigg(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\Bigg(\frac{3}{2}\left(\int\frac{\left(6a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)}{a^2}$$

$$\Big\downarrow 175$$

3.824. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(6a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\right.}{a^2}\right.$$

$$\Big\downarrow \text{39}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+6a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\right.}{a^2}\right.$$

$$\Big\downarrow \text{103}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-6\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\right.\right.}{a^2}\right.$$

$$\Big\downarrow \text{221}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-6a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\right.\right.}{a^2}\right.$$

$$\Big\downarrow \text{223}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\left(-\arcsin\left(\frac{1}{ax}\right)\right)-6a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\right.\right.}{a^2}\right.$$

input
```
Int[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

output
```
-(c^2*(-((1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x) - ((4*a*(1 - 1/(a*x))^(5
/2)*Sqrt[1 + 1/(a*x)])/3 + ((11*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/2
 + (3*(5*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] - a*ArcSin[1/(a*x)] - 6*a*A
rcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2)/3)/a^2))
```

3.824. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

3.824.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.824. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.824.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

method	result
risch	$\frac{(ax+1)\left(16a^2x^2-9ax+2\right)c^2\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax}}{a^4(ax-1)}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^2\left(-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+18\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\right.}{6(ax-1)\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)*(16*a^2*x^2-9*a*x+2)/x^3*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)+(-3*a
^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-1/2*a^3*arctan(1/(a
^2*x^2-1)^(1/2))+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4*((a*x-1)/(a*x+1))^(1
/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

3.824.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$=\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^2x^4+22\,a^3\right.}{6\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 7*a^2*c^2*x^2 - 7*a*c^2*x + 2
*c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

3.824.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$=\frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{2a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\right.\right.}{a^4}$$

input
```
integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + In
tegral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(
2*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-2
*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**4*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x**sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

3.824.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.15

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$=\frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17c^2\left(\frac{ax-1}{ax+1}\right.}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(a}{(a}}\right.$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (
21*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^2*((a*x - 1)/(a*x + 1))^(5/2) -
37*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/
(a*x + 1)^4 + a^2))
```

3.824.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.35

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx=\frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}$$
$$+\frac{3c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^2\operatorname{sgn}(ax+1)}{a}$$
$$+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|\operatorname{sgn}(ax+1)+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2\operatorname{sgn}(ax+1)+36\left(x|a|-\sqrt{a^2x^2-\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right.}$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2
*sgn(a*x + 1)/a + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a
*x + 1) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) + 36*(x*a
bs(a) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(a^2*x
^2 - 1))*c^2*abs(a)*sgn(a*x + 1) + 16*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sq
rt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

3.824.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.94

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{17\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}} + \frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^2*((a*x - 1)/(a*x + 1))^(3/2))/
3 + (17*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(
7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6
*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.825 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.825.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x-\frac{3c\csc^{-1}(ax)}{a}-\frac{3c\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

output
```
-3*c*arccsc(a*x)/a-3*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1-1/a
/x)^(3/2)*x*(1+1/a/x)^(1/2)
```

3.825.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)-3\arcsin\left(\frac{1}{ax}\right)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]))/a
```

3.825.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6748, 109, 27, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{-3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \ 6748$$

$$-c\int \frac{\left(1-\frac{1}{ax}\right)^{5/2} x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \ 109$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\int \frac{3\sqrt{1-\frac{1}{ax}}x}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\int \frac{\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)$$

$$\Big\downarrow \ 140$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)}{a}\right)$$

$$\Big\downarrow \ 39$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}\right)}{a}\right)$$

$$\Big\downarrow \ 103$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{a}\right)$$

$$\Big\downarrow \textbf{221}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a}\right)$$

$$\Big\downarrow \textbf{223}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(-\arcsin\left(\frac{1}{ax}\right)-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a}\right)$$

input

```
Int[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

output

```
-(c*(-((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) - (3*(-ArcSin[1/(a*x)] - A
rcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/a))
```

3.825.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.825.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\,a^{2}}+\frac{\left(-\frac{3a\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{\sqrt{a^{2}}}+\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^{2}c\bigg(-\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,a^{2}x^{2}+(a^{2}x^{2}-1)^{\frac{3}{2}}\sqrt{a^{2}}-3\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,ax+\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{2}x-3ax\sqrt{a^{2}}\,\arctan\bigg(}{(ax-1)\sqrt{(ax-1)(ax+1)}\,a^{2}x\sqrt{a^{2}}}$

input
```
int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+1)/x*c/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(-3*a*ln(a^2*x/(a^2)^(1/2)+(a
^2*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)-3*arctan(1/(a^2*x^2-1
)^(1/2)))*c*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

3.825.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2-c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
(6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c*x*log(sqrt((a*x - 1)/(a
*x + 1)) + 1) + 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 -
c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

3.825.6 Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\frac{a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

input
```
integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integr
al(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

3.825.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*
c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

3.825.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a} + \frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a} - \frac{2\,c\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c*log(abs(-x*
abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn
(a*x + 1)/a - 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs
(a))
```

3.825.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

input
```
int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x
 + 1)^2)
```

3.826 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.826.1 Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

output

```
-3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c+5/3*(1-1/a/x)^(1/2)/a/c/(1
+1/a/x)^(3/2)+x*(1-1/a/x)^(1/2)/c/(1+1/a/x)^(3/2)+14/3*(1-1/a/x)^(1/2)/a/c
/(1+1/a/x)^(1/2)
```

3.826.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(14+19ax+3a^2x^2)}{(1+ax)^2} - \frac{9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

input

```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

output

```
((Sqrt[1 - 1/(a^2*x^2)]*x*(14 + 19*a*x + 3*a^2*x^2))/(1 + a*x)^2 - (9*Log[
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

3.826.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6748, 110, 25, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \; 6748$$

$$-\frac{\int \frac{\sqrt{1-\frac{1}{ax}}x^2}{\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 110$$

$$-\frac{\int -\frac{\left(3a-\frac{2}{x}\right)x}{a^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\int \frac{\left(3a-\frac{2}{x}\right)x}{a^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{2}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{3}a\int \frac{\left(9a-\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{3}\int \frac{\left(9a-\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{169}$$

$$-\frac{-\frac{\frac{1}{3}\left(a\int \frac{9x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\frac{1}{3}\left(9a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{103}$$

$$-\frac{-\frac{\frac{1}{3}\left(\frac{14a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-9\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{\frac{1}{3}\left(\frac{14a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-9a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

output
```
-((-((Sqrt[1 - 1/(a*x)]*x)/(1 + 1/(a*x))^(3/2)) - ((5*a*Sqrt[1 - 1/(a*x)])
/(3*(1 + 1/(a*x))^(3/2)) + ((14*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 9
*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3)/a^2)/c)
```

3.826.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.826. $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.826.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.20

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}-\frac{2\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{3a^{5}\left(x+\frac{1}{a}\right)^{2}}+\frac{13\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{3a^{4}\left(x+\frac{1}{a}\right)}\right)a^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{\Big(-9\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{3}x^{3}+9\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right)a^{4}x^{3}+6\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}ax-27\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-2/3/a^5/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2
)+13/3/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^2/c*((a*x-1)/(a*x+
1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.826.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.67

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^{2}x^{2}}}\,dx = -\frac{9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(3\,a^{2}x^{2}+19\,ax+14\right)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^{2}cx+ac)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
-1/3*(9*(a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x + 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) - (3*a^2*x^2 + 19*a*x + 14)*sqrt((a*x - 1)/(a*
x + 1)))/(a^2*c*x + a*c)
```

3.826.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\left(\int \left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\right)dx + \int \frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx\right)}{c}$$

input

```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

output

```
a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x
**2 - a*x - 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
*3*x**3 + a**2*x**2 - a*x - 1), x))/c
```

3.826.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{1}{3}\,a\left(\frac{6\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c} - \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+12\sqrt{\frac{ax-1}{ax+1}}}{a^2c} + \frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c} - \frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input

```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output

```
-1/3*a*(6*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) -
(((a*x - 1)/(a*x + 1))^(3/2) + 12*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c) + 9*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c))
```

3.826.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a)) + sqrt(a
^2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

3.826.9 Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3\,a\,c}+\frac{\operatorname{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (4*((a
*x - 1)/(a*x + 1))^(1/2))/(a*c) + ((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c) + (a
tan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c)
```

3.827 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.827.1 Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

output
```
-3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2+6/5*(1-1/a/x)^(1/2)/a/c^
2/(1+1/a/x)^(5/2)+9/5*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(3/2)+x*(1-1/a/x)^(1
/2)/c^2/(1+1/a/x)^(5/2)+24/5*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

3.827.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.43

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(24+57ax+39a^2x^2+5a^3x^3\right)}{5(1+ax)^3}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 57*a*x + 39*a^2*x^2 + 5*a^3*x^3))/(5*(1
+ a*x)^3) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

3.827.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6748, 114, 27, 35, 110, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{3\left(a-\frac{1}{x}\right)x}{a^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{3\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{35}$$

$$-\frac{-\frac{3\int \frac{\sqrt{1-\frac{1}{ax}}x}{\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{110}$$

$$-\frac{-\frac{3\left(\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{2}{5}\int -\frac{\left(5a-\frac{4}{x}\right)x}{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{3\left(\frac{\int \frac{\left(5a-\frac{4}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow 169$$

$$-\frac{-\frac{3\left(\frac{\frac{1}{3}a\int \frac{3\left(5a-\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{3\left(\frac{\int \frac{\left(5a-\frac{3}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow 169$$

$$-\frac{-\frac{3\left(\frac{a\int \frac{5x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{3\left(\frac{5a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow 103$$

$$-\frac{-\frac{3\left(\frac{-5\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

3.827. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

$$\Big\downarrow 221$$

$$-\frac{-\frac{3\left(\frac{-5a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

output
```
-((-((Sqrt[1 - 1/(a*x)]*x)/(1 + 1/(a*x))^(5/2)) - (3*((2*Sqrt[1 - 1/(a*x)]
)/(5*(1 + 1/(a*x))^(5/2)) + ((3*a*Sqrt[1 - 1/(a*x)])/(1 + 1/(a*x))^(3/2) +
 (8*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 5*a*ArcTanh[Sqrt[1 - 1/(a*x)]
*Sqrt[1 + 1/(a*x)]])/(5*a)))/a)/c^2)
```

3.827.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

3.827. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.827.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.14

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{5a^8\left(x+\frac{1}{a}\right)^3}-\frac{6\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{5a^7\left(x+\frac{1}{a}\right)^2}+\frac{24\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{5a^6\left(x+\frac{1}{a}\right)}\right)a^4}{c^2(ax-1)}$
default	$-(120\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^5x^4-125\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4+480\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^4x^3+85((ax-1)(a$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

3.827. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-3/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+1/5/a^8/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1
/2)-6/5/a^7/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+24/5/a^6/(x+1/a)*(
a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^4/c^2*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)
*(a*x+1))^(1/2)/(a*x-1)
```

3.827.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx =$$
$$-\frac{15\left(a^2x^2+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^2x^2+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(5\,a^3x^3+39\,a^2x^2+\right.}{5\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
-1/5*(15*(a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*(a^
2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (5*a^3*x^3 + 39*a^
2*x^2 + 57*a*x + 24)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 + 2*a^2*c^2*x
 + a*c^2)
```

3.827.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx\right)}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x
**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x) + Integral(a*x**5*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**
2 + a*x + 1), x))/c**2
```

3.827.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.89

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{1}{20}\,a\left(\frac{40\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2} - \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+85\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2} + \frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2} - \frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}\right.}{a^2c^2}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/20*a*(40*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c
^2) - (((a*x - 1)/(a*x + 1))^(5/2) + 10*((a*x - 1)/(a*x + 1))^(3/2) + 85*s
qrt((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^2) - 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

3.827.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.33

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c^2|a|} + \frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^2*abs(a)) + sqrt
(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^2)
```

3.827.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}} + \frac{17\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2\,a\,c^2} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{20\,a\,c^2} + \frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^2,x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + (1
7*((a*x - 1)/(a*x + 1))^(1/2))/(4*a*c^2) + ((a*x - 1)/(a*x + 1))^(3/2)/(2*
a*c^2) + ((a*x - 1)/(a*x + 1))^(5/2)/(20*a*c^2) + (atan(((a*x - 1)/(a*x +
1))^(1/2)*1i)*6i)/(a*c^2)
```

3.827. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.828 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.828.1 Optimal result

Integrand size = 22, antiderivative size = 253

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}}\\ &\quad+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}}\\ &\quad+\frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^3}\end{aligned}$$

output
```
-3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-2/a/c^3/(1+1/a/x)^(7/2)/
(1-1/a/x)^(1/2)+x/c^3/(1+1/a/x)^(7/2)/(1-1/a/x)^(1/2)+11/7*(1-1/a/x)^(1/2)
/a/c^3/(1+1/a/x)^(7/2)+54/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(5/2)+71/35*(
1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+176/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x
)^(1/2)
```

3.828.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-176-423ax-125a^2x^2+368a^3x^3+286a^4x^4+35a^5x^5)}{35(-1+ax)(1+ax)^4}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-176 - 423*a*x - 125*a^2*x^2 + 368*a^3*x^3 +
286*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)*(1 + a*x)^4) - 3*Log[(1 + Sqrt[1
 - 1/(a^2*x^2)])*x])/(a*c^3)
```

3.828.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {6748, 114, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{\left(3a-\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{-\frac{a\left(-\int -\frac{\left(3a-\frac{8}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}\right)-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{a\int \frac{\left(3a-\frac{8}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{8}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{-\frac{\frac{1}{7}a\int \frac{3\left(7a-\frac{11}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{-\frac{\frac{3}{7}\int \frac{\left(7a-\frac{11}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}a\int \frac{\left(35a-\frac{36}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow 27$$

3.828. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\int \frac{\left(35a-\frac{36}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \color{blue}{169}$$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{\left(105a-\frac{71}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(105a-\frac{71}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \color{blue}{169}$$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\int \frac{105x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/}}}{c^3}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right.}}{c^3}$$

$$\Big\downarrow \color{blue}{103}$$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-105\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{}{\sqrt{1-}}}{c^3}$$

$$\Big\downarrow \color{blue}{221}$$

3.828. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-105a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-}{c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

output
```
-((-(x/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))) - ((-2*a)/(Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(7/2)) + (11*a*Sqrt[1 - 1/(a*x)])/(7*(1 + 1/(a*x))^(7/2))
 + (3*((18*a*Sqrt[1 - 1/(a*x)])/(5*(1 + 1/(a*x))^(5/2)) + ((71*a*Sqrt[1 -
1/(a*x)])/(3*(1 + 1/(a*x))^(3/2)) + ((176*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/
(a*x)] - 105*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3)/5))/7)/a^2
)/c^3)
```

3.828.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.828. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.828.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{14a^{11}\left(x+\frac{1}{a}\right)^4}+\frac{71\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{140a^{10}\left(x+\frac{1}{a}\right)^3}-\frac{477\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{280a^9\left(x+\frac{1}{a}\right)^2}+\frac{293}{}\right.}{c^3(ax-1)}$
default	$-\frac{\left(-3675\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7+3360\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^8x^7+2555((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5-11025\sqrt{(ax-1)}\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(-3/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-1/14/a^11/(x+1/a)^4*(a^2*(x+1/a)^2-2*a*(x+1/a))^
(1/2)+71/140/a^10/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-477/280/a^9/
(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+2931/560/a^8/(x+1/a)*(a^2*(x+1
/a)^2-2*a*(x+1/a))^(1/2)-1/16/a^8/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2
))*a^6/c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.828. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.828.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx =$$

$$-\frac{105\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-}{35\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
-1/35*(105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) - 105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (35*a^5*x^5 + 286*a^4*x^4 + 368*a^3*x^3 - 125*a^2*x^2 - 423*a*x -
 176)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*
x - a*c^3)
```

3.828.6 Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

$$=\frac{a^6\left(\int\left(-\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\right)dx+\int\frac{ax^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\,dx\right)}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*(Integral(-x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x
**6 - 3*a**5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x)
 + Integral(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**
6 - 3*a**5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x))/
c**3
```

3.828.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx =$$

$$-\frac{1}{560}a\left(\frac{35\left(\frac{33\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+56\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+350\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2520\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1680\log}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/560*a*(35*(33*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(
3/2) - a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - (5*((a*x - 1)/(a*x + 1))^(7/2)
 + 56*((a*x - 1)/(a*x + 1))^(5/2) + 350*((a*x - 1)/(a*x + 1))^(3/2) + 2520
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 1680*log(sqrt((a*x - 1)/(a*x + 1))
 + 1)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

3.828.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3, x)
```

3.828.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{33\,(a\,x-1)}{a\,x+1}-1}{16\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}+\frac{9\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3}+\frac{5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{112\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^3}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3,x)
```

output
```
((33*(a*x - 1))/(a*x + 1) - 1)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2) - 16*
a*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (9*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*
c^3) + (5*((a*x - 1)/(a*x + 1))^(3/2))/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(
5/2)/(10*a*c^3) + ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3) + (atan(((a*x -
1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c^3)
```

3.828. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

3.829 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.829.1 Optimal result

Integrand size = 22, antiderivative size = 327

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = &-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}\\ &+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(1+\frac{1}{ax}\right)^{5/2}}\\ &+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{1+\frac{1}{ax}}}\\ &+\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^4}\end{aligned}$$

output
```
-4/3/a/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^(9/2)+x/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)
^(9/2)-3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-5/a/c^4/(1+1/a/x)^
(9/2)/(1-1/a/x)^(1/2)+28/9*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(9/2)+139/63*(1
-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(7/2)+202/105*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x
)^(5/2)+719/315*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+1664/315*(1-1/a/x)^(
1/2)/a/c^4/(1+1/a/x)^(1/2)
```

3.829.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(1664+4047ax-339a^2x^2-7399a^3x^3-4029a^4x^4+2967a^5x^5+2669a^6x^6+315a^7x^7\right)}{315(-1+ax)^2(1+ax)^5}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

input

```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

output

```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(1664 + 4047*a*x - 339*a^2*x^2 - 7399*a^3*x^3
- 4029*a^4*x^4 + 2967*a^5*x^5 + 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*
x)^2*(1 + a*x)^5) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

3.829.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.96, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {6748, 114, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$\Big\downarrow \; 6748$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \; 114$$

$$-\frac{-\int \frac{\left(3a-\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{7}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{-\frac{1}{3}a\int -\frac{3\left(3a-\frac{8}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{8}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{a\left(-\int -\frac{\left(3a-\frac{25}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}\right)-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{a\int \frac{\left(3a-\frac{25}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{25}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{9}a\int \frac{\left(27a-\frac{112}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{9}\int \frac{\left(27a-\frac{112}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

3.829. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

$\downarrow$ 169

$$-\frac{-\frac{\frac{1}{9}\left(\frac{1}{7}a\int\frac{3\left(63a-\frac{139}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\int\frac{\left(63a-\frac{139}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$\downarrow$ 169

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}a\int\frac{\left(315a-\frac{404}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{}{(1-\frac{1}{ax}}}{c^4}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\int\frac{\left(315a-\frac{404}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)^{3}}}{c^4}$$

$\downarrow$ 169

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}a\int\frac{\left(945a-\frac{719}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}}{c^4}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{\left(945a-\frac{719}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}\right.}}{a^2}}{c^4}$$

$\downarrow$ 169

3.829. $\displaystyle\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

$$-\frac{-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(a\int\frac{945x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\Bigg)+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\Bigg)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\Bigg)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\Bigg)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-}{a^2}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(945a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\Bigg)+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\Bigg)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\Bigg)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\Bigg)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/}}}{a^2}}{c^4}$$

$$\Big\downarrow \; 103$$

$$-\frac{-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-945\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\Bigg)+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\Bigg)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\Bigg)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\Bigg)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{}{\sqrt{1-\frac{1}{ax}}(}}{a^2}}{c^4}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{\frac{1}{9}\Bigg(\frac{3}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-945a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\Bigg)+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\Bigg)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\Bigg)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\Bigg)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{}{\sqrt{1-\frac{1}{ax}}(}}{a^2}}{c^4}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

output
```
-((-(x/((1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2))) - ((-4*a)/(3*(1 - 1/(a*x
))^(3/2)*(1 + 1/(a*x))^(9/2)) - (5*a)/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/
2)) + (28*a*Sqrt[1 - 1/(a*x)])/(9*(1 + 1/(a*x))^(9/2)) + ((139*a*Sqrt[1 -
1/(a*x)])/(7*(1 + 1/(a*x))^(7/2)) + (3*((202*a*Sqrt[1 - 1/(a*x)])/(5*(1 +
1/(a*x))^(5/2)) + ((719*a*Sqrt[1 - 1/(a*x)])/(3*(1 + 1/(a*x))^(3/2)) + ((1
664*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 945*a*ArcTanh[Sqrt[1 - 1/(a*x
)]*Sqrt[1 + 1/(a*x)]])/3)/5))/7)/9)/a^2)/c^4)
```

3.829. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.829.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.829. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

3.829.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.09

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\Bigg(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{59\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{252a^{13}\left(x+\frac{1}{a}\right)^4}+\frac{1507\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{1680a^{12}\left(x+\frac{1}{a}\right)^3}-\frac{691\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{315a^{11}\left(x+\frac{1}{a}\right)^2}+}{}$
default	$-\frac{\Bigg(-138915\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^9x^9+120960\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right)a^{10}x^9+98595((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^7x^7-416745}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(-3/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-59/252/a^13/(x+1/a)^4*(a^2*(x+1/a)^2-2*a*(x+1/a)
)^(1/2)+1507/1680/a^12/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-691/315
/a^11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+113591/20160/a^10/(x+1/a
)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+1/36/a^14/(x+1/a)^5*(a^2*(x+1/a)^2-2*a
*(x+1/a))^(1/2)-1/96/a^11/(x-1/a)^2*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2)-31/1
92/a^10/(x-1/a)*((x-1/a)^2*a^2+2*(x-1/a)*a)^(1/2))*a^8/c^4*((a*x-1)/(a*x+1
))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

3.829.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.84

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx =$$

$$-\frac{945\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\,(a^6x^6+2\,a^5x^5-a^4x^4-}{315\,(a^7c^4x^6+}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
-1/315*(945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^
4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) -
(315*a^7*x^7 + 2669*a^6*x^6 + 2967*a^5*x^5 - 4029*a^4*x^4 - 7399*a^3*x^3 -
 339*a^2*x^2 + 4047*a*x + 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 +
2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x +
a*c^4)
```

3.829.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

output
```
Timed out
```

3.829.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$=\frac{1}{20160}a\left(\frac{105\left(\frac{29\,(ax-1)}{ax+1}-\frac{414\,(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{35\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+450\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+2961\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+14700\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a^2c^4}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
1/20160*a*(105*(29*(a*x - 1)/(a*x + 1) - 414*(a*x - 1)^2/(a*x + 1)^2 + 1)/
(a^2*c^4*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)
) + (35*((a*x - 1)/(a*x + 1))^(9/2) + 450*((a*x - 1)/(a*x + 1))^(7/2) + 29
61*((a*x - 1)/(a*x + 1))^(5/2) + 14700*((a*x - 1)/(a*x + 1))^(3/2) + 95445
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4) - 60480*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

3.829.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4, x)
```

3.829.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.69

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{303\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{29\,(a\,x-1)}{3\,(a\,x+1)}-\frac{138\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{35\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^4}+\frac{47\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}+\frac{5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{576\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4,x)
```

output
```
(303*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((29*(a*x - 1))/(3*(a*x + 1
)) - (138*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^
(3/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (35*((a*x - 1)/(a*x + 1))^
(3/2))/(48*a*c^4) + (47*((a*x - 1)/(a*x + 1))^(5/2))/(320*a*c^4) + (5*((a*
x - 1)/(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*
c^4) + (atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c^4)
```

3.830 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

3.830.1 Optimal result

Integrand size = 22, antiderivative size = 321

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx &= \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}\\ &-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}\\ &+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output

```
1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)+1/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)-3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/
(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+3/
2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+3*c^3*(c-c/a^2/x^2)^
(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1
/2)+c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.830.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6a^7x^6}+\frac{1}{5a^6x^5}-\frac{3}{4a^5x^4}-\frac{1}{a^4x^3}+\frac{3}{2a^3x^2}+\frac{3}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
((c - c/(a^2*x^2))^(7/2)*(1/(6*a^7*x^6) + 1/(5*a^6*x^5) - 3/(4*a^5*x^4) -
1/(a^4*x^3) + 3/(2*a^3*x^2) + 3/(a^2*x) + x + Log[x]/a))/(1 - 1/(a^2*x^2))
^(7/2)
```

3.830.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.31, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^3(ax+1)^4}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^3(ax+1)^4}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow 99$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \left(-a^7-\frac{a^6}{x}+\frac{3a^5}{x^2}+\frac{3a^4}{x^3}-\frac{3a^3}{x^4}-\frac{3a^2}{x^5}+\frac{a}{x^6}+\frac{1}{x^7}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow 2009$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(a^7(-x)-a^6\log(x)-\frac{3a^5}{x}-\frac{3a^4}{2x^2}+\frac{a^3}{x^3}+\frac{3a^2}{4x^4}-\frac{a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

output

```
-((c^3*Sqrt[c - c/(a^2*x^2)]*(-1/6*1/x^6 - a/(5*x^5) + (3*a^2)/(4*x^4) + a
^3/x^3 - (3*a^4)/(2*x^2) - (3*a^5)/x - a^7*x - a^6*Log[x]))/(a^7*Sqrt[1 -
1/(a^2*x^2)]))
```

3.830.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.830.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(60 a^{7} x^{7}+60 a^{6} \ln \left(x \right) x^{6}+180 a^{5} x^{5}+90 a^{4} x^{4}-60 a^{3} x^{3}-45 a^{2} x^{2}+12 a x +10\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} x}{60 \left(a x +1\right) \left(a^{2} x^{2}-1\right)^{3} \sqrt{\frac{a x -1}{a x +1}}}$	112

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(60*a^7*x^7+60*a^6*ln(x)*x^6+180*a^5*x^5+90*a^4*x^4-60*a^3*x^3-45*a^2
*x^2+12*a*x+10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)/(a^2*x^2-1)^3/((a*
x-1)/(a*x+1))^(1/2)
```

3.830.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \frac{(60\,a^7c^3x^7 + 60\,a^6c^3x^6\log\left(x\right) + 180\,a^5c^3x^5 + 90\,a^4c^3x^4 - 60\,a^3c^3x^3 - 45\,a^2c^3x^2 + 12\,ac^3x +}{60\,a^8x^6}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

output
```
1/60*(60*a^7*c^3*x^7 + 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 + 90*a^4*c^
3*x^4 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 12*a*c^3*x + 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

3.830.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

3.830.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.830.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.830.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.831 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$

3.831.1 Optimal result

Integrand size = 22, antiderivative size = 236

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = -\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3} + \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)-1/3*c^2*(c-c/a^2/
x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-
1/a^2/x^2)^(1/2)+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x
*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(
1-1/a^2/x^2)^(1/2)
```

3.831.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.31

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4a^5x^4}-\frac{1}{3a^4x^3}+\frac{1}{a^3x^2}+\frac{2}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
((c - c/(a^2*x^2))^(5/2)*(-1/4*1/(a^5*x^4) - 1/(3*a^4*x^3) + 1/(a^3*x^2) +
 2/(a^2*x) + x + Log[x]/a))/(1 - 1/(a^2*x^2))^(5/2)
```

3.831.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2} e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2(ax+1)^3}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^5+\frac{a^4}{x}-\frac{2a^3}{x^2}-\frac{2a^2}{x^3}+\frac{a}{x^4}+\frac{1}{x^5}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(a^5x+a^4\log(x)+\frac{2a^3}{x}+\frac{a^2}{x^2}-\frac{a}{3x^3}-\frac{1}{4x^4}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 - a/(3*x^3) + a^2/x^2 + (2*a^3)/x +
 a^5*x + a^4*Log[x]))/(a^5*Sqrt[1 - 1/(a^2*x^2)])
```

3.831.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.831.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\left(12 a^{5} x^{5}+12 \ln \left(x \right) x^{4} a^{4}+24 a^{3} x^{3}+12 a^{2} x^{2}-4 a x -3\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} x}{12 \left(a x +1\right) \left(a^{2} x^{2}-1\right)^{2} \sqrt{\frac{a x -1}{a x +1}}}$	96

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(12*a^5*x^5+12*ln(x)*x^4*a^4+24*a^3*x^3+12*a^2*x^2-4*a*x-3)*(c*(a^2*x
^2-1)/a^2/x^2)^(5/2)*x/(a*x+1)/(a^2*x^2-1)^2/((a*x-1)/(a*x+1))^(1/2)
```

3.831.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{(12\,a^5c^2x^5+12\,a^4c^2x^4\log(x)+24\,a^3c^2x^3+12\,a^2c^2x^2-4\,ac^2x-3\,c^2)\sqrt{a^2c}}{12\,a^6x^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

output
```
1/12*(12*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 + 12*a^2*c^2
*x^2 - 4*a*c^2*x - 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

3.831.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

3.831.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.831.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.831.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.832 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

3.832.1 Optimal result

Integrand size = 22, antiderivative size = 146

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2
)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c*
ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.832.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{3}{2a}+\frac{1}{2a^3x^2}+\frac{1}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(3/(2*a) + 1/(2*a^3*x^2) + 1/(a^2*x) + x + Log[x]
/a))/(1 - 1/(a^2*x^2))^(3/2)
```

3.832.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6751, 6747, 25, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}} \int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}} \int -\frac{(1-ax)(ax+1)^2}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}} \int \frac{(1-ax)(ax+1)^2}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 84}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}} \int \left(-a^3-\frac{a^2}{x}+\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(a^3(-x)-a^2\log(x)-\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
-((c*Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 - a/x - a^3*x - a^2*Log[x]))/(a^3*S
qrt[1 - 1/(a^2*x^2)]))
```

3.832.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.832.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\left(2 a^{3} x^{3}+2 a^{2} \ln \left(x \right) x^{2}+2 a x +1\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} x}{2 \left(a x +1\right) \left(a^{2} x^{2}-1\right) \sqrt{\frac{a x -1}{a x +1}}}$	80

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^3*x^3+2*a^2*ln(x)*x^2+2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a
*x+1)/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(1/2)
```

3.832.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(2\,a^3cx^3+2\,a^2cx^2\log\left(x\right)+2\,acx+c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(2*a^3*c*x^3 + 2*a^2*c*x^2*log(x) + 2*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

3.832.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

3.832.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.832.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.832.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.833 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.833.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)
```

3.833.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x + Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

3.833.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{ax+1}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.833.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.833.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{(ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)
```

3.833.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+\log{(x)})}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + log(x))/a^2
```

3.833.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)),
x)
```

3.833.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.833.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.833.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.834 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

3.834.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)+ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a
/(c-c/a^2/x^2)^(1/2)
```

3.834.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x+\frac{\log(1-ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*(x + Log[1 - a*x]/a))/Sqrt[c - c/(a^2*x^2)]
```

3.834.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x}{1-ax}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x}{1-ax}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{1}{(ax-1)a}-\frac{1}{a}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{\log(1-ax)}{a^2}-\frac{x}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((a*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a) - Log[1 - a*x]/a^2))/Sqrt[c - c/(a^2*x^
2)])
```

3.834. $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$

3.834.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.834.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{(ax-1)(ax+\ln(ax-1))}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\,a^{2}}$	57

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2*(a*x
+ln(a*x-1))
```

3.834.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.33

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{a^2c}(ax+\log{(ax-1)})}{a^2c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + log(a*x - 1))/(a^2*c)
```

3.834.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.834.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.834.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.834.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.835 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

3.835.1 Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} + \frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(-
a*x+1)/(c-c/a^2/x^2)^(1/2)+5/4*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2
/x^2)^(1/2)-1/4*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

3.835.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.41

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x+\frac{1}{2a-2a^2x}+\frac{5\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*(x + (2*a - 2*a^2*x)^(-1) + (5*Log[1 - a*x])/(4*a
) - Log[1 + a*x]/(4*a)))/(c - c/(a^2*x^2))^(3/2)
```

3.835.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3}{(1-ax)^2(ax+1)}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{1}{4a^3(ax+1)}+\frac{1}{a^3}+\frac{5}{4a^3(ax-1)}+\frac{1}{2a^3(ax-1)^2}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}+\frac{x}{a^3}\right)}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(a^3*Sqrt[1 - 1/(a^2*x^2)]*(x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 - a*x])
/(4*a^4) - Log[1 + a*x]/(4*a^4)))/(c*Sqrt[c - c/(a^2*x^2)])
```

3.835. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$

3.835.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.835.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{(ax-1)\left(-4a^{2}x^{2}+a\ln(ax+1)x-5a\ln(ax-1)x+4ax-\ln(ax+1)+5\ln(ax-1)+2\right)(ax+1)}{4\sqrt{\frac{ax-1}{ax+1}}\,a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(-4*a^2*x^2+a*ln(a*x+1)*x-5*a*ln(a*x-
1)*x+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)*(a*x+1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2
)^(3/2)
```

3.835.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(4\,a^2x^2-4\,ax-(ax-1)\log{(ax+1)}+5\,(ax-1)\log{(ax-1)}-2\right)\sqrt{a^2c}}{4\,(a^3c^2x-a^2c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

output
```
1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)*sqrt(a^2*c)/(a^3*c^2*x - a^2*c^2)
```

3.835.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

3.835.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.835.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.835.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.836 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.836.1 Optimal result

Integrand size = 22, antiderivative size = 263

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^
2/(-a*x+1)^2/(c-c/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)/(c-c/a
^2/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+23
/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-7/16*ln(a*x+1
)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

3.836.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(x-\frac{1}{8a(-1+ax)^2}+\frac{1}{a-a^2x}-\frac{1}{8a+8a^2x}+\frac{23\log(1-ax)}{16a}-\frac{7\log(1+ax)}{16a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*(x - 1/(8*a*(-1 + a*x)^2) + (a - a^2*x)^(-1) - (8
*a + 8*a^2*x)^(-1) + (23*Log[1 - a*x])/(16*a) - (7*Log[1 + a*x])/(16*a)))/
(c - c/(a^2*x^2))^(5/2)
```

3.836.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.43, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^5}{(1-ax)^3(ax+1)^2}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^5}{(1-ax)^3(ax+1)^2}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 99$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{7}{16a^5(ax+1)}-\frac{1}{8a^5(ax+1)^2}-\frac{1}{a^5}-\frac{23}{16a^5(ax-1)}-\frac{1}{a^5(ax-1)^2}-\frac{1}{4a^5(ax-1)^3}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(ax+1)}+\frac{1}{8a^6(1-ax)^2}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(ax+1)}{16a^6}-\frac{x}{a^5}\right)}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
-((a^5*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^5) + 1/(8*a^6*(1 - a*x)^2) - 1/(a^6*(1
 - a*x)) + 1/(8*a^6*(1 + a*x)) - (23*Log[1 - a*x])/(16*a^6) + (7*Log[1 + a
*x])/(16*a^6)))/(c^2*Sqrt[c - c/(a^2*x^2)]))
```

3.836.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.836. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.836.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

method	result
default	$-\frac{(ax-1)(ax+1)(-16a^4x^4+7a^3\ln(ax+1)x^3-23a^3\ln(ax-1)x^3+16a^3x^3-7a^2\ln(ax+1)x^2+23a^2\ln(ax-1)x^2+34a^2x^2-7a\ln(ax+1}{16\sqrt{\frac{ax-1}{ax+1}}\,a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-16*a^4*x^4+7*a^3*ln(a*x+1)
*x^3-23*a^3*ln(a*x-1)*x^3+16*a^3*x^3-7*a^2*ln(a*x+1)*x^2+23*a^2*ln(a*x-1)*
x^2+34*a^2*x^2-7*a*ln(a*x+1)*x+23*a*ln(a*x-1)*x-18*a*x+7*ln(a*x+1)-23*ln(a
*x-1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

3.836.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\,(a}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

output
```
1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) +
 12)*sqrt(a^2*c)/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

3.836.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

3.836.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.836.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.836.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.837 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.837.1 Optimal result

Integrand size = 22, antiderivative size = 359

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}\\
&-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}\\
&-\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)+1/24*(1-1/a^2/x^2)^(1/2)/a/c
^3/(-a*x+1)^3/(c-c/a^2/x^2)^(1/2)-11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)
^2/(c-c/a^2/x^2)^(1/2)+3/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c-c/a^2/x^2
)^(1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-5/16*
(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+51/32*ln(-a*x+1)*(1-
1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-19/32*ln(a*x+1)*(1-1/a^2/x^2)^(
1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)
```

3.837.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.33

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96x-\frac{4}{a(-1+ax)^3}-\frac{33}{a(-1+ax)^2}+\frac{3}{a(1+ax)^2}+\frac{144}{a-a^2x}-\frac{30}{a+a^2x}+\frac{153\log(1-ax)}{a}-\right.}{96\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(7/2)*(96*x - 4/(a*(-1 + a*x)^3) - 33/(a*(-1 + a*x)^2)
+ 3/(a*(1 + a*x)^2) + 144/(a - a^2*x) - 30/(a + a^2*x) + (153*Log[1 - a*x]
)/a - (57*Log[1 + a*x])/a))/(96*(c - c/(a^2*x^2))^(7/2))
```

3.837.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^7}{(1-ax)^4(ax+1)^3}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{19}{32a^7(ax+1)}+\frac{5}{16a^7(ax+1)^2}-\frac{1}{16a^7(ax+1)^3}+\frac{1}{a^7}+\frac{51}{32a^7(ax-1)}+\frac{3}{2a^7(ax-1)^2}+\frac{11}{16a^7(ax-1)^3}+\frac{1}{8a^7(ax-1)^4}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{3}{2a^8(1-ax)}-\frac{5}{16a^8(ax+1)}-\frac{11}{32a^8(1-ax)^2}+\frac{1}{32a^8(ax+1)^2}+\frac{1}{24a^8(1-ax)^3}+\frac{51\log(1-ax)}{32a^8}-\frac{19\log(ax+1)}{32a^8}+\frac{x}{a^7}\right)}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
(a^7*Sqrt[1 - 1/(a^2*x^2)]*(x/a^7 + 1/(24*a^8*(1 - a*x)^3) - 11/(32*a^8*(1
 - a*x)^2) + 3/(2*a^8*(1 - a*x)) + 1/(32*a^8*(1 + a*x)^2) - 5/(16*a^8*(1 +
 a*x)) + (51*Log[1 - a*x])/(32*a^8) - (19*Log[1 + a*x])/(32*a^8)))/(c^3*Sq
rt[c - c/(a^2*x^2)])
```

3.837.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.837. $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.837.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{(ax-1)(ax+1)\left(-96a^6x^6+57\ln(ax+1)x^5a^5-153\ln(ax-1)x^5a^5+96a^5x^5-57\ln(ax+1)x^4a^4+153\ln(ax-1)x^4a^4+366a^4x^4-114a^3}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-96*a^6*x^6+57*ln(a*x+1)*x^
5*a^5-153*ln(a*x-1)*x^5*a^5+96*a^5*x^5-57*ln(a*x+1)*x^4*a^4+153*ln(a*x-1)*
x^4*a^4+366*a^4*x^4-114*a^3*ln(a*x+1)*x^3+306*a^3*ln(a*x-1)*x^3-222*a^3*x^
3+114*a^2*ln(a*x+1)*x^2-306*a^2*ln(a*x-1)*x^2-338*a^2*x^2+57*a*ln(a*x+1)*x
-153*a*ln(a*x-1)*x+122*a*x-57*ln(a*x+1)+153*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*
x^2-1)/a^2/x^2)^(7/2)
```

3.837.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{(96\,a^6x^6-96\,a^5x^5-366\,a^4x^4+222\,a^3x^3+338\,a^2x^2-122\,ax-57\,(a^5x^5-a^4x^4-2\,}{96\,(a^7c^4x^5-a^6c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

output
```
1/96*(96*a^6*x^6 - 96*a^5*x^5 - 366*a^4*x^4 + 222*a^3*x^3 + 338*a^2*x^2 -
122*a*x - 57*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x
 + 1) + 153*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x
- 1) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 - a^6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*
c^4*x^2 + a^3*c^4*x - a^2*c^4)
```

3.837.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

3.837.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.837.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.837.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.838 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$

3.838.1 Optimal result

Integrand size = 24, antiderivative size = 372

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx &= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}\\ &+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}\\ &+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}\\ &+\frac{2a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\arcsin(ax)}{(1-ax)^{7/2}(1+ax)^{7/2}}+\frac{25a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{16(1-ax)^{7/2}(1+ax)^{7/2}}\end{aligned}$$

output
```
11/30*a^3*(c-c/a^2/x^2)^(7/2)*x^4/(-a*x+1)^3-57/16*a^6*(c-c/a^2/x^2)^(7/2)
*x^7/(-a*x+1)^3/(a*x+1)^3+41/24*a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a*x+1)^3/(a*
x+1)^2+57/80*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)-13/40*a^2*(c-c
/a^2/x^2)^(7/2)*x^3*(a*x+1)/(-a*x+1)^3+1/15*a*(c-c/a^2/x^2)^(7/2)*x^2*(a*x
+1)/(-a*x+1)^2+1/6*(c-c/a^2/x^2)^(7/2)*x*(a*x+1)/(-a*x+1)+2*a^6*(c-c/a^2/x
^2)^(7/2)*x^7*arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)+25/16*a^6*(c-c/a^2/
x^2)^(7/2)*x^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1
)^(7/2)
```

3.838.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.40

$$\int e^{2\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-40-96ax+70a^2x^2+352a^3x^3+105a^4x^4-736a^5x^5+240a^6\right.}{240a^6x^5\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 - 96*a*x + 70*a^2*x^2
+ 352*a^3*x^3 + 105*a^4*x^4 - 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*Arc
Tan[1/Sqrt[-1 + a^2*x^2]] + 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(2
40*a^6*x^5*Sqrt[-1 + a^2*x^2])
```

3.838.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.49, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6717, 6709, 540, 25, 27, 537, 25, 537, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2} e^{2\coth^{-1}(ax)} dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2} dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\int \frac{(ax+1)^2\left(1-a^2x^2\right)^{5/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}\int -\frac{a(5ax+12)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}\int \frac{a(5ax+12)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\int \frac{(5ax+12)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \; 537$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(\frac{1}{4}a^2\int -\frac{(25ax+48)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{(25ax+48)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\int \frac{(25ax+48)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{(25ax+48)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \; 537$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(\frac{1}{2}a^2\int -\frac{3(25ax+32)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(25ax+32)\left(1-a^2x^2\right)^{3/2}}{2x^3}\Bigg)-\frac{(25ax+48)\left(1-a^2x^2\right)^{5/2}}{20x^5}\Bigg)-(1-}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\int \frac{(25ax+32)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(25ax+32)\left(1-a^2x^2\right)^{3/2}}{2x^3}\Bigg)-\frac{(25ax+48)\left(1-a^2x^2\right)^{5/2}}{20x^5}\Bigg)-\frac{(1-a}{6}}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \; 536$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\Bigg(\int \frac{25a-32a^2x}{x\sqrt{1-a^2x^2}}dx-\frac{(32-25ax)\sqrt{1-a^2x^2}}{x}\Bigg)-\frac{(25ax+32)\left(1-a^2x^2\right)^{3/2}}{2x^3}\Bigg)-\frac{(25ax+48)(1-a^2}{20x^5}}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \; 538$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\Bigg(-32a^2\int \frac{1}{\sqrt{1-a^2x^2}}dx+25a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(32-25ax)\sqrt{1-a^2x^2}}{x}\Bigg)-\frac{(25ax+32)(1-}{2x^3}}{\left(1-a^2x^2\right)^{7/2}}$$

3.838. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

$$\Big\downarrow \text{223}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\left(25a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(32-25ax)}{x}-32a\arcsin(ax)\right)-\frac{(25ax+32)(1-a^2x^2)}{2x^3}}{(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\left(\frac{25}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(32-25ax)}{x}-32a\arcsin(ax)\right)-\frac{(25ax+32)(1-a^2x}{2x^3}}{(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\left(-\frac{25\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(32-25ax)}{x}-32a\arcsin(ax)\right)-\frac{(25ax+32)(1-}{2x^3}}{(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\left(-25a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(32-25ax)}{x}-32a\arcsin(ax)\right)-\frac{(25ax+3}{}}{(1-a^2x^2)^{7/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
-(((c - c/(a^2*x^2))^(7/2)*x^7*(-1/6*(1 - a^2*x^2)^(7/2)/x^6 + (a*(-1/20*(
(48 + 25*a*x)*(1 - a^2*x^2)^(5/2))/x^5 - (a^2*(-1/2*((32 + 25*a*x)*(1 - a^
2*x^2)^(3/2))/x^3 - (3*a^2*(-(((32 - 25*a*x)*Sqrt[1 - a^2*x^2])/x) - 32*a*
ArcSin[a*x] - 25*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/4))/6))/(1 - a^2*x^2)^
(7/2))
```

3.838. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

3.838.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

3.838. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.838.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{\left(736 a^{7} x^{7}-105 a^{6} x^{6}-1088 a^{5} x^{5}+35 a^{4} x^{4}+448 a^{3} x^{3}+110 a^{2} x^{2}-96 a x -40\right) c^{3} \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{240 x^{5} a^{6} \left(a^{2} x^{2}-1\right)}+\frac{\left(\frac{2 a^{7} \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{\sqrt{a^{2} c}}+\frac{25 a^{6}}{}\right.}{}$
default	$-\frac{\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} x \left(-2016 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{7}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{9} c \,x^{7}+2016 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{9}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{9} x^{5}+375 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{7}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{8} c \,x^{6}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(736*a^7*x^7-105*a^6*x^6-1088*a^5*x^5+35*a^4*x^4+448*a^3*x^3+110*a^
2*x^2-96*a*x-40)/x^5*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(2*
a^7*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+25/16*a^6/
(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)+a^6/c*(c*(a^2*x^2
-1))^(1/2))*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^
2-1))^(1/2)
```

3.838.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \left[-\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) - 375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-}{a^2x^2}}}{x^2}\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

output
```
[-1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^
2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(240*a^
6*c^3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c
^3*x^2 - 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5),
1/240*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)
*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 - 736*a^5*c^3
*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 - 96*a*c^3*x - 4
0*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

3.838.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.76 (sec) , antiderivative size = 1059, normalized size of antiderivative = 2.85

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(7/2),x)
```

output
```
c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1)
, (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*s
qrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**3*Piecewise((-a*sqrt
(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x*
*2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqr
t(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Pie
cewise((I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x*
*2))) - I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
 > 1), (-a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x)
, True))/a**2 - 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))*
*(3/2)/(3*c), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/
8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sq
rt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2)))
, 1/Abs(a**2*x**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*
x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) +
sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 + 2*c**3*Piecew
ise((2*a**3*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2
- 1)/(15*x**3) - sqrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) >
1), (2*I*a**3*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a...
```

3.838.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x - 1), x)
```

3.838.8 Giac [A] (verification not implemented)

Time = 41.99 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.51

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=$$

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{120\sqrt{a^2}}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
-1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x)/a^2 + 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*s
gn(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) + 1440*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^9*c^5*abs(a)*sgn(x) + 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)^8*a*c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*ab
s(a)*sgn(x) + 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(
x) + 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) + 6720*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) + 2976*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - c))*c^9*abs(a)*sgn(x) + 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

3.838.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.839 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

3.839.1 Optimal result

Integrand size = 24, antiderivative size = 294

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)}-\frac{2a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\arcsin(ax)}{(1-ax)^{5/2}(1+ax)^{5/2}}-\frac{9a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8(1-ax)^{5/2}(1+ax)^{5/2}}$$

output
```
-5/8*a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)^2+25/8*a^4*(c-c/a^2/x^2)^(5/2)*x
^5/(-a*x+1)^2/(a*x+1)^2-17/12*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+
1)+1/6*a*(c-c/a^2/x^2)^(5/2)*x^2*(a*x+1)/(-a*x+1)^2+1/4*(c-c/a^2/x^2)^(5/2
)*x*(a*x+1)/(-a*x+1)-2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5
/2)/(a*x+1)^(5/2)-9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(
a*x+1)^(1/2))/(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

3.839.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.46

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(6+16ax-3a^2x^2-64a^3x^3+24a^4x^4)+27a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a}}\right.\right.}{24a^4x^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 6
4*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 48*a^4
*x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

3.839.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.50, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.542$, Rules used = {6717, 6709, 540, 25, 27, 537, 25, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\int \frac{(ax+1)^2(1-a^2x^2)^{3/2}}{x^5}dx}{(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}\int -\frac{a(3ax+8)(1-a^2x^2)^{3/2}}{x^4}dx-\frac{(1-a^2x^2)^{5/2}}{4x^4}\right)}{(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}\int \frac{a(3ax+8)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\int \frac{(3ax+8)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \color{blue}{537}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(\frac{1}{2}a^2\int -\frac{(9ax+16)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\int \frac{(9ax+16)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \color{blue}{536}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(\int \frac{9a-16a^2x}{x\sqrt{1-a^2x^2}}dx-\frac{(16-9ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \color{blue}{538}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-16a^2\int \frac{1}{\sqrt{1-a^2x^2}}dx+9a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(16-9ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)\right.}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \color{blue}{223}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(9a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(16-9ax)}{x}-16a\arcsin(ax)\right)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-(1-\right.}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \color{blue}{243}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(\frac{9}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(16-9ax)}{x}-16a\arcsin(ax)\right)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\right.}{\left(1-a^2x^2\right)^{5/2}}$$

3.839. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

$$\Big\downarrow \text{73}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\Bigg(\frac{1}{4}a\Bigg(-\frac{1}{2}a^2\Bigg(-\frac{9\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(16-9ax)}{x}-16a\arcsin(ax)\Bigg)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\Bigg)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\Bigg(\frac{1}{4}a\Bigg(-\frac{1}{2}a^2\left(-9a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(16-9ax)}{x}-16a\arcsin(ax)\right)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/}}{6x^3}}{\left(1-a^2x^2\right)^{5/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
-(((c - c/(a^2*x^2))^(5/2)*x^5*(-1/4*(1 - a^2*x^2)^(5/2)/x^4 + (a*(-1/6*((
16 + 9*a*x)*(1 - a^2*x^2)^(3/2))/x^3 - (a^2*(-(((16 - 9*a*x)*Sqrt[1 - a^2*
x^2])/x) - 16*a*ArcSin[a*x] - 9*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/4))/(1
- a^2*x^2)^(5/2))
```

3.839.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

---

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

3.839. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.839.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\left(64 a^{5} x^{5}+3 a^{4} x^{4}-80 a^{3} x^{3}-9 a^{2} x^{2}+16 a x +6\right) c^{2} \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{24 x^{3} a^{4} \left(a^{2} x^{2}-1\right)}+\frac{\left(\frac{2 a^{5} \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{\sqrt{a^{2} c}}+\frac{9 a^{4} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right)}{8 \sqrt{-c}}+\frac{a^{4}}{}\right.}{a^{4} \left(a^{2} x^{2}-1\right)}$
default	$\frac{\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} x \left(-80 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{5}{2}} a^{7} c \,x^{5}+80 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{7}{2}} a^{7} x^{3}+48 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}\right)^{\frac{5}{2}} a^{6} c \,x^{4}+27 \sqrt{}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(64*a^5*x^5+3*a^4*x^4-80*a^3*x^3-9*a^2*x^2+16*a*x+6)/x^3*c^2/a^4*(c*
(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(2*a^5*ln(a^2*c*x/(a^2*c)^(1/2)+(a^
2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+9/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(
a^2*c*x^2-c)^(1/2))/x)+a^4/c*(c*(a^2*x^2-1))^(1/2))*c^2/a^4*(c*(a^2*x^2-1)
/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2-1))^(1/2)
```

3.839.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.34

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\left[-\frac{96\,a^{3}\sqrt{-c}c^{2}x^{3}\arctan\left(\frac{a^{2}\sqrt{-c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)-27\,a^{3}\sqrt{-c}c^{2}x^{3}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-}{x^{2}}\right.}{48\,a^{4}x^{3}}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 -
 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*c^
2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x
*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log
(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2
4*a^4*c^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt(
(a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

3.839.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a} - \frac{i\sqrt{c}\log(ax)}{a} + \frac{i\sqrt{c}\log(a^2x^2)}{2a} + \frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1 \\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a} + \frac{i\sqrt{c}\log(a^2x^2)}{2a} - \frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right) + \frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}} + \sqrt{c}\operatorname{acosh}(ax) + \frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}} - i\sqrt{c}\operatorname{asin}(ax) - \frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a} - \frac{2c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^3} - \frac{c^2\left(\begin{cases}\frac{ia^3\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8} - \frac{ia^2\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}} + \frac{3i\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{i\sqrt{c}}{4a^2x^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a^3\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8} + \frac{a^2\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c}}{4a^2x^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(5/2),x)
```

output
```
c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1)
, (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*s
qrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**2*Piecewise((-a*sqrt
(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x*
*2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqr
t(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - 2*c**2*P
iecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**
3 - c**2*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*
sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2)))
- I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1),
 (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2)
)) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqr
t(1 - 1/(a**2*x**2))), True))/a**4
```

3.839.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x - 1), x)
```

3.839.8 Giac [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.41

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=$$

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{12\sqrt{a^2cx^2}}{a}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*s
gn(x)/a^2 + 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c))*sgn(
x)/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) - 96*(sqrt(a^2*c)*x - sqrt(a^2*
c*x^2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
5*c^4*abs(a)*sgn(x) - 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2
)*sgn(x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) -
160*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) - 64*a*c^(13/2)*sgn(x))/((
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

3.839.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.840 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

3.840.1 Optimal result

Integrand size = 24, antiderivative size = 213

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx &= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}\\ &+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\arcsin(ax)}{(1-ax)^{3/2}(1+ax)^{3/2}}\\ &+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2(1-ax)^{3/2}(1+ax)^{3/2}}\end{aligned}$$

output
```
a*(c-c/a^2/x^2)^(3/2)*x^2/(-a*x+1)-5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1
)/(a*x+1)+1/2*(c-c/a^2/x^2)^(3/2)*x*(a*x+1)/(-a*x+1)+2*a^2*(c-c/a^2/x^2)^(
3/2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)+1/2*a^2*(c-c/a^2/x^2)^(3
/2)*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

3.840.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.54

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-1-4ax+2a^2x^2)+a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+4a^2x^2\log\left(ax+\right.\right.}{2a^2x\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 - 4*a*x + 2*a^2*x^2) + a^
2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2
]]))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

3.840.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.53, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6709, 540, 25, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\int \frac{(ax+1)^2\sqrt{1-a^2x^2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}\int -\frac{a(ax+4)\sqrt{1-a^2x^2}}{x^2}dx-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}\int \frac{a(ax+4)\sqrt{1-a^2x^2}}{x^2}dx-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\int \frac{(ax+4)\sqrt{1-a^2x^2}}{x^2}dx-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{536}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(\int \frac{a-4a^2x}{x\sqrt{1-a^2x^2}}dx-\frac{(4-ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(-4a^2\int \frac{1}{\sqrt{1-a^2x^2}}dx+a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(4-ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(4-ax)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(4-ax)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(4-ax)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(4-ax)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

input

```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

output

```
-(((c - c/(a^2*x^2))^(3/2)*x^3*(-1/2*(1 - a^2*x^2)^(3/2)/x^2 + (a*(-(((4 -
 a*x)*Sqrt[1 - a^2*x^2])/x) - 4*a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2
]]))/2))/(1 - a^2*x^2)^(3/2))
```

3.840. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

3.840.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.840. $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.840.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

method	result
risch	$\frac{(2a^4x^4-4a^3x^3-3a^2x^2+4ax+1)c\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x\,a^2(a^2x^2-1)}+\frac{\left(\frac{2a^3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{2\sqrt{-c}}\right)c\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\sqrt{}}{a^2(a^2x^2-1)}$
default	$\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{3}{2}}x\left(12\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^5c\,x^3-12\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\,a^5x+4\sqrt{-\frac{c}{a^2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^4c\,x^2-\left(\frac{c(a^2x^2}{a^2}\right.\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^4*x^4-4*a^3*x^3-3*a^2*x^2+4*a*x+1)/x*c/a^2*(c*(a^2*x^2-1)/a^2/x^2
)^(1/2)/(a^2*x^2-1)+(2*a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(
a^2*c)^(1/2)+1/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))
/x))*c/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-
1)
```

3.840.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.49

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\left[-\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-a\sqrt{-c}cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(}{4\,a^2x}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*
x^2))/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(2*a^2*c*x^2 - 4*a*c*x - c)
*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(
2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*
a^2*c*x^2 - 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

3.840.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.77

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log{(ax)}}{a}+\frac{i\sqrt{c}\log{(a^2x^2)}}{2a}+\frac{\sqrt{c}\operatorname{asin}{\left(\frac{1}{ax}\right)}}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log{(a^2x^2)}}{2a}-\frac{i\sqrt{c}\log{\left(\sqrt{-a^2x^2+1}+1\right)}}{a} & \text{otherwise}\end{cases}\right)+\frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}{(ax)}+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}{(ax)}-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}+\frac{c\left(\begin{cases}\frac{ia\sqrt{c}\operatorname{acosh}{\left(\frac{1}{ax}\right)}}{2}+\frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a\sqrt{c}\operatorname{asin}{\left(\frac{1}{ax}\right)}}{2}-\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(3/2),x)
```

output
```
c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt
(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (
I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c*Piecewise((-a*sqrt(c)*x/
sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1
)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I
*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I
*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/
a**2
```

3.840.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x - 1), x)
```

3.840.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.25

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=$$
$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(x}{a^2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
-(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^
2 + 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs
(a)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^3*c^2*abs(a)*sgn(x) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c
^(5/2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) -
4*a*c^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*a
bs(a)))*abs(a)
```

3.840.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

3.841 $\int e^{2 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2 x^2}} \, dx$

3.841.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{2 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2 x^2}} \, dx = \sqrt{c-\frac{c}{a^2 x^2}}x - \frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(
a*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.841.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{2 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2 x^2}} \, dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]
] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

3.841.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6709, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2(2ax+1)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2(2ax+1)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{2ax+1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(2a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 221$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

input

```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[1 - a^2*x^2] + 2*ArcSin[a*x] - ArcTanh[S
qrt[1 - a^2*x^2]]))/Sqrt[1 - a^2*x^2])
```

3.841.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.841.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^{2}}}\,a^{2}\sqrt{-\frac{c}{a^{2}}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^{2}}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^{2}}}-\sqrt{-\frac{c}{a^{2}}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{2}-c\ln\left(\frac{2\sqrt{-\frac{c}{a^{2}}}\right.\right.}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{2}\sqrt{-\frac{c}{a^{2}}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+c*x)/c^(1/2)
)*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^
(1/2)/a^2/(-c/a^2)^(1/2)
```

3.841.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.30

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a
^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a,
(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^
2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

3.841.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.841.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

3.841.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.841.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.842 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

3.842.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=-\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}+\frac{2\sqrt{1-ax}\sqrt{1+ax}\arcsin(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}$$

output
```
-2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2)^(1/2)-(a*x+1)^2/a^2/x/(c-c/a^2/x^2
)^(1/2)+2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/
2)
```

3.842.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{-3-2ax+a^2x^2+2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(-3 - 2*a*x + a^2*x^2 + 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]]
)/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

3.842.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6717, 6709, 527, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{\sqrt{1-a^2x^2}\int \frac{x(ax+1)^2}{(1-a^2x^2)^{3/2}}dx}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 527}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{2(ax+1)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{ax+2}{\sqrt{1-a^2x^2}}dx}{a}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 455}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{2(ax+1)}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}}{a}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{2(ax+1)}{a^2\sqrt{1-a^2x^2}}-\frac{\frac{2\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}}{a}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

input

```
Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output

```
-((Sqrt[1 - a^2*x^2]*((2*(1 + a*x))/(a^2*Sqrt[1 - a^2*x^2]) - (-(Sqrt[1 -
a^2*x^2]/a) + (2*ArcSin[a*x])/a)/a))/(Sqrt[c - c/(a^2*x^2)]*x))
```

3.842.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 527
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_S
ymbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a +
b*x^2])), x] + Simp[1/(b*d^(m - 2))   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(
2^(n - 1)*c^(m + n - 1) - d^m*x^m*(c + d*x)^(n - 1))/(c - d*x), x], x], x]
/; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && EqQ[b*c^2 + a*d^2,
 0]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.842.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

method	result
risch	$\frac{a^{2} x^{2}-1}{a^{2} x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}+\frac{\left(\frac{2 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\,x^{2}-c}\right)}{a \sqrt{a^{2} c}}-\frac{2 \sqrt{a^{2} c \left(x-\frac{1}{a}\right)^{2}+2 \left(x-\frac{1}{a}\right) a c}}{a^{3} c \left(x-\frac{1}{a}\right)}\right) \sqrt{c \left(a^{2} x^{2}-1\right)}}{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}$
default	$\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, \left(\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, \sqrt{c}\, a^{2} x+2 \ln \left(\sqrt{c}\, x+\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a c x-2 a \sqrt{\frac{c \left(a x-1\right) \left(a x+1\right)}{a^{2}}}\, \sqrt{c}-\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a \sqrt{c}-2 \ln \left(\sqrt{c}\, x+\sqrt{\frac{c\left(\right.}{}}\right.\right.}{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x\, c^{\frac{3}{2}} a \left(a x-1\right)}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a*ln(a^2*c*x/(a^2*c)^
(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-2/a^3/c/(x-1/a)*(a^2*c*(x-1/a)^2+
2*(x-1/a)*a*c)^(1/2))/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/
x
```

3.842.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(a^2x^2-3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac}, -\frac{2\,(ax-1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(a^2x^2-3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[((a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - c) + (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^2*c*x - a*c), -(2*(a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)))/(a^2*c*x - a*c)]
```

3.842.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)), x)
```

3.842.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a^2*x^2))), x)
```

3.842.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.842.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)), x)
```

3.843 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

3.843.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\arcsin(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

output
```
-1/3*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x+2/3*(-2*a*x+5)*(-a*x+1)*(a*x+1)^2
/a^4/(c-c/a^2/x^2)^(3/2)/x^3-2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^
4/(c-c/a^2/x^2)^(3/2)/x^3
```

3.843.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{10-4ax-11a^2x^2+3a^3x^3+6(-1+ax)\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{3a^2c\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*(-1 + a*x)*Sqrt[-1 + a^2*x^2]*Log
[a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x))
```

3.843.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6717, 6709, 529, 2166, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3(ax+1)^2}{\left(1-a^2x^2\right)^{5/2}}dx}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{(ax+1)^2}{3a^4\left(1-a^2x^2\right)^{3/2}}-\frac{1}{3}\int \frac{(ax+1)\left(\frac{3x^2}{a}+\frac{3x}{a^2}+\frac{2}{a^3}\right)}{\left(1-a^2x^2\right)^{3/2}}dx\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(\int \frac{3(ax+2)}{a^3\sqrt{1-a^2x^2}}dx-\frac{8(ax+1)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(ax+1)^2}{3a^4\left(1-a^2x^2\right)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(\frac{3\int \frac{ax+2}{\sqrt{1-a^2x^2}}dx}{a^3}-\frac{8(ax+1)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(ax+1)^2}{3a^4\left(1-a^2x^2\right)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(\frac{3\left(2\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^3}-\frac{8(ax+1)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(ax+1)^2}{3a^4\left(1-a^2x^2\right)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow 223$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{(ax+1)^2}{3a^4\left(1-a^2x^2\right)^{3/2}}+\frac{1}{3}\left(\frac{3\left(\frac{2\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^3}-\frac{8(ax+1)}{a^4\sqrt{1-a^2x^2}}\right)\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input

```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

output

```
-(((1 - a^2*x^2)^(3/2)*((1 + a*x)^2/(3*a^4*(1 - a^2*x^2)^(3/2)) + ((-8*(1
+ a*x))/(a^4*Sqrt[1 - a^2*x^2]) + (3*(-(Sqrt[1 - a^2*x^2]/a) + (2*ArcSin[a
*x])/a))/a^3)/3))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

3.843.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455

```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

3.843. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.843.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.74

method	result
risch	$\frac{a^{2} x^{2}-1}{a^{2} c x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}+\frac{\left(\frac{2 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\,x^{2}-c}\right)}{a^{3} \sqrt{a^{2} c}}-\frac{\sqrt{a^{2} c \left(x -\frac{1}{a}\right)^{2}+2 \left(x -\frac{1}{a}\right) a c}}{3 a^{6} c \left(x -\frac{1}{a}\right)^{2}}-\frac{8 \sqrt{a^{2} c \left(x -\frac{1}{a}\right)^{2}+2 \left(x -\frac{1}{a}\right) a c}}{3 a^{5} c \left(x -\frac{1}{a}\right)}\right) a^{2} \sqrt{c \left(a^{2} x^{2}-1\right)}}{c x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}$
default	$\frac{\left(3 c^{\frac{3}{2}} \sqrt{\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}}\, a^{3} x^{3}+4 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, c^{\frac{3}{2}} a^{2} x^{2}-15 x^{2} a^{2} c^{\frac{3}{2}} \sqrt{\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}}+6 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) \sqrt{\frac{c(ax-}{}}}{3 \sqrt{\frac{c(ax-1)}{a}}}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a^3*ln(a^2*c*x/(a^2
*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-1/3/a^6/c/(x-1/a)^2*(a^2*c*(x
-1/a)^2+2*(x-1/a)*a*c)^(1/2)-8/3/a^5/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*
a*c)^(1/2))*a^2/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

3.843. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

3.843.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.28

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \left[\frac{3\,(a^2x^2-2\,ax+1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(3\,a^3x^3-14\,a^2x^2+}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}\right.$$

$$\left.-\frac{6\,(a^2x^2-2\,ax+1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(3\,a^3x^3-14\,a^2x^2+10\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*
sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*
(6*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt(
(a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

3.843.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x - 1)), x
)
```

3.843.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

3.843.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.843.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)), x)
```

3.844 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.844.1 Optimal result

Integrand size = 24, antiderivative size = 203

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} \\ &+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} \\ &-\frac{2(1-ax)^3(1+ax)^2(28+43ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}+\frac{2(1-ax)^{5/2}(1+ax)^{5/2}\arcsin(ax)}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\end{aligned}$$

output
```
-1/5*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x+2/3*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a
^2/x^2)^(5/2)/x^2-58/15*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(5/2)/x^3-2
/15*(-a*x+1)^3*(a*x+1)^2*(43*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5+2*(-a*x+1
)^(5/2)*(a*x+1)^(5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

3.844.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{-56+82ax+32a^2x^2-76a^3x^3+15a^4x^4+30(-1+ax)^2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1}\right.}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
(-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*(-1 + a*x)^2*Sqr
t[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2
*x^2)]*x*(-1 + a*x)^2)
```

3.844.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6709, 529, 2166, 2345, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5(ax+1)^2}{\left(1-a^2x^2\right)^{7/2}}dx}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{(ax+1)^2}{5a^6\left(1-a^2x^2\right)^{5/2}}-\frac{1}{5}\int \frac{(ax+1)\left(\frac{5x^4}{a}+\frac{5x^3}{a^2}+\frac{5x^2}{a^3}+\frac{5x}{a^4}+\frac{2}{a^5}\right)}{\left(1-a^2x^2\right)^{5/2}}dx\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\frac{15x^3}{a^2}+\frac{30x^2}{a^3}+\frac{45x}{a^4}+\frac{16}{a^5}}{\left(1-a^2x^2\right)^{3/2}}dx-\frac{22(ax+1)}{3a^6\left(1-a^2x^2\right)^{3/2}}\right)+\frac{(ax+1)^2}{5a^6\left(1-a^2x^2\right)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \; 2345$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23ax+30)}{a^6\sqrt{1-a^2x^2}}-\int \frac{15(ax+2)}{a^5\sqrt{1-a^2x^2}}dx\right)-\frac{22(ax+1)}{3a^6\left(1-a^2x^2\right)^{3/2}}\right)+\frac{(ax+1)^2}{5a^6\left(1-a^2x^2\right)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23ax+30)}{a^6\sqrt{1-a^2x^2}}-\frac{15\int \frac{ax+2}{\sqrt{1-a^2x^2}}dx}{a^5}\right)-\frac{22(ax+1)}{3a^6\left(1-a^2x^2\right)^{3/2}}\right)+\frac{(ax+1)^2}{5a^6\left(1-a^2x^2\right)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23ax+30)}{a^6\sqrt{1-a^2x^2}}-\frac{15\left(2\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^5}\right)-\frac{22(ax+1)}{3a^6\left(1-a^2x^2\right)^{3/2}}\right)+\frac{(ax+1)^2}{5a^6\left(1-a^2x^2\right)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{(ax+1)^2}{5a^6\left(1-a^2x^2\right)^{5/2}}+\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23ax+30)}{a^6\sqrt{1-a^2x^2}}-\frac{15\left(\frac{2\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^5}\right)-\frac{22(ax+1)}{3a^6\left(1-a^2x^2\right)^{3/2}}\right)\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
-(((1 - a^2*x^2)^(5/2)*((1 + a*x)^2/(5*a^6*(1 - a^2*x^2)^(5/2)) + ((-22*(1
 + a*x))/(3*a^6*(1 - a^2*x^2)^(3/2)) + ((2*(30 + 23*a*x))/(a^6*Sqrt[1 - a^
2*x^2]) - (15*(-(Sqrt[1 - a^2*x^2]/a) + (2*ArcSin[a*x])/a))/a^5)/3)/5))/((
c - c/(a^2*x^2))^(5/2)*x^5))
```

3.844. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.844.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.844.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.47

method	result
risch	$\frac{a^{2} x^{2}-1}{a^{2} c^{2} x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}+\frac{\left(\frac{2 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\,x^{2}-c}\right)}{a^{5} \sqrt{a^{2} c}}+\frac{\sqrt{a^{2} c \left(x+\frac{1}{a}\right)^{2}-2 \left(x+\frac{1}{a}\right) a c}}{8 a^{7} c \left(x+\frac{1}{a}\right)}-\frac{41 \sqrt{a^{2} c \left(x-\frac{1}{a}\right)^{2}+2 \left(x-\frac{1}{a}\right) a c}}{60 a^{8} c \left(x-\frac{1}{a}\right)^{2}}-\frac{383 \sqrt{a^{2} c \left(x-\frac{1}{a}\right)^{2}+2 \left(x-\right.}}{120 a^{7} c \left(x-\frac{1}{a}\right)}\right.}{c^{2} x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}$
default	$\frac{\left(15 c^{\frac{5}{2}} \left(\frac{c \left(a x-1\right) \left(a x+1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{5} x^{5}-45 x^{4} c^{\frac{5}{2}} a^{4} \left(\frac{c \left(a x-1\right) \left(a x+1\right)}{a^{2}}\right)^{\frac{3}{2}}-16 c^{\frac{5}{2}} \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{4} x^{4}-60 c^{\frac{5}{2}} \left(\frac{c \left(a x-1\right) \left(a x+1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3} x^{3}+16 c^{\frac{5}{2}} \right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c^2/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a^5*ln(a^2*c*x/(a
^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+1/8/a^7/c/(x+1/a)*(a^2*c*(x
+1/a)^2-2*(x+1/a)*a*c)^(1/2)-41/60/a^8/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+2*(x-1
/a)*a*c)^(1/2)-383/120/a^7/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)
-1/10/a^9/c/(x-1/a)^3*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2))*a^4/c^2/x/(c*
(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

3.844.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.73

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \Bigg[\frac{15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(15\,a^5x^5}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3}$$

$$-\frac{30\,(a^4x^4-2\,a^3x^3+2\,ax-1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(15\,a^5x^5-76\,a^4x^4+32\,a^3x^3+82\,a^2x^2-}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^
2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 - 76*a^4*
x^4 + 32*a^3*x^3 + 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3), -1/15*(30*(a^4*x^4 - 2
*a^3*x^3 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) - (15*a^5*x^5 - 76*a^4*x^4 + 32*a^3*x^3 + 82
*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 - 2*a^4*c
^3*x^3 + 2*a^2*c^3*x - a*c^3)]
```

3.844.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x - 1)), x
)
```

3.844.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

3.844.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.844.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)), x)
```

3.845 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.845.1 Optimal result

Integrand size = 24, antiderivative size = 283

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}\\ &-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}+\frac{142(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}\\ &+\frac{2(1-ax)^4(1+ax)^3(72+107ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}-\frac{2(1-ax)^{7/2}(1+ax)^{7/2}\arcsin(ax)}{a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\end{aligned}$$

output
```
-1/7*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x+2/5*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a
^2/x^2)^(7/2)/x^2-124/105*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(7/2)/x^3
+782/105*(-a*x+1)^3*(a*x+1)^2/a^5/(c-c/a^2/x^2)^(7/2)/x^4+142/35*(-a*x+1)^
4*(a*x+1)^2/a^6/(c-c/a^2/x^2)^(7/2)/x^5+2/35*(-a*x+1)^4*(a*x+1)^3*(107*a*x
+72)/a^8/(c-c/a^2/x^2)^(7/2)/x^7-2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x
)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

3.845.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.47

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{432-654ax-636a^2x^2+1226a^3x^3+74a^4x^4-562a^5x^5+105a^6x^6+210(-1+ax)^3(}{105a^2c^3\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)^3(1+ax)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
(432 - 654*a*x - 636*a^2*x^2 + 1226*a^3*x^3 + 74*a^4*x^4 - 562*a^5*x^5 + 1
05*a^6*x^6 + 210*(-1 + a*x)^3*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[
-1 + a^2*x^2]])/(105*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x
))
```

3.845.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6717, 6709, 529, 2166, 2345, 2345, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7(ax+1)^2}{(1-a^2x^2)^{9/2}}dx}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}-\frac{1}{7}\int \frac{(ax+1)\left(\frac{7x^6}{a}+\frac{7x^5}{a^2}+\frac{7x^4}{a^3}+\frac{7x^3}{a^4}+\frac{7x^2}{a^5}+\frac{7x}{a^6}+\frac{2}{a^7}\right)}{(1-a^2x^2)^{7/2}}dx\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \; 2166$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\int \frac{\frac{35x^5}{a^2}+\frac{70x^4}{a^3}+\frac{105x^3}{a^4}+\frac{140x^2}{a^5}+\frac{175x}{a^6}+\frac{34}{a^7}}{(1-a^2x^2)^{5/2}}dx-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \; 2345$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}-\frac{1}{3}\int \frac{\frac{105x^3}{a^4}+\frac{210x^2}{a^5}+\frac{420x}{a^6}+\frac{142}{a^7}}{(1-a^2x^2)^{3/2}}dx\right)-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \; 2345$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{105(ax+2)}{a^7\sqrt{1-a^2x^2}}dx-\frac{352ax+525}{a^8\sqrt{1-a^2x^2}}\right)+\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{105\int \frac{ax+2}{\sqrt{1-a^2x^2}}dx}{a^7}-\frac{352ax+525}{a^8\sqrt{1-a^2x^2}}\right)+\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\Bigg(\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{1}{3}\Bigg(\frac{105\left(2\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^7}-\frac{352ax+525}{a^8\sqrt{1-a^2x^2}}\Bigg)+\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}\Bigg)-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\Bigg)+\frac{}{7a^8(}}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\Bigg(\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}+\frac{1}{7}\Bigg(\frac{1}{5}\Bigg(\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}+\frac{1}{3}\Bigg(\frac{105\left(\frac{2\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^7}-\frac{352ax+525}{a^8\sqrt{1-a^2x^2}}\Bigg)\Bigg)-\frac{44(ax+}{5a^8(1-a^2x}}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

3.845. $\displaystyle\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

output

```
-(((1 - a^2*x^2)^(7/2)*((1 + a*x)^2/(7*a^8*(1 - a^2*x^2)^(7/2)) + ((-44*(1
 + a*x))/(5*a^8*(1 - a^2*x^2)^(5/2)) + ((315 + 244*a*x)/(3*a^8*(1 - a^2*x^
2)^(3/2)) + (-((525 + 352*a*x)/(a^8*Sqrt[1 - a^2*x^2])) + (105*(-(Sqrt[1 -
 a^2*x^2]/a) + (2*ArcSin[a*x])/a))/a^7)/3)/5)/7))/((c - c/(a^2*x^2))^(7/2)
*x^7))
```

3.845.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455

```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.845. $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.845.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.35

method	result
risch	$\frac{a^{2} x^{2}-1}{a^{2} c^{3} x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}+\frac{\Bigg(\frac{2 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\, x^{2}-c}\right)}{a^{7} \sqrt{a^{2} c}}-\frac{\sqrt{a^{2} c \left(x-\frac{1}{a}\right)^{2}+2 \left(x-\frac{1}{a}\right) a c}}{28 a^{12} c \left(x-\frac{1}{a}\right)^{4}}-\frac{39 \sqrt{a^{2} c \left(x-\frac{1}{a}\right)^{2}+2 \left(x-\frac{1}{a}\right) a c}}{140 a^{11} c \left(x-\frac{1}{a}\right)^{3}}-\frac{1753 \sqrt{a^{2} c \left(x-\frac{1}{a}\right)^{2}+2 (x-}{1680 a^{10} c \left(x-\frac{1}{a}\right)^{2}}}{c^{3} x \sqrt{\frac{c(}{}}}$
default	$\Bigg(105 c^{\frac{7}{2}} \left(\frac{c (a x-1) (a x+1)}{a^{2}}\right)^{\frac{5}{2}} a^{7} x^{7}-553 x^{6} c^{\frac{7}{2}} a^{6} \left(\frac{c (a x-1) (a x+1)}{a^{2}}\right)^{\frac{5}{2}}+96 c^{\frac{7}{2}} \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{5}{2}} a^{6} x^{6}-392 c^{\frac{7}{2}} \left(\frac{c (a x-1) (a x+1)}{a^{2}}\right)^{\frac{5}{2}} a^{5} x^{5}-96 c^{\frac{7}{2}}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c^3/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a^7*ln(a^2*c*x/(a
^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-1/28/a^12/c/(x-1/a)^4*(a^2*
c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)-39/140/a^11/c/(x-1/a)^3*(a^2*c*(x-1/a)^2+
2*(x-1/a)*a*c)^(1/2)-1753/1680/a^10/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+2*(x-1/a)
*a*c)^(1/2)-3061/840/a^9/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)-1
/48/a^10/c/(x+1/a)^2*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)+7/24/a^9/c/(x+1
/a)*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2))*a^6/c^3/x/(c*(a^2*x^2-1)/a^2/x^
2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

3.845.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \left[\frac{105\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\right.}{105\left(a^7c^4x^6-2a^6c^4x^5-\right.}\right.$$

$$-\frac{210\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(105a^7x^7-562\right.}{105\left(a^7c^4x^6-2a^6c^4x^5-a^5c^4x^4+4a^4c^4x^3-a^3c^4x^2\right.}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

output
```
[1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - c) + (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4 - 636*
a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4
*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c
^4*x + a*c^4), -1/105*(210*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^
2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(a^2*c*x^2 - c)) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 122
6*a^4*x^4 - 636*a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)))/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c
^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

3.845.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x - 1)), x
)
```

3.845.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

3.845.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.845.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)), x)
```

3.846 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

3.846.1 Optimal result

Integrand size = 24, antiderivative size = 322

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/x^8/(1-1/a^2/x^2)^(1/2)+3/7*c^4*(c-c/a^2/x
^2)^(1/2)/a^8/x^7/(1-1/a^2/x^2)^(1/2)-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/x^5/
(1-1/a^2/x^2)^(1/2)-3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2
)+2*c^4*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+4*c^4*(c-c/a^2/x^2
)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c^4*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2
)^(1/2)+3*c^4*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.846.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(\frac{1}{8a^9x^8}+\frac{3}{7a^8x^7}-\frac{8}{5a^6x^5}-\frac{3}{2a^5x^4}+\frac{2}{a^4x^3}+\frac{4}{a^3x^2}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

output
```
((c - c/(a^2*x^2))^(9/2)*(1/(8*a^9*x^8) + 3/(7*a^8*x^7) - 8/(5*a^6*x^5) -
3/(2*a^5*x^4) + 2/(a^4*x^3) + 4/(a^3*x^2) + x + (3*Log[x])/a))/(1 - 1/(a^2
*x^2))^(9/2)
```

3.846.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{9/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^3(ax+1)^6}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^3(ax+1)^6}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow 99$$

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int\left(-a^9-\frac{3a^8}{x}+\frac{8a^6}{x^3}+\frac{6a^5}{x^4}-\frac{6a^4}{x^5}-\frac{8a^3}{x^6}+\frac{3a}{x^8}+\frac{1}{x^9}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow 2009$$

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(a^9(-x)-3a^8\log(x)-\frac{4a^6}{x^2}-\frac{2a^5}{x^3}+\frac{3a^4}{2x^4}+\frac{8a^3}{5x^5}-\frac{3a}{7x^7}-\frac{1}{8x^8}\right)}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

output
```
-((c^4*Sqrt[c - c/(a^2*x^2)]*(-1/8*1/x^8 - (3*a)/(7*x^7) + (8*a^3)/(5*x^5)
 + (3*a^4)/(2*x^4) - (2*a^5)/x^3 - (4*a^6)/x^2 - a^9*x - 3*a^8*Log[x]))/(a
^9*Sqrt[1 - 1/(a^2*x^2)]))
```

3.846.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.846. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx$

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.846.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(280 a^{9} x^{9}+840 a^{8} \ln \left(x \right) x^{8}+1120 a^{6} x^{6}+560 a^{5} x^{5}-420 a^{4} x^{4}-448 a^{3} x^{3}+120 a x +35\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{9}{2}} x}{280 \left(a x +1\right)^{3} \left(a^{2} x^{2}-1\right)^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	112

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/280*(280*a^9*x^9+840*a^8*ln(x)*x^8+1120*a^6*x^6+560*a^5*x^5-420*a^4*x^4-
448*a^3*x^3+120*a*x+35)*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a*x+1)^3/(a^2*x^2
-1)^3/((a*x-1)/(a*x+1))^(3/2)
```

3.846.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\frac{\left(280\,a^{9}c^{4}x^{9}+840\,a^{8}c^{4}x^{8}\log\left(x\right)+1120\,a^{6}c^{4}x^{6}+560\,a^{5}c^{4}x^{5}-420\,a^{4}c^{4}x^{4}-448\,a^{3}c^{4}x^{3}+1}{280\,a^{10}x^{8}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="fric
as")
```

output
```
1/280*(280*a^9*c^4*x^9 + 840*a^8*c^4*x^8*log(x) + 1120*a^6*c^4*x^6 + 560*a
^5*c^4*x^5 - 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqr
t(a^2*c)/(a^10*x^8)
```

3.846.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(9/2),x)
```

output
```
Timed out
```

3.846.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.846.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac
")
```

output
```
integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.846.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.847 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

3.847.1 Optimal result

Integrand size = 24, antiderivative size = 324

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx = &-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}\\&-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}\\&-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output

```
-1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)-3/5*c^3*(c-c/a^2/
x^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)-1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4
/(1-1/a^2/x^2)^(1/2)+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/
2)+5/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^
2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)
^(1/2)+3*c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.847.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6a^7x^6}-\frac{3}{5a^6x^5}-\frac{1}{4a^5x^4}+\frac{5}{3a^4x^3}+\frac{5}{2a^3x^2}-\frac{1}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
((c - c/(a^2*x^2))^(7/2)*(-1/6*1/(a^7*x^6) - 3/(5*a^6*x^5) - 1/(4*a^5*x^4)
 + 5/(3*a^4*x^3) + 5/(2*a^3*x^2) - 1/(a^2*x) + x + (3*Log[x])/a))/(1 - 1/(
a^2*x^2))^(7/2)
```

3.847.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2(ax+1)^5}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^7+\frac{3a^6}{x}+\frac{a^5}{x^2}-\frac{5a^4}{x^3}-\frac{5a^3}{x^4}+\frac{a^2}{x^5}+\frac{3a}{x^6}+\frac{1}{x^7}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(a^7x+3a^6\log(x)-\frac{a^5}{x}+\frac{5a^4}{2x^2}+\frac{5a^3}{3x^3}-\frac{a^2}{4x^4}-\frac{3a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a^2*x^2)]*(-1/6*1/x^6 - (3*a)/(5*x^5) - a^2/(4*x^4) + (5*
a^3)/(3*x^3) + (5*a^4)/(2*x^2) - a^5/x + a^7*x + 3*a^6*Log[x]))/(a^7*Sqrt[
1 - 1/(a^2*x^2)])
```

3.847.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.847. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$

3.847.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(60a^7x^7+180a^6\ln(x)x^6-60a^5x^5+150a^4x^4+100a^3x^3-15a^2x^2-36ax-10\right)\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}x}{60(ax+1)^3\left(a^2x^2-1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	112

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(60*a^7*x^7+180*a^6*ln(x)*x^6-60*a^5*x^5+150*a^4*x^4+100*a^3*x^3-15*a
^2*x^2-36*a*x-10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)^3/(a^2*x^2-1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

3.847.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx = \frac{(60\,a^7c^3x^7 + 180\,a^6c^3x^6\log(x) - 60\,a^5c^3x^5 + 150\,a^4c^3x^4 + 100\,a^3c^3x^3 - 15\,a^2c^3x^2 - 36\,ac^3}{60\,a^8x^6}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

output
```
1/60*(60*a^7*c^3*x^7 + 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 + 150*a^4*c
^3*x^4 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2 - 36*a*c^3*x - 10*c^3)*sqrt(a^2*
c)/(a^8*x^6)
```

3.847.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

3.847.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.847.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.847.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.848 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

3.848.1 Optimal result

Integrand size = 24, antiderivative size = 234

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^
(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2
/x^2)^(1/2)-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-c
/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+3*c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)
```

3.848.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{5}{4a}+\frac{1}{4a^5x^4}+\frac{1}{a^4x^3}+\frac{1}{a^3x^2}-\frac{2}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
((c - c/(a^2*x^2))^(5/2)*(5/(4*a) + 1/(4*a^5*x^4) + 1/(a^4*x^3) + 1/(a^3*x
^2) - 2/(a^2*x) + x + (3*Log[x])/a))/(1 - 1/(a^2*x^2))^(5/2)
```

3.848.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2} e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)(ax+1)^4}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)(ax+1)^4}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{84}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \left(-a^5-\frac{3a^4}{x}-\frac{2a^3}{x^2}+\frac{2a^2}{x^3}+\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(a^5(-x)-3a^4\log(x)+\frac{2a^3}{x}-\frac{a^2}{x^2}-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

3.848. $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2} dx$

output
```
-((c^2*Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 - a/x^3 - a^2/x^2 + (2*a^3)/x - a
^5*x - 3*a^4*Log[x]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]))
```

3.848.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.848.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\left(4 a^{5} x^{5}+12 \ln \left(x \right) x^{4} a^{4}-8 a^{3} x^{3}+4 a^{2} x^{2}+4 a x +1\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} x}{4 \left(a x +1\right)^{3} \left(a^{2} x^{2}-1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	96

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*a^5*x^5+12*ln(x)*x^4*a^4-8*a^3*x^3+4*a^2*x^2+4*a*x+1)*(c*(a^2*x^2-1
)/a^2/x^2)^(5/2)*x/(a*x+1)^3/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(3/2)
```

3.848.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.31

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{(4\,a^5c^2x^5+12\,a^4c^2x^4\log(x)-8\,a^3c^2x^3+4\,a^2c^2x^2+4\,ac^2x+c^2)\sqrt{a^2c}}{4\,a^6x^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

output
```
1/4*(4*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 + 4*a^2*c^2*x^2
 + 4*a*c^2*x + c^2)*sqrt(a^2*c)/(a^6*x^4)
```

3.848.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

3.848.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.848.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.848.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.849 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

3.849.1 Optimal result

Integrand size = 24, antiderivative size = 148

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = -\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2} - \frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-3*c*(c-c/a^2/x^2)^(
1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
+3*c*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.849.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2a^3x^2}-\frac{3}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(-1/2*1/(a^3*x^2) - 3/(a^2*x) + x + (3*Log[x])/a)
)/(1 - 1/(a^2*x^2))^(3/2)
```

3.849.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^3+\frac{3a^2}{x}+\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(a^3x+3a^2\log(x)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(c*Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 - (3*a)/x + a^3*x + 3*a^2*Log[x]))/(a
^3*Sqrt[1 - 1/(a^2*x^2)])
```

3.849.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.849.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{\left(2 a^{3} x^{3}+6 a^{2} \ln \left(x \right) x^{2}-6 a x -1\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} x}{2 \left(a x +1\right)^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	69

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^3*x^3+6*a^2*ln(x)*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a
*x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

3.849.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{(2\,a^3cx^3+6\,a^2cx^2\log{(x)}-6\,acx-c)\sqrt{a^2c}}{2\,a^4x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(2*a^3*c*x^3 + 6*a^2*c*x^2*log(x) - 6*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

3.849.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

3.849.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.849.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.849.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.850 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.850.1 Optimal result

Integrand size $=24$, antiderivative size $=109$

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.850.2 Mathematica [A] (verified)

Time $=0.04$ (sec) , antiderivative size $=51$, normalized size of antiderivative $=0.47$

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/Sqrt[1 - 1/(a^
2*x^2)]
```

3.850.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a}{ax-1}-a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(-ax-4\log(1-ax)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/(a*Sqrt[1 - 1
/(a^2*x^2)]))
```

3.850.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.850.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{\left(-ax+\ln\left(x\right)-4\ln\left(ax-1\right)\right)x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^
2/((a*x-1)/(a*x+1))^(3/2)
```

3.850.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+4\log(ax-1)-\log(x))}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

3.850.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.850.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.850.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.850.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.851 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

3.851.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)+2*(1-1/a^2/x^2)^(1/2)/a/(-a*x+1)
/(c-c/a^2/x^2)^(1/2)+3*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2
)
```

3.851.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x+\frac{2}{a(1-ax)}+\frac{3\log(1-ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*(x + 2/(a*(1 - a*x)) + (3*Log[1 - a*x])/a))/Sqrt[c
- c/(a^2*x^2)]
```

3.851.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x(ax+1)}{(1-ax)^2}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{3}{(ax-1)a}+\frac{2}{(ax-1)^2a}+\frac{1}{a}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{2}{a^2(1-ax)}+\frac{3\log(1-ax)}{a^2}+\frac{x}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(x/a + 2/(a^2*(1 - a*x)) + (3*Log[1 - a*x])/a^2))
/Sqrt[c - c/(a^2*x^2)]
```

3.851. $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$

3.851.3.1 Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.851.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{(ax-1)\left(a^{2}x^{2}+3a\ln(ax-1)x-ax-3\ln(ax-1)-2\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\,a^{2}}$	85

input

```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/
a^2*(a^2*x^2+3*a*ln(a*x-1)*x-a*x-3*ln(a*x-1)-2)
```

3.851.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{(a^2x^2-ax+3\,(ax-1)\log{(ax-1)}-2)\sqrt{a^2c}}{a^3cx-a^2c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
(a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)*sqrt(a^2*c)/(a^3*c*x - a^2*
c)
```

3.851.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.851.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.851.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.851.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.852 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

3.852.1 Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(-
a*x+1)^2/(c-c/a^2/x^2)^(1/2)+3*(1-1/a^2/x^2)^(1/2)/a/c/(-a*x+1)/(c-c/a^2/x
^2)^(1/2)+3*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

3.852.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.37

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x+\frac{5-6ax}{2a(-1+ax)^2}+\frac{3\log(1-ax)}{a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*(x + (5 - 6*a*x)/(2*a*(-1 + a*x)^2) + (3*Log[1 -
a*x])/a))/(c - c/(a^2*x^2))^(3/2)
```

3.852.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = $\{6751, 6747, 25, 49, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^3}{(1-ax)^3}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3}{(1-ax)^3}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{1}{a^3}-\frac{3}{a^3(ax-1)}-\frac{3}{a^3(ax-1)^2}-\frac{1}{a^3(ax-1)^3}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{3}{a^4(1-ax)}+\frac{1}{2a^4(1-ax)^2}-\frac{3\log(1-ax)}{a^4}-\frac{x}{a^3}\right)}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^3) + 1/(2*a^4*(1 - a*x)^2) - 3/(a^4*(1
 - a*x)) - (3*Log[1 - a*x])/a^4))/(c*Sqrt[c - c/(a^2*x^2)]))
```

3.852.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.852.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(ax-1)\left(2a^{3}x^{3}+6a^{2}\ln(ax-1)x^{2}-4a^{2}x^{2}-12a\ln(ax-1)x-4ax+6\ln(ax-1)+5\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(2*a^3*x^3+6*a^2*ln(a*x-1)*x^2-4*a^2*x
^2-12*a*ln(a*x-1)*x-4*a*x+6*ln(a*x-1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(
3/2)
```

3.852.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(2\,a^{3}x^{3} - 4\,a^{2}x^{2} - 4\,ax + 6\,(a^{2}x^{2} - 2\,ax + 1)\log\left(ax - 1\right) + 5\right)\sqrt{a^{2}c}}{2\,(a^{4}c^{2}x^{2} - 2\,a^{3}c^{2}x + a^{2}c^{2})}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(2*a^3*x^3 - 4*a^2*x^2 - 4*a*x + 6*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1)
+ 5)*sqrt(a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

3.852.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

3.852.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.852.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.852.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.853 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.853.1 Optimal result

Integrand size = 24, antiderivative size = 267

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}\\ &\quad-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}\\ &\quad+\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)+1/6*(1-1/a^2/x^2)^(1/2)/a/c^
2/(-a*x+1)^3/(c-c/a^2/x^2)^(1/2)-9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)^2/
(c-c/a^2/x^2)^(1/2)+31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)/(c-c/a^2/x^2)^
(1/2)+49/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-1/16*
ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

3.853.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48x-\frac{8}{a(-1+ax)^3}-\frac{54}{a(-1+ax)^2}+\frac{186}{a-a^2x}+\frac{147\log(1-ax)}{a}-\frac{3\log(1+ax)}{a}\right)}{48\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*(48*x - 8/(a*(-1 + a*x)^3) - 54/(a*(-1 + a*x)^2)
+ 186/(a - a^2*x) + (147*Log[1 - a*x])/a - (3*Log[1 + a*x])/a))/(48*(c - c
/(a^2*x^2))^(5/2))
```

3.853.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^5}{(1-ax)^4(ax+1)}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{1}{16a^5(ax+1)}+\frac{1}{a^5}+\frac{49}{16a^5(ax-1)}+\frac{31}{8a^5(ax-1)^2}+\frac{9}{4a^5(ax-1)^3}+\frac{1}{2a^5(ax-1)^4}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{31}{8a^6(1-ax)}-\frac{9}{8a^6(1-ax)^2}+\frac{1}{6a^6(1-ax)^3}+\frac{49\log(1-ax)}{16a^6}-\frac{\log(ax+1)}{16a^6}+\frac{x}{a^5}\right)}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
(a^5*Sqrt[1 - 1/(a^2*x^2)]*(x/a^5 + 1/(6*a^6*(1 - a*x)^3) - 9/(8*a^6*(1 -
a*x)^2) + 31/(8*a^6*(1 - a*x)) + (49*Log[1 - a*x])/(16*a^6) - Log[1 + a*x]
/(16*a^6)))/(c^2*Sqrt[c - c/(a^2*x^2)])
```

3.853.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.853. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.853.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

method	result
default	$-\frac{(ax-1)(ax+1)(-48a^4x^4+3a^3\ln(ax+1)x^3-147a^3\ln(ax-1)x^3+144a^3x^3-9a^2\ln(ax+1)x^2+441a^2\ln(ax-1)x^2+42a^2x^2+9a\ln(a}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

input

```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-48*a^4*x^4+3*a^3*ln(a*x+1)
*x^3-147*a^3*ln(a*x-1)*x^3+144*a^3*x^3-9*a^2*ln(a*x+1)*x^2+441*a^2*ln(a*x-
1)*x^2+42*a^2*x^2+9*a*ln(a*x+1)*x-441*a*ln(a*x-1)*x-270*a*x-3*ln(a*x+1)+14
7*ln(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

3.853.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.52

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{(48\,a^4x^4-144\,a^3x^3-42\,a^2x^2+270\,ax-3\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log(ax+1)+}{48\,(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

output

```
1/48*(48*a^4*x^4 - 144*a^3*x^3 - 42*a^2*x^2 + 270*a*x - 3*(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)*log(a*x + 1) + 147*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log
(a*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x -
a^2*c^3)
```

3.853.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

3.853.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.853.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.853.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.854 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.854.1 Optimal result

Integrand size = 24, antiderivative size = 360

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^4} \\ &+ \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3} - \frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2} + \frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)} \\ &- \frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}} - \frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)-1/16*(1-1/a^2/x^2)^(1/2)/a/c
^3/(-a*x+1)^4/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^3
/(c-c/a^2/x^2)^(1/2)-59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^2/(c-c/a^2/x
^2)^(1/2)+75/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c-c/a^2/x^2)^(1/2)-1/3
2*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+201/64*ln(-a*x+1)*
(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-9/64*ln(a*x+1)*(1-1/a^2/x^2)
^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)
```

3.854.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.32

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{2\left(104-207ax-59a^2x^2+309a^3x^3-87a^4x^4-96a^5x^5+32a^6x^6\right)}{(-1+ax)^4(1+ax)}+201\log(1-ax)-9\log\right.}{64a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(7/2)*((2*(104 - 207*a*x - 59*a^2*x^2 + 309*a^3*x^3 - 8
7*a^4*x^4 - 96*a^5*x^5 + 32*a^6*x^6))/((-1 + a*x)^4*(1 + a*x)) + 201*Log[1
 - a*x] - 9*Log[1 + a*x]))/(64*a*(c - c/(a^2*x^2))^(7/2))
```

3.854.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^7}{(1-ax)^5(ax+1)^2}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^7}{(1-ax)^5(ax+1)^2}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int\left(\frac{9}{64a^7(ax+1)}-\frac{1}{32a^7(ax+1)^2}-\frac{1}{a^7}-\frac{201}{64a^7(ax-1)}-\frac{75}{16a^7(ax-1)^2}-\frac{59}{16a^7(ax-1)^3}-\frac{3}{2a^7(ax-1)^4}-\frac{1}{4a^7(ax-1)^5}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{75}{16a^8(1-ax)}+\frac{1}{32a^8(ax+1)}+\frac{59}{32a^8(1-ax)^2}-\frac{1}{2a^8(1-ax)^3}+\frac{1}{16a^8(1-ax)^4}-\frac{201\log(1-ax)}{64a^8}+\frac{9\log(ax+1)}{64a^8}-\frac{x}{a^7}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
-((a^7*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^7) + 1/(16*a^8*(1 - a*x)^4) - 1/(2*a^8
*(1 - a*x)^3) + 59/(32*a^8*(1 - a*x)^2) - 75/(16*a^8*(1 - a*x)) + 1/(32*a^
8*(1 + a*x)) - (201*Log[1 - a*x])/(64*a^8) + (9*Log[1 + a*x])/(64*a^8)))/(
c^3*Sqrt[c - c/(a^2*x^2)]))
```

3.854.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.854. $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.854.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{(ax-1)(ax+1)\left(-64a^6x^6+9\ln(ax+1)x^5a^5-201\ln(ax-1)x^5a^5+192a^5x^5-27\ln(ax+1)x^4a^4+603\ln(ax-1)x^4a^4+174a^4x^4+18a^3\right.}{}$

input

```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-1/64/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-64*a^6*x^6+9*ln(a*x+1)*x^5
*a^5-201*ln(a*x-1)*x^5*a^5+192*a^5*x^5-27*ln(a*x+1)*x^4*a^4+603*ln(a*x-1)*
x^4*a^4+174*a^4*x^4+18*a^3*ln(a*x+1)*x^3-402*a^3*ln(a*x-1)*x^3-618*a^3*x^3
+18*a^2*ln(a*x+1)*x^2-402*a^2*ln(a*x-1)*x^2+118*a^2*x^2-27*a*ln(a*x+1)*x+6
03*a*ln(a*x-1)*x+414*a*x+9*ln(a*x+1)-201*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^
2-1)/a^2/x^2)^(7/2)
```

3.854.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\left(64\,a^6x^6-192\,a^5x^5-174\,a^4x^4+618\,a^3x^3-118\,a^2x^2-414\,ax-9\left(a^5x^5-3\,a^4x^4+\right.\right.}{64\left(a^7c^4x^5-3\,a\right.}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

output
```
1/64*(64*a^6*x^6 - 192*a^5*x^5 - 174*a^4*x^4 + 618*a^3*x^3 - 118*a^2*x^2 -
 414*a*x - 9*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log
(a*x + 1) + 201*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*
log(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x
^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)
```

3.854.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

3.854.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.854.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.854.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.855 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

3.855.1 Optimal result

Integrand size $=24$, antiderivative size $=322$

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)+1/5*c^3*(c-c/a^2/
x^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)+3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4
/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-3
/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+3*c^3*(c-c/a^2/x^2)
^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(
1/2)-c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.855.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6a^7x^6}+\frac{1}{5a^6x^5}+\frac{3}{4a^5x^4}-\frac{1}{a^4x^3}-\frac{3}{2a^3x^2}+\frac{3}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

output
```
((c - c/(a^2*x^2))^(7/2)*(-1/6*1/(a^7*x^6) + 1/(5*a^6*x^5) + 3/(4*a^5*x^4)
 - 1/(a^4*x^3) - 3/(2*a^3*x^2) + 3/(a^2*x) + x - Log[x]/a))/(1 - 1/(a^2*x^
2))^(7/2)
```

3.855.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^4(ax+1)^3}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^7-\frac{a^6}{x}-\frac{3a^5}{x^2}+\frac{3a^4}{x^3}+\frac{3a^3}{x^4}-\frac{3a^2}{x^5}-\frac{a}{x^6}+\frac{1}{x^7}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow 2009$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(a^7x-a^6\log(x)+\frac{3a^5}{x}-\frac{3a^4}{2x^2}-\frac{a^3}{x^3}+\frac{3a^2}{4x^4}+\frac{a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

output
```
(c^3*Sqrt[c - c/(a^2*x^2)]*(-1/6*1/x^6 + a/(5*x^5) + (3*a^2)/(4*x^4) - a^3
/x^3 - (3*a^4)/(2*x^2) + (3*a^5)/x + a^7*x - a^6*Log[x]))/(a^7*Sqrt[1 - 1/
(a^2*x^2)])
```

3.855.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.855. $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

3.855.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,x\left(-60a^{7}x^{7}+60a^{6}\ln\left(x\right)x^{6}-180a^{5}x^{5}+90a^{4}x^{4}+60a^{3}x^{3}-45a^{2}x^{2}-12ax+10\right)}{60\left(ax-1\right)\left(a^{2}x^{2}-1\right)^{3}}$	112

input
```
int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-60*a^7*x^7
+60*a^6*ln(x)*x^6-180*a^5*x^5+90*a^4*x^4+60*a^3*x^3-45*a^2*x^2-12*a*x+10)/
(a*x-1)/(a^2*x^2-1)^3
```

3.855.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^{2}x^{2}}\right)^{7/2}dx=\frac{\left(60\,a^{7}c^{3}x^{7}-60\,a^{6}c^{3}x^{6}\log\left(x\right)+180\,a^{5}c^{3}x^{5}-90\,a^{4}c^{3}x^{4}-60\,a^{3}c^{3}x^{3}+45\,a^{2}c^{3}x^{2}+12\,ac^{3}x\right.}{60\,a^{8}x^{6}}$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
1/60*(60*a^7*c^3*x^7 - 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 - 90*a^4*c^
3*x^4 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 12*a*c^3*x - 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

3.855.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.855.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.855.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.855.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int \left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.856 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

3.856.1 Optimal result

Integrand size = 24, antiderivative size = 238

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)-1/3*c^2*(c-c/a^2/x
^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1
/a^2/x^2)^(1/2)+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*
(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1
-1/a^2/x^2)^(1/2)
```

3.856.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4a^5x^4}-\frac{1}{3a^4x^3}-\frac{1}{a^3x^2}+\frac{2}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

output
```
((c - c/(a^2*x^2))^(5/2)*(1/(4*a^5*x^4) - 1/(3*a^4*x^3) - 1/(a^3*x^2) + 2/
(a^2*x) + x - Log[x]/a))/(1 - 1/(a^2*x^2))^(5/2)
```

3.856.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^3(ax+1)^2}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^3(ax+1)^2}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \left(-a^5+\frac{a^4}{x}+\frac{2a^3}{x^2}-\frac{2a^2}{x^3}-\frac{a}{x^4}+\frac{1}{x^5}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(a^5(-x)+a^4\log(x)-\frac{2a^3}{x}+\frac{a^2}{x^2}+\frac{a}{3x^3}-\frac{1}{4x^4}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

output

```
-((c^2*Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 + a/(3*x^3) + a^2/x^2 - (2*a^3)/x
 - a^5*x + a^4*Log[x]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]))
```

3.856.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.856.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,x\left(-12a^{5}x^{5}+12\ln\left(x\right)x^{4}a^{4}-24a^{3}x^{3}+12a^{2}x^{2}+4ax-3\right)}{12\left(ax-1\right)\left(a^{2}x^{2}-1\right)^{2}}$	96

input

```
int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-12*a^5*x^5
+12*ln(x)*x^4*a^4-24*a^3*x^3+12*a^2*x^2+4*a*x-3)/(a*x-1)/(a^2*x^2-1)^2
```

3.856.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{(12\,a^5c^2x^5-12\,a^4c^2x^4\log{(x)}+24\,a^3c^2x^3-12\,a^2c^2x^2-4\,ac^2x+3\,c^2)\sqrt{a^2c}}{12\,a^6x^4}$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
1/12*(12*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 - 12*a^2*c^2
*x^2 - 4*a*c^2*x + 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

3.856.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.856.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.856.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.856.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.857 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

3.857.1 Optimal result

Integrand size = 24, antiderivative size = 147

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/
2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-c
*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.857.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.43

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{3}{2a}-\frac{1}{2a^3x^2}+\frac{1}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(-3/(2*a) - 1/(2*a^3*x^2) + 1/(a^2*x) + x - Log[x
]/a))/(1 - 1/(a^2*x^2))^(3/2)
```

3.857.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \textbf{6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2(ax+1)}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{84}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^3-\frac{a^2}{x}-\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(a^3x-a^2\log(x)+\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```

output
```
(c*Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + a/x + a^3*x - a^2*Log[x]))/(a^3*Sqr
t[1 - 1/(a^2*x^2)])
```

3.857.3.1 Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.857.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} \sqrt{\frac{a x -1}{a x +1}}\, x \left(-2 a^{3} x^{3}+2 a^{2} \ln \left(x \right) x^{2}-2 a x +1\right)}{2 \left(a x -1\right) \left(a^{2} x^{2}-1\right)}$	80

input
```
int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-2*a^3*x^3+2
*a^2*ln(x)*x^2-2*a*x+1)/(a*x-1)/(a^2*x^2-1)
```

3.857.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(2\,a^3cx^3-2\,a^2cx^2\log\left(x\right)+2\,acx-c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
1/2*(2*a^3*c*x^3 - 2*a^2*c*x^2*log(x) + 2*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

3.857.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.857.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.857.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.857.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.858 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.858.1 Optimal result

Integrand size $=24$, antiderivative size $=68$

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)
```

3.858.2 Mathematica [A] (verified)

Time $=0.03$ (sec) , antiderivative size $=39$, normalized size of antiderivative $=0.57$

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

3.858.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{1-ax}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1-ax}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x}-a\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(\log(x)-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-(a*x) + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.858.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.858.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{(-ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	52

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x
-1)
```

3.858.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-\log{(x)})}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - log(x))/a^2
```

3.858.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.858.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.858.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.858.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.859 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

3.859.1 Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)-ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
(c-c/a^2/x^2)^(1/2)
```

3.859.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x-\frac{\log(1+ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*(x - Log[1 + a*x]/a))/Sqrt[c - c/(a^2*x^2)]
```

3.859.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x}{ax+1}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{1}{a}-\frac{1}{a(ax+1)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{x}{a}-\frac{\log(ax+1)}{a^2}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(x/a - Log[1 + a*x]/a^2))/Sqrt[c - c/(a^2*x^2)]
```

3.859.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.859.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(-ax+\ln(ax+1))}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\,a^{2}}$	59

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*x+ln(a*x+1))/(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)/x/a^2
```

3.859.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{a^2c}(ax-\log{(ax+1)})}{a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - log(a*x + 1))/(a^2*c)
```

3.859.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))),
x)
```

3.859.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

3.859.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Bad
Argument Type
```

3.859.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2), x)
```

3.860 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

3.860.1 Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}} - \frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(a
*x+1)/(c-c/a^2/x^2)^(1/2)+1/4*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/
x^2)^(1/2)-5/4*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

3.860.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.41

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4x-\frac{2}{a+a^2x}+\frac{\log(1-ax)}{a}-\frac{5\log(1+ax)}{a}\right)}{4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*(4*x - 2/(a + a^2*x) + Log[1 - a*x]/a - (5*Log[1
+ a*x])/a))/(4*(c - c/(a^2*x^2))^(3/2))
```

3.860.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.49, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 6751$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 6747$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^3}{(1-ax)(ax+1)^2}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3}{(1-ax)(ax+1)^2}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 99$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{5}{4a^3(ax+1)}-\frac{1}{2a^3(ax+1)^2}-\frac{1}{a^3}-\frac{1}{4a^3(ax-1)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{2a^4(ax+1)}-\frac{\log(1-ax)}{4a^4}+\frac{5\log(ax+1)}{4a^4}-\frac{x}{a^3}\right)}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

3.860. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$

output
```
-((a^3*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^3) + 1/(2*a^4*(1 + a*x)) - Log[1 - a*x
]/(4*a^4) + (5*Log[1 + a*x])/(4*a^4)))/(c*Sqrt[c - c/(a^2*x^2)]))
```

3.860.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.860.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-4a^{2}x^{2}+5a\ln(ax+1)x-a\ln(ax-1)x-4ax+5\ln(ax+1)-\ln(ax-1)+2\right)(ax-1)}{4a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	103

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-4*a^2*x^2+5*a*ln(a*x+1)*x-a*ln(a*x-
1)*x-4*a*x+5*ln(a*x+1)-ln(a*x-1)+2)*(a*x-1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2
)^(3/2)
```

3.860.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.38

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{(4\,a^2x^2+4\,ax-5\,(ax+1)\log\left(ax+1\right)+(ax+1)\log\left(ax-1\right)-2)\sqrt{a^2c}}{4\,(a^3c^2x+a^2c^2)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas
")
```

output
```
1/4*(4*a^2*x^2 + 4*a*x - 5*(a*x + 1)*log(a*x + 1) + (a*x + 1)*log(a*x - 1)
 - 2)*sqrt(a^2*c)/(a^3*c^2*x + a^2*c^2)
```

3.860.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

3.860.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(3/2), x)
```

3.860.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.860.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2), x)
```

3.861 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.861.1 Optimal result

Integrand size = 24, antiderivative size = 263

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^
2/(-a*x+1)/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)^2/(c-
c/a^2/x^2)^(1/2)-(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+7/1
6*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-23/16*ln(a*x+1)
*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

3.861.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(2\left(8x+\frac{1}{a(1+ax)^2}+\frac{1}{a-a^2x}-\frac{8}{a+a^2x}\right)+\frac{7\log(1-ax)}{a}-\frac{23\log(1+ax)}{a}\right)}{16\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*(2*(8*x + 1/(a*(1 + a*x)^2) + (a - a^2*x)^(-1) -
8/(a + a^2*x)) + (7*Log[1 - a*x])/a - (23*Log[1 + a*x])/a))/(16*(c - c/(a^
2*x^2))^(5/2))
```

3.861.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^5}{(1-ax)^2(ax+1)^3}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{23}{16a^5(ax+1)}+\frac{1}{a^5(ax+1)^2}-\frac{1}{4a^5(ax+1)^3}+\frac{1}{a^5}+\frac{7}{16a^5(ax-1)}+\frac{1}{8a^5(ax-1)^2}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{8a^6(1-ax)}-\frac{1}{a^6(ax+1)}+\frac{1}{8a^6(ax+1)^2}+\frac{7\log(1-ax)}{16a^6}-\frac{23\log(ax+1)}{16a^6}+\frac{x}{a^5}\right)}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
(a^5*Sqrt[1 - 1/(a^2*x^2)]*(x/a^5 + 1/(8*a^6*(1 - a*x)) + 1/(8*a^6*(1 + a*
x)^2) - 1/(a^6*(1 + a*x)) + (7*Log[1 - a*x])/(16*a^6) - (23*Log[1 + a*x])/
(16*a^6)))/(c^2*Sqrt[c - c/(a^2*x^2)])
```

3.861.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.861.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)\left(-16a^4x^4+23a^3\ln(ax+1)x^3-7a^3\ln(ax-1)x^3-16a^3x^3+23a^2\ln(ax+1)x^2-7a^2\ln(ax-1)x^2+34a^2x^2-23a}{16a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-16*a^4*x^4+23*a^3*ln(a*x+1
)*x^3-7*a^3*ln(a*x-1)*x^3-16*a^3*x^3+23*a^2*ln(a*x+1)*x^2-7*a^2*ln(a*x-1)*
x^2+34*a^2*x^2-23*a*ln(a*x+1)*x+7*a*ln(a*x-1)*x+18*a*x-23*ln(a*x+1)+7*ln(a
*x-1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

3.861.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{(16\,a^4x^4+16\,a^3x^3-34\,a^2x^2-18\,ax-23\,(a^3x^3+a^2x^2-ax-1)\log\left(ax+1\right)+7\,(a^3}{16\,(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas
")
```

output
```
1/16*(16*a^4*x^4 + 16*a^3*x^3 - 34*a^2*x^2 - 18*a*x - 23*(a^3*x^3 + a^2*x^
2 - a*x - 1)*log(a*x + 1) + 7*(a^3*x^3 + a^2*x^2 - a*x - 1)*log(a*x - 1) +
 12)*sqrt(a^2*c)/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

3.861.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

3.861.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(5/2), x)
```

3.861.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.861.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2), x)
```

3.862 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.862.1 Optimal result

Integrand size = 24, antiderivative size = 358

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}\\ &+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}+\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}\\ &-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output

```
x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)-1/32*(1-1/a^2/x^2)^(1/2)/a/c
^3/(-a*x+1)^2/(c-c/a^2/x^2)^(1/2)+5/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/
(c-c/a^2/x^2)^(1/2)-1/24*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^3/(c-c/a^2/x^2)
^(1/2)+11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-3/2*(
1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+19/32*ln(-a*x+1)*(1-1
/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-51/32*ln(a*x+1)*(1-1/a^2/x^2)^(1
/2)/a/c^3/(c-c/a^2/x^2)^(1/2)
```

3.862.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96x-\frac{3}{a(-1+ax)^2}-\frac{4}{a(1+ax)^3}+\frac{33}{a(1+ax)^2}+\frac{30}{a-a^2x}-\frac{144}{a+a^2x}+\frac{57\log(1-ax)}{a}-15\right.}{96\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(7/2)*(96*x - 3/(a*(-1 + a*x)^2) - 4/(a*(1 + a*x)^3) +
33/(a*(1 + a*x)^2) + 30/(a - a^2*x) - 144/(a + a^2*x) + (57*Log[1 - a*x])/
a - (153*Log[1 + a*x])/a))/(96*(c - c/(a^2*x^2))^(7/2))
```

3.862.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^7}{(1-ax)^3(ax+1)^4}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^7}{(1-ax)^3(ax+1)^4}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int\left(\frac{51}{32a^7(ax+1)}-\frac{3}{2a^7(ax+1)^2}+\frac{11}{16a^7(ax+1)^3}-\frac{1}{8a^7(ax+1)^4}-\frac{1}{a^7}-\frac{19}{32a^7(ax-1)}-\frac{5}{16a^7(ax-1)^2}-\frac{1}{16a^7(ax-1)^3}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{5}{16a^8(1-ax)}+\frac{3}{2a^8(ax+1)}+\frac{1}{32a^8(1-ax)^2}-\frac{11}{32a^8(ax+1)^2}+\frac{1}{24a^8(ax+1)^3}-\frac{19\log(1-ax)}{32a^8}+\frac{51\log(ax+1)}{32a^8}-\frac{x}{a^7}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
-((a^7*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^7) + 1/(32*a^8*(1 - a*x)^2) - 5/(16*a^
8*(1 - a*x)) + 1/(24*a^8*(1 + a*x)^3) - 11/(32*a^8*(1 + a*x)^2) + 3/(2*a^8
*(1 + a*x)) - (19*Log[1 - a*x])/(32*a^8) + (51*Log[1 + a*x])/(32*a^8)))/(c
^3*Sqrt[c - c/(a^2*x^2)]))
```

3.862.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.862. $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.862.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)\left(-96a^6x^6+153\ln(ax+1)x^5a^5-57\ln(ax-1)x^5a^5-96a^5x^5+153\ln(ax+1)x^4a^4-57\ln(ax-1)x^4a^4+366a^4x\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-96*a^6*x^6+153*ln(a*x+1)*x
^5*a^5-57*ln(a*x-1)*x^5*a^5-96*a^5*x^5+153*ln(a*x+1)*x^4*a^4-57*ln(a*x-1)*
x^4*a^4+366*a^4*x^4-306*a^3*ln(a*x+1)*x^3+114*a^3*ln(a*x-1)*x^3+222*a^3*x^
3-306*a^2*ln(a*x+1)*x^2+114*a^2*ln(a*x-1)*x^2-338*a^2*x^2+153*a*ln(a*x+1)*
x-57*a*ln(a*x-1)*x-122*a*x+153*ln(a*x+1)-57*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*
x^2-1)/a^2/x^2)^(7/2)
```

3.862.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.56

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\left(96\,a^6x^6+96\,a^5x^5-366\,a^4x^4-222\,a^3x^3+338\,a^2x^2+122\,ax-153\,\left(a^5x^5+a^4x^4-2\right.\right.}{96\,\left(a^7c^4x^5+a^6c^4x\right.}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas
")
```

output
```
1/96*(96*a^6*x^6 + 96*a^5*x^5 - 366*a^4*x^4 - 222*a^3*x^3 + 338*a^2*x^2 +
122*a*x - 153*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*
x + 1) + 57*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x
- 1) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 + a^6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*
c^4*x^2 + a^3*c^4*x + a^2*c^4)
```

3.862.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

3.862.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(7/2), x)
```

3.862.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.862.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2), x)
```

3.863 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

3.863.1 Optimal result

Integrand size = 24, antiderivative size = 375

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx &= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}\\
&+\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}\\
&+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}-\frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}\\
&-\frac{2a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\arcsin(ax)}{(1-ax)^{7/2}(1+ax)^{7/2}}+\frac{25a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{16(1-ax)^{7/2}(1+ax)^{7/2}}\end{aligned}$$

output
```
7/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(a*x+1)^3+3/8*a^5*(c-c/a^2/x^2
)^(7/2)*x^6/(-a*x+1)^3/(a*x+1)^2-1/15*a*(c-c/a^2/x^2)^(7/2)*x^2/(a*x+1)-19
/16*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)+2/3*a^3*(c-c/a^2/x^2)^(
7/2)*x^4/(-a*x+1)^2/(a*x+1)-23/120*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a*x+1)/(a
*x+1)+1/6*(c-c/a^2/x^2)^(7/2)*x*(-a*x+1)/(a*x+1)-2*a^6*(c-c/a^2/x^2)^(7/2)
*x^7*arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)+25/16*a^6*(c-c/a^2/x^2)^(7/2
)*x^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2)
```

3.863.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.40

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-40+96ax+70a^2x^2-352a^3x^3+105a^4x^4+736a^5x^5+240a^6\right.}{240a^6x^5\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 + 96*a*x + 70*a^2*x^2
- 352*a^3*x^3 + 105*a^4*x^4 + 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*Arc
Tan[1/Sqrt[-1 + a^2*x^2]] - 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(2
40*a^6*x^5*Sqrt[-1 + a^2*x^2])
```

3.863.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.49, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6717, 6709, 570, 540, 27, 537, 25, 537, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2} e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\int \frac{\left(1-a^2x^2\right)^{9/2}}{x^7(ax+1)^2}dx}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\int \frac{(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow 540$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}\int\frac{a(12-5ax)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow 27$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\int\frac{(12-5ax)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow 537$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(\frac{1}{4}a^2\int-\frac{(48-25ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{(48-25ax)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow 25$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\int\frac{(48-25ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{(48-25ax)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow 537$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(-\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\left(\frac{1}{2}a^2\int-\frac{3(32-25ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(32-25ax)\left(1-a^2x^2\right)^{3/2}}{2x^3}\right)-\frac{(48-25ax)\left(1-a^2x^2\right)^{5/2}}{20x^5}\Bigg)-(1}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow 27$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(-\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\int\frac{(32-25ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(32-25ax)\left(1-a^2x^2\right)^{3/2}}{2x^3}\right)-\frac{(48-25ax)\left(1-a^2x^2\right)^{5/2}}{20x^5}\Bigg)-(1-}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow 536$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(-\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\left(\int\frac{-32xa^2-25a}{x\sqrt{1-a^2x^2}}dx-\frac{(25ax+32)\sqrt{1-a^2x^2}}{x}\right)-\frac{(32-25ax)\left(1-a^2x^2\right)^{3/2}}{2x^3}\Bigg)-\frac{(48-25ax)(1}{20x}}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow 538$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(-\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\left(-32a^2\int\frac{1}{\sqrt{1-a^2x^2}}dx-25a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(25ax+32)\sqrt{1-a^2x^2}}{x}\right)-\frac{(32-25ax)(}{2x}}{\left(1-a^2x^2\right)^{7/2}}$$

3.863. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

$$\Big\downarrow 223$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Big(-\frac{1}{6}a\Big(-\frac{1}{4}a^2\Big(-\frac{3}{2}a^2\Big(-25a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(25ax+32)}{x}-32a\arcsin(ax)\Big)-\frac{(32-25ax)(1-a}{2x^3}}{(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow 243$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Big(-\frac{1}{6}a\Big(-\frac{1}{4}a^2\Big(-\frac{3}{2}a^2\Big(-\frac{25}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(25ax+32)}{x}-32a\arcsin(ax)\Big)-\frac{(32-25ax)(1-}{2x^3}}{(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow 73$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Bigg(-\frac{1}{6}a\Bigg(-\frac{1}{4}a^2\Bigg(-\frac{3}{2}a^2\Bigg(\frac{25\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(25ax+32)}{x}-32a\arcsin(ax)\Bigg)-\frac{(32-25ax)(1-}{2x^3}}{(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow 221$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\Big(-\frac{1}{6}a\Big(-\frac{1}{4}a^2\Big(-\frac{3}{2}a^2\Big(25a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(25ax+32)}{x}-32a\arcsin(ax)\Big)-\frac{(32-25a}{}}{(1-a^2x^2)^{7/2}}$$

input
```
Int[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((c - c/(a^2*x^2))^(7/2)*x^7*(-1/6*(1 - a^2*x^2)^(7/2)/x^6 - (a*(-1/20*(
(48 - 25*a*x)*(1 - a^2*x^2)^(5/2))/x^5 - (a^2*(-1/2*((32 - 25*a*x)*(1 - a^
2*x^2)^(3/2))/x^3 - (3*a^2*(-(((32 + 25*a*x)*Sqrt[1 - a^2*x^2])/x) - 32*a*
ArcSin[a*x] + 25*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/4))/6))/(1 - a^2*x^2)^
(7/2))
```

3.863. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$

3.863.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

3.863. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.863.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\left(736a^7x^7+105a^6x^6-1088a^5x^5-35a^4x^4+448a^3x^3-110a^2x^2-96ax+40\right)c^3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{240x^5a^6\left(a^2x^2-1\right)}+\frac{\left(-\frac{2a^7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{25a^6\ln}{}\right.}{}$
default	$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}x\left(-2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^9cx^7+2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\,a^9x^5-375\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^8cx^6+4\right.}{}$

input
```
int((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/240*(736*a^7*x^7+105*a^6*x^6-1088*a^5*x^5-35*a^4*x^4+448*a^3*x^3-110*a^2
*x^2-96*a*x+40)/x^5*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(-2*
a^7*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+25/16*a^6/
(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)+a^6/c*(c*(a^2*x^2
-1))^(1/2))*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^
2-1))^(1/2)
```

3.863.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.17

$$\int e^{-2\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \left[\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + 375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-}{x^2}\right.}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2
 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(240*a^6
*c^3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^
3*x^2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1
/240*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/(a^2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*
x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 + 736*a^5*c^3*
x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40
*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

3.863.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.11 (sec) , antiderivative size = 1059, normalized size of antiderivative = 2.82

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Too large to display}$$

input

```
integrate((c-c/a**2/x**2)**(7/2)*(a*x-1)/(a*x+1),x)
```

output

```
c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1)
, (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*s
qrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**3*Piecewise((-a*sqrt
(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x*
*2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqr
t(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Pie
cewise((I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x*
*2))) - I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
 > 1), (-a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x)
, True))/a**2 + 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))*
*(3/2)/(3*c), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/
8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sq
rt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2)))
, 1/Abs(a**2*x**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*
x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) +
sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 - 2*c**3*Piecew
ise((2*a**3*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2
- 1)/(15*x**3) - sqrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) >
1), (2*I*a**3*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a...
```

3.863.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

input

```
integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output

```
integrate((a*x - 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

3.863.8 Giac [A] (verification not implemented)

Time = 46.81 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.50

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx =$$

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}-\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-120\sqrt{a^2}\right.$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x)/a^2 - 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*s
gn(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) - 1440*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^9*c^5*abs(a)*sgn(x) - 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)^8*a*c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*ab
s(a)*sgn(x) - 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(
x) + 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) - 6720*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) - 2976*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - c))*c^9*abs(a)*sgn(x) - 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

3.863.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

3.864 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

3.864.1 Optimal result

Integrand size = 24, antiderivative size = 293

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx&=-\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}\\&+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)}\\&+\frac{2a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\arcsin(ax)}{(1-ax)^{5/2}(1+ax)^{5/2}}-\frac{9a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8(1-ax)^{5/2}(1+ax)^{5/2}}\end{aligned}$$

output

```
-7/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(a*x+1)^2-1/6*a*(c-c/a^2/x^2)^
(5/2)*x^2/(a*x+1)+2*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)-7/24*a^
2*(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)/(a*x+1)+1/4*(c-c/a^2/x^2)^(5/2)*x*(-a*x
+1)/(a*x+1)+2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/(a*x+
1)^(5/2)-9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1
/2))/(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

3.864.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.46

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\left(6-16ax-3a^2x^2+64a^3x^3+24a^4x^4\right)+27a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a}}\right.\right.}{24a^4x^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 - 16*a*x - 3*a^2*x^2 + 6
4*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 48*a^4
*x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

3.864.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.50, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6717, 6709, 570, 540, 27, 537, 25, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2} e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\int \frac{\left(1-a^2x^2\right)^{7/2}}{x^5(ax+1)^2}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\int \frac{(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 540}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}\int\frac{a(8-3ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow\ 27$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\int\frac{(8-3ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow\ 537$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(\frac{1}{2}a^2\int-\frac{(16-9ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow\ 25$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\int\frac{(16-9ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow\ 536$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(\int\frac{-16xa^2-9a}{x\sqrt{1-a^2x^2}}dx-\frac{(9ax+16)\sqrt{1-a^2x^2}}{x}\right)-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow\ 538$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\Bigg(-\frac{1}{4}a\Bigg(-\frac{1}{2}a^2\left(-16a^2\int\frac{1}{\sqrt{1-a^2x^2}}dx-9a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(9ax+16)\sqrt{1-a^2x^2}}{x}\right)-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow\ 223$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\Bigg(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-9a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(9ax+16)}{x}-16a\arcsin(ax)\right)-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow\ 243$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\Bigg(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-\frac{9}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(9ax+16)}{x}-16a\arcsin(ax)\right)-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow\ 73$$

3.864. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(\frac{9\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(9ax+16)}{x}-16a\arcsin(ax)\right)-\frac{(16-9ax)(1-a^2x^2)^{3/2}}{6x^3}\right)}{(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow 221$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(9a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(9ax+16)}{x}-16a\arcsin(ax)\right)-\frac{(16-9ax)(1-a^2x^2)^{3/}}{6x^3}\right.\right.}{(1-a^2x^2)^{5/2}}$$

input
```
Int[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((c - c/(a^2*x^2))^(5/2)*x^5*(-1/4*(1 - a^2*x^2)^(5/2)/x^4 - (a*(-1/6*((
16 - 9*a*x)*(1 - a^2*x^2)^(3/2))/x^3 - (a^2*(-(((16 + 9*a*x)*Sqrt[1 - a^2*
x^2])/x) - 16*a*ArcSin[a*x] + 9*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/4))/(1
- a^2*x^2)^(5/2))
```

3.864.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

3.864. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.864.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.80

method	result
risch	$\frac{\left(64 a^{5} x^{5}-3 a^{4} x^{4}-80 a^{3} x^{3}+9 a^{2} x^{2}+16 a x -6\right) c^{2} \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{24 x^{3} a^{4} \left(a^{2} x^{2}-1\right)}+\frac{\left(-\frac{2 a^{5} \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{\sqrt{a^{2} c}}+\frac{9 a^{4} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right)}{8 \sqrt{-c}}+\frac{a^{4}}{}\right.}{a^{4} \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} x \left(-80 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{5}{2}} a^{7} c \,x^{5}+80 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{7}{2}} a^{7} x^{3}-48 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}\right)^{\frac{5}{2}} a^{6} c \,x^{4}-27\right.}{}$

input
```
int((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(64*a^5*x^5-3*a^4*x^4-80*a^3*x^3+9*a^2*x^2+16*a*x-6)/x^3*c^2/a^4*(c*(
a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(-2*a^5*ln(a^2*c*x/(a^2*c)^(1/2)+(a^
2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+9/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(
a^2*c*x^2-c)^(1/2))/x)+a^4/c*(c*(a^2*x^2-1))^(1/2))*c^2/a^4*(c*(a^2*x^2-1)
/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2-1))^(1/2)
```

3.864.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.34

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\left[\frac{96\,a^{3}\sqrt{-c}c^{2}x^{3}\arctan\left(\frac{a^{2}\sqrt{-c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+27\,a^{3}\sqrt{-c}c^{2}x^{3}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right.}{48\,a^{4}x^{3}}\right.$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) + 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 -
2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*c^2
*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(
2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24
*a^4*c^2*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((
a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

3.864.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.97 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.71

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)-\frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}+\frac{2c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^3}-\frac{c^2\left(\begin{cases}\frac{ia^3\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{ia^2\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3i\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{4a^2x^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a^3\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{a^2\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c}}{4a^2x^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{a^4}$$

input
```
integrate((c-c/a**2/x**2)**(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1)
, (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*s
qrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**2*Piecewise((-a*sqrt
(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x*
*2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqr
t(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + 2*c**2*P
iecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**
3 - c**2*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*
sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2)))
- I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1),
 (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2)
)) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqr
t(1 - 1/(a**2*x**2))), True))/a**4
```

3.864.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)
```

3.864.8 Giac [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.42

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=$$

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2}}{a}\right.$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output

```
-1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*s
gn(x)/a^2 - 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(
x)/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) + 96*(sqrt(a^2*c)*x - sqrt(a^2*
c*x^2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
5*c^4*abs(a)*sgn(x) + 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2
)*sgn(x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) +
160*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) + 64*a*c^(13/2)*sgn(x))/((
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

3.864.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x-1)}{a\,x+1}\,dx$$

input

```
int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1),x)
```

output

```
int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

3.865 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

3.865.1 Optimal result

Integrand size = 24, antiderivative size = 213

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx &= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}\\ &+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\arcsin(ax)}{(1-ax)^{3/2}(1+ax)^{3/2}}\\ &+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2(1-ax)^{3/2}(1+ax)^{3/2}}\end{aligned}$$

output
```
-a*(c-c/a^2/x^2)^(3/2)*x^2/(a*x+1)-5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1
)/(a*x+1)+1/2*(c-c/a^2/x^2)^(3/2)*x*(-a*x+1)/(a*x+1)-2*a^2*(c-c/a^2/x^2)^(
3/2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)+1/2*a^2*(c-c/a^2/x^2)^(3
/2)*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

3.865.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.54

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-1+4ax+2a^2x^2)+a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-4a^2x^2\log\left(ax+\right.\right.}{2a^2x\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 + 4*a*x + 2*a^2*x^2) + a^
2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2
]]))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

3.865.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6717, 6709, 570, 540, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\int \frac{\left(1-a^2x^2\right)^{5/2}}{x^3(ax+1)^2}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\int \frac{(1-ax)^2\sqrt{1-a^2x^2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}\int \frac{a(4-ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\int \frac{(4-ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{536}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(\int\frac{-4xa^2-a}{x\sqrt{1-a^2x^2}}dx-\frac{(ax+4)\sqrt{1-a^2x^2}}{x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{538}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(-4a^2\int\frac{1}{\sqrt{1-a^2x^2}}dx-a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(ax+4)\sqrt{1-a^2x^2}}{x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{223}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(-a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(ax+4)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{243}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(-\frac{1}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(ax+4)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(\frac{\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(ax+4)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(ax+4)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

input

```
Int[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output

```
-(((c - c/(a^2*x^2))^(3/2)*x^3*(-1/2*(1 - a^2*x^2)^(3/2)/x^2 - (a*(-(((4 +
 a*x)*Sqrt[1 - a^2*x^2])/x) - 4*a*ArcSin[a*x] + a*ArcTanh[Sqrt[1 - a^2*x^2
]]))/2))/(1 - a^2*x^2)^(3/2))
```

3.865. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$

3.865.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.865. $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.865.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

method	result
risch	$\frac{\left(2 a^{4} x^{4}+4 a^{3} x^{3}-3 a^{2} x^{2}-4 a x +1\right) c \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{2 x \,a^{2} \left(a^{2} x^{2}-1\right)}+\frac{\left(-\frac{2 a^{3} \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{\sqrt{a^{2} c}}+\frac{a^{2} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right)}{2 \sqrt{-c}}\right) c \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}{a^{2} \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} x \left(12 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{5} c \,x^{3}-12 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{5}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{5} x -4 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{4} c \,x^{2}+\right.}{}$

input
```
int((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^4*x^4+4*a^3*x^3-3*a^2*x^2-4*a*x+1)/x*c/a^2*(c*(a^2*x^2-1)/a^2/x^2
)^(1/2)/(a^2*x^2-1)+(-2*a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/
(a^2*c)^(1/2)+1/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2)
)/x))*c/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2
-1)
```

3.865.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\left[\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+a\sqrt{-c}cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(2}{4\,a^2x}\right.$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c)) + a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(2*a^2*c*x^2 + 4*a*c*x - c)*
sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c
)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2
*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a
^2*c*x^2 + 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

3.865.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.77

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log\left(ax\right)}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}+\frac{\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{a} & \text{for}\ |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)-\frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\,\mathrm{acosh}\left(ax\right)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for}\ |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\,\mathrm{asin}\left(ax\right)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}+\frac{c\left(\begin{cases}\frac{ia\sqrt{c}\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for}\ \frac{1}{|a^2x^2|}>1\\ -\frac{a\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{2}-\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{a^2}$$

input
```
integrate((c-c/a**2/x**2)**(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt
(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (
I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c*Piecewise((-a*sqrt(c)*x/
sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1
)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I
*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I
*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/
a**2
```

3.865.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int \frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

3.865.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=$$

$$-\Bigg(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(x}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^
2 - 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs
(a)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^3*c^2*abs(a)*sgn(x) + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c
^(5/2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) +
4*a*c^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*a
bs(a)))*abs(a)
```

3.865.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.866 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.866.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(
a*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.866.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]
] - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

3.866.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6709, 570, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2(1-2ax)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2(1-2ax)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1-2ax}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-2a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 223$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[1 - a^2*x^2] - 2*ArcSin[a*x] - ArcTanh[S
qrt[1 - a^2*x^2]]))/Sqrt[1 - a^2*x^2])
```

3.866.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.866. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.866. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.866.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^{2}}}\,a^{2}\sqrt{-\frac{c}{a^{2}}}-2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^{2}}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^{2}}}-\sqrt{-\frac{c}{a^{2}}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{2}-c\ln\left(\frac{2\sqrt{-\frac{c}{a^{2}}}}{}\right.\right.}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{2}\sqrt{-\frac{c}{a^{2}}}}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)-2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+c*x)/c^(1/2)
)*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^
(1/2)/a^2/(-c/a^2)^(1/2)
```

3.866.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^{2}x^{2}}}\,dx$$

$$=\left[\frac{2\,ax\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}+4\sqrt{-c}\arctan\left(\frac{a^{2}\sqrt{-c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a
^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a,
(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^
2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

3.866.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.866.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

3.866.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.867 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

3.867.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=-\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}-\frac{2\sqrt{1-ax}\sqrt{1+ax}\arcsin(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}$$

output
```
-(-a*x+1)^2/a^2/x/(c-c/a^2/x^2)^(1/2)-2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^
2)^(1/2)-2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1
/2)
```

3.867.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{-3+2ax+a^2x^2-2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(-3 + 2*a*x + a^2*x^2 - 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]]
)/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

3.867.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6717, 6709, 563, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{\sqrt{1-a^2x^2}\int \frac{x\sqrt{1-a^2x^2}}{(ax+1)^2}dx}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 563}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{\int \frac{2-ax}{\sqrt{1-a^2x^2}}dx}{a}+\frac{2\sqrt{1-a^2x^2}}{a^2(ax+1)}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 455}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}}{a}+\frac{2\sqrt{1-a^2x^2}}{a^2(ax+1)}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{\frac{\sqrt{1-a^2x^2}}{a}+\frac{2\arcsin(ax)}{a}}{a}+\frac{2\sqrt{1-a^2x^2}}{a^2(ax+1)}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*((2*Sqrt[1 - a^2*x^2])/(a^2*(1 + a*x)) + (Sqrt[1 - a^
2*x^2]/a + (2*ArcSin[a*x])/a)/a))/(Sqrt[c - c/(a^2*x^2)]*x))
```

3.867.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.867.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.38

method	result
risch	$\frac{a^{2} x^{2}-1}{a^{2} x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}+\frac{\left(-\frac{2 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{a \sqrt{a^{2} c}}+\frac{2 \sqrt{a^{2} c \left(x+\frac{1}{a}\right)^{2}-2 \left(x+\frac{1}{a}\right) a c}}{a^{3} c \left(x+\frac{1}{a}\right)}\right) \sqrt{c \left(a^{2} x^{2}-1\right)}}{x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}$
default	$-\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, \Bigg(-\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, \sqrt{c}\, a^{2} x +2 \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a c x -2 a \sqrt{\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}}\, \sqrt{c}-\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a \sqrt{c}+2 \ln \Bigg(\sqrt{c}\, x +}{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x \,c^{\frac{3}{2}} a \left(a x +1\right)}$

input

```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/a^2*(a^2*x^2-1)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a*ln(a^2*c*x/(a^2*c)
^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+2/a^3/c/(x+1/a)*(a^2*c*(x+1/a)^2
-2*(x+1/a)*a*c)^(1/2))*(c*(a^2*x^2-1))^(1/2)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/
2)
```

3.867.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \left[\frac{(ax+1)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(a^2x^2+3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx+ac}, \frac{2\,(ax+1)\sqrt{-c}\arctan\left(\right.}{}\right.$$

input

```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output

```
[((a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - c) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^2*c*x + a*c), (2*(a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/(a^2*c*x + a*c)]
```

3.867.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

input

```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
```

output

```
Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)), x)
```

3.867.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

3.867.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.867.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{a\,x-1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)), x)
```

3.868 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

3.868.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\arcsin(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

output
```
-1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x+2/3*(-a*x+1)^2*(a*x+1)*(2*a*x+5)
/a^4/(c-c/a^2/x^2)^(3/2)/x^3+2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^
4/(c-c/a^2/x^2)^(3/2)/x^3
```

3.868.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{-10-4ax+11a^2x^2+3a^3x^3-6(1+ax)\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{3a^2c\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
(-10 - 4*a*x + 11*a^2*x^2 + 3*a^3*x^3 - 6*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log
[a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
```

3.868.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6717, 6709, 570, 529, 25, 2166, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(ax+1)^2\sqrt{1-a^2x^2}}dx}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3(1-ax)^2}{(1-a^2x^2)^{5/2}}dx}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}-\frac{1}{3}\int -\frac{(1-ax)\left(\frac{3x^2}{a}-\frac{3x}{a^2}+\frac{2}{a^3}\right)}{(1-a^2x^2)^{3/2}}dx\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\int \frac{(1-ax)\left(\frac{3x^2}{a}-\frac{3x}{a^2}+\frac{2}{a^3}\right)}{(1-a^2x^2)^{3/2}}dx+\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(-\int \frac{3(2-ax)}{a^3\sqrt{1-a^2x^2}}dx-\frac{8(1-ax)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(-\frac{3\int\frac{2-ax}{\sqrt{1-a^2x^2}}dx}{a^3}-\frac{8(1-ax)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(-\frac{3\left(2\int\frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^3}-\frac{8(1-ax)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}+\frac{1}{3}\left(-\frac{8(1-ax)}{a^4\sqrt{1-a^2x^2}}-\frac{3\left(\frac{\sqrt{1-a^2x^2}}{a}+\frac{2\arcsin(ax)}{a}\right)}{a^3}\right)\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input

```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

output

```
-(((1 - a^2*x^2)^(3/2)*((1 - a*x)^2/(3*a^4*(1 - a^2*x^2)^(3/2)) + ((-8*(1
- a*x))/(a^4*Sqrt[1 - a^2*x^2]) - (3*(Sqrt[1 - a^2*x^2]/a + (2*ArcSin[a*x]
)/a))/a^3)/3))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

3.868.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455

```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.868. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.868. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

3.868.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.63

method	result
risch	$\frac{a^{2} x^{2}-1}{a^{2} c x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}+\frac{\left(-\frac{2 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{a^{3} \sqrt{a^{2} c}}-\frac{\sqrt{a^{2} c \left(x+\frac{1}{a}\right)^{2}-2 \left(x+\frac{1}{a}\right) a c}}{3 a^{6} c \left(x+\frac{1}{a}\right)^{2}}+\frac{8 \sqrt{a^{2} c \left(x+\frac{1}{a}\right)^{2}-2 \left(x+\frac{1}{a}\right) a c}}{3 a^{5} c \left(x+\frac{1}{a}\right)}\right) a^{2} \sqrt{c \left(a^{2} x^{2}-1\right)}}{c x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}$
default	$\frac{\left(3 c^{\frac{3}{2}} \sqrt{\frac{c (a x-1) (a x+1)}{a^{2}}}\, a^{3} x^{3}+15 x^{2} a^{2} c^{\frac{3}{2}} \sqrt{\frac{c (a x-1) (a x+1)}{a^{2}}}-4 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, c^{\frac{3}{2}} a^{2} x^{2}-6 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, \ln \left(\sqrt{c}\, x+\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) \sqrt{\frac{c(ax-}{}}}{3\sqrt{\frac{c(ax-1)}{a}}}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a^3*ln(a^2*c*x/(a^
2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-1/3/a^6/c/(x+1/a)^2*(a^2*c*(
x+1/a)^2-2*(x+1/a)*a*c)^(1/2)+8/3/a^5/c/(x+1/a)*(a^2*c*(x+1/a)^2-2*(x+1/a)
*a*c)^(1/2))*a^2/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

3.868.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.25

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \left[\frac{3\left(a^2x^2+2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3+14\,a^2x^2+\right.}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*x^2 + 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*
sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2), 1/3*(
6*(a^2*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((
a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)]
```

3.868.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)), x
)
```

3.868.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

3.868.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.868.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a\,x+1\right)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)), x)
```

3.869 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.869.1 Optimal result

Integrand size = 24, antiderivative size = 195

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=-\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}-\frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}+\frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}-\frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}-\frac{2(1-ax)^{5/2}(1+ax)^{5/2}\arcsin(ax)}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

output
```
-(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x-2/5*(-a*x+1)^3/a^3/(c-c/a^2/x^2)^(5/
2)/x^2+2/15*(-a*x+1)^3*(a*x+1)/a^4/(c-c/a^2/x^2)^(5/2)/x^3-2/15*(-a*x+1)^3
*(a*x+1)^2*(13*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5-2*(-a*x+1)^(5/2)*(a*x+1
)^(5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

3.869.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{-56-82ax+32a^2x^2+76a^3x^3+15a^4x^4-30(1+ax)^2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1\right.}}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
(-56 - 82*a*x + 32*a^2*x^2 + 76*a^3*x^3 + 15*a^4*x^4 - 30*(1 + a*x)^2*Sqrt
[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*
x^2)]*x*(1 + a*x)^2)
```

3.869.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6709, 570, 529, 25, 2166, 2345, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \ 6709$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(ax+1)^2(1-a^2x^2)^{3/2}}dx}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \ 570$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5(1-ax)^2}{(1-a^2x^2)^{7/2}}dx}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \ 529$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}-\frac{1}{5}\int -\frac{(1-ax)\left(\frac{5x^4}{a}-\frac{5x^3}{a^2}+\frac{5x^2}{a^3}-\frac{5x}{a^4}+\frac{2}{a^5}\right)}{(1-a^2x^2)^{5/2}}dx\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\int \frac{(1-ax)\left(\frac{5x^4}{a}-\frac{5x^3}{a^2}+\frac{5x^2}{a^3}-\frac{5x}{a^4}+\frac{2}{a^5}\right)}{(1-a^2x^2)^{5/2}}dx+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(-\frac{1}{3}\int\frac{-\frac{15x^3}{a^2}+\frac{30x^2}{a^3}-\frac{45x}{a^4}+\frac{16}{a^5}}{(1-a^2x^2)^{3/2}}dx-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int\frac{15(2-ax)}{a^5\sqrt{1-a^2x^2}}dx+\frac{2(30-23ax)}{a^6\sqrt{1-a^2x^2}}\right)-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{15\int\frac{2-ax}{\sqrt{1-a^2x^2}}dx}{a^5}+\frac{2(30-23ax)}{a^6\sqrt{1-a^2x^2}}\right)-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{15\left(2\int\frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^5}+\frac{2(30-23ax)}{a^6\sqrt{1-a^2x^2}}\right)-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}+\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(30-23ax)}{a^6\sqrt{1-a^2x^2}}+\frac{15\left(\frac{\sqrt{1-a^2x^2}}{a}+\frac{2\arcsin(ax)}{a}\right)}{a^5}\right)-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
-(((1 - a^2*x^2)^(5/2)*((1 - a*x)^2/(5*a^6*(1 - a^2*x^2)^(5/2)) + ((-22*(1
 - a*x))/(3*a^6*(1 - a^2*x^2)^(3/2)) + ((2*(30 - 23*a*x))/(a^6*Sqrt[1 - a^
2*x^2]) + (15*(Sqrt[1 - a^2*x^2]/a + (2*ArcSin[a*x])/a))/a^5)/3)/5))/((c -
 c/(a^2*x^2))^(5/2)*x^5))
```

3.869. $\int\frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.869.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

3.869. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.869.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.47

method	result
risch	$\frac{a^2x^2-1}{a^2c^2x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^5\sqrt{a^2c}}-\frac{41\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{60a^8c\left(x+\frac{1}{a}\right)^2}+\frac{383\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{120a^7c\left(x+\frac{1}{a}\right)}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\right.}}{8a^7c\left(x-\frac{1}{a}\right)}\right.}{c^2x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}$
default	$\frac{\left(15c^{\frac{5}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^5x^5+45x^4c^{\frac{5}{2}}a^4\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}+16c^{\frac{5}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4x^4-60c^{\frac{5}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^3x^3+16c^{\frac{5}{2}}\left(\right.\right.}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c^2/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a^5*ln(a^2*c*x/(
a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-41/60/a^8/c/(x+1/a)^2*(a^2
*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)+383/120/a^7/c/(x+1/a)*(a^2*c*(x+1/a)^2-2
*(x+1/a)*a*c)^(1/2)-1/8/a^7/c/(x-1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2
)+1/10/a^9/c/(x+1/a)^3*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2))*a^4/c^2/x/(c
*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

3.869.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.80

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \left[\frac{15\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(15\,a^5x^5}{15\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac\right.}\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/15*(15*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^
2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 + 76*a^4*
x^4 + 32*a^3*x^3 - 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3), 1/15*(30*(a^4*x^4 + 2*
a^3*x^3 - 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) + (15*a^5*x^5 + 76*a^4*x^4 + 32*a^3*x^3 - 82*
a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 + 2*a^4*c^
3*x^3 - 2*a^2*c^3*x - a*c^3)]
```

3.869.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x + 1)), x
)
```

3.869.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

3.869.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)), x)
```

3.870 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.870.1 Optimal result

Integrand size = 24, antiderivative size = 270

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}\\ &+\frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}+\frac{2(1-ax)^4(1+ax)^2}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}\\ &+\frac{2(1-ax)^4(1+ax)^3(72+37ax)}{35a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1-ax)^{7/2}(1+ax)^{7/2}\arcsin(ax)}{a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\end{aligned}$$

output
```
-1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x+10/3*(-a*x+1)^3/a^3/(c-c/a^2/x^2
)^(7/2)/x^2+12/7*(-a*x+1)^4/a^4/(c-c/a^2/x^2)^(7/2)/x^3+82/105*(-a*x+1)^4*
(a*x+1)/a^5/(c-c/a^2/x^2)^(7/2)/x^4+2/35*(-a*x+1)^4*(a*x+1)^2/a^6/(c-c/a^2
/x^2)^(7/2)/x^5+2/35*(-a*x+1)^4*(a*x+1)^3*(37*a*x+72)/a^8/(c-c/a^2/x^2)^(7
/2)/x^7+2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8/(c-c/a^2/x^2)^(7/2)
/x^7
```

3.870.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.49

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{432+654ax-636a^2x^2-1226a^3x^3+74a^4x^4+562a^5x^5+105a^6x^6-210(-1+ax)(}{105a^2\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)(c+acx)^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
(432 + 654*a*x - 636*a^2*x^2 - 1226*a^3*x^3 + 74*a^4*x^4 + 562*a^5*x^5 + 1
05*a^6*x^6 - 210*(-1 + a*x)*(1 + a*x)^3*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[
-1 + a^2*x^2]])/(105*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)*(c + a*c*x)^3)
```

3.870.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6709, 570, 529, 25, 2166, 2345, 2345, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(ax+1)^2\left(1-a^2x^2\right)^{5/2}}dx}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7(1-ax)^2}{\left(1-a^2x^2\right)^{9/2}}dx}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}-\frac{1}{7}\int -\frac{(1-ax)\left(\frac{7x^6}{a}-\frac{7x^5}{a^2}+\frac{7x^4}{a^3}-\frac{7x^3}{a^4}+\frac{7x^2}{a^5}-\frac{7x}{a^6}+\frac{2}{a^7}\right)}{(1-a^2x^2)^{7/2}}dx\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 25$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\int \frac{(1-ax)\left(\frac{7x^6}{a}-\frac{7x^5}{a^2}+\frac{7x^4}{a^3}-\frac{7x^3}{a^4}+\frac{7x^2}{a^5}-\frac{7x}{a^6}+\frac{2}{a^7}\right)}{(1-a^2x^2)^{7/2}}dx+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 2166$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(-\frac{1}{5}\int \frac{-\frac{35x^5}{a^2}+\frac{70x^4}{a^3}-\frac{105x^3}{a^4}+\frac{140x^2}{a^5}-\frac{175x}{a^6}+\frac{34}{a^7}}{(1-a^2x^2)^{5/2}}dx-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 2345$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{-\frac{105x^3}{a^4}+\frac{210x^2}{a^5}-\frac{420x}{a^6}+\frac{142}{a^7}}{(1-a^2x^2)^{3/2}}dx+\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 2345$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\int \frac{105(2-ax)}{a^7\sqrt{1-a^2x^2}}dx-\frac{525-352ax}{a^8\sqrt{1-a^2x^2}}\right)+\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 27$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{105\int \frac{2-ax}{\sqrt{1-a^2x^2}}dx}{a^7}-\frac{525-352ax}{a^8\sqrt{1-a^2x^2}}\right)+\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 455$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{105\left(2\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^7}-\frac{525-352ax}{a^8\sqrt{1-a^2x^2}}\right)+\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 223$$

3.870. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}+\frac{1}{7}\left(\frac{1}{5}\left(\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}+\frac{1}{3}\left(-\frac{525-352ax}{a^8\sqrt{1-a^2x^2}}-\frac{105\left(\frac{\sqrt{1-a^2x^2}}{a}+\frac{2\arcsin(ax)}{a}\right)}{a^7}\right)\right)-\frac{44(1-}{5a^8(1-a}\right.\right.}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
-(((1 - a^2*x^2)^(7/2)*((1 - a*x)^2/(7*a^8*(1 - a^2*x^2)^(7/2)) + ((-44*(1
 - a*x))/(5*a^8*(1 - a^2*x^2)^(5/2)) + ((315 - 244*a*x)/(3*a^8*(1 - a^2*x^
2)^(3/2)) + (-((525 - 352*a*x)/(a^8*Sqrt[1 - a^2*x^2])) - (105*(Sqrt[1 - a
^2*x^2]/a + (2*ArcSin[a*x])/a))/a^7)/3)/5)/7))/((c - c/(a^2*x^2))^(7/2)*x^
7))
```

3.870.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.870. $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.870.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.37

method	result
risch	$\frac{a^2x^2-1}{a^2c^3x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^7\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{28a^{12}c\left(x+\frac{1}{a}\right)^4}+\frac{39\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{140a^{11}c\left(x+\frac{1}{a}\right)^3}-\frac{1753\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2(}}{1680a^{10}c\left(x+\frac{1}{a}\right)}}{c^3x\sqrt{}}$
default	$-\frac{\left(-105c^{\frac{7}{2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}a^7x^7+96c^{\frac{7}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}a^6x^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}+96c^{\frac{7}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}a^5x^5+392}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c^3/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a^7*ln(a^2*c*x/(
a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-1/28/a^12/c/(x+1/a)^4*(a^2
*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)+39/140/a^11/c/(x+1/a)^3*(a^2*c*(x+1/a)^2
-2*(x+1/a)*a*c)^(1/2)-1753/1680/a^10/c/(x+1/a)^2*(a^2*c*(x+1/a)^2-2*(x+1/a
)*a*c)^(1/2)+3061/840/a^9/c/(x+1/a)*(a^2*c*(x+1/a)^2-2*(x+1/a)*a*c)^(1/2)-
1/48/a^10/c/(x-1/a)^2*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2)-7/24/a^9/c/(x-
1/a)*(a^2*c*(x-1/a)^2+2*(x-1/a)*a*c)^(1/2))*a^6/c^3/x/(c*(a^2*x^2-1)/a^2/x
^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

3.870.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.83

$$\int\frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\left[\frac{105\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{}\right.}{105\left(a^7c^4x^6+2\,a^6c^4x^5-\right.}\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

output
```
[1/105*(105*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - c) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4 - 636*
a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4
*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c
^4*x + a*c^4), 1/105*(210*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2
*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^2*c*x^2 - c)) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226
*a^4*x^4 - 636*a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)))/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^
4*x^2 + 2*a^2*c^4*x + a*c^4)]
```

3.870.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x + 1)), x
)
```

3.870.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

3.870.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.870.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)), x)
```

3.871 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

3.871.1 Optimal result

Integrand size = 24, antiderivative size = 322

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx &= -\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}\\ &\quad-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}\\ &\quad-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
-1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/x^8/(1-1/a^2/x^2)^(1/2)+3/7*c^4*(c-c/a^2/
x^2)^(1/2)/a^8/x^7/(1-1/a^2/x^2)^(1/2)-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/x^5
/(1-1/a^2/x^2)^(1/2)+3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/
2)+2*c^4*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-4*c^4*(c-c/a^2/x^
2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c^4*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^
2)^(1/2)-3*c^4*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.871.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(-\frac{1}{8a^9x^8}+\frac{3}{7a^8x^7}-\frac{8}{5a^6x^5}+\frac{3}{2a^5x^4}+\frac{2}{a^4x^3}-\frac{4}{a^3x^2}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^(9/2)*(-1/8*1/(a^9*x^8) + 3/(7*a^8*x^7) - 8/(5*a^6*x^5)
 + 3/(2*a^5*x^4) + 2/(a^4*x^3) - 4/(a^3*x^2) + x - (3*Log[x])/a))/(1 - 1/(
a^2*x^2))^(9/2)
```

3.871.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{9/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^6(ax+1)^3}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^9-\frac{3a^8}{x}+\frac{8a^6}{x^3}-\frac{6a^5}{x^4}-\frac{6a^4}{x^5}+\frac{8a^3}{x^6}-\frac{3a}{x^8}+\frac{1}{x^9}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\big\downarrow \text{2009}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(a^9x-3a^8\log(x)-\frac{4a^6}{x^2}+\frac{2a^5}{x^3}+\frac{3a^4}{2x^4}-\frac{8a^3}{5x^5}+\frac{3a}{7x^7}-\frac{1}{8x^8}\right)}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*Sqrt[c - c/(a^2*x^2)]*(-1/8*1/x^8 + (3*a)/(7*x^7) - (8*a^3)/(5*x^5) +
 (3*a^4)/(2*x^4) + (2*a^5)/x^3 - (4*a^6)/x^2 + a^9*x - 3*a^8*Log[x]))/(a^9
*Sqrt[1 - 1/(a^2*x^2)])
```

3.871.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.871. $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

3.871.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\left(-280 a^{9} x^{9}+840 a^{8} \ln \left(x \right) x^{8}+1120 a^{6} x^{6}-560 a^{5} x^{5}-420 a^{4} x^{4}+448 a^{3} x^{3}-120 a x +35\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{9}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{280 \left(a x -1\right)^{3} \left(a^{2} x^{2}-1\right)^{3}}$	112

input
```
int((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/280*(-280*a^9*x^9+840*a^8*ln(x)*x^8+1120*a^6*x^6-560*a^5*x^5-420*a^4*x^
4+448*a^3*x^3-120*a*x+35)*x*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1)
)^(3/2)/(a*x-1)^3/(a^2*x^2-1)^3
```

3.871.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\frac{(280\,a^9c^4x^9-840\,a^8c^4x^8\log(x)-1120\,a^6c^4x^6+560\,a^5c^4x^5+420\,a^4c^4x^4-448\,a^3c^4x^3+1}{280\,a^{10}x^8}$$

input
```
integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
1/280*(280*a^9*c^4*x^9 - 840*a^8*c^4*x^8*log(x) - 1120*a^6*c^4*x^6 + 560*a
^5*c^4*x^5 + 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqr
t(a^2*c)/(a^10*x^8)
```

3.871.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.871.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.871.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.871.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input

```
int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output

```
int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.872 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

3.872.1 Optimal result

Integrand size = 24, antiderivative size = 324

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx&=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}\\&\quad+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}\\&\quad-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)-3/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)+1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/
(1-1/a^2/x^2)^(1/2)+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2
)-5/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2
)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^
(1/2)-3*c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.872.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6a^7x^6}-\frac{3}{5a^6x^5}+\frac{1}{4a^5x^4}+\frac{5}{3a^4x^3}-\frac{5}{2a^3x^2}-\frac{1}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^(7/2)*(1/(6*a^7*x^6) - 3/(5*a^6*x^5) + 1/(4*a^5*x^4) +
5/(3*a^4*x^3) - 5/(2*a^3*x^2) - 1/(a^2*x) + x - (3*Log[x])/a))/(1 - 1/(a^2
*x^2))^(7/2)
```

3.872.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^5(ax+1)^2}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^5(ax+1)^2}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow 99$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int\left(-a^7+\frac{3a^6}{x}-\frac{a^5}{x^2}-\frac{5a^4}{x^3}+\frac{5a^3}{x^4}+\frac{a^2}{x^5}-\frac{3a}{x^6}+\frac{1}{x^7}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow 2009$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(a^7(-x)+3a^6\log(x)+\frac{a^5}{x}+\frac{5a^4}{2x^2}-\frac{5a^3}{3x^3}-\frac{a^2}{4x^4}+\frac{3a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^3*Sqrt[c - c/(a^2*x^2)]*(-1/6*1/x^6 + (3*a)/(5*x^5) - a^2/(4*x^4) - (
5*a^3)/(3*x^3) + (5*a^4)/(2*x^2) + a^5/x - a^7*x + 3*a^6*Log[x]))/(a^7*Sqr
t[1 - 1/(a^2*x^2)]))
```

3.872.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.872.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\left(-60 a^{7} x^{7}+180 a^{6} \ln \left(x \right) x^{6}+60 a^{5} x^{5}+150 a^{4} x^{4}-100 a^{3} x^{3}-15 a^{2} x^{2}+36 a x -10\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{60 \left(a x -1\right)^{3} \left(a^{2} x^{2}-1\right)^{2}}$	112

input
```
int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-60*a^7*x^7+180*a^6*ln(x)*x^6+60*a^5*x^5+150*a^4*x^4-100*a^3*x^3-15
*a^2*x^2+36*a*x-10)*x*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2
)/(a*x-1)^3/(a^2*x^2-1)^2
```

3.872.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{\left(60\,a^{7}c^{3}x^{7}-180\,a^{6}c^{3}x^{6}\log\left(x\right)-60\,a^{5}c^{3}x^{5}-150\,a^{4}c^{3}x^{4}+100\,a^{3}c^{3}x^{3}+15\,a^{2}c^{3}x^{2}-36\,ac^{3}}{60\,a^{8}x^{6}}$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
1/60*(60*a^7*c^3*x^7 - 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 - 150*a^4*c
^3*x^4 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2 - 36*a*c^3*x + 10*c^3)*sqrt(a^2*
c)/(a^8*x^6)
```

3.872.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.872.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.872.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.872.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.873 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

3.873.1 Optimal result

Integrand size $=24$, antiderivative size $=235$

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)
^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^
2/x^2)^(1/2)-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-
c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-3*c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-
1/a^2/x^2)^(1/2)
```

3.873.2 Mathematica [A] (verified)

Time $=0.07$ (sec) , antiderivative size $=79$, normalized size of antiderivative $=0.34$

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{5}{4a}-\frac{1}{4a^5x^4}+\frac{1}{a^4x^3}-\frac{1}{a^3x^2}-\frac{2}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^(5/2)*(-5/(4*a) - 1/(4*a^5*x^4) + 1/(a^4*x^3) - 1/(a^3*
x^2) - 2/(a^2*x) + x - (3*Log[x])/a))/(1 - 1/(a^2*x^2))^(5/2)
```

3.873.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2} e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^4(ax+1)}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{84}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^5-\frac{3a^4}{x}+\frac{2a^3}{x^2}+\frac{2a^2}{x^3}-\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(a^5x-3a^4\log(x)-\frac{2a^3}{x}-\frac{a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output

```
(c^2*Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 + a/x^3 - a^2/x^2 - (2*a^3)/x + a^5
*x - 3*a^4*Log[x]))/(a^5*Sqrt[1 - 1/(a^2*x^2)])
```

3.873.3.1 Defintions of rubi rules used

rule 84

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.873.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(-4 a^{5} x^{5}+12 \ln \left(x \right) x^{4} a^{4}+8 a^{3} x^{3}+4 a^{2} x^{2}-4 a x +1\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{4 \left(a x -1\right)^{3} \left(a^{2} x^{2}-1\right)}$	96

input

```
int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*a^5*x^5+12*ln(x)*x^4*a^4+8*a^3*x^3+4*a^2*x^2-4*a*x+1)*x*(c*(a^2*x
^2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a^2*x^2-1)
```

3.873.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{(4a^5c^2x^5-12a^4c^2x^4\log(x)-8a^3c^2x^3-4a^2c^2x^2+4ac^2x-c^2)\sqrt{a^2c}}{4a^6x^4}$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
1/4*(4*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 - 4*a^2*c^2*x^2
 + 4*a*c^2*x - c^2)*sqrt(a^2*c)/(a^6*x^4)
```

3.873.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.873.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.873.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.873.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.874 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$

3.874.1 Optimal result

Integrand size = 24, antiderivative size = 148

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2} - \frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x} + \frac{c\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-3*c*(c-c/a^2/x^2)^(1
/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-
3*c*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.874.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^3x^2}-\frac{3}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(1/(2*a^3*x^2) - 3/(a^2*x) + x - (3*Log[x])/a))/(
1 - 1/(a^2*x^2))^(3/2)
```

3.874.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \left(-a^3+\frac{3a^2}{x}-\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(a^3(-x)+3a^2\log(x)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c*Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + (3*a)/x - a^3*x + 3*a^2*Log[x]))/
(a^3*Sqrt[1 - 1/(a^2*x^2)]))
```

3.874.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.874.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{\left(-2 a^{3} x^{3}+6 a^{2} \ln \left(x \right) x^{2}+6 a x -1\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 \left(a x -1\right)^{3}}$	69

input
```
int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*a^3*x^3+6*a^2*ln(x)*x^2+6*a*x-1)*x*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*
((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3
```

3.874.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{(2\,a^3cx^3-6\,a^2cx^2\log{(x)}-6\,acx+c)\sqrt{a^2c}}{2\,a^4x^2}$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
1/2*(2*a^3*c*x^3 - 6*a^2*c*x^2*log(x) - 6*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

3.874.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.874.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.874.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.874.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.875 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.875.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)-4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.875.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

3.875.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a}{ax+1}+a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax-4\log(ax+1)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

3.875.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.875.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{\left(-ax+4\ln\left(ax+1\right)-\ln\left(x\right)\right)x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(ax-1\right)^{2}}$	67

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+4*ln(a*x+1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)
/(a*x+1))^(3/2)/(a*x-1)^2
```

3.875.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.23

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-4\log(ax+1)+\log(x))}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

3.875.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.875.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.875.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.875.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.876 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

3.876.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

output

```
x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)-2*(1-1/a^2/x^2)^(1/2)/a/(a*x+1)/
(c-c/a^2/x^2)^(1/2)-3*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2)
```

3.876.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x-\frac{2}{a(1+ax)}-\frac{3\log(1+ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input

```
Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

output

```
(Sqrt[1 - 1/(a^2*x^2)]*(x - 2/(a*(1 + a*x)) - (3*Log[1 + a*x])/a))/Sqrt[c
 - c/(a^2*x^2)]
```

3.876.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x(1-ax)}{(ax+1)^2}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x(1-ax)}{(ax+1)^2}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{3}{(ax+1)a}-\frac{2}{(ax+1)^2a}-\frac{1}{a}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{2}{a^2(ax+1)}+\frac{3\log(ax+1)}{a^2}-\frac{x}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
-((a*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a) + 2/(a^2*(1 + a*x)) + (3*Log[1 + a*x])/
a^2))/Sqrt[c - c/(a^2*x^2)])
```

3.876. $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$

3.876.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.876.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1)\left(-a^{2}x^{2}+3a\ln(ax+1)x-ax+3\ln(ax+1)+2\right)}{(ax-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\, x\, a^{2}}$	87

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)*(-a^2*x^2+3*a*ln(a*x+1)*x-a*x+3*l
n(a*x+1)+2)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

3.876.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\left(a^2x^2+ax-3\,(ax+1)\log\left(ax+1\right)-2\right)\sqrt{a^2c}}{a^3cx+a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas
")
```

output
```
(a^2*x^2 + a*x - 3*(a*x + 1)*log(a*x + 1) - 2)*sqrt(a^2*c)/(a^3*c*x + a^2*
c)
```

3.876.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.876.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a^2*x^2)), x)
```

3.876.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.876.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2), x)
```

3.877 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

3.877.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(a
*x+1)^2/(c-c/a^2/x^2)^(1/2)-3*(1-1/a^2/x^2)^(1/2)/a/c/(a*x+1)/(c-c/a^2/x^2
)^(1/2)-3*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

3.877.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.38

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x-\frac{5+6ax}{2a(1+ax)^2}-\frac{3\log(1+ax)}{a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*(x - (5 + 6*a*x)/(2*a*(1 + a*x)^2) - (3*Log[1 + a
*x])/a))/(c - c/(a^2*x^2))^(3/2)
```

3.877.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3}{(ax+1)^3}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{1}{a^3}-\frac{3}{a^3(ax+1)}+\frac{3}{a^3(ax+1)^2}-\frac{1}{a^3(ax+1)^3}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{3}{a^4(ax+1)}+\frac{1}{2a^4(ax+1)^2}-\frac{3\log(ax+1)}{a^4}+\frac{x}{a^3}\right)}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
(a^3*Sqrt[1 - 1/(a^2*x^2)]*(x/a^3 + 1/(2*a^4*(1 + a*x)^2) - 3/(a^4*(1 + a*
x)) - (3*Log[1 + a*x])/a^4))/(c*Sqrt[c - c/(a^2*x^2)])
```

3.877.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.877.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(-2a^{3}x^{3}+6a^{2}\ln\left(ax+1\right)x^{2}-4a^{2}x^{2}+12a\ln\left(ax+1\right)x+4ax+6\ln\left(ax+1\right)+5\right)}{2a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-2*a^3*x^3+6*a^2*ln(a*x+1)*x^2-4*a^2
*x^2+12*a*ln(a*x+1)*x+4*a*x+6*ln(a*x+1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)
^(3/2)
```

3.877.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(2\,a^3x^3+4\,a^2x^2-4\,ax-6\,\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)-5\right)\sqrt{a^2c}}{2\,\left(a^4c^2x^2+2\,a^3c^2x+a^2c^2\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas
")
```

output
```
1/2*(2*a^3*x^3 + 4*a^2*x^2 - 4*a*x - 6*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
- 5)*sqrt(a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

3.877.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

3.877.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

3.877.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.877.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

3.878 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.878.1 Optimal result

Integrand size = 24, antiderivative size = 264

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}\\&\quad+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}\\&\quad+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)-1/6*(1-1/a^2/x^2)^(1/2)/a/c^
2/(a*x+1)^3/(c-c/a^2/x^2)^(1/2)+9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)^2/(c
-c/a^2/x^2)^(1/2)-31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/
2)+1/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-49/16*ln(
a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

3.878.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48x-\frac{8}{a(1+ax)^3}+\frac{54}{a(1+ax)^2}-\frac{186}{a+a^2x}+\frac{3\log(1-ax)}{a}-\frac{147\log(1+ax)}{a}\right)}{48\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*(48*x - 8/(a*(1 + a*x)^3) + 54/(a*(1 + a*x)^2) -
186/(a + a^2*x) + (3*Log[1 - a*x])/a - (147*Log[1 + a*x])/a))/(48*(c - c/(
a^2*x^2))^(5/2))
```

3.878.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.43, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^5}{(1-ax)(ax+1)^4}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^5}{(1-ax)(ax+1)^4}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 99$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{49}{16a^5(ax+1)}-\frac{31}{8a^5(ax+1)^2}+\frac{9}{4a^5(ax+1)^3}-\frac{1}{2a^5(ax+1)^4}-\frac{1}{a^5}-\frac{1}{16a^5(ax-1)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{31}{8a^6(ax+1)}-\frac{9}{8a^6(ax+1)^2}+\frac{1}{6a^6(ax+1)^3}-\frac{\log(1-ax)}{16a^6}+\frac{49\log(ax+1)}{16a^6}-\frac{x}{a^5}\right)}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
-((a^5*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^5) + 1/(6*a^6*(1 + a*x)^3) - 9/(8*a^6*
(1 + a*x)^2) + 31/(8*a^6*(1 + a*x)) - Log[1 - a*x]/(16*a^6) + (49*Log[1 +
a*x])/(16*a^6)))/(c^2*Sqrt[c - c/(a^2*x^2)]))
```

3.878.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.878. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

3.878.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1)(ax-1)\left(-48a^4x^4+147a^3\ln(ax+1)x^3-3a^3\ln(ax-1)x^3-144a^3x^3+441a^2\ln(ax+1)x^2-9a^2\ln(ax-1)x^2+42a^2x^2}{48a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)*(-48*a^4*x^4+147*a^3*ln(a*x+
1)*x^3-3*a^3*ln(a*x-1)*x^3-144*a^3*x^3+441*a^2*ln(a*x+1)*x^2-9*a^2*ln(a*x-
1)*x^2+42*a^2*x^2+441*a*ln(a*x+1)*x-9*a*ln(a*x-1)*x+270*a*x+147*ln(a*x+1)-
3*ln(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

3.878.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(48\,a^4x^4+144\,a^3x^3-42\,a^2x^2-270\,ax-147\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\log\left(ax+1\right)}{48\left(a^5c^3x^3+3\,a^4c^3x^2+3\,a^3c^3x+\right.}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas
")
```

output
```
1/48*(48*a^4*x^4 + 144*a^3*x^3 - 42*a^2*x^2 - 270*a*x - 147*(a^3*x^3 + 3*a
^2*x^2 + 3*a*x + 1)*log(a*x + 1) + 3*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*log
(a*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x +
a^2*c^3)
```

3.878.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

3.878.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

3.878.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.878.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}\,dx$$

input

```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2),x)
```

output

```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

3.879 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.879.1 Optimal result

Integrand size = 24, antiderivative size = 357

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}\\ &+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^4}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}\\ &-\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c
^3/(-a*x+1)/(c-c/a^2/x^2)^(1/2)+1/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^4/(
c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^3/(c-c/a^2/x^2)^(
1/2)+59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-75/16*(
1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+9/64*ln(-a*x+1)*(1-1/
a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-201/64*ln(a*x+1)*(1-1/a^2/x^2)^(1
/2)/a/c^3/(c-c/a^2/x^2)^(1/2)
```

3.879.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(64x+\frac{208+478ax+74a^2x^2-490a^3x^3-302a^4x^4}{a(-1+ax)(1+ax)^4}+\frac{9\log(1-ax)}{a}-\frac{201\log(1+ax)}{a}\right)}{64\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input

```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

output

```
((1 - 1/(a^2*x^2))^(7/2)*(64*x + (208 + 478*a*x + 74*a^2*x^2 - 490*a^3*x^3
 - 302*a^4*x^4)/(a*(-1 + a*x)*(1 + a*x)^4) + (9*Log[1 - a*x])/a - (201*Log
[1 + a*x])/a))/(64*(c - c/(a^2*x^2))^(7/2))
```

3.879.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^7}{(1-ax)^2(ax+1)^5}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{201}{64a^7(ax+1)}+\frac{75}{16a^7(ax+1)^2}-\frac{59}{16a^7(ax+1)^3}+\frac{3}{2a^7(ax+1)^4}-\frac{1}{4a^7(ax+1)^5}+\frac{1}{a^7}+\frac{9}{64a^7(ax-1)}+\frac{1}{32a^7(ax-1)^2}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{32a^8(1-ax)}-\frac{75}{16a^8(ax+1)}+\frac{59}{32a^8(ax+1)^2}-\frac{1}{2a^8(ax+1)^3}+\frac{1}{16a^8(ax+1)^4}+\frac{9\log(1-ax)}{64a^8}-\frac{201\log(ax+1)}{64a^8}+\frac{x}{a^7}\right)}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
(a^7*Sqrt[1 - 1/(a^2*x^2)]*(x/a^7 + 1/(32*a^8*(1 - a*x)) + 1/(16*a^8*(1 +
a*x)^4) - 1/(2*a^8*(1 + a*x)^3) + 59/(32*a^8*(1 + a*x)^2) - 75/(16*a^8*(1
+ a*x)) + (9*Log[1 - a*x])/(64*a^8) - (201*Log[1 + a*x])/(64*a^8)))/(c^3*S
qrt[c - c/(a^2*x^2)])
```

3.879.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.879. $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

3.879.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)(ax-1)\left(-64a^{6}x^{6}+201\ln(ax+1)x^{5}a^{5}-9\ln(ax-1)x^{5}a^{5}-192a^{5}x^{5}+603\ln(ax+1)x^{4}a^{4}-27\ln(ax-1)x^{4}a^{4}+174a^{4}\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)*(-64*a^6*x^6+201*ln(a*x+1)*x
^5*a^5-9*ln(a*x-1)*x^5*a^5-192*a^5*x^5+603*ln(a*x+1)*x^4*a^4-27*ln(a*x-1)*
x^4*a^4+174*a^4*x^4+402*a^3*ln(a*x+1)*x^3-18*a^3*ln(a*x-1)*x^3+618*a^3*x^3
-402*a^2*ln(a*x+1)*x^2+18*a^2*ln(a*x-1)*x^2+118*a^2*x^2-603*a*ln(a*x+1)*x+
27*a*ln(a*x-1)*x-414*a*x-201*ln(a*x+1)+9*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^
2-1)/a^2/x^2)^(7/2)
```

3.879.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\left(64\,a^{6}x^{6}+192\,a^{5}x^{5}-174\,a^{4}x^{4}-618\,a^{3}x^{3}-118\,a^{2}x^{2}+414\,ax-201\,(a^{5}x^{5}+3\,a^{4}x^{4}\right.}{64\,(a^{7}c^{4}x^{5}+3\,a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas
")
```

output
```
1/64*(64*a^6*x^6 + 192*a^5*x^5 - 174*a^4*x^4 - 618*a^3*x^3 - 118*a^2*x^2 +
 414*a*x - 201*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*l
og(a*x + 1) + 9*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*
log(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x
^3 - 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

3.879.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

3.879.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima
")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

3.879.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.879.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

3.880 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx$

3.880.1 Optimal result

Integrand size = 25, antiderivative size = 80

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x^m*(c-c/a^2/x^2)^(1/2)/a/m/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(c-c/a^2/x^2)^(1/2
)/(1+m)/(1-1/a^2/x^2)^(1/2)
```

3.880.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x^m}{am}+\frac{x^{1+m}}{1+m}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x^m/(a*m) + x^(1 + m)/(1 + m)))/Sqrt[1 - 1/(a^2*x^
2)]
```

3.880.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6751, 6747, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-\frac{c}{a^2x^2}} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \int e^{\coth^{-1}(ax)} \sqrt{1-\frac{1}{a^2x^2}} x^m dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \int x^{m-1}(ax+1)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 53$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \int \left(x^{m-1}+ax^m\right) dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \left(\frac{ax^{m+1}}{m+1}+\frac{x^m}{m}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x^m/m + (a*x^(1 + m))/(1 + m)))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

3.880.3.1 Defintions of rubi rules used

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.880.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{x^{1+m}(amx+m+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{m(1+m)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	63
risch	$\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\, x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{(ax-1)(ax+1)}}\,(ax-1)(amx+m+1)x^{m}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{c}\,(a^{2}x^{2}-1)(1+m)m}$	101

input

```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERB
OSE)
```

output

```
x^(1+m)/m/(1+m)/(a*x+1)*(a*m*x+m+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/((a*x-1)
/(a*x+1))^(1/2)
```

3.880.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=-\frac{(amx^2+(m+1)x)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-(am^2+am)x+m}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

output
```
-(a*m*x^2 + (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/
(a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

3.880.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***m*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.880.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.55

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{\left(a\sqrt{c}mx+\sqrt{c}(m+1)\right)(ax+1)x^m}{(m^2+m)a^2x+(m^2+m)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

output
```
(a*sqrt(c)*m*x + sqrt(c)*(m + 1))*(a*x + 1)*x^m/((m^2 + m)*a^2*x + (m^2 +
m)*a)
```

3.880.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.880.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\int\frac{x^m\,\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x^m*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x^m*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.881 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

3.881.1 Optimal result

Integrand size $=25$, antiderivative size $=76$

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
1/2*x^2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

3.881.2 Mathematica [A] (verified)

Time $=0.03$ (sec) , antiderivative size $=45$, normalized size of antiderivative $=0.59$

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(3+2ax)}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output

```
(Sqrt[c - c/(a^2*x^2)]*x^2*(3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.881.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(ax+1)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(ax^2+x\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{ax^3}{3}+\frac{x^2}{2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x^2/2 + (a*x^3)/3))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.881.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.881.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^{3}(2ax+3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53
default	$\frac{x^{3}(2ax+3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/6*x^3*(2*a*x+3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^
(1/2)
```

3.881.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,ax^3+3\,x^2)\sqrt{a^2c}}{6\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

output
```
1/6*(2*a*x^3 + 3*x^2)*sqrt(a^2*c)/a^2
```

3.881.6 Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.881.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.881.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.881.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x+3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

input
```
int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x + 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(
a*x - 1))
```

3.882 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

3.882.1 Optimal result

Integrand size = 23, antiderivative size = 71

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)
```

3.882.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x}{a}+\frac{x^2}{2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x/a + x^2/2))/Sqrt[1 - 1/(a^2*x^2)]
```

3.882.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6751, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int (ax+1)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(2*a^2*Sqrt[1 - 1/(a^2*x^2)])
```

3.882.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.882.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{x^{2}(ax+2)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	52
default	$\frac{x^{2}(ax+2)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	52

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/2*x^2*(a*x+2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)
```

3.882.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^{2}x^{2}}}x\,dx=\frac{\sqrt{a^{2}c}(ax^{2}+2\,x)}{2\,a^{2}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

output
```
1/2*sqrt(a^2*c)*(a*x^2 + 2*x)/a^2
```

3.882.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1))
, x)
```

3.882.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.882.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.882.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,\left(a\,x-1\right)}$$

input
```
int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*
x - 1))
```

3.883 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.883.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)
```

3.883.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x + Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

3.883.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{ax+1}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.883.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.883.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{(ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)
```

3.883.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+\log(x))}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + log(x))/a^2
```

3.883.6 Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)),
x)
```

3.883.7 Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.883.8 Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.883.9 Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.884 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.884.1 Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)
```

3.884.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-(1/(a*x)) + Log[x]))/Sqrt[1 - 1/(a^2*x^2)]
```

3.884.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \; 6751$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 6747$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{ax+1}{x^2}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{a}{x}+\frac{1}{x^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(a\log(x)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.884.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.884.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{(a\ln(x)x-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOS
E)
```

output
```
(a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)
```

3.884.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{a^2c}(ax\log{(x)}-1)}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fr
icas")
```

output
```
sqrt(a^2*c)*(a*x*log(x) - 1)/(a^2*x)
```

3.884.6 Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1)
)), x)
```

3.884.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.884.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.884.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.885 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

3.885.1 Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

output
```
-1/2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)
```

3.885.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2ax^2}-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/2*1/(a*x^2) - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)]
```

3.885.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6751, 6747, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}}{x^2} \, dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \int \frac{ax+1}{x^3}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
-1/2*(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

3.885.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.885.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

method	result	size
gosper	$-\frac{(2ax+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53
default	$-\frac{(2ax+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/2*(2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/(a*x+1)/((a*x-1)/(a*x+1))^(
1/2)
```

3.885.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{a^2c}(2\,ax+1)}{2\,a^2x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
fricas")
```

output
```
-1/2*sqrt(a^2*c)*(2*a*x + 1)/(a^2*x^2)
```

3.885.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

output
```
Timed out
```

3.885.7 Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.885.8 Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.885.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\left(x\sqrt{c-\frac{c}{a^2\,x^2}}+\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

input

```
int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output

```
((x*(c - c/(a^2*x^2))^(1/2) + (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a
*x + 1))^(1/2))/(x/a - x^2)
```

3.886 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

3.886.1 Optimal result

Integrand size = 27, antiderivative size = 160

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx&=\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{24a^3}\\&\quad+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1+ax)^2}{4a^2}\\&\quad-\frac{7\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{8a^3\sqrt{1-ax}\sqrt{1+ax}}\end{aligned}$$

output
```
7/8*x*(c-c/a^2/x^2)^(1/2)/a^3+7/24*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3+1/6*x
*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3+1/4*x^2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a
^2-7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.886.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.58

$$\begin{aligned}&\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx\\&=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(32+21ax+16a^2x^2+6a^3x^3)+21\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^3\sqrt{-1+a^2x^2}}\end{aligned}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

output

```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6
*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

3.886.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6709, 541, 25, 27, 533, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x^2(ax+1)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int-\frac{a^2x^2(8ax+7)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int\frac{a^2x^2(8ax+7)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\int\frac{x^2(8ax+7)}{\sqrt{1-a^2x^2}}dx-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\int\frac{ax(21ax+16)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\int\frac{x(21ax+16)}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 533$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\frac{\int\frac{a(32ax+21)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\frac{\int\frac{32ax+21}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\frac{21\int\frac{1}{\sqrt{1-a^2x^2}}dx-\frac{32\sqrt{1-a^2x^2}}{a}}{2a}-\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{x\left(\frac{1}{4}\left(\frac{\frac{\frac{21\arcsin(ax)}{a}-\frac{32\sqrt{1-a^2x^2}}{a}}{2a}-\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input

```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/4*(x^3*Sqrt[1 - a^2*x^2]) + ((-8*x^2*Sqrt[1
 - a^2*x^2])/(3*a) + ((-21*x*Sqrt[1 - a^2*x^2])/(2*a) + ((-32*Sqrt[1 - a^2
*x^2])/a + (21*ArcSin[a*x])/a)/(2*a))/(3*a))/4))/Sqrt[1 - a^2*x^2])
```

3.886. $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

3.886.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.886. $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

3.886.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

method	result
risch	$\frac{\left(6 a^{3} x^{3}+16 a^{2} x^{2}+21 a x +32\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}{24 a^{3}}+\frac{7 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\,x^{2}-c}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{8 a^{2} \sqrt{a^{2} c}\, \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x \left(-6 x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{4}-16 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3}-27 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{2} c x +27 c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right)-48 c^{\frac{3}{2}} \ln \right.}{24 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, c\, a^{4}}$

input
```
int(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3+16*a^2*x^2+21*a*x+32)/a^3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+
7/8/a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^
2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.886.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.39

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\left[\frac{2\,(6\,a^4x^4+16\,a^3x^3+21\,a^2x^2+32\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},\ (6\,a^4x\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(2*(6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 + 32*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2
 + 32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

3.886.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x
)
```

3.886.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^3}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x - 1), x)
```

3.886.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\operatorname{sgn}(x)}{a^2}+\frac{8\operatorname{sgn}(x)}{a^3}\right)+\frac{21\operatorname{sgn}(x)}{a^4}\right)x+\frac{32\operatorname{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x\right.\right.}{a^4}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 + 8*sgn(x)/a^3) + 21*sgn
(x)/a^4)*x + 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2
*c*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) - 64*sqrt(-c
)*abs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

3.886.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.887 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

3.887.1 Optimal result

Integrand size = 27, antiderivative size = 123

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(a*x
+1)^2*(c-c/a^2/x^2)^(1/2)/a^2-x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x+
1)^(1/2)/(a*x+1)^(1/2)
```

3.887.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(5+3ax+a^2x^2)+3\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{3a^2\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 + 3*a*x + a^2*x^2) + 3*Log
[a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

3.887.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6709, 541, 25, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(ax+1)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2x(6ax+5)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2x(6ax+5)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\int \frac{x(6ax+5)}{\sqrt{1-a^2x^2}}dx-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\left(\frac{\int \frac{2a(5ax+3)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3x\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\left(\frac{\int\frac{5ax+3}{\sqrt{1-a^2x^2}}dx}{a}-\frac{3x\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\left(\frac{3\int\frac{1}{\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{a}}{a}-\frac{3x\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{x\left(\frac{1}{3}\left(\frac{\frac{3\arcsin(ax)}{a}-\frac{5\sqrt{1-a^2x^2}}{a}}{a}-\frac{3x\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input

```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/3*(x^2*Sqrt[1 - a^2*x^2]) + ((-3*x*Sqrt[1 -
 a^2*x^2])/a + ((-5*Sqrt[1 - a^2*x^2])/a + (3*ArcSin[a*x])/a)/a)/3))/Sqrt[
1 - a^2*x^2])
```

3.887.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455

```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.887.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01

method	result
risch	$\frac{\left(a^{2} x^{2}+3 a x +5\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}{3 a^{2}}+\frac{\ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right) x \sqrt{c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{a \sqrt{a^{2} c}\, \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x \left(-\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3}-3 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{2} c x +3 c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right)-6 c^{\frac{3}{2}} \ln \left(\frac{\sqrt{c}\, \sqrt{\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}}+c x}{\sqrt{c}}\right)-}{3 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c}$

input
```
int(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a^2*x^2+3*a*x+5)/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+1/a*ln(a^2*c*x/(
a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)*(c
*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)
```

3.887.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.66

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$

$$=\left[\frac{2\left(a^3x^3+3a^2x^2+5ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6a^3},\ (a^3x^3+3a^2x^2+5ax\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(2*(a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*
sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
) - c))/a^3, 1/3*((a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
/(a^2*c*x^2 - c)))/a^3]
```

3.887.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x
)
```

3.887.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^2}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x - 1), x)
```

3.887.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}+\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 + 3*sgn(x)/a^3) + 5*sgn(x)/a^4
) - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*a
bs(a)) + (3*a*sqrt(c)*log(abs(c)) - 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)
))*abs(a)
```

3.887.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.888 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

3.888.1 Optimal result

Integrand size = 25, antiderivative size = 98

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx\\ =\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}\end{aligned}$$

output
```
3/2*x*(c-c/a^2/x^2)^(1/2)/a+1/2*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a-3/2*x*arcs
in(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.888.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 -
a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

3.888.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6717, 6709, 469, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6717$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}xdx$$

$$\Big\downarrow \; 6709$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 469$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3}{2}\int \frac{ax+1}{\sqrt{1-a^2x^2}}dx-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{x\left(\frac{3}{2}\left(\frac{\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/2*((1 + a*x)*Sqrt[1 - a^2*x^2])/a + (3*(-(S
qrt[1 - a^2*x^2]/a) + ArcSin[a*x]/a))/2))/Sqrt[1 - a^2*x^2])
```

3.888.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.888.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

method	result
risch	$\frac{(ax+4)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x}{2a}+\frac{3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2+\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-4\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)-4\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a\right)}{2\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2}$

input
```
int(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+4)/a*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+3/2*ln(a^2*c*x/(a^2*c)^(1/2)
+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*
x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.888.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$= \left[\frac{2\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2}, \frac{\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-}}{2\,a}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(
2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2,
 1/2*((a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arcta
n(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

3.888.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.888.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x/(a*x - 1), x)
```

3.888.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx\\&=\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\operatorname{sgn}(x)}{a^2}+\frac{4\operatorname{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^2|a|}+\frac{(3\,a\sqrt{c}\log\left(|c|\right)}{}\right.\end{aligned}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 + 4*sgn(x)/a^3) - 6*sqrt(c)*log(a
bs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(
c)*log(abs(c)) - 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

3.888.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.889 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.889.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(
a*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.889.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]
] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

3.889.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6709, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2(2ax+1)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2(2ax+1)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{2ax+1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(2a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{221}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

input

```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[1 - a^2*x^2] + 2*ArcSin[a*x] - ArcTanh[S
qrt[1 - a^2*x^2]]))/Sqrt[1 - a^2*x^2])
```

3.889.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.889. $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.889.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\, x \left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\, a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\, a^2-c\ln\left(\frac{2\sqrt{-\frac{c}{a^2}}}{}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\, a^2\sqrt{-\frac{c}{a^2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+c*x)/c^(1/2)
)*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^
(1/2)/a^2/(-c/a^2)^(1/2)
```

3.889.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.30

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a
^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a,
(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^
2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

3.889.6 Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.889.7 Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

3.889.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.889.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.890 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.890.1 Optimal result

Integrand size = 27, antiderivative size = 117

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\,\mathrm{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
(c-c/a^2/x^2)^(1/2)-a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*
x+1)^(1/2)+2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)
/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.890.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}-2ax\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+ax\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] - 2*a*x*ArcTan[1/Sqrt[-1 + a^2*
x^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

3.890.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6709, 540, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^2\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\int -\frac{a(ax+2)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{a(ax+2)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\int \frac{ax+2}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(a\int \frac{1}{\sqrt{1-a^2x^2}}dx+2\int \frac{1}{x\sqrt{1-a^2x^2}}dx\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(2\int \frac{1}{x\sqrt{1-a^2x^2}}dx+\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2+\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(\arcsin(ax)-\frac{2\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(\arcsin(ax)-2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-(Sqrt[1 - a^2*x^2]/x) + a*(ArcSin[a*x] - 2*Ar
cTanh[Sqrt[1 - a^2*x^2]])))/Sqrt[1 - a^2*x^2])
```

3.890.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.890. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.890.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24

method	result
risch	$\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}+\frac{\left(\frac{a^{2}\ln\left(\frac{a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-c}\right)}{\sqrt{a^{2}c}}-\frac{2a\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^{2}c\,x^{2}-c}}{x}\right)}{\sqrt{-c}}\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{c(a^{2}x^{2}-1)}\,x}{a^{2}x^{2}-1}$
default	$-\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(-\sqrt{-\frac{c}{a^{2}}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{3}c\,x^{2}+a^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^{2}}}+c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)\sqrt{-\frac{c}{a^{2}}}\,ax-2c^{\frac{3}{2}}\sqrt{-\frac{c}{a^{2}}}\,\ln\right.}{a\sqrt{\frac{c\left(a^{2}\right.}{}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^
(1/2))/(a^2*c)^(1/2)-2*a/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1
/2))/x))*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.890.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.15

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x}\,dx = \Bigg[&-\sqrt{-c}\arctan\left(\frac{a^{2}\sqrt{-c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)\\ &+\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)\\ &+\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}},-2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)\\ &+\frac{1}{2}\sqrt{c}\log\left(2\,a^{2}cx^{2}+2\,a^{2}\sqrt{c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)\\ &+\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}\Bigg]\end{aligned}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*
x^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), -2*sqrt(c)*arct
an(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt
(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
c) + sqrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

3.890.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x*(a*x - 1)), x)
```

3.890.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x), x)
```

3.890.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$$
$$=\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}-\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}+\frac{2\,c^{\frac{3}{2}}\mathrm{s}}{\left(\left(\sqrt{a^2}cx-\sqrt{a^2}\right.\right.}\right.$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="giac")
```

output

```
(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
 - sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) +
2*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*a
bs(a)
```

3.890.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x\,\left(a\,x-1\right)}\,dx$$

input

```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

output

```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

3.891 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

3.891.1 Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
3/2*a*(c-c/a^2/x^2)^(1/2)+1/2*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x+3/2*a^2*x*arct
anh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+
1)^(1/2)
```

3.891.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left((1+4ax)\sqrt{-1+a^2x^2}-3a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{2x\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((1 + 4*a*x)*Sqrt[-1 + a^2*x^2] - 3*a^2*x^2*ArcTan[
1/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

3.891.3 Rubi [A] (verified)

Time $=0.60$ (sec) , antiderivative size $=97$, normalized size of antiderivative $=0.87$, number of steps used $=10$, number of rules used $=9$, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used $=\{6717, 6709, 540, 25, 27, 534, 243, 73, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \color{blue}{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}dx$$

$$\Big\downarrow \color{blue}{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^3\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}\int -\frac{a(3ax+4)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{a(3ax+4)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}a\int \frac{3ax+4}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{534}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}a\left(3a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}a\left(\frac{3}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \color{blue}{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}a\left(-\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$-\frac{x\left(\frac{1}{2}a\left(-3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/2*Sqrt[1 - a^2*x^2]/x^2 + (a*((-4*Sqrt[1 -
a^2*x^2])/x - 3*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/Sqrt[1 - a^2*x^2])
```

3.891.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.891. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.891.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.28

method	result
risch	$\frac{\left(4 a^{3} x^{3}+a^{2} x^{2}-4 a x -1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{2 x \left(a^{2} x^{2}-1\right)}-\frac{3 a^{2} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{2 \sqrt{-c}\, \left(a^{2} x^{2}-1\right)}$
default	$-\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \Bigg(-4 \sqrt{-\frac{c}{a^{2}}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c \,x^{3}+4 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3} x +4 \sqrt{-\frac{c}{a^{2}}}\, c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a \,x^{2}-4 \sqrt{-}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

3.891. $\int \frac{e^{2 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2x^2}}}{x^2} \, dx$

output
```
1/2*(4*a^3*x^3+a^2*x^2-4*a*x-1)/x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1
)-3/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2
*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.891.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.59

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x}, -\frac{3\,a\sqrt{c}x\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)) - 2*c)/x^2) + 2*(4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x
, -1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*c*x^2 - c)) - (4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

3.891.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^2\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)),
 x)
```

3.891. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

3.891.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^2}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^2), x)
```

3.891.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(91) = 182$.

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= \left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x) - \frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3 ac\operatorname{sgn}(x) - 4\left(\sqrt{a^2c}x-\sqrt{a^2}\right.}{\left(\left(\sqrt{a^2}\right.\right.}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
(3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
 ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) - 4*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^2*sgn(x) - 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

3.891.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^2\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

3.892 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

3.892.1 Optimal result

Integrand size = 27, antiderivative size = 137

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{3x^2}+\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
a^2*(c-c/a^2/x^2)^(1/2)+1/3*a*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x+1/3*(a*x+1)^2*
(c-c/a^2/x^2)^(1/2)/x^2+a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a
^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.892.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(1+3ax+5a^2x^2)-3a^3x^3\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{3x^2\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 + 3*a*x + 5*a^2*x^2) - 3*a^3
*x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

3.892.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6709, 540, 25, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^4\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}\int -\frac{a(5ax+6)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\int \frac{a(5ax+6)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\int \frac{5ax+6}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(-\frac{1}{2}\int -\frac{2a(3ax+5)}{x^2\sqrt{1-a^2x^2}}dx-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(a\int\frac{3ax+5}{x^2\sqrt{1-a^2x^2}}dx-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 534$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(a\left(3a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(a\left(\frac{3}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(a\left(-\frac{3\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 221$$

$$-\frac{x\left(\frac{1}{3}a\left(a\left(-3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/3*Sqrt[1 - a^2*x^2]/x^3 + (a*((-3*Sqrt[1 -
a^2*x^2])/x^2 + a*((-5*Sqrt[1 - a^2*x^2])/x - 3*a*ArcTanh[Sqrt[1 - a^2*x^2
]])))/3))/Sqrt[1 - a^2*x^2])
```

3.892.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

---

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0]
```

3.892. $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.892.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.10

method	result
risch	$\frac{\left(5 a^{4} x^{4}+3 a^{3} x^{3}-4 a^{2} x^{2}-3 a x -1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{3 x^{2} \left(a^{2} x^{2}-1\right)}-\frac{a^{3} \ln\left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{\sqrt{-c}\, \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, a \left(-6 \sqrt{-\frac{c}{a^{2}}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c \,x^{4}+6 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3} x^{2}+6 \ln\left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) \sqrt{-\frac{c}{a^{2}}}\, c^{\frac{3}{2}} a \,x^{3}-6 \sqrt{\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(5*a^4*x^4+3*a^3*x^3-4*a^2*x^2-3*a*x-1)/x^2*(c*(a^2*x^2-1)/a^2/x^2)^(1
/2)/(a^2*x^2-1)-a^3/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/
x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.892.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.47

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \left[\frac{3\,a^{2}\sqrt{-c}x^{2}\log\left(-\frac{a^{2}cx^{2}+2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)+2\,(5\,a^{2}x^{2}+3\,ax+1)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{6\,x^{2}},\ -\frac{3\,a^{2}\sqrt{c}x^{2}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)-(5\,a^{2}x^{2}+3\,ax+1)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{3\,x^{2}}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output

```
[1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
- c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/x^2, -1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c
*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/x^2]
```

3.892.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^3\,(ax-1)}\,dx$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

output

```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)),
 x)
```

3.892.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^3}\,dx$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output

```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^3), x)
```

3.892.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.69

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$

$$=\frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5ac\,\mathrm{sgn}(x)-3\left(\sqrt{a^2c}x-\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) - 3*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) - 12*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))*a*c^3*sgn(x) - 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

3.892.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x^3\,\left(a\,x-1\right)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

3.893 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

3.893.1 Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\,\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8\sqrt{1-ax}\sqrt{1+ax}}$$

output

```
4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3+2/3*a*(c-c/a^2/x^2
)^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x+7/8*a^4*x*arctanh((-a*x+1)^(1/2)
*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.893.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(6+16ax+21a^2x^2+32a^3x^3)-21a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{24x^3\sqrt{-1+a^2x^2}}$$

input

```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a
^3*x^3) - 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*
x^2])
```

3.893.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6717, 6709, 540, 25, 27, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^5\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}\int -\frac{a(7ax+8)}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\int \frac{a(7ax+8)}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\int \frac{7ax+8}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(-\frac{1}{3}\int -\frac{a(16ax+21)}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}\int \frac{a(16ax+21)}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\int \frac{16ax+21}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 539$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{1}{2}\int -\frac{a(21ax+32)}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}\int \frac{a(21ax+32)}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\int \frac{21ax+32}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 534$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(21a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(\frac{21}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{21\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 221$$

3.893. $\displaystyle\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

$$-\frac{x\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(-21a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/4*Sqrt[1 - a^2*x^2]/x^4 + (a*((-8*Sqrt[1 -
a^2*x^2])/(3*x^3) + (a*((-21*Sqrt[1 - a^2*x^2])/(2*x^2) + (a*((-32*Sqrt[1
- a^2*x^2])/x - 21*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/3))/4))/Sqrt[1 - a^2
*x^2])
```

3.893.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.893.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02

method	result
risch	$\frac{\left(32 a^{5} x^{5}+21 a^{4} x^{4}-16 a^{3} x^{3}-15 a^{2} x^{2}-16 a x -6\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{24 x^{3} \left(a^{2} x^{2}-1\right)}-\frac{7 a^{4} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{8 \sqrt{-c}\, \left(a^{2} x^{2}-1\right)}$
default	$-\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, a^{2} \Bigg(-48 \sqrt{-\frac{c}{a^{2}}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c \,x^{5}+48 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3} x^{3}+48 \sqrt{-\frac{c}{a^{2}}}\, c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a \,x^{4}-$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/24*(32*a^5*x^5+21*a^4*x^4-16*a^3*x^3-15*a^2*x^2-16*a*x-6)/x^3*(c*(a^2*x^
2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-7/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(
a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
/(a^2*x^2-1)*x
```

3.893.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.39

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \left[\frac{21\,a^3\sqrt{-c}x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},\; -\frac{21\,a^3\sqrt{c}x^3\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{24\,x^3}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sq
rt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (32*a^3*x^3 + 2
1*a^2*x^2 + 16*a*x + 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

3.893.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^4\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)),
 x)
```

3.893.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^4}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^4), x)
```

3.893.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(128) = 256.

Time = 2.08 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.03

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$
$$= \frac{1}{12}\left(21\,a^2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x) - \frac{21\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7 a^2c\,\mathrm{sgn}(x) + 45\left(\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) - 96*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*a^2*c^3*sgn(x) - 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*
a^2*c^4*sgn(x) - 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - c))^2 + c)^4)*abs(a)
```

3.893.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^4\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

3.894 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

3.894.1 Optimal result

Integrand size = 27, antiderivative size = 181

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{4\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4+1/2*a*(c-c/a^2/x^2
)^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2+3/4*a^3*(c-c/a^2/x^2)^(1/2)/x+
3/4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+
1)^(1/2)/(a*x+1)^(1/2)
```

3.894.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(4+10ax+12a^2x^2+15a^3x^3+24a^4x^4)-15a^5x^5\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{20x^4\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 + 10*a*x + 12*a^2*x^2 + 15*a
^3*x^3 + 24*a^4*x^4) - 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*S
qrt[-1 + a^2*x^2])
```

3.894.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {6717, 6709, 540, 25, 27, 539, 27, 539, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\,e^{2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^6\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}\int -\frac{a(9ax+10)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\int \frac{a(9ax+10)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\int \frac{9ax+10}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(-\frac{1}{4}\int -\frac{6a(5ax+6)}{x^4\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\int\frac{5ax+6}{x^4\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 539$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(-\frac{1}{3}\int-\frac{3a(4ax+5)}{x^3\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\int\frac{4ax+5}{x^3\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 539$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(-\frac{1}{2}\int-\frac{a(5ax+8)}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 25$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}\int\frac{a(5ax+8)}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\int\frac{5ax+8}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 534$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\left(5a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\left(\frac{5}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right.}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\left(-\frac{5\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\right.}{\sqrt{1-a^2x^2}}$$

3.894. $\int\frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

$$\Big\downarrow \text{221}$$

$$-\frac{x\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\left(-5a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)\sqrt{\ }}{\sqrt{1-a^2x^2}}$$

input

```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/5*Sqrt[1 - a^2*x^2]/x^5 + (a*((-5*Sqrt[1 -
a^2*x^2])/(2*x^4) + (3*a*((-2*Sqrt[1 - a^2*x^2])/x^3 + a*((-5*Sqrt[1 - a^2
*x^2])/(2*x^2) + (a*((-8*Sqrt[1 - a^2*x^2])/x - 5*a*ArcTanh[Sqrt[1 - a^2*x
^2]]))/2)))/2))/5))/Sqrt[1 - a^2*x^2])
```

3.894.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.894.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\left(24a^6x^6+15a^5x^5-12a^4x^4-5a^3x^3-8a^2x^2-10ax-4\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{20x^4\left(a^2x^2-1\right)}-\frac{3a^5\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{4\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a^2\left(-40\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^4c\,x^6+40\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4x^4+40\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)a^2x^5\right.}{}$

3.894. $\int\frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/20*(24*a^6*x^6+15*a^5*x^5-12*a^4*x^4-5*a^3*x^3-8*a^2*x^2-10*a*x-4)/x^4*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-3/4*a^5/(-c)^(1/2)*ln((-2*c+2*(-c
)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-
1))^(1/2)/(a^2*x^2-1)*x
```

3.894.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.29

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$= \left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4},\right.$$
$$\left.-\frac{15\,a^4\sqrt{c}x^4\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{20\,x^4}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 +
10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, -1/20*(15*a^4*sqrt(c)*x^
4*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (2
4*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)))/x^4]
```

3.894.6 Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^5\left(ax-1\right)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)),
 x)
```

3.894.7 Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^5}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^5), x)
```

3.894.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(149) = 298.

Time = 2.36 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$=\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\operatorname{sgn}(x)+70\,(}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) - 40*(sqrt(a^2*c)*x
 - sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) - 200*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*a^3*c^4*sgn(x) - 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)*a^3*c^5*sgn(x) - 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^2 + c)^5)*abs(a)
```

3.894.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^5\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

3.895 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

3.895.1 Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
4*x*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x^2*(c-c/a^2/x^2)^(1/2)/
a^2/(1-1/a^2/x^2)^(1/2)+x^3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*
x^4*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/
2)/a^4/(1-1/a^2/x^2)^(1/2)
```

3.895.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4x}{a^3}+\frac{2x^2}{a^2}+\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1-ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

output

```
(Sqrt[c - c/(a^2*x^2)]*((4*x)/a^3 + (2*x^2)/a^2 + x^3/a + x^4/4 + (4*Log[1
 - a*x])/a^4))/Sqrt[1 - 1/(a^2*x^2)]
```

3.895.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{x^2(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-ax^3-3x^2-\frac{4x}{a}-\frac{4}{a^2(ax-1)}-\frac{4}{a^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(1-ax)}{a^3}-\frac{4x}{a^2}-\frac{ax^4}{4}-\frac{2x^2}{a}-x^3\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*((-4*x)/a^2 - (2*x^2)/a - x^3 - (a*x^4)/4 - (4*Lo
g[1 - a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.895.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.895.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(a^{4} x^{4}+4 a^{3} x^{3}+8 a^{2} x^{2}+16 a x +16 \ln \left(a x -1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{4 a^{3} \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	89

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/4*(a^4*x^4+4*a^3*x^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2
/x^2)^(1/2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.895.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log{(ax-1)})\sqrt{a^2c}}{4\,a^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

output
```
1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(a^2*
c)/a^5
```

3.895.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.895.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.895.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.895.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.896 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

3.896.1 Optimal result

Integrand size = 27, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
4*x*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x^2*(c-c/a^2/x^2)^(1/2
)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*
ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)
```

3.896.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(ax(24+9ax+2a^2x^2)+24\log(1-ax)\right)}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(
6*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

3.896.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{x(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-ax^2-3x-\frac{4}{a}-\frac{4}{a(ax-1)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(1-ax)}{a^2}-\frac{ax^3}{3}-\frac{4x}{a}-\frac{3x^2}{2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*((-4*x)/a - (3*x^2)/2 - (a*x^3)/3 - (4*Log[1 - a*
x])/a^2))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.896.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.896.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(2a^3x^3+9a^2x^2+24ax+24\ln(ax-1)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax-1)}{6a^2(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	82

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/6*(2*a^3*x^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1
/2)*(a*x-1)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.896.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log{(ax-1)})\sqrt{a^2c}}{6\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

output
```
1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(a^2*c)/a^4
```

3.896.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.896.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.896.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.896.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.897 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

3.897.1 Optimal result

Integrand size = 25, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
3*x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/
(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/
2)
```

3.897.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/Sqrt[1 -
1/(a^2*x^2)]
```

3.897.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \textbf{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-ax+\frac{4}{1-ax}-3\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/(a*Sqrt[
1 - 1/(a^2*x^2)]))
```

3.897.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.897.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(a^{2} x^{2}+6 a x +8 \ln \left(a x -1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{2 a \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	73

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/a/
(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.897.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{(a^2x^2+6\,ax+8\,\log{(ax-1)})\sqrt{a^2c}}{2\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

output
```
1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(a^2*c)/a^3
```

3.897.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.897.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.897.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.897.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.898 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.898.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.898.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/Sqrt[1 - 1/(a^
2*x^2)]
```

3.898.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a}{ax-1}-a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(-ax-4\log(1-ax)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/(a*Sqrt[1 - 1
/(a^2*x^2)]))
```

3.898.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.898.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{\left(-ax+\ln\left(x\right)-4\ln\left(ax-1\right)\right)x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^
2/((a*x-1)/(a*x+1))^(3/2)
```

3.898.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+4\log(ax-1)-\log(x))}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

3.898.6 Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

3.898.7 Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.898.8 Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.898.9 Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.899 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.899.1 Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.899.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}-3\log(x)+4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(a*x) - 3*Log[x] + 4*Log[1 - a*x]))/Sqrt[1 - 1/(
a^2*x^2)]
```

3.899.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^2(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^2(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^2}{ax-1}+\frac{3a}{x}+\frac{1}{x^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(3a\log(x)-4a\log(1-ax)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-x^(-1) + 3*a*Log[x] - 4*a*Log[1 - a*x]))/(a*Sqr
t[1 - 1/(a^2*x^2)]))
```

3.899. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}dx$

3.899.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.899.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{(3a\ln(x)x-4a\ln(ax-1)x-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,(ax-1)}{(ax+1)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	67

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOS
E)
```

output
```
-(3*a*ln(x)*x-4*a*ln(a*x-1)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*
x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

3.899. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.899.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{a^2c}(4\,ax\log{(ax-1)}-3\,ax\log{(x)}+1)}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fr
icas")
```

output
```
sqrt(a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a^2*x)
```

3.899.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

output
```
Timed out
```

3.899.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.899.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gi
ac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.899.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.900 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

3.900.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2} + \frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
1/2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+3*(c-c/a^2/x^2)^(1/2)/x/
(1-1/a^2/x^2)^(1/2)-4*a*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a*
ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.900.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.41

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2ax^2}+\frac{3}{x}-4a\log(x)+4a\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output

```
(Sqrt[c - c/(a^2*x^2)]*(1/(2*a*x^2) + 3/x - 4*a*Log[x] + 4*a*Log[1 - a*x])
)/Sqrt[1 - 1/(a^2*x^2)]
```

3.900.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^3(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^3(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^3}{ax-1}+\frac{4a^2}{x}+\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(1-ax)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 - (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[
1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.900. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}dx$

3.900.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.900.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{\left(8 a^{2} \ln \left(x \right) x^{2}-8 a^{2} \ln \left(a x -1\right) x^{2}-6 a x -1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{2 \left(a x +1\right)^{2} x \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	82

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/2*(8*a^2*ln(x)*x^2-8*a^2*ln(a*x-1)*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)
^(1/2)*(a*x-1)/(a*x+1)^2/x/((a*x-1)/(a*x+1))^(3/2)
```

3.900.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{8\,a^3\sqrt{c}x^2\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\sqrt{a^2c}(6\,ax+1)}{2\,a^2x^2}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
fricas")
```

output

```
1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + sqrt(a^2*c)*(6*a*x + 1))/(a^2*x^2)
```

3.900.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

output

```
Timed out
```

3.900.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
maxima")
```

output

```
integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.900.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.900.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.901 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

3.901.1 Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/3*(c-c/a^2/x^2)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)/
x^2/(1-1/a^2/x^2)^(1/2)+4*a*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^
2*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(-a*x+1)*(c-c/a^2/
x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.901.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3ax^3}+\frac{3}{2x^2}+\frac{4a}{x}-4a^2\log(x)+4a^2\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(3*a*x^3) + 3/(2*x^2) + (4*a)/x - 4*a^2*Log[x] +
 4*a^2*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

3.901.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^4(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^4(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^4}{ax-1}+\frac{4a^3}{x}+\frac{4a^2}{x^2}+\frac{3a}{x^3}+\frac{1}{x^4}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^3\log(x)-4a^3\log(1-ax)-\frac{4a^2}{x}-\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

3.901. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}dx$

output

```
-((Sqrt[c - c/(a^2*x^2)]*(-1/3*1/x^3 - (3*a)/(2*x^2) - (4*a^2)/x + 4*a^3*L
og[x] - 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.901.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.901.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.48

method	result	size
default	$-\frac{\left(24 a^{3} \ln \left(x \right) x^{3}-24 a^{3} \ln \left(a x -1\right) x^{3}-24 a^{2} x^{2}-9 a x -2\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{6 \left(a x +1\right)^{2} x^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	90

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x,method=_RETURNVERB
OSE)
```

output
```
-1/6*(24*a^3*ln(x)*x^3-24*a^3*ln(a*x-1)*x^3-24*a^2*x^2-9*a*x-2)*(c*(a^2*x^
2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^2/((a*x-1)/(a*x+1))^(3/2)
```

3.901.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.49

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{24\,a^4\sqrt{c}x^3\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+(24\,a^2x^2+9\,ax+2)\sqrt{a^2c}}{6\,a^2x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="
fricas")
```

output
```
1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
- 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(a^2*c))/(
a^2*x^3)
```

3.901.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

output
```
Timed out
```

3.901.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.901.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="
giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.901.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.902 $\int \frac{e^{3 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2x^2}}}{x^4} \, dx$

3.902.1 Optimal result

Integrand size = 27, antiderivative size = 222

$$\begin{aligned}\int \frac{e^{3 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2x^2}}}{x^4} \, dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^4} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^3} \\ &\quad + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2} + \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x} \\ &\quad - \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
1/4*(c-c/a^2/x^2)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)/x^3/
(1-1/a^2/x^2)^(1/2)+2*a*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^2*
(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(x)*(c-c/a^2/x^2)^(1/2)/
(1-1/a^2/x^2)^(1/2)+4*a^3*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/
2)
```

3.902.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.35

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4ax^4}+\frac{1}{x^3}+\frac{2a}{x^2}+\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(4*a*x^4) + x^(-3) + (2*a)/x^2 + (4*a^2)/x - 4*a
^3*Log[x] + 4*a^3*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

3.902.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^5(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^5(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^5}{ax-1}+\frac{4a^4}{x}+\frac{4a^3}{x^2}+\frac{4a^2}{x^3}+\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^4\log(x)-4a^4\log(1-ax)-\frac{4a^3}{x}-\frac{2a^2}{x^2}-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 - a/x^3 - (2*a^2)/x^2 - (4*a^3)/x + 4
*a^4*Log[x] - 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.902.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.902. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.902.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{\left(16\ln\left(x\right)x^{4}a^{4}-16\ln\left(ax-1\right)x^{4}a^{4}-16a^{3}x^{3}-8a^{2}x^{2}-4ax-1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{4\left(ax+1\right)^{2}x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	98

input

```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x,method=_RETURNVERB
OSE)
```

output

```
-1/4*(16*ln(x)*x^4*a^4-16*ln(a*x-1)*x^4*a^4-16*a^3*x^3-8*a^2*x^2-4*a*x-1)*
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^3/((a*x-1)/(a*x+1))^(3/2
)
```

3.902.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{16\,a^{5}\sqrt{c}x^{4}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax-1)\sqrt{c}+ac}{ax^{2}-x}\right)+\left(16\,a^{3}x^{3}+8\,a^{2}x^{2}+4\,ax+1\right)\sqrt{a^{2}c}}{4\,a^{2}x^{4}}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="
fricas")
```

output

```
1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
- 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*sq
rt(a^2*c))/(a^2*x^4)
```

3.902.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

output
```
Timed out
```

3.902.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.902.8 Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="
giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.902.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.903 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

3.903.1 Optimal result

Integrand size = 27, antiderivative size = 264

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}x^3}\\ &\quad+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &\quad-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
1/5*(c-c/a^2/x^2)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+3/4*(c-c/a^2/x^2)^(1/2)/
x^4/(1-1/a^2/x^2)^(1/2)+4/3*a*(c-c/a^2/x^2)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+
2*a^2*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^3*(c-c/a^2/x^2)^(1/2
)/x/(1-1/a^2/x^2)^(1/2)-4*a^4*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2
)+4*a^4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.903.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.34

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5ax^5}+\frac{3}{4x^4}+\frac{4a}{3x^3}+\frac{2a^2}{x^2}+\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(5*a*x^5) + 3/(4*x^4) + (4*a)/(3*x^3) + (2*a^2)/
x^2 + (4*a^3)/x - 4*a^4*Log[x] + 4*a^4*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)
]
```

3.903.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x^5}dx$$
$$\Big\downarrow \text{6751}$$
$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$\Big\downarrow \text{6747}$$
$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^6(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^6(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^6}{ax-1}+\frac{4a^5}{x}+\frac{4a^4}{x^2}+\frac{4a^3}{x^3}+\frac{4a^2}{x^4}+\frac{3a}{x^5}+\frac{1}{x^6}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^5\log(x)-4a^5\log(1-ax)-\frac{4a^4}{x}-\frac{2a^3}{x^2}-\frac{4a^2}{3x^3}-\frac{3a}{4x^4}-\frac{1}{5x^5}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*(-1/5*1/x^5 - (3*a)/(4*x^4) - (4*a^2)/(3*x^3) - (
2*a^3)/x^2 - (4*a^4)/x + 4*a^5*Log[x] - 4*a^5*Log[1 - a*x]))/(a*Sqrt[1 - 1
/(a^2*x^2)]))
```

3.903.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

3.903. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.903.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{\left(240 a^{5} \ln \left(x\right) x^{5}-240 \ln \left(a x -1\right) x^{5} a^{5}-240 a^{4} x^{4}-120 a^{3} x^{3}-80 a^{2} x^{2}-45 a x -12\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{60 \left(a x +1\right)^{2} x^{4} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	106

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x,method=_RETURNVERB
OSE)
```

output
```
-1/60*(240*a^5*ln(x)*x^5-240*ln(a*x-1)*x^5*a^5-240*a^4*x^4-120*a^3*x^3-80*
a^2*x^2-45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^4/((a
*x-1)/(a*x+1))^(3/2)
```

3.903.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.41

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{240\,a^{6}\sqrt{c}x^{5}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax-1)\sqrt{c}+ac}{ax^{2}-x}\right)+(240\,a^{4}x^{4}+120\,a^{3}x^{3}+80\,a^{2}x^{2}+45\,ax+12)\sqrt{a^{2}c}}{60\,a^{2}x^{5}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="
fricas")
```

output
```
1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*
x - 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (240*a^4*x^4 + 120*a^3*x^3 + 80*a^2*x
^2 + 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

3.903.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

output
```
Timed out
```

3.903.7 Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.903.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="
giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.903.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.903. $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

3.904 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx$

3.904.1 Optimal result

Integrand size = 27, antiderivative size = 81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
-x^m*(c-c/a^2/x^2)^(1/2)/a/m/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(c-c/a^2/x^2)^(1/
2)/(1+m)/(1-1/a^2/x^2)^(1/2)
```

3.904.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^m\left(-\frac{1}{am}+\frac{x}{1+m}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

output

```
(Sqrt[c - c/(a^2*x^2)]*x^m*(-(1/(a*m)) + x/(1 + m)))/Sqrt[1 - 1/(a^2*x^2)]
```

3.904.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^mdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -x^{m-1}(1-ax)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{m-1}(1-ax)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{53}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(x^{m-1}-ax^m\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x^m}{m}-\frac{ax^{m+1}}{m+1}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(x^m/m - (a*x^(1 + m))/(1 + m)))/(a*Sqrt[1 - 1/(a
^2*x^2)]))
```

3.904.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.904.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^{1+m} \left(a m x -m -1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{\frac{a x -1}{a x +1}}}{m \left(1+m \right) \left(a x -1\right)}$	65
risch	$\frac{\sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{\left(a x -1\right) \left(a x +1\right)}}\, \left(a x +1\right) \left(a m x -m -1\right) x^{m}}{\sqrt{c}\, \left(a^{2} x^{2}-1\right) \left(1+m \right) m}$	103

input
```
int(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
x^(1+m)/m/(1+m)/(a*x-1)*(a*m*x-m-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)
/(a*x+1))^(1/2)
```

3.904.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx = -\frac{(amx^2-(m+1)x)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-(am^2+am)x+m}$$

input
```
integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

output
```
-(a*m*x^2 - (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/
(a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

3.904.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx = \text{Timed out}$$

input
```
integrate(x**m*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.904.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx = \frac{\left(a\sqrt{c}mx-\sqrt{c}(m+1)\right)(ax-1)x^m}{(m^2+m)a^2x-(m^2+m)a}$$

input
```
integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

output
```
(a*sqrt(c)*m*x - sqrt(c)*(m + 1))*(a*x - 1)*x^m/((m^2 + m)*a^2*x - (m^2 +
m)*a)
```

3.904.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.904.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^m\,dx=\int x^m\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x^m*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^m*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.905 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

3.905.1 Optimal result

Integrand size = 27, antiderivative size = 76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*x^2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(
1/2)/(1-1/a^2/x^2)^(1/2)
```

3.905.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(-3+2ax)}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x^2*(-3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

3.905.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -x(1-ax)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(1-ax)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(x-ax^2\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{x^2}{2}-\frac{ax^3}{3}\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(x^2/2 - (a*x^3)/3))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.905.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.905.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^{3}(2ax-3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	53
default	$\frac{x^{3}(2ax-3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	53

input
```
int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/6*x^3*(2*a*x-3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a
*x-1)
```

3.905.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,ax^3-3\,x^2)\sqrt{a^2c}}{6\,a^2}$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

output
```
1/6*(2*a*x^3 - 3*x^2)*sqrt(a^2*c)/a^2
```

3.905.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.905.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.905.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.905.9 Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

input
```
int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(
a*x - 1))
```

3.906 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

3.906.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(
1-1/a^2/x^2)^(1/2)
```

3.906.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{x}{a}+\frac{x^2}{2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-(x/a) + x^2/2))/Sqrt[1 - 1/(a^2*x^2)]
```

3.906.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = $\{6751, 6747, 17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int (ax-1)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a^2*Sqrt[1 - 1/(a^2*x^2)])
```

3.906.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.906.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x^{2}(ax-2)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	52
default	$\frac{x^{2}(ax-2)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	52

input

```
int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/2*x^2*(a*x-2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x
-1)
```

3.906.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.29

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^{2}x^{2}}}x\,dx=\frac{\sqrt{a^{2}c}(ax^{2}-2\,x)}{2\,a^{2}}$$

input

```
integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fric
as")
```

output

```
1/2*sqrt(a^2*c)*(a*x^2 - 2*x)/a^2
```

3.906.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\text{Timed out}$$

input
```
integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.906.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.906.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.906.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,\left(a\,x-1\right)}$$

input
```
int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*
x - 1))
```

3.907 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.907.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)
```

3.907.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

3.907.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{1-ax}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1-ax}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x}-a\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(\log(x)-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-(a*x) + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.907.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.907.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{(-ax+\ln(x))x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	52

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x
-1)
```

3.907.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-\log{(x)})}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - log(x))/a^2
```

3.907.6 Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

3.907.7 Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.907.8 Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.907.9 Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.908 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.908.1 Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

3.908.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(a*x) + Log[x]))/Sqrt[1 - 1/(a^2*x^2)]
```

3.908.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = $\{6751, 6747, 25, 49, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{1-ax}{x^2}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1-ax}{x^2}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^2}-\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-a\log(x)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-x^(-1) - a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

3.908. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}dx$

3.908.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.908.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{(a\ln(x)x+1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	50

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(a*ln(x)*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)
```

3.908.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{a^2c}(ax\log{(x)}+1)}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x*log(x) + 1)/(a^2*x)
```

3.908.6 Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/x
, x)
```

3.908.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.908.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac
")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.908.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.909 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

3.909.1 Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

output
```
1/2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)
```

3.909.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2ax^2}-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(2*a*x^2) - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)]
```

3.909.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6751, 6747, 25, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{1-ax}{x^3}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1-ax}{x^3}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{48}$$

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

3.909.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.909.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

method	result	size
gosper	$-\frac{(2ax-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{2(ax-1)x}$	53
default	$-\frac{(2ax-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{2(ax-1)x}$	53

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/2*(2*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)/x
```

3.909. $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x^{2}}\,dx$

3.909.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{a^2c}(2\,ax-1)}{2\,a^2x^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fr
icas")
```

output
```
-1/2*sqrt(a^2*c)*(2*a*x - 1)/(a^2*x^2)
```

3.909.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Timed out
```

3.909.7 Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.909.8 Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

3.909.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\left(x\,\sqrt{c-\frac{c}{a^2\,x^2}}-\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

output
```
((x*(c - c/(a^2*x^2))^(1/2) - (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a
*x + 1))^(1/2))/(x/a - x^2)
```

3.910 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

3.910.1 Optimal result

Integrand size = 27, antiderivative size = 163

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=-\frac{7\sqrt{c-\frac{c}{a^2x^2}}x}{8a^3}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{24a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(1-ax)^2}{4a^2}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{8a^3\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
-7/8*x*(c-c/a^2/x^2)^(1/2)/a^3-7/24*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3-1/6
*x*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3+1/4*x^2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/
2)/a^2-7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1
/2)
```

3.910.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(-32+21ax-16a^2x^2+6a^3x^3)+21\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(-32 + 21*a*x - 16*a^2*x^2 +
6*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2]
)
```

3.910.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6717, 6709, 570, 541, 25, 27, 533, 25, 27, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x^2\left(1-a^2x^2\right)^{3/2}}{(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x^2(1-ax)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int-\frac{a^2x^2(7-8ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int\frac{a^2x^2(7-8ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\int \frac{x^2(7-8ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 533$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\int -\frac{ax(16-21ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\int \frac{ax(16-21ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\int \frac{x(16-21ax)}{\sqrt{1-a^2x^2}}dx}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 533$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{\int -\frac{a(21-32ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{21x\sqrt{1-a^2x^2}}{2a}-\frac{\int \frac{a(21-32ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{21x\sqrt{1-a^2x^2}}{2a}-\frac{\int \frac{21-32ax}{\sqrt{1-a^2x^2}}dx}{2a}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 455$$

3.910. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{21x\sqrt{1-a^2x^2}}{2a}-\frac{21\int\frac{1}{\sqrt{1-a^2x^2}}dx+\frac{32\sqrt{1-a^2x^2}}{a}}{2a}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 223$$

$$-\frac{x\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{21x\sqrt{1-a^2x^2}}{2a}-\frac{\frac{32\sqrt{1-a^2x^2}}{a}+\frac{21\arcsin(ax)}{a}}{2a}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/4*(x^3*Sqrt[1 - a^2*x^2]) + ((8*x^2*Sqrt[1
- a^2*x^2])/(3*a) - ((21*x*Sqrt[1 - a^2*x^2])/(2*a) - ((32*Sqrt[1 - a^2*x^
2])/a + (21*ArcSin[a*x])/a)/(2*a))/(3*a))/4))/Sqrt[1 - a^2*x^2])
```

3.910.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.910. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.910.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.82

method	result
risch	$\frac{(6a^3x^3-16a^2x^2+21ax-32)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{24a^3}+\frac{7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{8a^2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\Bigg(-6x\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4+16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+27c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-48c^{\frac{3}{2}}\ln}{24\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c\,a^4}$

input
```
int(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3-16*a^2*x^2+21*a*x-32)/a^3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+
7/8/a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^
2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.910.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.36

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$= \left[\frac{2\left(6\,a^{4}x^{4}-16\,a^{3}x^{3}+21\,a^{2}x^{2}-32\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}+21\,\sqrt{c}\log\left(2\,a^{2}cx^{2}+2\,a^{2}\sqrt{c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)}{48\,a^{4}},\ (6\,a^{4}x\right.$$

input
```
integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(2*(6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 - 32*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2
 - 32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

3.910.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^{3}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x**3*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x
)
```

3.910.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x^3}{ax+1}\,dx$$

input
```
integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1), x)
```

3.910.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.79

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\mathrm{sgn}(x)}{a^2}-\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x-\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x-\right.\right.}{a^4}\right.$$

input
```
integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 - 8*sgn(x)/a^3) + 21*sgn
(x)/a^4)*x - 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2
*c*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) + 64*sqrt(-c
)*abs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

3.910.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.911 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

3.911.1 Optimal result

Integrand size = 27, antiderivative size = 124

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)^2}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{a^2\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(-a
*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^2+x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*
x+1)^(1/2)/(a*x+1)^(1/2)
```

3.911.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(5-3ax+a^2x^2)-3\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{3a^2\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 - 3*a*x + a^2*x^2) - 3*Log
[a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

3.911.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6717, 6709, 571, 466, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x\left(1-a^2x^2\right)^{3/2}}{(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 571}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2\int\frac{\left(1-a^2x^2\right)^{3/2}}{ax+1}dx}{a}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(ax+1)^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 466}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2\left(\int\sqrt{1-a^2x^2}dx+\frac{\left(1-a^2x^2\right)^{3/2}}{3a}\right)}{a}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(ax+1)^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 211}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2\left(\frac{1}{2}\int\frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}x\sqrt{1-a^2x^2}\right)}{a}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(ax+1)^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{x\left(-\frac{2\left(\frac{(1-a^2x^2)^{3/2}}{3a}+\frac{1}{2}x\sqrt{1-a^2x^2}+\frac{\arcsin(ax)}{2a}\right)}{a}-\frac{(1-a^2x^2)^{5/2}}{a^2(ax+1)^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-((1 - a^2*x^2)^(5/2)/(a^2*(1 + a*x)^2)) - (2*
((x*Sqrt[1 - a^2*x^2])/2 + (1 - a^2*x^2)^(3/2)/(3*a) + ArcSin[a*x]/(2*a)))
/a))/Sqrt[1 - a^2*x^2])
```

3.911.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

3.911. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.911.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

method	result
risch	$\frac{(a^2x^2-3ax+5)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{3a^2}-\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)x\sqrt{c(a^2x^2-1)}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{a\sqrt{a^2c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2cx+3c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)+6\sqrt{}\right.}{3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3c}$

input
```
int(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a^2*x^2-3*a*x+5)/a^2*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)-1/a*ln(a^2*c*x/(
a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)*(c
*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)
```

3.911.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.65

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\left[\frac{2\,(a^3x^3-3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\frac{(a^3x^3-3\,a^2x^2+5\,ax}{}\right.$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/6*(2*(a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*
sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
) - c))/a^3, 1/3*((a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) + 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
/(a^2*c*x^2 - c)))/a^3]
```

3.911.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x**2*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x
)
```

3.911.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x^2}{ax+1}\,dx$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1), x)
```

3.911.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}-\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)+\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}\right.$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 - 3*sgn(x)/a^3) + 5*sgn(x)/a^4
) + 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*a
bs(a)) - (3*a*sqrt(c)*log(abs(c)) + 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)
))*abs(a)
```

3.911.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.912 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

3.912.1 Optimal result

Integrand size = 25, antiderivative size = 99

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x(1-ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{2a\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
-3/2*x*(c-c/a^2/x^2)^(1/2)/a-1/2*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a-3/2*x*ar
csin(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.912.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{1+ax}(4-5ax+a^2x^2)-6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
-1/2*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqr
t[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x
^2])
```

3.912.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6717, 6709, 466, 466, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}xdx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{466}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3}{2}\int \frac{\sqrt{1-a^2x^2}}{ax+1}dx+\frac{(1-a^2x^2)^{3/2}}{2a(ax+1)}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{466}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}\right)+\frac{(1-a^2x^2)^{3/2}}{2a(ax+1)}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\left(\frac{3}{2}\left(\frac{\sqrt{1-a^2x^2}}{a}+\frac{\arcsin(ax)}{a}\right)+\frac{(1-a^2x^2)^{3/2}}{2a(ax+1)}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*((1 - a^2*x^2)^(3/2)/(2*a*(1 + a*x)) + (3*(Sqrt
[1 - a^2*x^2]/a + ArcSin[a*x]/a))/2))/Sqrt[1 - a^2*x^2])
```

3.912.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.912.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(ax-4)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x}{2a}+\frac{3\ln\left(\frac{a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-c}\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{c(a^{2}x^{2}-1)}\,x}{2\sqrt{a^{2}c}\,(a^{2}x^{2}-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\left(-x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{2}+\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)-4\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^{2}}}+cx}{\sqrt{c}}\right)+4\sqrt{\frac{c(ax-1)(ax+1)}{a^{2}}}\,a\right)}{2\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{2}}$

input
```
int(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-4)/a*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+3/2*ln(a^2*c*x/(a^2*c)^(1/2)
+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*
x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.912.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.90

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$

$$=\left[\frac{2\,(a^2x^2-4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\frac{(a^2x^2-4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-}}{2\,a}\right.$$

input
```
integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(
2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2,
 1/2*((a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arcta
n(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

3.912.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.912.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x}{ax+1}\,dx$$

input
```
integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1), x)
```

3.912.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}(x)}{a^2}-\frac{4\,\mathrm{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^2|a|}+\frac{\left(3\,a\sqrt{c}\log\left(|c|\right)\right.}{}\right.$$

input
```
integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 - 4*sgn(x)/a^3) - 6*sqrt(c)*log(a
bs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(
c)*log(abs(c)) + 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

3.912.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

input
```
int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.913 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.913.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(
a*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.913.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]
] - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

3.913.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6709, 570, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2(1-2ax)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2(1-2ax)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1-2ax}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-2a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 223$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 221$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[1 - a^2*x^2] - 2*ArcSin[a*x] - ArcTanh[S
qrt[1 - a^2*x^2]]))/Sqrt[1 - a^2*x^2])
```

3.913.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.913. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.913. $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.913.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-c\ln\left(2\sqrt{-\frac{c}{a^2}}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)-2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+c*x)/c^(1/2)
)*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^
(1/2)/a^2/(-c/a^2)^(1/2)
```

3.913.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a
^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a,
(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^
2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

3.913.6 Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.913.7 Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

3.913.8 Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.913.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.914 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.914.1 Optimal result

Integrand size = 27, antiderivative size = 117

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\arcsin(ax)}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\,\mathrm{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
(c-c/a^2/x^2)^(1/2)-a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*
x+1)^(1/2)-2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)
/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.914.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}+2ax\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+ax\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2*
x^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

3.914.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6709, 570, 540, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x^2(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^2\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\int \frac{a(2-ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\int \frac{2-ax}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(2\int \frac{1}{x\sqrt{1-a^2x^2}}dx-a\int \frac{1}{\sqrt{1-a^2x^2}}dx\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(2\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \textbf{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(-\frac{2\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(-2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-(Sqrt[1 - a^2*x^2]/x) - a*(-ArcSin[a*x] - 2*A
rcTanh[Sqrt[1 - a^2*x^2]])))/Sqrt[1 - a^2*x^2])
```

3.914.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.914. $\int\frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.914. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.914.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24

method	result
risch	$\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}+\frac{\left(\frac{a^{2}\ln\left(\frac{a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-c}\right)}{\sqrt{a^{2}c}}+\frac{2a\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^{2}c\,x^{2}-c}}{x}\right)}{\sqrt{-c}}\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{c(a^{2}x^{2}-1)}\,x}{a^{2}x^{2}-1}$
default	$-\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\left(-\sqrt{-\frac{c}{a^{2}}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{3}c\,x^{2}+a^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^{2}}}+c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)\sqrt{-\frac{c}{a^{2}}}\,ax-2c^{\frac{3}{2}}\sqrt{-\frac{c}{a^{2}}}\,\ln\right.}{a\sqrt{\frac{c\left(a^{2}\right.}{}}}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^
(1/2))/(a^2*c)^(1/2)+2*a/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1
/2))/x))*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.914.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.15

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x}\,dx = \left[-\sqrt{-c}\arctan\left(\frac{a^{2}\sqrt{-c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right) + \sqrt{-c}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right) + \sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}, 2\,\sqrt{c}\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right) + \frac{1}{2}\,\sqrt{c}\log\left(2\,a^{2}cx^{2}+2\,a^{2}\sqrt{c}x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right) + \sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}\right]$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
[-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*
x^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), 2*sqrt(c)*arcta
n(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(
c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c
) + sqrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

3.914.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x\,(ax+1)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x*(a*x + 1)), x)
```

3.914.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

3.914.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx =$$

$$-\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}+\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}-\frac{2}{\left(\left(\sqrt{a^2c}x-\right.\right.}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
-(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/
a + sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) -
 2*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*
abs(a)
```

3.914.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.915 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

3.915.1 Optimal result

Integrand size = 27, antiderivative size = 112

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{2\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
-3/2*a*(c-c/a^2/x^2)^(1/2)+1/2*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x+3/2*a^2*x*ar
ctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*
x+1)^(1/2)
```

3.915.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((-1+4ax)\sqrt{-1+a^2x^2}+3a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{2x\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output

```
-1/2*(Sqrt[c - c/(a^2*x^2)]*((-1 + 4*a*x)*Sqrt[-1 + a^2*x^2] + 3*a^2*x^2*A
rcTan[1/Sqrt[-1 + a^2*x^2]]))/(x*Sqrt[-1 + a^2*x^2])
```

3.915.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6709, 570, 540, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x^3(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^3\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}\int \frac{a(4-3ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}a\int \frac{4-3ax}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}a\left(-3a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}a\left(-\frac{3}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}a\left(\frac{3\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{x\left(-\frac{1}{2}a\left(3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/2*Sqrt[1 - a^2*x^2]/x^2 - (a*((-4*Sqrt[1 -
a^2*x^2])/x + 3*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/Sqrt[1 - a^2*x^2])
```

3.915.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.915.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{\left(4 a^{3} x^{3}-a^{2} x^{2}-4 a x +1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{2 x \left(a^{2} x^{2}-1\right)}-\frac{3 a^{2} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{2 \sqrt{-c}\, \left(a^{2} x^{2}-1\right)}$
default	$\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(-4 \sqrt{-\frac{c}{a^{2}}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c \,x^{3}+4 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3} x +4 \sqrt{-\frac{c}{a^{2}}}\, c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a \,x^{2}-4 \sqrt{-\frac{c}{a^{2}}}$

3.915. $\int \frac{e^{-2 \coth^{-1}(a x)} \sqrt{c-\frac{c}{a^{2} x^{2}}}}{x^{2}}\, dx$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(4*a^3*x^3-a^2*x^2-4*a*x+1)/x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-
1)-3/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^
2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.915.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.57

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x}, -\frac{3\,a\sqrt{c}x\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)) - 2*c)/x^2) - 2*(4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x
, -1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*c*x^2 - c)) + (4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

3.915.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)),
 x)
```

3.915.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

3.915.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(91) = 182$.

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.73

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$
$$= \Bigg(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x) - \frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3 ac\,\mathrm{sgn}(x) + 4\Big(\sqrt{a^2c}x-\sqrt{a^2}}{\Big(\Big(\sqrt{a^2}}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
(3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
 ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) + 4*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^2*sgn(x) + 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

3.915.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^2\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.916 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

3.916.1 Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{3x^2} - \frac{a^3\sqrt{c-\frac{c}{a^2x^2}}x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
a^2*(c-c/a^2/x^2)^(1/2)-1/3*a*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x+1/3*(-a*x+1)^
2*(c-c/a^2/x^2)^(1/2)/x^2-a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c
/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.916.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(1-3ax+5a^2x^2)+3a^3x^3\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{3x^2\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 - 3*a*x + 5*a^2*x^2) + 3*a^3
*x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

3.916.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = $\{6717, 6709, 570, 540, 27, 539, 27, 534, 243, 73, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^4(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^4\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}\int \frac{a(6-5ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\int \frac{6-5ax}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-\frac{1}{2}\int \frac{2a(5-3ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-a\int \frac{5-3ax}{x^2\sqrt{1-a^2x^2}}dx-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-a\left(-3a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-a\left(-\frac{3}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-a\left(\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x\left(-\frac{1}{3}a\left(-a\left(3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (a*((-3*Sqrt[1 -
a^2*x^2])/x^2 - a*((-5*Sqrt[1 - a^2*x^2])/x + 3*a*ArcTanh[Sqrt[1 - a^2*x^2
]])))/3))/Sqrt[1 - a^2*x^2])
```

3.916.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.916. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

3.916. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.916.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

method	result
risch	$\frac{\left(5 a^{4} x^{4}-3 a^{3} x^{3}-4 a^{2} x^{2}+3 a x -1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{3 x^{2} \left(a^{2} x^{2}-1\right)}+\frac{a^{3} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{\sqrt{-c}\, \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, a \Bigg(-6 \sqrt{-\frac{c}{a^{2}}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c \,x^{4}+6 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3} x^{2}+6 \sqrt{-\frac{c}{a^{2}}}\, c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a \,x^{3}-6 \sqrt{-}}{}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(5*a^4*x^4-3*a^3*x^3-4*a^2*x^2+3*a*x-1)/x^2*(c*(a^2*x^2-1)/a^2/x^2)^(1
/2)/(a^2*x^2-1)+a^3/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/
x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

3.916.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$
$$=\left[\frac{3\,a^{2}\sqrt{-c}x^{2}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)+2\,(5\,a^{2}x^{2}-3\,ax+1)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{6\,x^{2}},\ \frac{3\,a^{2}\sqrt{c}x^{2}\arctan\left(\frac{a\sqrt{c}x\sqrt{}}{a^{2}cx}\right.}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
[1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
- c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/x^2, 1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)))/x^2]
```

3.916.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^3\left(ax+1\right)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)),
 x)
```

3.916.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^3}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^3), x)
```

3.916.8 Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.65

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=$$

$$-\frac{2}{3}\Bigg(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5 ac\operatorname{sgn}(x)+3\left(\sqrt{a^2c}x\right.}{}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
-2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*s
gn(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) + 3*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) + 12*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))*a*c^3*sgn(x) + 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

3.916.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^3\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.917 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

3.917.1 Optimal result

Integrand size = 27, antiderivative size = 156

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = {} & -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} \\ & -\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} \\ & +\frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{8\sqrt{1-ax}\sqrt{1+ax}}\end{aligned}$$

output
```
-4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3-2/3*a*(c-c/a^2/x^
2)^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x+7/8*a^4*x*arctanh((-a*x+1)^(1/2
)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

3.917.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\begin{aligned}&\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx \\ &= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-6+16ax-21a^2x^2+32a^3x^3)+21a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{24x^3\sqrt{-1+a^2x^2}}\end{aligned}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
-1/24*(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-6 + 16*a*x - 21*a^2*x^2
 + 32*a^3*x^3) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(x^3*Sqrt[-1 +
a^2*x^2])
```

3.917.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {6717, 6709, 570, 540, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^5(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^5\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}\int \frac{a(8-7ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\int \frac{8-7ax}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}\int\frac{a(21-16ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\int\frac{21-16ax}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 539$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}\int\frac{a(32-21ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\int\frac{32-21ax}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 534$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(-21a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(-\frac{21}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(\frac{21\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 221$$

$$-\frac{x\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(21a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input

```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

3.917. $\int\frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/4*Sqrt[1 - a^2*x^2]/x^4 - (a*((-8*Sqrt[1 -
a^2*x^2])/(3*x^3) - (a*((-21*Sqrt[1 - a^2*x^2])/(2*x^2) - (a*((-32*Sqrt[1
- a^2*x^2])/x + 21*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/3))/4))/Sqrt[1 - a^2
*x^2])
```

3.917.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.917. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.917.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\left(32 a^{5} x^{5}-21 a^{4} x^{4}-16 a^{3} x^{3}+15 a^{2} x^{2}-16 a x +6\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{24 x^{3} \left(a^{2} x^{2}-1\right)}-\frac{7 a^{4} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{8 \sqrt{-c}\, \left(a^{2} x^{2}-1\right)}$
default	$\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, a^{2} \left(-48 \sqrt{-\frac{c}{a^{2}}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c \,x^{5}+48 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3} x^{3}+48 \sqrt{-\frac{c}{a^{2}}}\, c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a \,x^{4}-4\right.}{}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

3.917. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

output
```
-1/24*(32*a^5*x^5-21*a^4*x^4-16*a^3*x^3+15*a^2*x^2-16*a*x+6)/x^3*(c*(a^2*x
^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-7/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*
(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2
)/(a^2*x^2-1)*x
```

3.917.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$
$$= \left[\frac{21\,a^3\sqrt{-c}x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},\right.$$
$$\left.-\frac{21\,a^3\sqrt{c}x^3\arctan\left(\frac{a\sqrt{c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{24\,x^3}\right]$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sq
rt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 2
1*a^2*x^2 + 16*a*x - 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

3.917.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^4\,(ax+1)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)),
 x)
```

3.917.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^4}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^4), x)
```

3.917.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(128) = 256$.

Time = 1.77 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.03

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= \frac{1}{12}\Bigg(21\,a^2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x) - \frac{21\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7 a^2c\,\mathrm{sgn}(x)+45\,\Big(\sqrt{c}}{}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) + 96*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*a^2*c^3*sgn(x) + 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*
a^2*c^4*sgn(x) + 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - c))^2 + c)^4)*abs(a)
```

3.917.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^4\,\left(a\,x+1\right)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.918 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

3.918.1 Optimal result

Integrand size = 27, antiderivative size = 181

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\,\mathrm{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{4\sqrt{1-ax}\sqrt{1+ax}}$$

output
```
6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4-1/2*a*(c-c/a^2/x^2
)^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2-3/4*a^3*(c-c/a^2/x^2)^(1/2)/x-
3/4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+
1)^(1/2)/(a*x+1)^(1/2)
```

3.918.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(4-10ax+12a^2x^2-15a^3x^3+24a^4x^4)+15a^5x^5\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{20x^4\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 - 10*a*x + 12*a^2*x^2 - 15*a
^3*x^3 + 24*a^4*x^4) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*S
qrt[-1 + a^2*x^2])
```

3.918.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6717, 6709, 570, 540, 27, 539, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^6(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^6\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}\int \frac{a(10-9ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\int \frac{10-9ax}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{1}{4}\int \frac{6a(6-5ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\int\frac{6-5ax}{x^4\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 539$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-\frac{1}{3}\int\frac{3a(5-4ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\int\frac{5-4ax}{x^3\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 539$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}\int\frac{a(8-5ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\int\frac{8-5ax}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 534$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\left(-5a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\right.}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\left(-\frac{5}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\right.}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\left(\frac{5\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right.\right.}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow 221$$

3.918. $\displaystyle\int\frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

$$-\frac{x\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\left(5a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x}}{5x^5}}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/5*Sqrt[1 - a^2*x^2]/x^5 - (a*((-5*Sqrt[1 -
a^2*x^2])/(2*x^4) - (3*a*((-2*Sqrt[1 - a^2*x^2])/x^3 - a*((-5*Sqrt[1 - a^2
*x^2])/(2*x^2) - (a*((-8*Sqrt[1 - a^2*x^2])/x + 5*a*ArcTanh[Sqrt[1 - a^2*x
^2]]))/2)))/2))/5))/Sqrt[1 - a^2*x^2])
```

3.918.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.918. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

3.918.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\left(24a^6x^6-15a^5x^5-12a^4x^4+5a^3x^3-8a^2x^2+10ax-4\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{20x^4\left(a^2x^2-1\right)}+\frac{3a^5\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{4\sqrt{-c}\,\left(a^2x^2-1\right)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a^2\left(-40\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^4c\,x^6+40\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4x^4+40\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)a^2x^5\right.}{}$

3.918. $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/20*(24*a^6*x^6-15*a^5*x^5-12*a^4*x^4+5*a^3*x^3-8*a^2*x^2+10*a*x-4)/x^4*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+3/4*a^5/(-c)^(1/2)*ln((-2*c+2*(-c
)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-
1))^(1/2)/(a^2*x^2-1)*x
```

3.918.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.28

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$

$$= \left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\quad 1}{40\,x^4}, -\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

output
```
[1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 -
10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, 1/20*(15*a^4*sqrt(c)*x^4
*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (24
*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)))/x^4]
```

3.918.6 Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^5\,(ax+1)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)),
 x)
```

3.918.7 Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^5}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^5), x)
```

3.918.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(149) = 298$.

Time = 2.57 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx =$$

$$-\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\,\mathrm{sgn}(x)+70\,(\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

output
```
-1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c
))*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) + 40*(sqrt(a^2*c)*x
 - sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) + 200*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^3*a^3*c^4*sgn(x) + 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))*a^3*c^5*sgn(x) + 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - c))^2 + c)^5)*abs(a)
```

3.918.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.919 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

3.919.1 Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=-\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-4*x*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x^2*(c-c/a^2/x^2)^(1/2)
/a^2/(1-1/a^2/x^2)^(1/2)-x^3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4
*x^4*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/
2)/a^4/(1-1/a^2/x^2)^(1/2)
```

3.919.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4x}{a^3}+\frac{2x^2}{a^2}-\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1+ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((-4*x)/a^3 + (2*x^2)/a^2 - x^3/a + x^4/4 + (4*Log[
1 + a*x])/a^4))/Sqrt[1 - 1/(a^2*x^2)]
```

3.919.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(1-ax)^2}{ax+1}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(ax^3-3x^2+\frac{4x}{a}+\frac{4}{a^2(ax+1)}-\frac{4}{a^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4\log(ax+1)}{a^3}-\frac{4x}{a^2}+\frac{ax^4}{4}+\frac{2x^2}{a}-x^3\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((-4*x)/a^2 + (2*x^2)/a - x^3 + (a*x^4)/4 + (4*Log[
1 + a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.919.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.919.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(a^{4} x^{4}-4 a^{3} x^{3}+8 a^{2} x^{2}-16 a x +16 \ln \left(a x +1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{4 a^{3} \left(a x -1\right)^{2}}$	89

input
```
int(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/4*(a^4*x^4-4*a^3*x^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2
/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

3.919.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log{(ax+1)})\sqrt{a^2c}}{4\,a^5}$$

input
```
integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

output
```
1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(a^2*
c)/a^5
```

3.919.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\text{Timed out}$$

input
```
integrate(x**3*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.919.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int \sqrt{c-\frac{c}{a^2x^2}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.919.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.919.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.920 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

3.920.1 Optimal result

Integrand size = 27, antiderivative size = 151

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
4*x*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x^2*(c-c/a^2/x^2)^(1/2
)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*
ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)
```

3.920.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.41

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax(24-9ax+2a^2x^2)-24\log(1+ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(
6*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

3.920.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6751, 6747, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(1-ax)^2}{ax+1}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(ax^2-3x+\frac{4}{a}-\frac{4}{a(ax+1)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(ax+1)}{a^2}+\frac{ax^3}{3}+\frac{4x}{a}-\frac{3x^2}{2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((4*x)/a - (3*x^2)/2 + (a*x^3)/3 - (4*Log[1 + a*x])
/a^2))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.920.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.920.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\left(-2 a^{3} x^{3}+9 a^{2} x^{2}-24 a x +24 \ln \left(a x +1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{6 a^{2} \left(a x -1\right)^{2}}$	82

input
```
int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/6*(-2*a^3*x^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

3.920.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,a^3x^3-9\,a^2x^2+24\,ax-24\,\log{(ax+1)})\sqrt{a^2c}}{6\,a^4}$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

output
```
1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(a^2*c)/a^4
```

3.920.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.920.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.920.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.920.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.921 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

3.921.1 Optimal result

Integrand size = 25, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-3*x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)
/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/
2)
```

3.921.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/Sqrt[1 -
 1/(a^2*x^2)]
```

3.921.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{ax+1}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(ax+\frac{4}{ax+1}-3\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/(a*Sqrt[1
- 1/(a^2*x^2)])
```

3.921.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.921.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(a^{2} x^{2}-6 a x+8 \ln \left(a x+1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{2 \left(a x-1\right)^{2} a}$	73

input
```
int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((
a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/a
```

3.921.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{(a^2x^2-6\,ax+8\,\log{(ax+1)})\sqrt{a^2c}}{2\,a^3}$$

input
```
integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(a^2*c)/a^3
```

3.921.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\text{Timed out}$$

input
```
integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.921.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.921.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.921.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.922 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.922.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1
/a^2/x^2)^(1/2)-4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

3.922.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

3.922.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 93$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a}{ax+1}+a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax-4\log(ax+1)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

3.922.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.922.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{\left(-ax+4\ln\left(ax+1\right)-\ln\left(x\right)\right)x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(ax-1\right)^{2}}$	67

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+4*ln(a*x+1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)
/(a*x+1))^(3/2)/(a*x-1)^2
```

3.922.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.23

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-4\log(ax+1)+\log(x))}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

3.922.6 Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

3.922.7 Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.922.8 Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.922.9 Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.923 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

3.923.1 Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(c-c/a^2/x^2)^(1/2)/(
1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.923.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{ax}-3\log(x)+4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-(1/(a*x)) - 3*Log[x] + 4*Log[1 + a*x]))/Sqrt[1 -
1/(a^2*x^2)]
```

3.923.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^2(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4a^2}{ax+1}-\frac{3a}{x}+\frac{1}{x^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[
1 - 1/(a^2*x^2)])
```

3.923. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}dx$

3.923.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.923.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{(4a\ln(ax+1)x-3a\ln(x)x-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^{2}}$	66

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(4*a*ln(a*x+1)*x-3*a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*
x-1)/(a*x+1))^(3/2)/(a*x-1)^2
```

3.923.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{a^2c}(4\,ax\log{(ax+1)}-3\,ax\log{(x)}-1)}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a^2*x)
```

3.923.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Timed out
```

3.923.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.923.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

3.923.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.924 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

3.924.1 Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+3*(c-c/a^2/x^2)^(1/2)/x
/(1-1/a^2/x^2)^(1/2)+4*a*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*a
*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.924.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2ax^2}+\frac{3}{x}+4a\log(x)-4a\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/2*1/(a*x^2) + 3/x + 4*a*Log[x] - 4*a*Log[1 + a*
x]))/Sqrt[1 - 1/(a^2*x^2)]
```

3.924.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^3(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^3}{ax+1}+\frac{4a^2}{x}-\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1
+ a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.924. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}dx$

3.924.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.924.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{\left(8 a^{2} \ln \left(a x +1\right) x^{2}-8 a^{2} \ln \left(x \right) x^{2}-6 a x +1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 \left(a x -1\right)^{2} x}$	82

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/2*(8*a^2*ln(a*x+1)*x^2-8*a^2*ln(x)*x^2-6*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)
^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x
```

3.924.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{8\,a^3\sqrt{c}x^2\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)+\sqrt{a^2c}(6\,ax-1)}{2\,a^2x^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fr
icas")
```

output
```
1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) + sqrt(a^2*c)*(6*a*x - 1))/(a^2*x^2)
```

3.924.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Timed out
```

3.924.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.924.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

3.924.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.925 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

3.925.1 Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/3*(c-c/a^2/x^2)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)
/x^2/(1-1/a^2/x^2)^(1/2)-4*a*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a
^2*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(a*x+1)*(c-c/a^2/
x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.925.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3ax^3}+\frac{3}{2x^2}-\frac{4a}{x}-4a^2\log(x)+4a^2\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/3*1/(a*x^3) + 3/(2*x^2) - (4*a)/x - 4*a^2*Log[x
] + 4*a^2*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

3.925.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \textit{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textit{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^4(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textit{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4a^4}{ax+1}-\frac{4a^3}{x}+\frac{4a^2}{x^2}-\frac{3a}{x^3}+\frac{1}{x^4}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textit{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^3\log(x)+4a^3\log(ax+1)-\frac{4a^2}{x}+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log
[x] + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.925. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}dx$

3.925.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.925.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(24a^{3}\ln\left(ax+1\right)x^{3}-24a^{3}\ln\left(x\right)x^{3}-24a^{2}x^{2}+9ax-2\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6\left(ax-1\right)^{2}x^{2}}$	90

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOS
E)
```

output
```
1/6*(24*a^3*ln(a*x+1)*x^3-24*a^3*ln(x)*x^3-24*a^2*x^2+9*a*x-2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

3.925.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{24\,a^4\sqrt{c}x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)-(24\,a^2x^2-9\,ax+2)\sqrt{a^2c}}{6\,a^2x^3}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fr
icas")
```

output
```
1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x
+ 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(a^2*c))/(
a^2*x^3)
```

3.925.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Timed out
```

3.925.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.925.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

3.925.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^3}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.926 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

3.926.1 Optimal result

Integrand size = 27, antiderivative size = 221

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = {} & -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3} \\ & -\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\ & +\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
-1/4*(c-c/a^2/x^2)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)/x^3
/(1-1/a^2/x^2)^(1/2)-2*a*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^2
*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(x)*(c-c/a^2/x^2)^(1/2)
/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/
2)
```

3.926.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.34

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4ax^4}+\frac{1}{x^3}-\frac{2a}{x^2}+\frac{4a^2}{x}+4a^3\log(x)-4a^3\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/4*1/(a*x^4) + x^(-3) - (2*a)/x^2 + (4*a^2)/x +
4*a^3*Log[x] - 4*a^3*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

3.926.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.36, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x^4}dx$$
$$\Big\downarrow \text{6751}$$
$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$\Big\downarrow \text{6747}$$
$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^5(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-\frac{4a^5}{ax+1}+\frac{4a^4}{x}-\frac{4a^3}{x^2}+\frac{4a^2}{x^3}-\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^4\log(x)-4a^4\log(ax+1)+\frac{4a^3}{x}-\frac{2a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a
^4*Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

3.926.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.926. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

3.926.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{\left(16 \ln \left(a x+1\right) x^{4} a^{4}-16 \ln \left(x \right) x^{4} a^{4}-16 a^{3} x^{3}+8 a^{2} x^{2}-4 a x+1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{4 \left(a x-1\right)^{2} x^{3}}$	98

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/4*(16*ln(a*x+1)*x^4*a^4-16*ln(x)*x^4*a^4-16*a^3*x^3+8*a^2*x^2-4*a*x+1)*
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^
3
```

3.926.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{16\,a^{5}\sqrt{c}x^{4}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax+1)\sqrt{c}+ac}{ax^{2}+x}\right)+\left(16\,a^{3}x^{3}-8\,a^{2}x^{2}+4\,ax-1\right)\sqrt{a^{2}c}}{4\,a^{2}x^{4}}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fr
icas")
```

output
```
1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
+ 1)*sqrt(c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*sq
rt(a^2*c))/(a^2*x^4)
```

3.926.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Timed out
```

3.926.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.926.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

3.926. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

3.926.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.927 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

3.927.1 Optimal result

Integrand size = 27, antiderivative size = 263

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = &-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}\\ &-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
-1/5*(c-c/a^2/x^2)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+3/4*(c-c/a^2/x^2)^(1/2)
/x^4/(1-1/a^2/x^2)^(1/2)-4/3*a*(c-c/a^2/x^2)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)
+2*a^2*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4*a^3*(c-c/a^2/x^2)^(1/
2)/x/(1-1/a^2/x^2)^(1/2)-4*a^4*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/
2)+4*a^4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

3.927.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.34

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5ax^5}+\frac{3}{4x^4}-\frac{4a}{3x^3}+\frac{2a^2}{x^2}-\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/5*1/(a*x^5) + 3/(4*x^4) - (4*a)/(3*x^3) + (2*a^
2)/x^2 - (4*a^3)/x - 4*a^4*Log[x] + 4*a^4*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x
^2)]
```

3.927.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.35, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^6(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4a^6}{ax+1}-\frac{4a^5}{x}+\frac{4a^4}{x^2}-\frac{4a^3}{x^3}+\frac{4a^2}{x^4}-\frac{3a}{x^5}+\frac{1}{x^6}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^5\log(x)+4a^5\log(ax+1)-\frac{4a^4}{x}+\frac{2a^3}{x^2}-\frac{4a^2}{3x^3}+\frac{3a}{4x^4}-\frac{1}{5x^5}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/5*1/x^5 + (3*a)/(4*x^4) - (4*a^2)/(3*x^3) + (2*
a^3)/x^2 - (4*a^4)/x - 4*a^5*Log[x] + 4*a^5*Log[1 + a*x]))/(a*Sqrt[1 - 1/(
a^2*x^2)])
```

3.927.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.927. $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

3.927.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\left(240 \ln \left(a x +1\right) x^{5} a^{5}-240 a^{5} \ln \left(x \right) x^{5}-240 a^{4} x^{4}+120 a^{3} x^{3}-80 a^{2} x^{2}+45 a x -12\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{60 \left(a x -1\right)^{2} x^{4}}$	106

input

```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOS
E)
```

output

```
1/60*(240*ln(a*x+1)*x^5*a^5-240*a^5*ln(x)*x^5-240*a^4*x^4+120*a^3*x^3-80*a
^2*x^2+45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^
(3/2)/(a*x-1)^2/x^4
```

3.927.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{240\,a^{6}\sqrt{c}x^{5}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx+\sqrt{a^{2}c}(2\,ax+1)\sqrt{c}+ac}{ax^{2}+x}\right)-(240\,a^{4}x^{4}-120\,a^{3}x^{3}+80\,a^{2}x^{2}-45\,ax+12)\sqrt{a^{2}c}}{60\,a^{2}x^{5}}$$

input

```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fr
icas")
```

output

```
1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*
x + 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (240*a^4*x^4 - 120*a^3*x^3 + 80*a^2*x
^2 - 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

3.927.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Timed out
```

3.927.7 Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.927.8 Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

3.927.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.928 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

3.928.1 Optimal result

Integrand size = 20, antiderivative size = 154

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx &= \frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}\\ &\quad-\frac{2^{1+\frac{n}{2}}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}\end{aligned}$$

output
```
4*c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2
*n],(a-1/x)/(a+1/x))/a/(2-n)-2^(1+1/2*n)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([
-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/a/(2-n)
```

3.928.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$

$$= \frac{ce^{n\coth^{-1}(ax)}\left(2ax+anx+e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\,\text{Hype}\right.}{a(}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
(c*E^(n*ArcCoth[a*x])*(2*a*x + a*n*x + E^(2*ArcCoth[a*x])*n*Hypergeometric
2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*Hypergeometric2F1[1
, n/2, 1 + n/2, E^(2*ArcCoth[a*x])] + 4*E^(2*ArcCoth[a*x])*Hypergeometric2
F1[2, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]))/(a*(2 + n))
```

3.928.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6748, 139, 27, 88, 79, 168, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{n \coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6748}$$

$$-c\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{n+2}{2}} x^2d\frac{1}{x}$$

$$\Big\downarrow \text{139}$$

$$-c\left(\frac{\int \frac{\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}}{a}d\frac{1}{x}}{a^2}+\int \frac{\left(a+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2}{a}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{\int \left(3a+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}d\frac{1}{x}}{a^3}+\frac{\int \left(a+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2d\frac{1}{x}}{a}\right)$$

$$\Big\downarrow \text{88}$$

$$-c\left(\frac{-\frac{an\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}d\frac{1}{x}}{2-n}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}}{2-n}}{a^3}+\frac{\int \left(a+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2d\frac{1}{x}}{a}\right)$$

$$\Big\downarrow \text{79}$$

$$-c\left(\frac{\int \left(a+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2d\frac{1}{x}}{a}+\frac{\frac{a^2 2^{n/2}n\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(2-n)(4-n)}-\frac{2a^2\left(1-\frac{1}{ax}\right.}{}}{a^3}\right.$$

$$\Big\downarrow \ 168$$

$$-c\left(\frac{-\int -n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}xd\frac{1}{x}-ax\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}}{a}+\frac{\frac{a^2 2^{n/2}n\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2}\right.}{(2-n)(4-n)}}{}\right.$$

$$\Big\downarrow \ 25$$

$$-c\left(\frac{\int n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}xd\frac{1}{x}-ax\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{a}+\frac{\frac{a^2 2^{n/2}n\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2-\right.}{(2-n)(4-n)}}{a^3}\right.$$

$$\Big\downarrow \ 27$$

$$-c\left(\frac{n\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}xd\frac{1}{x}-ax\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{a}+\frac{\frac{a^2 2^{n/2}n\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2-\right.}{(2-n)(4-n)}}{a^3}\right.$$

$$\Big\downarrow \ 141$$

$$-c\left(\frac{\frac{a^2 2^{n/2}n\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(2-n)(4-n)}-\frac{2a^2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{2-n}}{a^3}+\frac{2n\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hype}}{2-}\right.$$

input

```
Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

3.928. $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

output
```
-(c*((-(a*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x) + (2*n*(1
- 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[1, (-2
 + n)/2, n/2, (a + x^(-1))/(a - x^(-1))])/(2 - n))/a + ((-2*a^2*(1 - 1/(a*
x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(2 - n) + (2^(n/2)*a^2*n*(1 - 1/
(a*x))^(2 - n/2)*Hypergeometric2F1[(2 - n)/2, 2 - n/2, 3 - n/2, (a - x^(-1
))/(2*a)])/((2 - n)*(4 - n)))/a^3))
```

3.928.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 139
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[f^(p - 1)/d^p   Int[(a + b*x)^m*((d*e*p - c*f*(p - 1) +
 d*f*x)/(c + d*x)^(m + 1)), x], x] + Simp[f^(p - 1)   Int[(a + b*x)^m*((e +
 f*x)^p/(c + d*x)^(m + 1))*ExpandToSum[f^(-p + 1)*(c + d*x)^(-p + 1) - (d*e
*p - c*f*(p - 1) + d*f*x)/(d^p*(e + f*x)^p), x], x], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && EqQ[m + n + p, 0] && ILtQ[p, 0] && (LtQ[m, 0] || SumS
implerQ[m, 1] ||  !(LtQ[n, 0] || SumSimplerQ[n, 1]))
```

3.928. $\int e^{n \coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.928.4 Maple [F]

$$\int \mathrm{e}^{n \operatorname{arccoth}(ax)} \left(c - \frac{c}{a^2 x^2}\right) dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

3.928.5 Fricas [F]

$$\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{a^2 x^2}\right) dx = \int \left(c - \frac{c}{a^2 x^2}\right) \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*x^2), x)
```

3.928.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\int a^2e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-\frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^2}\right)\,dx\right)}{a^2}$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2),x)
```

output
```
c*(Integral(a**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x**2,
 x))/a**2
```

3.928.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.928.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.928.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)), x)
```

3.929 $\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

3.929.1 Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn}+\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c}+\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac}$$

output
```
-(1+n)*(1+1/a/x)^(1/2*n)/a/c/n/((1-1/a/x)^(1/2*n))+(1+1/a/x)^(1/2*n)*x/c/(
(1-1/a/x)^(1/2*n))+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n],(a+1
/x)/(a-1/x))/a/c/((1-1/a/x)^(1/2*n))
```

3.929.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{e^{n \coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n^2\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+n\,\mathrm{Hy}\right.\right.}{acn(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output

```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2,
2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + n*Hypergeometric2F1[1
, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

3.929.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6748, 144, 25, 27, 172, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x^2d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 144}$$

$$-\frac{x\left(-\left(\frac{1}{ax}+1\right)^{n/2}\right)\left(1-\frac{1}{ax}\right)^{-n/2}-\int -\frac{\left(an+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x}{a^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\left(an+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\int \left(an+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{ 172}$$

$$-\frac{\frac{\frac{a(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{n}-\frac{a\int -n^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{n}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\frac{a\int n^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{n}+\frac{a(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{n}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{an\int\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}+\frac{a(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{n}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{141}$$

$$-\frac{\frac{\frac{a(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{n}-2a\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

input

```
Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output

```
-((-(((1 + 1/(a*x))^(n/2)*x)/(1 - 1/(a*x))^(n/2)) + ((a*(1 + n)*(1 + 1/(a*
x))^(n/2))/(n*(1 - 1/(a*x))^(n/2)) - (2*a*(1 + 1/(a*x))^(n/2)*Hypergeometr
ic2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(1 - 1/(a*x))^(n/2))/
a^2)/c)
```

3.929.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

3.929. $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.929.4 Maple [F]

$$\int \frac{\mathrm{e}^{n \operatorname{arccoth}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

3.929.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{a^2x^2}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

3.929.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2\int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2),x)
```

output
```
a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 1), x)/c
```

3.929.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{a^2x^2}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

3.929.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{a^2x^2}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

3.929.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a^2\,x^2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)), x)
```

3.930 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.930.1 Optimal result

Integrand size = 22, antiderivative size = 289

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{(6+4n+n^2)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} + \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac^2}$$

output
```
-(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/(2+n)+(-n^3-n^2+4*n
+6)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/n/(-n^2+4)-(n^2+4*n+6)*
(1+1/a/x)^(-1+1/2*n)/a/c^2/n/(2+n)/((1-1/a/x)^(1/2*n))+(1-1/a/x)^(-1-1/2*n
)*(1+1/a/x)^(-1+1/2*n)*x/c^2+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1
/2*n],(a+1/x)/(a-1/x))/a/c^2/((1-1/a/x)^(1/2*n))
```

3.930.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$=\frac{e^{n\coth^{-1}(ax)}\Big(-6+n^2+6anx-an^3x+6a^2x^2-2a^2n^2x^2-4a^3nx^3+a^3n^3x^3+e^{2\coth^{-1}(ax)}(-2+n)n^2(-1}{ac^2}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
(E^(n*ArcCoth[a*x])*(-6 + n^2 + 6*a*n*x - a*n^3*x + 6*a^2*x^2 - 2*a^2*n^2*
x^2 - 4*a^3*n*x^3 + a^3*n^3*x^3 + E^(2*ArcCoth[a*x])*(-2 + n)*n^2*(-1 + a^
2*x^2)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + n*(-4
+ n^2)*(-1 + a^2*x^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x]
)]))/(a*c^2*(-2 + n)*n*(2 + n)*(-1 + a^2*x^2))
```

3.930.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6748, 144, 25, 27, 172, 25, 27, 172, 25, 27, 172, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{144}$$

$$-\frac{x\left(-\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}-\int -\frac{\left(an+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x}{a^2}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(an+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\int \left(an+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow 172$$

$$-\frac{\frac{\frac{a(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n+2}-a\int -\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(an(n+2)+\frac{2(n+3)}{x}\right)x}{\frac{a}{n+2}}d\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(an(n+2)+\frac{2(n+3)}{x}\right)x}{\frac{a}{n+2}}d\frac{1}{x}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(an(n+2)+\frac{2(n+3)}{x}\right)xd\frac{1}{x}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow 172$$

$$-\frac{\frac{\frac{\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}-a\int -\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(a(n+2)n^2+\frac{n^2+4n+6}{x}\right)x}{\frac{a}{n}}d\frac{1}{x}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\Big(1}{c^2}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(a(n+2)n^2+\frac{n^2+4n+6}{x}\right)x}{\frac{a}{n}}d\frac{1}{x}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\Big(1-}{c^2}$$

$$\Big\downarrow 27$$

3.930. $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$

$$-\frac{\frac{\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(a(n+2)n^2+\frac{n^2+4n+6}{x}\right)xd\frac{1}{x}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x(1-}{c^2}$$

$$\Big\downarrow \textcolor{blue}{172}$$

$$-\frac{\frac{\frac{\frac{\frac{a\int n^2\left(4-n^2\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{2-n}-\frac{a\left(-n^3-n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{2-n}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+\right.}{}}{a^2}}{c^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\frac{\frac{\frac{\frac{an^2\left(4-n^2\right)\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{2-n}-\frac{a\left(-n^3-n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{2-n}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+\right.}{}}{a^2}}{c^2}$$

$$\Big\downarrow \textcolor{blue}{141}$$

$$-\frac{\frac{\frac{\frac{-\frac{2an\left(4-n^2\right)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{2-n}-\frac{a\left(-n^3-n^2+4n+6\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{2-n}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n}}{n}}{n+2}}{a^2}}{c^2}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
-((-((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x) + ((a*(3 + n)*
(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(2 + n) + ((a*(6 + 4*
n + n^2)*(1 + 1/(a*x))^((-2 + n)/2))/(n*(1 - 1/(a*x))^(n/2)) + (-((a*(6 +
4*n - n^2 - n^3)*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(2 -
n)) - (2*a*n*(4 - n^2)*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, n/2, (2 +
n)/2, (a + x^(-1))/(a - x^(-1))])/((2 - n)*(1 - 1/(a*x))^(n/2)))/n)/(2 + n
))/a^2)/c^2)
```

3.930. $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.930.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.930. $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

3.930.4 Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

3.930.5 Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
integral(a^4*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^
2 + c^2), x)
```

3.930.6 Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{a^4\int \frac{x^4e^{n\operatorname{acoth}(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*Integral(x**4*exp(n*acoth(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**
2
```

3.930.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

3.930.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

3.930.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a^2\,x^2}\right)^2}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2,x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2, x)
```

3.931 $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.931.1 Optimal result

Integrand size = 24, antiderivative size = 295

$$\begin{aligned}
\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{1-\frac{1}{a^2x^2}}} \\
&\quad+\frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}} \\
&\quad-\frac{2^{\frac{1+n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

output
```
(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2
/x^2)^(1/2)+2*n*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*hypergeom([1,
 1/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))*(c-c/a^2/x^2)^(1/2)/a/(1-n)/(1-1/
a^2/x^2)^(1/2)-2^(1/2+1/2*n)*(1-1/a/x)^(1/2-1/2*n)*hypergeom([1/2-1/2*n, 1
/2-1/2*n],[3/2-1/2*n],1/2*(a-1/x)/a)*(c-c/a^2/x^2)^(1/2)/a/(1-n)/(1-1/a^2/
x^2)^(1/2)
```

3.931.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.49

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{ae^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}x^2\left(a(1+n)\sqrt{1-\frac{1}{a^2x^2}}x+2e^{\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\right.\right.}{(1+n)\left(-1+a^2x^2\right)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2*(a*(
1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E^ArcCoth[a*x]*Hypergeometric2F1[1, (1
+ n)/2, (3 + n)/2, -E^(2*ArcCoth[a*x])] + 2*E^ArcCoth[a*x]*n*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]))/((1 + n)*(-1 + a^2*x^2
))
```

3.931.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6751, 6748, 140, 79, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{n\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6751}$$
$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$\Big\downarrow \text{6748}$$
$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n+1}{2}}x^2d\frac{1}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$\Big\downarrow \text{140}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^2d\frac{1}{x}-\frac{\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}d\frac{1}{x}}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 79$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^2d\frac{1}{x}+\frac{2^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 107$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\Bigg(\frac{n\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}xd\frac{1}{x}}{a}+\frac{2^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)}+x\Big(-\big(\frac{1}{ax}+1\big)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 141$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\Bigg(-\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)}+\frac{2^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\Big(\frac{1-n}{2},\frac{1-n}{2}}{a(1-n)}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*(-((1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 +
n)/2)*x) - (2*n*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hyper
geometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a*(1 -
n)) + (2^((1 + n)/2)*(1 - 1/(a*x))^((1 - n)/2)*Hypergeometric2F1[(1 - n)/2
, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(2*a)])/(a*(1 - n))))/Sqrt[1 - 1/(a^2
*x^2)])
```

3.931. $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

3.931.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.931. $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.931.4 Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

3.931.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)), x)
```

3.931.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

3.931.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.931.8 Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.931.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-\frac{c}{a^2\,x^2}}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2), x)
```

3.932 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

3.932.1 Optimal result

Integrand size = 24, antiderivative size = 183

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx&=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}}\\&\quad+\frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2
/x^2)^(1/2)+2*n*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*hypergeom([1,
 1/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/a/(1-n)/(c-c/
a^2/x^2)^(1/2)
```

3.932.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$
$$= \frac{e^{n\coth^{-1}(ax)}(-1+a^2x^2)\left(a(1+n)\sqrt{1-\frac{1}{a^2x^2}}x+2e^{\coth^{-1}(ax)}n\,\mathrm{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e^{2\coth^{-1}(ax}\right.\right.}{a^3(1+n)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}x^2}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(E^(n*ArcCoth[a*x])*(-1 + a^2*x^2)*(a*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*
E^ArcCoth[a*x]*n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a
*x])]))/(a^3*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2)
```

3.932.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6748, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$
$$\Big\downarrow \ 6751$$
$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$
$$\Big\downarrow \ 6748$$
$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^2d\frac{1}{x}}{\sqrt{c-\frac{c}{a^2x^2}}}$$
$$\Big\downarrow \ 107$$

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{n\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}xd\frac{1}{x}}{a}-x\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow 141$$

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}-\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((Sqrt[1 - 1/(a^2*x^2)]*(-((1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 +
n)/2)*x) - (2*n*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hyper
geometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a*(1 -
n))))/Sqrt[c - c/(a^2*x^2)])
```

3.932.3.1 Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

3.932. $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.932.4 Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

3.932.5 Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c), x)
```

3.932.6 Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{e^{n \operatorname{acoth}(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x)
```

3.932.7 Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

3.932.8 Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

3.932.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2), x)
```

3.933 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

3.933.1 Optimal result

Integrand size = 22, antiderivative size = 116

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}\operatorname{AppellF1}\left(1+\frac{n}{2}+p,\frac{1}{2}(n-2p),2,2+\frac{n}{2}+p,\frac{a+\frac{1}{x}}{2a},1+\right.}{a(2+n+2p)}$$

output
```
-2^(1-1/2*n+p)*(c-c/a^2/x^2)^p*(1+1/a/x)^(1+1/2*n+p)*AppellF1(1+1/2*n+p,1/
2*n-p,2,2+1/2*n+p,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n+2*p)/((1-1/a^2/x^2)^p)
```

3.933.2 Mathematica [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = \int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

output
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

3.933.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6751, 6748, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$

$$\Big\downarrow \text{6748}$$

$$-\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n}{2}+p}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{-\frac{n}{2}+p+1}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}\text{AppellF1}\left(\frac{n}{2}+p+1,\frac{1}{2}(n-2p),2,\frac{n}{2}+p+2,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2p+2)}$$

input

```
Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

output

```
-((2^(1 - n/2 + p)*(c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + n/2 + p)*AppellF
1[1 + n/2 + p, (n - 2*p)/2, 2, 2 + n/2 + p, (a + x^(-1))/(2*a), 1 + 1/(a*x
)])/(a*(2 + n + 2*p)*(1 - 1/(a^2*x^2))^p))
```

3.933.3.1 Defintions of rubi rules used

rule 153

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.933.4 Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

3.933.5 Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

3.933.6 Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^pe^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

3.933.7 Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.933.8 Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

3.933.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

3.934 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

3.934.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1-\frac{1}{ax}\right)^{1+2p}\text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1-\frac{1}{ax}\right)}{a(1+2p)}$$

output
```
(c-c/a^2/x^2)^p*(1-1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2*p+2],1-1/a/x)/a/
(1+2*p)/((1-1/a^2/x^2)^p)
```

3.934.2 Mathematica [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = \int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

input
```
Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]), x]
```

3.934.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6751, 6748, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p e^{-2p\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6751$$

$$\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$

$$\Big\downarrow \; 6748$$

$$-\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int \left(1-\frac{1}{ax}\right)^{2p}x^2d\frac{1}{x}$$

$$\Big\downarrow \; 75$$

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}\left(2,2p+1,2(p+1),1-\frac{1}{ax}\right)}{a(2p+1)}$$

input
```
Int[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^p*(1 - 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p,
 2*(1 + p), 1 - 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

3.934.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.934.4 Maple [F]

$$\int \left(c-\frac{c}{a^{2} x^{2}}\right)^{p} {\mathrm e}^{-2 p \,\operatorname{arccoth}\left(a x \right)}d x$$

input
```
int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
```

output
```
int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
```

3.934.5 Fricas [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \frac{\left(c-\frac{c}{a^{2} x^{2}}\right)^{p}}{\left(\frac{a x+1}{a x-1}\right)^{p}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

output
```
integral(((a^2*c*x^2 - c)/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.934.6 Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \left(- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)\right)^{p} e^{- 2 p \operatorname{acoth}{\left(a x \right)}}\, dx$$

input
```
integrate((c-c/a**2/x**2)**p/exp(2*p*acoth(a*x)),x)
```

output
```
Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

3.934.7 Maxima [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

output
```
integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.934.8 Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

3.934.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \mathrm{e}^{-2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

input
```
int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

output
```
int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

3.935 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

3.935.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx \\ =-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+2p}\text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{1}{ax}\right)}{a(1+2p)}$$

output
```
-(c-c/a^2/x^2)^p*(1+1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2*p+2],1+1/a/x)/a
/(1+2*p)/((1-1/a^2/x^2)^p)
```

3.935.2 Mathematica [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

input
```
Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

output
```
Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

3.935.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6751, 6748, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p e^{2p\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6751}$$

$$\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$

$$\Big\downarrow \text{ 6748}$$

$$-\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int \left(1+\frac{1}{ax}\right)^{2p}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}\left(2,2p+1,2(p+1),1+\frac{1}{ax}\right)}{a(2p+1)}$$

input
```
Int[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

output
```
-(((c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*
p, 2*(1 + p), 1 + 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p))
```

3.935.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

3.935.4 Maple [F]

$$\int \mathrm{e}^{2p\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

input
```
int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

output
```
int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

3.935.5 Fricas [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int \left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^p*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

3.935.6 Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{2p\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(2*p*acoth(a*x))*(c-c/a**2/x**2)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

3.935.7 Maxima [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.935.8 Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

3.935.9 Mupad [F(-1)]

Timed out.

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int e^{2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

input
```
int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

output
```
int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != −1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2∗leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2∗leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(l
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```