Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.5-Inverse-hyperbolic-secant/201-7.5.2-Inverse-hyperbolic-secant-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [100]. This is test number [201].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (100)	0.00 (0)
Mathematica	99.00 (99)	1.00 (1)
Maple	83.00 (83)	17.00 (17)
Fricas	73.00 (73)	27.00 (27)
Mupad	56.00 (56)	44.00 (44)
Maxima	21.00 (21)	79.00 (79)
Giac	4.00 (4)	96.00 (96)
Sympy	3.00 (3)	97.00 (97)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

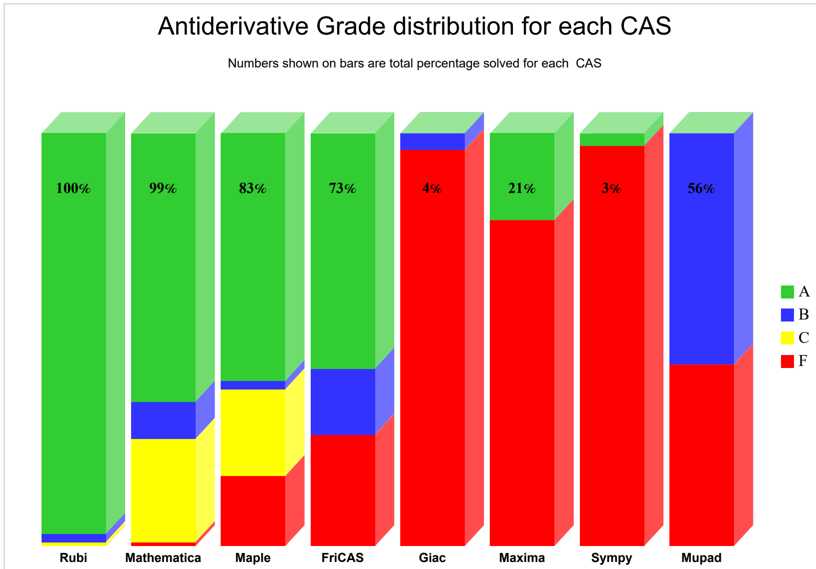
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

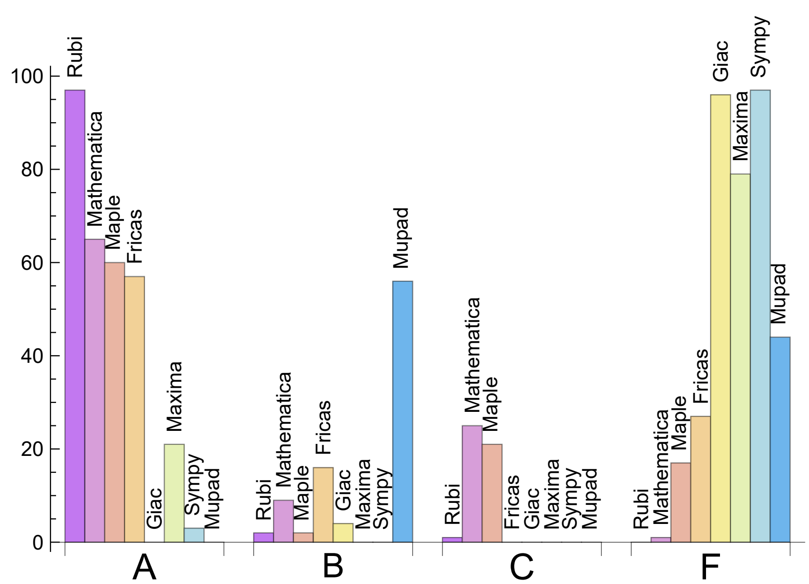
System	% A grade	% B grade	% C grade	% F grade
Rubi	89.000	2.000	9.000	0.000
Mathematica	65.000	9.000	25.000	1.000
Maple	60.000	2.000	21.000	17.000
Fricas	57.000	16.000	0.000	27.000
Maxima	21.000	0.000	0.000	79.000
Sympy	3.000	0.000	0.000	97.000
Giac	0.000	4.000	0.000	96.000
Mupad	0.000	56.000	0.000	44.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	17	100.00	0.00	0.00
Fricas	27	77.78	0.00	22.22
Mupad	44	0.00	100.00	0.00
Maxima	79	93.67	0.00	6.33
Giac	96	89.58	0.00	10.42
Sympy	97	98.97	1.03	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.22
Fricas	0.25
Maple	0.47
Rubi	0.49
Giac	0.66
Mathematica	1.11
Sympy	3.21
Mupad	13.15

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	49.76	0.58	46.00	0.67
Sympy	78.33	1.23	82.00	0.95
Fricas	136.37	1.55	87.00	1.05
Rubi	138.98	1.11	107.00	1.00
Maple	177.34	1.83	104.00	1.15
Giac	194.75	2.59	197.50	2.62
Mathematica	222.89	1.49	101.00	1.09
Mupad	329.71	2.76	88.50	1.74

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

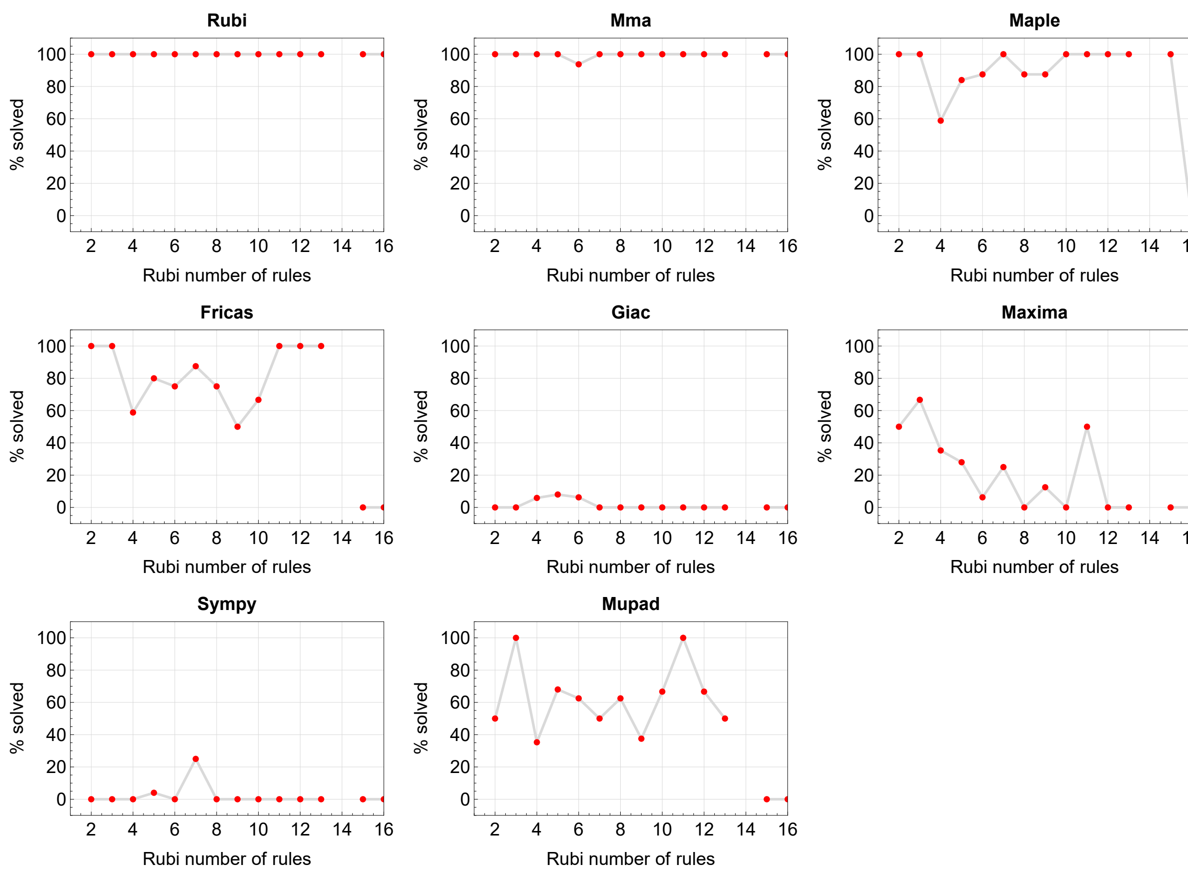


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

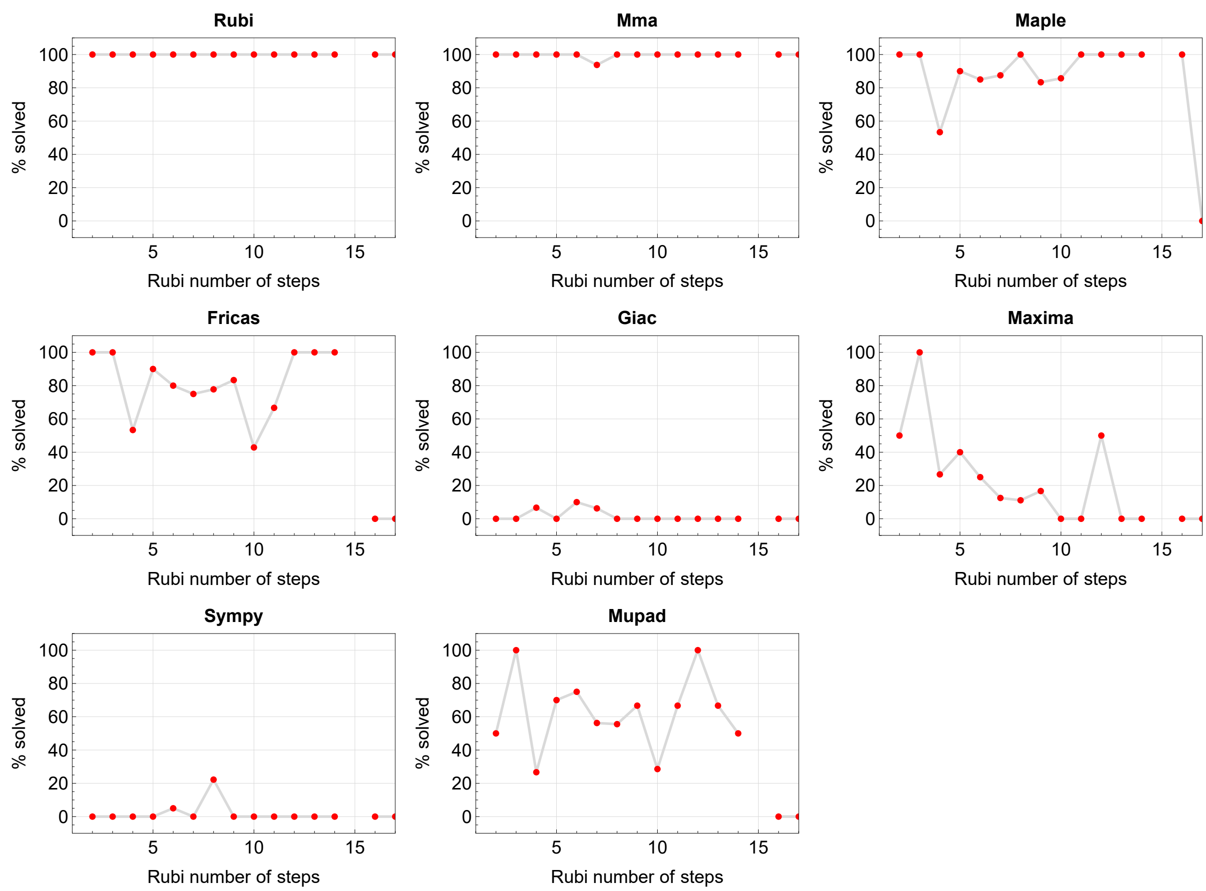


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

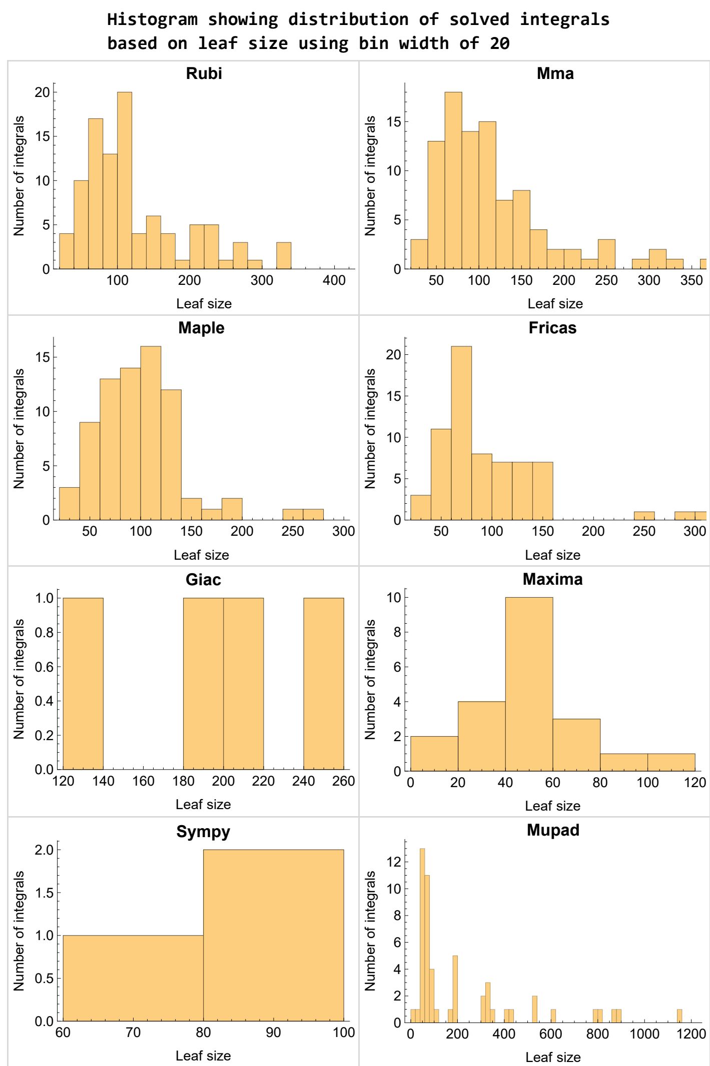


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

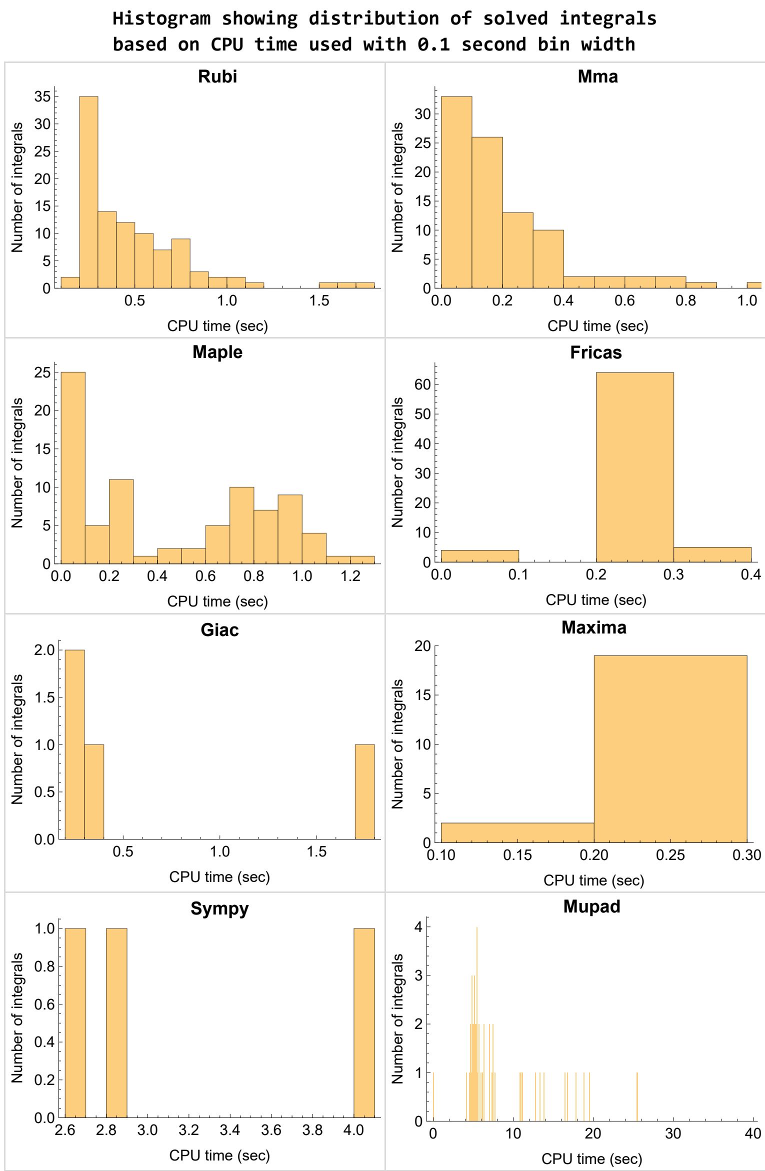


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

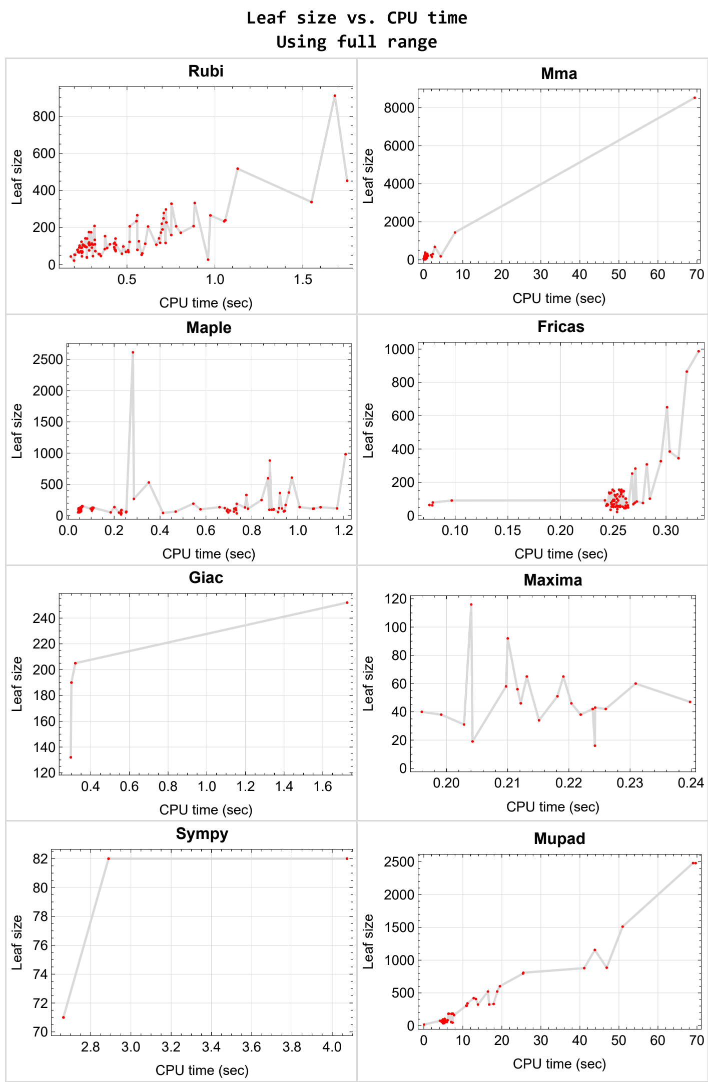


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{24, 29, 30, 31, 37, 47, 53, 63, 98\}$

Mathematica $\{18, 57, 59, 60\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

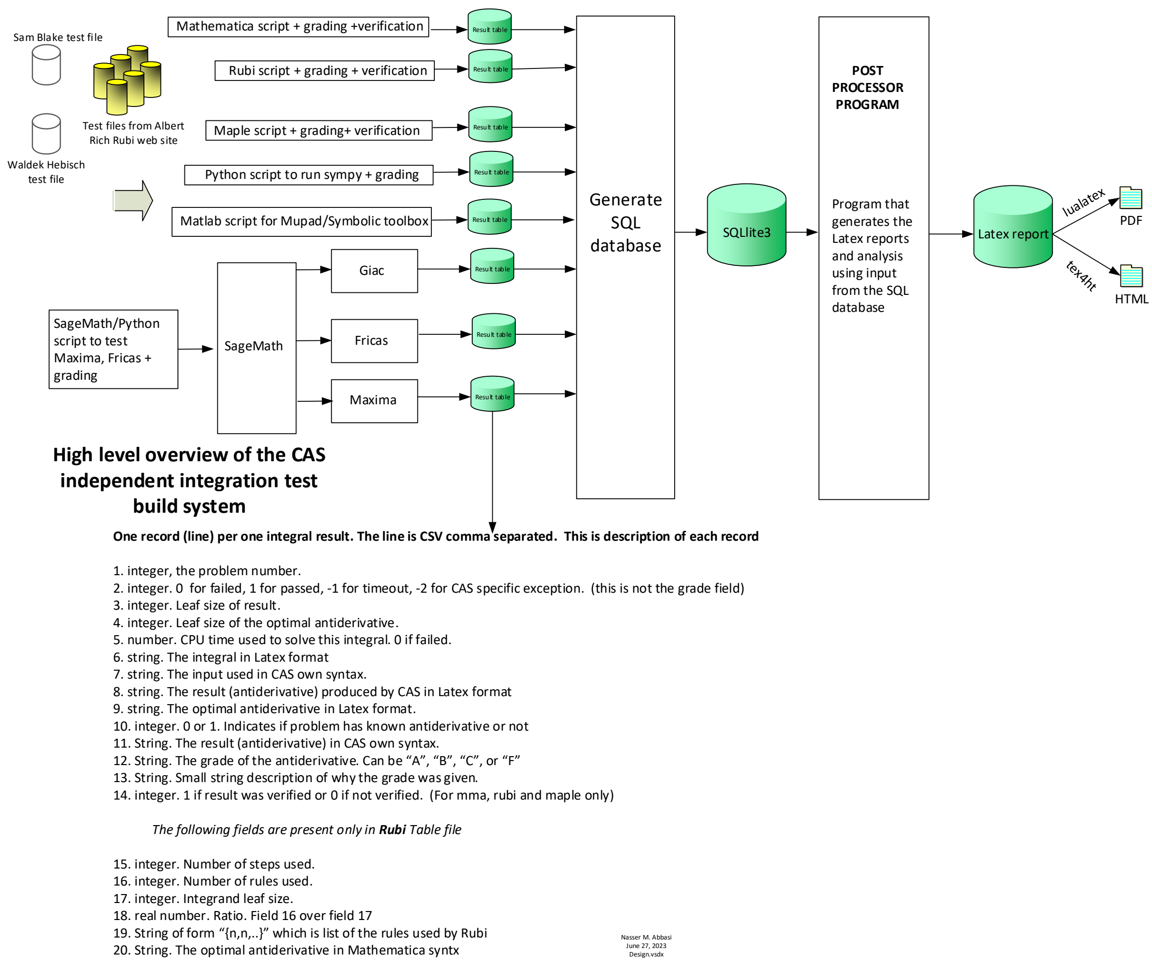
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100 }

B grade { 38, 97 }

C grade { 5, 12, 17, 24, 29, 30, 31, 39, 98 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 8, 9, 10, 11, 12, 15, 16, 17, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 34, 39, 40, 41, 42, 43, 44, 47, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 83, 84, 85, 86, 87, 88, 89, 91, 93, 94, 95, 96, 98, 99, 100 }

B grade { 4, 6, 7, 23, 29, 31, 36, 38, 97 }

C grade { 1, 2, 3, 5, 13, 14, 18, 33, 35, 37, 45, 46, 48, 49, 50, 52, 54, 63, 65, 67, 77, 79, 81, 90, 92 }

F normal fail { 19 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 9, 10, 11, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 66, 68, 70, 71, 72, 73, 74, 75, 82, 83, 84, 85, 86, 87, 89, 91, 93, 95, 97, 98, 99 }

B grade { 6, 80 }

C grade { 5, 7, 8, 33, 35, 37, 51, 55, 63, 65, 67, 69, 76, 77, 78, 79, 81, 90, 92, 94, 96 }

F normal fail { 12, 15, 16, 17, 18, 19, 30, 56, 57, 58, 59, 60, 61, 62, 64, 88, 100 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 20, 21, 22, 23, 25, 26, 27, 28, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 91, 94, 96, 97 }

B grade { 2, 3, 4, 6, 7, 8, 36, 38, 51, 53, 90, 92, 93, 95, 99, 100 }

C grade { }

F normal fail { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 30, 52, 56, 57, 58, 59, 88, 98 }

F(-1) timedout fail { }

F(-2) exception fail { 29, 31, 60, 61, 62, 64 }

2.1.5 Maxima

A grade { 4, 20, 21, 22, 23, 25, 26, 27, 28, 32, 34, 39, 41, 43, 47, 66, 71, 73, 75, 99, 100 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 29, 30, 31, 33, 35, 36, 37, 38, 40, 42, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 63, 64, 65, 67, 68, 69, 70, 72, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

F(-1) timedout fail { }

F(-2) exception fail { 56, 57, 60, 61, 62 }

2.1.6 Giac

A grade { }

B grade { 45, 47, 49, 53 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 52, 54, 56, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

F(-1) timedout fail { }

F(-2) exception fail { 32, 33, 34, 46, 48, 50, 51, 55, 57, 65 }

2.1.7 Mupad

A grade { }

B grade { 4, 25, 28, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 49, 51, 53, 55, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 29, 30, 31, 46, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 88, 98 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 38, 55, 70 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99 }

F(-1) timedout fail { 100 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	206	225	250	0	345	0	0	0
N.S.	1	1.01	1.11	1.23	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.526	0.552	0.842	0.000	0.312	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	200	189	0	327	0	0	0
N.S.	1	1.00	1.31	1.24	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.393	0.249	0.734	0.000	0.295	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	106	176	111	0	308	0	0	0
N.S.	1	0.99	1.64	1.04	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.437	0.184	0.723	0.000	0.282	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	97	44	31	253	0	0	43
N.S.	1	1.00	2.20	1.00	0.70	5.75	0.00	0.00	0.98
time (sec)	N/A	0.243	0.276	0.413	0.203	0.268	0.000	0.000	4.983

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	233	332	882	0	0	0	0	0
N.S.	1	1.37	1.95	5.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.051	0.314	0.878	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	72	244	171	0	651	0	0	0
N.S.	1	1.03	3.49	2.44	0.00	9.30	0.00	0.00	0.00
time (sec)	N/A	0.425	0.269	0.947	0.000	0.301	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	159	315	370	0	865	0	0	0
N.S.	1	1.20	2.37	2.78	0.00	6.50	0.00	0.00	0.00
time (sec)	N/A	0.761	0.897	0.961	0.000	0.320	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	239	368	608	0	987	0	0	0
N.S.	1	1.21	1.87	3.09	0.00	5.01	0.00	0.00	0.00
time (sec)	N/A	1.067	0.360	0.974	0.000	0.331	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	266	305	599	0	0	0	0	0
N.S.	1	0.95	1.09	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.540	2.278	0.870	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	140	172	331	0	0	0	0	0
N.S.	1	0.94	1.15	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	0.523	0.776	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	77	105	192	0	0	0	0	0
N.S.	1	0.96	1.31	2.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	0.240	0.545	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	337	280	0	0	0	0	0	0
N.S.	1	1.23	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.561	0.336	0.000	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	227	678	362	0	0	0	0	0
N.S.	1	1.01	3.03	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.714	2.832	0.922	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	517	1439	982	0	0	0	0	0
N.S.	1	0.96	2.68	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.206	7.997	1.208	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	234	254	0	0	0	0	0	0
N.S.	1	0.90	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.562	0.653	0.000	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	125	153	0	0	0	0	0	0
N.S.	1	0.92	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.572	0.138	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	378	452	384	0	0	0	0	0	0
N.S.	1	1.20	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.832	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	330	332	8527	0	0	0	0	0	0
N.S.	1	1.01	25.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.921	69.418	0.000	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	965	911	0	0	0	0	0	0	0
N.S.	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.788	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	103	84	54	58	57	0	0	0
N.S.	1	0.63	0.51	0.33	0.35	0.35	0.00	0.00	0.00
time (sec)	N/A	0.251	0.050	0.252	0.210	0.255	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	92	72	49	46	52	0	0	0
N.S.	1	0.73	0.57	0.39	0.37	0.41	0.00	0.00	0.00
time (sec)	N/A	0.246	0.037	0.221	0.212	0.250	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	79	56	42	34	45	0	0	0
N.S.	1	0.90	0.64	0.48	0.39	0.51	0.00	0.00	0.00
time (sec)	N/A	0.227	0.030	0.231	0.215	0.260	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	118	36	19	39	0	0	0
N.S.	1	1.00	2.74	0.84	0.44	0.91	0.00	0.00	0.00
time (sec)	N/A	0.184	0.127	0.230	0.204	0.254	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	58	45	65	0	0	0	0	0
N.S.	1	1.26	0.98	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	0.058	0.468	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	79	111	64	65	45	0	0	40
N.S.	1	0.81	1.13	0.65	0.66	0.46	0.00	0.00	0.41
time (sec)	N/A	0.219	0.083	0.253	0.219	0.262	0.000	0.000	4.901

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	102	125	79	92	54	0	0	0
N.S.	1	0.75	0.92	0.58	0.68	0.40	0.00	0.00	0.00
time (sec)	N/A	0.229	0.123	0.230	0.210	0.259	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	123	140	91	116	60	0	0	0
N.S.	1	0.72	0.81	0.53	0.67	0.35	0.00	0.00	0.00
time (sec)	N/A	0.242	0.137	0.232	0.204	0.262	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	25	20	16	22	0	0	17
N.S.	1	1.00	1.19	0.95	0.76	1.05	0.00	0.00	0.81
time (sec)	N/A	0.194	0.022	0.230	0.224	0.254	0.000	0.000	0.070

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	77	219	111	0	0	0	0	0
N.S.	1	1.26	3.59	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.504	1.086	1.066	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	68	49	0	0	0	0	0	0
N.S.	1	1.26	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	97	249	135	0	0	0	0	0
N.S.	1	1.26	3.23	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	1.859	1.099	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	107	65	64	47	65	0	0	75
N.S.	1	1.67	1.02	1.00	0.73	1.02	0.00	0.00	1.17
time (sec)	N/A	0.251	0.099	0.047	0.240	0.245	0.000	0.000	4.127

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	90	97	118	0	95	0	0	521
N.S.	1	1.07	1.15	1.40	0.00	1.13	0.00	0.00	6.20
time (sec)	N/A	0.243	0.143	0.045	0.000	0.250	0.000	0.000	16.438

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	52	48	54	38	54	0	0	55
N.S.	1	1.37	1.26	1.42	1.00	1.42	0.00	0.00	1.45
time (sec)	N/A	0.204	0.068	0.049	0.222	0.254	0.000	0.000	5.112

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	75	92	0	79	0	0	303
N.S.	1	1.00	1.42	1.74	0.00	1.49	0.00	0.00	5.72
time (sec)	N/A	0.208	0.067	0.048	0.000	0.253	0.000	0.000	10.972

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	39	79	80	0	115	0	0	182
N.S.	1	1.62	3.29	3.33	0.00	4.79	0.00	0.00	7.58
time (sec)	N/A	0.274	0.053	0.103	0.000	0.259	0.000	0.000	6.311

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	65	75	92	0	77	0	0	184
N.S.	1	1.35	1.56	1.92	0.00	1.60	0.00	0.00	3.83
time (sec)	N/A	0.232	0.056	0.048	0.000	0.270	0.000	0.000	6.371

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	102	93	91	0	128	71	0	71
N.S.	1	2.91	2.66	2.60	0.00	3.66	2.03	0.00	2.03
time (sec)	N/A	0.265	0.066	0.046	0.000	0.254	2.665	0.000	4.878

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	107	43	53	43	52	0	0	58
N.S.	1	1.95	0.78	0.96	0.78	0.95	0.00	0.00	1.05
time (sec)	N/A	0.250	0.054	0.043	0.224	0.262	0.000	0.000	4.587

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	140	110	110	0	138	0	0	602
N.S.	1	1.06	0.83	0.83	0.00	1.05	0.00	0.00	4.56
time (sec)	N/A	0.279	0.087	0.046	0.000	0.246	0.000	0.000	19.502

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	141	60	63	51	60	0	0	76
N.S.	1	1.23	0.52	0.55	0.44	0.52	0.00	0.00	0.66
time (sec)	N/A	0.269	0.073	0.048	0.218	0.250	0.000	0.000	4.644

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	174	129	132	0	148	0	0	878
N.S.	1	1.07	0.79	0.81	0.00	0.91	0.00	0.00	5.39
time (sec)	N/A	0.300	0.133	0.053	0.000	0.260	0.000	0.000	41.119

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	175	76	71	60	69	0	0	95
N.S.	1	1.20	0.52	0.49	0.41	0.47	0.00	0.00	0.65
time (sec)	N/A	0.300	0.098	0.056	0.231	0.248	0.000	0.000	4.862

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	208	145	152	0	156	0	0	1155
N.S.	1	1.07	0.75	0.78	0.00	0.80	0.00	0.00	5.95
time (sec)	N/A	0.311	0.137	0.060	0.000	0.258	0.000	0.000	43.823

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	95	111	137	0	116	0	205	521
N.S.	1	0.86	1.00	1.23	0.00	1.05	0.00	1.85	4.69
time (sec)	N/A	0.262	0.183	0.201	0.000	0.252	0.000	0.320	18.815

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	99	143	114	0	79	0	0	0
N.S.	1	0.86	1.24	0.99	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.264	0.350	1.070	0.000	0.079	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	71	56	60	42	60	0	190	57
N.S.	1	1.22	0.97	1.03	0.72	1.03	0.00	3.28	0.98
time (sec)	N/A	0.241	0.110	0.052	0.226	0.257	0.000	0.300	5.472

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	91	140	136	0	91	0	0	0
N.S.	1	0.81	1.25	1.21	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.312	0.460	0.659	0.000	0.097	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	92	112	0	102	0	132	306
N.S.	1	1.00	1.46	1.78	0.00	1.62	0.00	2.10	4.86
time (sec)	N/A	0.241	0.103	0.098	0.000	0.262	0.000	0.297	10.881

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	112	102	0	62	0	0	0
N.S.	1	1.00	1.67	1.52	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.225	0.273	0.576	0.000	0.078	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	100	127	0	133	0	0	182
N.S.	1	1.00	1.47	1.87	0.00	1.96	0.00	0.00	2.68
time (sec)	N/A	0.249	0.069	0.110	0.000	0.247	0.000	0.000	7.093

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	107	135	132	0	0	0	0	0
N.S.	1	0.73	0.92	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.328	0.770	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	71	22	103	0	102	0	252	185
N.S.	1	0.89	0.28	1.29	0.00	1.28	0.00	3.15	2.31
time (sec)	N/A	0.242	0.086	0.106	0.000	0.254	0.000	1.726	7.400

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	94	101	104	0	64	0	0	0
N.S.	1	0.82	0.88	0.90	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.269	0.227	0.895	0.000	0.076	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	97	105	129	0	146	82	0	71
N.S.	1	0.82	0.89	1.09	0.00	1.24	0.69	0.00	0.60
time (sec)	N/A	0.271	0.397	0.105	0.000	0.260	4.073	0.000	5.193

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	113	187	0	0	0	0	0	0
N.S.	1	1.04	1.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	0.612	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	107	113	159	0	0	0	0	0	0
N.S.	1	1.06	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	2.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	145	0	0	0	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.368	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	139	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.713	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	186	0	0	0	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	4.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	159	0	0	0	0	0	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.327	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	164	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.251	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	76	96	116	0	102	0	0	0
N.S.	1	0.87	1.10	1.33	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.289	0.152	1.171	0.000	0.285	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	166	0	0	0	0	0	0
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	265	105	136	0	103	0	0	808
N.S.	1	1.31	0.52	0.67	0.00	0.51	0.00	0.00	3.98
time (sec)	N/A	1.005	0.159	0.060	0.000	0.262	0.000	0.000	25.515

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	171	52	72	42	62	0	0	63
N.S.	1	1.46	0.44	0.62	0.36	0.53	0.00	0.00	0.54
time (sec)	N/A	0.832	0.075	0.051	0.224	0.244	0.000	0.000	4.678

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	205	86	105	0	87	0	0	420
N.S.	1	1.21	0.51	0.62	0.00	0.51	0.00	0.00	2.49
time (sec)	N/A	0.626	0.078	0.054	0.000	0.249	0.000	0.000	12.778

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	116	89	98	0	124	0	0	56
N.S.	1	1.36	1.05	1.15	0.00	1.46	0.00	0.00	0.66
time (sec)	N/A	0.713	0.069	0.045	0.000	0.251	0.000	0.000	7.010

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	82	75	111	0	85	0	0	162
N.S.	1	1.44	1.32	1.95	0.00	1.49	0.00	0.00	2.84
time (sec)	N/A	0.426	0.107	0.055	0.000	0.272	0.000	0.000	7.778

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	107	86	110	0	138	82	0	323
N.S.	1	1.24	1.00	1.28	0.00	1.60	0.95	0.00	3.76
time (sec)	N/A	0.651	0.067	0.047	0.000	0.248	2.889	0.000	13.879

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	60	52	73	46	61	0	0	67
N.S.	1	1.05	0.91	1.28	0.81	1.07	0.00	0.00	1.18
time (sec)	N/A	0.582	0.073	0.046	0.220	0.253	0.000	0.000	4.762

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	141	121	131	0	146	0	0	885
N.S.	1	0.96	0.82	0.89	0.00	0.99	0.00	0.00	6.02
time (sec)	N/A	0.667	0.129	0.056	0.000	0.257	0.000	0.000	46.860

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	172	69	84	56	69	0	0	86
N.S.	1	0.94	0.38	0.46	0.31	0.38	0.00	0.00	0.47
time (sec)	N/A	0.691	0.092	0.050	0.212	0.269	0.000	0.000	5.425

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	249	137	153	0	156	0	0	2480
N.S.	1	0.93	0.51	0.57	0.00	0.58	0.00	0.00	9.29
time (sec)	N/A	0.729	0.157	0.062	0.000	0.255	0.000	0.000	68.991

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	278	85	92	65	78	0	0	105
N.S.	1	0.92	0.28	0.31	0.22	0.26	0.00	0.00	0.35
time (sec)	N/A	0.725	0.108	0.056	0.213	0.248	0.000	0.000	5.340

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	207	65	531	0	65	0	0	73
N.S.	1	1.41	0.44	3.61	0.00	0.44	0.00	0.00	0.50
time (sec)	N/A	0.888	0.096	0.351	0.000	0.250	0.000	0.000	5.348

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	206	97	120	0	95	0	0	795
N.S.	1	1.26	0.60	0.74	0.00	0.58	0.00	0.00	4.88
time (sec)	N/A	0.797	0.128	0.918	0.000	0.251	0.000	0.000	25.409

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	117	48	269	0	54	0	0	57
N.S.	1	1.56	0.64	3.59	0.00	0.72	0.00	0.00	0.76
time (sec)	N/A	0.709	0.069	0.286	0.000	0.248	0.000	0.000	4.887

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	122	75	94	0	79	0	0	407
N.S.	1	1.30	0.80	1.00	0.00	0.84	0.00	0.00	4.33
time (sec)	N/A	0.517	0.079	0.876	0.000	0.263	0.000	0.000	13.356

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	93	72	2612	0	115	0	0	47
N.S.	1	1.43	1.11	40.18	0.00	1.77	0.00	0.00	0.72
time (sec)	N/A	0.426	0.045	0.282	0.000	0.256	0.000	0.000	7.361

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	52	74	96	0	76	0	0	184
N.S.	1	1.13	1.61	2.09	0.00	1.65	0.00	0.00	4.00
time (sec)	N/A	0.568	0.044	0.894	0.000	0.278	0.000	0.000	7.480

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	79	92	96	0	128	0	0	323
N.S.	1	1.10	1.28	1.33	0.00	1.78	0.00	0.00	4.49
time (sec)	N/A	0.568	0.063	0.886	0.000	0.257	0.000	0.000	16.763

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	112	43	58	0	52	0	0	58
N.S.	1	0.97	0.37	0.50	0.00	0.45	0.00	0.00	0.50
time (sec)	N/A	0.613	0.059	0.914	0.000	0.256	0.000	0.000	5.728

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	189	110	115	0	138	0	0	1511
N.S.	1	0.94	0.55	0.58	0.00	0.69	0.00	0.00	7.56
time (sec)	N/A	0.677	0.119	0.931	0.000	0.258	0.000	0.000	50.942

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	220	60	68	0	60	0	0	75
N.S.	1	0.94	0.26	0.29	0.00	0.26	0.00	0.00	0.32
time (sec)	N/A	0.699	0.073	0.939	0.000	0.243	0.000	0.000	5.173

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	297	129	137	0	148	0	0	2479
N.S.	1	0.93	0.40	0.43	0.00	0.46	0.00	0.00	7.75
time (sec)	N/A	0.721	0.158	1.008	0.000	0.251	0.000	0.000	69.659

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	328	76	76	0	69	0	0	91
N.S.	1	0.93	0.22	0.22	0.00	0.20	0.00	0.00	0.26
time (sec)	N/A	0.742	0.105	0.943	0.000	0.251	0.000	0.000	5.232

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	95	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	0.795	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	113	69	74	0	69	0	0	76
N.S.	1	1.28	0.78	0.84	0.00	0.78	0.00	0.00	0.86
time (sec)	N/A	0.424	0.220	0.691	0.000	0.244	0.000	0.000	5.423

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	83	110	122	0	91	0	0	340
N.S.	1	1.11	1.47	1.63	0.00	1.21	0.00	0.00	4.53
time (sec)	N/A	0.376	0.199	0.682	0.000	0.255	0.000	0.000	11.191

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	52	0	49	0	0	44
N.S.	1	1.00	0.98	1.16	0.00	1.09	0.00	0.00	0.98
time (sec)	N/A	0.311	0.135	0.697	0.000	0.264	0.000	0.000	5.245

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	68	97	0	53	0	0	84
N.S.	1	1.00	1.84	2.62	0.00	1.43	0.00	0.00	2.27
time (sec)	N/A	0.269	0.200	0.694	0.000	0.252	0.000	0.000	6.105

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	72	73	82	0	92	0	0	59
N.S.	1	1.01	1.03	1.15	0.00	1.30	0.00	0.00	0.83
time (sec)	N/A	0.313	0.165	0.704	0.000	0.242	0.000	0.000	5.954

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	59	65	0	62	0	0	37
N.S.	1	1.10	1.40	1.55	0.00	1.48	0.00	0.00	0.88
time (sec)	N/A	0.357	0.213	0.722	0.000	0.264	0.000	0.000	5.083

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	117	108	119	0	156	0	0	331
N.S.	1	1.08	1.00	1.10	0.00	1.44	0.00	0.00	3.06
time (sec)	N/A	0.435	0.216	0.729	0.000	0.249	0.000	0.000	17.892

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	109	90	90	0	89	0	0	75
N.S.	1	1.28	1.06	1.06	0.00	1.05	0.00	0.00	0.88
time (sec)	N/A	0.409	0.323	0.731	0.000	0.251	0.000	0.000	5.484

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	26	28	36	0	35	0	0	76
N.S.	1	2.17	2.33	3.00	0.00	2.92	0.00	0.00	6.33
time (sec)	N/A	0.973	0.441	0.734	0.000	0.247	0.000	0.000	5.778

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	67	52	111	0	0	0	0	0
N.S.	1	1.10	0.85	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.508	0.101	0.783	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	56	106	53	38	283	0	0	56
N.S.	1	0.98	1.86	0.93	0.67	4.96	0.00	0.00	0.98
time (sec)	N/A	0.328	0.174	0.185	0.199	0.271	0.000	0.000	5.585

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	56	106	0	40	385	0	0	54
N.S.	1	0.97	1.83	0.00	0.69	6.64	0.00	0.00	0.93
time (sec)	N/A	0.342	0.218	0.000	0.196	0.304	0.000	0.000	5.009

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [5] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	1.01	10	0.800
2	A	6	5	1.00	10	0.500
3	A	10	9	0.99	8	1.125
4	A	5	4	1.00	6	0.667
5	C	16	15	1.37	10	1.500
6	A	8	7	1.03	10	0.700
7	A	13	12	1.20	10	1.200
8	A	14	13	1.21	10	1.300
9	A	6	5	0.95	12	0.417
10	A	7	6	0.94	10	0.600
11	A	8	7	0.96	8	0.875
12	C	17	16	1.23	12	1.333
13	A	6	5	1.01	12	0.417
14	A	7	6	0.96	12	0.500
15	A	7	6	0.90	10	0.600
16	A	9	8	0.92	8	1.000
17	C	17	16	1.20	12	1.333
18	A	6	5	1.01	12	0.417
19	A	7	6	0.94	12	0.500
20	A	4	4	0.63	10	0.400
21	A	4	4	0.73	10	0.400
22	A	4	4	0.90	8	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	6	0.333
24	C	10	9	1.26	10	0.900
25	A	6	5	0.81	10	0.500
26	A	7	6	0.75	10	0.600
27	A	8	7	0.72	10	0.700
28	A	3	3	1.00	4	0.750
29	C	10	9	1.26	10	0.900
30	C	10	9	1.26	10	0.900
31	C	10	9	1.26	10	0.900
32	A	5	5	1.67	10	0.500
33	A	6	6	1.07	10	0.600
34	A	3	3	1.37	10	0.300
35	A	4	4	1.00	8	0.500
36	A	4	3	1.62	6	0.500
37	A	6	6	1.35	10	0.600
38	B	8	7	2.91	10	0.700
39	C	5	5	1.95	10	0.500
40	A	10	9	1.06	10	0.900
41	A	7	7	1.23	10	0.700
42	A	12	11	1.07	10	1.100
43	A	9	9	1.20	10	0.900
44	A	14	13	1.07	10	1.300
45	A	7	6	0.86	12	0.500
46	A	5	5	0.86	12	0.417
47	A	4	4	1.22	12	0.333
48	A	7	7	0.81	12	0.583
49	A	6	5	1.00	12	0.417
50	A	4	4	1.00	12	0.333
51	A	7	6	1.00	10	0.600
52	A	8	8	0.73	8	1.000
53	A	6	5	0.89	12	0.417
54	A	5	5	0.82	12	0.417
55	A	8	7	0.82	12	0.583
56	A	4	4	1.04	12	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	4	1.06	12	0.333
58	A	4	4	1.00	10	0.400
59	A	6	5	1.00	12	0.417
60	A	4	4	1.00	12	0.333
61	A	4	4	1.00	10	0.400
62	A	4	4	1.00	8	0.500
63	A	7	6	0.87	12	0.500
64	A	4	4	1.00	12	0.333
65	A	13	12	1.31	12	1.000
66	A	12	11	1.46	12	0.917
67	A	8	7	1.21	12	0.583
68	A	11	10	1.36	10	1.000
69	A	7	6	1.44	8	0.750
70	A	6	5	1.24	12	0.417
71	A	5	4	1.05	12	0.333
72	A	6	5	0.96	12	0.417
73	A	6	5	0.94	12	0.417
74	A	7	6	0.93	12	0.500
75	A	6	5	0.92	12	0.417
76	A	13	12	1.41	12	1.000
77	A	10	9	1.26	12	0.750
78	A	9	8	1.56	12	0.667
79	A	7	6	1.30	10	0.600
80	A	7	6	1.43	8	0.750
81	A	6	5	1.13	12	0.417
82	A	6	5	1.10	12	0.417
83	A	5	4	0.97	12	0.333
84	A	7	6	0.94	12	0.500
85	A	6	5	0.94	12	0.417
86	A	7	6	0.93	12	0.500
87	A	6	5	0.93	12	0.417
88	A	5	5	1.00	24	0.208
89	A	9	8	1.28	22	0.364
90	A	8	8	1.11	22	0.364

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	4	4	1.00	22	0.182
92	A	5	5	1.00	20	0.250
93	A	9	8	1.01	19	0.421
94	A	5	5	1.10	22	0.227
95	A	11	10	1.08	22	0.455
96	A	8	8	1.28	22	0.364
97	B	2	2	2.17	25	0.080
98	C	11	10	1.10	19	0.526
99	A	6	5	0.98	12	0.417
100	A	6	5	0.97	14	0.357

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 \text{sech}^{-1}(a+bx)\,dx$

3.1.1 Optimal result

Integrand size = 10, antiderivative size = 203

$$\int x^3 \text{sech}^{-1}(a+bx)\,dx = -\frac{(2+17a^2)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^4} - \frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{3b^4} - \frac{a^4\text{sech}^{-1}(a+bx)}{4b^4} + \frac{1}{4}x^4\text{sech}^{-1}(a+bx) + \frac{a(1+2a^2)\arctan\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{a+bx}\right)}{2b^4}$$

output
```
-1/4*a^4*arcsech(b*x+a)/b^4+1/4*x^4*arcsech(b*x+a)+1/2*a*(2*a^2+1)*arctan(
(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^4-1/12*(17*a^2+2)*(b*x+a
+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^4-1/12*x^2*(b*x+a+1)*((-b*x-a+1)/(b*x+a
+1))^(1/2)/b^2+1/3*a*(b*x+a)*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^4
```

3.1.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.11

$$\int x^3\text{sech}^{-1}(a+bx)\,dx =$$

$$-\frac{\sqrt{-\frac{-1+a+bx}{1+a+bx}}(2+2a+13a^2+13a^3+(2-4a+9a^2)bx+(1-3a)b^2x^2+b^3x^3) - 3b^4x^4\text{sech}^{-1}(a+bx)}{}$$

input
```
Integrate[x^3*ArcSech[a + b*x],x]
```

output
```
-1/12*(Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(2 + 2*a + 13*a^2 + 13*a^3 +
(2 - 4*a + 9*a^2)*b*x + (1 - 3*a)*b^2*x^2 + b^3*x^3) - 3*b^4*x^4*ArcSech[a
 + b*x] - 3*a^4*Log[a + b*x] + 3*a^4*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a
+ b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a +
b*x)/(1 + a + b*x))]] + (6*I)*a*(1 + 2*a^2)*Log[(-2*I)*(a + b*x) + 2*Sqrt[
-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)])/b^4
```

3.1.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6875, 25, 5991, 3042, 4269, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{sech}^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{6875}$$

$$-\frac{\int b^3x^3(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)d\text{sech}^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -b^3x^3(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)d\text{sech}^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{5991}$$

$$-\frac{\frac{1}{4}\int b^4x^4d\text{sech}^{-1}(a+bx)-\frac{1}{4}b^4x^4\text{sech}^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}b^4x^4\text{sech}^{-1}(a+bx)+\frac{1}{4}\int \left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^4d\text{sech}^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{4269}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{3}\int -bx\left(3a^3+8(a+bx)^2a-\left(9a^2+2\right)(a+bx)\right)d\text{sech}^{-1}(a+bx)+\frac{1}{3}b^2x^2\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\right)-\frac{1}{4}b^4x^4\text{s}}{b^4}$$

$\downarrow$ 3042

$$-\frac{-\frac{1}{4}b^4x^4\text{sech}^{-1}(a+bx)+\frac{1}{4}\left(\frac{1}{3}b^2x^2\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)+\frac{1}{3}\int\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)\left(3a^3+8\csc\left(i s\right.\right.\right.}{b^4}$$

$\downarrow$ 4536

$$-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(6a^4-12\left(2a^2+1\right)(a+bx)a+2\left(17a^2+2\right)(a+bx)^2\right)d\text{sech}^{-1}(a+bx)-4a(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+b\right.\right.}{b^4}$$

$\downarrow$ 2009

$$-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(6a^4\text{sech}^{-1}(a+bx)-12\left(2a^2+1\right)a\arctan\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)+2\left(17a^2+2\right)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1\right.\right.\right.}{b^4}$$

input
```
Int[x^3*ArcSech[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcSech[a + b*x]) + ((b^2*x^2*Sqrt[(1 - a - b*x)/(1 + a +
 b*x)]*(1 + a + b*x))/3 + (-4*a*(a + b*x)*Sqrt[(1 - a - b*x)/(1 + a + b*x)
]*(1 + a + b*x) + (2*(2 + 17*a^2)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a
 + b*x) + 6*a^4*ArcSech[a + b*x] - 12*a*(1 + 2*a^2)*ArcTan[(Sqrt[(1 - a -
b*x)/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x)])/2)/3)/4)/b^4)
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.1.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsech}(bx+a)a^4}{4}-\operatorname{arcsech}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsech}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsech}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsech}(bx+a)(bx+a)^4}{4}-$
default	$\frac{\frac{\operatorname{arcsech}(bx+a)a^4}{4}-\operatorname{arcsech}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsech}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsech}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsech}(bx+a)(bx+a)^4}{4}-$
parts	$\frac{x^4\operatorname{arcsech}(bx+a)}{4}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(3\operatorname{csgn}(b)\operatorname{arctanh}\left(\frac{1}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)a^4+\operatorname{csgn}(b)b^2x^2\sqrt{-b^2}\right.}{}$

input
```
int(x^3*arcsech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/4*arcsech(b*x+a)*a^4-arcsech(b*x+a)*a^3*(b*x+a)+3/2*arcsech(b*x+a
)*a^2*(b*x+a)^2-arcsech(b*x+a)*a*(b*x+a)^3+1/4*arcsech(b*x+a)*(b*x+a)^4-1/
12*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(3*a^4*arc
tanh(1/(1-(b*x+a)^2)^(1/2))+12*a^3*arcsin(b*x+a)+18*a^2*(1-(b*x+a)^2)^(1/2
)-6*a*(b*x+a)*(1-(b*x+a)^2)^(1/2)+(1-(b*x+a)^2)^(1/2)*(b*x+a)^2+6*a*arcsin
(b*x+a)+2*(1-(b*x+a)^2)^(1/2))/(1-(b*x+a)^2)^(1/2))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.70

$$\int x^3 \operatorname{sech}^{-1}(a+bx)\,dx$$
$$= \frac{6\,b^4x^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right) - 3\,a^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right) + 3\,a^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx}{b^2x^2+2\,ab}}}{x}\right.}{}$$

input
```
integrate(x^3*arcsech(b*x+a),x, algorithm="fricas")
```

output
```
1/24*(6*b^4*x^4*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^
2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - 3*a^4*log(((b*x + a)*sqrt(-(b^2*x^2
+ 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + 3*a^4*log(((b*x
+ a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x
) + 12*(2*a^3 + a)*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b
*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) -
2*(b^3*x^3 - 3*a*b^2*x^2 + 13*a^3 + (9*a^2 + 2)*b*x + 2*a)*sqrt(-(b^2*x^2
+ 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^4
```

3.1.6 Sympy [F]

$$\int x^3 \text{sech}^{-1}(a+bx)\,dx = \int x^3\,\text{asech}\,(a+bx)\,dx$$

input
```
integrate(x**3*asech(b*x+a),x)
```

output
```
Integral(x**3*asech(a + b*x), x)
```

3.1.7 Maxima [F]

$$\int x^3 \text{sech}^{-1}(a+bx)\,dx = \int x^3\,\text{arsech}\,(bx+a)\,\,dx$$

input
```
integrate(x^3*arcsech(b*x+a),x, algorithm="maxima")
```

output
```
1/8*(2*b^4*x^4*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
 + 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^4*x^4*log(b*x + a) - b^2*x^2 +
 6*a*b*x - (a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1) - 2*(b^4*x^4 -
 a^4)*log(b*x + a) - (a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(-b*x - a + 1))/b^
4 + integrate(1/4*(b^2*x^5 + a*b*x^4)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2
+ 2*a*b*x + a^2 - 1)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1)) - 1)
, x)
```

3.1.8 Giac [F]

$$\int x^3 \text{sech}^{-1}(a+bx)\,dx = \int x^3\,\text{arsech}\,(bx+a)\,\,dx$$

input
```
integrate(x^3*arcsech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*arcsech(b*x + a), x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{sech}^{-1}(a+bx)\,dx = \int x^3\,\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)\,dx$$

input
```
int(x^3*acosh(1/(a + b*x)),x)
```

output
```
int(x^3*acosh(1/(a + b*x)), x)
```

3.2 $\int x^2 \text{sech}^{-1}(a+bx)\,dx$

3.2.1 Optimal result

Integrand size = 10, antiderivative size = 153

$$\int x^2 \text{sech}^{-1}(a+bx)\,dx = \frac{5a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^3} - \frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3\text{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\text{sech}^{-1}(a+bx) - \frac{(1+6a^2)\arctan\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{a+bx}\right)}{6b^3}$$

output
```
1/3*a^3*arcsech(b*x+a)/b^3+1/3*x^3*arcsech(b*x+a)-1/6*(6*a^2+1)*arctan((b*
x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^3+5/6*a*(b*x+a+1)*((-b*x-a+
1)/(b*x+a+1))^(1/2)/b^3-1/6*x*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

3.2.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.31

$$\int x^2 \text{sech}^{-1}(a+bx)\,dx = \frac{\sqrt{-\frac{-1+a+bx}{1+a+bx}}(5a^2-bx(1+bx)+a(5+4bx)) + 2b^3x^3\text{sech}^{-1}(a+bx) - 2a^3\log(a+bx) + 2a^3\log\left(1+\sqrt{-\right.}{6b}$$

input
```
Integrate[x^2*ArcSech[a + b*x],x]
```

output
```
(Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(5*a^2 - b*x*(1 + b*x) + a*(5 + 4*b
*x)) + 2*b^3*x^3*ArcSech[a + b*x] - 2*a^3*Log[a + b*x] + 2*a^3*Log[1 + Sqr
t[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x)
)] + b*x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]] + I*(1 + 6*a^2)*Log[(-2*I)
*(a + b*x) + 2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)])/(6*b^
3)
```

3.2.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6875, 5991, 3042, 4269, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{sech}^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{6875}$$

$$-\frac{\int b^2x^2(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)d\text{sech}^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{5991}$$

$$-\frac{-\frac{1}{3}\int -b^3x^3d\text{sech}^{-1}(a+bx)-\frac{1}{3}b^3x^3\text{sech}^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{3}b^3x^3\text{sech}^{-1}(a+bx)-\frac{1}{3}\int \left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^3d\text{sech}^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{4269}$$

$$-\frac{\frac{1}{3}\left(\frac{1}{2}bx\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)-\frac{1}{2}\int \left(2a^3+5(a+bx)^2a-\left(6a^2+1\right)(a+bx)\right)d\text{sech}^{-1}(a+bx)\right)-\frac{1}{3}b^3x^3\text{sech}^{-1}}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}\left(\frac{1}{2}\left(-2a^3\mathrm{sech}^{-1}(a+bx)+(6a^2+1)\arctan\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)-5a\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\right)+\frac{1}{2}bx\sqrt{\frac{-a-}{a+b}}\right.}{b^3}$$

input
```
Int[x^2*ArcSech[a + b*x],x]
```

output
```
-((-1/3*(b^3*x^3*ArcSech[a + b*x]) + ((b*x*Sqrt[(1 - a - b*x)/(1 + a + b*x
)]*(1 + a + b*x))/2 + (-5*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x
) - 2*a^3*ArcSech[a + b*x] + (1 + 6*a^2)*ArcTan[(Sqrt[(1 - a - b*x)/(1 + a
 + b*x)]*(1 + a + b*x))/(a + b*x)])/2)/3)/b^3)
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.2.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsech}(bx+a)a^3}{3}+\operatorname{arcsech}(bx+a)a^2(bx+a)-\operatorname{arcsech}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsech}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+}{bx}}}{}}{b^3}$
default	$\frac{-\frac{\operatorname{arcsech}(bx+a)a^3}{3}+\operatorname{arcsech}(bx+a)a^2(bx+a)-\operatorname{arcsech}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsech}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+}{bx}}}{}}{b^3}$
parts	$\frac{x^3\operatorname{arcsech}(bx+a)}{3}+\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(2\operatorname{arctanh}\left(\frac{1}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)\operatorname{csgn}(b)a^3-\sqrt{-b^2x^2-2abx-a^2}\right.}{}$

input
```
int(x^2*arcsech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(-1/3*arcsech(b*x+a)*a^3+arcsech(b*x+a)*a^2*(b*x+a)-arcsech(b*x+a)*a
*(b*x+a)^2+1/3*arcsech(b*x+a)*(b*x+a)^3+1/6*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*
x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(2*a^3*arctanh(1/(1-(b*x+a)^2)^(1/2))+6*a^2
*arcsin(b*x+a)+6*a*(1-(b*x+a)^2)^(1/2)-(b*x+a)*(1-(b*x+a)^2)^(1/2)+arcsin(
b*x+a))/(1-(b*x+a)^2)^(1/2))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(131) = 262$.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.14

$$\int x^2\operatorname{sech}^{-1}(a+bx)\,dx$$
$$= \frac{2\,b^3x^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right)+a^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right)-a^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2}{b^2x^2+2\,abx+a}}}{x}\right.}{6\,b^3}$$

input
```
integrate(x^2*arcsech(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
 + 2*a*b*x + a^2)) + 1)/(b*x + a)) + a^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2
*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - a^3*log(((b*x + a)*
sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - (
6*a^2 + 1)*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2
 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (b^2*x^2
 - 4*a*b*x - 5*a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x
 + a^2)))/b^3
```

3.2.6 Sympy [F]

$$\int x^2\text{sech}^{-1}(a+bx)\,dx = \int x^2\,\text{asech}\,(a+bx)\,dx$$

input
```
integrate(x**2*asech(b*x+a),x)
```

output
```
Integral(x**2*asech(a + b*x), x)
```

3.2.7 Maxima [F]

$$\int x^2\text{sech}^{-1}(a+bx)\,dx = \int x^2\,\text{arsech}\,(bx+a)\;dx$$

input
```
integrate(x^2*arcsech(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(2*b^3*x^3*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
 + 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^3*x^3*log(b*x + a) - 2*b*x + (
a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1) - 2*(b^3*x^3 + a^3)*log(b*x + a) +
 (a^3 - 3*a^2 + 3*a - 1)*log(-b*x - a + 1))/b^3 + integrate(1/3*(b^2*x^4 +
 a*b*x^3)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(1/2*
log(b*x + a + 1) + 1/2*log(-b*x - a + 1)) - 1), x)
```

3.2.8 Giac [F]

$$\int x^2\text{sech}^{-1}(a+bx)\,dx=\int x^2\,\text{arsech}\,(bx+a)\,\,dx$$

input
```
integrate(x^2*arcsech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*arcsech(b*x + a), x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{sech}^{-1}(a+bx)\,dx=\int x^2\,\text{acosh}\left(\frac{1}{a+b\,x}\right)\,dx$$

input
```
int(x^2*acosh(1/(a + b*x)),x)
```

output
```
int(x^2*acosh(1/(a + b*x)), x)
```

3.3 $\int x\text{sech}^{-1}(a+bx)\,dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 107

$$\int x\text{sech}^{-1}(a+bx)\,dx=-\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2b^2}-\frac{a^2\text{sech}^{-1}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)+\frac{a\arctan\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{a+bx}\right)}{b^2}$$

output
```
-1/2*a^2*arcsech(b*x+a)/b^2+1/2*x^2*arcsech(b*x+a)+a*arctan((b*x+a+1)*((-b
*x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^2-1/2*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))
^(1/2)/b^2
```

3.3.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.64

$$\int x\text{sech}^{-1}(a+bx)\,dx$$
$$=\frac{-\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)+b^2x^2\text{sech}^{-1}(a+bx)+a^2\log(a+bx)-a^2\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a}{1+a}}\right.}{2b^2}$$

input
```
Integrate[x*ArcSech[a + b*x],x]
```

output
```
(-(Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)) + b^2*x^2*ArcSech[
a + b*x] + a^2*Log[a + b*x] - a^2*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b
*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x
)/(1 + a + b*x))]] - (2*I)*a*Log[(-2*I)*(a + b*x) + 2*Sqrt[-((-1 + a + b*x
)/(1 + a + b*x))]*(1 + a + b*x)])/(2*b^2)
```

3.3.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6875, 25, 5991, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{sech}^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6875}$$

$$-\frac{\int bx(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)d\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -bx(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)d\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 5991}$$

$$-\frac{\frac{1}{2}\int b^2x^2d\text{sech}^{-1}(a+bx)-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)+\frac{1}{2}\int\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2d\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 4260}$$

$$-\frac{\frac{1}{2}\left(-2a\int(a+bx)d\text{sech}^{-1}(a+bx)+\int(a+bx)^2d\text{sech}^{-1}(a+bx)+a^2\text{sech}^{-1}(a+bx)\right)-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)+\frac{1}{2}\left(-2a\int\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(a+bx)+\int\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)^2 ds}{b^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)+\frac{1}{2}\left(i\int 1d\left(-i\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\right)-2a\int\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(a+bx}{b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)+\frac{1}{2}\left(-2a\int\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(a+bx)+a^2\text{sech}^{-1}(a+bx)+\sqrt{\frac{-a-bx+1}{a+bx+1}}\right.}{b^2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{1}{2}\left(a^2\text{sech}^{-1}(a+bx)-2a\arctan\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)+\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\right)-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)}{b^2}$$

input
```
Int[x*ArcSech[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcSech[a + b*x]) + (Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1
 + a + b*x) + a^2*ArcSech[a + b*x] - 2*a*ArcTan[(Sqrt[(1 - a - b*x)/(1 + a
 + b*x)]*(1 + a + b*x))/(a + b*x)])/2)/b^2)
```

3.3.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.3.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{-\operatorname{arcsech}(bx+a)a(bx+a)+\frac{\operatorname{arcsech}(bx+a)(bx+a)^2}{2}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(2a\arcsin(bx+a)+\sqrt{1-(bx+a)^2}\right)}{2\sqrt{1-(bx+a)^2}}}{b^2}$
default	$\frac{-\operatorname{arcsech}(bx+a)a(bx+a)+\frac{\operatorname{arcsech}(bx+a)(bx+a)^2}{2}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(2a\arcsin(bx+a)+\sqrt{1-(bx+a)^2}\right)}{2\sqrt{1-(bx+a)^2}}}{b^2}$
parts	$\frac{x^2\operatorname{arcsech}(bx+a)}{2}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\operatorname{csgn}(b)\operatorname{arctanh}\left(\frac{1}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)a^2+\operatorname{csgn}(b)\sqrt{-b^2x^2-2a\right.}{2b^2\sqrt{-b^2x^2-2abx-a^2+1}}$

input
```
int(x*arcsech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-arcsech(b*x+a)*a*(b*x+a)+1/2*arcsech(b*x+a)*(b*x+a)^2-1/2*(-(b*x+a
-1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(2*a*arcsin(b*x+a)+(1
-(b*x+a)^2)^(1/2))/(1-(b*x+a)^2)^(1/2))
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(93) = 186$.

Time = 0.28 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.88

$$\int x\operatorname{sech}^{-1}(a+bx)\,dx = \frac{2\,b^2x^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right) - a^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right) + a^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2\cdots}{b^2x^2+2\,abx+a\cdots}}\cdots}{x}\right.\cdots}{4\,b^2}$$

input
```
integrate(x*arcsech(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - a^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2
*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + a^2*log(((b*x + a)*
sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 4
*a*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b
^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(b*x + a)*sqrt
(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^2
```

3.3.6 Sympy [F]

$$\int x\operatorname{sech}^{-1}(a+bx)\,dx = \int x\operatorname{asech}(a+bx)\,dx$$

input
```
integrate(x*asech(b*x+a),x)
```

output
```
Integral(x*asech(a + b*x), x)
```

3.3.7 Maxima [F]

$$\int x\text{sech}^{-1}(a+bx)\,dx=\int x\,\text{arsech}\,(bx+a)\,\,dx$$

input
```
integrate(x*arcsech(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(2*b^2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
 + 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^2*x^2*log(b*x + a) - (a^2 + 2*
a + 1)*log(b*x + a + 1) - 2*(b^2*x^2 - a^2)*log(b*x + a) - (a^2 - 2*a + 1)
*log(-b*x - a + 1))/b^2 + integrate(1/2*(b^2*x^3 + a*b*x^2)/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(1/2*log(b*x + a + 1) + 1/2*l
og(-b*x - a + 1)) - 1), x)
```

3.3.8 Giac [F]

$$\int x\text{sech}^{-1}(a+bx)\,dx=\int x\,\text{arsech}\,(bx+a)\,\,dx$$

input
```
integrate(x*arcsech(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*arcsech(b*x + a), x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{-1}(a+bx)\,dx=\int x\,\text{acosh}\left(\frac{1}{a+b\,x}\right)\,dx$$

input
```
int(x*acosh(1/(a + b*x)),x)
```

output
```
int(x*acosh(1/(a + b*x)), x)
```

3.4 $\int \operatorname{sech}^{-1}(a+bx)\,dx$

3.4.1 Optimal result

Integrand size = 6, antiderivative size = 44

$$\int \operatorname{sech}^{-1}(a+bx)\,dx=\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-\frac{2\arctan\left(\sqrt{\frac{1-a-bx}{1+a+bx}}\right)}{b}$$

output
```
(b*x+a)*arcsech(b*x+a)/b-2*arctan(((-b*x-a+1)/(b*x+a+1))^(1/2))/b
```

3.4.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(44) = 88.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.20

$$\int \operatorname{sech}^{-1}(a+bx)\,dx=x\operatorname{sech}^{-1}(a+bx)-\frac{2\sqrt{-\frac{-1+a+bx}{1+a+bx}}\left(-a\arctan\left(\sqrt{\frac{-1+a+bx}{1+a+bx}}\right)+\operatorname{arctanh}\left(\sqrt{\frac{-1+a+bx}{1+a+bx}}\right)\right)}{b\sqrt{\frac{-1+a+bx}{1+a+bx}}}$$

input
```
Integrate[ArcSech[a + b*x],x]
```

output
```
x*ArcSech[a + b*x] - (2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(-(a*ArcTan[
Sqrt[(-1 + a + b*x)/(1 + a + b*x)]]) + ArcTanh[Sqrt[(-1 + a + b*x)/(1 + a
+ b*x)]]))/(b*Sqrt[(-1 + a + b*x)/(1 + a + b*x)])
```

3.4.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6867, 2055, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6867}$$

$$\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}}{-a-bx+1}dx+\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 2055}$$

$$\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-4b\int \frac{1}{2b^2\left(\frac{-a-bx+1}{a+bx+1}+1\right)}d\sqrt{\frac{-a-bx+1}{a+bx+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-\frac{2\int \frac{1}{\frac{-a-bx+1}{a+bx+1}+1}d\sqrt{\frac{-a-bx+1}{a+bx+1}}}{b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-\frac{2\arctan\left(\sqrt{\frac{-a-bx+1}{a+bx+1}}\right)}{b}$$

input
```
Int[ArcSech[a + b*x],x]
```

output
```
((a + b*x)*ArcSech[a + b*x])/b - (2*ArcTan[Sqrt[(1 - a - b*x)/(1 + a + b*x
)]])/b
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

rule 6867
```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSech[c + d*
x]/d), x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

3.4.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{(bx+a)\,\mathrm{arcsech}(bx+a)-\arctan\left(\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)}{b}$
default	$\frac{(bx+a)\,\mathrm{arcsech}(bx+a)-\arctan\left(\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)}{b}$
parts	$x\,\mathrm{arcsech}\,(bx+a)+\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\,\left(\mathrm{csgn}(b)\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)a+\arctan\left(\frac{\mathrm{cs}}{\sqrt{-(bx}}\right.\right.}{b\sqrt{-b^2x^2-2abx-a^2+1}}$

input
```
int(arcsech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arcsech(b*x+a)-arctan((1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)
))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.75

$$\int \operatorname{sech}^{-1}(a+bx)\,dx$$
$$= \frac{2\,bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right)+a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right)-a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right.}{2\,b}$$

input
```
integrate(arcsech(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2
*a*b*x + a^2)) + 1)/(b*x + a)) + a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x
 + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - a*log(((b*x + a)*sqrt(-(b
^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - 2*arctan(
(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2
*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

3.4.6 Sympy [F]

$$\int \operatorname{sech}^{-1}(a+bx)\,dx = \int \operatorname{asech}(a+bx)\,dx$$

input
```
integrate(asech(b*x+a),x)
```

output
```
Integral(asech(a + b*x), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \operatorname{sech}^{-1}(a+bx)\,dx = \frac{(bx+a)\operatorname{arsech}(bx+a) - \arctan\left(\sqrt{\frac{1}{(bx+a)^2}-1}\right)}{b}$$

input
```
integrate(arcsech(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*arcsech(b*x + a) - arctan(sqrt(1/(b*x + a)^2 - 1)))/b
```

3.4.8 Giac [F]

$$\int \operatorname{sech}^{-1}(a+bx)\,dx = \int \operatorname{arsech}(bx+a)\,dx$$

input
```
integrate(arcsech(b*x+a),x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a), x)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \operatorname{sech}^{-1}(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{a+b\,x}-1}\,\sqrt{\frac{1}{a+b\,x}+1}}\right) + \operatorname{acosh}\left(\frac{1}{a+b\,x}\right)\,(a+b\,x)}{b}$$

input
```
int(acosh(1/(a + b*x)),x)
```

output
```
(atan(1/((1/(a + b*x) - 1)^(1/2)*(1/(a + b*x) + 1)^(1/2))) + acosh(1/(a +
b*x))*(a + b*x))/b
```

3.5 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x} dx$

3.5.1 Optimal result

Integrand size = 10, antiderivative size = 170

$$\begin{aligned}\int \frac{\operatorname{sech}^{-1}(a+bx)}{x} dx &= \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\ &+\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\ &-\operatorname{sech}^{-1}(a+bx)\log\left(1+e^{2\operatorname{sech}^{-1}(a+bx)}\right)\\ &+\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\ &-\frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(a+bx)}\right)\end{aligned}$$

output
```
-arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2
)+arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)
)/(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)
*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-1/2*polylog(2,-(1/(b*x+a)+(1/(b*
x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))+polylog(2,a*(1/(b*x+a)+(1/
(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

3.5.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.95

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx = -4i\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{(1+a)\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)\log\left(1+e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) + 2i\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) - 2i\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) - \operatorname{PolyLog}\left(2,-\frac{\left(-1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) - \operatorname{PolyLog}\left(2,\frac{\left(1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right)$$

input
```
Integrate[ArcSech[a + b*x]/x,x]
```

output
```
(-4*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*ArcTanh[((1 + a)*Tanh[ArcSech[a +
b*x]/2])/Sqrt[1 - a^2]] - ArcSech[a + b*x]*Log[1 + E^(-2*ArcSech[a + b*x])
] + ArcSech[a + b*x]*Log[1 + (-1 + Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x])]
+ (2*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + (-1 + Sqrt[1 - a^2])/(a*E
^ArcSech[a + b*x])] + ArcSech[a + b*x]*Log[1 - (1 + Sqrt[1 - a^2])/(a*E^Ar
cSech[a + b*x])] - (2*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 - (1 + Sqr
t[1 - a^2])/(a*E^ArcSech[a + b*x])] + PolyLog[2, -E^(-2*ArcSech[a + b*x])]
/2 - PolyLog[2, -((-1 + Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x]))] - PolyLog[
2, (1 + Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x])]
```

3.5.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.37, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {6875, 25, 6129, 6104, 25, 3042, 26, 4201, 2620, 2715, 2838, 6096, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx$$

$$\Big\downarrow \; 6875$$

$$-\int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)}{bx}d\operatorname{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 25$$

$$\int -\frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)}{bx}d\operatorname{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 6129$$

$$\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)}{\frac{a}{a+bx}-1}d\operatorname{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 6104$$

$$a\int -\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\operatorname{sech}^{-1}(a+bx)-\int \sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)d\operatorname{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 25$$

$$-\int \sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)d\operatorname{sech}^{-1}(a+bx)-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\operatorname{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 3042$$

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)-\int -i\text{sech}^{-1}(a+bx)\tan\left(i\text{sech}^{-1}(a+bx)\right)d\text{sech}^{-1}(a+bx)$$

$\downarrow$ 26

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\int \text{sech}^{-1}(a+bx)\tan\left(i\text{sech}^{-1}(a+bx)\right)d\text{sech}^{-1}(a+bx)$$

$\downarrow$ 4201

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\left(2i\int \frac{e^{2\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)}{1+e^{2\text{sech}^{-1}(a+bx)}}d\text{sech}^{-1}(a+bx)-\frac{1}{2}i\text{sech}^{-1}(a+bx)^2\right)$$

$\downarrow$ 2620

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+bx)\right)-\frac{1}{2}i\text{sech}^{-1}(a+$$

$\downarrow$ 2715

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{1}{4}\int e^{-2\text{sech}^{-1}(a+bx)}\log\left(1+e^{2\text{sech}^{-1}(a+bx)}\right)de^{2\text{sech}^{-1}(a+bx)}\right)-\frac{1}{2}$$

$\downarrow$ 2838

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\text{sech}^{-1}(a+bx)\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)\right)-\frac{1}{2}i\text{sech}^{-1}(a+bx)^2\right)$$

$\downarrow$ 6096

3.5. $\int \frac{\text{sech}^{-1}(a+bx)}{x}dx$

$$-a\left(\int \frac{e^{\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)}{-e^{\text{sech}^{-1}(a+bx)}a-\sqrt{1-a^2}+1}d\text{sech}^{-1}(a+bx)+\int \frac{e^{\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)}{-e^{\text{sech}^{-1}(a+bx)}a+\sqrt{1-a^2}+1}d\text{sech}^{-1}(a+bx)+\frac{\text{sec}}{}\right.$$
$$i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\text{sech}^{-1}(a+bx)\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)\right)-\frac{1}{2}i\text{sech}^{-1}(a+bx)^2\right)$$

$\downarrow$ 2620

$$-a\left(\frac{\int \log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\text{sech}^{-1}(a+bx)}{a}+\frac{\int \log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)d\text{sech}^{-1}(a+bx)}{a}-\frac{\text{sech}^{-1}(a+bx)\log}{}\right.$$
$$i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\text{sech}^{-1}(a+bx)\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)\right)-\frac{1}{2}i\text{sech}^{-1}(a+bx)^2\right)$$

$\downarrow$ 2715

$$-a\left(\frac{\int e^{-\text{sech}^{-1}(a+bx)}\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)de^{\text{sech}^{-1}(a+bx)}}{a}+\frac{\int e^{-\text{sech}^{-1}(a+bx)}\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)de^{\text{sech}^{-1}(a+}}{a}\right.$$
$$i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\text{sech}^{-1}(a+bx)\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)\right)-\frac{1}{2}i\text{sech}^{-1}(a+bx)^2\right)$$

$\downarrow$ 2838

$$-a\left(-\frac{\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a}-\frac{\text{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{\text{sec}}{}\right.$$
$$i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\text{sech}^{-1}(a+bx)\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)\right)-\frac{1}{2}i\text{sech}^{-1}(a+bx)^2\right)$$

input
```
Int[ArcSech[a + b*x]/x,x]
```

output
```
-(a*(ArcSech[a + b*x]^2/(2*a) - (ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a +
 b*x])/(1 - Sqrt[1 - a^2])])/a - (ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a
+ b*x])/(1 + Sqrt[1 - a^2])])/a - PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - S
qrt[1 - a^2])]/a - PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]/
a)) + I*((-1/2*I)*ArcSech[a + b*x]^2 + (2*I)*((ArcSech[a + b*x]*Log[1 + E^
(2*ArcSech[a + b*x])])/2 + PolyLog[2, -E^(2*ArcSech[a + b*x])]/4))
```

3.5. $\int \frac{\text{sech}^{-1}(a+bx)}{x}\,dx$

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

3.5. $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx$

rule 6104
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Tanh[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Sinh[c + d*x]*(Tanh[c + d*x
]^(n - 1)/(a + b*Cosh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6129
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[(e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cosh[c + d*x
])), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.5.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 882, normalized size of antiderivative = 5.19

method	result
derivativedivides	$\frac{\operatorname{arcsech}(bx+a)\ln\left(\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{-a^2+1}-1}{-1+\sqrt{-a^2+1}}\right)}{2}+\frac{\operatorname{arcsech}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\sqrt{\frac{1}{bx+a}+\cdots}\right.}{1+\sqrt{-a^2+1}}\right.}{2}$
default	$\frac{\operatorname{arcsech}(bx+a)\ln\left(\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{-a^2+1}-1}{-1+\sqrt{-a^2+1}}\right)}{2}+\frac{\operatorname{arcsech}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\sqrt{\frac{1}{bx+a}+\cdots}\right.}{1+\sqrt{-a^2+1}}\right.}{2}$

input
```
int(arcsech(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2
))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))+1/2*arcsech(b*x+a)*ln((-a*(1/(b*
x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1
)^(1/2)))+1/2*(-a^2+1)^(1/2)/(a^2-1)*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b
*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))
-1/2*(-a^2+1)^(1/2)/(a^2-1)*arcsech(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+dilog((a
*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1
+(-a^2+1)^(1/2)))+dilog((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(
1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-arcsech(b*x+a)*ln(1+I*(1/(b*x+
a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-arcsech(b*x+a)*ln(1-I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-dilog(1+I*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2)))+1/2*(a^2+(-a^2+1)^(1/2)-1)*arcsech(b*x+a)*(ln((-a*
(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(
-a^2+1)^(1/2)))*a^2+ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/
2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))*a^2-2*ln((a*(1/(b*x+a)+(1/(b*x+
a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-2*
ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1
)/(-1+(-a^2+1)^(1/2)))*(-a^2+1)^(1/2))/a^2/(a^2-1)
```

3.5.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsech}(bx+a)}{x}\,dx$$

input
```
integrate(arcsech(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)/x, x)
```

3.5.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{asech}(a+bx)}{x}\,dx$$

input
```
integrate(asech(b*x+a)/x,x)
```

output
```
Integral(asech(a + b*x)/x, x)
```

3.5.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsech}(bx+a)}{x}\,dx$$

input
```
integrate(arcsech(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(arcsech(b*x + a)/x, x)
```

3.5.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsech}(bx+a)}{x}\,dx$$

input
```
integrate(arcsech(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)/x, x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x}\,dx$$

input
```
int(acosh(1/(a + b*x))/x,x)
```

output
```
int(acosh(1/(a + b*x))/x, x)
```

3.6 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2} dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2} dx = -\frac{b\operatorname{sech}^{-1}(a+bx)}{a} - \frac{\operatorname{sech}^{-1}(a+bx)}{x} + \frac{2b\operatorname{arctanh}\left(\frac{\sqrt{1+a}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}$$

output
```
-b*arcsech(b*x+a)/a-arcsech(b*x+a)/x+2*b*arctanh((1+a)^(1/2)*tanh(1/2*arcs
ech(b*x+a))/(1-a)^(1/2))/a/(-a^2+1)^(1/2)
```

3.6.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.49

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2} dx = -\frac{\operatorname{sech}^{-1}(a+bx)}{x} + \frac{b\left(-\log(x)+\sqrt{1-a^2}\log(a+bx)-\sqrt{1-a^2}\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a+bx}{1+a+bx}}+bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)\right.}{a\sqrt{1-}}$$

input
```
Integrate[ArcSech[a + b*x]/x^2,x]
```

output
```
-(ArcSech[a + b*x]/x) + (b*(-Log[x] + Sqrt[1 - a^2]*Log[a + b*x] - Sqrt[1
- a^2]*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a +
b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]] + Log[1
- a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sq
rt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*x*Sqrt
[-((-1 + a + b*x)/(1 + a + b*x))]]))/(a*Sqrt[1 - a^2])
```

3.6.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6875, 5991, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{6875}$$

$$-b \int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{b^2x^2}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{5991}$$

$$-b\left(\int -\frac{1}{bx}d\text{sech}^{-1}(a+bx)+\frac{\text{sech}^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{3042}$$

$$-b\left(\frac{\text{sech}^{-1}(a+bx)}{bx}+\int \frac{1}{a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{4270}$$

$$-b\left(-\frac{\int \frac{1}{1-\frac{a}{a+bx}}d\text{sech}^{-1}(a+bx)}{a}+\frac{\text{sech}^{-1}(a+bx)}{a}+\frac{\text{sech}^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{3042}$$

$$-b\left(-\frac{\int \frac{1}{1-a\sin\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}d\text{sech}^{-1}(a+bx)}{a}+\frac{\text{sech}^{-1}(a+bx)}{a}+\frac{\text{sech}^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{3138}$$

$$-b\left(-\frac{2\int \frac{1}{-\left((a+1)\tanh^2\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)\right)-a+1}d\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{a}+\frac{\text{sech}^{-1}(a+bx)}{a}+\frac{\text{sech}^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{221}$$

$$-b\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}+\frac{\text{sech}^{-1}(a+bx)}{a}+\frac{\text{sech}^{-1}(a+bx)}{bx}\right)$$

input
```
Int[ArcSech[a + b*x]/x^2,x]
```

output
```
-(b*(ArcSech[a + b*x]/a + ArcSech[a + b*x]/(b*x) - (2*ArcTanh[(Sqrt[1 + a]
*Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2])))
```

3.6.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(62) = 124$.

Time = 0.95 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.44

method	result
derivativedivides	$b\left(-\frac{\operatorname{arcsech}(bx+a)}{bx}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(bx+a)^2}}\right)a^2+\sqrt{-a^2+1}\,\ln\left(\frac{2\sqrt{-a^2+1}\,\sqrt{1-(bx-\cdots}{bx}\right.\right.}{\sqrt{1-(bx+a)^2}\,a(-1+a)(1+a)}\right.$
default	$b\left(-\frac{\operatorname{arcsech}(bx+a)}{bx}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(bx+a)^2}}\right)a^2+\sqrt{-a^2+1}\,\ln\left(\frac{2\sqrt{-a^2+1}\,\sqrt{1-(bx-\cdots}{bx}\right.\right.}{\sqrt{1-(bx+a)^2}\,a(-1+a)(1+a)}\right.$
parts	$-\frac{\operatorname{arcsech}(bx+a)}{x}-\frac{b\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)a^2+\sqrt{-a^2+1}\,\ln\left(\frac{-2a^2+2-2abx\cdots}{\cdots}\right.\right.}{\sqrt{-b^2x^2-2abx-a^2+1}\,(1+a)\cdots}$

input
```
int(arcsech(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-1/b/x*arcsech(b*x+a)-(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1)/(b*
x+a))^(1/2)*(arctanh(1/(1-(b*x+a)^2)^(1/2))*a^2+(-a^2+1)^(1/2)*ln(2*((-a^2
+1)^(1/2)*(1-(b*x+a)^2)^(1/2)-(b*x+a)*a+1)/b/x)-arctanh(1/(1-(b*x+a)^2)^(1
/2)))/(1-(b*x+a)^2)^(1/2)/a/(-1+a)/(1+a))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(62) = 124$.

Time = 0.30 (sec) , antiderivative size = 651, normalized size of antiderivative = 9.30

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2}\,dx$$

$$= \left[-\frac{(a^2-1)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right) - (a^2-1)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right) + \sqrt{-a^2+1}b}{}\right.$$

$$-\frac{(a^2-1)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right) - (a^2-1)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right) + 2\sqrt{a^2-1}bx}{2\,(a^3-}$$

input
```
integrate(arcsech(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
[-1/2*((a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b
^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2
*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + sqrt(-a^2 +
 1)*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*(a*
b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b
*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 2)/x^2) + 2*(a^3 - a)*log(((b*x
 + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/
(b*x + a)))/((a^3 - a)*x), -1/2*((a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2*x
^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^2 - 1)*b*x
*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a
^2)) - 1)/x) + 2*sqrt(a^2 - 1)*b*x*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b
*x - a)*sqrt(a^2 - 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b
*x + a^2))/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + 2*(a
^3 - a)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*
b*x + a^2)) + 1)/(b*x + a)))/((a^3 - a)*x)]
```

3.6.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{asech}(a+bx)}{x^2}\,dx$$

input
```
integrate(asech(b*x+a)/x**2,x)
```

output
```
Integral(asech(a + b*x)/x**2, x)
```

3.6.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{arsech}(bx+a)}{x^2}\,dx$$

input
```
integrate(arcsech(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
b*log(x)/(a^3 - a) - 1/2*((a^2*b - a*b)*x*log(b*x + a + 1) + (a^2*b + a*b)
*x*log(-b*x - a + 1) + 2*(a^3 - a)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1
)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*(a^3 + (a^2*
b - b)*x - a)*log(b*x + a) - 2*(a^3 - a)*log(b*x + a))/((a^3 - a)*x) - int
egrate((b^2*x + a*b)/(b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x + (b^2*x^3 + 2*a*b
*x^2 + (a^2 - 1)*x)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))), x)
```

3.6.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{arsech}(bx+a)}{x^2}\,dx$$

input
```
integrate(arcsech(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)/x^2, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

input
```
int(acosh(1/(a + b*x))/x^2,x)
```

output
```
int(acosh(1/(a + b*x))/x^2, x)
```

3.7 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3} \, dx$

3.7.1 Optimal result

Integrand size = 10, antiderivative size = 133

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3} \, dx = \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{(1-2a^2)\,b^2\operatorname{arctanh}\left(\frac{\sqrt{1+a}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2(1-a^2)^{3/2}}$$

output
```
1/2*b^2*arcsech(b*x+a)/a^2-1/2*arcsech(b*x+a)/x^2-(-2*a^2+1)*b^2*arctanh((
1+a)^(1/2)*tanh(1/2*arcsech(b*x+a))/(1-a)^(1/2))/a^2/(-a^2+1)^(3/2)+1/2*b*
(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/x
```

3.7.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(133) = 266.

Time = 0.90 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.37

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3}\,dx$$
$$= \frac{1}{2}\left(-\frac{b\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)}{(-1+a)a(1+a)x} - \frac{\operatorname{sech}^{-1}(a+bx)}{x^2} - \frac{(-1+2a^2)\,b^2\log(x)}{a^2\,(1-a^2)^{3/2}}\right.$$
$$- \frac{b^2\log(a+bx)}{a^2} + \frac{b^2\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a+bx}{1+a+bx}}+bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)}{a^2}$$
$$+ \frac{(-1+2a^2)\,b^2\log\left(1-a^2-abx+\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}}+\sqrt{1-a^2}bx\sqrt{-\frac{-1+a}{1+a+}}\right.}{a^2\,(1-a^2)^{3/2}}$$

input
```
Integrate[ArcSech[a + b*x]/x^3,x]
```

output
```
(-((b*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x))/((-1 + a)*a*(1
+ a)*x)) - ArcSech[a + b*x]/x^2 - ((-1 + 2*a^2)*b^2*Log[x])/(a^2*(1 - a^2)
^(3/2)) - (b^2*Log[a + b*x])/a^2 + (b^2*Log[1 + Sqrt[-((-1 + a + b*x)/(1 +
 a + b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a
 + b*x)/(1 + a + b*x))]])/a^2 + ((-1 + 2*a^2)*b^2*Log[1 - a^2 - a*b*x + Sq
rt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[1 - a^2]*Sqrt[-
((-1 + a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*x*Sqrt[-((-1 + a + b*x)/
(1 + a + b*x))]])/(a^2*(1 - a^2)^(3/2)))/2
```

3.7.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6875, 25, 5991, 3042, 4272, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3}\,dx$$
$$\Big\downarrow \; 6875$$

$$-b^2\int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{b^3x^3}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$b^2\int -\frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{b^3x^3}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{5991}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)}{2b^2x^2}-\frac{1}{2}\int \frac{1}{b^2x^2}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)}{2b^2x^2}-\frac{1}{2}\int \frac{1}{\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{4272}$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\int -\frac{-a^2-(a+bx)a+1}{bx}d\text{sech}^{-1}(a+bx)}{a(1-a^2)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}\right)+\frac{\text{sech}^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)}{2b^2x^2}+\frac{1}{2}\left(-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}-\frac{\int \frac{-a^2-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)a+1}{a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}d\text{sech}^{-1}(a+bx)}{a(1-a^2)}\right)\right)$$

$$\Big\downarrow \text{4407}$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{(1-2a^2)\int -\frac{a+bx}{bx}d\text{sech}^{-1}(a+bx)}{a}+\frac{(1-a^2)\text{sech}^{-1}(a+bx)}{a}}{a(1-a^2)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}\right)+\frac{\text{sech}^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)}{2b^2x^2}+\frac{1}{2}\left(-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}-\frac{\frac{(1-a^2)\text{sech}^{-1}(a+bx)}{a}+\frac{(1-2a^2)\int \frac{\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}{a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}d\text{sech}}{a}}{a(1-a^2)}\right.\right.$$

$$\Big\downarrow \text{4318}$$

3.7. $\int \frac{\text{sech}^{-1}(a+bx)}{x^3}\,dx$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{(1-a^2)\text{sech}^{-1}(a+bx)}{a}-\frac{(1-2a^2)\int\frac{1}{1-\frac{a}{a+bx}}d\text{sech}^{-1}(a+bx)}{a}}{a\left(1-a^2\right)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a\left(1-a^2\right)bx}\right)+\frac{\text{sech}^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)}{2b^2x^2}+\frac{1}{2}\left(-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a\left(1-a^2\right)bx}-\frac{\frac{(1-a^2)\text{sech}^{-1}(a+bx)}{a}-\frac{(1-2a^2)\int\frac{1}{1-a\sin\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}d\text{secl}}{a}}{a\left(1-a^2\right)}\right.\right.$$

$$\Big\downarrow \text{ 3138}$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{(1-a^2)\text{sech}^{-1}(a+bx)}{a}-\frac{2(1-2a^2)\int\frac{1}{-\left((a+1)\tanh^2\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)\right)-a+1}d\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{a}}{a\left(1-a^2\right)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+}{a\left(1-a^2\right)}\right.\right.$$

$$\Big\downarrow \text{ 221}$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{(1-a^2)\text{sech}^{-1}(a+bx)}{a}-\frac{2(1-2a^2)\text{arctanh}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}}{a\left(1-a^2\right)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a\left(1-a^2\right)bx}\right)+\frac{\text{sech}^{-1}}{2b}\right.$$

input
```
Int[ArcSech[a + b*x]/x^3,x]
```

output
```
-(b^2*(ArcSech[a + b*x]/(2*b^2*x^2) + (-((Sqrt[(1 - a - b*x)/(1 + a + b*x)
]*(1 + a + b*x))/(a*(1 - a^2)*b*x)) - (((1 - a^2)*ArcSech[a + b*x])/a - (2
*(1 - 2*a^2)*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/
(a*Sqrt[1 - a^2]))/(a*(1 - a^2)))/2))
```

3.7. $\int \frac{\text{sech}^{-1}(a+bx)}{x^3}\,dx$

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.7.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.96 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.78

method	result
parts	$-\frac{\operatorname{arcsech}(bx+a)}{2x^2} - \frac{b\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\ \operatorname{csgn}(b)^2\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)a^4bx-2\sqrt{-a^2+1}\ \ln(\right.}{}$
derivativedivides	$b^2\left(-\frac{\operatorname{arcsech}(bx+a)}{2b^2x^2} - \frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(bx+a)^2}}\right)a^5-\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(bx+a)^2}}\right)a^4(bx-\right.}{}\right.$
default	$b^2\left(-\frac{\operatorname{arcsech}(bx+a)}{2b^2x^2} - \frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(bx+a)^2}}\right)a^5-\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(bx+a)^2}}\right)a^4(bx-\right.}{}\right.$

input
```
int(arcsech(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsech(b*x+a)/x^2-1/2*b*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1
)/(b*x+a))^(1/2)*csgn(b)^2*(-arctanh(1/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))*a^4
*b*x-2*(-a^2+1)^(1/2)*ln(2*(-a*b*x+(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)
^(1/2)-a^2+1)/x)*a^2*b*x+2*arctanh(1/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))*a^2*b
*x+a^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+(-a^2+1)^(1/2)*ln(2*(-a*b*x+(-a^2+1)
^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a^2+1)/x)*b*x-arctanh(1/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2))*b*x-(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)/(1+a)/(-1+a)/a^2/(a^2-1)/x
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(112) = 224$.

Time = 0.32 (sec) , antiderivative size = 865, normalized size of antiderivative = 6.50

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3}\,dx$$

$$= \left[-\frac{(2a^2-1)\sqrt{-a^2+1}b^2x^2\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-4a^2+2(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{-a^2+1}\sqrt{-\frac{b^2x^2+2abx+a^2}{b^2x^2+2abx+}}}{x^2}\right.}{}\right.$$

input
```
integrate(arcsech(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
[-1/4*((2*a^2 - 1)*sqrt(-a^2 + 1)*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4
 + 4*(a^3 - a)*b*x - 4*a^2 + 2*(a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqr
t(-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))
 + 2)/x^2) - (a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + (a^4 - 2*a^2 + 1)*b^2
*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x
 + a^2)) - 1)/x) + 2*(a^6 - 2*a^4 + a^2)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2
*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) + 2*((a^3 - a
)*b^2*x^2 + (a^4 - a^2)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
+ 2*a*b*x + a^2)))/((a^6 - 2*a^4 + a^2)*x^2), 1/4*(2*(2*a^2 - 1)*sqrt(a^2
- 1)*b^2*x^2*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(a^2 - 1)*
sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/((a^2 - 1)*
b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^4 - 2*a^2 + 1)*b^2*x^2*
log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^
2)) + 1)/x) - (a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*
a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - 2*(a^6 - 2*a^4 + a^2
)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x +
a^2)) + 1)/(b*x + a)) - 2*((a^3 - a)*b^2*x^2 + (a^4 - a^2)*b*x)*sqrt(-(b^2
*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a^6 - 2*a^4 + a^2)
*x^2)]
```

3.7.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3} dx = \int \frac{\operatorname{asech}(a+bx)}{x^3} dx$$

input
```
integrate(asech(b*x+a)/x**3,x)
```

output
```
Integral(asech(a + b*x)/x**3, x)
```

3.7.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3} dx = \int \frac{\operatorname{arsech}(bx+a)}{x^3} dx$$

input
```
integrate(arcsech(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(3*a^2*b^2 - b^2)*log(x)/(a^6 - 2*a^4 + a^2) + 1/4*((a^4*b^2 - 2*a^3*
b^2 + a^2*b^2)*x^2*log(b*x + a + 1) + (a^4*b^2 + 2*a^3*b^2 + a^2*b^2)*x^2*
log(-b*x - a + 1) - 2*(a^3*b - a*b)*x - 2*(a^6 - 2*a^4 + a^2)*log(sqrt(b*x
 + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a
+ b*x + a) + 2*(a^6 - 2*a^4 - (a^4*b^2 - 2*a^2*b^2 + b^2)*x^2 + a^2)*log(b
*x + a) + 2*(a^6 - 2*a^4 + a^2)*log(b*x + a))/((a^6 - 2*a^4 + a^2)*x^2) -
integrate(1/2*(b^2*x + a*b)/(b^2*x^4 + 2*a*b*x^3 + (a^2 - 1)*x^2 + (b^2*x^
4 + 2*a*b*x^3 + (a^2 - 1)*x^2)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a
+ 1))), x)
```

3.7.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3} dx = \int \frac{\operatorname{arsech}(bx+a)}{x^3} dx$$

input
```
integrate(arcsech(b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)/x^3, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

input
```
int(acosh(1/(a + b*x))/x^3,x)
```

output
```
int(acosh(1/(a + b*x))/x^3, x)
```

3.8 $\int \frac{\text{sech}^{-1}(a+bx)}{x^4}\,dx$

3.8.1 Optimal result

Integrand size = 10, antiderivative size = 197

$$\int \frac{\text{sech}^{-1}(a+bx)}{x^4}\,dx = \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\text{sech}^{-1}(a+bx)}{3a^3} - \frac{\text{sech}^{-1}(a+bx)}{3x^3} + \frac{(2-5a^2+6a^4)b^3\text{arctanh}\left(\frac{\sqrt{1+a}\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{3a^3(1-a^2)^{5/2}}$$

output
```
-1/3*b^3*arcsech(b*x+a)/a^3-1/3*arcsech(b*x+a)/x^3+1/3*(6*a^4-5*a^2+2)*b^3
*arctanh((1+a)^(1/2)*tanh(1/2*arcsech(b*x+a))/(1-a)^(1/2))/a^3/(-a^2+1)^(5
/2)+1/6*b*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/x^2-1/6*(-5*a^
2+2)*b^2*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a^2/(-a^2+1)^2/x
```

3.8.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx$$
$$= \frac{1}{6}\left(\frac{b\sqrt{-\frac{-1+a+bx}{1+a+bx}}(a-a^4-abx-2bx(1+bx)+a^3(-1+4bx)+a^2(1+5bx+5b^2x^2))}{(-1+a)^2a^2(1+a)^2x^2}\right.$$
$$-\frac{2\operatorname{sech}^{-1}(a+bx)}{x^3}-\frac{(2-5a^2+6a^4)\,b^3\log(x)}{a^3\,(1-a^2)^{5/2}}+\frac{2b^3\log(a+bx)}{a^3}$$
$$-\frac{2b^3\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a+bx}{1+a+bx}}+bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)}{a^3}$$
$$+\frac{(2-5a^2+6a^4)\,b^3\log\left(1-a^2-abx+\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}}+\sqrt{1-a^2}bx\sqrt{-\right.}{a^3\,(1-a^2)^{5/2}}$$

input
```
Integrate[ArcSech[a + b*x]/x^4,x]
```

output
```
((b*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(a - a^4 - a*b*x - 2*b*x*(1 + b*
x) + a^3*(-1 + 4*b*x) + a^2*(1 + 5*b*x + 5*b^2*x^2)))/((-1 + a)^2*a^2*(1 +
 a)^2*x^2) - (2*ArcSech[a + b*x])/x^3 - ((2 - 5*a^2 + 6*a^4)*b^3*Log[x])/(
a^3*(1 - a^2)^(5/2)) + (2*b^3*Log[a + b*x])/a^3 - (2*b^3*Log[1 + Sqrt[-((-
1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b
*x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]])/a^3 + ((2 - 5*a^2 + 6*a^4)*b^3*
Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]
+ a*Sqrt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*
x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]])/(a^3*(1 - a^2)^(5/2)))/6
```

3.8.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {6875, 5991, 3042, 4272, 3042, 4548, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(a+bx)}{x^4}\,dx$$

$$\Big\downarrow \text{6875}$$

$$-b^3\int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)}{b^4x^4}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{5991}$$

$$-b^3\left(\frac{1}{3}\int -\frac{1}{b^3x^3}d\text{sech}^{-1}(a+bx)+\frac{\text{sech}^{-1}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$-b^3\left(\frac{\text{sech}^{-1}(a+bx)}{3b^3x^3}+\frac{1}{3}\int \frac{1}{\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^3}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{4272}$$

$$-b^3\left(\frac{1}{3}\left(\frac{\int \frac{-(a+bx)^2-2a(a+bx)+2(1-a^2)}{b^2x^2}d\text{sech}^{-1}(a+bx)}{2a(1-a^2)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{2a(1-a^2)b^2x^2}\right)+\frac{\text{sech}^{-1}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$-b^3\left(\frac{\text{sech}^{-1}(a+bx)}{3b^3x^3}+\frac{1}{3}\left(-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{2a(1-a^2)b^2x^2}+\frac{\int \frac{-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)^2-2a\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)+2(1-a^2)}{\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2}}{2a(1-a^2)}\right.\right.$$

$$\Big\downarrow \text{4548}$$

$$-b^3\left(\frac{1}{3}\left(\frac{\frac{\int -\frac{2(1-a^2)^2-a(1-4a^2)(a+bx)}{bx}d\text{sech}^{-1}(a+bx)}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}}{2a(1-a^2)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{2a(1-a^2)b^2x^2}\right)+\frac{\text{sech}^{-1}}{3b}\right.$$

$$\Big\downarrow \text{3042}$$

$$-b^3\left(\frac{\text{sech}^{-1}(a+bx)}{3b^3x^3}+\frac{1}{3}\left(-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{2a(1-a^2)b^2x^2}+\frac{\frac{(2-5a^2)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}+\frac{\int \frac{2(1-a^2)^2-a(1-4a^2)\csc\left(i\text{sech}^{-1}(a\right.}{a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}}{a(1-a^2)}}{2a(1-a^2)}\right.\right.$$

3.8. $\int \frac{\text{sech}^{-1}(a+bx)}{x^4}\,dx$

$$\Big\downarrow \text{4407}$$

$$-b^3\left(\frac{1}{3}\left(\frac{\frac{\left(6a^4-5a^2+2\right)\int -\frac{\frac{a+bx}{bx}}{a}d\,\text{sech}^{-1}(a+bx)+\frac{2\left(1-a^2\right)^2\text{sech}^{-1}(a+bx)}{a}}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}}{2a\left(1-a^2\right)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1}{2a\left(1-a^2\right)b^2x^2}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-b^3\left(\frac{\text{sech}^{-1}(a+bx)}{3b^3x^3}+\frac{1}{3}\left(-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{2a\left(1-a^2\right)b^2x^2}+\frac{\frac{(2-5a^2)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}+\frac{\frac{2\left(1-a^2\right)^2\text{sech}^{-1}(a+bx)}{a}+\frac{\left(6a^4-5a^2+\right.}{}}{}}{2a\left(1-a^2\right)}\right.\right.$$

$$\Big\downarrow \text{4318}$$

$$-b^3\left(\frac{1}{3}\left(\frac{\frac{\frac{2\left(1-a^2\right)^2\text{sech}^{-1}(a+bx)}{a}-\frac{\left(6a^4-5a^2+2\right)\int \frac{1}{1-\frac{a}{a+bx}}d\,\text{sech}^{-1}(a+bx)}{a}}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}}{2a\left(1-a^2\right)}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+}{2a\left(1-a^2\right)b^2x^2}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-b^3\left(\frac{\text{sech}^{-1}(a+bx)}{3b^3x^3}+\frac{1}{3}\left(-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{2a\left(1-a^2\right)b^2x^2}+\frac{\frac{(2-5a^2)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}+\frac{\frac{2\left(1-a^2\right)^2\text{sech}^{-1}(a+bx)}{a}-\frac{\left(6a^4-5a^2+\right.}{}}{}}{2a\left(1-a^2\right)}\right.\right.$$

$$\Big\downarrow \text{3138}$$

$$-b^3\left(\frac{1}{3}\left(\frac{\frac{\frac{2\left(1-a^2\right)^2\text{sech}^{-1}(a+bx)}{a}-\frac{2\left(6a^4-5a^2+2\right)\int \frac{1}{-\left((a+1)\tanh^2\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)\right)-a+1}d\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{a}}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{\frac{-a-bx+1}{a+bx+1}}}{a(1-a^2)bx}}{2a\left(1-a^2\right)}\right.\right.$$

$$\Big\downarrow \text{221}$$

3.8. $\quad \int \frac{\text{sech}^{-1}(a+bx)}{x^4}\,dx$

$$-b^3\left(\frac{1}{3}\left(\frac{\frac{(2-5a^2)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a(1-a^2)bx}+\frac{\frac{2(1-a^2)^2\text{sech}^{-1}(a+bx)}{a}-\frac{2(6a^4-5a^2+2)\text{arctanh}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}}{a(1-a^2)}}{2a(1-a^2)}-\frac{\sqrt{\frac{-a-b}{a+bx}}}{2a(}\right.\right.$$

input
```
Int[ArcSech[a + b*x]/x^4,x]
```

output
```
-(b^3*(ArcSech[a + b*x]/(3*b^3*x^3) + (-1/2*(Sqrt[(1 - a - b*x)/(1 + a + b
*x)]*(1 + a + b*x))/(a*(1 - a^2)*b^2*x^2) + (((2 - 5*a^2)*Sqrt[(1 - a - b*
x)/(1 + a + b*x)]*(1 + a + b*x))/(a*(1 - a^2)*b*x) + ((2*(1 - a^2)^2*ArcSe
ch[a + b*x])/a - (2*(2 - 5*a^2 + 6*a^4)*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[
a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2]))/(a*(1 - a^2)))/(2*a*(1 - a^2
)))/3))
```

3.8.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.8. $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4} \, dx$

3.8.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.97 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.09

method	result
parts	$-\frac{\operatorname{arcsech}(bx+a)}{3x^3}-\frac{b\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\ \operatorname{csgn}(b)^2\left(2\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)a^6b^2x^2+6\sqrt{-a^2+1}\ \mathrm{l}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arcsech(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsech(b*x+a)/x^3-1/6*b*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1
)/(b*x+a))^(1/2)*csgn(b)^2*(2*arctanh(1/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))*a^
6*b^2*x^2+6*(-a^2+1)^(1/2)*ln(2*(-a*b*x+(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)-a^2+1)/x)*a^4*b^2*x^2-6*arctanh(1/(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2))*a^4*b^2*x^2-5*(-a^2+1)^(1/2)*ln(2*(-a*b*x+(-a^2+1)^(1/2)*(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)-a^2+1)/x)*a^2*b^2*x^2-5*a^5*b*x*(-b^2*x^2-2*a*b*x-a^2+1)
^(1/2)+6*arctanh(1/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))*a^2*b^2*x^2+a^6*(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)+2*(-a^2+1)^(1/2)*ln(2*(-a*b*x+(-a^2+1)^(1/2)*(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)-a^2+1)/x)*b^2*x^2+7*(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2)*a^3*b*x-2*arctanh(1/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))*b^2*x^2-2*(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)*a^4-2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a*b*x+(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)*a^2)/x^2/(a^2-1)^2/(-1+a)/(1+a)/a^3/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(167) = 334.

Time = 0.33 (sec) , antiderivative size = 987, normalized size of antiderivative = 5.01

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx$$

$$= \left[-\frac{(6a^4-5a^2+2)\sqrt{-a^2+1}b^3x^3\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-4a^2-2(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{-a^2+1}\sqrt{-\frac{b^2x^2}{b^2x}}}{x^2}\right.}{} \right.$$

$$-\frac{(6a^4-5a^2+2)\sqrt{a^2-1}b^3x^3\arctan\left(\frac{(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{a^2-1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{(a^2-1)b^2x^2+a^4+2(a^3-a)bx-2a^2+1}\right)+(a^6-3a^4+3a^2}{}$$

input
```
integrate(arcsech(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
[-1/12*((6*a^4 - 5*a^2 + 2)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^
2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*(a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x
 - a)*sqrt(-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*
x + a^2)) + 2)/x^2) + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*s
qrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - 2*
(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x
+ a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 4*(a^9 - 3*a^7 + 3*a^5 - a
^3)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x
+ a^2)) + 1)/(b*x + a)) - 2*((5*a^5 - 7*a^3 + 2*a)*b^3*x^3 + (4*a^6 - 5*a^
4 + a^2)*b^2*x^2 - (a^7 - 2*a^5 + a^3)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2
 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a^9 - 3*a^7 + 3*a^5 - a^3)*x^3), -1/6*
((6*a^4 - 5*a^2 + 2)*sqrt(a^2 - 1)*b^3*x^3*arctan((a*b^2*x^2 + a^3 + (2*a^
2 - 1)*b*x - a)*sqrt(a^2 - 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
 + 2*a*b*x + a^2))/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)
) + (a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*
b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^6 - 3*a^4 + 3*a^2 -
 1)*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 +
2*a*b*x + a^2)) - 1)/x) + 2*(a^9 - 3*a^7 + 3*a^5 - a^3)*log(((b*x + a)*sqr
t(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)
) - ((5*a^5 - 7*a^3 + 2*a)*b^3*x^3 + (4*a^6 - 5*a^4 + a^2)*b^2*x^2 - (a...
```

3.8. $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx$

3.8.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{asech}(a+bx)}{x^4}\,dx$$

input
```
integrate(asech(b*x+a)/x**4,x)
```

output
```
Integral(asech(a + b*x)/x**4, x)
```

3.8.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{arsech}(bx+a)}{x^4}\,dx$$

input
```
integrate(arcsech(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
1/3*(6*a^4*b^3 - 3*a^2*b^3 + b^3)*log(x)/(a^9 - 3*a^7 + 3*a^5 - a^3) - 1/6
*((a^6*b^3 - 3*a^5*b^3 + 3*a^4*b^3 - a^3*b^3)*x^3*log(b*x + a + 1) + (a^6*
b^3 + 3*a^5*b^3 + 3*a^4*b^3 + a^3*b^3)*x^3*log(-b*x - a + 1) - 2*(3*a^5*b^
2 - 4*a^3*b^2 + a*b^2)*x^2 + (a^6*b - 2*a^4*b + a^2*b)*x + 2*(a^9 - 3*a^7
+ 3*a^5 - a^3)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
 + 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*(a^9 - 3*a^7 + 3*a^5 + (a^6*b^3
- 3*a^4*b^3 + 3*a^2*b^3 - b^3)*x^3 - a^3)*log(b*x + a) - 2*(a^9 - 3*a^7 +
3*a^5 - a^3)*log(b*x + a))/((a^9 - 3*a^7 + 3*a^5 - a^3)*x^3) - integrate(1
/3*(b^2*x + a*b)/(b^2*x^5 + 2*a*b*x^4 + (a^2 - 1)*x^3 + (b^2*x^5 + 2*a*b*x
^4 + (a^2 - 1)*x^3)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))), x)
```

3.8.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{arsech}(bx+a)}{x^4}\,dx$$

input
```
integrate(arcsech(b*x+a)/x^4,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)/x^4, x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx=\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x^4}\,dx$$

input
```
int(acosh(1/(a + b*x))/x^4,x)
```

output
```
int(acosh(1/(a + b*x))/x^4, x)
```

3.9 $\int x^2\text{sech}^{-1}(a+bx)^2\,dx$

3.9.1 Optimal result

Integrand size = 12, antiderivative size = 279

$$\begin{aligned}\int x^2\text{sech}^{-1}(a+bx)^2\,dx =& -\frac{x}{3b^2}+\frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3}\\ &-\frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)}{3b^3}\\ &+\frac{a^3\text{sech}^{-1}(a+bx)^2}{3b^3}+\frac{1}{3}x^3\text{sech}^{-1}(a+bx)^2\\ &-\frac{2\text{sech}^{-1}(a+bx)\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)}{3b^3}\\ &-\frac{4a^2\text{sech}^{-1}(a+bx)\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)}{b^3}\\ &+\frac{2a\log(a+bx)}{b^3}+\frac{i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)}{3b^3}\\ &+\frac{2ia^2\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^3}\\ &-\frac{i\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(a+bx)}\right)}{3b^3}-\frac{2ia^2\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(a+bx)}\right)}{b^3}\end{aligned}$$

output
```
-1/3*x/b^2+1/3*a^3*arcsech(b*x+a)^2/b^3+1/3*x^3*arcsech(b*x+a)^2-2/3*arcse
ch(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/b^3-4*
a^2*arcsech(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2
))/b^3+2*a*ln(b*x+a)/b^3+1/3*I*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)
*(1/(b*x+a)+1)^(1/2)))/b^3+2*I*a^2*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(
1/2)*(1/(b*x+a)+1)^(1/2)))/b^3-1/3*I*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^
(1/2)*(1/(b*x+a)+1)^(1/2)))/b^3-2*I*a^2*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-
1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^3+2*a*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1
)/(b*x+a+1))^(1/2)/b^3-1/3*(b*x+a)*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b
*x+a+1))^(1/2)/b^3
```

3.9.2 Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.09

$$\int x^2 \operatorname{sech}^{-1}(a+bx)^2 \, dx =$$

$$-\frac{2(a+bx)\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)+6a(a+bx)^2\operatorname{sech}^{-1}(a+bx)^2-2(a+bx)^3\operatorname{sech}^{-1}(a+}{}$$

input
```
Integrate[x^2*ArcSech[a + b*x]^2,x]
```

output
```
-1/6*(2*(a + b*x)*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcS
ech[a + b*x] + 6*a*(a + b*x)^2*ArcSech[a + b*x]^2 - 2*(a + b*x)^3*ArcSech[
a + b*x]^2 + 2*(a + b*x - 6*a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a
 + b*x)*ArcSech[a + b*x] - 3*a^2*(a + b*x)*ArcSech[a + b*x]^2) + 12*a*Log[
(a + b*x)^(-1)] - (1 + 6*a^2)*(Pi*Log[1 - I*E^ArcSech[a + b*x]] - (2*I)*Ar
cSech[a + b*x]*Log[1 - I*E^ArcSech[a + b*x]] - Pi*Log[1 + I*E^ArcSech[a +
b*x]] + (2*I)*ArcSech[a + b*x]*Log[1 + I*E^ArcSech[a + b*x]] - Pi*Log[Cot[
(Pi + (2*I)*ArcSech[a + b*x])/4]] + (2*I)*PolyLog[2, (-I)*E^ArcSech[a + b*
x]] - (2*I)*PolyLog[2, I*E^ArcSech[a + b*x]]))/b^3
```

3.9.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6875, 5991, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{sech}^{-1}(a+bx)^2 \, dx$$

$$\Big\downarrow \ 6875$$

$$-\frac{\int b^2x^2(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2 d\text{sech}^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \ 5991$$

$$-\frac{-\frac{2}{3}\int -b^3x^3\text{sech}^{-1}(a+bx)d\text{sech}^{-1}(a+bx)-\frac{1}{3}b^3x^3\text{sech}^{-1}(a+bx)^2}{b^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{1}{3}b^3x^3\text{sech}^{-1}(a+bx)^2-\frac{2}{3}\int \text{sech}^{-1}(a+bx)\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^3 d\text{sech}^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \ 4678$$

$$-\frac{-\frac{2}{3}\int \left(\text{sech}^{-1}(a+bx)a^3-3(a+bx)\text{sech}^{-1}(a+bx)a^2+3(a+bx)^2\text{sech}^{-1}(a+bx)a-(a+bx)^3\text{sech}^{-1}(a+bx)\right)}{b^3}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{1}{3}b^3x^3\text{sech}^{-1}(a+bx)^2-\frac{2}{3}\left(\frac{1}{2}a^3\text{sech}^{-1}(a+bx)^2-6a^2\text{sech}^{-1}(a+bx)\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)+3ia^2\,\text{PolyLog}\left(2\right.\right.}{}$$

input
```
Int[x^2*ArcSech[a + b*x]^2,x]
```

output
```
-((-1/3*(b^3*x^3*ArcSech[a + b*x]^2) - (2*((-a - b*x)/2 + 3*a*Sqrt[(1 - a
- b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x] - ((a + b*x)*Sqrt[(1
- a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x])/2 + (a^3*ArcSech
[a + b*x]^2)/2 - ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]] - 6*a^2*ArcSe
ch[a + b*x]*ArcTan[E^ArcSech[a + b*x]] - 3*a*Log[(a + b*x)^(-1)] + (I/2)*P
olyLog[2, (-I)*E^ArcSech[a + b*x]] + (3*I)*a^2*PolyLog[2, (-I)*E^ArcSech[a
 + b*x]] - (I/2)*PolyLog[2, I*E^ArcSech[a + b*x]] - (3*I)*a^2*PolyLog[2, I
*E^ArcSech[a + b*x]]))/3)/b^3)
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.9.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\operatorname{arcsech}(bx+a)^2 a^2 (bx+a) - \operatorname{arcsech}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arcsech}(bx+a)^2 (bx+a)^3}{3} + 2 \operatorname{arcsech}(bx+a) \sqrt{-\frac{bx+a-1}{bx+a}} \sqrt{\frac{bx+a+1}{bx+a}}$
default	$\operatorname{arcsech}(bx+a)^2 a^2 (bx+a) - \operatorname{arcsech}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arcsech}(bx+a)^2 (bx+a)^3}{3} + 2 \operatorname{arcsech}(bx+a) \sqrt{-\frac{bx+a-1}{bx+a}} \sqrt{\frac{bx+a+1}{bx+a}}$

input
```
int(x^2*arcsech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(arcsech(b*x+a)^2*a^2*(b*x+a)-arcsech(b*x+a)^2*a*(b*x+a)^2+1/3*arcse
ch(b*x+a)^2*(b*x+a)^3+2*arcsech(b*x+a)*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+
1)/(b*x+a))^(1/2)*a*(b*x+a)-1/3*arcsech(b*x+a)*(-(b*x+a-1)/(b*x+a))^(1/2)*
((b*x+a+1)/(b*x+a))^(1/2)*(b*x+a)^2-2*a*arcsech(b*x+a)-1/3*b*x-1/3*a+1/3*I
*arcsech(b*x+a)*ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))
)-1/3*I*dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I
*a^2*dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*a^
2*dilog(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*ln(1+(1
/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)*a+4*a*ln(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))-1/3*I*arcsech(b*x+a)*ln(1-I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I*a^2*arcsech(b*x+a)*ln(1-
I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*a^2*arcsech(b*x
+a)*ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+1/3*I*dilo
g(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))))
```

3.9.5 Fricas [F]

$$\int x^2 \operatorname{sech}^{-1}(a+bx)^2 \, dx = \int x^2 \operatorname{arsech}(bx+a)^2 \, dx$$

input
```
integrate(x^2*arcsech(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^2*arcsech(b*x + a)^2, x)
```

3.9.6 Sympy [F]

$$\int x^2 \operatorname{sech}^{-1}(a+bx)^2\,dx = \int x^2 \operatorname{asech}^2(a+bx)\,dx$$

input
```
integrate(x**2*asech(b*x+a)**2,x)
```

output
```
Integral(x**2*asech(a + b*x)**2, x)
```

3.9.7 Maxima [F]

$$\int x^2 \operatorname{sech}^{-1}(a+bx)^2\,dx = \int x^2 \operatorname{arsech}(bx+a)^2\,dx$$

input
```
integrate(x^2*arcsech(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*s
qrt(-b*x - a + 1)*a + b*x + a)^2 - integrate(-2/3*(6*(b^3*x^5 + 3*a*b^2*x^
4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1
)*log(b*x + a)^2 + 6*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a
)*x^2)*log(b*x + a)^2 - (b^3*x^5 + 2*a*b^2*x^4 + (a^2*b - b)*x^3 + 6*(b^3*
x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a) + (3*(
b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a +
1)*log(b*x + a) + (2*b^3*x^5 + 4*a*b^2*x^4 + (2*a^2*b - b)*x^3 + 3*(b^3*x^
5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a))*sqrt(b*
x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b
*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2
*x^2 + a^3 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x
+ a + 1)*sqrt(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

3.9.8 Giac [F]

$$\int x^2 \mathrm{sech}^{-1}(a+bx)^2\,dx=\int x^2\,\mathrm{arsech}\,(bx+a)^2\,\,dx$$

input
```
integrate(x^2*arcsech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arcsech(b*x + a)^2, x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{sech}^{-1}(a+bx)^2\,dx=\int x^2\,\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

input
```
int(x^2*acosh(1/(a + b*x))^2,x)
```

output
```
int(x^2*acosh(1/(a + b*x))^2, x)
```

3.10 $\int x\text{sech}^{-1}(a+bx)^2\,dx$

3.10.1 Optimal result

Integrand size = 10, antiderivative size = 149

$$\int x\text{sech}^{-1}(a+bx)^2\,dx=-\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)}{b^2}-\frac{a^2\text{sech}^{-1}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)^2+\frac{4a\text{sech}^{-1}(a+bx)\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)}{b^2}-\frac{\log(a+bx)}{b^2}-\frac{2ia\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{2ia\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}$$

output
```
-1/2*a^2*arcsech(b*x+a)^2/b^2+1/2*x^2*arcsech(b*x+a)^2+4*a*arcsech(b*x+a)*
arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/b^2-ln(b*x+a)/b^
2-2*I*a*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/
b^2+2*I*a*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))
/b^2-(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

3.10.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.15

$$\int x\text{sech}^{-1}(a+bx)^2\,dx$$
$$=\frac{-2\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)-2a(a+bx)\text{sech}^{-1}(a+bx)^2+(a+bx)^2\text{sech}^{-1}(a+bx)^2-4i}{}$$

input
```
Integrate[x*ArcSech[a + b*x]^2,x]
```

output
```
(-2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcSech[a + b*x] -
 2*a*(a + b*x)*ArcSech[a + b*x]^2 + (a + b*x)^2*ArcSech[a + b*x]^2 - (4*I)
*a*ArcSech[a + b*x]*(Log[1 - I/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a
 + b*x]]) + 2*Log[(a + b*x)^(-1)] - (4*I)*a*(PolyLog[2, (-I)/E^ArcSech[a +
 b*x]] - PolyLog[2, I/E^ArcSech[a + b*x]]))/(2*b^2)
```

3.10.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6875, 25, 5991, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{sech}^{-1}(a+bx)^2\,dx$$
$$\Big\downarrow \text{6875}$$
$$-\frac{\int bx(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2d\text{sech}^{-1}(a+bx)}{b^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int -bx(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2d\text{sech}^{-1}(a+bx)}{b^2}$$
$$\Big\downarrow \text{5991}$$
$$-\frac{\int b^2x^2\text{sech}^{-1}(a+bx)d\text{sech}^{-1}(a+bx)-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)^2}{b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)^2+\int \text{sech}^{-1}(a+bx)\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2d\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \; 4678$$

$$-\frac{\int \left(\text{sech}^{-1}(a+bx)a^2-2(a+bx)\text{sech}^{-1}(a+bx)a+(a+bx)^2\text{sech}^{-1}(a+bx)\right)d\text{sech}^{-1}(a+bx)-\frac{1}{2}b^2x^2\text{sech}^{-1}(a}{b^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{2}a^2\text{sech}^{-1}(a+bx)^2-4a\text{sech}^{-1}(a+bx)\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)^2+2ia\,\text{PolyLog}\left(2,-ie^{\text{se}}\right.}{b^2}$$

input
```
Int[x*ArcSech[a + b*x]^2,x]
```

output
```
-((Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x] + (a^2
*ArcSech[a + b*x]^2)/2 - (b^2*x^2*ArcSech[a + b*x]^2)/2 - 4*a*ArcSech[a +
b*x]*ArcTan[E^ArcSech[a + b*x]] - Log[(a + b*x)^(-1)] + (2*I)*a*PolyLog[2,
 (-I)*E^ArcSech[a + b*x]] - (2*I)*a*PolyLog[2, I*E^ArcSech[a + b*x]])/b^2)
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.10.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.22

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsech}(bx+a)\left(2\operatorname{arcsech}(bx+a)a(bx+a)-\operatorname{arcsech}(bx+a)(bx+a)^2+2\sqrt{-\frac{bx+a-1}{bx+a}}\sqrt{\frac{bx+a+1}{bx+a}}(bx+a)-2\right)}{2}-2\ln\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-\right.}{}$
default	$\frac{-\frac{\operatorname{arcsech}(bx+a)\left(2\operatorname{arcsech}(bx+a)a(bx+a)-\operatorname{arcsech}(bx+a)(bx+a)^2+2\sqrt{-\frac{bx+a-1}{bx+a}}\sqrt{\frac{bx+a+1}{bx+a}}(bx+a)-2\right)}{2}-2\ln\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-\right.}{}$

input
```
int(x*arcsech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-1/2*arcsech(b*x+a)*(2*arcsech(b*x+a)*a*(b*x+a)-arcsech(b*x+a)*(b*x
+a)^2+2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*(b*x+a)-2)-2*
ln(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+ln(1+(1/(b*x+a)+(1/(
b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-2*I*a*arcsech(b*x+a)*ln(1+I*(1/(b*
x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*a*arcsech(b*x+a)*ln(1-I
*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I*a*dilog(1+I*(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*a*dilog(1-I*(1/(b*x+a
)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))))
```

3.10.5 Fricas [F]

$$\int x\text{sech}^{-1}(a+bx)^2\,dx = \int x\,\text{arsech}\,(bx+a)^2\,dx$$

input
```
integrate(x*arcsech(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x*arcsech(b*x + a)^2, x)
```

3.10.6 Sympy [F]

$$\int x\text{sech}^{-1}(a+bx)^2\,dx = \int x\,\text{asech}^2\,(a+bx)\,dx$$

input
```
integrate(x*asech(b*x+a)**2,x)
```

output
```
Integral(x*asech(a + b*x)**2, x)
```

3.10.7 Maxima [F]

$$\int x\text{sech}^{-1}(a+bx)^2\,dx = \int x\,\text{arsech}\,(bx+a)^2\,dx$$

input
```
integrate(x*arcsech(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*s
qrt(-b*x - a + 1)*a + b*x + a)^2 - integrate(-(4*(b^3*x^4 + 3*a*b^2*x^3 +
(3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(
b*x + a)^2 + 4*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*l
og(b*x + a)^2 - (b^3*x^4 + 2*a*b^2*x^3 + (a^2*b - b)*x^2 + 4*(b^3*x^4 + 3*
a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a) + (2*(b^3*x^4 +
3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*log(b*x +
 a) + (2*b^3*x^4 + 4*a*b^2*x^3 + (2*a^2*b - b)*x^2 + 2*(b^3*x^4 + 3*a*b^2*
x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a))*sqrt(b*x + a + 1))*sq
rt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x
+ a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (
b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(
-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

3.10.8 Giac [F]

$$\int x\text{sech}^{-1}(a+bx)^2\,dx=\int x\,\text{arsech}\left(bx+a\right)^2\,dx$$

input
```
integrate(x*arcsech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*arcsech(b*x + a)^2, x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{-1}(a+bx)^2\,dx=\int x\,\text{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

input
```
int(x*acosh(1/(a + b*x))^2,x)
```

output
```
int(x*acosh(1/(a + b*x))^2, x)
```

3.11 $\int \operatorname{sech}^{-1}(a+bx)^2\,dx$

3.11.1 Optimal result

Integrand size = 8, antiderivative size = 80

$$\int \operatorname{sech}^{-1}(a+bx)^2\,dx = \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\arctan\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{2i\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} - \frac{2i\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}$$

output
```
(b*x+a)*arcsech(b*x+a)^2/b-4*arcsech(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))/b+2*I*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2)))/b-2*I*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)
*(1/(b*x+a)+1)^(1/2)))/b
```

3.11.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \operatorname{sech}^{-1}(a+bx)^2\,dx = \frac{i\left(\operatorname{sech}^{-1}(a+bx)\left(-i(a+bx)\operatorname{sech}^{-1}(a+bx) + 2\log\left(1-ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - 2\log\left(1+ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right) +}{b}$$

input
```
Integrate[ArcSech[a + b*x]^2,x]
```

output
```
(I*(ArcSech[a + b*x]*((-I)*(a + b*x)*ArcSech[a + b*x] + 2*Log[1 - I/E^ArcS
ech[a + b*x]] - 2*Log[1 + I/E^ArcSech[a + b*x]]) + 2*PolyLog[2, (-I)/E^Arc
Sech[a + b*x]] - 2*PolyLog[2, I/E^ArcSech[a + b*x]]))/b
```

3.11.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6869, 6833, 5941, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^{-1}(a+bx)^2\,dx$$

$$\Big\downarrow \text{ 6869}$$

$$\frac{\int \text{sech}^{-1}(a+bx)^2d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6833}$$

$$-\frac{\int (a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2d\text{sech}^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 5941}$$

$$-\frac{2\int (a+bx)\text{sech}^{-1}(a+bx)d\text{sech}^{-1}(a+bx)-(a+bx)\text{sech}^{-1}(a+bx)^2}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^2+2\int \text{sech}^{-1}(a+bx)\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 4668}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^2+2\Big(-i\int \log\left(1-ie^{\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+bx)+i\int \log\left(1+ie^{\text{sech}^{-1}(a+bx)}\right)d\text{sech}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^2+2\Big(-i\int e^{-\text{sech}^{-1}(a+bx)}\log\left(1-ie^{\text{sech}^{-1}(a+bx)}\right)de^{\text{sech}^{-1}(a+bx)}+i\int e^{-\text{sech}^{-1}(a+bx)}\log$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^2+2\left(2\text{sech}^{-1}(a+bx)\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)-i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)+i\,\text{Poly}}{b}$$

input
```
Int[ArcSech[a + b*x]^2,x]
```

output
```
-((-((a + b*x)*ArcSech[a + b*x]^2) + 2*(2*ArcSech[a + b*x]*ArcTan[E^ArcSec
h[a + b*x]] - I*PolyLog[2, (-I)*E^ArcSech[a + b*x]] + I*PolyLog[2, I*E^Arc
Sech[a + b*x]]))/b)
```

3.11.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6833
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

rule 6869
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSech[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

3.11.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{\operatorname{arcsech}(bx+a)^2(bx+a)+2i\operatorname{arcsech}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\sqrt{\frac{1}{bx+a}+1}\right)\right)-2i\operatorname{arcsech}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+\right.\right.}{}$
default	$\frac{\operatorname{arcsech}(bx+a)^2(bx+a)+2i\operatorname{arcsech}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\sqrt{\frac{1}{bx+a}+1}\right)\right)-2i\operatorname{arcsech}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+\right.\right.}{}$

input
```
int(arcsech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(arcsech(b*x+a)^2*(b*x+a)+2*I*arcsech(b*x+a)*ln(1+I*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I*arcsech(b*x+a)*ln(1-I*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*dilog(1+I*(1/(b*x+a)+(1/(b*x+a
)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I*dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))))
```

3.11.5 Fricas [F]

$$\int \operatorname{sech}^{-1}(a+bx)^2\,dx = \int \operatorname{arsech}(bx+a)^2\,dx$$

input
```
integrate(arcsech(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)^2, x)
```

3.11.6 Sympy [F]

$$\int \operatorname{sech}^{-1}(a+bx)^2\,dx = \int \operatorname{asech}^2(a+bx)\,dx$$

input
```
integrate(asech(b*x+a)**2,x)
```

output
```
Integral(asech(a + b*x)**2, x)
```

3.11.7 Maxima [F]

$$\int \operatorname{sech}^{-1}(a+bx)^2\,dx = \int \operatorname{arsech}(bx+a)^2\,dx$$

input
```
integrate(arcsech(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b
*x - a + 1)*a + b*x + a)^2 - integrate(-2*(2*(b^3*x^3 + 3*a*b^2*x^2 + a^3
+ (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2
 + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 -
(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*log(b*x + a) + ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2
*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^3 + 4*a*b^2*x^2 +
 (2*a^2*b - b)*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log
(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/
(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b
)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

3.11.8 Giac [F]

$$\int \operatorname{sech}^{-1}(a+bx)^2\,dx = \int \operatorname{arsech}\left(bx+a\right)^2\,dx$$

input
```
integrate(arcsech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)^2, x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{-1}(a+bx)^2\,dx = \int \operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

input
```
int(acosh(1/(a + b*x))^2,x)
```

output
```
int(acosh(1/(a + b*x))^2, x)
```

3.12 $\int \frac{\text{sech}^{-1}(a+bx)^2}{x} dx$

3.12.1 Optimal result

Integrand size = 12, antiderivative size = 274

$$\begin{aligned}\int \frac{\text{sech}^{-1}(a+bx)^2}{x}\,dx = {} & \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\ & + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\ & - \text{sech}^{-1}(a+bx)^2\log\left(1+e^{2\text{sech}^{-1}(a+bx)}\right) \\ & + 2\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\ & + 2\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\ & - \text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right) \\ & - 2\,\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) - 2\,\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\ & + \frac{1}{2}\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(a+bx)}\right)\end{aligned}$$

output
```
-arcsech(b*x+a)^2*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))
^2)+arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(
1/2))/(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-arcsech(b*x+a)*polylog(2,-
(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+2*arcsech(b*x+a)*po
lylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^
(1/2)))+2*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*
x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))+1/2*polylog(3,-(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))-2*polylog(3,a*(1/(b*x+a)+(1/(b*
x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

3.12.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x}\,dx = & -\frac{2}{3}\operatorname{sech}^{-1}(a+bx)^3-\operatorname{sech}^{-1}(a+bx)^2\log\left(1+e^{-2\operatorname{sech}^{-1}(a+bx)}\right)\\ & +\operatorname{sech}^{-1}(a+bx)^2\log\left(1+\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)\\ & +\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\ & +\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right)\\ & +2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)\\ & +2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\ & +\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right)\\ & -2\operatorname{PolyLog}\left(3,-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)-2\operatorname{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\end{aligned}$$

input
```
Integrate[ArcSech[a + b*x]^2/x,x]
```

output
```
(-2*ArcSech[a + b*x]^3)/3 - ArcSech[a + b*x]^2*Log[1 + E^(-2*ArcSech[a + b
*x])] + ArcSech[a + b*x]^2*Log[1 + (a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a
^2])] + ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^
2])] + ArcSech[a + b*x]*PolyLog[2, -E^(-2*ArcSech[a + b*x])] + 2*ArcSech[a
 + b*x]*PolyLog[2, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))] + 2*Arc
Sech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + Pol
yLog[3, -E^(-2*ArcSech[a + b*x])]/2 - 2*PolyLog[3, -((a*E^ArcSech[a + b*x]
)/(-1 + Sqrt[1 - a^2]))] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1
 - a^2])]
```

3.12.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.23, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6875, 25, 6129, 6104, 25, 3042, 26, 4201, 2620, 3011, 2720, 6096, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(a+bx)^2}{x} \, dx$$

$$\Big\downarrow \text{6875}$$

$$-\int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{bx}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$\int -\frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{bx}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{6129}$$

$$\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{\frac{a}{a+bx}-1}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{6104}$$

$$a\int -\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)-\int \sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2d\text{sech}^{-1}(a+bx)$$

$\downarrow$ 25

$$-\int \sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2d\text{sech}^{-1}(a+bx)-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)$$

$\downarrow$ 3042

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)-\int -i\text{sech}^{-1}(a+bx)^2\tan\left(i\text{sech}^{-1}(a+bx)\right)d\text{sech}^{-1}(a+bx)$$

$\downarrow$ 26

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\int \text{sech}^{-1}(a+bx)^2\tan\left(i\text{sech}^{-1}(a+bx)\right)d\text{sech}^{-1}(a+bx)$$

$\downarrow$ 4201

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\left(2i\int \frac{e^{2\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)^2}{1+e^{2\text{sech}^{-1}(a+bx)}}d\text{sech}^{-1}(a+bx)-\frac{1}{3}i\text{sech}^{-1}(a+bx)^3\right)$$

$\downarrow$ 2620

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^2\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\int \text{sech}^{-1}(a+bx)\log\left(1+e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+bx)\right)-\right.$$

$\downarrow$ 3011

3.12. $\int \frac{\text{sech}^{-1}(a+bx)^2}{x}\,dx$

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+$$
$$i\left(2i\left(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+bx)+\frac{1}{2}\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\text{se}\right.\right.$$

$\downarrow$ 2720

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+$$
$$i\left(2i\left(-\frac{1}{4}\int e^{-2\text{sech}^{-1}(a+bx)}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)de^{2\text{sech}^{-1}(a+bx)}+\frac{1}{2}\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(\right.\right.\right.$$

$\downarrow$ 6096

$$-a\left(\int \frac{e^{\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)^2}{-e^{\text{sech}^{-1}(a+bx)}a-\sqrt{1-a^2}+1}d\text{sech}^{-1}(a+bx)+\int \frac{e^{\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)^2}{-e^{\text{sech}^{-1}(a+bx)}a+\sqrt{1-a^2}+1}d\text{sech}^{-1}(a+bx)+\frac{\text{sec}}{}\right.$$
$$i\left(2i\left(-\frac{1}{4}\int e^{-2\text{sech}^{-1}(a+bx)}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)de^{2\text{sech}^{-1}(a+bx)}+\frac{1}{2}\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(\right.\right.\right.$$

$\downarrow$ 2620

$$-a\left(\frac{2\int \text{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\text{sech}^{-1}(a+bx)}{a}+\frac{2\int \text{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)d\text{se}}{a}\right.$$
$$i\left(2i\left(-\frac{1}{4}\int e^{-2\text{sech}^{-1}(a+bx)}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)de^{2\text{sech}^{-1}(a+bx)}+\frac{1}{2}\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(\right.\right.\right.$$

$\downarrow$ 3011

$$-a\left(\frac{2\left(\int \text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\text{sech}^{-1}(a+bx)-\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\right)}{a}+\frac{2\left(\int \text{Poly}\right.}{}\right.$$
$$i\left(2i\left(-\frac{1}{4}\int e^{-2\text{sech}^{-1}(a+bx)}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)de^{2\text{sech}^{-1}(a+bx)}+\frac{1}{2}\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(\right.\right.\right.$$

$\downarrow$ 2720

3.12. $\int \frac{\text{sech}^{-1}(a+bx)^2}{x}dx$

$$-a\left(\frac{2\left(\int e^{-\text{sech}^{-1}(a+bx)}\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)de^{\text{sech}^{-1}(a+bx)}-\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\right)}{a}\right.$$

$$i\left(2i\left(-\frac{1}{4}\int e^{-2\text{sech}^{-1}(a+bx)}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)de^{2\text{sech}^{-1}(a+bx)}+\frac{1}{2}\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(\right.\right.\right.$$

$$\downarrow 7143$$

$$-a\left(\frac{2\left(\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\right)}{a}+\frac{2\left(\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx}}{\sqrt{1-a^2}+1}\right.\right.}{}\right.$$

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\text{sech}^{-1}(a+bx)^2\log\left(e^{2}\right.\right.\right.$$

input
```
Int[ArcSech[a + b*x]^2/x,x]
```

output
```
-(a*(ArcSech[a + b*x]^3/(3*a) - (ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a
 + b*x])/(1 - Sqrt[1 - a^2])])/a - (ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSec
h[a + b*x])/(1 + Sqrt[1 - a^2])])/a + (2*(-(ArcSech[a + b*x]*PolyLog[2, (a
*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])]) + PolyLog[3, (a*E^ArcSech[a + b
*x])/(1 - Sqrt[1 - a^2])]))/a + (2*(-(ArcSech[a + b*x]*PolyLog[2, (a*E^Arc
Sech[a + b*x])/(1 + Sqrt[1 - a^2])]) + PolyLog[3, (a*E^ArcSech[a + b*x])/(
1 + Sqrt[1 - a^2])]))/a)) + I*((-1/3*I)*ArcSech[a + b*x]^3 + (2*I)*((ArcSe
ch[a + b*x]^2*Log[1 + E^(2*ArcSech[a + b*x])])/2 + (ArcSech[a + b*x]*PolyL
og[2, -E^(2*ArcSech[a + b*x])])/2 - PolyLog[3, -E^(2*ArcSech[a + b*x])]/4)
)
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

3.12. $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} \, dx$

rule 6104
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Tanh[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Sinh[c + d*x]*(Tanh[c + d*x
]^(n - 1)/(a + b*Cosh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6129
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[(e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cosh[c + d*x
])), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.12.4 Maple [F]

$$\int \frac{\operatorname{arcsech}(bx+a)^2}{x}dx$$

input
```
int(arcsech(b*x+a)^2/x,x)
```

output
```
int(arcsech(b*x+a)^2/x,x)
```

3.12.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsech}(bx+a)^2}{x}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)^2/x, x)
```

3.12.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{asech}^2(a+bx)}{x}\,dx$$

input
```
integrate(asech(b*x+a)**2/x,x)
```

output
```
Integral(asech(a + b*x)**2/x, x)
```

3.12.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsech}(bx+a)^2}{x}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(arcsech(b*x + a)^2/x, x)
```

3.12.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)^2/x, x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2}{x}\,dx$$

input
```
int(acosh(1/(a + b*x))^2/x,x)
```

output
```
int(acosh(1/(a + b*x))^2/x, x)
```

3.13 $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx$

3.13.1 Optimal result

Integrand size = 12, antiderivative size = 224

$$\begin{aligned}\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx =& -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a}-\frac{\operatorname{sech}^{-1}(a+bx)^2}{x}\\ &+\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\\ &-\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\\ &+\frac{2b\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2b\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\end{aligned}$$

output
```
-b*arcsech(b*x+a)^2/a-arcsech(b*x+a)^2/x+2*b*arcsech(b*x+a)*ln(1-a*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1
)^(1/2)-2*b*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a
)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*b*polylog(2,a*(1/(b*x+a
)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^
(1/2)-2*b*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/
(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

3.13.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 678, normalized size of antiderivative = 3.03

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx$$

$$= \frac{-\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b\left(2\operatorname{sech}^{-1}(a+bx)\arctan\left(\frac{(-1+a)\coth\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{-1+a^2}}\right) - 2i\arccos\left(\frac{1}{a}\right)\arctan\left(\frac{(1+a)\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+\right.}{\sqrt{-1+a^2}}\right.}{}}{}$$

input
```
Integrate[ArcSech[a + b*x]^2/x^2,x]
```

output
```
(-(((a + b*x)*ArcSech[a + b*x]^2)/x) + (2*b*(2*ArcSech[a + b*x]*ArcTan[((-
1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-1)]*Ar
cTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)]
+ 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[(
(1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/(Sq
rt[2]*Sqrt[a]*E^(ArcSech[a + b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[
a^(-1)] - 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] +
ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 +
 a^2]*E^(ArcSech[a + b*x]/2))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])]
- (ArcCos[a^(-1)] + 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 +
a^2]])*Log[-(((-1 + a)*(1 + a - I*Sqrt[-1 + a^2])*(-1 + Tanh[ArcSech[a + b
*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2])))] - (Arc
Cos[a^(-1)] - 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])
*Log[((-1 + a)*(1 + a + I*Sqrt[-1 + a^2])*(1 + Tanh[ArcSech[a + b*x]/2]))/
(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] + I*(PolyLog[2,
((-1 - I*Sqrt[-1 + a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/
2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] - PolyLog[2
, ((I + Sqrt[-1 + a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2
]))/(a*((-I)*(-1 + a) + Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))])))/Sqrt
[-1 + a^2])/a
```

3.13.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6875, 5991, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(a+bx)^2}{x^2} \, dx$$

$$\Big\downarrow \ 6875$$

$$-b\int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{b^2x^2}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \ 5991$$

$$-b\left(2\int -\frac{\text{sech}^{-1}(a+bx)}{bx}d\text{sech}^{-1}(a+bx)+\frac{\text{sech}^{-1}(a+bx)^2}{bx}\right)$$

$$\Big\downarrow \ 3042$$

$$-b\left(\frac{\text{sech}^{-1}(a+bx)^2}{bx}+2\int \frac{\text{sech}^{-1}(a+bx)}{a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \ 4679$$

$$-b\left(2\int \left(\frac{\text{sech}^{-1}(a+bx)}{a}+\frac{\text{sech}^{-1}(a+bx)}{a\left(\frac{a}{a+bx}-1\right)}\right)d\text{sech}^{-1}(a+bx)+\frac{\text{sech}^{-1}(a+bx)^2}{bx}\right)$$

$$\Big\downarrow \ 2009$$

$$-b\left(2\left(-\frac{\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{\text{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\right.\right.$$

input
```
Int[ArcSech[a + b*x]^2/x^2,x]
```

output
```
-(b*(ArcSech[a + b*x]^2/(b*x) + 2*(ArcSech[a + b*x]^2/(2*a) - (ArcSech[a +
 b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2
]) + (ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]
)/(a*Sqrt[1 - a^2]) - PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2]
)]/(a*Sqrt[1 - a^2]) + PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2
])]/(a*Sqrt[1 - a^2]))))
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.13.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.62

method	result
derivativedivides	$b\left(-\frac{(bx+a)\,\mathrm{arcsech}(bx+a)^2}{abx}+\frac{2\sqrt{-a^2+1}\,\mathrm{arcsech}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{a(a^2-1)}-\cdots\right.$
default	$b\left(-\frac{(bx+a)\,\mathrm{arcsech}(bx+a)^2}{abx}+\frac{2\sqrt{-a^2+1}\,\mathrm{arcsech}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{a(a^2-1)}-\cdots\right.$

input
```
int(arcsech(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-(b*x+a)*arcsech(b*x+a)^2/a/b/x+2*(-a^2+1)^(1/2)/a/(a^2-1)*arcsech(b*x+
a)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/
2)+1)/(1+(-a^2+1)^(1/2)))-2*(-a^2+1)^(1/2)/a/(a^2-1)*arcsech(b*x+a)*ln((a*
(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+
(-a^2+1)^(1/2)))+2*(-a^2+1)^(1/2)/a/(a^2-1)*dilog((-a*(1/(b*x+a)+(1/(b*x+a
)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-2*(-
a^2+1)^(1/2)/a/(a^2-1)*dilog((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+
1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2))))
```

3.13.5 Fricas [F]

$$\int \frac{\mathrm{sech}^{-1}(a+bx)^2}{x^2}\,dx=\int \frac{\mathrm{arsech}\,(bx+a)^2}{x^2}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)^2/x^2, x)
```

3.13.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{asech}^2(a+bx)}{x^2}\,dx$$

input
```
integrate(asech(b*x+a)**2/x**2,x)
```

output
```
Integral(asech(a + b*x)**2/x**2, x)
```

3.13.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arsech}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1)*a + b*x + a)^2/x - integrate(-2*(2*(b^3*x^3 + 3*a*b^2*x^2 + a^3
 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^
2 + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 +
 (b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
 (3*a^2*b - b)*x - a)*log(b*x + a) - ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^
2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2
+ (2*a^2*b - b)*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*lo
g(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*s
qrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))
/(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2 + (b^3*x^5 + 3
*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1)), x)
```

3.13.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)^2/x^2, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2}{x^2}\,dx$$

input
```
int(acosh(1/(a + b*x))^2/x^2,x)
```

output
```
int(acosh(1/(a + b*x))^2/x^2, x)
```

3.14 $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx$

3.14.1 Optimal result

Integrand size = 12, antiderivative size = 537

$$\begin{aligned}\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx &= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)}+\frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2}\\
&\quad-\frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2}+\frac{b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}\\
&\quad-\frac{2b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}\\
&\quad-\frac{b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}\\
&\quad+\frac{2b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}+\frac{b^2\log\left(\frac{x}{a+bx}\right)}{a^2\left(1-a^2\right)}\\
&\quad+\frac{b^2\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}-\frac{2b^2\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}\\
&\quad-\frac{b^2\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{2b^2\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}\end{aligned}$$

output
```
1/2*b^2*arcsech(b*x+a)^2/a^2-1/2*arcsech(b*x+a)^2/x^2+b^2*ln(x/(b*x+a))/a^
2/(-a^2+1)+b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*
x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-b^2*arcsech(b*x+a)*l
n(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2
)))/a^2/(-a^2+1)^(3/2)+b^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(
b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-b^2*polylog(2,a*(1
/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/
(-a^2+1)^(3/2)-2*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+2*b^2*arcsech(
b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2
+1)^(1/2)))/a^2/(-a^2+1)^(1/2)-2*b^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^
(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+2*b^2*po
lylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^
(1/2)))/a^2/(-a^2+1)^(1/2)+b^2*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a
+1))^(1/2)/a/(-a^2+1)/(b*x+a)/(1-a/(b*x+a))
```

3.14.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.00 (sec) , antiderivative size = 1439, normalized size of antiderivative = 2.68

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3} dx = \text{Too large to display}$$

input
```
Integrate[ArcSech[a + b*x]^2/x^3,x]
```

output
```
-1/2*((a + b*x)^2*ArcSech[a + b*x]^2)/(a^2*x^2) + (b*ArcSech[a + b*x]*(-(a
*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)) + (-1 + a^2)*(a + b*
x)*ArcSech[a + b*x]))/((-1 + a)*a^2*(1 + a)*x) + (b^2*Log[(b*x)/(a + b*x)]
)/(a^2 - a^4) - (2*b^2*(2*ArcSech[a + b*x]*ArcTan[((-1 + a)*Coth[ArcSech[a
 + b*x]/2])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-1)]*ArcTan[((1 + a)*Tanh[Ar
cSech[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] + 2*(ArcTan[((-1 + a)
*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[((1 + a)*Tanh[ArcSech[
a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sqrt[a]*E^(ArcS
ech[a + b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)] - 2*(ArcTan[(
(-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[((1 + a)*Tanh[
ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a^2]*E^(ArcSech[a +
b*x]/2))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (ArcCos[a^(-1)] + 2
*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[-(((-1 + a
)*(1 + a - I*Sqrt[-1 + a^2])*(-1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a +
 I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2])))] - (ArcCos[a^(-1)] - 2*ArcTa
n[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[((-1 + a)*(1 + a
 + I*Sqrt[-1 + a^2])*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-
1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] + I*(PolyLog[2, ((-1 - I*Sqrt[-1 + a^
2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*S
qrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] - PolyLog[2, ((I + Sqrt[-1 + ...
```

3.14.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 517, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6875, 25, 5991, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(a+bx)^2}{x^3} \, dx$$

$$\Big\downarrow \text{6875}$$

$$-b^2 \int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{b^3x^3}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$b^2 \int -\frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^2}{b^3x^3}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{5991}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)^2}{2b^2x^2}-\int \frac{\text{sech}^{-1}(a+bx)}{b^2x^2}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)^2}{2b^2x^2}-\int \frac{\text{sech}^{-1}(a+bx)}{\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{4679}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)^2}{2b^2x^2}-\int \left(\frac{2\text{sech}^{-1}(a+bx)}{a^2\left(\frac{a}{a+bx}-1\right)}+\frac{\text{sech}^{-1}(a+bx)}{a^2}+\frac{\text{sech}^{-1}(a+bx)}{a^2\left(\frac{a}{a+bx}-1\right)^2}\right)d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{2009}$$

$$-b^2\left(\frac{2\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}-\frac{\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}-\frac{2\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2\sqrt{1-a^2}}+\frac{\text{PolyLog}\left(2,\right.}{a^2\left(1\right.}\right.$$

input
```
Int[ArcSech[a + b*x]^2/x^3,x]
```

output
```
-(b^2*(-((Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x]
)/(a*(1 - a^2)*(a + b*x)*(1 - a/(a + b*x)))) - ArcSech[a + b*x]^2/(2*a^2)
+ ArcSech[a + b*x]^2/(2*b^2*x^2) - (ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[
a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (2*ArcSech[a + b*x
]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2])
 + (ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/
(a^2*(1 - a^2)^(3/2)) - (2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])
/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - Log[1 - a/(a + b*x)]/(a^2*(1
- a^2)) - PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])]/(a^2*(1 -
 a^2)^(3/2)) + (2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/
(a^2*Sqrt[1 - a^2]) + PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2]
)]/(a^2*(1 - a^2)^(3/2)) - (2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[
1 - a^2])])/(a^2*Sqrt[1 - a^2])))
```

3.14. $\int \frac{\text{sech}^{-1}(a+bx)^2}{x^3}\,dx$

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.14.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 982, normalized size of antiderivative = 1.83

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arcsech}(bx+a)\left(2\operatorname{arcsech}(bx+a)a^3(bx+a)-\operatorname{arcsech}(bx+a)a^2(bx+a)^2-2\sqrt{-\frac{bx+a-1}{bx+a}}\sqrt{\frac{bx+a+1}{bx+a}}\,a^2(bx+a)+2\sqrt{}\right.}{2a^2(a^2}\right.$
default	$b^2\left(-\frac{\operatorname{arcsech}(bx+a)\left(2\operatorname{arcsech}(bx+a)a^3(bx+a)-\operatorname{arcsech}(bx+a)a^2(bx+a)^2-2\sqrt{-\frac{bx+a-1}{bx+a}}\sqrt{\frac{bx+a+1}{bx+a}}\,a^2(bx+a)+2\sqrt{}\right.}{2a^2(a^2}\right.$

input
```
int(arcsech(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*(-1/2*arcsech(b*x+a)*(2*arcsech(b*x+a)*a^3*(b*x+a)-arcsech(b*x+a)*a^2*
(b*x+a)^2-2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*a^2*(b*x+
a)+2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*a*(b*x+a)^2-2*ar
csech(b*x+a)*a*(b*x+a)+arcsech(b*x+a)*(b*x+a)^2+2*a^2-4*(b*x+a)*a+2*(b*x+a
)^2)/a^2/(a^2-1)/b^2/x^2+2/a^2/(a^2-1)*ln(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1
/(b*x+a)+1)^(1/2))-1/a^2/(a^2-1)*ln(a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b
*x+a)+1)^(1/2))^2+a-2/(b*x+a)-2*(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(
-a^2+1)^(1/2)/a^2/(a^2-1)^2*arcsech(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-(-a^2+1)
^(1/2)/a^2/(a^2-1)^2*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))+(-a^2+1)^(1/2)/
a^2/(a^2-1)^2*dilog((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)
)+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-(-a^2+1)^(1/2)/a^2/(a^2-1)^2*dilog
((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/
(-1+(-a^2+1)^(1/2)))-2*(-a^2+1)^(1/2)/(a^2-1)^2*arcsech(b*x+a)*ln((-a*(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2
+1)^(1/2)))+2*(-a^2+1)^(1/2)/(a^2-1)^2*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/
(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)
))-2*(-a^2+1)^(1/2)/(a^2-1)^2*dilog((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/
(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+2*(-a^2+1)^(1/2...
```

3.14.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^2}{x^3}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)^2/x^3, x)
```

3.14.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{asech}^2\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(asech(b*x+a)**2/x**3,x)
```

output
```
Integral(asech(a + b*x)**2/x**3, x)
```

3.14.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^2}{x^3}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt
(-b*x - a + 1)*a + b*x + a)^2/x^2 - integrate(-(4*(b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x +
 a)^2 + 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)
^2 + (b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 4*(b^3*x^3 + 3*a*b^2*x^2 + a
^3 + (3*a^2*b - b)*x - a)*log(b*x + a) - (2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
 (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^
2*x^2 + (2*a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x
 - a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x +
a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b
*x + a))/(b^3*x^6 + 3*a*b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3 + (b^3
*x^6 + 3*a*b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3)*sqrt(b*x + a + 1)*
sqrt(-b*x - a + 1)), x)
```

3.14.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx=\int \frac{\operatorname{arsech}\left(bx+a\right)^2}{x^3}\,dx$$

input
```
integrate(arcsech(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)^2/x^3, x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx=\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2}{x^3}\,dx$$

input
```
int(acosh(1/(a + b*x))^2/x^3,x)
```

output
```
int(acosh(1/(a + b*x))^2/x^3, x)
```

3.15 $\int x\text{sech}^{-1}(a+bx)^3\,dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 260

$$\begin{aligned}\int x\text{sech}^{-1}(a+bx)^3\,dx =& -\frac{3\text{sech}^{-1}(a+bx)^2}{2b^2}-\frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)^2}{2b^2}\\
&-\frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)^3\\
&+\frac{6a\text{sech}^{-1}(a+bx)^2\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)}{b^2}\\
&+\frac{3\text{sech}^{-1}(a+bx)\log\left(1+e^{2\text{sech}^{-1}(a+bx)}\right)}{b^2}\\
&-\frac{6ia\text{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}\\
&+\frac{6ia\text{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}\\
&+\frac{3\operatorname{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)}{2b^2}\\
&+\frac{6ia\operatorname{PolyLog}\left(3,-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}\\
&-\frac{6ia\operatorname{PolyLog}\left(3,ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}\end{aligned}$$

output
```
-3/2*arcsech(b*x+a)^2/b^2-1/2*a^2*arcsech(b*x+a)^3/b^2+1/2*x^2*arcsech(b*x
+a)^3+6*a*arcsech(b*x+a)^2*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)
+1)^(1/2))/b^2+3*arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*
x+a)+1)^(1/2))^2)/b^2-6*I*a*arcsech(b*x+a)*polylog(2,-I*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2+6*I*a*arcsech(b*x+a)*polylog(2,I*(1
/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2+3/2*polylog(2,-(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)/b^2+6*I*a*polylog(3,-I*
(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2-6*I*a*polylog(3,I
*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2-3/2*(b*x+a+1)*ar
csech(b*x+a)^2*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

3.15.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.98

$$\int x\text{sech}^{-1}(a+bx)^3\,dx$$
$$= \frac{-3\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)^2-2a(a+bx)\text{sech}^{-1}(a+bx)^3+(a+bx)^2\text{sech}^{-1}(a+bx)^3+3}{}$$

input
```
Integrate[x*ArcSech[a + b*x]^3,x]
```

output
```
(-3*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcSech[a + b*x]^2
 - 2*a*(a + b*x)*ArcSech[a + b*x]^3 + (a + b*x)^2*ArcSech[a + b*x]^3 + 3*A
rcSech[a + b*x]*(ArcSech[a + b*x] + 2*Log[1 + E^(-2*ArcSech[a + b*x])]) -
3*PolyLog[2, -E^(-2*ArcSech[a + b*x])] + (6*I)*a*(-(ArcSech[a + b*x]^2*(Lo
g[1 - I/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a + b*x]])) - 2*ArcSech[
a + b*x]*(PolyLog[2, (-I)/E^ArcSech[a + b*x]] - PolyLog[2, I/E^ArcSech[a +
 b*x]]) - 2*PolyLog[3, (-I)/E^ArcSech[a + b*x]] + 2*PolyLog[3, I/E^ArcSech
[a + b*x]]))/(2*b^2)
```

3.15.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6875, 25, 5991, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{sech}^{-1}(a+bx)^3\,dx$$

$$\Big\downarrow \text{ 6875}$$

$$-\frac{\int bx(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3d\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -bx(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3d\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 5991}$$

$$-\frac{\frac{3}{2}\int b^2x^2\text{sech}^{-1}(a+bx)^2d\text{sech}^{-1}(a+bx)-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)^3}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)^3+\frac{3}{2}\int \text{sech}^{-1}(a+bx)^2\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2d\text{sech}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 4678}$$

$$-\frac{\frac{3}{2}\int\left(a^2\text{sech}^{-1}(a+bx)^2+(a+bx)^2\text{sech}^{-1}(a+bx)^2-2a(a+bx)\text{sech}^{-1}(a+bx)^2\right)d\text{sech}^{-1}(a+bx)-\frac{1}{2}b^2x^2\text{sech}}{b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{sech}^{-1}(a+bx)^3+\frac{3}{2}\left(\frac{1}{3}a^2\text{sech}^{-1}(a+bx)^3-4a\text{sech}^{-1}(a+bx)^2\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)+4ia\text{sech}^{-1}(a+b\right.}{}$$

input
```
Int[x*ArcSech[a + b*x]^3,x]
```

output
```
-((-1/2*(b^2*x^2*ArcSech[a + b*x]^3) + (3*(ArcSech[a + b*x]^2 + Sqrt[(1 -
a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x]^2 + (a^2*ArcSech[a
+ b*x]^3)/3 - 4*a*ArcSech[a + b*x]^2*ArcTan[E^ArcSech[a + b*x]] - 2*ArcSec
h[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])] + (4*I)*a*ArcSech[a + b*x]*Poly
Log[2, (-I)*E^ArcSech[a + b*x]] - (4*I)*a*ArcSech[a + b*x]*PolyLog[2, I*E^
ArcSech[a + b*x]] - PolyLog[2, -E^(2*ArcSech[a + b*x])] - (4*I)*a*PolyLog[
3, (-I)*E^ArcSech[a + b*x]] + (4*I)*a*PolyLog[3, I*E^ArcSech[a + b*x]]))/2
)/b^2)
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.15.4 Maple [F]

$$\int x \operatorname{arcsech}(bx+a)^3\, dx$$

input
```
int(x*arcsech(b*x+a)^3,x)
```

output
```
int(x*arcsech(b*x+a)^3,x)
```

3.15.5 Fricas [F]

$$\int x \operatorname{sech}^{-1}(a+bx)^3\, dx = \int x \operatorname{arsech}(bx+a)^3\, dx$$

input
```
integrate(x*arcsech(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x*arcsech(b*x + a)^3, x)
```

3.15.6 Sympy [F]

$$\int x \operatorname{sech}^{-1}(a+bx)^3\, dx = \int x \operatorname{asech}^3(a+bx)\, dx$$

input
```
integrate(x*asech(b*x+a)**3,x)
```

output
```
Integral(x*asech(a + b*x)**3, x)
```

3.15.7 Maxima [F]

$$\int x\text{sech}^{-1}(a+bx)^3\,dx=\int x\,\text{arsech}\,(bx+a)^3\,dx$$

input
```
integrate(x*arcsech(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*s
qrt(-b*x - a + 1)*a + b*x + a)^3 - integrate(1/2*(16*(b^3*x^4 + 3*a*b^2*x^
3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*
log(b*x + a)^3 + 16*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)
*x)*log(b*x + a)^3 + 3*(b^3*x^4 + 2*a*b^2*x^3 + (a^2*b - b)*x^2 + 4*(b^3*x
^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a) + (2*(b^3
*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*lo
g(b*x + a) + (2*b^3*x^4 + 4*a*b^2*x^3 + (2*a^2*b - b)*x^2 + 2*(b^3*x^4 + 3
*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a))*sqrt(b*x + a +
 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sq
rt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 24*((b^3*x^4 + 3*a*b^2
*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a +
1)*log(b*x + a)^2 + (b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)
*x)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*
x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
 (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

3.15.8 Giac [F]

$$\int x\text{sech}^{-1}(a+bx)^3\,dx=\int x\,\text{arsech}\,(bx+a)^3\,dx$$

input
```
integrate(x*arcsech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*arcsech(b*x + a)^3, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{-1}(a+bx)^3\,dx=\int x\,\text{acosh}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

input
```
int(x*acosh(1/(a + b*x))^3,x)
```

output
```
int(x*acosh(1/(a + b*x))^3, x)
```

3.16 $\int \operatorname{sech}^{-1}(a+bx)^3\,dx$

3.16.1 Optimal result

Integrand size = 8, antiderivative size = 136

$$\int \operatorname{sech}^{-1}(a+bx)^3\,dx = \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\arctan\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} - \frac{6i\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} - \frac{6i\operatorname{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{PolyLog}\left(3,ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}$$

output
```
(b*x+a)*arcsech(b*x+a)^3/b-6*arcsech(b*x+a)^2*arctan(1/(b*x+a)+(1/(b*x+a)-
1)^(1/2)*(1/(b*x+a)+1)^(1/2))/b+6*I*arcsech(b*x+a)*polylog(2,-I*(1/(b*x+a)
+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b-6*I*arcsech(b*x+a)*polylog(2,
I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b-6*I*polylog(3,-I*
(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b+6*I*polylog(3,I*(1/
(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b
```

3.16.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \operatorname{sech}^{-1}(a+bx)^3\,dx = \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{3i\Big(-\operatorname{sech}^{-1}(a+bx)^2\left(\log\left(1-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-\log\left(1+ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)-2\operatorname{sech}^{-1}(a+bx)\left(\text{PolyL}\right.}{}$$

input
```
Integrate[ArcSech[a + b*x]^3,x]
```

output
```
((a + b*x)*ArcSech[a + b*x]^3)/b - ((3*I)*(-(ArcSech[a + b*x]^2*(Log[1 - I
/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a + b*x]])) - 2*ArcSech[a + b*x
]*(PolyLog[2, (-I)/E^ArcSech[a + b*x]] - PolyLog[2, I/E^ArcSech[a + b*x]])
 - 2*(PolyLog[3, (-I)/E^ArcSech[a + b*x]] - PolyLog[3, I/E^ArcSech[a + b*x
]])))/b
```

3.16.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6869, 6833, 5941, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^{-1}(a+bx)^3\,dx$$

$$\Big\downarrow \text{ 6869}$$

$$\frac{\int \operatorname{sech}^{-1}(a+bx)^3 d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6833}$$

$$-\frac{\int (a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)^3 d\operatorname{sech}^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 5941}$$

$$-\frac{3\int (a+bx)\operatorname{sech}^{-1}(a+bx)^2 d\operatorname{sech}^{-1}(a+bx)-(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^3+3\int \text{sech}^{-1}(a+bx)^2\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)d\text{sech}^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^3+3\Big(-2i\int \text{sech}^{-1}(a+bx)\log\left(1-ie^{\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+bx)+2i\int \text{sech}^{-1}(a+$$ $$b$$

$$\Big\downarrow \text{3011}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^3+3\Big(2i\Big(\int \text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+bx)-\text{sech}^{-1}(a+bx)\,\text{PolyLog}\Big(2$$

$$\Big\downarrow \text{2720}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^3+3\Big(2i\Big(\int e^{-\text{sech}^{-1}(a+bx)}\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)de^{\text{sech}^{-1}(a+bx)}-\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{7143}$$

$$-\frac{-(a+bx)\text{sech}^{-1}(a+bx)^3+3\Big(2\text{sech}^{-1}(a+bx)^2\arctan\left(e^{\text{sech}^{-1}(a+bx)}\right)+2i\Big(\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(a+bx)}\right)-\text{sec}$$

input
```
Int[ArcSech[a + b*x]^3,x]
```

output
```
-((-((a + b*x)*ArcSech[a + b*x]^3) + 3*(2*ArcSech[a + b*x]^2*ArcTan[E^ArcS
ech[a + b*x]] + (2*I)*(-(ArcSech[a + b*x]*PolyLog[2, (-I)*E^ArcSech[a + b*
x]]) + PolyLog[3, (-I)*E^ArcSech[a + b*x]]) - (2*I)*(-(ArcSech[a + b*x]*Po
lyLog[2, I*E^ArcSech[a + b*x]]) + PolyLog[3, I*E^ArcSech[a + b*x]])))/b)
```

3.16.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5941
```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

rule 6833
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

rule 6869
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSech[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.16. $\int \operatorname{sech}^{-1}(a+bx)^3 \, dx$

3.16.4 Maple [F]

$$\int \text{arcsech}\,(bx+a)^3\,dx$$

input
```
int(arcsech(b*x+a)^3,x)
```

output
```
int(arcsech(b*x+a)^3,x)
```

3.16.5 Fricas [F]

$$\int \text{sech}^{-1}(a+bx)^3\,dx = \int \text{arsech}\,(bx+a)^3\,dx$$

input
```
integrate(arcsech(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)^3, x)
```

3.16.6 Sympy [F]

$$\int \text{sech}^{-1}(a+bx)^3\,dx = \int \text{asech}^3\,(a+bx)\,dx$$

input
```
integrate(asech(b*x+a)**3,x)
```

output
```
Integral(asech(a + b*x)**3, x)
```

3.16.7 Maxima [F]

$$\int \operatorname{sech}^{-1}(a+bx)^3\,dx = \int \operatorname{arsech}(bx+a)^3\,dx$$

input
```
integrate(arcsech(b*x+a)^3,x, algorithm="maxima")
```

output
```
x*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b
*x - a + 1)*a + b*x + a)^3 - integrate((8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (
3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^3 +
8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^3 + 3*(
b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (
3*a^2*b - b)*x - a)*log(b*x + a) + ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*
b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^3 + 4*a*b^2*x^2 +
(2*a^2*b - b)*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(
b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2
- 12*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1
)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^
2*b - b)*x - a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b
*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2
*x^2 + a^3 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x
+ a + 1)*sqrt(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

3.16.8 Giac [F]

$$\int \operatorname{sech}^{-1}(a+bx)^3\,dx = \int \operatorname{arsech}(bx+a)^3\,dx$$

input
```
integrate(arcsech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)^3, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{-1}(a+bx)^3\,dx = \int \operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

input
```
int(acosh(1/(a + b*x))^3,x)
```

output
```
int(acosh(1/(a + b*x))^3, x)
```

3.17 $\int \frac{\text{sech}^{-1}(a+bx)^3}{x} dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 378

$$\begin{aligned}\int \frac{\text{sech}^{-1}(a+bx)^3}{x}\,dx &= \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&+\text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&-\text{sech}^{-1}(a+bx)^3\log\left(1+e^{2\text{sech}^{-1}(a+bx)}\right)\\
&+3\text{sech}^{-1}(a+bx)^2\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&+3\text{sech}^{-1}(a+bx)^2\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&-\frac{3}{2}\text{sech}^{-1}(a+bx)^2\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)\\
&-6\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&-6\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&+\frac{3}{2}\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(a+bx)}\right)\\
&+6\,\text{PolyLog}\left(4,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+6\,\text{PolyLog}\left(4,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&-\frac{3}{4}\,\text{PolyLog}\left(4,-e^{2\text{sech}^{-1}(a+bx)}\right)\end{aligned}$$

output
```
-arcsech(b*x+a)^3*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))
^2)+arcsech(b*x+a)^3*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(
1/2))/(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)^3*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-3/2*arcsech(b*x+a)^2*polyl
og(2,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+3*arcsech(b*x
+a)^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(
-a^2+1)^(1/2)))+3*arcsech(b*x+a)^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))+3/2*arcsech(b*x+a)*polylog(3,
-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-6*arcsech(b*x+a)*p
olylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)
^(1/2)))-6*arcsech(b*x+a)*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b
*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-3/4*polylog(4,-(1/(b*x+a)+(1/(b*x+a)-1
)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+6*polylog(4,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/
2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))+6*polylog(4,a*(1/(b*x+a)+(1/(b
*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

3.17. $\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\,dx$

3.17.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\,dx = -\frac{1}{2}\operatorname{sech}^{-1}(a+bx)^4 - \operatorname{sech}^{-1}(a+bx)^3\log\left(1+e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1+\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \frac{3}{2}\operatorname{sech}^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + 3\operatorname{sech}^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + 3\operatorname{sech}^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \frac{3}{2}\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(3,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) - 6\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(3,-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) - 6\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \frac{3}{4}\operatorname{PolyLog}\left(4,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + 6\operatorname{PolyLog}\left(4,-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + 6\operatorname{PolyLog}\left(4,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

input
```
Integrate[ArcSech[a + b*x]^3/x,x]
```

output
```
-1/2*ArcSech[a + b*x]^4 - ArcSech[a + b*x]^3*Log[1 + E^(-2*ArcSech[a + b*x
])] + ArcSech[a + b*x]^3*Log[1 + (a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2
])] + ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2]
)] + (3*ArcSech[a + b*x]^2*PolyLog[2, -E^(-2*ArcSech[a + b*x])])/2 + 3*Arc
Sech[a + b*x]^2*PolyLog[2, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))]
 + 3*ArcSech[a + b*x]^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^
2])] + (3*ArcSech[a + b*x]*PolyLog[3, -E^(-2*ArcSech[a + b*x])])/2 - 6*Arc
Sech[a + b*x]*PolyLog[3, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))] -
 6*ArcSech[a + b*x]*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]
 + (3*PolyLog[4, -E^(-2*ArcSech[a + b*x])])/4 + 6*PolyLog[4, -((a*E^ArcSec
h[a + b*x])/(-1 + Sqrt[1 - a^2]))] + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(
1 + Sqrt[1 - a^2])]
```

3.17.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6875, 25, 6129, 6104, 25, 3042, 26, 4201, 2620, 3011, 6096, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(a+bx)^3}{x} \, dx$$

$$\Big\downarrow \text{6875}$$

$$-\int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{bx}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$\int -\frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{bx}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{6129}$$

$$\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{\frac{a}{a+bx}-1}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{6104}$$

$$a\int -\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)-\int \sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 25$$

$$-\int \sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3d\text{sech}^{-1}(a+bx)-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 3042$$

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)-\int -i\text{sech}^{-1}(a+bx)^3\tan\left(i\text{sech}^{-1}(a+bx)\right)d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 26$$

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\int \text{sech}^{-1}(a+bx)^3\tan\left(i\text{sech}^{-1}(a+bx)\right)d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \; 4201$$

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\left(2i\int \frac{e^{2\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)^3}{1+e^{2\text{sech}^{-1}(a+bx)}}d\text{sech}^{-1}(a+bx)-\frac{1}{4}i\text{sech}^{-1}(a+bx)^4\right)$$

$$\Big\downarrow \; 2620$$

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^3\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{3}{2}\int \text{sech}^{-1}(a+bx)^2\log\left(1+e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+bx)\right)\right.$$

$$\Big\downarrow \; 3011$$

3.17. $\int \frac{\text{sech}^{-1}(a+bx)^3}{x}\,dx$

$$-a\int \frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{(a+bx)\left(1-\frac{a}{a+bx}\right)}d\text{sech}^{-1}(a+bx)+$$
$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^3\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{3}{2}\left(\int \text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+\right.\right.\right.$$

$$\Big\downarrow \text{6096}$$

$$-a\left(\int \frac{e^{\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)^3}{-e^{\text{sech}^{-1}(a+bx)}a-\sqrt{1-a^2}+1}d\text{sech}^{-1}(a+bx)+\int \frac{e^{\text{sech}^{-1}(a+bx)}\text{sech}^{-1}(a+bx)^3}{-e^{\text{sech}^{-1}(a+bx)}a+\sqrt{1-a^2}+1}d\text{sech}^{-1}(a+bx)+\frac{\text{sec}}{}\right.$$
$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^3\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{3}{2}\left(\int \text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+\right.\right.\right.$$

$$\Big\downarrow \text{2620}$$

$$-a\left(\frac{3\int \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\text{sech}^{-1}(a+bx)}{a}+\frac{3\int \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)d}{a}\right.$$
$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^3\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{3}{2}\left(\int \text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$-a\left(\frac{3\left(2\int \text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\text{sech}^{-1}(a+bx)-\text{sech}^{-1}(a+bx)^2\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+}}{1-\sqrt{1-a^2}}\right.\right.}{a}\right.$$
$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^3\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{3}{2}\left(\int \text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+\right.\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$-a\left(\frac{3\left(2\left(\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-\int \text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\text{sech}^{-1}(a+bx)\right)-\text{sech}^{-1}\right.}{a}\right.$$
$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^3\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{3}{2}\left(-\frac{1}{2}\int \text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(a+bx)}\right)d\text{sech}^{-1}(a+bx)-\frac{1}{2}\text{sec}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

3.17. $\int \frac{\text{sech}^{-1}(a+bx)^3}{x}\,dx$

$$-a\left(\frac{3\left(2\left(\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-\int e^{-\text{sech}^{-1}(a+bx)}\,\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)de^{\text{sech}^{-1}(a+bx)}\right.\right.}{a}\right.$$

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^3\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{3}{2}\left(-\frac{1}{4}\int e^{-2\text{sech}^{-1}(a+bx)}\,\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(a+bx)}\right)de^{2\text{sech}^{-1}}\right.\right.\right.$$

$$\Big\downarrow \textcolor{blue}{7143}$$

$$-a\left(\frac{3\left(2\left(\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-\text{PolyLog}\left(4,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\right)-\text{sech}^{-1}(a+bx)^2\,\text{PolyLog}\right.}{a}\right.$$

$$i\left(2i\left(\frac{1}{2}\text{sech}^{-1}(a+bx)^3\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)-\frac{3}{2}\left(-\frac{1}{2}\text{sech}^{-1}(a+bx)^2\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\text{sech}^{-1}\right.\right.\right.$$

input
```
Int[ArcSech[a + b*x]^3/x,x]
```

output
```
-(a*(ArcSech[a + b*x]^4/(4*a) - (ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a
 + b*x])/(1 - Sqrt[1 - a^2])])/a - (ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSec
h[a + b*x])/(1 + Sqrt[1 - a^2])])/a + (3*(-(ArcSech[a + b*x]^2*PolyLog[2,
(a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])]) + 2*(ArcSech[a + b*x]*PolyLog
[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] - PolyLog[4, (a*E^ArcSech[
a + b*x])/(1 - Sqrt[1 - a^2])])))/a + (3*(-(ArcSech[a + b*x]^2*PolyLog[2,
(a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]) + 2*(ArcSech[a + b*x]*PolyLog
[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - PolyLog[4, (a*E^ArcSech[
a + b*x])/(1 + Sqrt[1 - a^2])])))/a)) + I*((-1/4*I)*ArcSech[a + b*x]^4 + (
2*I)*((ArcSech[a + b*x]^3*Log[1 + E^(2*ArcSech[a + b*x])])/2 - (3*(-1/2*(A
rcSech[a + b*x]^2*PolyLog[2, -E^(2*ArcSech[a + b*x])]) + (ArcSech[a + b*x]
*PolyLog[3, -E^(2*ArcSech[a + b*x])])/2 - PolyLog[4, -E^(2*ArcSech[a + b*x
])]/4))/2))
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

3.17. $\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} \, dx$

rule 6104
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Tanh[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Sinh[c + d*x]*(Tanh[c + d*x
]^(n - 1)/(a + b*Cosh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6129
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[(e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cosh[c + d*x
])), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.17.4 Maple [F]

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x} dx$$

input
```
int(arcsech(b*x+a)^3/x,x)
```

output
```
int(arcsech(b*x+a)^3/x,x)
```

3.17.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(arcsech(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)^3/x, x)
```

3.17.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{asech}^3\left(a+bx\right)}{x}\,dx$$

input
```
integrate(asech(b*x+a)**3/x,x)
```

output
```
Integral(asech(a + b*x)**3/x, x)
```

3.17.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(arcsech(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
integrate(arcsech(b*x + a)^3/x, x)
```

3.17.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} \, dx = \int \frac{\operatorname{arsech}(bx+a)^3}{x} \, dx$$

input
```
integrate(arcsech(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)^3/x, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} \, dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x} \, dx$$

input
```
int(acosh(1/(a + b*x))^3/x,x)
```

output
```
int(acosh(1/(a + b*x))^3/x, x)
```

3.18 $\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx$

3.18.1 Optimal result

Integrand size = 12, antiderivative size = 330

$$\begin{aligned}\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx &= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a}-\frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\\
&\quad+\frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\\
&\quad-\frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\\
&\quad+\frac{6b\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\\
&\quad-\frac{6b\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\\
&\quad-\frac{6b\operatorname{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6b\operatorname{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\end{aligned}$$

output
```
-b*arcsech(b*x+a)^3/a-arcsech(b*x+a)^3/x+3*b*arcsech(b*x+a)^2*ln(1-a*(1/(b
*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2
+1)^(1/2)-3*b*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b
*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+6*b*arcsech(b*x+a)*po
lylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^
(1/2)))/a/(-a^2+1)^(1/2)-6*b*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)-6*b
*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+
1)^(1/2)))/a/(-a^2+1)^(1/2)+6*b*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)
*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

3.18.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 69.42 (sec) , antiderivative size = 8527, normalized size of antiderivative = 25.84

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[ArcSech[a + b*x]^3/x^2,x]
```

output
```
Result too large to show
```

3.18.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6875, 5991, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx$$

$$\Big\downarrow \text{6875}$$

$$-b\int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\operatorname{sech}^{-1}(a+bx)^3}{b^2x^2}d\operatorname{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{5991}$$

$$-b\left(3\int -\frac{\text{sech}^{-1}(a+bx)^2}{bx}d\text{sech}^{-1}(a+bx)+\frac{\text{sech}^{-1}(a+bx)^3}{bx}\right)$$

$$\Big\downarrow \text{3042}$$

$$-b\left(\frac{\text{sech}^{-1}(a+bx)^3}{bx}+3\int \frac{\text{sech}^{-1}(a+bx)^2}{a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{4679}$$

$$-b\left(3\int \left(\frac{\text{sech}^{-1}(a+bx)^2}{a}+\frac{\text{sech}^{-1}(a+bx)^2}{a\left(\frac{a}{a+bx}-1\right)}\right)d\text{sech}^{-1}(a+bx)+\frac{\text{sech}^{-1}(a+bx)^3}{bx}\right)$$

$$\Big\downarrow \text{2009}$$

$$-b\left(3\left(-\frac{2\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2\text{sech}^{-1}(a+bx)\,\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}+\frac{2\,\text{PolyLo}}{}\right.\right.$$

input
```
Int[ArcSech[a + b*x]^3/x^2,x]
```

output
```
-(b*(ArcSech[a + b*x]^3/(b*x) + 3*(ArcSech[a + b*x]^3/(3*a) - (ArcSech[a +
 b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a
^2]) + (ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^
2])])/(a*Sqrt[1 - a^2]) - (2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a +
b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (2*ArcSech[a + b*x]*PolyLo
g[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (2*P
olyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) -
 (2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^
2]))))
```

3.18. $\int \frac{\text{sech}^{-1}(a+bx)^3}{x^2}\,dx$

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.18.4 Maple [F]

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x^2} dx$$

input
```
int(arcsech(b*x+a)^3/x^2,x)
```

output
```
int(arcsech(b*x+a)^3/x^2,x)
```

3.18.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(arcsech(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)^3/x^2, x)
```

3.18.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{asech}^3\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(asech(b*x+a)**3/x**2,x)
```

output
```
Integral(asech(a + b*x)**3/x**2, x)
```

3.18.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(arcsech(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1)*a + b*x + a)^3/x - integrate((8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^3 +
 8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^3 - 3*
(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*log(b*x + a) - ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2
*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 +
 (2*a^2*b - b)*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log
(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2
 - 12*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a +
1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a
^2*b - b)*x - a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*
b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^5 + 3*a*b^
2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2 + (b^3*x^5 + 3*a*b^2*x^4 + (3*a^
2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)), x)
```

3.18.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(arcsech(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)^3/x^2, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x^2}\,dx$$

input
```
int(acosh(1/(a + b*x))^3/x^2,x)
```

output
```
int(acosh(1/(a + b*x))^3/x^2, x)
```

3.19 $\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3} \, dx$

3.19.1 Optimal result

Integrand size = 12, antiderivative size = 965

$$
\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = & -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2(1-a^2)}+\frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a(1-a^2)(a+bx)\left(1-\frac{a}{a+bx}\right)} \\
& +\frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2}-\frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} \\
& +\frac{3b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2(1-a^2)} \\
& +\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{2a^2(1-a^2)^{3/2}} \\
& -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}} \\
& +\frac{3b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2(1-a^2)} \\
& -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{2a^2(1-a^2)^{3/2}} \\
& +\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}} \\
& +\frac{3b^2\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2(1-a^2)} \\
& +\frac{3b^2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2(1-a^2)^{3/2}} \\
& -\frac{6b^2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}} \\
& +\frac{3b^2\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2(1-a^2)} \\
& -\frac{3b^2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2(1-a^2)^{3/2}} \\
& +\frac{6b^2\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}} \\
& -\frac{3b^2\operatorname{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2(1-a^2)^{3/2}}+\frac{6b^2\operatorname{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}} \\
& +\frac{3b^2\operatorname{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2(1-a^2)^{3/2}}-\frac{6b^2\operatorname{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}
\end{aligned}
$$

output
```
-3/2*b^2*arcsech(b*x+a)^2/a^2/(-a^2+1)+1/2*b^2*arcsech(b*x+a)^3/a^2-1/2*ar
csech(b*x+a)^3/x^2+3*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)+3/2*b^2*arcsech(
b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a
^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)-3/
2*b^2*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)
^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*polylog(2,a*(1/(b*x+a
)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1
)+3*b^2*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+
a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*polylog(2,a*(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-
a^2+1)-3*b^2*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/
(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-3*b^2*polylog(3,a
*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a
^2/(-a^2+1)^(3/2)+3*b^2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x
+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-3*b^2*arcsech(b*x+a)^
2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(
1/2)))/a^2/(-a^2+1)^(1/2)+3*b^2*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2...
```

3.19.2 Mathematica [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx$$

input
```
Integrate[ArcSech[a + b*x]^3/x^3,x]
```

output
```
Integrate[ArcSech[a + b*x]^3/x^3, x]
```

3.19.3 Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 911, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6875, 25, 5991, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(a+bx)^3}{x^3} \, dx$$

$$\Big\downarrow \text{ 6875}$$

$$-b^2 \int \frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{b^3x^3}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{ 25}$$

$$b^2 \int -\frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)\text{sech}^{-1}(a+bx)^3}{b^3x^3}d\text{sech}^{-1}(a+bx)$$

$$\Big\downarrow \text{ 5991}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\int \frac{\text{sech}^{-1}(a+bx)^2}{b^2x^2}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\int \frac{\text{sech}^{-1}(a+bx)^2}{\left(a-\csc\left(i\text{sech}^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2}d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{ 4679}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\int \left(\frac{2\text{sech}^{-1}(a+bx)^2}{a^2\left(\frac{a}{a+bx}-1\right)}+\frac{\text{sech}^{-1}(a+bx)^2}{a^2}+\frac{\text{sech}^{-1}(a+bx)^2}{a^2\left(\frac{a}{a+bx}-1\right)^2}\right)d\text{sech}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-b^2\left(\frac{\text{sech}^{-1}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(\frac{\text{sech}^{-1}(a+bx)^3}{3a^2}-\frac{2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\text{sech}^{-1}(a+bx)^2}{a^2\sqrt{1-a^2}}+\frac{\log\left(1-\frac{ae^{\text{sech}^{-1}(a+b}}{1-\sqrt{1-a^2}}\right.}{a^2(1-}\right.\right.$$

input
```
Int[ArcSech[a + b*x]^3/x^3,x]
```

output
```
-(b^2*(ArcSech[a + b*x]^3/(2*b^2*x^2) - (3*(-(ArcSech[a + b*x]^2/(a^2*(1 -
 a^2))) + (Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x
]^2)/(a*(1 - a^2)*(a + b*x)*(1 - a/(a + b*x))) + ArcSech[a + b*x]^3/(3*a^2
) + (2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])
])/(a^2*(1 - a^2)) + (ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1
 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (2*ArcSech[a + b*x]^2*Log[1 -
(a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (2*ArcS
ech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1
- a^2)) - (ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 -
 a^2])])/(a^2*(1 - a^2)^(3/2)) + (2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSec
h[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (2*PolyLog[2, (a*E
^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)) + (2*ArcSech[a +
b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2
)^(3/2)) - (4*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt
[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1
 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)) - (2*ArcSech[a + b*x]*PolyLog[2, (a*E^
ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (4*ArcSech
[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqr
t[1 - a^2]) - (2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(
a^2*(1 - a^2)^(3/2)) + (4*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1...
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 5991
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6875
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Sech[x]*T
anh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.19.4 Maple [F]

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x^3}dx$$

input
```
int(arcsech(b*x+a)^3/x^3,x)
```

output
```
int(arcsech(b*x+a)^3/x^3,x)
```

3.19.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsech}(bx+a)^3}{x^3}\,dx$$

input
```
integrate(arcsech(b*x+a)^3/x^3,x, algorithm="fricas")
```

output
```
integral(arcsech(b*x + a)^3/x^3, x)
```

3.19.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{asech}^3\left(a+bx\right)}{x^3}\,dx$$

input
```
integrate(asech(b*x+a)**3/x**3,x)
```

output
```
Integral(asech(a + b*x)**3/x**3, x)
```

3.19.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^3}{x^3}\,dx$$

input
```
integrate(arcsech(b*x+a)^3/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt
(-b*x - a + 1)*a + b*x + a)^3/x^2 - integrate(1/2*(16*(b^3*x^3 + 3*a*b^2*x
^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b
*x + a)^3 + 16*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x
 + a)^3 - 3*(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 4*(b^3*x^3 + 3*a*b^2*
x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a) - (2*(b^3*x^3 + 3*a*b^2*x^2
+ a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 +
 4*a*b^2*x^2 + (2*a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b
 - b)*x - a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt
(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1
)*a + b*x + a)^2 - 24*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)
*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*
x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(
b^3*x^6 + 3*a*b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3 + (b^3*x^6 + 3*a
*b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3)*sqrt(b*x + a + 1)*sqrt(-b*x
- a + 1)), x)
```

3.19.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsech}\left(bx+a\right)^3}{x^3}\,dx$$

input
```
integrate(arcsech(b*x+a)^3/x^3,x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)^3/x^3, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x^3}\,dx$$

input
```
int(acosh(1/(a + b*x))^3/x^3,x)
```

output
```
int(acosh(1/(a + b*x))^3/x^3, x)
```

3.20 $\int x^3 \mathrm{sech}^{-1}(\sqrt{x})\,dx$

3.20.1 Optimal result

Integrand size = 10, antiderivative size = 164

$$\int x^3 \mathrm{sech}^{-1}(\sqrt{x})\,dx = -\frac{1-x}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^2}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} - \frac{3(1-x)^3}{20\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^4}{28\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{1}{4}x^4\mathrm{sech}^{-1}(\sqrt{x})$$

output
```
1/4*x^4*arcsech(x^(1/2))+1/4*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1
/2))^(1/2)+1/4*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)-3/
20*(1-x)^3/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+1/28*(1-x)^4/x
^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.51

$$\int x^3 \mathrm{sech}^{-1}(\sqrt{x})\,dx = -\frac{1}{140}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(16+16\sqrt{x}+8x+8x^{3/2}+6x^2+6x^{5/2}+5x^3+5x^{7/2}\right) + \frac{1}{4}x^4\mathrm{sech}^{-1}(\sqrt{x})$$

input
```
Integrate[x^3*ArcSech[Sqrt[x]],x]
```

output
```
-1/140*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(16 + 16*Sqrt[x] + 8*x + 8*x^(3/
2) + 6*x^2 + 6*x^(5/2) + 5*x^3 + 5*x^(7/2))) + (x^4*ArcSech[Sqrt[x]])/4
```

3.20.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6899, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{sech}^{-1}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{6899}$$

$$\frac{\sqrt{1-x}\int \frac{x^3}{2\sqrt{1-x}}dx}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{4}x^4\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{1-x}\int \frac{x^3}{\sqrt{1-x}}dx}{8\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{4}x^4\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{1-x}\int \left(-(1-x)^{5/2}+3(1-x)^{3/2}-3\sqrt{1-x}+\frac{1}{\sqrt{1-x}}\right)dx}{8\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{4}x^4\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4\text{sech}^{-1}\left(\sqrt{x}\right)+\frac{\left(\frac{2}{7}(1-x)^{7/2}-\frac{6}{5}(1-x)^{5/2}+2(1-x)^{3/2}-2\sqrt{1-x}\right)\sqrt{1-x}}{8\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

input
```
Int[x^3*ArcSech[Sqrt[x]],x]
```

output
```
((-2*Sqrt[1 - x] + 2*(1 - x)^(3/2) - (6*(1 - x)^(5/2))/5 + (2*(1 - x)^(7/2
))/7)*Sqrt[1 - x])/(8*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) +
(x^4*ArcSech[Sqrt[x]])/4
```

3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6899
```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Simp[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]))   Int[SimplifyIntegrand[
(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
 + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

3.20.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

method	result	size
derivativedivides	$\frac{x^4 \operatorname{arcsech}\left(\sqrt{x}\right)}{4}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(5x^3+6x^2+8x+16\right)}{140}$	54
default	$\frac{x^4 \operatorname{arcsech}\left(\sqrt{x}\right)}{4}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(5x^3+6x^2+8x+16\right)}{140}$	54
parts	$\frac{x^4 \operatorname{arcsech}\left(\sqrt{x}\right)}{4}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(5x^3+6x^2+8x+16\right)}{140}$	54

input
```
int(x^3*arcsech(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arcsech(x^(1/2))-1/140*(-(x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*((x^(1
/2)+1)/x^(1/2))^(1/2)*(5*x^3+6*x^2+8*x+16)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.35

$$\int x^3 \text{sech}^{-1}(\sqrt{x})\, dx$$
$$= \frac{1}{4}\, x^4 \log\left(\frac{x\sqrt{-\frac{x-1}{x}} + \sqrt{x}}{x}\right) - \frac{1}{140}\left(5\, x^3 + 6\, x^2 + 8\, x + 16\right)\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

input
```
integrate(x^3*arcsech(x^(1/2)),x, algorithm="fricas")
```

output
```
1/4*x^4*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/140*(5*x^3 + 6*x^2 + 8*x
 + 16)*sqrt(x)*sqrt(-(x - 1)/x)
```

3.20.6 Sympy [F]

$$\int x^3 \text{sech}^{-1}(\sqrt{x})\, dx = \int x^3 \operatorname{asech}\left(\sqrt{x}\right) dx$$

input
```
integrate(x**3*asech(x**(1/2)),x)
```

output
```
Integral(x**3*asech(sqrt(x)), x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

$$\int x^3 \operatorname{sech}^{-1}(\sqrt{x})\, dx = \frac{1}{28}\,x^{\frac{7}{2}}\left(\frac{1}{x}-1\right)^{\frac{7}{2}} - \frac{3}{20}\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}} + \frac{1}{4}\,x^4 \operatorname{arsech}\left(\sqrt{x}\right) + \frac{1}{4}\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}} - \frac{1}{4}\,\sqrt{x}\sqrt{\frac{1}{x}-1}$$

input
```
integrate(x^3*arcsech(x^(1/2)),x, algorithm="maxima")
```

output
```
1/28*x^(7/2)*(1/x - 1)^(7/2) - 3/20*x^(5/2)*(1/x - 1)^(5/2) + 1/4*x^4*arcs
ech(sqrt(x)) + 1/4*x^(3/2)*(1/x - 1)^(3/2) - 1/4*sqrt(x)*sqrt(1/x - 1)
```

3.20.8 Giac [F]

$$\int x^3 \operatorname{sech}^{-1}(\sqrt{x})\, dx = \int x^3 \operatorname{arsech}\left(\sqrt{x}\right)\, dx$$

input
```
integrate(x^3*arcsech(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^3*arcsech(sqrt(x)), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{sech}^{-1}(\sqrt{x})\, dx = \int x^3 \operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)\, dx$$

input
```
int(x^3*acosh(1/x^(1/2)),x)
```

output
```
int(x^3*acosh(1/x^(1/2)), x)
```

3.21 $\int x^2 \text{sech}^{-1}(\sqrt{x})\,dx$

3.21.1 Optimal result

Integrand size = 10, antiderivative size = 126

$$\int x^2 \text{sech}^{-1}(\sqrt{x})\,dx = -\frac{1-x}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{2(1-x)^2}{9\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} - \frac{(1-x)^3}{15\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{1}{3}x^3\text{sech}^{-1}(\sqrt{x})$$

output
```
1/3*x^3*arcsech(x^(1/2))+1/3*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1
/2))^(1/2)+2/9*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)-1/
15*(1-x)^3/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$\int x^2 \text{sech}^{-1}(\sqrt{x})\,dx = -\frac{1}{45}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(8+8\sqrt{x}+4x+4x^{3/2}+3x^2+3x^{5/2}\right)+\frac{1}{3}x^3\text{sech}^{-1}(\sqrt{x})$$

input
```
Integrate[x^2*ArcSech[Sqrt[x]],x]
```

output
```
-1/45*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(8 + 8*Sqrt[x] + 4*x + 4*x^(3/2)
+ 3*x^2 + 3*x^(5/2))) + (x^3*ArcSech[Sqrt[x]])/3
```

3.21.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6899, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{sech}^{-1}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{ 6899}$$

$$\frac{\sqrt{1-x}\int \frac{x^2}{2\sqrt{1-x}}dx}{3\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{3}x^3\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{1-x}\int \frac{x^2}{\sqrt{1-x}}dx}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{3}x^3\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\sqrt{1-x}\int \left((1-x)^{3/2}-2\sqrt{1-x}+\frac{1}{\sqrt{1-x}}\right)dx}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{3}x^3\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3\text{sech}^{-1}\left(\sqrt{x}\right)+\frac{\left(-\frac{2}{5}(1-x)^{5/2}+\frac{4}{3}(1-x)^{3/2}-2\sqrt{1-x}\right)\sqrt{1-x}}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

input
```
Int[x^2*ArcSech[Sqrt[x]],x]
```

output
```
((-2*Sqrt[1 - x] + (4*(1 - x)^(3/2))/3 - (2*(1 - x)^(5/2))/5)*Sqrt[1 - x])
/(6*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (x^3*ArcSech[Sqrt[
x]])/3
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6899
```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Simp[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]))   Int[SimplifyIntegrand[
(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
 + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

3.21.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{x^{3} \operatorname{arcsech}\left(\sqrt{x}\right)}{3}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\, \sqrt{x}\, \sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\, \left(3 x^{2}+4 x +8\right)}{45}$	49
default	$\frac{x^{3} \operatorname{arcsech}\left(\sqrt{x}\right)}{3}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\, \sqrt{x}\, \sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\, \left(3 x^{2}+4 x +8\right)}{45}$	49
parts	$\frac{x^{3} \operatorname{arcsech}\left(\sqrt{x}\right)}{3}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\, \sqrt{x}\, \sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\, \left(3 x^{2}+4 x +8\right)}{45}$	49

input
```
int(x^2*arcsech(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arcsech(x^(1/2))-1/45*(-(x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*((x^(1/
2)+1)/x^(1/2))^(1/2)*(3*x^2+4*x+8)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int x^2 \operatorname{sech}^{-1}(\sqrt{x})\,dx = \frac{1}{3}\,x^3\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right) - \frac{1}{45}\left(3\,x^2+4\,x+8\right)\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

input
```
integrate(x^2*arcsech(x^(1/2)),x, algorithm="fricas")
```

output
```
1/3*x^3*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/45*(3*x^2 + 4*x + 8)*sqr
t(x)*sqrt(-(x - 1)/x)
```

3.21.6 Sympy [F]

$$\int x^2 \operatorname{sech}^{-1}(\sqrt{x})\,dx = \int x^2 \operatorname{asech}\left(\sqrt{x}\right)dx$$

input
```
integrate(x**2*asech(x**(1/2)),x)
```

output
```
Integral(x**2*asech(sqrt(x)), x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int x^2 \operatorname{sech}^{-1}(\sqrt{x})\,dx = -\frac{1}{15}\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}+\frac{1}{3}\,x^3\operatorname{arsech}\left(\sqrt{x}\right)+\frac{2}{9}\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}\sqrt{\frac{1}{x}-1}$$

input
```
integrate(x^2*arcsech(x^(1/2)),x, algorithm="maxima")
```

output
```
-1/15*x^(5/2)*(1/x - 1)^(5/2) + 1/3*x^3*arcsech(sqrt(x)) + 2/9*x^(3/2)*(1/
x - 1)^(3/2) - 1/3*sqrt(x)*sqrt(1/x - 1)
```

3.21.8 Giac [F]

$$\int x^2\text{sech}^{-1}(\sqrt{x})\,dx=\int x^2\,\text{arsech}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x^2*arcsech(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^2*arcsech(sqrt(x)), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{sech}^{-1}(\sqrt{x})\,dx=\int x^2\,\text{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

input
```
int(x^2*acosh(1/x^(1/2)),x)
```

output
```
int(x^2*acosh(1/x^(1/2)), x)
```

3.22 $\int x\text{sech}^{-1}(\sqrt{x})\,dx$

3.22.1 Optimal result

Integrand size = 8, antiderivative size = 88

$$\int x\text{sech}^{-1}(\sqrt{x})\,dx = -\frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^2}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{1}{2}x^2\text{sech}^{-1}(\sqrt{x})$$

output
```
1/2*x^2*arcsech(x^(1/2))+1/2*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1
/2))^(1/2)+1/6*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int x\text{sech}^{-1}(\sqrt{x})\,dx = -\frac{1}{6}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(2+2\sqrt{x}+x+x^{3/2}\right) + \frac{1}{2}x^2\text{sech}^{-1}(\sqrt{x})$$

input
```
Integrate[x*ArcSech[Sqrt[x]],x]
```

output
```
-1/6*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(2 + 2*Sqrt[x] + x + x^(3/2)) + (
x^2*ArcSech[Sqrt[x]])/2
```

3.22.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6899, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{sech}^{-1}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{ 6899}$$

$$\frac{\sqrt{1-x}\int \frac{x}{2\sqrt{1-x}}dx}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{2}x^2\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{1-x}\int \frac{x}{\sqrt{1-x}}dx}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{2}x^2\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\sqrt{1-x}\int \left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{1}{2}x^2\text{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2\text{sech}^{-1}\left(\sqrt{x}\right)+\frac{\left(\frac{2}{3}(1-x)^{3/2}-2\sqrt{1-x}\right)\sqrt{1-x}}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

input
```
Int[x*ArcSech[Sqrt[x]],x]
```

output
```
((-2*Sqrt[1 - x] + (2*(1 - x)^(3/2))/3)*Sqrt[1 - x])/(4*Sqrt[-1 + 1/Sqrt[x
]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (x^2*ArcSech[Sqrt[x]])/2
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6899
```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Simp[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]))   Int[SimplifyIntegrand[
(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
 + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

3.22.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

method	result	size
derivativedivides	$\frac{x^{2} \operatorname{arcsech}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(x+2\right)}{6}$	42
default	$\frac{x^{2} \operatorname{arcsech}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(x+2\right)}{6}$	42
parts	$\frac{x^{2} \operatorname{arcsech}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(x+2\right)}{6}$	42

input
```
int(x*arcsech(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arcsech(x^(1/2))-1/6*(-(x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*((x^(1/2
)+1)/x^(1/2))^(1/2)*(x+2)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int x\operatorname{sech}^{-1}(\sqrt{x})\,dx = \frac{1}{2}\,x^2\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right) - \frac{1}{6}\,(x+2)\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

input
```
integrate(x*arcsech(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x^2*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/6*(x + 2)*sqrt(x)*sqrt(-
(x - 1)/x)
```

3.22.6 Sympy [F]

$$\int x\operatorname{sech}^{-1}(\sqrt{x})\,dx = \int x\operatorname{asech}\left(\sqrt{x}\right)dx$$

input
```
integrate(x*asech(x**(1/2)),x)
```

output
```
Integral(x*asech(sqrt(x)), x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.39

$$\int x\operatorname{sech}^{-1}(\sqrt{x})\,dx = \frac{1}{6}\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}} + \frac{1}{2}\,x^2\operatorname{arsech}\left(\sqrt{x}\right) - \frac{1}{2}\,\sqrt{x}\sqrt{\frac{1}{x}-1}$$

input
```
integrate(x*arcsech(x^(1/2)),x, algorithm="maxima")
```

output
```
1/6*x^(3/2)*(1/x - 1)^(3/2) + 1/2*x^2*arcsech(sqrt(x)) - 1/2*sqrt(x)*sqrt(
1/x - 1)
```

3.22.8 Giac [F]

$$\int x\text{sech}^{-1}(\sqrt{x})\ dx = \int x\,\text{arsech}\left(\sqrt{x}\right)\ dx$$

input
```
integrate(x*arcsech(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(x*arcsech(sqrt(x)), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{-1}(\sqrt{x})\ dx = \int x\,\text{acosh}\left(\frac{1}{\sqrt{x}}\right)\ dx$$

input
```
int(x*acosh(1/x^(1/2)),x)
```

output
```
int(x*acosh(1/x^(1/2)), x)
```

3.23 $\int \operatorname{sech}^{-1}(\sqrt{x})\,dx$

3.23.1 Optimal result

Integrand size = 6, antiderivative size = 43

$$\int \operatorname{sech}^{-1}(\sqrt{x})\,dx=-\frac{1-x}{\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+x\operatorname{sech}^{-1}(\sqrt{x})$$

output
```
x*arcsech(x^(1/2))+(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

3.23.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(43) = 86.

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.74

$$\int \operatorname{sech}^{-1}(\sqrt{x})\,dx=-\frac{2\left(-1+\sqrt{1-\sqrt{x}}\right)^2\left(-1+\sqrt{1+\sqrt{x}}\right)^2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\sqrt{1+\sqrt{x}}}{\left(-2+\sqrt{1-\sqrt{x}}+\sqrt{1+\sqrt{x}}\right)^2\sqrt{1-\sqrt{x}}}+x\operatorname{sech}^{-1}(\sqrt{x})$$

input
```
Integrate[ArcSech[Sqrt[x]],x]
```

output
```
(-2*(-1 + Sqrt[1 - Sqrt[x]])^2*(-1 + Sqrt[1 + Sqrt[x]])^2*Sqrt[(1 - Sqrt[x
])/(1 + Sqrt[x])]*Sqrt[1 + Sqrt[x]])/((-2 + Sqrt[1 - Sqrt[x]] + Sqrt[1 + S
qrt[x]])^2*Sqrt[1 - Sqrt[x]]) + x*ArcSech[Sqrt[x]]
```

3.23.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6897, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \ 6897$$

$$\frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}}dx}{\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+x\operatorname{sech}^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \ 17$$

$$x\operatorname{sech}^{-1}\left(\sqrt{x}\right)-\frac{1-x}{\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

input
```
Int[ArcSech[Sqrt[x]],x]
```

output
```
-((1 - x)/(Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])) + x*ArcSech[
Sqrt[x]]
```

3.23.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6897
```
Int[ArcSech[u_], x_Symbol] :> Simp[x*ArcSech[u], x] + Simp[Sqrt[1 - u^2]/(u
*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u])   Int[SimplifyIntegrand[x*(D[u, x]/(u*Sqrt[1
 - u^2])), x], x], x] /; InverseFunctionFreeQ[u, x] &&  !FunctionOfExponent
ialQ[u, x]
```

3.23.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$x\,\mathrm{arcsech}\left(\sqrt{x}\right)-\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}$	36
default	$x\,\mathrm{arcsech}\left(\sqrt{x}\right)-\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}$	36
parts	$x\,\mathrm{arcsech}\left(\sqrt{x}\right)-\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}$	36

input
```
int(arcsech(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arcsech(x^(1/2))-(-(x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*((x^(1/2)+1)/x^(1/
2))^(1/2)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \operatorname{sech}^{-1}\left(\sqrt{x}\right)\,dx = x\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)-\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

input
```
integrate(arcsech(x^(1/2)),x, algorithm="fricas")
```

output
```
x*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - sqrt(x)*sqrt(-(x - 1)/x)
```

3.23.6 Sympy [F]

$$\int \operatorname{sech}^{-1}\left(\sqrt{x}\right)\,dx = \int \operatorname{asech}\left(\sqrt{x}\right)dx$$

input
```
integrate(asech(x**(1/2)),x)
```

output
```
Integral(asech(sqrt(x)), x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.44

$$\int \operatorname{sech}^{-1}\left(\sqrt{x}\right)\,dx = x \operatorname{arsech}\left(\sqrt{x}\right) - \sqrt{x}\sqrt{\frac{1}{x} - 1}$$

input
```
integrate(arcsech(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arcsech(sqrt(x)) - sqrt(x)*sqrt(1/x - 1)
```

3.23.8 Giac [F]

$$\int \operatorname{sech}^{-1}\left(\sqrt{x}\right)\,dx = \int \operatorname{arsech}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(arcsech(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(arcsech(sqrt(x)), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{-1}\left(\sqrt{x}\right)\,dx = \int \operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

input
```
int(acosh(1/x^(1/2)),x)
```

output
```
int(acosh(1/x^(1/2)), x)
```

3.24 $\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} \, dx$

3.24.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} \, dx = \operatorname{sech}^{-1}(\sqrt{x})^2 - 2\operatorname{sech}^{-1}(\sqrt{x}) \log\left(1 + e^{2\operatorname{sech}^{-1}(\sqrt{x})}\right) - \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(\sqrt{x})}\right)$$

output
```
arcsech(x^(1/2))^2-2*arcsech(x^(1/2))*ln(1+(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)
*(1+1/x^(1/2))^(1/2))^2)-polylog(2,-(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*(1+1/x
^(1/2))^(1/2))^2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} \, dx = -\operatorname{sech}^{-1}(\sqrt{x}) \left(\operatorname{sech}^{-1}(\sqrt{x}) + 2\log\left(1 + e^{-2\operatorname{sech}^{-1}(\sqrt{x})}\right)\right) + \operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(\sqrt{x})}\right)$$

input
```
Integrate[ArcSech[Sqrt[x]]/x,x]
```

output
```
-(ArcSech[Sqrt[x]]*(ArcSech[Sqrt[x]] + 2*Log[1 + E^(-2*ArcSech[Sqrt[x]])])
) + PolyLog[2, -E^(-2*ArcSech[Sqrt[x]])]
```

3.24.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {7267, 6835, 6297, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}\left(\sqrt{x}\right)}{x} \, dx$$

$$\Big\downarrow \text{ 7267}$$

$$2 \int \frac{\text{sech}^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} d\sqrt{x}$$

$$\Big\downarrow \text{ 6835}$$

$$-2 \int \frac{\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{x}} d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 6297}$$

$$-2 \int \sqrt{\frac{\frac{1}{\sqrt{x}}-1}{1+\frac{1}{\sqrt{x}}}} \left(1+\frac{1}{\sqrt{x}}\right) \sqrt{x} \text{arccosh}\left(\frac{1}{\sqrt{x}}\right) d\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-2 \int -i\text{arccosh}\left(\frac{1}{\sqrt{x}}\right) \tan\left(i\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)\right) d\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 26}$$

$$2i \int \text{arccosh}\left(\frac{1}{\sqrt{x}}\right) \tan\left(i\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)\right) d\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 4201}$$

$$2i \left(2i \int \frac{e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)} \text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}{1+e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}} d\text{arccosh}\left(\frac{1}{\sqrt{x}}\right) - \frac{ix}{2}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$2i \left(2i \left(\frac{1}{2}\text{arccosh}\left(\frac{1}{\sqrt{x}}\right) \log\left(e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}+1\right) - \frac{1}{2}\int \log\left(1+e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}\right) d\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)\right) - \frac{ix}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$2i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)\log\left(e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}+1\right)-\frac{1}{4}\int e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}\log\left(1+e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}\right)de^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}\right)-\frac{i}{2}$$

$$\Big\downarrow \text{2838}$$

$$2i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}\right)+\frac{1}{2}\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)\log\left(e^{2\text{arccosh}\left(\frac{1}{\sqrt{x}}\right)}+1\right)\right)-\frac{ix}{2}\right)$$

input
```
Int[ArcSech[Sqrt[x]]/x,x]
```

output
```
(2*I)*((-1/2*I)*x + (2*I)*((ArcCosh[1/Sqrt[x]]*Log[1 + E^(2*ArcCosh[1/Sqrt
[x]])])/2 + PolyLog[2, -E^(2*ArcCosh[1/Sqrt[x]])]/4))
```

3.24.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6835
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.24.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\operatorname{arcsech}\left(\sqrt{x}\right)^2-2\operatorname{arcsech}\left(\sqrt{x}\right)\ln\left(1+\left(\frac{1}{\sqrt{x}}+\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\right)^2\right)-\operatorname{polylog}\left(\right.$
default	$\operatorname{arcsech}\left(\sqrt{x}\right)^2-2\operatorname{arcsech}\left(\sqrt{x}\right)\ln\left(1+\left(\frac{1}{\sqrt{x}}+\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\right)^2\right)-\operatorname{polylog}\left(\right.$

input
```
int(arcsech(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
arcsech(x^(1/2))^2-2*arcsech(x^(1/2))*ln(1+(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)
*(1+1/x^(1/2))^(1/2))^2)-polylog(2,-(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*(1+1/x
^(1/2))^(1/2))^2)
```

3.24.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arsech}(\sqrt{x})}{x}\,dx$$

input
```
integrate(arcsech(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arcsech(sqrt(x))/x, x)
```

3.24.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{asech}(\sqrt{x})}{x}\,dx$$

input
```
integrate(asech(x**(1/2))/x,x)
```

output
```
Integral(asech(sqrt(x))/x, x)
```

3.24.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arsech}(\sqrt{x})}{x}\,dx$$

input
```
integrate(arcsech(x^(1/2))/x,x, algorithm="maxima")
```

output
```
-1/4*log(x)^2 + log(x)*log(sqrt(sqrt(x) + 1)*sqrt(-sqrt(x) + 1) + 1) - log
(sqrt(x) + 1)*log(sqrt(x)) - log(sqrt(x))*log(-sqrt(x) + 1) - dilog(-sqrt(
x)) - dilog(sqrt(x)) + integrate(1/2*log(x)/((x - 1)*e^(1/2*log(sqrt(x) +
1) + 1/2*log(-sqrt(x) + 1)) + x - 1), x)
```

3.24.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arsech}(\sqrt{x})}{x}\,dx$$

input
```
integrate(arcsech(x^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arcsech(sqrt(x))/x, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x}\,dx$$

input
```
int(acosh(1/x^(1/2))/x,x)
```

output
```
int(acosh(1/x^(1/2))/x, x)
```

3.25 $\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2} \, dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 98

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2} \, dx = \frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} + \frac{\sqrt{1-x}\operatorname{arctanh}(\sqrt{1-x})}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

output
```
-arcsech(x^(1/2))/x+1/2*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(
1/2)+1/2*arctanh((1-x)^(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+
1/x^(1/2))^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2} \, dx = \frac{\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}(1+\sqrt{x}) - 2\operatorname{sech}^{-1}(\sqrt{x}) + x\log\left(1+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\right) - \frac{1}{2}x\log(x)}{2x}$$

input
```
Integrate[ArcSech[Sqrt[x]]/x^2,x]
```

output
```
(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(1 + Sqrt[x]) - 2*ArcSech[Sqrt[x]] + x*
Log[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x
])]*Sqrt[x]] - (x*Log[x])/2)/(2*x)
```

3.25.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6899, 27, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2}dx$$

$$\Big\downarrow \text{ 6899}$$

$$-\frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}x^2}dx}{\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}x^2}dx}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{\sqrt{1-x}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x}x}dx-\frac{\sqrt{1-x}}{x}\right)}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\sqrt{1-x}\left(-\int \frac{1}{x}d\sqrt{1-x}-\frac{\sqrt{1-x}}{x}\right)}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\sqrt{1-x}\left(-\operatorname{arctanh}(\sqrt{1-x})-\frac{\sqrt{1-x}}{x}\right)}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}$$

input
```
Int[ArcSech[Sqrt[x]]/x^2,x]
```

output
```
-(ArcSech[Sqrt[x]]/x) - (Sqrt[1 - x]*(-(Sqrt[1 - x]/x) - ArcTanh[Sqrt[1 -
x]]))/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6899
```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Simp[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]))   Int[SimplifyIntegrand[
(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
 + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

3.25. $\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2} \, dx$

3.25.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x+\sqrt{1-x}\right)}{2\sqrt{x}\,\sqrt{1-x}}$	64
default	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x+\sqrt{1-x}\right)}{2\sqrt{x}\,\sqrt{1-x}}$	64
parts	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x+\sqrt{1-x}\right)}{2\sqrt{x}\,\sqrt{1-x}}$	64

input
```
int(arcsech(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsech(x^(1/2))/x+1/2*(-(x^(1/2)-1)/x^(1/2))^(1/2)/x^(1/2)*((x^(1/2)+1)/
x^(1/2))^(1/2)*(arctanh(1/(1-x)^(1/2))*x+(1-x)^(1/2))/(1-x)^(1/2)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x^2}\,dx=\frac{(x-2)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)+\sqrt{x}\sqrt{-\frac{x-1}{x}}}{2\,x}$$

input
```
integrate(arcsech(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
1/2*((x - 2)*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) + sqrt(x)*sqrt(-(x - 1)
/x))/x
```

3.25.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2}\,dx = \int \frac{\operatorname{asech}(\sqrt{x})}{x^2}\,dx$$

input
```
integrate(asech(x**(1/2))/x**2,x)
```

output
```
Integral(asech(sqrt(x))/x**2, x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2}\,dx = -\frac{\sqrt{x}\sqrt{\frac{1}{x}-1}}{2\left(x\left(\frac{1}{x}-1\right)-1\right)} - \frac{\operatorname{arsech}(\sqrt{x})}{x} + \frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}+1\right) - \frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}-1\right)$$

input
```
integrate(arcsech(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(x)*sqrt(1/x - 1)/(x*(1/x - 1) - 1) - arcsech(sqrt(x))/x + 1/4*lo
g(sqrt(x)*sqrt(1/x - 1) + 1) - 1/4*log(sqrt(x)*sqrt(1/x - 1) - 1)
```

3.25.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2}\,dx = \int \frac{\operatorname{arsech}(\sqrt{x})}{x^2}\,dx$$

input
```
integrate(arcsech(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
integrate(arcsech(sqrt(x))/x^2, x)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^2}\,dx = \frac{\sqrt{\frac{1}{\sqrt{x}}-1}\,\sqrt{\frac{1}{\sqrt{x}}+1}}{2\,\sqrt{x}} - \frac{2\,\mathrm{acosh}\left(\frac{1}{\sqrt{x}}\right)\,\left(\frac{1}{2\,\sqrt{x}}-\frac{\sqrt{x}}{4}\right)}{\sqrt{x}}$$

input
```
int(acosh(1/x^(1/2))/x^2,x)
```

output
```
((1/x^(1/2) - 1)^(1/2)*(1/x^(1/2) + 1)^(1/2))/(2*x^(1/2)) - (2*acosh(1/x^(
1/2))*(1/(2*x^(1/2)) - x^(1/2)/4))/x^(1/2)
```

3.26 $\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx$

3.26.1 Optimal result

Integrand size = 10, antiderivative size = 136

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx = \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{5/2}} + \frac{3(1-x)}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} + \frac{3\sqrt{1-x}\operatorname{arctanh}(\sqrt{1-x})}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

output
```
-1/2*arcsech(x^(1/2))/x^2+1/8*(1-x)/x^(5/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1
/2))^(1/2)+3/16*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+3/1
6*arctanh((1-x)^(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx = \frac{1}{16}\left(\frac{\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(2+2\sqrt{x}+3x+3x^{3/2}\right)}{x^2} - \frac{8\operatorname{sech}^{-1}(\sqrt{x})}{x^2} + 3\log\left(1+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\right) - \frac{3\log(x)}{2}\right)$$

input
```
Integrate[ArcSech[Sqrt[x]]/x^3,x]
```

output
```
((Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(2 + 2*Sqrt[x] + 3*x + 3*x^(3/2)))/x^2
 - (8*ArcSech[Sqrt[x]])/x^2 + 3*Log[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]
+ Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]] - (3*Log[x])/2)/16
```

3.26.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6899, 27, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx$$

$$\Big\downarrow \ 6899$$

$$-\frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}x^3}dx}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}x^3}dx}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2}$$

$$\Big\downarrow \ 52$$

$$-\frac{\sqrt{1-x}\left(\frac{3}{4}\int \frac{1}{\sqrt{1-x}x^2}dx-\frac{\sqrt{1-x}}{2x^2}\right)}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2}$$

$$\Big\downarrow \ 52$$

$$-\frac{\sqrt{1-x}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x}x}dx-\frac{\sqrt{1-x}}{x}\right)-\frac{\sqrt{1-x}}{2x^2}\right)}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2}$$

$$\Big\downarrow \ 73$$

$$-\frac{\sqrt{1-x}\left(\frac{3}{4}\left(-\int \frac{1}{x}d\sqrt{1-x}-\frac{\sqrt{1-x}}{x}\right)-\frac{\sqrt{1-x}}{2x^2}\right)}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2}$$

$$\Big\downarrow 219$$

$$-\frac{\sqrt{1-x}\left(\frac{3}{4}\left(-\operatorname{arctanh}\left(\sqrt{1-x}\right)-\frac{\sqrt{1-x}}{x}\right)-\frac{\sqrt{1-x}}{2x^2}\right)}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcSech[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcSech[Sqrt[x]]/x^2 - (Sqrt[1 - x]*(-1/2*Sqrt[1 - x]/x^2 + (3*(-(Sqr
t[1 - x]/x) - ArcTanh[Sqrt[1 - x]]))/4))/(4*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 +
1/Sqrt[x]]*Sqrt[x])
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6899
```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Simp[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]))   Int[SimplifyIntegrand[
(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
 + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

3.26.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{2}+3\sqrt{1-x}\,x+2\sqrt{1-x}\right)}{16x^{\frac{3}{2}}\sqrt{1-x}}$	79
default	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{2}+3\sqrt{1-x}\,x+2\sqrt{1-x}\right)}{16x^{\frac{3}{2}}\sqrt{1-x}}$	79
parts	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{2}+3\sqrt{1-x}\,x+2\sqrt{1-x}\right)}{16x^{\frac{3}{2}}\sqrt{1-x}}$	79

input
```
int(arcsech(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsech(x^(1/2))/x^2+1/16*(-(x^(1/2)-1)/x^(1/2))^(1/2)/x^(3/2)*((x^(1
/2)+1)/x^(1/2))^(1/2)*(3*arctanh(1/(1-x)^(1/2))*x^2+3*(1-x)^(1/2)*x+2*(1-x
)^(1/2))/(1-x)^(1/2)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.40

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx=\frac{(3\,x+2)\sqrt{x}\sqrt{-\frac{x-1}{x}}+(3\,x^2-8)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)}{16\,x^2}$$

input
```
integrate(arcsech(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/16*((3*x + 2)*sqrt(x)*sqrt(-(x - 1)/x) + (3*x^2 - 8)*log((x*sqrt(-(x - 1
)/x) + sqrt(x))/x))/x^2
```

3.26.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx = \int \frac{\operatorname{asech}(\sqrt{x})}{x^3}\,dx$$

input
```
integrate(asech(x**(1/2))/x**3,x)
```

output
```
Integral(asech(sqrt(x))/x**3, x)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx = -\frac{3\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}} - 5\,\sqrt{x}\sqrt{\frac{1}{x}-1}}{16\left(x^2\left(\frac{1}{x}-1\right)^2 - 2\,x\left(\frac{1}{x}-1\right)+1\right)} - \frac{\operatorname{arsech}(\sqrt{x})}{2\,x^2} + \frac{3}{32}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}+1\right) - \frac{3}{32}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}-1\right)$$

input
```
integrate(arcsech(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/16*(3*x^(3/2)*(1/x - 1)^(3/2) - 5*sqrt(x)*sqrt(1/x - 1))/(x^2*(1/x - 1)
^2 - 2*x*(1/x - 1) + 1) - 1/2*arcsech(sqrt(x))/x^2 + 3/32*log(sqrt(x)*sqrt
(1/x - 1) + 1) - 3/32*log(sqrt(x)*sqrt(1/x - 1) - 1)
```

3.26.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx = \int \frac{\operatorname{arsech}(\sqrt{x})}{x^3}\,dx$$

input
```
integrate(arcsech(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
integrate(arcsech(sqrt(x))/x^3, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x^3}\,dx$$

input
```
int(acosh(1/x^(1/2))/x^3,x)
```

output
```
int(acosh(1/x^(1/2))/x^3, x)
```

3.27 $\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4} \, dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 172

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4} \, dx = \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{5/2}} + \frac{5(1-x)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} + \frac{5\sqrt{1-x}\operatorname{arctanh}\left(\sqrt{1-x}\right)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

output
```
-1/3*arcsech(x^(1/2))/x^3+1/18*(1-x)/x^(7/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(
1/2))^(1/2)+5/72*(1-x)/x^(5/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+5/
48*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+5/48*arctanh((1-
x)^(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx$$
$$= \frac{\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(8+8\sqrt{x}+10x+10x^{3/2}+15x^2+15x^{5/2}\right)-48\operatorname{sech}^{-1}(\sqrt{x})+15x^3\log\left(1+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\sqrt{\right.}{144x^3}$$

input
```
Integrate[ArcSech[Sqrt[x]]/x^4,x]
```

output
```
(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(8 + 8*Sqrt[x] + 10*x + 10*x^(3/2) + 15
*x^2 + 15*x^(5/2)) - 48*ArcSech[Sqrt[x]] + 15*x^3*Log[1 + Sqrt[(1 - Sqrt[x
])/(1 + Sqrt[x])] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]] - (15*x^3*L
og[x])/2)/(144*x^3)
```

3.27.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6899, 27, 52, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx$$
$$\Big\downarrow \; 6899$$
$$-\frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}x^4}dx}{3\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3}$$
$$\Big\downarrow \; 27$$
$$-\frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}x^4}dx}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3}$$
$$\Big\downarrow \; 52$$

$$-\frac{\sqrt{1-x}\left(\frac{5}{6}\int \frac{1}{\sqrt{1-x}x^3}dx-\frac{\sqrt{1-x}}{3x^3}\right)}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\sqrt{1-x}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{1}{\sqrt{1-x}x^2}dx-\frac{\sqrt{1-x}}{2x^2}\right)-\frac{\sqrt{1-x}}{3x^3}\right)}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\sqrt{1-x}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x}x}dx-\frac{\sqrt{1-x}}{x}\right)-\frac{\sqrt{1-x}}{2x^2}\right)-\frac{\sqrt{1-x}}{3x^3}\right)}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{1-x}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\int \frac{1}{x}d\sqrt{1-x}-\frac{\sqrt{1-x}}{x}\right)-\frac{\sqrt{1-x}}{2x^2}\right)-\frac{\sqrt{1-x}}{3x^3}\right)}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{1-x}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\text{arctanh}\left(\sqrt{1-x}\right)-\frac{\sqrt{1-x}}{x}\right)-\frac{\sqrt{1-x}}{2x^2}\right)-\frac{\sqrt{1-x}}{3x^3}\right)}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

input
```
Int[ArcSech[Sqrt[x]]/x^4,x]
```

output
```
-1/3*ArcSech[Sqrt[x]]/x^3 - (Sqrt[1 - x]*(-1/3*Sqrt[1 - x]/x^3 + (5*(-1/2*
Sqrt[1 - x]/x^2 + (3*(-(Sqrt[1 - x]/x) - ArcTanh[Sqrt[1 - x]]))/4))/6))/(6
*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6899
```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Simp[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]))   Int[SimplifyIntegrand[
(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
 + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

3.27.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(15\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{3}+15\sqrt{1-x}\,x^{2}+10\sqrt{1-x}\,x+8\sqrt{1-x}\right)}{144x^{\frac{5}{2}}\sqrt{1-x}}$	91
default	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(15\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{3}+15\sqrt{1-x}\,x^{2}+10\sqrt{1-x}\,x+8\sqrt{1-x}\right)}{144x^{\frac{5}{2}}\sqrt{1-x}}$	91
parts	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(15\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{3}+15\sqrt{1-x}\,x^{2}+10\sqrt{1-x}\,x+8\sqrt{1-x}\right)}{144x^{\frac{5}{2}}\sqrt{1-x}}$	91

input
```
int(arcsech(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsech(x^(1/2))/x^3+1/144*(-(x^(1/2)-1)/x^(1/2))^(1/2)/x^(5/2)*((x^(
1/2)+1)/x^(1/2))^(1/2)*(15*arctanh(1/(1-x)^(1/2))*x^3+15*(1-x)^(1/2)*x^2+1
0*(1-x)^(1/2)*x+8*(1-x)^(1/2))/(1-x)^(1/2)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx = \frac{(15\,x^2+10\,x+8)\sqrt{x}\sqrt{-\frac{x-1}{x}}+3\,(5\,x^3-16)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)}{144\,x^3}$$

input
```
integrate(arcsech(x^(1/2))/x^4,x, algorithm="fricas")
```

output
```
1/144*((15*x^2 + 10*x + 8)*sqrt(x)*sqrt(-(x - 1)/x) + 3*(5*x^3 - 16)*log((
x*sqrt(-(x - 1)/x) + sqrt(x))/x))/x^3
```

3.27.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx = \int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x^4}\,dx$$

input
```
integrate(asech(x**(1/2))/x**4,x)
```

output
```
Integral(asech(sqrt(x))/x**4, x)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx = -\frac{15\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}-40\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}+33\,\sqrt{x}\sqrt{\frac{1}{x}-1}}{144\left(x^3\left(\frac{1}{x}-1\right)^3-3\,x^2\left(\frac{1}{x}-1\right)^2+3\,x\left(\frac{1}{x}-1\right)-1\right)} - \frac{\operatorname{arsech}\left(\sqrt{x}\right)}{3\,x^3} + \frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}+1\right) - \frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}-1\right)$$

input
```
integrate(arcsech(x^(1/2))/x^4,x, algorithm="maxima")
```

output
```
-1/144*(15*x^(5/2)*(1/x - 1)^(5/2) - 40*x^(3/2)*(1/x - 1)^(3/2) + 33*sqrt(
x)*sqrt(1/x - 1))/(x^3*(1/x - 1)^3 - 3*x^2*(1/x - 1)^2 + 3*x*(1/x - 1) - 1
) - 1/3*arcsech(sqrt(x))/x^3 + 5/96*log(sqrt(x)*sqrt(1/x - 1) + 1) - 5/96*
log(sqrt(x)*sqrt(1/x - 1) - 1)
```

3.27.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx = \int \frac{\operatorname{arsech}(\sqrt{x})}{x^4}\,dx$$

input
```
integrate(arcsech(x^(1/2))/x^4,x, algorithm="giac")
```

output
```
integrate(arcsech(sqrt(x))/x^4, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x^4}\,dx$$

input
```
int(acosh(1/x^(1/2))/x^4,x)
```

output
```
int(acosh(1/x^(1/2))/x^4, x)
```

3.28 $\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) \, dx$

3.28.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) \, dx = -\sqrt{-1+x}\sqrt{1+x} + x\operatorname{arccosh}(x)$$

output
```
x*arccosh(x)-(-1+x)^(1/2)*(1+x)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) \, dx = -\sqrt{\frac{-1+x}{1+x}}(1+x) + x\operatorname{sech}^{-1}\left(\frac{1}{x}\right)$$

input
```
Integrate[ArcSech[x^(-1)],x]
```

output
```
-(Sqrt[(-1 + x)/(1 + x)]*(1 + x)) + x*ArcSech[x^(-1)]
```

3.28.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6881, 6294, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx$$

$$\Big\downarrow \text{ 6881}$$

$$\int \operatorname{arccosh}(x)dx$$

$$\Big\downarrow \text{ 6294}$$

$$x\operatorname{arccosh}(x)-\int \frac{x}{\sqrt{x-1}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 83}$$

$$x\operatorname{arccosh}(x)-\sqrt{x-1}\sqrt{x+1}$$

input
```
Int[ArcSech[x^(-1)],x]
```

output
```
-(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]
```

3.28.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6881
```
Int[ArcSech[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCosh[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.28.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
parts	$x \operatorname{arcsech}\left(\frac{1}{x}\right)-\sqrt{x-1}\,\sqrt{1+x}$	20
derivativedivides	$x \operatorname{arcsech}\left(\frac{1}{x}\right)-\sqrt{-\left(-1+\frac{1}{x}\right) x}\,\sqrt{\left(1+\frac{1}{x}\right) x}$	29
default	$x \operatorname{arcsech}\left(\frac{1}{x}\right)-\sqrt{-\left(-1+\frac{1}{x}\right) x}\,\sqrt{\left(1+\frac{1}{x}\right) x}$	29

input
```
int(arcsech(1/x),x,method=_RETURNVERBOSE)
```

output
```
x*arcsech(1/x)-(x-1)^(1/2)*(1+x)^(1/2)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right)\,dx = x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

input
```
integrate(arcsech(1/x),x, algorithm="fricas")
```

output
```
x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

3.28.6 Sympy [F]

$$\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx = \int \operatorname{asech}\left(\frac{1}{x}\right) dx$$

input
```
integrate(asech(1/x),x)
```

output
```
Integral(asech(1/x), x)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx = x \operatorname{arsech}\left(\frac{1}{x}\right) - \sqrt{x^2 - 1}$$

input
```
integrate(arcsech(1/x),x, algorithm="maxima")
```

output
```
x*arcsech(1/x) - sqrt(x^2 - 1)
```

3.28.8 Giac [F]

$$\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx = \int \operatorname{arsech}\left(\frac{1}{x}\right) dx$$

input
```
integrate(arcsech(1/x),x, algorithm="giac")
```

output
```
integrate(arcsech(1/x), x)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx = x \operatorname{acosh}(x) - \sqrt{x-1}\sqrt{x+1}$$

input
```
int(acosh(x),x)
```

output
```
x*acosh(x) - (x - 1)^(1/2)*(x + 1)^(1/2)
```

3.29 $\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx = \frac{\operatorname{sech}^{-1}(ax^n)^2}{2n} - \frac{\operatorname{sech}^{-1}(ax^n)\log\left(1+e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{n} - \frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{2n}$$

output
```
1/2*arcsech(a*x^n)^2/n-arcsech(a*x^n)*ln(1+(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*
(1/a/(x^n)+1)^(1/2))^2)/n-1/2*polylog(2,-(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(1
/a/(x^n)+1)^(1/2))^2)/n
```

3.29.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 219 vs. 2(61) = 122.

Time = 1.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.59

$$\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx = \operatorname{sech}^{-1}(ax^n)\log(x) + \frac{\sqrt{\frac{1-ax^n}{1+ax^n}}\left(4\sqrt{-1+a^2x^{2n}}\arctan\left(\sqrt{-1+a^2x^{2n}}\right)\left(2n\log(x)-\log\left(a^2x^{2n}\right)\right)+\sqrt{1-a^2x^{2n}}\left(\log^2\left(a^2x^{2n}\right)-\right.\right.}{8\,(n-}$$

input
```
Integrate[ArcSech[a*x^n]/x,x]
```

output
```
ArcSech[a*x^n]*Log[x] + (Sqrt[(1 - a*x^n)/(1 + a*x^n)]*(4*Sqrt[-1 + a^2*x^
(2*n)]*ArcTan[Sqrt[-1 + a^2*x^(2*n)]]*(2*n*Log[x] - Log[a^2*x^(2*n)]) + Sq
rt[1 - a^2*x^(2*n)]*(Log[a^2*x^(2*n)]^2 - 4*Log[a^2*x^(2*n)]*Log[(1 + Sqrt
[1 - a^2*x^(2*n)])/2] + 2*Log[(1 + Sqrt[1 - a^2*x^(2*n)])/2]^2 - 4*PolyLog
[2, 1/2 - Sqrt[1 - a^2*x^(2*n)]/2])))/(8*(n - a*n*x^n))
```

3.29.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {7282, 6835, 6297, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(ax^n)}{x} \, dx$$

$$\Big\downarrow \text{ 7282}$$

$$\frac{\int x^{-n}\text{sech}^{-1}(ax^n) \, dx^n}{n}$$

$$\Big\downarrow \text{ 6835}$$

$$-\frac{\int x^{-n}\text{arccosh}\left(\frac{x^{-n}}{a}\right) \, dx^{-n}}{n}$$

$$\Big\downarrow \text{ 6297}$$

$$-\frac{\int ax^n\sqrt{\frac{\frac{x^{-n}}{a}-1}{\frac{x^{-n}}{a}+1}}\left(\frac{x^{-n}}{a}+1\right)\text{arccosh}\left(\frac{x^{-n}}{a}\right) \, d\text{arccosh}\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int -i\text{arccosh}\left(\frac{x^{-n}}{a}\right)\tan\left(i\text{arccosh}\left(\frac{x^{-n}}{a}\right)\right) \, d\text{arccosh}\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int \text{arccosh}\left(\frac{x^{-n}}{a}\right)\tan\left(i\text{arccosh}\left(\frac{x^{-n}}{a}\right)\right)d\text{arccosh}\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \text{4201}$$

$$\frac{i\left(2i\int \frac{e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}\text{arccosh}\left(\frac{x^{-n}}{a}\right)}{1+e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}}d\text{arccosh}\left(\frac{x^{-n}}{a}\right)-\frac{1}{2}ix^{2n}\right)}{n}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{x^{-n}}{a}\right)\log\left(e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}\right)d\text{arccosh}\left(\frac{x^{-n}}{a}\right)\right)-\frac{1}{2}ix^{2n}\right)}{n}$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{x^{-n}}{a}\right)\log\left(e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}+1\right)-\frac{1}{4}\int e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}\log\left(1+e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}\right)de^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}\right)-\right.}{n}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}\right)+\frac{1}{2}\text{arccosh}\left(\frac{x^{-n}}{a}\right)\log\left(e^{2\text{arccosh}\left(\frac{x^{-n}}{a}\right)}+1\right)\right)-\frac{1}{2}ix^{2n}\right)}{n}$$

input
```
Int[ArcSech[a*x^n]/x,x]
```

output
```
(I*((-1/2*I)*x^(2*n) + (2*I)*((ArcCosh[1/(a*x^n)]*Log[1 + E^(2*ArcCosh[1/(
a*x^n)])])/2 + PolyLog[2, -E^(2*ArcCosh[1/(a*x^n)])]/4)))/n
```

3.29. $\int \frac{\text{sech}^{-1}(ax^n)}{x}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6835
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

3.29. $\int \frac{\operatorname{sech}^{-1}(ax^n)}{x} dx$

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.29.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsech}(a x^n)^2}{2}-\operatorname{arcsech}(a x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\,\sqrt{\frac{x^{-n}}{a}+1}\right)^2\right)-\frac{\operatorname{polylog}\left(2,-\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\,\sqrt{\frac{x^{-n}}{a}+1}\right)^2\right)}{2}}{n}$
default	$\frac{\frac{\operatorname{arcsech}(a x^n)^2}{2}-\operatorname{arcsech}(a x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\,\sqrt{\frac{x^{-n}}{a}+1}\right)^2\right)-\frac{\operatorname{polylog}\left(2,-\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\,\sqrt{\frac{x^{-n}}{a}+1}\right)^2\right)}{2}}{n}$

input
```
int(arcsech(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/2*arcsech(a*x^n)^2-arcsech(a*x^n)*ln(1+(1/a/(x^n)+(1/a/(x^n)-1)^(1/
2)*(1/a/(x^n)+1)^(1/2))^2)-1/2*polylog(2,-(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(
1/a/(x^n)+1)^(1/2))^2))
```

3.29.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsech(a*x^n)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.29.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{asech}(ax^n)}{x}\,dx$$

input
```
integrate(asech(a*x**n)/x,x)
```

output
```
Integral(asech(a*x**n)/x, x)
```

3.29.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arsech}(ax^n)}{x}\,dx$$

input
```
integrate(arcsech(a*x^n)/x,x, algorithm="maxima")
```

output
```
a^2*n*integrate(x^(2*n)*log(x)/(a^2*x*x^(2*n) + (a^2*x*x^(2*n) - x)*sqrt(a
*x^n + 1)*sqrt(-a*x^n + 1) - x), x) + n*integrate(1/2*log(x)/(a*x*x^n + x)
, x) - n*integrate(1/2*log(x)/(a*x*x^n - x), x) + log(sqrt(a*x^n + 1)*sqrt
(-a*x^n + 1) + 1)*log(x) - log(a)*log(x) - log(x)*log(x^n)
```

3.29.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arsech}(ax^n)}{x}\,dx$$

input
```
integrate(arcsech(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x^n)/x, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x^n}\right)}{x}\,dx$$

input
```
int(acosh(1/(a*x^n))/x,x)
```

output
```
int(acosh(1/(a*x^n))/x, x)
```

3.30 $\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx$

3.30.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx = \frac{1}{10}\operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5}\operatorname{sech}^{-1}(ax^5)\log\left(1+e^{2\operatorname{sech}^{-1}(ax^5)}\right) - \frac{1}{10}\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(ax^5)}\right)$$

output
```
1/10*arcsech(a*x^5)^2-1/5*arcsech(a*x^5)*ln(1+(1/a/x^5+(1/a/x^5-1)^(1/2)*(
1/a/x^5+1)^(1/2))^2)-1/10*polylog(2,-(1/a/x^5+(1/a/x^5-1)^(1/2)*(1/a/x^5+1
)^(1/2))^2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx = \frac{1}{10}\left(-\operatorname{sech}^{-1}(ax^5)\left(\operatorname{sech}^{-1}(ax^5)+2\log\left(1+e^{-2\operatorname{sech}^{-1}(ax^5)}\right)\right) + \operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(ax^5)}\right)\right)$$

input
```
Integrate[ArcSech[a*x^5]/x,x]
```

output
```
(-(ArcSech[a*x^5]*(ArcSech[a*x^5] + 2*Log[1 + E^(-2*ArcSech[a*x^5])])) + P
olyLog[2, -E^(-2*ArcSech[a*x^5])])/10
```

3.30.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {7282, 6835, 6297, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{-1}\left(ax^5\right)}{x}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{1}{5}\int \frac{\operatorname{sech}^{-1}\left(ax^5\right)}{x^5}dx^5$$

$$\Big\downarrow \text{6835}$$

$$-\frac{1}{5}\int \frac{\operatorname{arccosh}\left(\frac{1}{ax^5}\right)}{x^5}d\frac{1}{x^5}$$

$$\Big\downarrow \text{6297}$$

$$-\frac{1}{5}\int a\sqrt{\frac{\frac{1}{ax^5}-1}{1+\frac{1}{x^5a}}}\left(1+\frac{1}{x^5a}\right)x^5\operatorname{arccosh}\left(\frac{1}{ax^5}\right)d\operatorname{arccosh}\left(\frac{1}{ax^5}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\int -i\operatorname{arccosh}\left(\frac{1}{ax^5}\right)\tan\left(i\operatorname{arccosh}\left(\frac{1}{ax^5}\right)\right)d\operatorname{arccosh}\left(\frac{1}{ax^5}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{5}i\int \operatorname{arccosh}\left(\frac{1}{ax^5}\right)\tan\left(i\operatorname{arccosh}\left(\frac{1}{ax^5}\right)\right)d\operatorname{arccosh}\left(\frac{1}{ax^5}\right)$$

$$\Big\downarrow \text{4201}$$

$$\frac{1}{5}i\left(2i\int \frac{e^{2\operatorname{arccosh}\left(\frac{1}{ax^5}\right)}\operatorname{arccosh}\left(\frac{1}{ax^5}\right)}{1+e^{2\operatorname{arccosh}\left(\frac{1}{ax^5}\right)}}d\operatorname{arccosh}\left(\frac{1}{ax^5}\right)-\frac{ix^{10}}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{5}i\left(2i\left(\frac{1}{2}\operatorname{arccosh}\left(\frac{1}{ax^5}\right)\log\left(e^{2\operatorname{arccosh}\left(\frac{1}{ax^5}\right)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\operatorname{arccosh}\left(\frac{1}{ax^5}\right)}\right)d\operatorname{arccosh}\left(\frac{1}{ax^5}\right)\right)-\frac{ix^{10}}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{5}i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{1}{ax^5}\right)\log\left(e^{2\text{arccosh}\left(\frac{1}{ax^5}\right)}+1\right)-\frac{1}{4}\int e^{2\text{arccosh}\left(\frac{1}{ax^5}\right)}\log\left(1+e^{2\text{arccosh}\left(\frac{1}{ax^5}\right)}\right)de^{2\text{arccosh}\left(\frac{1}{ax^5}\right)}\right)-$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{5}i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}\left(\frac{1}{ax^5}\right)}\right)+\frac{1}{2}\text{arccosh}\left(\frac{1}{ax^5}\right)\log\left(e^{2\text{arccosh}\left(\frac{1}{ax^5}\right)}+1\right)\right)-\frac{ix^{10}}{2}\right)$$

input
```
Int[ArcSech[a*x^5]/x,x]
```

output
```
(I/5)*((-1/2*I)*x^10 + (2*I)*((ArcCosh[1/(a*x^5)]*Log[1 + E^(2*ArcCosh[1/(
a*x^5)])])/2 + PolyLog[2, -E^(2*ArcCosh[1/(a*x^5)])]/4))
```

3.30.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6835
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.30.4 Maple [F]

$$\int \frac{\operatorname{arcsech}\left(a\,x^{5}\right)}{x}dx$$

input
```
int(arcsech(a*x^5)/x,x)
```

output
```
int(arcsech(a*x^5)/x,x)
```

3.30.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arsech}\left(ax^5\right)}{x}\,dx$$

input
```
integrate(arcsech(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arcsech(a*x^5)/x, x)
```

3.30.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{asech}(ax^5)}{x}\,dx$$

input
```
integrate(asech(a*x**5)/x,x)
```

output
```
Integral(asech(a*x**5)/x, x)
```

3.30.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arsech}(ax^5)}{x}\,dx$$

input
```
integrate(arcsech(a*x^5)/x,x, algorithm="maxima")
```

output
```
integrate(arcsech(a*x^5)/x, x)
```

3.30.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arsech}(ax^5)}{x}\,dx$$

input
```
integrate(arcsech(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arcsech(a*x^5)/x, x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x^5}\right)}{x}\,dx$$

input
```
int(acosh(1/(a*x^5))/x,x)
```

output
```
int(acosh(1/(a*x^5))/x, x)
```

3.31 $\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right)\,dx$

3.31.1 Optimal result

Integrand size = 10, antiderivative size = 77

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right)\,dx=\frac{\operatorname{sech}^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\operatorname{sech}^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\operatorname{sech}^{-1}\left(ce^{a+bx}\right)}\right)}{b}-\frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

output
```
1/2*arcsech(c*exp(b*x+a))^2/b-arcsech(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+(
1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b-1/2*polylog(2,-(1/c
/exp(b*x+a)+(1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b
```

3.31.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(77) = 154.

Time = 1.86 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.23

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right)\,dx=x\operatorname{sech}^{-1}\left(ce^{a+bx}\right)-\frac{\sqrt{\frac{1-ce^{a+bx}}{1+ce^{a+bx}}}\sqrt{1+ce^{a+bx}}\left(\operatorname{arctanh}\left(\sqrt{1-c^2e^{2(a+bx)}}\right)\left(8bx-4\log\left(c^2e^{2(a+bx)}\right)\right)-\log^2\left(c^2e^{2(a+bx)}\right)+4\log\right.}{}$$

input
```
Integrate[ArcSech[c*E^(a + b*x)],x]
```

output
```
x*ArcSech[c*E^(a + b*x)] - (Sqrt[(1 - c*E^(a + b*x))/(1 + c*E^(a + b*x))]*
Sqrt[1 + c*E^(a + b*x)]*(ArcTanh[Sqrt[1 - c^2*E^(2*(a + b*x))]]*(8*b*x - 4
*Log[c^2*E^(2*(a + b*x))]) - Log[c^2*E^(2*(a + b*x))]^2 + 4*Log[c^2*E^(2*(
a + b*x))]*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2] - 2*Log[(1 + Sqrt[1
- c^2*E^(2*(a + b*x))])/2]^2 + 4*PolyLog[2, (1 - Sqrt[1 - c^2*E^(2*(a + b*
x))])/2]))/(8*b*Sqrt[1 - c*E^(a + b*x)])
```

3.31.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2720, 6835, 6297, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^{-1}\left(ce^{a+bx}\right)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int e^{-a-bx}\text{sech}^{-1}\left(ce^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \ 6835$$

$$-\frac{\int e^{-a-bx}\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)de^{-a-bx}}{b}$$

$$\Big\downarrow \ 6297$$

$$-\frac{\int ce^{a+bx}\sqrt{\frac{\frac{e^{-a-bx}}{c}-1}{1+\frac{e^{-a-bx}}{c}}}\left(1+\frac{e^{-a-bx}}{c}\right)\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)d\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int -i\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)\tan\left(i\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)\right)d\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{i\int \text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)\tan\left(i\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)\right)d\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$$\Big\downarrow \text{4201}$$

$$\frac{i\left(2i\int \frac{e^{a+bx+2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}}{1+e^{2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}}d\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)\log\left(e^{2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}\right)d\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)\right)-\frac{1}{2}ie^{2\cdots}\right.}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)\log\left(e^{2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}+1\right)-\frac{1}{4}\int e^{2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}\log\left(1+e^{2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}\right)de^{2\text{arccosh}\cdots}\right.\right.}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,-e^{2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}\right)+\frac{1}{2}\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)\log\left(e^{2\text{arccosh}\left(\frac{e^{-a-bx}}{c}\right)}+1\right)\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

input
```
Int[ArcSech[c*E^(a + b*x)],x]
```

output
```
(I*((-1/2*I)*E^(2*a + 2*b*x) + (2*I)*((ArcCosh[E^(-a - b*x)/c]*Log[1 + E^(
2*ArcCosh[E^(-a - b*x)/c])])/2 + PolyLog[2, -E^(2*ArcCosh[E^(-a - b*x)/c])
]/4)))/b
```

3.31.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6835
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

3.31. $\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right)\, dx$

3.31.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsech}\left(e^{bx+a}c\right)^{2}}{2}-\operatorname{arcsech}\left(e^{bx+a}c\right)\ln\left(1+\left(\frac{e^{-bx-a}}{c}+\sqrt{\frac{e^{-bx-a}}{c}-1}\,\sqrt{\frac{e^{-bx-a}}{c}+1}\right)^{2}\right)-\frac{\operatorname{polylog}\left(2,-\left(\frac{e^{-bx-a}}{c}+\sqrt{\frac{e^{-bx-}}{c}}\right.\right.}{2}}{b}$
default	$\frac{\frac{\operatorname{arcsech}\left(e^{bx+a}c\right)^{2}}{2}-\operatorname{arcsech}\left(e^{bx+a}c\right)\ln\left(1+\left(\frac{e^{-bx-a}}{c}+\sqrt{\frac{e^{-bx-a}}{c}-1}\,\sqrt{\frac{e^{-bx-a}}{c}+1}\right)^{2}\right)-\frac{\operatorname{polylog}\left(2,-\left(\frac{e^{-bx-a}}{c}+\sqrt{\frac{e^{-bx-}}{c}}\right.\right.}{2}}{b}$

input
```
int(arcsech(exp(b*x+a)*c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*arcsech(exp(b*x+a)*c)^2-arcsech(exp(b*x+a)*c)*ln(1+(1/c/exp(b*x+a
)+(1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)-1/2*polylog(2,-(1/
c/exp(b*x+a)+(1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2))
```

3.31.5 Fricas [F(-2)]

Exception generated.

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsech(c*exp(b*x+a)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.31.6 Sympy [F]

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right)\,dx = \int \operatorname{asech}\left(ce^{a+bx}\right)dx$$

input
```
integrate(asech(c*exp(b*x+a)),x)
```

output
```
Integral(asech(c*exp(a + b*x)), x)
```

3.31.7 Maxima [F]

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{arsech}\left(ce^{(bx+a)}\right) dx$$

input
```
integrate(arcsech(c*exp(b*x+a)),x, algorithm="maxima")
```

output
```
b*c^2*integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2
*a) - 1)*e^(1/2*log(c*e^(b*x + a) + 1) + 1/2*log(-c*e^(b*x + a) + 1)) - 1)
, x) - 1/2*b*x^2 - (a + log(c))*x + x*log(sqrt(c*e^(b*x + a) + 1)*sqrt(-c*
e^(b*x + a) + 1) + 1) - 1/2*(b*x*log(c*e^(b*x + a) + 1) + dilog(-c*e^(b*x
+ a)))/b - 1/2*(b*x*log(-c*e^(b*x + a) + 1) + dilog(c*e^(b*x + a)))/b
```

3.31.8 Giac [F]

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{arsech}\left(ce^{(bx+a)}\right) dx$$

input
```
integrate(arcsech(c*exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arcsech(c*e^(b*x + a)), x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{acosh}\left(\frac{e^{-a-bx}}{c}\right) dx$$

input
```
int(acosh(exp(- a - b*x)/c),x)
```

output
```
int(acosh(exp(- a - b*x)/c), x)
```

3.32 $\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx$

3.32.1 Optimal result

Integrand size = 10, antiderivative size = 64

$$\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx=-\frac{2e^{\operatorname{sech}^{-1}(ax)}x}{15a^4}+\frac{x^2}{15a^3}-\frac{e^{\operatorname{sech}^{-1}(ax)}x^3}{15a^2}+\frac{x^4}{20a}+\frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5$$

output
```
-2/15*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a^4+1/15*x^2/a^3-1/15*(1/a
/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3/a^2+1/20*x^4/a+1/5*(1/a/x+(1/a/x-1
)^(1/2)*(1+1/a/x)^(1/2))*x^5
```

3.32.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx=\frac{15a^4x^4+4\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2(-2+2ax-3a^2x^2+3a^3x^3)}{60a^5}$$

input
```
Integrate[E^ArcSech[a*x]*x^4,x]
```

output
```
(15*a^4*x^4 + 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-2 + 2*a*x - 3*a^2*
x^2 + 3*a^3*x^3))/(60*a^5)
```

3.32.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6889, 15, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{\text{sech}^{-1}(ax)} \, dx$$

$$\Big\downarrow \ 6889$$

$$\frac{\int x^3 dx}{5a}+\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x^3}{\sqrt{1-ax}\sqrt{ax+1}}dx}{5a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}(ax)}$$

$$\Big\downarrow \ 15$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x^3}{\sqrt{1-ax}\sqrt{ax+1}}dx}{5a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}(ax)}+\frac{x^4}{20a}$$

$$\Big\downarrow \ 111$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{\int -\frac{2x}{\sqrt{1-ax}\sqrt{ax+1}}dx}{3a^2}-\frac{x^2\sqrt{1-ax}\sqrt{ax+1}}{3a^2}\right)}{5a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}(ax)}+\frac{x^4}{20a}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{2\int \frac{x}{\sqrt{1-ax}\sqrt{ax+1}}dx}{3a^2}-\frac{x^2\sqrt{1-ax}\sqrt{ax+1}}{3a^2}\right)}{5a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}(ax)}+\frac{x^4}{20a}$$

$$\Big\downarrow \ 83$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{2\sqrt{1-ax}\sqrt{ax+1}}{3a^4}-\frac{x^2\sqrt{1-ax}\sqrt{ax+1}}{3a^2}\right)}{5a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}(ax)}+\frac{x^4}{20a}$$

input
```
Int[E^ArcSech[a*x]*x^4,x]
```

output
```
x^4/(20*a) + (E^ArcSech[a*x]*x^5)/5 + (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*
((-2*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(3*a^4) - (x^2*Sqrt[1 - a*x]*Sqrt[1 + a*
x])/(3*a^2)))/(5*a)
```

3.32.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.32.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\, x\sqrt{-\frac{ax-1}{ax}}\, (a^2x^2-1)(3a^2x^2+2)}{15a^4}+\frac{x^4}{4a}$	64

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)*(3*a^2*x^2+2)/
a^4+1/4*x^4/a
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx=\frac{15\,a^3x^4+4\,(3\,a^4x^5-a^2x^3-2\,x)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{60\,a^4}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="fricas
")
```

output
```
1/60*(15*a^3*x^4 + 4*(3*a^4*x^5 - a^2*x^3 - 2*x)*sqrt((a*x + 1)/(a*x))*sqr
t(-(a*x - 1)/(a*x)))/a^4
```

3.32.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx=\frac{\int x^3\,dx+\int ax^4\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**4,x)
```

output
```
(Integral(x**3, x) + Integral(a*x**4*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)),
 x))/a
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx=\frac{x^4}{4\,a}+\frac{(3\,a^4x^4-a^2x^2-2)\sqrt{ax+1}\sqrt{-ax+1}}{15\,a^5}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="maxima
")
```

output
```
1/4*x^4/a + 1/15*(3*a^4*x^4 - a^2*x^2 - 2)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^
5
```

3.32.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,3,2,2,0,0]%%%}+%%%{1,[0,2,0,1,1,1]%%%} / %%%{1,[0,0,2
,3,0,0]%%
```

3.32.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx = \frac{x^4}{4\,a}-\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,x\,\sqrt{\frac{1}{a\,x}+1}}{15\,a^4}-\frac{x^5\,\sqrt{\frac{1}{a\,x}+1}}{5}+\frac{x^3\,\sqrt{\frac{1}{a\,x}+1}}{15\,a^2}\right)$$

input
```
int(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
x^4/(4*a) - (1/(a*x) - 1)^(1/2)*((2*x*(1/(a*x) + 1)^(1/2))/(15*a^4) - (x^5
*(1/(a*x) + 1)^(1/2))/5 + (x^3*(1/(a*x) + 1)^(1/2))/(15*a^2))
```

3.33 $\int e^{\text{sech}^{-1}(ax)}x^3\,dx$

3.33.1 Optimal result

Integrand size = 10, antiderivative size = 84

$$\int e^{\text{sech}^{-1}(ax)}x^3\,dx=\frac{x^3}{12a}+\frac{1}{4}e^{\text{sech}^{-1}(ax)}x^4-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{1+ax}}}+\frac{\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\arcsin(ax)}{8a^4}$$

output
```
1/12*x^3/a+1/4*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4-1/8*x*(-a*x+1)^
(1/2)/a^3/(1/(a*x+1))^(1/2)+1/8*arcsin(a*x)*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2
)/a^4
```

3.33.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int e^{\text{sech}^{-1}(ax)}x^3\,dx$$
$$=\frac{8a^3x^3-3a\sqrt{\frac{1-ax}{1+ax}}(x+ax^2-2a^2x^3-2a^3x^4)+3i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{24a^4}$$

input
```
Integrate[E^ArcSech[a*x]*x^3,x]
```

output
```
(8*a^3*x^3 - 3*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^2*x^3 - 2*a^3*
x^4) + (3*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(24*
a^4)
```

3.33.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6889, 15, 101, 25, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\text{sech}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{\int x^2 dx}{4a} + \frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x^2}{\sqrt{1-ax}\sqrt{ax+1}}dx}{4a} + \frac{1}{4}x^4 e^{\text{sech}^{-1}(ax)}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x^2}{\sqrt{1-ax}\sqrt{ax+1}}dx}{4a} + \frac{1}{4}x^4 e^{\text{sech}^{-1}(ax)} + \frac{x^3}{12a}$$

$$\Big\downarrow \text{101}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{\int -\frac{1}{\sqrt{1-ax}\sqrt{ax+1}}dx}{2a^2} - \frac{x\sqrt{1-ax}\sqrt{ax+1}}{2a^2}\right)}{4a} + \frac{1}{4}x^4 e^{\text{sech}^{-1}(ax)} + \frac{x^3}{12a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{\int \frac{1}{\sqrt{1-ax}\sqrt{ax+1}}dx}{2a^2} - \frac{x\sqrt{1-ax}\sqrt{ax+1}}{2a^2}\right)}{4a} + \frac{1}{4}x^4 e^{\text{sech}^{-1}(ax)} + \frac{x^3}{12a}$$

$$\Big\downarrow \text{39}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2} - \frac{x\sqrt{1-ax}\sqrt{ax+1}}{2a^2}\right)}{4a} + \frac{1}{4}x^4 e^{\text{sech}^{-1}(ax)} + \frac{x^3}{12a}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-ax}\sqrt{ax+1}}{2a^2}\right)}{4a} + \frac{1}{4}x^4 e^{\text{sech}^{-1}(ax)} + \frac{x^3}{12a}$$

input
```
Int[E^ArcSech[a*x]*x^3,x]
```

output
```
x^3/(12*a) + (E^ArcSech[a*x]*x^4)/4 + (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*
(-1/2*(x*Sqrt[1 - a*x]*Sqrt[1 + a*x])/a^2 + ArcSin[a*x]/(2*a^3)))/(4*a)
```

3.33.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.33.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\, x \sqrt{-\frac{ax-1}{ax}}\left(2\,\mathrm{csgn}(a) a^3 x^3 \sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}\, x\, \mathrm{csgn}(a) a+\arctan\left(\frac{\mathrm{csgn}(a) a x}{\sqrt{-a^2x^2+1}}\right)\right)\mathrm{csgn}(a)}{8\sqrt{-a^2x^2+1}\, a^3}+\frac{x^3}{3a}$	118

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(2*csgn(a)*a^3*x^3*(-a^2*x^
2+1)^(1/2)-(-a^2*x^2+1)^(1/2)*x*csgn(a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^
(1/2)))*csgn(a)/(-a^2*x^2+1)^(1/2)/a^3+1/3*x^3/a
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13

$$\int e^{\mathrm{sech}^{-1}(ax)} x^3\,dx = \frac{8\,a^3x^3+3\left(2\,a^4x^4-a^2x^2\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{24\,a^4}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="fricas
")
```

output
```
1/24*(8*a^3*x^3 + 3*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
 - 1)/(a*x)) - 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^4
```

3.33.6 Sympy [F]

$$\int e^{\mathrm{sech}^{-1}(ax)} x^3\,dx = \frac{\int x^2\,dx+\int ax^3\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**3,x)
```

output
```
(Integral(x**2, x) + Integral(a*x**3*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)),
 x))/a
```

3.33.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}x^3\,dx = \int x^3\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="maxima
")
```

output
```
1/3*x^3/a + integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2, x)/a
```

3.33.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,2,2,0,0]%%%}+%%%{1,[0,1,0,1,1,1]%%%} / %%%{1,[0,0,2
,3,0,0]%%
```

3.33.9 Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 521, normalized size of antiderivative = 6.20

$$\int e^{\operatorname{sech}^{-1}(ax)}x^3\,dx = \frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{8\,a^4} - \frac{\frac{1\mathrm{i}}{1024\,a^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{128\,a^4\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4\,11\mathrm{i}}{512\,a^4\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6\,7\mathrm{i}}{256\,a^4\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8\,239\mathrm{i}}{1024\,a^4\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}\,1\mathrm{i}}{256\,a^4\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}} - \frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{8\,a^4} + \frac{x^3}{3\,a} - \frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{256\,a^4\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2} - \frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4\,1\mathrm{i}}{1024\,a^4\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}$$

input
```
int(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
(log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i)/(8*
a^4) - (1i/(1024*a^4) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(128*a^4*((1/(a*
x) + 1)^(1/2) - 1)^2) + (((1/(a*x) - 1)^(1/2) - 1i)^4*11i)/(512*a^4*((1/(a
*x) + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)^(1/2) - 1i)^6*7i)/(256*a^4*((1/(a
*x) + 1)^(1/2) - 1)^6) - (((1/(a*x) - 1)^(1/2) - 1i)^8*239i)/(1024*a^4*((1
/(a*x) + 1)^(1/2) - 1)^8) + (((1/(a*x) - 1)^(1/2) - 1i)^10*1i)/(256*a^4*((
1/(a*x) + 1)^(1/2) - 1)^10))/(((1/(a*x) - 1)^(1/2) - 1i)^4/((1/(a*x) + 1)^
(1/2) - 1)^4 + (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^
6 + (6*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 + (4*((1/
(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + ((1/(a*x) - 1)^(
1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12) - (log(((1/(a*x) - 1)^(1/2) -
1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(8*a^4) + x^3/(3*a) - (((1/(a*x) - 1)^(
1/2) - 1i)^2*1i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(
1/2) - 1i)^4*1i)/(1024*a^4*((1/(a*x) + 1)^(1/2) - 1)^4)
```

3.34 $\int e^{\operatorname{sech}^{-1}(ax)} x^2 \, dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int e^{\operatorname{sech}^{-1}(ax)} x^2 \, dx = -\frac{e^{\operatorname{sech}^{-1}(ax)} x}{3a^2} + \frac{x^2}{6a} + \frac{1}{3} e^{\operatorname{sech}^{-1}(ax)} x^3$$

output
```
-1/3*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a^2+1/6*x^2/a+1/3*(1/a/x+(1
/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3
```

3.34.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int e^{\operatorname{sech}^{-1}(ax)} x^2 \, dx = \frac{3a^2x^2 + 2(-1 + ax)\sqrt{\frac{1-ax}{1+ax}}(1 + ax)^2}{6a^3}$$

input
```
Integrate[E^ArcSech[a*x]*x^2,x]
```

output
```
(3*a^2*x^2 + 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(6*a^3)
```

3.34.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6889, 15, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\text{sech}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{\int xdx}{3a}+\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x}{\sqrt{1-ax}\sqrt{ax+1}}dx}{3a}+\frac{1}{3}x^3 e^{\text{sech}^{-1}(ax)}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x}{\sqrt{1-ax}\sqrt{ax+1}}dx}{3a}+\frac{1}{3}x^3 e^{\text{sech}^{-1}(ax)}+\frac{x^2}{6a}$$

$$\Big\downarrow \text{83}$$

$$-\frac{\sqrt{1-ax}}{3a^3\sqrt{\frac{1}{ax+1}}}+\frac{1}{3}x^3 e^{\text{sech}^{-1}(ax)}+\frac{x^2}{6a}$$

input
```
Int[E^ArcSech[a*x]*x^2,x]
```

output
```
x^2/(6*a) + (E^ArcSech[a*x]*x^3)/3 - Sqrt[1 - a*x]/(3*a^3*Sqrt[(1 + a*x)^(
-1)])
```

3.34.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.34.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\, x \sqrt{-\frac{ax-1}{ax}}\, \left(a^{2} x^{2}-1\right)}{3 a^{2}}+\frac{x^{2}}{2 a}$	54

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)/a^2+1/2*x^2/a
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int e^{\operatorname{sech}^{-1}(ax)} x^2\, dx = \frac{3\, a x^2 + 2\, \left(a^2 x^3 - x\right) \sqrt{\frac{ax+1}{ax}} \sqrt{-\frac{ax-1}{ax}}}{6\, a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="fricas
")
```

output
```
1/6*(3*a*x^2 + 2*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)
))/a^2
```

3.34.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}x^2\,dx = \frac{\int x\,dx + \int ax^2\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**2,x)
```

output
```
(Integral(x, x) + Integral(a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x)
)/a
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int e^{\operatorname{sech}^{-1}(ax)}x^2\,dx = \frac{x^2}{2\,a} + \frac{(a^2x^2-1)\sqrt{ax+1}\sqrt{-ax+1}}{3\,a^3}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="maxima
")
```

output
```
1/2*x^2/a + 1/3*(a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^3
```

3.34.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,2,0,0]%%%}+%%%{1,[0,0,0,1,1,1]%%%} / %%%{1,[0,0,2
,3,0,0]%%
```

3.34.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int e^{\operatorname{sech}^{-1}(ax)}x^2\,dx=\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x^3\,\sqrt{\frac{1}{a\,x}+1}}{3}-\frac{x\,\sqrt{\frac{1}{a\,x}+1}}{3\,a^2}\right)+\frac{x^2}{2\,a}$$

input
```
int(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
(1/(a*x) - 1)^(1/2)*((x^3*(1/(a*x) + 1)^(1/2))/3 - (x*(1/(a*x) + 1)^(1/2))
/(3*a^2)) + x^2/(2*a)
```

3.35 $\int e^{\text{sech}^{-1}(ax)}x\,dx$

3.35.1 Optimal result

Integrand size = 8, antiderivative size = 53

$$\int e^{\text{sech}^{-1}(ax)}x\,dx=\frac{x}{2a}+\frac{1}{2}e^{\text{sech}^{-1}(ax)}x^2+\frac{\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\arcsin(ax)}{2a^2}$$

output
```
1/2*x/a+1/2*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2+1/2*arcsin(a*x)*(1
/(a*x+1))^(1/2)*(a*x+1)^(1/2)/a^2
```

3.35.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int e^{\text{sech}^{-1}(ax)}x\,dx=\frac{2ax+ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)+i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{2a^2}$$

input
```
Integrate[E^ArcSech[a*x]*x,x]
```

output
```
(2*a*x + a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + I*Log[(-2*I)*a*x + 2*Sq
rt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(2*a^2)
```

3.35.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6889, 24, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\text{sech}^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6889}$$

$$\frac{\int 1dx}{2a}+\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{\sqrt{1-ax}\sqrt{ax+1}}dx}{2a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax)}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{\sqrt{1-ax}\sqrt{ax+1}}dx}{2a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax)}+\frac{x}{2a}$$

$$\Big\downarrow \text{ 39}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax)}+\frac{x}{2a}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\arcsin(ax)}{2a^2}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax)}+\frac{x}{2a}$$

input
```
Int[E^ArcSech[a*x]*x,x]
```

output
```
x/(2*a) + (E^ArcSech[a*x]*x^2)/2 + (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*Arc
Sin[a*x])/(2*a^2)
```

3.35.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.35.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\, x \sqrt{-\frac{ax-1}{ax}}\left(\sqrt{-a^{2}x^{2}+1}\, x \operatorname{csgn}(a) a+\arctan\left(\frac{\operatorname{csgn}(a) a x}{\sqrt{-a^{2}x^{2}+1}}\right)\right) \operatorname{csgn}(a)}{2\sqrt{-a^{2}x^{2}+1}\, a}+\frac{x}{a}$	92

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*((-a^2*x^2+1)^(1/2)*x*csgn(
a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)*csgn(a)/a+
x/a
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int e^{\operatorname{sech}^{-1}(ax)}x\,dx=\frac{a^2x^2\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+2\,ax-\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{2\,a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2*a*x - arctan
(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^2
```

3.35.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}x\,dx=\frac{\int 1\,dx+\int ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x,x)
```

output
```
(Integral(1, x) + Integral(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a
```

3.35.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}x\,dx=\int x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="maxima")
```

output
```
x/a + integrate(sqrt(a*x + 1)*sqrt(-a*x + 1), x)/a
```

3.35.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}x\,dx=\int x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="giac")
```

output
```
integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 303, normalized size of antiderivative = 5.72

$$\int e^{\operatorname{sech}^{-1}(ax)}x\,dx=\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2\,a^2}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{2\,a^2}+\frac{\frac{1\mathrm{i}}{32\,a^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}+\frac{x}{a}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}$$

input
```
int(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
(log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i)/(2*
a^2) - (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(2*a
^2) + (1i/(32*a^2) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^2*((1/(a*x) +
 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^2*((1/(a*x) +
 1)^(1/2) - 1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)
^2 + (2*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a
*x) - 1)^(1/2) - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) + x/a + (((1/(a*x) - 1
)^(1/2) - 1i)^2*1i)/(32*a^2*((1/(a*x) + 1)^(1/2) - 1)^2)
```

3.36 $\int e^{\text{sech}^{-1}(ax)}\,dx$

3.36.1 Optimal result

Integrand size = 6, antiderivative size = 24

$$\int e^{\text{sech}^{-1}(ax)}\,dx = e^{\text{sech}^{-1}(ax)}x - \frac{\text{sech}^{-1}(ax)}{a} + \frac{\log(x)}{a}$$

output
```
(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x-arcsech(a*x)/a+ln(x)/a
```

3.36.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(24) = 48.

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.29

$$\int e^{\text{sech}^{-1}(ax)}\,dx = \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax) + 2\log(ax) - \log\left(1+\sqrt{\frac{1-ax}{1+ax}} + ax\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

input
```
Integrate[E^ArcSech[a*x],x]
```

output
```
(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 2*Log[a*x] - Log[1 + Sqrt[(1 - a*x)
/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/a
```

3.36.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6883, 2056, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\operatorname{sech}^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6883}$$

$$\frac{\int \frac{\sqrt{\frac{1-ax}{ax+1}}}{x(1-ax)}dx}{a}+\frac{\log(x)}{a}+xe^{\operatorname{sech}^{-1}(ax)}$$

$$\Big\downarrow \text{ 2056}$$

$$-4\int \frac{1}{2a-\frac{2a(1-ax)}{ax+1}}d\sqrt{\frac{1-ax}{ax+1}}+\frac{\log(x)}{a}+xe^{\operatorname{sech}^{-1}(ax)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{2\text{arctanh}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a}+\frac{\log(x)}{a}+xe^{\operatorname{sech}^{-1}(ax)}$$

input
```
Int[E^ArcSech[a*x],x]
```

output
```
E^ArcSech[a*x]*x - (2*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/a + Log[x]/a
```

3.36.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2056
```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c
- a*d)/n)   Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)
^((m + 1)/n - 1)/(b*e - d*x^q)^((m + 1)/n + 1))*(u /. x -> ((-a)*e + c*x^q)
^(1/n)/(b*e - d*x^q)^(1/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/
q)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p]
&& IntegerQ[1/n] && IntegersQ[m, r]
```

rule 6883
```
Int[E^ArcSech[(a_.)*(x_)], x_Symbol] :> Simp[x*E^ArcSech[a*x], x] + (Simp[L
og[x]/a, x] + Simp[1/a   Int[(1/(x*(1 - a*x)))*Sqrt[(1 - a*x)/(1 + a*x)], x
], x]) /; FreeQ[a, x]
```

3.36.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\sqrt{-\frac{ax-1}{ax}}\,x\sqrt{\frac{ax+1}{ax}}\left(-\sqrt{-a^{2}x^{2}+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\right)}{\sqrt{-a^{2}x^{2}+1}}$	80

input
```
int(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(-(-a^2*x^2+1)^(1/2)+ar
ctanh(1/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 4.79

$$\int e^{\operatorname{sech}^{-1}(ax)}\,dx = \frac{2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right) + \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right) + 2\,\log\left(x\right)}{2\,a}$$

input
```
integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - log(a*x*sqrt((a*
x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) + log(a*x*sqrt((a*x + 1)/(a*x))*
sqrt(-(a*x - 1)/(a*x)) - 1) + 2*log(x))/a
```

3.36.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}\,dx = \frac{\int \frac{1}{x}\,dx + \int a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

input
```
integrate(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2),x)
```

output
```
(Integral(1/x, x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a
```

3.36.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}\,dx = \int \sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\,dx$$

input
```
integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x), x)
```

3.36.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}(ax)}\,dx = \int \sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\,dx$$

input
```
integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x), x)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 6.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 7.58

$$\int e^{\operatorname{sech}^{-1}(ax)}\,dx = \frac{\ln(x)}{a} - \frac{4\operatorname{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{a} + \frac{\frac{5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1}{\frac{4\,a\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}+\frac{4\,a\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}} + \frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{4\,a\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}$$

input
```
int((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x),x)
```

output
```
log(x)/a - (4*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))
/a + ((5*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)/((
4*a*((1/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1) + (4*a*((1/(a*x)
 - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3) + ((1/(a*x) - 1)^(1/2) -
 1i)/(4*a*((1/(a*x) + 1)^(1/2) - 1))
```

3.37 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x}\,dx$

3.37.1 Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x}\,dx = -\frac{2}{1-\sqrt{\frac{1-ax}{1+ax}}}+2\arctan\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

output
```
2*arctan(((-a*x+1)/(a*x+1))^(1/2))-2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

3.37.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x}\,dx = -\frac{1}{ax}+\left(-1-\frac{1}{ax}\right)\sqrt{\frac{1-ax}{1+ax}}-i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)$$

input
```
Integrate[E^ArcSech[a*x]/x,x]
```

output
```
-(1/(a*x)) + (-1 - 1/(a*x))*Sqrt[(1 - a*x)/(1 + a*x)] - I*Log[(-2*I)*a*x +
 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)]
```

3.37.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6888, 108, 25, 27, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} \, dx$$

$$\Big\downarrow \; 6888$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{\sqrt{1-ax}\sqrt{ax+1}}{x^2}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 108$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\int -\frac{a^2}{\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{x}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\int \frac{a^2}{\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{x}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(a^2\left(-\int \frac{1}{\sqrt{1-ax}\sqrt{ax+1}}dx\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{x}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 39$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(a^2\left(-\int \frac{1}{\sqrt{1-a^2x^2}}dx\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{x}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 223$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-a\arcsin(ax)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{x}\right)}{a}-\frac{1}{ax}$$

input
```
Int[E^ArcSech[a*x]/x,x]
```

output
```
-(1/(a*x)) + (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*(-((Sqrt[1 - a*x]*Sqrt[1
+ a*x])/x) - a*ArcSin[a*x]))/a
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6888
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Simp[(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^p)]   Int[Sqrt[1 + a*x^p]*(Sqrt
[1 - a*x^p]/x^(p + 1)), x], x] /; FreeQ[{a, p}, x]
```

3.37.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.92

method	result	size
default	$-\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\left(\arctan\left(\frac{\operatorname{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)ax+\operatorname{csgn}(a)\sqrt{-a^2x^2+1}\right)\operatorname{csgn}(a)}{\sqrt{-a^2x^2+1}}-\frac{1}{ax}$	92

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-((a*x+1)/a/x)^(1/2)*(-(a*x-1)/a/x)^(1/2)*(arctan(csgn(a)*a*x/(-a^2*x^2+1)
^(1/2))*a*x+csgn(a)*(-a^2*x^2+1)^(1/2))*csgn(a)/(-a^2*x^2+1)^(1/2)-1/a/x
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x}\,dx = -\frac{ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)+1}{ax}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="fricas")
```

output
```
-(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - a*x*arctan(sqrt((a*x
+ 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) + 1)/(a*x)
```

3.37.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x}\,dx = \frac{\int \frac{1}{x^2}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x,x)
```

output
```
(Integral(x**(-2), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x,
 x))/a
```

3.37.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^2, x)/a - 1/(a*x)
```

3.37.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x, x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.83

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x}\,dx = -\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)\,1\mathrm{i}+\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i} - \frac{1}{a\,x}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 8\mathrm{i}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2\left(1+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}\right)}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x,x)
```

output
```
log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i - log(((1/(a*
x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i - 1/(a*x) + (((1
/(a*x) - 1)^(1/2) - 1i)^2*8i)/((((1/(a*x) + 1)^(1/2) - 1)^2*(((1/(a*x) - 1)
^(1/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 - (2*((1/(a*x) - 1)^(1/2) - 1i)
^2)/((1/(a*x) + 1)^(1/2) - 1)^2 + 1))
```

3.38 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^2} \, dx$

3.38.1 Optimal result

Integrand size = 10, antiderivative size = 35

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^2} \, dx = -\frac{e^{\text{sech}^{-1}(ax)}}{2x} + a \operatorname{arctanh}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

output
```
-1/2*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x+a*arctanh(((-a*x+1)/(a*x+1)
)^(1/2))
```

3.38.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(35) = 70.

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.66

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^2} \, dx = \frac{1}{2}\left(-\frac{1}{ax^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax^2} - a\log(x) + a\log\left(1+\sqrt{\frac{1-ax}{1+ax}} + ax\sqrt{\frac{1-ax}{1+ax}}\right)\right)$$

input
```
Integrate[E^ArcSech[a*x]/x^2,x]
```

output
```
(-(1/(a*x^2)) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x^2) - a*Log[x] +
 a*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/2
```

3.38.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6889, 15, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^2} dx$$

$$\Big\downarrow \text{6889}$$

$$-\frac{\int \frac{1}{x^3}dx}{a}-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^3\sqrt{1-ax}\sqrt{ax+1}}dx}{a}-\frac{e^{\text{sech}^{-1}(ax)}}{x}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^3\sqrt{1-ax}\sqrt{ax+1}}dx}{a}+\frac{1}{2ax^2}-\frac{e^{\text{sech}^{-1}(ax)}}{x}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{2}\int -\frac{a^2}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)}{a}+\frac{1}{2ax^2}-\frac{e^{\text{sech}^{-1}(ax)}}{x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{1}{2}\int \frac{a^2}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)}{a}+\frac{1}{2ax^2}-\frac{e^{\text{sech}^{-1}(ax)}}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{1}{2}a^2\int \frac{1}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)}{a}+\frac{1}{2ax^2}-\frac{e^{\text{sech}^{-1}(ax)}}{x}$$

$$\Big\downarrow \text{103}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{2}a^3\int \frac{1}{a-a(1-ax)(ax+1)}d\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)}{a}+\frac{1}{2ax^2}-\frac{e^{\text{sech}^{-1}(ax)}}{x}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{2}a^2\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)}{a}+\frac{1}{2ax^2}-\frac{e^{\text{sech}^{-1}(ax)}}{x}$$

input
```
Int[E^ArcSech[a*x]/x^2,x]
```

output
```
1/(2*a*x^2) - E^ArcSech[a*x]/x - (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*(-1/2
*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^2 - (a^2*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a
*x]])/2))/a
```

3.38.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.38.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(a^2x^2 \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\sqrt{-a^2x^2+1}\right)}{2x\sqrt{-a^2x^2+1}}-\frac{1}{2a\,x^2}$	91

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+1)/a/x)^(1/2)/x*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2*arctanh(1/(-a^2*x^
2+1)^(1/2))-(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/2/a/x^2
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.66

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = \frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-2}{4\,ax^2}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="fricas
")
```

output
```
1/4*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a
^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a*x*s
qrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2)/(a*x^2)
```

3.38.6 Sympy [A] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = -a\left(2\sqrt{-1+\frac{1}{ax}}\left(\frac{\left(1+\frac{1}{ax}\right)^{\frac{3}{2}}}{4}-\frac{\sqrt{1+\frac{1}{ax}}}{4}\right)-\log\left(2\sqrt{-1+\frac{1}{ax}}+2\sqrt{1+\frac{1}{ax}}\right)\right)-\frac{1}{2ax^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**2,x)
```

output
```
-a*(2*sqrt(-1 + 1/(a*x))*((1 + 1/(a*x))**(3/2)/4 - sqrt(1 + 1/(a*x))/4) -
log(2*sqrt(-1 + 1/(a*x)) + 2*sqrt(1 + 1/(a*x)))) - 1/(2*a*x**2)
```

3.38.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^2}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3, x)/a - 1/2/(a*x^2)
```

3.38.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^2}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^2, x)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = \frac{a\ln\left(\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}+\frac{1}{a\,x}\right)}{2}-\frac{1}{2\,a\,x^2}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{2\,x}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^2,x)
```

output
```
(a*log((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)))/2 - 1/(2*a*x^2)
 - ((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/(2*x)
```

3.39 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^3} \, dx$

3.39.1 Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^3} \, dx = -\frac{1}{3ax^3} - \frac{8a^2\left(\frac{1-ax}{1+ax}\right)^{3/2}}{3\left(1-\frac{1-ax}{1+ax}\right)^3}$$

output
```
-1/3/a/x^3-8/3*a^2*((-a*x+1)/(a*x+1))^(3/2)/(1+(a*x-1)/(a*x+1))^3
```

3.39.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^3} \, dx = \frac{-1+(-1+ax)\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{3ax^3}$$

input
```
Integrate[E^ArcSech[a*x]/x^3,x]
```

output
```
(-1 + (-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a*x^3)
```

3.39.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6889, 15, 114, 27, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^3} \, dx$$

$$\Big\downarrow \ 6889$$

$$-\frac{\int \frac{1}{x^4}dx}{2a}-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^4\sqrt{1-ax}\sqrt{ax+1}}dx}{2a}-\frac{e^{\text{sech}^{-1}(ax)}}{2x^2}$$

$$\Big\downarrow \ 15$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^4\sqrt{1-ax}\sqrt{ax+1}}dx}{2a}+\frac{1}{6ax^3}-\frac{e^{\text{sech}^{-1}(ax)}}{2x^2}$$

$$\Big\downarrow \ 114$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{3}\int -\frac{2a^2}{x^2\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)}{2a}+\frac{1}{6ax^3}-\frac{e^{\text{sech}^{-1}(ax)}}{2x^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)}{2a}+\frac{1}{6ax^3}-\frac{e^{\text{sech}^{-1}(ax)}}{2x^2}$$

$$\Big\downarrow \ 106$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{2a^2\sqrt{1-ax}\sqrt{ax+1}}{3x}-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)}{2a}+\frac{1}{6ax^3}-\frac{e^{\text{sech}^{-1}(ax)}}{2x^2}$$

input
```
Int[E^ArcSech[a*x]/x^3,x]
```

output
```
1/(6*a*x^3) - E^ArcSech[a*x]/(2*x^2) - (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]
*(-1/3*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^3 - (2*a^2*Sqrt[1 - a*x]*Sqrt[1 + a
*x])/(3*x)))/(2*a)
```

3.39.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.39.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)}{3x^2}-\frac{1}{3a\,x^3}$	53

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*((a*x+1)/a/x)^(1/2)/x^2*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)-1/3/a/x^3
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \frac{(a^3x^3 - ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 1}{3\,ax^3}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="fricas
")
```

output
```
1/3*((a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1)/(a*
x^3)
```

3.39.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \frac{\int \frac{1}{x^4}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^3}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**3,x)
```

output
```
(Integral(x**(-4), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x*
*3, x))/a
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \frac{(a^2x^3 - x)\sqrt{ax+1}\sqrt{-ax+1}}{3\,ax^4} - \frac{1}{3\,ax^3}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="maxima
")
```

output
```
1/3*(a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*x^4) - 1/3/(a*x^3)
```

3.39.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^3}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^3, x)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = -\frac{1}{3\,a\,x^3} - \frac{\left(\frac{\sqrt{\frac{1}{a\,x}+1}}{3} - \frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{3}\right)\sqrt{\frac{1}{a\,x}-1}}{x^2}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^3,x)
```

output
```
- 1/(3*a*x^3) - (((1/(a*x) + 1)^(1/2)/3 - (a^2*x^2*(1/(a*x) + 1)^(1/2))/3)
*(1/(a*x) - 1)^(1/2))/x^2
```

3.40 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^4}\,dx$

3.40.1 Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^4}\,dx = \frac{1}{12ax^4} - \frac{e^{\text{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{8}a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)$$

output
```
1/12/a/x^4-1/3*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3+1/12*(-a*x+1)^(
1/2)/a/x^4/(1/(a*x+1))^(1/2)+1/8*a*(-a*x+1)^(1/2)/x^2/(1/(a*x+1))^(1/2)+1/
8*a^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2
)
```

3.40.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^4}\,dx = \frac{-2+\sqrt{\frac{1-ax}{1+ax}}(-2-2ax+a^2x^2+a^3x^3)-a^4x^4\log(x)+a^4x^4\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{8ax^4}$$

input
```
Integrate[E^ArcSech[a*x]/x^4,x]
```

output
```
(-2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-2 - 2*a*x + a^2*x^2 + a^3*x^3) - a^4*x^4
*Log[x] + a^4*x^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(
1 + a*x)]])/(8*a*x^4)
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6889, 15, 114, 27, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^4} \, dx$$

$$\Big\downarrow \text{ 6889}$$

$$-\frac{\int \frac{1}{x^5}dx}{3a}-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^5\sqrt{1-ax}\sqrt{ax+1}}dx}{3a}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^5\sqrt{1-ax}\sqrt{ax+1}}dx}{3a}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{4}\int -\frac{3a^2}{x^3\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)}{3a}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{3}{4}a^2\int \frac{1}{x^3\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)}{3a}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{3}{4}a^2\left(-\frac{1}{2}\int -\frac{a^2}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)}{3a}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{3}{4}a^2\left(\frac{1}{2}\int\frac{a^2}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)}{3a}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{1}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)}{3a}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

$\Big\downarrow$ 103

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{3}{4}a^2\left(-\frac{1}{2}a^3\int\frac{1}{a-a(1-ax)(ax+1)}d\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)}{3a}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

$\Big\downarrow$ 221

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{3}{4}a^2\left(-\frac{1}{2}a^2\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)}{3a}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

input
```
Int[E^ArcSech[a*x]/x^4,x]
```

output
```
1/(12*a*x^4) - E^ArcSech[a*x]/(3*x^3) - (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x
]*(-1/4*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^4 + (3*a^2*(-1/2*(Sqrt[1 - a*x]*Sq
rt[1 + a*x])/x^2 - (a^2*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/2))/4))/(3*a
)
```

3.40.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.40.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)a^{4}x^{4}+a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-2\sqrt{-a^{2}x^{2}+1}\right)}{8x^{3}\sqrt{-a^{2}x^{2}+1}}-\frac{1}{4a\,x^{4}}$	110

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*x+1)/a/x)^(1/2)/x^3*(-(a*x-1)/a/x)^(1/2)*(arctanh(1/(-a^2*x^2+1)^(
1/2))*a^4*x^4+a^2*x^2*(-a^2*x^2+1)^(1/2)-2*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1
)^(1/2)-1/4/a/x^4
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(a^3x^3-2\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{16\,ax^4}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="fricas
")
```

output
```
1/16*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(a^3
*x^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 4)/(a*x^4)
```

3.40.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = \frac{\int \frac{1}{x^5}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^4}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**4,x)
```

output
```
(Integral(x**(-5), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x*
*4, x))/a
```

3.40.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^4}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^5, x)/a - 1/4/(a*x^4)
```

3.40.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^4}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x)) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^4, x)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 19.50 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.56

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = \frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{2} - \frac{\frac{35\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{273\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{715\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{715\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^9}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^9}+\frac{273\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{11}}+\frac{35\,a^3\left(\sqrt{\cdots}\right.}{2\left(\sqrt{\cdots}\right.}}{1+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{56\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{70\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{56\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}-\frac{8\left(\sqrt{\frac{1}{a\,x}}-1\cdots\right.}{\left(\sqrt{\frac{1}{a\,x}+1}-\cdots\right.}} - \frac{1}{4\,a\,x^4}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^4,x)
```

output
```
(a^3*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/2 - ((35
*a^3*((1/(a*x) - 1)^(1/2) - 1i)^3)/(2*((1/(a*x) + 1)^(1/2) - 1)^3) + (273*
a^3*((1/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) + (715*a
^3*((1/(a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (715*a^
3*((1/(a*x) - 1)^(1/2) - 1i)^9)/(2*((1/(a*x) + 1)^(1/2) - 1)^9) + (273*a^3
*((1/(a*x) - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (35*a^3
*((1/(a*x) - 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) + (a^3*((
1/(a*x) - 1)^(1/2) - 1i)^15)/(2*((1/(a*x) + 1)^(1/2) - 1)^15) + (a^3*((1/(
a*x) - 1)^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((28*((1/(a*x) - 1)^
(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (8*((1/(a*x) - 1)^(1/2) - 1i)
^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a
*x) + 1)^(1/2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(
1/2) - 1)^8 - (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)
^10 + (28*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (8
*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + ((1/(a*x) -
 1)^(1/2) - 1i)^16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1) - 1/(4*a*x^4)
```

3.40. $\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4} \, dx$

3.41 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}\,dx$

3.41.1 Optimal result

Integrand size = 10, antiderivative size = 115

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}\,dx = \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} + \frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{1+ax}}}$$

output
```
1/20/a/x^5-1/4*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4+1/20*(-a*x+1)^(
1/2)/a/x^5/(1/(a*x+1))^(1/2)+1/15*a*(-a*x+1)^(1/2)/x^3/(1/(a*x+1))^(1/2)+2
/15*a^3*(-a*x+1)^(1/2)/x/(1/(a*x+1))^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}\,dx = \frac{-3 + \sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(-3+3ax-2a^2x^2+2a^3x^3\right)}{15ax^5}$$

input
```
Integrate[E^ArcSech[a*x]/x^5,x]
```

output
```
(-3 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 + 2*a^
3*x^3))/(15*a*x^5)
```

3.41.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6889, 15, 114, 27, 114, 27, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6889}$$

$$-\frac{\int \frac{1}{x^6}dx}{4a}-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^6\sqrt{1-ax}\sqrt{ax+1}}dx}{4a}-\frac{e^{\text{sech}^{-1}(ax)}}{4x^4}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^6\sqrt{1-ax}\sqrt{ax+1}}dx}{4a}+\frac{1}{20ax^5}-\frac{e^{\text{sech}^{-1}(ax)}}{4x^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{5}\int -\frac{4a^2}{x^4\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)}{4a}+\frac{1}{20ax^5}-\frac{e^{\text{sech}^{-1}(ax)}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{4}{5}a^2\int \frac{1}{x^4\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)}{4a}+\frac{1}{20ax^5}-\frac{e^{\text{sech}^{-1}(ax)}}{4x^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{4}{5}a^2\left(-\frac{1}{3}\int -\frac{2a^2}{x^2\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)}{4a}+\frac{1}{20ax^5}-\frac{e^{\text{sech}^{-1}(ax)}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{4}{5}a^2\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)}{4a}+\frac{1}{20ax^5}-\frac{e^{\text{sech}^{-1}(ax)}}{4x^4}$$

$$\Big\downarrow \text{106}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-ax}\sqrt{ax+1}}{3x}-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)}{4a}+\frac{1}{20ax^5}-\frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4}$$

input
```
Int[E^ArcSech[a*x]/x^5,x]
```

output
```
1/(20*a*x^5) - E^ArcSech[a*x]/(4*x^4) - (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x
]*(-1/5*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^5 + (4*a^2*(-1/3*(Sqrt[1 - a*x]*Sq
rt[1 + a*x])/x^3 - (2*a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(3*x)))/5))/(4*a)
```

3.41.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.41.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)\,(2a^2x^2+3)}{15x^4}-\frac{1}{5a\,x^5}$	63

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/15*((a*x+1)/a/x)^(1/2)/x^4*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)*(2*a^2*x^2+3
)-1/5/a/x^5
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}\,dx=\frac{(2\,a^5x^5+a^3x^3-3\,ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-3}{15\,ax^5}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="fricas
")
```

output
```
1/15*((2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/
(a*x)) - 3)/(a*x^5)
```

3.41.6 Sympy [F]

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}\,dx=\frac{\int \frac{1}{x^6}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^5}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**5,x)
```

output
```
(Integral(x**(-6), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x*
*5, x))/a
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = \frac{(2\,a^4x^5 + a^2x^3 - 3\,x)\sqrt{ax+1}\sqrt{-ax+1}}{15\,ax^6} - \frac{1}{5\,ax^5}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="maxima
")
```

output
```
1/15*(2*a^4*x^5 + a^2*x^3 - 3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*x^6) - 1/
5/(a*x^5)
```

3.41.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^5}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^5, x)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = \frac{\sqrt{\frac{1}{a\,x}-1}\left(\frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{15} - \frac{\sqrt{\frac{1}{a\,x}+1}}{5} + \frac{2\,a^4\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{15}\right)}{x^4} - \frac{1}{5\,a\,x^5}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^5,x)
```

output
```
((1/(a*x) - 1)^(1/2)*((a^2*x^2*(1/(a*x) + 1)^(1/2))/15 - (1/(a*x) + 1)^(1/
2)/5 + (2*a^4*x^4*(1/(a*x) + 1)^(1/2))/15))/x^4 - 1/(5*a*x^5)
```

3.42 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^6}\,dx$

3.42.1 Optimal result

Integrand size = 10, antiderivative size = 163

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^6}\,dx = \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{16}a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)$$

output
```
1/30/a/x^6-1/5*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5+1/30*(-a*x+1)^(
1/2)/a/x^6/(1/(a*x+1))^(1/2)+1/24*a*(-a*x+1)^(1/2)/x^4/(1/(a*x+1))^(1/2)+1
/16*a^3*(-a*x+1)^(1/2)/x^2/(1/(a*x+1))^(1/2)+1/16*a^5*arctanh((-a*x+1)^(1/
2)*(a*x+1)^(1/2))*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.79

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^6}\,dx = \frac{-8 + \sqrt{\frac{1-ax}{1+ax}}(-8 - 8ax + 2a^2x^2 + 2a^3x^3 + 3a^4x^4 + 3a^5x^5) - 3a^6x^6\log(x) + 3a^6x^6\log\left(1 + \sqrt{\frac{1-ax}{1+ax}} + ax \cdots\right.}{48ax^6}$$

input
```
Integrate[E^ArcSech[a*x]/x^6,x]
```

output
```
(-8 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3*x^3 + 3*a^
4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(1 - a*x)/(
1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(48*a*x^6)
```

3.42.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6889, 15, 114, 27, 114, 27, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^6} \, dx$$

$$\Big\downarrow \text{6889}$$

$$-\frac{\int \frac{1}{x^7}dx}{5a}-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^7\sqrt{1-ax}\sqrt{ax+1}}dx}{5a}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^7\sqrt{1-ax}\sqrt{ax+1}}dx}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{6}\int -\frac{5a^2}{x^5\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{5}{6}a^2\int \frac{1}{x^5\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{5}{6}a^2\left(-\frac{1}{4}\int -\frac{3a^2}{x^3\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\int\frac{1}{x^3\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(-\frac{1}{2}\int-\frac{a^2}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}\int\frac{a^2}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{1}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{103}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(-\frac{1}{2}a^3\int\frac{1}{a-a(1-ax)(ax+1)}d\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(-\frac{1}{2}a^2\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)}{5a}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}$$

input
```
Int[E^ArcSech[a*x]/x^6,x]
```

3.42. $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^6}\,dx$

output
```
1/(30*a*x^6) - E^ArcSech[a*x]/(5*x^5) - (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x
]*(-1/6*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^6 + (5*a^2*(-1/4*(Sqrt[1 - a*x]*Sq
rt[1 + a*x])/x^4 + (3*a^2*(-1/2*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^2 - (a^2*A
rcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/2))/4))/6))/(5*a)
```

3.42.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.42.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)a^6x^6+3\sqrt{-a^2x^2+1}\,a^4x^4+2a^2x^2\sqrt{-a^2x^2+1}-8\sqrt{-a^2x^2+1}\right)}{48x^5\sqrt{-a^2x^2+1}}-\frac{1}{6a\,x^6}$	132

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*x+1)/a/x)^(1/2)/x^5*(-(a*x-1)/a/x)^(1/2)*(3*arctanh(1/(-a^2*x^2+1
)^(1/2))*a^6*x^6+3*(-a^2*x^2+1)^(1/2)*a^4*x^4+2*a^2*x^2*(-a^2*x^2+1)^(1/2)
-8*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/6/a/x^6
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^6}\,dx$$
$$= \frac{3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(3\,a^5x^5+2\,a^3x^3-8\,ax\right)\sqrt{\cdots}}{96\,ax^6}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="fricas
")
```

output
```
1/96*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)
- 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*
(3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x
)) - 16)/(a*x^6)
```

3.42.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \frac{\int \frac{1}{x^7}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^6}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**6,x)
```

output
```
(Integral(x**(-7), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x*
*6, x))/a
```

3.42.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^6}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^7, x)/a - 1/6/(a*x^6)
```

3.42.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^6}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^6, x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 41.12 (sec) , antiderivative size = 878, normalized size of antiderivative = 5.39

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \frac{\frac{35\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{12\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{757\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{7339\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{41929\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^9}{6\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^9}+\frac{25661\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{11}}+\frac{25661}{2(}}{1+\frac{66\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{220\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{495\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{792\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}-\frac{79}{}} + \frac{a^5\,\operatorname{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{4}-\frac{1}{6\,a\,x^6}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^6,x)
```

output
```
((35*a^5*((1/(a*x) - 1)^(1/2) - 1i)^3)/(12*((1/(a*x) + 1)^(1/2) - 1)^3) +
(757*a^5*((1/(a*x) - 1)^(1/2) - 1i)^5)/(4*((1/(a*x) + 1)^(1/2) - 1)^5) + (
7339*a^5*((1/(a*x) - 1)^(1/2) - 1i)^7)/(4*((1/(a*x) + 1)^(1/2) - 1)^7) + (
41929*a^5*((1/(a*x) - 1)^(1/2) - 1i)^9)/(6*((1/(a*x) + 1)^(1/2) - 1)^9) +
(25661*a^5*((1/(a*x) - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11)
 + (25661*a^5*((1/(a*x) - 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^
13) + (41929*a^5*((1/(a*x) - 1)^(1/2) - 1i)^15)/(6*((1/(a*x) + 1)^(1/2) -
1)^15) + (7339*a^5*((1/(a*x) - 1)^(1/2) - 1i)^17)/(4*((1/(a*x) + 1)^(1/2)
- 1)^17) + (757*a^5*((1/(a*x) - 1)^(1/2) - 1i)^19)/(4*((1/(a*x) + 1)^(1/2)
 - 1)^19) + (35*a^5*((1/(a*x) - 1)^(1/2) - 1i)^21)/(12*((1/(a*x) + 1)^(1/2
) - 1)^21) - (a^5*((1/(a*x) - 1)^(1/2) - 1i)^23)/(4*((1/(a*x) + 1)^(1/2) -
 1)^23) - (a^5*((1/(a*x) - 1)^(1/2) - 1i))/(4*((1/(a*x) + 1)^(1/2) - 1)))/
((66*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (12*((1/(
a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (220*((1/(a*x) - 1)
^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (495*((1/(a*x) - 1)^(1/2) -
1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (792*((1/(a*x) - 1)^(1/2) - 1i)^10)/(
(1/(a*x) + 1)^(1/2) - 1)^10 + (924*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x
) + 1)^(1/2) - 1)^12 - (792*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^
(1/2) - 1)^14 + (495*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) -
 1)^16 - (220*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^...
```

3.43 $\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7}\,dx$

3.43.1 Optimal result

Integrand size = 10, antiderivative size = 146

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7}\,dx=\frac{1}{42ax^7}-\frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6}+\frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}}+\frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}}+\frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}}+\frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{1+ax}}}$$

output
```
1/42/a/x^7-1/6*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6+1/42*(-a*x+1)^(
1/2)/a/x^7/(1/(a*x+1))^(1/2)+1/35*a*(-a*x+1)^(1/2)/x^5/(1/(a*x+1))^(1/2)+4
/105*a^3*(-a*x+1)^(1/2)/x^3/(1/(a*x+1))^(1/2)+8/105*a^5*(-a*x+1)^(1/2)/x/(
1/(a*x+1))^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7}\,dx=\frac{-15+\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(-15+15ax-12a^2x^2+12a^3x^3-8a^4x^4+8a^5x^5\right)}{105ax^7}$$

input
```
Integrate[E^ArcSech[a*x]/x^7,x]
```

output
```
(-15 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*x^2 +
12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a*x^7)
```

3.43.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6889, 15, 114, 27, 114, 27, 114, 27, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^7}dx$$

$$\Big\downarrow \text{ 6889}$$

$$-\frac{\int \frac{1}{x^8}dx}{6a}-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^8\sqrt{1-ax}\sqrt{ax+1}}dx}{6a}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^8\sqrt{1-ax}\sqrt{ax+1}}dx}{6a}+\frac{1}{42ax^7}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{7}\int -\frac{6a^2}{x^6\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{7x^7}\right)}{6a}+\frac{1}{42ax^7}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{6}{7}a^2\int \frac{1}{x^6\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{7x^7}\right)}{6a}+\frac{1}{42ax^7}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{6}{7}a^2\left(-\frac{1}{5}\int -\frac{4a^2}{x^4\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{7x^7}\right)}{6a}+\frac{1}{42ax^7}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{6}{7}a^2\left(\frac{4}{5}a^2\int\frac{1}{x^4\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{7x^7}\right)}{6a}+\frac{1}{42ax^7}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

$$\Big\downarrow \; 114$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{6}{7}a^2\left(\frac{4}{5}a^2\left(-\frac{1}{3}\int-\frac{2a^2}{x^2\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{7x^7}\right)}{6a}+\frac{1}{42ax^7}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{6}{7}a^2\left(\frac{4}{5}a^2\left(\frac{2}{3}a^2\int\frac{1}{x^2\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{7x^7}\right)}{6a}+\frac{1}{42ax^7}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

$$\Big\downarrow \; 106$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{6}{7}a^2\left(\frac{4}{5}a^2\left(-\frac{2a^2\sqrt{1-ax}\sqrt{ax+1}}{3x}-\frac{\sqrt{1-ax}\sqrt{ax+1}}{3x^3}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{5x^5}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{7x^7}\right)}{6a}+\frac{1}{42ax^7}-\frac{e^{\text{sech}^{-1}(ax)}}{6x^6}$$

input
```
Int[E^ArcSech[a*x]/x^7,x]
```

output
```
1/(42*a*x^7) - E^ArcSech[a*x]/(6*x^6) - (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x
]*(-1/7*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^7 + (6*a^2*(-1/5*(Sqrt[1 - a*x]*Sq
rt[1 + a*x])/x^5 + (4*a^2*(-1/3*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^3 - (2*a^2
*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(3*x)))/5))/7))/(6*a)
```

3.43.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.43.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(a^2x^2-1\right)\left(8a^4x^4+12a^2x^2+15\right)}{105x^6}-\frac{1}{7a\,x^7}$	71

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/105*((a*x+1)/a/x)^(1/2)/x^6*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)*(8*a^4*x^4+
12*a^2*x^2+15)-1/7/a/x^7
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7} dx = \frac{(8a^7x^7 + 4a^5x^5 + 3a^3x^3 - 15ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 15}{105ax^7}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="fricas
")
```

output
```
1/105*((8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*x + 1)/(a*x))*
sqrt(-(a*x - 1)/(a*x)) - 15)/(a*x^7)
```

3.43.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7} dx = \frac{\int \frac{1}{x^8} dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^7} dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**7,x)
```

output
```
(Integral(x**(-8), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x*
*7, x))/a
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.41

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7} dx = \frac{(8a^6x^7 + 4a^4x^5 + 3a^2x^3 - 15x)\sqrt{ax+1}\sqrt{-ax+1}}{105ax^8} - \frac{1}{7ax^7}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="maxima
")
```

output
```
1/105*(8*a^6*x^7 + 4*a^4*x^5 + 3*a^2*x^3 - 15*x)*sqrt(a*x + 1)*sqrt(-a*x +
 1)/(a*x^8) - 1/7/(a*x^7)
```

3.43.8 Giac [F]

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^7}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^7}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^7, x)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^7}\,dx = \frac{\sqrt{\frac{1}{a\,x}-1}\left(\frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{35}-\frac{\sqrt{\frac{1}{a\,x}+1}}{7}+\frac{4\,a^4\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{105}+\frac{8\,a^6\,x^6\,\sqrt{\frac{1}{a\,x}+1}}{105}\right)}{x^6}-\frac{1}{7\,a\,x^7}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^7,x)
```

output
```
((1/(a*x) - 1)^(1/2)*((a^2*x^2*(1/(a*x) + 1)^(1/2))/35 - (1/(a*x) + 1)^(1/
2)/7 + (4*a^4*x^4*(1/(a*x) + 1)^(1/2))/105 + (8*a^6*x^6*(1/(a*x) + 1)^(1/2
))/105))/x^6 - 1/(7*a*x^7)
```

3.44 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx$

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 194

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx = \frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}+\frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}}+\frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}}+\frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}}+\frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}}+\frac{5}{128}a^7\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\,\text{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)$$

output
```
1/56/a/x^8-1/7*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7+1/56*(-a*x+1)^(
1/2)/a/x^8/(1/(a*x+1))^(1/2)+1/48*a*(-a*x+1)^(1/2)/x^6/(1/(a*x+1))^(1/2)+5
/192*a^3*(-a*x+1)^(1/2)/x^4/(1/(a*x+1))^(1/2)+5/128*a^5*(-a*x+1)^(1/2)/x^2
/(1/(a*x+1))^(1/2)+5/128*a^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(1/(a*x
+1))^(1/2)*(a*x+1)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.75

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx = \frac{-48+\sqrt{\frac{1-ax}{1+ax}}(-48-48ax+8a^2x^2+8a^3x^3+10a^4x^4+10a^5x^5+15a^6x^6+15a^7x^7)-15a^8x^8\log(x)+1\ldots}{384ax^8}$$

input
```
Integrate[E^ArcSech[a*x]/x^8,x]
```

output
```
(-48 + Sqrt[(1 - a*x)/(1 + a*x)]*(-48 - 48*a*x + 8*a^2*x^2 + 8*a^3*x^3 + 1
0*a^4*x^4 + 10*a^5*x^5 + 15*a^6*x^6 + 15*a^7*x^7) - 15*a^8*x^8*Log[x] + 15
*a^8*x^8*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]
])/(384*a*x^8)
```

3.44.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {6889, 15, 114, 27, 114, 27, 114, 27, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}dx$$

$$\Big\downarrow \text{6889}$$

$$-\frac{\int \frac{1}{x^9}dx}{7a}-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^9\sqrt{1-ax}\sqrt{ax+1}}dx}{7a}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{1}{x^9\sqrt{1-ax}\sqrt{ax+1}}dx}{7a}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(-\frac{1}{8}\int -\frac{7a^2}{x^7\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{8x^8}\right)}{7a}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\int \frac{1}{x^7\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{8x^8}\right)}{7a}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow \text{114}$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(-\frac{1}{6}\int -\frac{5a^2}{x^5\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{8x^8}\right)}{7a}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow 27$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(\frac{5}{6}a^2\int\frac{1}{x^5\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{8x^8}\right)}{7a}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow 114$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(\frac{5}{6}a^2\left(-\frac{1}{4}\int-\frac{3a^2}{x^3\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{8x^8}\right)}{7a}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow 27$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\int\frac{1}{x^3\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{8x^8}\right)}{7a}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow 114$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(-\frac{1}{2}\int-\frac{a^2}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)-\frac{\sqrt{1-}}{}\right.}{7a}$$

$$\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow 25$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}\int\frac{a^2}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)-\frac{\sqrt{1-ax}\sqrt{}}{8x^8}\right.}{7a}$$

$$\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow 27$$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{1}{x\sqrt{1-ax}\sqrt{ax+1}}dx-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{6x^6}\right)-\frac{\sqrt{1-a}}{}\right.}{7a}$$

$$\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$$\Big\downarrow 103$$

3.44. $\int\frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx$

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(-\frac{1}{2}a^3\int\frac{1}{a-a(1-ax)(ax+1)}d\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\right.\right.}{7a}$$
$$\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

$\Big\downarrow$ 221

$$-\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\left(\frac{7}{8}a^2\left(\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(-\frac{1}{2}a^2\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{2x^2}\right)-\frac{\sqrt{1-ax}\sqrt{ax+1}}{4x^4}\right)-\frac{\sqrt{1-ax}\sqrt{ax+}}{6x^6}\right.\right.}{7a}$$
$$\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}$$

input
```
Int[E^ArcSech[a*x]/x^8,x]
```

output
```
1/(56*a*x^8) - E^ArcSech[a*x]/(7*x^7) - (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x
]*(-1/8*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^8 + (7*a^2*(-1/6*(Sqrt[1 - a*x]*Sq
rt[1 + a*x])/x^6 + (5*a^2*(-1/4*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^4 + (3*a^2
*(-1/2*(Sqrt[1 - a*x]*Sqrt[1 + a*x])/x^2 - (a^2*ArcTanh[Sqrt[1 - a*x]*Sqrt
[1 + a*x]])/2))/4))/6))/8))/(7*a)
```

3.44.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.44.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.78

method	result
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(15\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)a^{8}x^{8}+15\sqrt{-a^{2}x^{2}+1}\,a^{6}x^{6}+10\sqrt{-a^{2}x^{2}+1}\,a^{4}x^{4}+8a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-48\sqrt{-a^{2}x^{2}+1}\right)}{384x^{7}\sqrt{-a^{2}x^{2}+1}}$

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/384*((a*x+1)/a/x)^(1/2)/x^7*(-(a*x-1)/a/x)^(1/2)*(15*arctanh(1/(-a^2*x^2
+1)^(1/2))*a^8*x^8+15*(-a^2*x^2+1)^(1/2)*a^6*x^6+10*(-a^2*x^2+1)^(1/2)*a^4
*x^4+8*a^2*x^2*(-a^2*x^2+1)^(1/2)-48*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2
)-1/8/a/x^8
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^8}\,dx = \frac{15\,a^8x^8\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-15\,a^8x^8\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\,(15\,a^7x^7+10\,a^5x^5+8\,a^3}{768\,ax^8}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="fricas
")
```

output
```
1/768*(15*a^8*x^8*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1
) - 15*a^8*x^8*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) +
 2*(15*a^7*x^7 + 10*a^5*x^5 + 8*a^3*x^3 - 48*a*x)*sqrt((a*x + 1)/(a*x))*sq
rt(-(a*x - 1)/(a*x)) - 96)/(a*x^8)
```

3.44.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^8}\,dx = \frac{\int \frac{1}{x^9}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^8}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**8,x)
```

output
```
(Integral(x**(-9), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x*
*8, x))/a
```

3.44.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^8}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^8}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^9, x)/a - 1/8/(a*x^8)
```

3.44.8 Giac [F]

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx = \int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^8}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^8, x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 43.82 (sec) , antiderivative size = 1155, normalized size of antiderivative = 5.95

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx = \text{Too large to display}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^8,x)
```

output
```
(5*a^7*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/32 - (
(1723*a^7*((1/(a*x) - 1)^(1/2) - 1i)^5)/(96*((1/(a*x) + 1)^(1/2) - 1)^5) -
 (235*a^7*((1/(a*x) - 1)^(1/2) - 1i)^3)/(96*((1/(a*x) + 1)^(1/2) - 1)^3) +
 (72283*a^7*((1/(a*x) - 1)^(1/2) - 1i)^7)/(32*((1/(a*x) + 1)^(1/2) - 1)^7)
 + (848801*a^7*((1/(a*x) - 1)^(1/2) - 1i)^9)/(32*((1/(a*x) + 1)^(1/2) - 1)
^9) + (4181067*a^7*((1/(a*x) - 1)^(1/2) - 1i)^11)/(32*((1/(a*x) + 1)^(1/2)
 - 1)^11) + (10994181*a^7*((1/(a*x) - 1)^(1/2) - 1i)^13)/(32*((1/(a*x) + 1
)^(1/2) - 1)^13) + (17457599*a^7*((1/(a*x) - 1)^(1/2) - 1i)^15)/(32*((1/(a
*x) + 1)^(1/2) - 1)^15) + (17457599*a^7*((1/(a*x) - 1)^(1/2) - 1i)^17)/(32
*((1/(a*x) + 1)^(1/2) - 1)^17) + (10994181*a^7*((1/(a*x) - 1)^(1/2) - 1i)^
19)/(32*((1/(a*x) + 1)^(1/2) - 1)^19) + (4181067*a^7*((1/(a*x) - 1)^(1/2)
- 1i)^21)/(32*((1/(a*x) + 1)^(1/2) - 1)^21) + (848801*a^7*((1/(a*x) - 1)^(
1/2) - 1i)^23)/(32*((1/(a*x) + 1)^(1/2) - 1)^23) + (72283*a^7*((1/(a*x) -
1)^(1/2) - 1i)^25)/(32*((1/(a*x) + 1)^(1/2) - 1)^25) + (1723*a^7*((1/(a*x)
 - 1)^(1/2) - 1i)^27)/(96*((1/(a*x) + 1)^(1/2) - 1)^27) - (235*a^7*((1/(a*
x) - 1)^(1/2) - 1i)^29)/(96*((1/(a*x) + 1)^(1/2) - 1)^29) + (5*a^7*((1/(a*
x) - 1)^(1/2) - 1i)^31)/(32*((1/(a*x) + 1)^(1/2) - 1)^31) + (5*a^7*((1/(a*
x) - 1)^(1/2) - 1i))/(32*((1/(a*x) + 1)^(1/2) - 1)))/((120*((1/(a*x) - 1)^
(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (16*((1/(a*x) - 1)^(1/2) - 1i
)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (560*((1/(a*x) - 1)^(1/2) - 1i)^6)/(...
```

3.45 $\int e^{\text{sech}^{-1}(ax^2)} x^7 \, dx$

3.45.1 Optimal result

Integrand size = 12, antiderivative size = 111

$$\int e^{\text{sech}^{-1}(ax^2)} x^7 \, dx = \frac{x^6}{24a} + \frac{1}{8} e^{\text{sech}^{-1}(ax^2)} x^8 - \frac{x^2 \sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \sqrt{1-a^2x^4}}{16a^3} + \frac{\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \arcsin(ax^2)}{16a^4}$$

output
```
1/24*x^6/a+1/8*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^8+1/16*arcs
in(a*x^2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^4-1/16*x^2*(1/(a*x^2+1))^(
1/2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a^3
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int e^{\text{sech}^{-1}(ax^2)} x^7 \, dx = \frac{8a^3x^6 - 3a\sqrt{\frac{1-ax^2}{1+ax^2}}(x^2+ax^4-2a^2x^6-2a^3x^8) + 3i\log\left(-2iax^2+2\sqrt{\frac{1-ax^2}{1+ax^2}}(1+ax^2)\right)}{48a^4}$$

input
```
Integrate[E^ArcSech[a*x^2]*x^7,x]
```

output
```
(8*a^3*x^6 - 3*a*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^2 + a*x^4 - 2*a^2*x^6 -
2*a^3*x^8) + (3*I)*Log[(-2*I)*a*x^2 + 2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 +
 a*x^2)])/(48*a^4)
```

3.45.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6889, 15, 335, 807, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 e^{\text{sech}^{-1}(ax^2)} \, dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{\int x^5dx}{4a}+\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^5}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{4a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^5}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{4a}+\frac{x^6}{24a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{335}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^5}{\sqrt{1-a^2x^4}}dx}{4a}+\frac{x^6}{24a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^4}{\sqrt{1-a^2x^4}}dx^2}{8a}+\frac{x^6}{24a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{\int \frac{1}{\sqrt{1-a^2x^4}}dx^2}{2a^2}-\frac{x^2\sqrt{1-a^2x^4}}{2a^2}\right)}{8a}+\frac{x^6}{24a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{\arcsin(ax^2)}{2a^3}-\frac{x^2\sqrt{1-a^2x^4}}{2a^2}\right)}{8a}+\frac{x^6}{24a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

input
```
Int[E^ArcSech[a*x^2]*x^7,x]
```

output
```
x^6/(24*a) + (E^ArcSech[a*x^2]*x^8)/8 + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a
*x^2]*(-1/2*(x^2*Sqrt[1 - a^2*x^4])/a^2 + ArcSin[a*x^2]/(2*a^3)))/(8*a)
```

3.45.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.45.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,x^{2}\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\left(2x^{6}\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}\,a^{4}-x^{2}\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}\,a^{2}+\arctan\left(\frac{x^{2}}{\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}}\right)\right)}{16\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}\,a^{4}}+\frac{x^{6}}{6a}$	137

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x,method=_RETURNVERB
OSE)
```

output
```
1/16*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(2*x^6*(-(a^2*x^
4-1)/a^2)^(1/2)*a^4-x^2*(-(a^2*x^4-1)/a^2)^(1/2)*a^2+arctan(x^2/(-(a^2*x^4
-1)/a^2)^(1/2)))/(-(a^2*x^4-1)/a^2)^(1/2)/a^4+1/6/a*x^6
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^7\,dx$$
$$=\frac{8\,a^{3}x^{6}+3\left(2\,a^{4}x^{8}-a^{2}x^{4}\right)\sqrt{\frac{ax^{2}+1}{ax^{2}}}\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-6\,\arctan\left(\frac{ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-1}{ax^{2}}\right)}{48\,a^{4}}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="
fricas")
```

output
```
1/48*(8*a^3*x^6 + 3*(2*a^4*x^8 - a^2*x^4)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-
(a*x^2 - 1)/(a*x^2)) - 6*arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*
x^2 - 1)/(a*x^2)) - 1)/(a*x^2)))/a^4
```

3.45.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^7\,dx = \frac{\int x^5\,dx + \int ax^7\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**7,x)
```

output
```
(Integral(x**5, x) + Integral(a*x**7*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x
**2)), x))/a
```

3.45.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^7\,dx = \int x^7\left(\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="
maxima")
```

output
```
1/6*x^6/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^5, x)/a
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(76) = 152.

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.85

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^7\,dx$$

$$= \frac{8\,a^2x^6 + 4\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}\left((a^2x^2+a)\left(\frac{2\,(a^2x^2+a)}{a^4}-\frac{7}{a^3}\right)+\frac{9}{a^2}\right) + \left(\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}\left((a^2x^2\right.\right.}{48\,a^3}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="
giac")
```

output
```
1/48*(8*a^2*x^6 + 4*sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*((a^2*x^2 + a)*(2
*(a^2*x^2 + a)/a^4 - 7/a^3) + 9/a^2) + (sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 +
a)*((a^2*x^2 + a)*(2*(a^2*x^2 + a)*(3*(a^2*x^2 + a)/a^6 - 13/a^5) + 43/a^4
) - 39/a^3) - 18*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/a^2)*a + 24
*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/a)/a^3
```

3.45.9 Mupad [B] (verification not implemented)

Time = 18.82 (sec) , antiderivative size = 521, normalized size of antiderivative = 4.69

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^7\,dx = \frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{16\,a^4} - \frac{\frac{1\mathrm{i}}{2048\,a^4}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4 11\mathrm{i}}{1024\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^6 7\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^6}-\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^8 239\mathrm{i}}{2048\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^8}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^{10} 1\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^{10}}}{\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^{12}}} - \frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)1\mathrm{i}}{16\,a^4}+\frac{x^6}{6\,a}-\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4 1\mathrm{i}}{2048\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}$$

input
```
int(x^7*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
(log(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)
/(16*a^4) - (1i/(2048*a^4) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(256*a^4*
((1/(a*x^2) + 1)^(1/2) - 1)^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^4*11i)/(102
4*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^4) + (((1/(a*x^2) - 1)^(1/2) - 1i)^6*7i)
/(512*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^6) - (((1/(a*x^2) - 1)^(1/2) - 1i)^8
*239i)/(2048*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^8) + (((1/(a*x^2) - 1)^(1/2)
- 1i)^10*1i)/(512*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^10))/(((1/(a*x^2) - 1)^(
1/2) - 1i)^4/((1/(a*x^2) + 1)^(1/2) - 1)^4 + (4*((1/(a*x^2) - 1)^(1/2) - 1
i)^6)/((1/(a*x^2) + 1)^(1/2) - 1)^6 + (6*((1/(a*x^2) - 1)^(1/2) - 1i)^8)/(
(1/(a*x^2) + 1)^(1/2) - 1)^8 + (4*((1/(a*x^2) - 1)^(1/2) - 1i)^10)/((1/(a*
x^2) + 1)^(1/2) - 1)^10 + ((1/(a*x^2) - 1)^(1/2) - 1i)^12/((1/(a*x^2) + 1)
^(1/2) - 1)^12) - (log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2)
 - 1))*1i)/(16*a^4) + x^6/(6*a) - (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(512
*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^2) - (((1/(a*x^2) - 1)^(1/2) - 1i)^4*1i)/
(2048*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^4)
```

3.46 $\int e^{\text{sech}^{-1}(ax^2)}x^6\,dx$

3.46.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int e^{\text{sech}^{-1}(ax^2)}x^6\,dx=\frac{2x^5}{35a}+\frac{1}{7}e^{\text{sech}^{-1}(ax^2)}x^7-\frac{2x\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{21a^3}+\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{21a^{7/2}}$$

output
```
2/35*x^5/a+1/7*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7+2/21*Elli
pticF(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(7/2)-2/21*x*(1/(
a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a^3
```

3.46.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.24

$$\int e^{\text{sech}^{-1}(ax^2)}x^6\,dx=$$
$$-\frac{2\sqrt{2}\sqrt{\frac{e^{\text{sech}^{-1}(ax^2)}}{1+e^{2\text{sech}^{-1}(ax^2)}}}x^5\left(-5-17e^{2\text{sech}^{-1}(ax^2)}-67e^{4\text{sech}^{-1}(ax^2)}+5e^{6\text{sech}^{-1}(ax^2)}+5\left(1+e^{2\text{sech}^{-1}(ax^2)}\right)^{7/2}\cdots}{105a\left(1+e^{2\text{sech}^{-1}(ax^2)}\right)^3(ax^2)^{5/2}}$$

input
```
Integrate[E^ArcSech[a*x^2]*x^6,x]
```

output
```
(-2*Sqrt[2]*Sqrt[E^ArcSech[a*x^2]/(1 + E^(2*ArcSech[a*x^2]))]*x^5*(-5 - 17
*E^(2*ArcSech[a*x^2]) - 67*E^(4*ArcSech[a*x^2]) + 5*E^(6*ArcSech[a*x^2]) +
 5*(1 + E^(2*ArcSech[a*x^2]))^(7/2)*Hypergeometric2F1[1/4, 1/2, 5/4, -E^(2
*ArcSech[a*x^2])]))/(105*a*(1 + E^(2*ArcSech[a*x^2]))^3*(a*x^2)^(5/2))
```

3.46.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6889, 15, 335, 843, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 e^{\text{sech}^{-1}\left(ax^2\right)} \, dx$$

$$\Big\downarrow \text{ 6889}$$

$$\frac{2 \int x^4dx}{7a}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^4}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{7a}+\frac{1}{7}x^7e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^4}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{7a}+\frac{2x^5}{35a}+\frac{1}{7}x^7e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{ 335}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^4}{\sqrt{1-a^2x^4}}dx}{7a}+\frac{2x^5}{35a}+\frac{1}{7}x^7e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{\int \frac{1}{\sqrt{1-a^2x^4}}dx}{3a^2}-\frac{x\sqrt{1-a^2x^4}}{3a^2}\right)}{7a}+\frac{2x^5}{35a}+\frac{1}{7}x^7e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{3a^{5/2}}-\frac{x\sqrt{1-a^2x^4}}{3a^2}\right)}{7a}+\frac{2x^5}{35a}+\frac{1}{7}x^7e^{\text{sech}^{-1}\left(ax^2\right)}$$

input
```
Int[E^ArcSech[a*x^2]*x^6,x]
```

output
```
(2*x^5)/(35*a) + (E^ArcSech[a*x^2]*x^7)/7 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt
[1 + a*x^2]*(-1/3*(x*Sqrt[1 - a^2*x^4])/a^2 + EllipticF[ArcSin[Sqrt[a]*x],
 -1]/(3*a^(5/2))))/(7*a)
```

3.46.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.46.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,x^{2}\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(3a^{\frac{9}{2}}x^{9}-5a^{\frac{5}{2}}x^{5}-2\,\mathrm{EllipticF}\left(x\sqrt{a},i\right)\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}+2x\sqrt{a}\right)}{21a^{\frac{5}{2}}\left(x^{4}a^{2}-1\right)}+\frac{x^{5}}{5a}$	114

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x,method=_RETURNVERB
OSE)
```

output
```
1/21*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(3*a^(9/2)*x^9-5
*a^(5/2)*x^5-2*EllipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)+2*x
*a^(1/2))/a^(5/2)/(a^2*x^4-1)+1/5/a*x^5
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^6\,dx=\frac{21\,a^2x^5+5\left(3\,a^3x^7-2\,ax^3\right)\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+\frac{10i\,F\left(\arcsin\left(\frac{1}{\sqrt{a}x}\right)|-1\right)}{\sqrt{a}}}{105\,a^3}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="
fricas")
```

output
```
1/105*(21*a^2*x^5 + 5*(3*a^3*x^7 - 2*a*x^3)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt
(-(a*x^2 - 1)/(a*x^2)) + 10*I*elliptic_f(arcsin(1/(sqrt(a)*x)), -1)/sqrt(a
))/a^3
```

3.46.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^6\,dx=\frac{\int x^4\,dx+\int ax^6\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**6,x)
```

output
```
(Integral(x**4, x) + Integral(a*x**6*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x
**2)), x))/a
```

3.46.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^6\,dx = \int x^6\left(\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="
maxima")
```

output
```
1/5*x^5/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^4, x)/a
```

3.46.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^6\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,4,2,1,1,1]%%%}+%%%{1,[0,4,0,0,0,2]%%%} / %%%{1,[0,0,0
,0,0,3]%%
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^6\,dx = \int x^6\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)\,dx$$

input
```
int(x^6*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
int(x^6*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.47 $\int e^{\text{sech}^{-1}(ax^2)} x^5 \, dx$

3.47.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int e^{\text{sech}^{-1}(ax^2)} x^5 \, dx = \frac{x^4}{12a} + \frac{1}{6} e^{\text{sech}^{-1}(ax^2)} x^6 - \frac{\sqrt{1-ax^2}}{6a^3 \sqrt{\frac{1}{1+ax^2}}}$$

output
```
1/12*x^4/a+1/6*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6-1/6*(-a*x
^2+1)^(1/2)/a^3/(1/(a*x^2+1))^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int e^{\text{sech}^{-1}(ax^2)} x^5 \, dx = \frac{x^4}{4a} + \frac{(-1+ax^2) \sqrt{\frac{1-ax^2}{1+ax^2}} (1+ax^2)^2}{6a^3}$$

input
```
Integrate[E^ArcSech[a*x^2]*x^5,x]
```

output
```
x^4/(4*a) + ((-1 + a*x^2)*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)^2)/(6*
a^3)
```

3.47.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6889, 15, 335, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 e^{\text{sech}^{-1}(ax^2)} dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{\int x^3 dx}{3a} + \frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^3}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{3a} + \frac{1}{6}x^6 e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^3}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{3a} + \frac{x^4}{12a} + \frac{1}{6}x^6 e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{335}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^3}{\sqrt{1-a^2x^4}}dx}{3a} + \frac{x^4}{12a} + \frac{1}{6}x^6 e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{793}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{6a^3} + \frac{x^4}{12a} + \frac{1}{6}x^6 e^{\text{sech}^{-1}(ax^2)}$$

input
```
Int[E^ArcSech[a*x^2]*x^5,x]
```

output
```
x^4/(12*a) + (E^ArcSech[a*x^2]*x^6)/6 - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a
*x^2]*Sqrt[1 - a^2*x^4])/(6*a^3)
```

3.47.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.47.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,x^2\sqrt{\frac{a\,x^2+1}{a\,x^2}}\,(x^4a^2-1)}{6a^2}+\frac{x^4}{4a}$	60

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x,method=_RETURNVERB
OSE)
```

output
```
1/6*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^2*x^4-1)/a^2+1
/4*x^4/a
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^5\,dx = \frac{3\,ax^4 + 2\,(a^2x^6 - x^2)\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}}{12\,a^2}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="
fricas")
```

output
```
1/12*(3*a*x^4 + 2*(a^2*x^6 - x^2)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 -
 1)/(a*x^2)))/a^2
```

3.47.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^5\,dx = \frac{\int x^3\,dx + \int ax^5\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**5,x)
```

output
```
(Integral(x**3, x) + Integral(a*x**5*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x
**2)), x))/a
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^5\,dx = \frac{x^4}{4\,a} + \frac{(a^2x^4 - 1)\sqrt{ax^2 + 1}\sqrt{-ax^2 + 1}}{6\,a^3}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="
maxima")
```

output
```
1/4*x^4/a + 1/6*(a^2*x^4 - 1)*sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/a^3
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(69) = 138$.

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.28

$$\int e^{\operatorname{sech}^{-1}(ax^2)} x^5 \, dx$$
$$= \frac{\left(\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}\left((a^2x^2+a)\left(\frac{2\,(a^2x^2+a)}{a^4}-\frac{7}{a^3}\right)+\frac{9}{a^2}\right)+\frac{6\arcsin\left(\frac{\sqrt{2}\sqrt{a^2x^2+a}}{2\sqrt{a}}\right)}{a}\right)a-\frac{3\left(2\,a^2\arcsin\left(\frac{\sqrt{2}\sqrt{a^2x}}{2\sqrt{a}}\right.\right.}{}}{12\,a^3}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="
giac")
```

output
```
1/12*((sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*((a^2*x^2 + a)*(2*(a^2*x^2 + a
)/a^4 - 7/a^3) + 9/a^2) + 6*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/
a)*a - 3*(2*a^2*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a)) - sqrt(a^2*x
^2 + a)*(a^2*x^2 - 2*a)*sqrt(-a^2*x^2 + a))/a^2 + 3*((a^2*x^2 + a)^2 - 2*(
a^2*x^2 + a)*a)/a^2)/a^3
```

3.47.9 Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int e^{\operatorname{sech}^{-1}(ax^2)} x^5 \, dx = \sqrt{\frac{1}{a\,x^2}-1}\left(\frac{x^6\sqrt{\frac{1}{a\,x^2}+1}}{6}-\frac{x^2\sqrt{\frac{1}{a\,x^2}+1}}{6\,a^2}\right)+\frac{x^4}{4\,a}$$

input
```
int(x^5*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
(1/(a*x^2) - 1)^(1/2)*((x^6*(1/(a*x^2) + 1)^(1/2))/6 - (x^2*(1/(a*x^2) + 1
)^(1/2))/(6*a^2)) + x^4/(4*a)
```

3.48 $\int e^{\operatorname{sech}^{-1}(ax^2)} x^4 \, dx$

3.48.1 Optimal result

Integrand size = 12, antiderivative size = 112

$$\int e^{\operatorname{sech}^{-1}(ax^2)} x^4 \, dx = \frac{2x^3}{15a} + \frac{1}{5} e^{\operatorname{sech}^{-1}(ax^2)} x^5 + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\, E\left(\arcsin\left(\sqrt{a}x\right)\middle| -1\right)}{5a^{5/2}} - \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\, \operatorname{EllipticF}\left(\arcsin\left(\sqrt{a}x\right), -1\right)}{5a^{5/2}}$$

output
```
2/15*x^3/a+1/5*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5+2/5*Ellip
ticE(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(5/2)-2/5*Elliptic
F(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(5/2)
```

3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.25

$$\int e^{\operatorname{sech}^{-1}(ax^2)} x^4 \, dx = \frac{1}{15}\left(\frac{5x^3}{a} + \frac{3\sqrt{\frac{1-ax^2}{1+ax^2}}(x^3+ax^5)}{a} + \frac{6i\sqrt{\frac{1-ax^2}{1+ax^2}}\sqrt{1-a^2x^4}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-a}x\right)\middle| -1\right) - \operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-a}x\right), -1\right)\right)}{(-a)^{5/2}(-1+ax^2)}\right)$$

input
```
Integrate[E^ArcSech[a*x^2]*x^4,x]
```

output
```
((5*x^3)/a + (3*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^3 + a*x^5))/a + ((6*I)*Sq
rt[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[-a
]*x], -1] - EllipticF[I*ArcSinh[Sqrt[-a]*x], -1]))/((-a)^(5/2)*(-1 + a*x^2
)))/15
```

3.48.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6889, 15, 335, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{\text{sech}^{-1}\left(ax^2\right)} \, dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{2 \int x^2dx}{5a}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^2}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{5a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^2}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{5a}+\frac{2x^3}{15a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{335}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^2}{\sqrt{1-a^2x^4}}dx}{5a}+\frac{2x^3}{15a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{836}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{\int \frac{ax^2+1}{\sqrt{1-a^2x^4}}dx}{a}-\frac{\int \frac{1}{\sqrt{1-a^2x^4}}dx}{a}\right)}{5a}+\frac{2x^3}{15a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{762}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{\int \frac{ax^2+1}{\sqrt{1-a^2x^4}}dx}{a}-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right)}{5a}+\frac{2x^3}{15a}+\frac{1}{5}x^5 e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{1388}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{\int \frac{\sqrt{ax^2+1}}{\sqrt{1-ax^2}}dx}{a}-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right)}{5a}+\frac{2x^3}{15a}+\frac{1}{5}x^5e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{327}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{E\left(\arcsin\left(\sqrt{a}x\right)|-1\right)}{a^{3/2}}-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right)}{5a}+\frac{2x^3}{15a}+\frac{1}{5}x^5e^{\text{sech}^{-1}(ax^2)}$$

input
```
Int[E^ArcSech[a*x^2]*x^4,x]
```

output
```
(2*x^3)/(15*a) + (E^ArcSech[a*x^2]*x^5)/5 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt
[1 + a*x^2]*(EllipticE[ArcSin[Sqrt[a]*x], -1]/a^(3/2) - EllipticF[ArcSin[S
qrt[a]*x], -1]/a^(3/2)))/(5*a)
```

3.48.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.48.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,x^{2}\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(a^{\frac{7}{2}}x^{7}-x^{3}a^{\frac{3}{2}}+2\,\operatorname{EllipticF}\left(x\sqrt{a},i\right)\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}-2\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,\operatorname{EllipticE}\left(x\sqrt{a},i\right)\right)}{5(x^{4}a^{2}-1)a^{\frac{3}{2}}}+\frac{x^{3}}{3a}$

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x,method=_RETURNVERB
OSE)
```

output
```
1/5*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^(7/2)*x^7-x^3*
a^(3/2)+2*EllipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)-2*(-a*x^
2+1)^(1/2)*(a*x^2+1)^(1/2)*EllipticE(x*a^(1/2),I))/(a^2*x^4-1)/a^(3/2)+1/3
*x^3/a
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^4\,dx = \frac{5\,a^2x^3+3\,(a^3x^5-2\,ax)\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-\frac{6i\,E\left(\arcsin\left(\frac{1}{\sqrt{a}x}\right)\,|\,-1\right)}{\sqrt{a}}+\frac{6i\,F\left(\arcsin\left(\frac{1}{\sqrt{a}x}\right)\,|\,-1\right)}{\sqrt{a}}}{15\,a^3}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="
fricas")
```

output
```
1/15*(5*a^2*x^3 + 3*(a^3*x^5 - 2*a*x)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x
^2 - 1)/(a*x^2)) - 6*I*elliptic_e(arcsin(1/(sqrt(a)*x)), -1)/sqrt(a) + 6*I
*elliptic_f(arcsin(1/(sqrt(a)*x)), -1)/sqrt(a))/a^3
```

3.48.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^4\,dx = \frac{\int x^2\,dx+\int ax^4\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**4,x)
```

output
```
(Integral(x**2, x) + Integral(a*x**4*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x
**2)), x))/a
```

3.48.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^4\,dx = \int x^4\left(\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="
maxima")
```

output
```
1/3*x^3/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^2, x)/a
```

3.48.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^4\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,2,1,1,1]%%%}+%%%{1,[0,2,0,0,0,2]%%%} / %%%{1,[0,0,0
,0,0,3]%%
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^4\,dx = \int x^4\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)\,dx$$

input
```
int(x^4*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
int(x^4*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.49 $\int e^{\text{sech}^{-1}(ax^2)}x^3\,dx$

3.49.1 Optimal result

Integrand size = 12, antiderivative size = 63

$$\int e^{\text{sech}^{-1}(ax^2)}x^3\,dx=\frac{x^2}{4a}+\frac{1}{4}e^{\text{sech}^{-1}(ax^2)}x^4+\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\arcsin(ax^2)}{4a^2}$$

output
```
1/4*x^2/a+1/4*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4+1/4*arcsin
(a*x^2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^2
```

3.49.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

$$\int e^{\text{sech}^{-1}(ax^2)}x^3\,dx=\frac{2ax^2+a\sqrt{\frac{1-ax^2}{1+ax^2}}(x^2+ax^4)+i\log\left(-2iax^2+2\sqrt{\frac{1-ax^2}{1+ax^2}}(1+ax^2)\right)}{4a^2}$$

input
```
Integrate[E^ArcSech[a*x^2]*x^3,x]
```

output
```
(2*a*x^2 + a*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^2 + a*x^4) + I*Log[(-2*I)*a*
x^2 + 2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)])/(4*a^2)
```

3.49.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6889, 15, 335, 807, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\text{sech}^{-1}(ax^2)} \, dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{2a}+\frac{\int xdx}{2a}+\frac{1}{4}x^4 e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{2a}+\frac{x^2}{4a}+\frac{1}{4}x^4 e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{335}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x}{\sqrt{1-a^2x^4}}dx}{2a}+\frac{x^2}{4a}+\frac{1}{4}x^4 e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{\sqrt{1-a^2x^4}}dx^2}{4a}+\frac{x^2}{4a}+\frac{1}{4}x^4 e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\arcsin\left(ax^2\right)}{4a^2}+\frac{x^2}{4a}+\frac{1}{4}x^4 e^{\text{sech}^{-1}(ax^2)}$$

input
```
Int[E^ArcSech[a*x^2]*x^3,x]
```

output
```
x^2/(4*a) + (E^ArcSech[a*x^2]*x^4)/4 + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*
x^2]*ArcSin[a*x^2])/(4*a^2)
```

3.49.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.49.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\, x^{2} \sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\, \left(x^{2} \sqrt{-\frac{x^{4} a^{2}-1}{a^{2}}}\, a^{2}+\arctan\left(\frac{x^{2}}{\sqrt{-\frac{x^{4} a^{2}-1}{a^{2}}}}\right)\right)}{4 \sqrt{-\frac{x^{4} a^{2}-1}{a^{2}}}\, a^{2}}+\frac{x^{2}}{2 a}$	112

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x,method=_RETURNVERB
OSE)
```

output
```
1/4*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(x^2*(-(a^2*x^4-1
)/a^2)^(1/2)*a^2+arctan(x^2/(-(a^2*x^4-1)/a^2)^(1/2)))/(-(a^2*x^4-1)/a^2)^
(1/2)/a^2+1/2*x^2/a
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^3\,dx = \frac{a^2x^4\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} + 2\,ax^2 - 2\,\arctan\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-1}{ax^2}\right)}{4\,a^2}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="
fricas")
```

output
```
1/4*(a^2*x^4*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 2*a*x^
2 - 2*arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) -
 1)/(a*x^2)))/a^2
```

3.49.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^3\,dx = \frac{\int x\,dx + \int ax^3\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**3,x)
```

output
```
(Integral(x, x) + Integral(a*x**3*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
)), x))/a
```

3.49.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^3\,dx = \int x^3\left(\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="
maxima")
```

output
```
1/2*x^2/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x, x)/a
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(56) = 112$.

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^3\,dx$$
$$= \frac{2\,a^2x^2 + 4\,a\arcsin\left(\frac{\sqrt{2}\sqrt{a^2x^2+a}}{2\,\sqrt{a}}\right) + 2\,\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a} + 2\,a - \frac{2\,a^2\arcsin\left(\frac{\sqrt{2}\sqrt{a^2x^2+a}}{2\,\sqrt{a}}\right) - \sqrt{a^2x^2+a}\,(a^2x^2-2\,a)\surd}{a}}{4\,a^3}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="
giac")
```

output
```
1/4*(2*a^2*x^2 + 4*a*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a)) + 2*sqr
t(a^2*x^2 + a)*sqrt(-a^2*x^2 + a) + 2*a - (2*a^2*arcsin(1/2*sqrt(2)*sqrt(a
^2*x^2 + a)/sqrt(a)) - sqrt(a^2*x^2 + a)*(a^2*x^2 - 2*a)*sqrt(-a^2*x^2 + a
))/a)/a^3
```

3.49.9 Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.86

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^3\,dx = \frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{4\,a^2} - \frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)\,1\mathrm{i}}{4\,a^2} + \frac{\frac{1\mathrm{i}}{64\,a^2}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{64\,a^2\,\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+\frac{2\,\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^6}} + \frac{x^2}{2\,a} + \frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{64\,a^2\,\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}$$

input
```
int(x^3*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
(log(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)
/(4*a^2) - (log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1))*
1i)/(4*a^2) + (1i/(64*a^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(32*a^2*(
(1/(a*x^2) + 1)^(1/2) - 1)^2) - (((1/(a*x^2) - 1)^(1/2) - 1i)^4*15i)/(64*a
^2*((1/(a*x^2) + 1)^(1/2) - 1)^4))/(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*
x^2) + 1)^(1/2) - 1)^2 + (2*((1/(a*x^2) - 1)^(1/2) - 1i)^4)/((1/(a*x^2) +
1)^(1/2) - 1)^4 + ((1/(a*x^2) - 1)^(1/2) - 1i)^6/((1/(a*x^2) + 1)^(1/2) -
1)^6) + x^2/(2*a) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(64*a^2*((1/(a*x^2
) + 1)^(1/2) - 1)^2)
```

3.50 $\int e^{\text{sech}^{-1}(ax^2)} x^2 \, dx$

3.50.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int e^{\text{sech}^{-1}(ax^2)} x^2 \, dx = \frac{2x}{3a} + \frac{1}{3} e^{\text{sech}^{-1}(ax^2)} x^3 + \frac{2\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \, \text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right), -1\right)}{3a^{3/2}}$$

output
```
2/3*x/a+1/3*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3+2/3*Elliptic
F(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(3/2)
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int e^{\text{sech}^{-1}(ax^2)} x^2 \, dx = -\frac{2\sqrt{2} e^{-\text{sech}^{-1}(ax^2)} \left(\frac{e^{\text{sech}^{-1}(ax^2)}}{1+e^{2\text{sech}^{-1}(ax^2)}}\right)^{3/2} x \left(-1 - 2e^{2\text{sech}^{-1}(ax^2)} + \left(1+e^{2\text{sech}^{-1}(ax^2)}\right)^{3/2} \text{Hypergeometric2F1}\right.}{3a\sqrt{ax^2}}$$

input
```
Integrate[E^ArcSech[a*x^2]*x^2,x]
```

output
```
(-2*Sqrt[2]*(E^ArcSech[a*x^2]/(1 + E^(2*ArcSech[a*x^2])))^(3/2)*x*(-1 - 2*
E^(2*ArcSech[a*x^2]) + (1 + E^(2*ArcSech[a*x^2]))^(3/2)*Hypergeometric2F1[
1/4, 1/2, 5/4, -E^(2*ArcSech[a*x^2])]))/(3*a*E^ArcSech[a*x^2]*Sqrt[a*x^2])
```

3.50.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6889, 24, 284, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\text{sech}^{-1}\left(ax^2\right)} \, dx$$

$$\Big\downarrow \text{ 6889}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{3a}+\frac{2\int 1dx}{3a}+\frac{1}{3}x^3 e^{\text{sech}^{-1}\left(ax^2\right)}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{3a}+\frac{1}{3}x^3 e^{\text{sech}^{-1}\left(ax^2\right)}+\frac{2x}{3a}$$

$$\Big\downarrow \text{ 284}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{\sqrt{1-a^2x^4}}dx}{3a}+\frac{1}{3}x^3 e^{\text{sech}^{-1}\left(ax^2\right)}+\frac{2x}{3a}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{3a^{3/2}}+\frac{1}{3}x^3 e^{\text{sech}^{-1}\left(ax^2\right)}+\frac{2x}{3a}$$

input
```
Int[E^ArcSech[a*x^2]*x^2,x]
```

output
```
(2*x)/(3*a) + (E^ArcSech[a*x^2]*x^3)/3 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1
+ a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(3*a^(3/2))
```

3.50.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.50.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,x^2\sqrt{\frac{a\,x^2+1}{a\,x^2}}\left(a^{\frac{5}{2}}x^5-2\,\mathrm{EllipticF}\left(x\sqrt{a},i\right)\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}-x\sqrt{a}\right)}{3\left(x^4a^2-1\right)\sqrt{a}}+\frac{x}{a}$	102

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x,method=_RETURNVERB
OSE)
```

output
```
1/3*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^(5/2)*x^5-2*El
lipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)-x*a^(1/2))/(a^2*x^4-
1)/a^(1/2)+x/a
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^2\,dx = \frac{ax^3\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+3\,x+\frac{2i\,F\left(\arcsin\left(\frac{1}{\sqrt{a}x}\right)\,|\,-1\right)}{\sqrt{a}}}{3\,a}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="
fricas")
```

output
```
1/3*(a*x^3*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 3*x + 2*
I*elliptic_f(arcsin(1/(sqrt(a)*x)), -1)/sqrt(a))/a
```

3.50.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^2\,dx = \frac{\int 1\,dx+\int ax^2\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**2,x)
```

output
```
(Integral(1, x) + Integral(a*x**2*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
)), x))/a
```

3.50.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^2\,dx = \int x^2\left(\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="
maxima")
```

output
```
x/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), x)/a
```

3.50.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,1,1]%%%}+%%%{1,[0,0,0,0,2]%%%} / %%%{1,[0,0,0,0,3
]%%%} Err
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^2\,dx = \int x^2\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)\,dx$$

input
```
int(x^2*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
int(x^2*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.51 $\int e^{\text{sech}^{-1}(ax^2)} x \, dx$

3.51.1 Optimal result

Integrand size = 10, antiderivative size = 68

$$\int e^{\text{sech}^{-1}(ax^2)} x \, dx = \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 - \frac{\sqrt{\frac{1}{1+ax^2}} \sqrt{1+ax^2} \operatorname{arctanh}\left(\sqrt{1-a^2x^4}\right)}{2a} + \frac{\log(x)}{a}$$

output
```
1/2*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2+ln(x)/a-1/2*arctanh(
(-a^2*x^4+1)^(1/2))*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a
```

3.51.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.47

$$\int e^{\text{sech}^{-1}(ax^2)} x \, dx = \frac{\sqrt{\frac{1-ax^2}{1+ax^2}}(1+ax^2) + 2\log(ax^2) - \log\left(1 + \sqrt{\frac{1-ax^2}{1+ax^2}} + ax^2\sqrt{\frac{1-ax^2}{1+ax^2}}\right)}{2a}$$

input
```
Integrate[E^ArcSech[a*x^2]*x,x]
```

output
```
(Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2) + 2*Log[a*x^2] - Log[1 + Sqrt[(
1 - a*x^2)/(1 + a*x^2)] + a*x^2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]])/(2*a)
```

3.51.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6889, 14, 335, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\text{sech}^{-1}(ax^2)}\,dx$$

$$\Big\downarrow \text{ 6889}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a}+\frac{\int \frac{1}{x}dx}{a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{ 335}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x\sqrt{1-a^2x^4}}dx}{a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^4\sqrt{1-a^2x^4}}dx^4}{4a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{\frac{1}{a^2}-\frac{x^8}{a^2}}d\sqrt{1-a^2x^4}}{2a^3}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\text{arctanh}\left(\sqrt{1-a^2x^4}\right)}{2a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}+\frac{\log(x)}{a}$$

input
```
Int[E^ArcSech[a*x^2]*x,x]
```

output
```
(E^ArcSech[a*x^2]*x^2)/2 - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcTanh
[Sqrt[1 - a^2*x^4]])/(2*a) + Log[x]/a
```

3.51.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.51.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\, x^{2} \sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\, \left(\mathrm{csgn}\left(\frac{1}{a}\right) a \sqrt{-\frac{x^{4} a^{2}-1}{a^{2}}}-\ln\left(\frac{2\,\mathrm{csgn}\left(\frac{1}{a}\right) a \sqrt{-\frac{x^{4} a^{2}-1}{a^{2}}}+2}{a^{2} x^{2}}\right)\right) \mathrm{csgn}\left(\frac{1}{a}\right)}{2 a \sqrt{-\frac{x^{4} a^{2}-1}{a^{2}}}}+\frac{\ln\left(x\right)}{a}$	127

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x,method=_RETURNVERBOS
E)
```

output
```
1/2*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(csgn(1/a)*a*(-(a
^2*x^4-1)/a^2)^(1/2)-ln(2*(csgn(1/a)*a*(-(a^2*x^4-1)/a^2)^(1/2)+1)/a^2/x^2
))*csgn(1/a)/a/(-(a^2*x^4-1)/a^2)^(1/2)+ln(x)/a
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(61) = 122$.

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.96

$$\int e^{\mathrm{sech}^{-1}(ax^2)} x\, dx = \frac{2\, a x^{2} \sqrt{\frac{a x^{2}+1}{a x^{2}}} \sqrt{-\frac{a x^{2}-1}{a x^{2}}} - \log\left(a x^{2} \sqrt{\frac{a x^{2}+1}{a x^{2}}} \sqrt{-\frac{a x^{2}-1}{a x^{2}}} + 1\right) + \log\left(a x^{2} \sqrt{\frac{a x^{2}+1}{a x^{2}}} \sqrt{-\frac{a x^{2}-1}{a x^{2}}} - 1\right) + 4 \log\left(x\right)}{4\, a}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="fr
icas")
```

output
```
1/4*(2*a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - log(a*
x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1) + log(a*x^2*
sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1) + 4*log(x))/a
```

3.51.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x\,dx = \frac{\int \frac{1}{x}\,dx + \int ax\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x,x)
```

output
```
(Integral(1/x, x) + Integral(a*x*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)
), x))/a
```

3.51.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x\,dx = \int x\left(\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x, x)/a + log(x)/a
```

3.51.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.51.9 Mupad [B] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.68

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x\,dx=\frac{\ln(x)}{a}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)}{a}+\frac{\frac{5\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1}{\frac{8\,a\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x^2}+1}-1}+\frac{8\,a\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^3}}+\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{8\,a\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)}$$

input
```
int(x*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
log(x)/a - (2*atanh(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) -
1)))/a + ((5*((1/(a*x^2) - 1)^(1/2) - 1i)^2)/((1/(a*x^2) + 1)^(1/2) - 1)^2
 + 1)/((8*a*((1/(a*x^2) - 1)^(1/2) - 1i))/((1/(a*x^2) + 1)^(1/2) - 1) + (8
*a*((1/(a*x^2) - 1)^(1/2) - 1i)^3)/((1/(a*x^2) + 1)^(1/2) - 1)^3) + ((1/(a
*x^2) - 1)^(1/2) - 1i)/(8*a*((1/(a*x^2) + 1)^(1/2) - 1))
```

3.52 $\int e^{\text{sech}^{-1}(ax^2)}\,dx$

3.52.1 Optimal result

Integrand size = 8, antiderivative size = 147

$$\int e^{\text{sech}^{-1}(ax^2)}\,dx = -\frac{2}{ax}+e^{\text{sech}^{-1}(ax^2)}x-\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{ax} - \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}E\left(\arcsin\left(\sqrt{a}x\right)|-1\right)}{\sqrt{a}} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{\sqrt{a}}$$

output
```
-2/a/x+(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x-2*EllipticE(x*a^(1/
2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(1/2)+2*EllipticF(x*a^(1/2),I)
*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(1/2)-2*(1/(a*x^2+1))^(1/2)*(a*x^2+
1)^(1/2)*(-a^2*x^4+1)^(1/2)/a/x
```

3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int e^{\operatorname{sech}^{-1}(ax^2)}\,dx$$
$$= -\frac{1}{ax} + \left(-\frac{1}{ax} - x\right)\sqrt{\frac{1-ax^2}{1+ax^2}} - \frac{2i\sqrt{\frac{1-ax^2}{1+ax^2}}\sqrt{1-a^2x^4}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-a}x\right)\middle|-1\right) - \operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-a}x\right),-1\right)\right)}{\sqrt{-a}\left(-1+ax^2\right)}$$

input
```
Integrate[E^ArcSech[a*x^2],x]
```

output
```
-(1/(a*x)) + (-(1/(a*x)) - x)*Sqrt[(1 - a*x^2)/(1 + a*x^2)] - ((2*I)*Sqrt[
(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[-a]*x
], -1] - EllipticF[I*ArcSinh[Sqrt[-a]*x], -1]))/(Sqrt[-a]*(-1 + a*x^2))
```

3.52.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6884, 15, 335, 847, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}\,dx$$
$$\Big\downarrow \text{6884}$$
$$\frac{2\int \frac{1}{x^2}dx}{a} + \frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^2\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a} + xe^{\operatorname{sech}^{-1}(ax^2)}$$
$$\Big\downarrow \text{15}$$
$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^2\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a} + xe^{\operatorname{sech}^{-1}(ax^2)} - \frac{2}{ax}$$
$$\Big\downarrow \text{335}$$
$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^2\sqrt{1-a^2x^4}}dx}{a} + xe^{\operatorname{sech}^{-1}(ax^2)} - \frac{2}{ax}$$

$$\Big\downarrow \ 847$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(a^2\left(-\int \frac{x^2}{\sqrt{1-a^2x^4}}dx\right)-\frac{\sqrt{1-a^2x^4}}{x}\right)}{a}+xe^{\text{sech}^{-1}(ax^2)}-\frac{2}{ax}$$

$$\Big\downarrow \ 836$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(-\left(a^2\left(\frac{\int \frac{ax^2+1}{\sqrt{1-a^2x^4}}dx}{a}-\frac{\int \frac{1}{\sqrt{1-a^2x^4}}dx}{a}\right)\right)-\frac{\sqrt{1-a^2x^4}}{x}\right)}{a}+xe^{\text{sech}^{-1}(ax^2)}-\frac{2}{ax}$$

$$\Big\downarrow \ 762$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(-\left(a^2\left(\frac{\int \frac{ax^2+1}{\sqrt{1-a^2x^4}}dx}{a}-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right)\right)-\frac{\sqrt{1-a^2x^4}}{x}\right)}{a}+xe^{\text{sech}^{-1}(ax^2)}-\frac{2}{ax}$$

$$\Big\downarrow \ 1388$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(-\left(a^2\left(\frac{\int \frac{\sqrt{ax^2+1}}{\sqrt{1-ax^2}}dx}{a}-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right)\right)-\frac{\sqrt{1-a^2x^4}}{x}\right)}{a}+xe^{\text{sech}^{-1}(ax^2)}-\frac{2}{ax}$$

$$\Big\downarrow \ 327$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(-\frac{\sqrt{1-a^2x^4}}{x}-\left(a^2\left(\frac{E\left(\arcsin\left(\sqrt{a}x\right)|-1\right)}{a^{3/2}}-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right)\right)\right)}{a}+xe^{\text{sech}^{-1}(ax^2)}-\frac{2}{ax}$$

input
```
Int[E^ArcSech[a*x^2],x]
```

output
```
-2/(a*x) + E^ArcSech[a*x^2]*x + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*
(-(Sqrt[1 - a^2*x^4]/x) - a^2*(EllipticE[ArcSin[Sqrt[a]*x], -1]/a^(3/2) -
EllipticF[ArcSin[Sqrt[a]*x], -1]/a^(3/2))))/a
```

3.52. $\int e^{\text{sech}^{-1}(ax^2)}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 6884
```
Int[E^ArcSech[(a_.)*(x_)^(p_)], x_Symbol] :> Simp[x*E^ArcSech[a*x^p], x] +
(Simp[p/a   Int[1/x^p, x], x] + Simp[p*(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^
p)]   Int[1/(x^p*Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, p},
 x]
```

3.52.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

method	result
default	$-\frac{1}{ax}-\frac{\sqrt{-\frac{a x^2-1}{a x^2}}\, x \sqrt{\frac{a x^2+1}{a x^2}}\left(x^4 a^2+2\sqrt{-a x^2+1}\,\sqrt{a x^2+1}\, x\, \mathrm{EllipticF}\left(x\sqrt{a}, i\right)\sqrt{a}-2\sqrt{-a x^2+1}\,\sqrt{a x^2+1}\, x\, \mathrm{EllipticE}\left(x\sqrt{a}, i\right)\sqrt{a}-\right.}{x^4 a^2-1}$

input
```
int(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x-(-(a*x^2-1)/a/x^2)^(1/2)*x*((a*x^2+1)/a/x^2)^(1/2)*(x^4*a^2+2*(-a*x
^2+1)^(1/2)*(a*x^2+1)^(1/2)*x*EllipticF(x*a^(1/2),I)*a^(1/2)-2*(-a*x^2+1)^
(1/2)*(a*x^2+1)^(1/2)*x*EllipticE(x*a^(1/2),I)*a^(1/2)-1)/(a^2*x^4-1)
```

3.52.5 Fricas [F]

$$\int e^{\operatorname{sech}^{-1}\left(ax^2\right)}\, dx = \int \sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\, dx$$

input
```
integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="fricas
")
```

output
```
integral((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/
(a*x^2), x)
```

3.52.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}\,dx = \frac{\int \frac{1}{x^2}\,dx + \int a\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate(1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2),x)
```

output
```
(Integral(x**(-2), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**
2)), x))/a
```

3.52.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}\,dx = \int \sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\,dx$$

input
```
integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^2, x)/a - 1/(a*x)
```

3.52.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}\,dx = \int \sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\,dx$$

input
```
integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}\,dx = \int \sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\,dx$$

input
```
int((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2),x)
```

output
```
int((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2), x)
```

3.53 $\int \frac{e^{\text{sech}^{-1}\left(ax^2\right)}}{x}\,dx$

3.53.1 Optimal result

Integrand size = 12, antiderivative size = 80

$$\int \frac{e^{\text{sech}^{-1}\left(ax^2\right)}}{x}\,dx=-\frac{1}{2ax^2}-\frac{\sqrt{1-ax^2}}{2ax^2\sqrt{\frac{1}{1+ax^2}}}-\frac{1}{2}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\arcsin\left(ax^2\right)$$

output
```
-1/2/a/x^2-1/2*(-a*x^2+1)^(1/2)/a/x^2/(1/(a*x^2+1))^(1/2)-1/2*arcsin(a*x^2
)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.28

$$\int \frac{e^{\text{sech}^{-1}\left(ax^2\right)}}{x}\,dx=-\frac{1}{2}e^{\text{sech}^{-1}\left(ax^2\right)}+\arctan\left(e^{\text{sech}^{-1}\left(ax^2\right)}\right)$$

input
```
Integrate[E^ArcSech[a*x^2]/x,x]
```

output
```
-1/2*E^ArcSech[a*x^2] + ArcTan[E^ArcSech[a*x^2]]
```

3.53.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6888, 335, 807, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}\,dx$$

$$\Big\downarrow \text{6888}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{\sqrt{1-ax^2}\sqrt{ax^2+1}}{x^3}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{335}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{\sqrt{1-a^2x^4}}{x^3}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{\sqrt{1-a^2x^4}}{x^4}dx^2}{2a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{247}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(a^2\left(-\int \frac{1}{\sqrt{1-a^2x^4}}dx^2\right)-\frac{\sqrt{1-a^2x^4}}{x^2}\right)}{2a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(-\frac{\sqrt{1-a^2x^4}}{x^2}-a\arcsin\left(ax^2\right)\right)}{2a}-\frac{1}{2ax^2}$$

input
```
Int[E^ArcSech[a*x^2]/x,x]
```

output
```
-1/2*1/(a*x^2) + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*(-(Sqrt[1 - a^2*x
^4]/x^2) - a*ArcSin[a*x^2]))/(2*a)
```

3.53.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6888
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Simp[(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^p)]   Int[Sqrt[1 + a*x^p]*(Sqrt
[1 - a*x^p]/x^(p + 1)), x], x] /; FreeQ[{a, p}, x]
```

3.53.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(\arctan\left(\frac{x^{2}}{\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}}\right)x^{2}+\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}\right)}{2\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}}-\frac{1}{2a\,x^{2}}$	103

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x,method=_RETURNVERBOS
E)
```

output
```
-1/2*(-(a*x^2-1)/a/x^2)^(1/2)*((a*x^2+1)/a/x^2)^(1/2)*(arctan(x^2/(-(a^2*x
^4-1)/a^2)^(1/2))*x^2+(-(a^2*x^4-1)/a^2)^(1/2))/(-(a^2*x^4-1)/a^2)^(1/2)-1
/2/a/x^2
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(44) = 88$.

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x}\,dx = -\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} - 2\,ax^2\arctan\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-1}{ax^2}\right)+1}{2\,ax^2}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="fr
icas")
```

output
```
-1/2*(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 2*a*x^2
*arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1)/(
a*x^2)) + 1)/(a*x^2)
```

3.53.6 Sympy [F]

$$\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x}\,dx = \frac{\int \frac{1}{x^3}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}}{x}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x,x)
```

output
```
(Integral(x**(-3), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**
2))/x, x))/a
```

3.53.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}\,dx = \int \frac{\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}}{x}\,dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^3, x)/a - 1/2/(a*x^2)
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(44) = 88$.

Time = 1.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.15

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}\,dx = -\frac{\left(\pi+2\arctan\left(\frac{\sqrt{a^2x^2+a}\left(\frac{\left(\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}\right)^2}{a^2x^2+a}-1\right)}{2\left(\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}\right)}\right)\right)a^3+\frac{4a^3\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}}{\sqrt{a^2x^2+a}}-\frac{\sqrt{a^2x^2+a}}{\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}}\right)}{\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}}{\sqrt{a^2x^2+a}}-\frac{\sqrt{a^2x^2+a}}{\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}}\right)^2-4}+\frac{a^2}{x^2}}{2a^3}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="gi
ac")
```

output
```
-1/2*((pi + 2*arctan(1/2*sqrt(a^2*x^2 + a)*((sqrt(2)*sqrt(a) - sqrt(-a^2*x
^2 + a))^2/(a^2*x^2 + a) - 1)/(sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a))))*a^3
 + 4*a^3*((sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a))/sqrt(a^2*x^2 + a) - sqrt(
a^2*x^2 + a)/(sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a)))/(((sqrt(2)*sqrt(a) -
sqrt(-a^2*x^2 + a))/sqrt(a^2*x^2 + a) - sqrt(a^2*x^2 + a)/(sqrt(2)*sqrt(a)
 - sqrt(-a^2*x^2 + a)))^2 - 4) + a^2/x^2)/a^3
```

3.53.9 Mupad [B] (verification not implemented)

Time = 7.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.31

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}\,dx = -\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2} + \frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)\,1\mathrm{i}}{2} - \frac{1}{2\,a\,x^2} + \frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2 8\mathrm{i}}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2\left(2+\frac{2\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}\right)}$$

input
```
int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x,x)
```

output
```
(log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1))*1i)/2 - (lo
g(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)/2
- 1/(2*a*x^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*8i)/(((1/(a*x^2) + 1)^(1/2
) - 1)^2*((2*((1/(a*x^2) - 1)^(1/2) - 1i)^4)/((1/(a*x^2) + 1)^(1/2) - 1)^4
 - (4*((1/(a*x^2) - 1)^(1/2) - 1i)^2)/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 2))
```

3.54 $\int \frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{x^2}\,dx$

3.54.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2}\,dx = \frac{2}{3ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{3ax^3} - \frac{2}{3}\sqrt{a}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)$$

output
```
2/3/a/x^3-(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x-2/3*EllipticF(x*
a^(1/2),I)*a^(1/2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)+2/3*(1/(a*x^2+1))^(
1/2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a/x^3
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2}\,dx = -\frac{a\sqrt{1+e^{2\operatorname{sech}^{-1}(ax^2)}}\sqrt{\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2+2e^{2\operatorname{sech}^{-1}(ax^2)}}}\,x\left(\sqrt{1+e^{2\operatorname{sech}^{-1}(ax^2)}} - 4\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2\operatorname{sech}^{-1}(ax^2)}\right.\right.}{3\sqrt{ax^2}}$$

input
```
Integrate[E^ArcSech[a*x^2]/x^2,x]
```

output
```
-1/3*(a*Sqrt[1 + E^(2*ArcSech[a*x^2])]*Sqrt[E^ArcSech[a*x^2]/(2 + 2*E^(2*A
rcSech[a*x^2]))]*x*(Sqrt[1 + E^(2*ArcSech[a*x^2])] - 4*Hypergeometric2F1[1
/4, 1/2, 5/4, -E^(2*ArcSech[a*x^2])]))/Sqrt[a*x^2]
```

3.54.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6889, 15, 335, 847, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x^2}dx$$

$$\Big\downarrow \text{6889}$$

$$-\frac{2\int \frac{1}{x^4}dx}{a}-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^4\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a}-\frac{e^{\text{sech}^{-1}(ax^2)}}{x}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^4\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a}+\frac{2}{3ax^3}-\frac{e^{\text{sech}^{-1}(ax^2)}}{x}$$

$$\Big\downarrow \text{335}$$

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^4\sqrt{1-a^2x^4}}dx}{a}+\frac{2}{3ax^3}-\frac{e^{\text{sech}^{-1}(ax^2)}}{x}$$

$$\Big\downarrow \text{847}$$

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{1}{3}a^2\int \frac{1}{\sqrt{1-a^2x^4}}dx-\frac{\sqrt{1-a^2x^4}}{3x^3}\right)}{a}+\frac{2}{3ax^3}-\frac{e^{\text{sech}^{-1}(ax^2)}}{x}$$

$$\Big\downarrow \text{762}$$

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{1}{3}a^{3/2}\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)-\frac{\sqrt{1-a^2x^4}}{3x^3}\right)}{a}+\frac{2}{3ax^3}-\frac{e^{\text{sech}^{-1}(ax^2)}}{x}$$

input
```
Int[E^ArcSech[a*x^2]/x^2,x]
```

output
```
2/(3*a*x^3) - E^ArcSech[a*x^2]/x - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^
2]*(-1/3*Sqrt[1 - a^2*x^4]/x^3 + (a^(3/2)*EllipticF[ArcSin[Sqrt[a]*x], -1]
)/3))/a
```

3.54.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.54.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(2\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\ \mathrm{EllipticF}\left(x\sqrt{a},i\right)x^{3}a^{\frac{3}{2}}-x^{4}a^{2}+1\right)}{3x\left(x^{4}a^{2}-1\right)}-\frac{1}{3a\,x^{3}}$	104

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x,method=_RETURNVERB
OSE)
```

output
```
1/3*(-(a*x^2-1)/a/x^2)^(1/2)/x*((a*x^2+1)/a/x^2)^(1/2)*(2*(-a*x^2+1)^(1/2)
*(a*x^2+1)^(1/2)*EllipticF(x*a^(1/2),I)*x^3*a^(3/2)-x^4*a^2+1)/(a^2*x^4-1)
-1/3/a/x^3
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.56

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2}\,dx = -\frac{2\,a^{\frac{3}{2}}x^3F(\arcsin\left(\sqrt{a}x\right)\mid -1)+ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+1}{3\,ax^3}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="
fricas")
```

output
```
-1/3*(2*a^(3/2)*x^3*elliptic_f(arcsin(sqrt(a)*x), -1) + a*x^2*sqrt((a*x^2
+ 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/(a*x^3)
```

3.54.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2}\,dx = \frac{\int \frac{1}{x^4}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}}{x^2}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x**2,x)
```

output
```
(Integral(x**(-4), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**
2))/x**2, x))/a
```

3.54.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}}{x^2}\,dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^4, x)/a - 1/3/(a*x^3)
```

3.54.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}}{x^2}\,dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="
giac")
```

output
```
integrate((sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2))/x^2, x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}}{x^2}\,dx$$

input
```
int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^2,x)
```

output
```
int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^2, x)
```

3.55 $\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x^3}\,dx$

3.55.1 Optimal result

Integrand size = 12, antiderivative size = 118

$$\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x^3}\,dx = \frac{1}{4ax^4} - \frac{e^{\text{sech}^{-1}(ax^2)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} + \frac{1}{4}a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\operatorname{arctanh}\left(\sqrt{1-a^2x^4}\right)$$

output
```
1/4/a/x^4-1/2*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2+1/4*a*arct
anh((-a^2*x^4+1)^(1/2))*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)+1/4*(1/(a*x^2+
1))^(1/2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a/x^4
```

3.55.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x^3}\,dx = -\frac{\frac{1}{x^4} + \frac{\sqrt{\frac{1-ax^2}{1+ax^2}}(1+ax^2)}{x^4} - \frac{a^2\sqrt{\frac{1-ax^2}{1+ax^2}}(1+ax^2)\arctan\left(\sqrt{-1+a^2x^4}\right)}{\sqrt{-1+a^2x^4}}}{4a}$$

input
```
Integrate[E^ArcSech[a*x^2]/x^3,x]
```

output
```
-1/4*(x^(-4) + (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2))/x^4 - (a^2*Sqrt
[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)*ArcTan[Sqrt[-1 + a^2*x^4]])/Sqrt[-1
+ a^2*x^4])/a
```

3.55.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6889, 15, 335, 798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x^3}dx$$

$$\Big\downarrow \text{6889}$$

$$-\frac{\int \frac{1}{x^5}dx}{a}-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^5\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a}-\frac{e^{\text{sech}^{-1}(ax^2)}}{2x^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^5\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a}+\frac{1}{4ax^4}-\frac{e^{\text{sech}^{-1}(ax^2)}}{2x^2}$$

$$\Big\downarrow \text{335}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^5\sqrt{1-a^2x^4}}dx}{a}+\frac{1}{4ax^4}-\frac{e^{\text{sech}^{-1}(ax^2)}}{2x^2}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{1}{x^8\sqrt{1-a^2x^4}}dx^4}{4a}+\frac{1}{4ax^4}-\frac{e^{\text{sech}^{-1}(ax^2)}}{2x^2}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(\frac{1}{2}a^2\int \frac{1}{x^4\sqrt{1-a^2x^4}}dx^4-\frac{\sqrt{1-a^2x^4}}{x^4}\right)}{4a}+\frac{1}{4ax^4}-\frac{e^{\text{sech}^{-1}(ax^2)}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(-\int \frac{1}{\frac{1}{a^2}-\frac{x^8}{a^2}}d\sqrt{1-a^2x^4}-\frac{\sqrt{1-a^2x^4}}{x^4}\right)}{4a}+\frac{1}{4ax^4}-\frac{e^{\text{sech}^{-1}(ax^2)}}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^4}\right)\right)-\frac{\sqrt{1-a^2x^4}}{x^4}\right)}{4a}+\frac{1}{4ax^4}-\frac{e^{\text{sech}^{-1}(ax^2)}}{2x^2}$$

input
```
Int[E^ArcSech[a*x^2]/x^3,x]
```

output
```
1/(4*a*x^4) - E^ArcSech[a*x^2]/(2*x^2) - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 +
a*x^2]*(-(Sqrt[1 - a^2*x^4]/x^4) - a^2*ArcTanh[Sqrt[1 - a^2*x^4]]))/(4*a)
```

3.55.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.55.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(\ln\left(\frac{2\,\mathrm{csgn}\left(\frac{1}{a}\right)a\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}+2}{a^{2}x^{2}}\right)x^{4}a-\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}\,\mathrm{csgn}\left(\frac{1}{a}\right)\right)\mathrm{csgn}\left(\frac{1}{a}\right)}{4x^{2}\sqrt{-\frac{x^{4}a^{2}-1}{a^{2}}}}-\frac{1}{4a\,x^{4}}$	129

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x,method=_RETURNVERB
OSE)
```

output
```
1/4*(-(a*x^2-1)/a/x^2)^(1/2)/x^2*((a*x^2+1)/a/x^2)^(1/2)*(ln(2*(csgn(1/a)*
a*(-(a^2*x^4-1)/a^2)^(1/2)+1)/a^2/x^2)*x^4*a-(-(a^2*x^4-1)/a^2)^(1/2)*csgn
(1/a))*csgn(1/a)/(-(a^2*x^4-1)/a^2)^(1/2)-1/4/a/x^4
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^3}\,dx$$
$$= \frac{a^2x^4\log\left(ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+1\right)-a^2x^4\log\left(ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-1\right)-2\,ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-}{8\,ax^4}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="
fricas")
```

output
```
1/8*(a^2*x^4*log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)
) + 1) - a^2*x^4*log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*
x^2)) - 1) - 2*a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2))
- 2)/(a*x^4)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^3}\,dx = -\frac{a\left(2\sqrt{-1+\frac{1}{ax^2}}\left(\frac{\left(1+\frac{1}{ax^2}\right)^{\frac{3}{2}}}{4}-\frac{\sqrt{1+\frac{1}{ax^2}}}{4}\right)-\log\left(2\sqrt{-1+\frac{1}{ax^2}}+2\sqrt{1+\frac{1}{ax^2}}\right)\right)}{2}-\frac{1}{4ax^4}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x**3,x)
```

output
```
-a*(2*sqrt(-1 + 1/(a*x**2))*((1 + 1/(a*x**2))**(3/2)/4 - sqrt(1 + 1/(a*x**
2))/4) - log(2*sqrt(-1 + 1/(a*x**2)) + 2*sqrt(1 + 1/(a*x**2))))/2 - 1/(4*a
*x**4)
```

3.55.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^3}\,dx = \int \frac{\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}}{x^3}\,dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="
maxima")
```

output
```
integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^5, x)/a - 1/4/(a*x^4)
```

3.55.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.55.9 Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^3}\,dx = \frac{a\,\ln\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)}{4} - \frac{1}{4\,a\,x^4} - \frac{\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}}{4\,x^2}$$

input
```
int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^3,x)
```

output
```
(a*log((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)))/4 - 1/(4*
a*x^4) - ((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2))/(4*x^2)
```

3.56 $\int e^{\text{sech}^{-1}(ax^3)}x^m\,dx$

3.56.1 Optimal result

Integrand size = 12, antiderivative size = 109

$$\int e^{\text{sech}^{-1}(ax^3)}x^m\,dx = -\frac{3x^{-2+m}}{a(2+m-m^2)}+\frac{e^{\text{sech}^{-1}(ax^3)}x^{1+m}}{1+m} - \frac{3x^{-2+m}\sqrt{\frac{1}{1+ax^3}}\sqrt{1+ax^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-2+m),\frac{4+m}{6},a^2x^6\right)}{a(2+m-m^2)}$$

output
```
-3*x^(-2+m)/a/(-m^2+m+2)+(1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^(
1+m)/(1+m)-3*x^(-2+m)*hypergeom([1/2, -1/3+1/6*m],[2/3+1/6*m],a^2*x^6)*(1/
(a*x^3+1))^(1/2)*(a*x^3+1)^(1/2)/a/(-m^2+m+2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.72

$$\int e^{\text{sech}^{-1}(ax^3)}x^m\,dx = \frac{2^{\frac{1+m}{3}}e^{\text{sech}^{-1}(ax^3)}\left(\frac{e^{\text{sech}^{-1}(ax^3)}}{1+e^{2\text{sech}^{-1}(ax^3)}}\right)^{\frac{1+m}{3}}\left(1+e^{2\text{sech}^{-1}(ax^3)}\right)^{\frac{1+m}{3}}x^{1+m}(ax^3)^{\frac{1}{3}(-1-m)}\left((10+m)\,\text{Hypergeometric}\right.}{(4 -}$$

input
```
Integrate[E^ArcSech[a*x^3]*x^m,x]
```

output
```
(2^((1 + m)/3)*E^ArcSech[a*x^3]*(E^ArcSech[a*x^3]/(1 + E^(2*ArcSech[a*x^3]
)))^((1 + m)/3)*(1 + E^(2*ArcSech[a*x^3]))^((1 + m)/3)*x^(1 + m)*(a*x^3)^(
(-1 - m)/3)*((10 + m)*Hypergeometric2F1[(4 + m)/6, (4 + m)/3, (10 + m)/6,
-E^(2*ArcSech[a*x^3])] - E^(2*ArcSech[a*x^3])*(4 + m)*Hypergeometric2F1[(4
 + m)/3, (10 + m)/6, (16 + m)/6, -E^(2*ArcSech[a*x^3])]))/((4 + m)*(10 + m
))
```

3.56.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6889, 15, 791, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\text{sech}^{-1}(ax^3)} dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{3\int x^{m-3}dx}{a(m+1)}+\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}\int \frac{x^{m-3}}{\sqrt{1-ax^3}\sqrt{ax^3+1}}dx}{a(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^3)}}{m+1}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}\int \frac{x^{m-3}}{\sqrt{1-ax^3}\sqrt{ax^3+1}}dx}{a(m+1)}-\frac{3x^{m-2}}{a(2-m)(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^3)}}{m+1}$$

$$\Big\downarrow \text{791}$$

$$\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}\int \frac{x^{m-3}}{\sqrt{1-a^2x^6}}dx}{a(m+1)}-\frac{3x^{m-2}}{a(2-m)(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^3)}}{m+1}$$

$$\Big\downarrow \text{888}$$

$$-\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}x^{m-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-2}{6},\frac{m+4}{6},a^2x^6\right)}{a(2-m)(m+1)}-\frac{3x^{m-2}}{a(2-m)(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^3)}}{m+1}$$

input
```
Int[E^ArcSech[a*x^3]*x^m,x]
```

output
```
(-3*x^(-2 + m))/(a*(2 - m)*(1 + m)) + (E^ArcSech[a*x^3]*x^(1 + m))/(1 + m)
 - (3*x^(-2 + m)*Sqrt[(1 + a*x^3)^(-1)]*Sqrt[1 + a*x^3]*Hypergeometric2F1[
1/2, (-2 + m)/6, (4 + m)/6, a^2*x^6])/(a*(2 - m)*(1 + m))
```

3.56.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 791
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.56.4 Maple [F]

$$\int \left(\frac{1}{a\,x^{3}}+\sqrt{\frac{1}{a\,x^{3}}-1}\,\sqrt{\frac{1}{a\,x^{3}}+1}\right) x^{m}dx$$

input
```
int((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x)
```

output
```
int((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x)
```

3.56.5 Fricas [F]

$$\int e^{\operatorname{sech}^{-1}(ax^3)}x^m\,dx = \int x^m\left(\sqrt{\frac{1}{ax^3}+1}\sqrt{\frac{1}{ax^3}-1}+\frac{1}{ax^3}\right)dx$$

input
```
integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="
fricas")
```

output
```
integral((a*x^3*x^m*sqrt((a*x^3 + 1)/(a*x^3))*sqrt(-(a*x^3 - 1)/(a*x^3)) +
 x^m)/(a*x^3), x)
```

3.56.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^3)}x^m\,dx = \frac{\int \frac{x^m}{x^3}\,dx + \int ax^m\sqrt{-1+\frac{1}{ax^3}}\sqrt{1+\frac{1}{ax^3}}\,dx}{a}$$

input
```
integrate((1/a/x**3+(1/a/x**3-1)**(1/2)*(1/a/x**3+1)**(1/2))*x**m,x)
```

output
```
(Integral(x**m/x**3, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x**3))*sqrt(1 + 1
/(a*x**3)), x))/a
```

3.56.7 Maxima [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^3)}x^m\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-3>0)', see `assume?` for more
details)Is
```

3.56.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}(ax^3)}x^m\,dx = \int x^m\left(\sqrt{\frac{1}{ax^3}+1}\sqrt{\frac{1}{ax^3}-1}+\frac{1}{ax^3}\right)dx$$

input
```
integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="
giac")
```

output
```
integrate(x^m*(sqrt(1/(a*x^3) + 1)*sqrt(1/(a*x^3) - 1) + 1/(a*x^3)), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^3)}x^m\,dx = \int x^m\left(\sqrt{\frac{1}{a\,x^3}-1}\,\sqrt{\frac{1}{a\,x^3}+1}+\frac{1}{a\,x^3}\right)dx$$

input
```
int(x^m*((1/(a*x^3) - 1)^(1/2)*(1/(a*x^3) + 1)^(1/2) + 1/(a*x^3)),x)
```

output
```
int(x^m*((1/(a*x^3) - 1)^(1/2)*(1/(a*x^3) + 1)^(1/2) + 1/(a*x^3)), x)
```

3.57 $\int e^{\text{sech}^{-1}(ax^2)} x^m \, dx$

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 107

$$\int e^{\text{sech}^{-1}(ax^2)} x^m \, dx = -\frac{2x^{-1+m}}{a(1-m^2)} + \frac{e^{\text{sech}^{-1}(ax^2)} x^{1+m}}{1+m} - \frac{2x^{-1+m}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+m),\frac{3+m}{4},a^2x^4\right)}{a(1-m^2)}$$

output
```
-2*x^(-1+m)/a/(-m^2+1)+(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^(1+
m)/(1+m)-2*x^(-1+m)*hypergeom([1/2, -1/4+1/4*m],[3/4+1/4*m],a^2*x^4)*(1/(a
*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a/(-m^2+1)
```

3.57.2 Mathematica [A] (warning: unable to verify)

Time = 2.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.49

$$\int e^{\text{sech}^{-1}(ax^2)} x^m \, dx = \frac{2^{\frac{1+m}{2}} e^{\text{sech}^{-1}(ax^2)} \left(\frac{e^{\text{sech}^{-1}(ax^2)}}{1+e^{2\text{sech}^{-1}(ax^2)}}\right)^{\frac{1+m}{2}} x^{1+m}(ax^2)^{\frac{1}{2}(-1-m)} \left((7+m)\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{7+m}{4},-e^{2\text{se}}\right.\right.}{(3+m)(7+m)}$$

input
```
Integrate[E^ArcSech[a*x^2]*x^m,x]
```

output
```
(2^((1 + m)/2)*E^ArcSech[a*x^2]*(E^ArcSech[a*x^2]/(1 + E^(2*ArcSech[a*x^2]
)))^((1 + m)/2)*x^(1 + m)*(a*x^2)^((-1 - m)/2)*((7 + m)*Hypergeometric2F1[
1, (1 - m)/4, (7 + m)/4, -E^(2*ArcSech[a*x^2])] - E^(2*ArcSech[a*x^2])*(3
+ m)*Hypergeometric2F1[1, (5 - m)/4, (11 + m)/4, -E^(2*ArcSech[a*x^2])]))/
((3 + m)*(7 + m))
```

3.57.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6889, 15, 335, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\text{sech}^{-1}(ax^2)} \, dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{2\int x^{m-2}dx}{a(m+1)}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^{m-2}}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^2)}}{m+1}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^{m-2}}{\sqrt{1-ax^2}\sqrt{ax^2+1}}dx}{a(m+1)}-\frac{2x^{m-1}}{a(1-m)(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^2)}}{m+1}$$

$$\Big\downarrow \text{335}$$

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\int \frac{x^{m-2}}{\sqrt{1-a^2x^4}}dx}{a(m+1)}-\frac{2x^{m-1}}{a(1-m)(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^2)}}{m+1}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}x^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-1}{4},\frac{m+3}{4},a^2x^4\right)}{a(1-m)(m+1)}-\frac{2x^{m-1}}{a(1-m)(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^2)}}{m+1}$$

input
```
Int[E^ArcSech[a*x^2]*x^m,x]
```

output
```
(-2*x^(-1 + m))/(a*(1 - m)*(1 + m)) + (E^ArcSech[a*x^2]*x^(1 + m))/(1 + m)
 - (2*x^(-1 + m)*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Hypergeometric2F1[
1/2, (-1 + m)/4, (3 + m)/4, a^2*x^4])/(a*(1 - m)*(1 + m))
```

3.57.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.57.4 Maple [F]

$$\int \left(\frac{1}{a\,x^{2}}+\sqrt{\frac{1}{a\,x^{2}}-1}\,\sqrt{\frac{1}{a\,x^{2}}+1}\right) x^{m}dx$$

input
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x)
```

output
```
int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x)
```

3.57.5 Fricas [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^m\,dx=\int x^m\left(\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="
fricas")
```

output
```
integral((a*x^2*x^m*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) +
 x^m)/(a*x^2), x)
```

3.57.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^m\,dx=\frac{\int \frac{x^m}{x^2}\,dx+\int ax^m\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

input
```
integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**m,x)
```

output
```
(Integral(x**m/x**2, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1
/(a*x**2)), x))/a
```

3.57.7 Maxima [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^m\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

3.57.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^2)} x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^2)} x^m\,dx = \int x^m \left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)\,dx$$

input
```
int(x^m*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
int(x^m*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.58 $\int e^{\text{sech}^{-1}(ax)} x^m \, dx$

3.58.1 Optimal result

Integrand size = 10, antiderivative size = 91

$$\int e^{\text{sech}^{-1}(ax)} x^m \, dx = \frac{x^m}{am(1+m)} + \frac{e^{\text{sech}^{-1}(ax)} x^{1+m}}{1+m} + \frac{x^m \sqrt{\frac{1}{1+ax}} \sqrt{1+ax} \, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, a^2x^2\right)}{am(1+m)}$$

output
```
x^m/a/m/(1+m)+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^(1+m)/(1+m)+x^m*hy
pergeom([1/2, 1/2*m],[1+1/2*m],a^2*x^2)*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)/a/
m/(1+m)
```

3.58.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.59

$$\int e^{\text{sech}^{-1}(ax)} x^m \, dx = -\frac{2^{1+m} e^{2\text{sech}^{-1}(ax)} \left(\frac{e^{\text{sech}^{-1}(ax)}}{1+e^{2\text{sech}^{-1}(ax)}}\right)^m \left(1+e^{2\text{sech}^{-1}(ax)}\right)^m x^m (ax)^{-m} \left(-\left((4+m)\,\text{Hypergeometric2F1}\left(1+\right.\right.\right.}{a(2}$$

input
```
Integrate[E^ArcSech[a*x]*x^m,x]
```

output
```
-((2^(1 + m)*E^(2*ArcSech[a*x])*(E^ArcSech[a*x]/(1 + E^(2*ArcSech[a*x])))^
m*(1 + E^(2*ArcSech[a*x]))^m*x^m*(-((4 + m)*Hypergeometric2F1[1 + m/2, 2 +
 m, 2 + m/2, -E^(2*ArcSech[a*x])]) + E^(2*ArcSech[a*x])*(2 + m)*Hypergeome
tric2F1[2 + m/2, 2 + m, 3 + m/2, -E^(2*ArcSech[a*x])]))/(a*(2 + m)*(4 + m)
*(a*x)^m))
```

3.58.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6889, 15, 135, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\text{sech}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{\int x^{m-1}dx}{a(m+1)}+\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x^{m-1}}{\sqrt{1-ax}\sqrt{ax+1}}dx}{a(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax)}}{m+1}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x^{m-1}}{\sqrt{1-ax}\sqrt{ax+1}}dx}{a(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax)}}{m+1}+\frac{x^m}{am(m+1)}$$

$$\Big\downarrow \text{135}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\int \frac{x^{m-1}}{\sqrt{1-a^2x^2}}dx}{a(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax)}}{m+1}+\frac{x^m}{am(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}x^m \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{m+2}{2},a^2x^2\right)}{am(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax)}}{m+1}+\frac{x^m}{am(m+1)}$$

input
```
Int[E^ArcSech[a*x]*x^m,x]
```

output
```
x^m/(a*m*(1 + m)) + (E^ArcSech[a*x]*x^(1 + m))/(1 + m) + (x^m*Sqrt[(1 + a*
x)^(-1)]*Sqrt[1 + a*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, a^2*x^2])/(a
*m*(1 + m))
```

3.58.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 135
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f, m, n, p},
 x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.58.4 Maple [F]

$$\int \left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^m dx$$

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x)
```

output
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x)
```

3.58.5 Fricas [F]

$$\int e^{\text{sech}^{-1}(ax)}x^m\,dx=\int x^m\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="fricas
")
```

output
```
integral((a*x*x^m*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + x^m)/(a*x
), x)
```

3.58.6 Sympy [F]

$$\int e^{\text{sech}^{-1}(ax)}x^m\,dx=\frac{\int \frac{x^m}{x}\,dx+\int ax^m\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**m,x)
```

output
```
(Integral(x**m/x, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)
), x))/a
```

3.58.7 Maxima [F]

$$\int e^{\text{sech}^{-1}(ax)}x^m\,dx=\int x^m\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^m/x, x)/a + x^m/(a*m)
```

3.58.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}(ax)} x^m \, dx = \int x^m \left(\sqrt{\frac{1}{ax} + 1} \sqrt{\frac{1}{ax} - 1} + \frac{1}{ax} \right) dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax)} x^m \, dx = \int x^m \left(\sqrt{\frac{1}{a\,x} - 1} \sqrt{\frac{1}{a\,x} + 1} + \frac{1}{a\,x} \right) dx$$

input
```
int(x^m*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
int(x^m*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)), x)
```

3.59 $\int e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx$

3.59.1 Optimal result

Integrand size = 12, antiderivative size = 109

$$\int e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx = \frac{e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{x^{2+m}}{a\left(2+3m+m^2\right)} - \frac{\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\, x^{2+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-2-m), -\frac{m}{2}, \frac{a^2}{x^2}\right)}{a\left(2+3m+m^2\right)}$$

output
```
(x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^(1+m)/(1+m)-x^(2+m)/a/(m^2+3*m+2)-x^(
2+m)*hypergeom([1/2, -1-1/2*m],[-1/2*m],a^2/x^2)*(1/(1+a/x))^(1/2)*(1+a/x)
^(1/2)/a/(m^2+3*m+2)
```

3.59.2 Mathematica [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx = -\frac{2^{-1-m} a e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} \left(\frac{e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{1+e^{2\text{sech}^{-1}\left(\frac{a}{x}\right)}}\right)^{-1-m} \left(\frac{a}{x}\right)^m x^m \left(-\left((-2+m)\text{Hypergeometric2F1}\left(1, 1+\frac{m}{2}, 1-\frac{m}{2}, -\right.\right.\right.}{(-2+m)m}$$

input
```
Integrate[E^ArcSech[a/x]*x^m,x]
```

output
```
-((2^(-1 - m)*a*E^ArcSech[a/x]*(E^ArcSech[a/x]/(1 + E^(2*ArcSech[a/x])))^(
-1 - m)*(a/x)^m*x^m*(-((-2 + m)*Hypergeometric2F1[1, 1 + m/2, 1 - m/2, -E^
(2*ArcSech[a/x])]) + E^(2*ArcSech[a/x])*m*Hypergeometric2F1[1, 2 + m/2, 2
- m/2, -E^(2*ArcSech[a/x])]))/((-2 + m)*m))
```

3.59.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6889, 15, 791, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} \, dx$$

$$\Big\downarrow \ 6889$$

$$-\frac{\int x^{m+1}dx}{a(m+1)}-\frac{\sqrt{\frac{1}{\frac{a}{x}+1}}\sqrt{\frac{a}{x}+1}\int \frac{x^{m+1}}{\sqrt{1-\frac{a}{x}}\sqrt{\frac{a}{x}+1}}dx}{a(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

$$\Big\downarrow \ 15$$

$$-\frac{\sqrt{\frac{1}{\frac{a}{x}+1}}\sqrt{\frac{a}{x}+1}\int \frac{x^{m+1}}{\sqrt{1-\frac{a}{x}}\sqrt{\frac{a}{x}+1}}dx}{a(m+1)}-\frac{x^{m+2}}{a(m+1)(m+2)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

$$\Big\downarrow \ 791$$

$$-\frac{\sqrt{\frac{1}{\frac{a}{x}+1}}\sqrt{\frac{a}{x}+1}\int \frac{x^{m+1}}{\sqrt{1-\frac{a^2}{x^2}}}dx}{a(m+1)}-\frac{x^{m+2}}{a(m+1)(m+2)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

$$\Big\downarrow \ 862$$

$$\frac{\sqrt{\frac{1}{\frac{a}{x}+1}}\sqrt{\frac{a}{x}+1}\left(\frac{1}{x}\right)^m x^m \int \frac{\left(\frac{1}{x}\right)^{-m-3}}{\sqrt{1-\frac{a^2}{x^2}}}d\frac{1}{x}}{a(m+1)}-\frac{x^{m+2}}{a(m+1)(m+2)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

$$\Big\downarrow \ 278$$

$$-\frac{\sqrt{\frac{1}{\frac{a}{x}+1}}\sqrt{\frac{a}{x}+1}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-2),-\frac{m}{2},\frac{a^2}{x^2}\right)}{a(m+1)(m+2)}-\frac{x^{m+2}}{a(m+1)(m+2)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

input
```
Int[E^ArcSech[a/x]*x^m,x]
```

output
```
(E^ArcSech[a/x]*x^(1 + m))/(1 + m) - x^(2 + m)/(a*(1 + m)*(2 + m)) - (Sqrt
[(1 + a/x)^(-1)]*Sqrt[1 + a/x]*x^(2 + m)*Hypergeometric2F1[1/2, (-2 - m)/2
, -1/2*m, a^2/x^2])/(a*(1 + m)*(2 + m))
```

3.59.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 791
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.59.4 Maple [F]

$$\int \left(\frac{x}{a}+\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\right)x^m dx$$

input
```
int((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x)
```

output
```
int((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x)
```

3.59.5 Fricas [F]

$$\int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}x^m\,dx=\int x^m\left(\sqrt{\frac{x}{a}+1}\sqrt{\frac{x}{a}-1}+\frac{x}{a}\right)dx$$

input
```
integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="fricas")
```

output
```
integral((a*x^m*sqrt((a + x)/a)*sqrt(-(a - x)/a) + x*x^m)/a, x)
```

3.59.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}x^m\,dx=\frac{\int xx^m\,dx+\int ax^m\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\,dx}{a}$$

input
```
integrate((x/a+(-1+x/a)**(1/2)*(1+x/a)**(1/2))*x**m,x)
```

output
```
(Integral(x*x**m, x) + Integral(a*x**m*sqrt(-1 + x/a)*sqrt(1 + x/a), x))/a
```

3.59.7 Maxima [F]

$$\int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx = \int x^m \left(\sqrt{\frac{x}{a}+1}\sqrt{\frac{x}{a}-1}+\frac{x}{a}\right) dx$$

input
```
integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="maxima")
```

output
```
x^2*x^m/(a*(m + 2)) + integrate(sqrt(a + x)*sqrt(-a + x)*x^m, x)/a
```

3.59.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx = \int x^m \left(\sqrt{\frac{x}{a}+1}\sqrt{\frac{x}{a}-1}+\frac{x}{a}\right) dx$$

input
```
integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*(sqrt(x/a + 1)*sqrt(x/a - 1) + x/a), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx = \int x^m \left(\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}+\frac{x}{a}\right) dx$$

input
```
int(x^m*((x/a - 1)^(1/2)*(x/a + 1)^(1/2) + x/a),x)
```

output
```
int(x^m*((x/a - 1)^(1/2)*(x/a + 1)^(1/2) + x/a), x)
```

3.60 $\int e^{\text{sech}^{-1}(ax^p)}x^m\,dx$

3.60.1 Optimal result

Integrand size = 12, antiderivative size = 133

$$\int e^{\text{sech}^{-1}(ax^p)}x^m\,dx = \frac{e^{\text{sech}^{-1}(ax^p)}x^{1+m}}{1+m}+\frac{px^{1+m-p}}{a(1+m)(1+m-p)} + \frac{px^{1+m-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m-p}{2p},\frac{1+m+p}{2p},a^2x^{2p}\right)}{a(1+m)(1+m-p)}$$

output
```
(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^(1+m)/(1+m)+p*x^(1+m
-p)/a/(1+m)/(1+m-p)+p*x^(1+m-p)*hypergeom([1/2, 1/2*(1+m-p)/p],[1/2*(1+m+p
)/p],a^2*x^(2*p))*(1/(1+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(1+m)/(1+m-p)
```

3.60.2 Mathematica [A] (warning: unable to verify)

Time = 4.33 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.40

$$\int e^{\text{sech}^{-1}(ax^p)}x^m\,dx = \frac{2^{\frac{1+m}{p}}e^{\text{sech}^{-1}(ax^p)}\left(\frac{e^{\text{sech}^{-1}(ax^p)}}{1+e^{2\text{sech}^{-1}(ax^p)}}\right)^{\frac{1+m}{p}}x^{1+m}(ax^p)^{-\frac{1+m}{p}}\left(-e^{2\text{sech}^{-1}(ax^p)}(1+m+p)\,\text{Hypergeometric2F1}\left(1,-\right.\right.}{(1+m+p)}$$

input
```
Integrate[E^ArcSech[a*x^p]*x^m,x]
```

output
```
(2^((1 + m)/p)*E^ArcSech[a*x^p]*(E^ArcSech[a*x^p]/(1 + E^(2*ArcSech[a*x^p]
)))^((1 + m)/p)*x^(1 + m)*(-(E^(2*ArcSech[a*x^p])*(1 + m + p)*Hypergeometr
ic2F1[1, -1/2*(1 + m - 3*p)/p, (1 + m + 5*p)/(2*p), -E^(2*ArcSech[a*x^p])]
) + (1 + m + 3*p)*Hypergeometric2F1[1, 1 - (1 + m + p)/(2*p), (1 + m + 3*p
)/(2*p), -E^(2*ArcSech[a*x^p])]))/((1 + m + p)*(1 + m + 3*p)*(a*x^p)^((1 +
 m)/p))
```

3.60.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6889, 15, 791, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\text{sech}^{-1}(ax^p)} \, dx$$

$$\Big\downarrow \ 6889$$

$$\frac{p\int x^{m-p}dx}{a(m+1)}+\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{m-p}}{\sqrt{1-ax^p}\sqrt{ax^p+1}}dx}{a(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^p)}}{m+1}$$

$$\Big\downarrow \ 15$$

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{m-p}}{\sqrt{1-ax^p}\sqrt{ax^p+1}}dx}{a(m+1)}+\frac{px^{m-p+1}}{a(m+1)(m-p+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^p)}}{m+1}$$

$$\Big\downarrow \ 791$$

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{m-p}}{\sqrt{1-a^2x^{2p}}}dx}{a(m+1)}+\frac{px^{m-p+1}}{a(m+1)(m-p+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^p)}}{m+1}$$

$$\Big\downarrow \ 888$$

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}x^{m-p+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-p+1}{2p},\frac{m+p+1}{2p},a^2x^{2p}\right)}{a(m+1)(m-p+1)}+\frac{px^{m-p+1}}{a(m+1)(m-p+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^p)}}{m+1}$$

input
```
Int[E^ArcSech[a*x^p]*x^m,x]
```

output
```
(E^ArcSech[a*x^p]*x^(1 + m))/(1 + m) + (p*x^(1 + m - p))/(a*(1 + m)*(1 + m
 - p)) + (p*x^(1 + m - p)*Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeom
etric2F1[1/2, (1 + m - p)/(2*p), (1 + m + p)/(2*p), a^2*x^(2*p)])/(a*(1 +
m)*(1 + m - p))
```

3.60.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 791
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.60.4 Maple [F]

$$\int \left(\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}\right)x^m dx$$

input
```
int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x)
```

output
```
int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x)
```

3.60.5 Fricas [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algor
ithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.60.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x^m\,dx = \frac{\int x^m x^{-p}\,dx + \int ax^m\sqrt{-1+\frac{x^{-p}}{a}}\sqrt{1+\frac{x^{-p}}{a}}\,dx}{a}$$

input
```
integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))*x**m,x)
```

output
```
(Integral(x**m/x**p, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1
/(a*x**p)), x))/a
```

3.60.7 Maxima [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^p)} x^m \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-p>0)', see `assume?` for more
details)Is
```

3.60.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}(ax^p)} x^m \, dx = \int x^m \left(\sqrt{\frac{1}{ax^p} + 1} \sqrt{\frac{1}{ax^p} - 1} + \frac{1}{ax^p} \right) dx$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algor
ithm="giac")
```

output
```
integrate(x^m*(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p)), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^p)} x^m \, dx = \int x^m \left(\sqrt{\frac{1}{a\,x^p} - 1} \sqrt{\frac{1}{a\,x^p} + 1} + \frac{1}{a\,x^p} \right) dx$$

input
```
int(x^m*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)),x)
```

output
```
int(x^m*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)), x)
```

3.61 $\int e^{\text{sech}^{-1}(ax^p)}x\,dx$

3.61.1 Optimal result

Integrand size = 10, antiderivative size = 119

$$\int e^{\text{sech}^{-1}(ax^p)}x\,dx = \frac{1}{2}e^{\text{sech}^{-1}(ax^p)}x^2 + \frac{px^{2-p}}{2a(2-p)} + \frac{px^{2-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-1+\frac{2}{p}\right),\frac{1}{2}\left(1+\frac{2}{p}\right),a^2x^{2p}\right)}{2a(2-p)}$$

output
```
1/2*(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^2+1/2*p*x^(2-p)/
a/(2-p)+1/2*p*x^(2-p)*hypergeom([1/2, -1/2+1/p],[1/2+1/p],a^2*x^(2*p))*(1/
(1+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(2-p)
```

3.61.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.34

$$\int e^{\text{sech}^{-1}(ax^p)}x\,dx = \frac{x^{2-p}\left(-1-\sqrt{\frac{1-ax^p}{1+ax^p}}-ax^p\sqrt{\frac{1-ax^p}{1+ax^p}}+\frac{a^2px^{2p}\sqrt{\frac{1-ax^p}{1+ax^p}}\sqrt{1-a^2x^{2p}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{p},\frac{3}{2}+\frac{1}{p},a^2x^{2p}\right)}{(2+p)(-1+ax^p)}\right)}{a(-2+p)}$$

input
```
Integrate[E^ArcSech[a*x^p]*x,x]
```

output
```
(x^(2 - p)*(-1 - Sqrt[(1 - a*x^p)/(1 + a*x^p)] - a*x^p*Sqrt[(1 - a*x^p)/(1
 + a*x^p)] + (a^2*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(
2*p)]*Hypergeometric2F1[1/2, 1/2 + p^(-1), 3/2 + p^(-1), a^2*x^(2*p)])/((2
 + p)*(-1 + a*x^p))))/(a*(-2 + p))
```

3.61.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6889, 15, 791, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\text{sech}^{-1}(ax^p)}\,dx$$

$$\Big\downarrow \text{6889}$$

$$\frac{p\int x^{1-p}dx}{2a}+\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{1-p}}{\sqrt{1-ax^p}\sqrt{ax^p+1}}dx}{2a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^p)}$$

$$\Big\downarrow \text{15}$$

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{1-p}}{\sqrt{1-ax^p}\sqrt{ax^p+1}}dx}{2a}+\frac{px^{2-p}}{2a(2-p)}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^p)}$$

$$\Big\downarrow \text{791}$$

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{1-p}}{\sqrt{1-a^2x^{2p}}}dx}{2a}+\frac{px^{2-p}}{2a(2-p)}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^p)}$$

$$\Big\downarrow \text{888}$$

$$\frac{px^{2-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{2}{p}-1\right),\frac{1}{2}\left(1+\frac{2}{p}\right),a^2x^{2p}\right)}{2a(2-p)}+\frac{px^{2-p}}{2a(2-p)}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^p)}$$

input
```
Int[E^ArcSech[a*x^p]*x,x]
```

output
```
(E^ArcSech[a*x^p]*x^2)/2 + (p*x^(2 - p))/(2*a*(2 - p)) + (p*x^(2 - p)*Sqrt
[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + 2/p)/2, (1
 + 2/p)/2, a^2*x^(2*p)])/(2*a*(2 - p))
```

3.61.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 791
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.61.4 Maple [F]

$$\int \left(\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}\right)x\,dx$$

input
```
int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x)
```

output
```
int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x)
```

3.61.5 Fricas [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorit
hm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.61.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x\,dx = \frac{\int xx^{-p}\,dx + \int ax\sqrt{-1+\frac{x^{-p}}{a}}\sqrt{1+\frac{x^{-p}}{a}}\,dx}{a}$$

input
```
integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))*x,x)
```

output
```
(Integral(x/x**p, x) + Integral(a*x*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/(a*x*
*p)), x))/a
```

3.61.7 Maxima [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-p>0)', see `assume?` for more
details)Is
```

3.61.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x\,dx = \int x\left(\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}\right)dx$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorit
hm="giac")
```

output
```
integrate(x*(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p)), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x\,dx = \int x\left(\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}\right)dx$$

input
```
int(x*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)),x)
```

output
```
int(x*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)), x)
```

3.62 $\int e^{\text{sech}^{-1}(ax^p)}\,dx$

3.62.1 Optimal result

Integrand size = 8, antiderivative size = 105

$$\int e^{\text{sech}^{-1}(ax^p)}\,dx = e^{\text{sech}^{-1}(ax^p)}x + \frac{px^{1-p}}{a(1-p)} + \frac{px^{1-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-1+\frac{1}{p}\right),\frac{1+p}{2p},a^2x^{2p}\right)}{a(1-p)}$$

output
```
(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x+p*x^(1-p)/a/(1-p)+p*
x^(1-p)*hypergeom([1/2, -1/2+1/2/p],[1/2*(p+1)/p],a^2*x^(2*p))*(1/(1+a*x^p
))^(1/2)*(1+a*x^p)^(1/2)/a/(1-p)
```

3.62.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.56

$$\int e^{\text{sech}^{-1}(ax^p)}\,dx = \frac{x^{1-p}\left(-1-\sqrt{\frac{1-ax^p}{1+ax^p}}-ax^p\sqrt{\frac{1-ax^p}{1+ax^p}}+\frac{a^2px^{2p}\sqrt{\frac{1-ax^p}{1+ax^p}}\sqrt{1-a^2x^{2p}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2p},\frac{1}{2}\left(3+\frac{1}{p}\right),a^2x^{2p}\right)}{(1+p)(-1+ax^p)}\right)}{a(-1+p)}$$

input
```
Integrate[E^ArcSech[a*x^p],x]
```

output
```
(x^(1 - p)*(-1 - Sqrt[(1 - a*x^p)/(1 + a*x^p)] - a*x^p*Sqrt[(1 - a*x^p)/(1
 + a*x^p)] + (a^2*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(
2*p)]*Hypergeometric2F1[1/2, (1 + p)/(2*p), (3 + p^(-1))/2, a^2*x^(2*p)])/
((1 + p)*(-1 + a*x^p))))/(a*(-1 + p))
```

3.62.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6884, 15, 791, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\text{sech}^{-1}(ax^p)}\,dx$$

$$\Big\downarrow \text{6884}$$

$$\frac{p\int x^{-p}dx}{a}+\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{-p}}{\sqrt{1-ax^p}\sqrt{ax^p+1}}dx}{a}+xe^{\text{sech}^{-1}(ax^p)}$$

$$\Big\downarrow \text{15}$$

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{-p}}{\sqrt{1-ax^p}\sqrt{ax^p+1}}dx}{a}+\frac{px^{1-p}}{a(1-p)}+xe^{\text{sech}^{-1}(ax^p)}$$

$$\Big\downarrow \text{791}$$

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{-p}}{\sqrt{1-a^2x^{2p}}}dx}{a}+\frac{px^{1-p}}{a(1-p)}+xe^{\text{sech}^{-1}(ax^p)}$$

$$\Big\downarrow \text{888}$$

$$\frac{px^{1-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{p}-1\right),\frac{p+1}{2p},a^2x^{2p}\right)}{a(1-p)}+\frac{px^{1-p}}{a(1-p)}+xe^{\text{sech}^{-1}(ax^p)}$$

input
```
Int[E^ArcSech[a*x^p],x]
```

output
```
E^ArcSech[a*x^p]*x + (p*x^(1 - p))/(a*(1 - p)) + (p*x^(1 - p)*Sqrt[(1 + a*
x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + p^(-1))/2, (1 + p)
/(2*p), a^2*x^(2*p)])/(a*(1 - p))
```

3.62.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 791
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6884
```
Int[E^ArcSech[(a_.)*(x_)^(p_)], x_Symbol] :> Simp[x*E^ArcSech[a*x^p], x] +
(Simp[p/a   Int[1/x^p, x], x] + Simp[p*(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^
p)]   Int[1/(x^p*Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, p},
 x]
```

3.62.4 Maple [F]

$$\int \left(\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}\right)dx$$

input
```
int(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x)
```

output
```
int(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x)
```

3.62.5 Fricas [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^p)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.62.6 Sympy [F]

$$\int e^{\operatorname{sech}^{-1}(ax^p)} dx = \frac{\int x^{-p} dx + \int a\sqrt{-1 + \frac{x^{-p}}{a}}\sqrt{1 + \frac{x^{-p}}{a}} dx}{a}$$

input
```
integrate(1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2),x)
```

output
```
(Integral(x**(-p), x) + Integral(a*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/(a*x**
p)), x))/a
```

3.62.7 Maxima [F(-2)]

Exception generated.

$$\int e^{\operatorname{sech}^{-1}(ax^p)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-p>0)', see `assume?` for more d
etails)Is
```

3.62.8 Giac [F]

$$\int e^{\operatorname{sech}^{-1}(ax^p)}\,dx = \int \sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}\,dx$$

input
```
integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{sech}^{-1}(ax^p)}\,dx = \int \sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}\,dx$$

input
```
int((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p),x)
```

output
```
int((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p), x)
```

3.63 $\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x}\,dx$

3.63.1 Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x}\,dx=-\frac{x^{-p}}{ap}-\frac{x^{-p}\sqrt{1-ax^p}}{ap\sqrt{\frac{1}{1+ax^p}}}-\frac{\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\arcsin(ax^p)}{p}$$

output
```
-1/a/p/(x^p)-(1-a*x^p)^(1/2)/a/p/(x^p)/(1/(1+a*x^p))^(1/2)-arcsin(a*x^p)*(
1/(1+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/p
```

3.63.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x}\,dx=-\frac{i\left(-ix^{-p}-i(a+x^{-p})\sqrt{\frac{1-ax^p}{1+ax^p}}+a\log\left(-2iax^p+2\sqrt{\frac{1-ax^p}{1+ax^p}}(1+ax^p)\right)\right)}{ap}$$

input
```
Integrate[E^ArcSech[a*x^p]/x,x]
```

output
```
((-I)*((-I)/x^p - I*(a + x^(-p))*Sqrt[(1 - a*x^p)/(1 + a*x^p)] + a*Log[(-2
*I)*a*x^p + 2*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*(1 + a*x^p)]))/(a*p)
```

3.63.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6888, 791, 868, 773, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x}dx$$

$$\Big\downarrow \text{6888}$$

$$\frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int x^{-p-1}\sqrt{1-ax^p}\sqrt{ax^p+1}dx}{a}-\frac{x^{-p}}{ap}$$

$$\Big\downarrow \text{791}$$

$$\frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int x^{-p-1}\sqrt{1-a^2x^{2p}}dx}{a}-\frac{x^{-p}}{ap}$$

$$\Big\downarrow \text{868}$$

$$-\frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \sqrt{1-a^2x^{2p}}dx^{-p}}{ap}-\frac{x^{-p}}{ap}$$

$$\Big\downarrow \text{773}$$

$$\frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int x^{2p}\sqrt{1-a^2x^{-2p}}dx^p}{ap}-\frac{x^{-p}}{ap}$$

$$\Big\downarrow \text{247}$$

$$\frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\left(x^p\left(-\sqrt{1-a^2x^{-2p}}\right)-a^2\int \frac{1}{\sqrt{1-a^2x^{-2p}}}dx^p\right)}{ap}-\frac{x^{-p}}{ap}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\left(x^p\left(-\sqrt{1-a^2x^{-2p}}\right)-a\arcsin\left(ax^p\right)\right)}{ap}-\frac{x^{-p}}{ap}$$

input
```
Int[E^ArcSech[a*x^p]/x,x]
```

output
```
-(1/(a*p*x^p)) + (Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*(-(x^p*Sqrt[1 - a
^2/x^(2*p)]) - a*ArcSin[a*x^p]))/(a*p)
```

3.63.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 791
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 6888
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Simp[(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^p)]   Int[Sqrt[1 + a*x^p]*(Sqrt
[1 - a*x^p]/x^(p + 1)), x], x] /; FreeQ[{a, p}, x]
```

3.63.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{-\frac{(a\,x^{p}-1)x^{-p}}{a}}\,\sqrt{\frac{(1+a\,x^{p})x^{-p}}{a}}\,\left(\arctan\left(\frac{\mathrm{csgn}(a)a\,x^{p}}{\sqrt{-a^{2}x^{2p}+1}}\right)a\,x^{p}+\sqrt{-a^{2}x^{2p}+1}\,\mathrm{csgn}(a)\right)\mathrm{csgn}(a)}{\sqrt{-a^{2}x^{2p}+1}}-\frac{x^{-p}}{a}}{p}$	116
default	$\frac{-\frac{\sqrt{-\frac{(a\,x^{p}-1)x^{-p}}{a}}\,\sqrt{\frac{(1+a\,x^{p})x^{-p}}{a}}\,\left(\arctan\left(\frac{\mathrm{csgn}(a)a\,x^{p}}{\sqrt{-a^{2}x^{2p}+1}}\right)a\,x^{p}+\sqrt{-a^{2}x^{2p}+1}\,\mathrm{csgn}(a)\right)\mathrm{csgn}(a)}{\sqrt{-a^{2}x^{2p}+1}}-\frac{x^{-p}}{a}}{p}$	116

input
```
int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x,method=_RETURN
VERBOSE)
```

output
```
1/p*(-(-(a*x^p-1)/a/(x^p))^(1/2)*((1+a*x^p)/a/(x^p))^(1/2)*(arctan(csgn(a)
*a*x^p/(-(x^p)^2*a^2+1)^(1/2))*a*x^p+(-(x^p)^2*a^2+1)^(1/2)*csgn(a))*csgn(
a)/(-(x^p)^2*a^2+1)^(1/2)-1/a/(x^p))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x}\,dx = -\frac{ax^p\sqrt{\frac{ax^p+1}{ax^p}}\sqrt{-\frac{ax^p-1}{ax^p}} - ax^p\arctan\left(\sqrt{\frac{ax^p+1}{ax^p}}\sqrt{-\frac{ax^p-1}{ax^p}}\right)+1}{apx^p}$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorit
hm="fricas")
```

output
```
-(a*x^p*sqrt((a*x^p + 1)/(a*x^p))*sqrt(-(a*x^p - 1)/(a*x^p)) - a*x^p*arcta
n(sqrt((a*x^p + 1)/(a*x^p))*sqrt(-(a*x^p - 1)/(a*x^p))) + 1)/(a*p*x^p)
```

3.63.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x}\,dx = \frac{\int \frac{x^{-p}}{x}\,dx + \int \frac{a\sqrt{-1+\frac{x^{-p}}{a}}\sqrt{1+\frac{x^{-p}}{a}}}{x}\,dx}{a}$$

input
```
integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))/x,x)
```

output
```
(Integral(1/(x*x**p), x) + Integral(a*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/(a*
x**p))/x, x))/a
```

3.63.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x}\,dx = \int \frac{\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}}{x}\,dx$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorit
hm="maxima")
```

output
```
integrate(sqrt(a*x^p + 1)*sqrt(-a*x^p + 1)/(x*x^p), x)/a - 1/(a*p*x^p)
```

3.63.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x}\,dx = \int \frac{\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}}{x}\,dx$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorit
hm="giac")
```

output
```
integrate((sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p))/x, x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x}\,dx = \int \frac{\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}}{x}\,dx$$

input
```
int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x,x)
```

output
```
int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x, x)
```

3.64 $\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2}\,dx$

3.64.1 Optimal result

Integrand size = 12, antiderivative size = 107

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2}\,dx = -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} + \frac{px^{-1-p}}{a(1+p)} + \frac{px^{-1-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1+p}{2p},-\frac{1-p}{2p},a^2x^{2p}\right)}{a(1+p)}$$

output
```
-(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x+p*x^(-1-p)/a/(p+1)+
p*x^(-1-p)*hypergeom([1/2, 1/2*(-1-p)/p],[1/2*(-1+p)/p],a^2*x^(2*p))*(1/(1
+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(p+1)
```

3.64.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.55

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2}\,dx = \frac{x^{-1-p}\left(-1-\sqrt{\frac{1-ax^p}{1+ax^p}}-ax^p\sqrt{\frac{1-ax^p}{1+ax^p}}+\frac{a^2px^{2p}\sqrt{\frac{1-ax^p}{1+ax^p}}\sqrt{1-a^2x^{2p}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-1+p}{2p},\frac{3}{2}-\frac{1}{2p},a^2x^{2p}\right)}{(-1+p)(-1+ax^p)}\right)}{a(1+p)}$$

input
```
Integrate[E^ArcSech[a*x^p]/x^2,x]
```

output
```
(x^(-1 - p)*(-1 - Sqrt[(1 - a*x^p)/(1 + a*x^p)] - a*x^p*Sqrt[(1 - a*x^p)/(
1 + a*x^p)] + (a^2*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^
(2*p)]*Hypergeometric2F1[1/2, (-1 + p)/(2*p), 3/2 - 1/(2*p), a^2*x^(2*p)])
/((-1 + p)*(-1 + a*x^p))))/(a*(1 + p))
```

3.64.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6889, 15, 791, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x^2} \, dx$$

$$\Big\downarrow \text{6889}$$

$$-\frac{p\int x^{-p-2}dx}{a}-\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{-p-2}}{\sqrt{1-ax^p}\sqrt{ax^p+1}}dx}{a}-\frac{e^{\text{sech}^{-1}(ax^p)}}{x}$$

$$\Big\downarrow \text{15}$$

$$-\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{-p-2}}{\sqrt{1-ax^p}\sqrt{ax^p+1}}dx}{a}+\frac{px^{-p-1}}{a(p+1)}-\frac{e^{\text{sech}^{-1}(ax^p)}}{x}$$

$$\Big\downarrow \text{791}$$

$$-\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\int \frac{x^{-p-2}}{\sqrt{1-a^2x^{2p}}}dx}{a}+\frac{px^{-p-1}}{a(p+1)}-\frac{e^{\text{sech}^{-1}(ax^p)}}{x}$$

$$\Big\downarrow \text{888}$$

$$\frac{px^{-p-1}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p+1}{2p},-\frac{1-p}{2p},a^2x^{2p}\right)}{a(p+1)}+\frac{px^{-p-1}}{a(p+1)}-\frac{e^{\text{sech}^{-1}(ax^p)}}{x}$$

input
```
Int[E^ArcSech[a*x^p]/x^2,x]
```

output
```
-(E^ArcSech[a*x^p]/x) + (p*x^(-1 - p))/(a*(1 + p)) + (p*x^(-1 - p)*Sqrt[(1
 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, -1/2*(1 + p)/p, -1/
2*(1 - p)/p, a^2*x^(2*p)])/(a*(1 + p))
```

3.64.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 791
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6889
```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Simp[p/(a*(m + 1))   Int[x^(m - p), x], x] +
 Simp[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)]   Int[x^(m - p)/(
Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m,
-1]
```

3.64.4 Maple [F]

$$\int \frac{\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}}{x^{2}}dx$$

input
```
int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x)
```

output
```
int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x)
```

3.64.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algor
ithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.64.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2}\,dx = \frac{\int \frac{x^{-p}}{x^2}\,dx + \int \frac{a\sqrt{-1+\frac{x^{-p}}{a}}\sqrt{1+\frac{x^{-p}}{a}}}{x^2}\,dx}{a}$$

input
```
integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))/x**2,x)
```

output
```
(Integral(1/(x**2*x**p), x) + Integral(a*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/
(a*x**p))/x**2, x))/a
```

3.64.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}}{x^2}\,dx$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algor
ithm="maxima")
```

output
```
integrate(sqrt(a*x^p + 1)*sqrt(-a*x^p + 1)/(x^2*x^p), x)/a - x^(-p - 1)/(a
*(p + 1))
```

3.64.8 Giac [F]

$$\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}}{x^2}\,dx$$

input
```
integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algor
ithm="giac")
```

output
```
integrate((sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p))/x^2, x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}}{x^2}\,dx$$

input
```
int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x^2,x)
```

output
```
int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x^2, x)
```

3.65 $\int e^{2\text{sech}^{-1}(ax)}x^4\,dx$

3.65.1 Optimal result

Integrand size = 12, antiderivative size = 203

$$\int e^{2\text{sech}^{-1}(ax)}x^4\,dx=\frac{5\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{4a^5}+\frac{(1-ax)(1+ax)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5}+\frac{(1+ax)\left(4-\sqrt{\frac{1-ax}{1+ax}}\right)}{4a^5}-\frac{(1+ax)^3\left(4+45\sqrt{\frac{1-ax}{1+ax}}\right)}{30a^5}-\frac{\arctan\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^5}$$

output
```
1/5*(-a*x+1)*(a*x+1)^4/a^5-1/2*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^5+1/4*(a
*x+1)*(4-((-a*x+1)/(a*x+1))^(1/2))/a^5+5/4*(a*x+1)^2*((-a*x+1)/(a*x+1))^(1
/2)/a^5+1/10*(a*x+1)^4*(5-6*((-a*x+1)/(a*x+1))^(1/2))*((-a*x+1)/(a*x+1))^(
1/2)/a^5-1/30*(a*x+1)^3*(4+45*((-a*x+1)/(a*x+1))^(1/2))/a^5
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52

$$\int e^{2\text{sech}^{-1}(ax)}x^4\,dx=\frac{40a^3x^3-12a^5x^5-15a\sqrt{\frac{1-ax}{1+ax}}(x+ax^2-2a^2x^3-2a^3x^4)+15i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{60a^5}$$

input
```
Integrate[E^(2*ArcSech[a*x])*x^4,x]
```

output
```
(40*a^3*x^3 - 12*a^5*x^5 - 15*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a
^2*x^3 - 2*a^3*x^4) + (15*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*
(1 + a*x)])/(60*a^5)
```

3.65.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6891, 7268, 2335, 27, 2342, 2335, 27, 2345, 27, 2345, 454, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{2\text{sech}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6891}$$

$$\int x^4 \left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax} + \sqrt{\frac{1-ax}{ax+1}} + \frac{1}{ax}\right)^2 dx$$

$$\Big\downarrow \text{7268}$$

$$-\frac{4 \int \frac{\sqrt{\frac{1-ax}{ax+1}} \left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2 \left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^6}{\left(\frac{1-ax}{ax+1}+1\right)^6} d\sqrt{\frac{1-ax}{ax+1}}}{a^5}$$

$$\Big\downarrow \text{2335}$$

$$-\frac{4 \left(-\frac{1}{10} \int -\frac{2 \left(5 \left(\frac{1-ax}{ax+1}\right)^{7/2}+15 \left(\frac{1-ax}{ax+1}\right)^{5/2}-65 \left(\frac{1-ax}{ax+1}\right)^{3/2}+21 \sqrt{\frac{1-ax}{ax+1}}+\frac{20(1-ax)}{ax+1}-\frac{40(1-ax)^2}{(ax+1)^2}+\frac{20(1-ax)^3}{(ax+1)^3}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^5} d\sqrt{\frac{1-ax}{ax+1}} - \frac{8(1-ax)}{5(ax+1)\left(\frac{1-ax}{ax+1}+\right.}\right.}{a^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4 \left(\frac{1}{5} \int \frac{5 \left(\frac{1-ax}{ax+1}\right)^{7/2}+15 \left(\frac{1-ax}{ax+1}\right)^{5/2}-65 \left(\frac{1-ax}{ax+1}\right)^{3/2}+21 \sqrt{\frac{1-ax}{ax+1}}+\frac{20(1-ax)}{ax+1}-\frac{40(1-ax)^2}{(ax+1)^2}+\frac{20(1-ax)^3}{(ax+1)^3}}{\left(\frac{1-ax}{ax+1}+1\right)^5} d\sqrt{\frac{1-ax}{ax+1}} - \frac{8(1-ax)}{5(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \text{2342}$$

$$-\frac{4\left(\frac{1}{5}\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{5(1-ax)^3}{(ax+1)^3}+\frac{15(1-ax)^2}{(ax+1)^2}-\frac{65(1-ax)}{ax+1}+20\left(\frac{1-ax}{ax+1}\right)^{5/2}-40\left(\frac{1-ax}{ax+1}\right)^{3/2}+20\sqrt{\frac{1-ax}{ax+1}}+21\right)}{\left(\frac{1-ax}{ax+1}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}-\frac{8(1-ax)}{5(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$\Big\downarrow$ 2335

$$-\frac{4\left(\frac{1}{5}\left(-\frac{1}{8}\int -\frac{8\left(5\left(\frac{1-ax}{ax+1}\right)^{5/2}+10\left(\frac{1-ax}{ax+1}\right)^{3/2}-3\sqrt{\frac{1-ax}{ax+1}}-\frac{60(1-ax)}{ax+1}+\frac{20(1-ax)^2}{(ax+1)^2}+10\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}-\frac{2\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}\right)-\frac{8\cdots}{5(ax+1\cdots}\cdots\right)}{a^5}$$

$\Big\downarrow$ 27

$$-\frac{4\left(\frac{1}{5}\left(\int \frac{5\left(\frac{1-ax}{ax+1}\right)^{5/2}+10\left(\frac{1-ax}{ax+1}\right)^{3/2}-3\sqrt{\frac{1-ax}{ax+1}}-\frac{60(1-ax)}{ax+1}+\frac{20(1-ax)^2}{(ax+1)^2}+10}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}-\frac{2\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}\right)-\frac{8(1-ax)}{5(ax+1)\left(\frac{1-ax}{ax+1}+1\right)\cdots}\cdots\right)}{a^5}$$

$\Big\downarrow$ 2345

$$-\frac{4\left(\frac{1}{5}\left(-\frac{1}{6}\int \frac{30\left(-\left(\frac{1-ax}{ax+1}\right)^{3/2}-\sqrt{\frac{1-ax}{ax+1}}-\frac{4(1-ax)}{ax+1}+1\right)}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}-\frac{2\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{45\sqrt{\frac{1-ax}{ax+1}}+4}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8(1-ax)}{5(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^5}\cdots\right)}{a^5}$$

$\Big\downarrow$ 27

$$-\frac{4\left(\frac{1}{5}\left(-5\int \frac{-\left(\frac{1-ax}{ax+1}\right)^{3/2}-\sqrt{\frac{1-ax}{ax+1}}-\frac{4(1-ax)}{ax+1}+1}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}-\frac{2\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{45\sqrt{\frac{1-ax}{ax+1}}+4}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8(1-ax)}{5(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$\Big\downarrow$ 2345

$$-\frac{4\left(\frac{1}{5}\left(-5\left(\frac{5\sqrt{\frac{1-ax}{ax+1}}}{4\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{1}{4}\int \frac{4\sqrt{\frac{1-ax}{ax+1}}+1}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{2\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{45\sqrt{\frac{1-ax}{ax+1}}+4}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8(1-ax)}{5(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$\Big\downarrow$ 454

$$-\frac{4\left(\frac{1}{5}\left(-5\left(\frac{1}{4}\left(\frac{4-\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)}-\frac{1}{2}\int \frac{1}{\frac{1-ax}{ax+1}+1}d\sqrt{\frac{1-ax}{ax+1}}\right)+\frac{5\sqrt{\frac{1-ax}{ax+1}}}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)-\frac{2\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{45\sqrt{\frac{1-ax}{ax+1}}+4}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8\cdots}{5(ax+1\cdots}\cdots\right)}{a^5}$$

$\Big\downarrow$ 216

3.65. $\int e^{2\text{sech}^{-1}(ax)}x^4\,dx$

$$-\frac{4\left(\frac{1}{5}\left(-5\left(\frac{1}{4}\left(\frac{4-\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)}-\frac{1}{2}\arctan\left(\sqrt{\frac{1-ax}{ax+1}}\right)\right)+\frac{5\sqrt{\frac{1-ax}{ax+1}}}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)-\frac{2\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{45\sqrt{\frac{1-ax}{ax+1}}+4}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8(}{5(ax+1)}}{a^5}$$

input
```
Int[E^(2*ArcSech[a*x])*x^4,x]
```

output
```
(-4*((-8*(1 - a*x))/(5*(1 + a*x)*(1 + (1 - a*x)/(1 + a*x))^5) + ((-2*Sqrt[
(1 - a*x)/(1 + a*x)]*(5 - 6*Sqrt[(1 - a*x)/(1 + a*x)]))/(1 + (1 - a*x)/(1
+ a*x))^4 + (4 + 45*Sqrt[(1 - a*x)/(1 + a*x)])/(3*(1 + (1 - a*x)/(1 + a*x)
)^3) - 5*((5*Sqrt[(1 - a*x)/(1 + a*x)])/(4*(1 + (1 - a*x)/(1 + a*x))^2) +
((4 - Sqrt[(1 - a*x)/(1 + a*x)])/(2*(1 + (1 - a*x)/(1 + a*x))) - ArcTan[Sq
rt[(1 - a*x)/(1 + a*x)]]/2)/4))/5))/a^5
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2342
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.65.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

method	result
default	$\frac{-\frac{1}{5}a^2x^5+\frac{1}{3}x^3}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\left(2\,\mathrm{csgn}(a)a^3x^3\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}\,x\,\mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\mathrm{csgn}(a)}{4a^4\sqrt{-a^2x^2+1}}+\frac{x^3}{3a^2}$

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/5*a^2*x^5+1/3*x^3)+1/4/a^4*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^
(1/2)*(2*csgn(a)*a^3*x^3*(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)*x*csgn(a)*a
+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))*csgn(a)/(-a^2*x^2+1)^(1/2)+1/3*x^
3/a^2
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.51

$$\int e^{2\operatorname{sech}^{-1}(ax)}x^4\,dx$$
$$= -\frac{12\,a^5x^5 - 40\,a^3x^3 - 15\,(2\,a^4x^4 - a^2x^2)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 15\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{60\,a^5}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="fric
as")
```

output
```
-1/60*(12*a^5*x^5 - 40*a^3*x^3 - 15*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(
a*x))*sqrt(-(a*x - 1)/(a*x)) + 15*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
- 1)/(a*x))))/a^5
```

3.65.6 Sympy [F]

$$\int e^{2\operatorname{sech}^{-1}(ax)}x^4\,dx = \frac{\int 2x^2\,dx + \int(-a^2x^4)\,dx + \int 2ax^3\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**4,x)
```

output
```
(Integral(2*x**2, x) + Integral(-a**2*x**4, x) + Integral(2*a*x**3*sqrt(-1
 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a**2
```

3.65.7 Maxima [F]

$$\int e^{2\operatorname{sech}^{-1}(ax)}x^4\,dx = \int x^4\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="maxi
ma")
```

output
```
2/3*x^3/a^2 + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2, x)/a^2 - integ
rate(x^4, x)
```

3.65.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\operatorname{sech}^{-1}(ax)}x^4\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,4,0,6,0,0]%%%}+%%%{1,[0,2,4,4,0,0]%%%}+%%%{1,[0,2,0,
4,0,0]%%%
```

3.65.9 Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 808, normalized size of antiderivative = 3.98

$$\int e^{2\operatorname{sech}^{-1}(ax)}x^4\,dx =$$

$$-\frac{\frac{1\mathrm{i}}{512\,a^5}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 3\mathrm{i}}{64\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 53\mathrm{i}}{256\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6 87\mathrm{i}}{128\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8 657\mathrm{i}}{512\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10} 121\mathrm{i}}{128\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}\cdots}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}}$$

$$-\frac{\frac{1\mathrm{i}}{16\,a^5}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{8\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 15\mathrm{i}}{16\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}-\frac{x^5\left(\frac{a^2}{5}-\frac{2}{3\,x^2}\right)}{a^2}$$

$$-\frac{\ln\left(\frac{a\sqrt{\frac{1}{a\,x}+1}-\frac{1}{x}+a\sqrt{\frac{1}{a\,x}-1}\,1\mathrm{i}}{2\,a-2\,a\sqrt{\frac{1}{a\,x}+1}+\frac{1}{x}}\right)3\mathrm{i}}{4\,a^5}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)1\mathrm{i}}{4\,a^5}+\frac{\ln\left(\frac{2\,a\sqrt{\frac{a+\frac{1}{x}}{a}}-\frac{2}{x}+a\sqrt{-\frac{a-\frac{1}{x}}{a}}\,2\mathrm{i}}{2\,a+\frac{1}{x}-2\,a\sqrt{\frac{a+\frac{1}{x}}{a}}}\right)1\mathrm{i}}{a^5}$$

$$-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{128\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 1\mathrm{i}}{512\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}$$

input
```
int(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
(log((a*(-(a - 1/x)/a)^(1/2)*2i - 2/x + 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/
x - 2*a*((a + 1/x)/a)^(1/2)))*1i)/a^5 - (1i/(512*a^5) - (((1/(a*x) - 1)^(1
/2) - 1i)^2*3i)/(64*a^5*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/
2) - 1i)^4*53i)/(256*a^5*((1/(a*x) + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)^(1
/2) - 1i)^6*87i)/(128*a^5*((1/(a*x) + 1)^(1/2) - 1)^6) + (((1/(a*x) - 1)^(
1/2) - 1i)^8*657i)/(512*a^5*((1/(a*x) + 1)^(1/2) - 1)^8) + (((1/(a*x) - 1)
^(1/2) - 1i)^10*121i)/(128*a^5*((1/(a*x) + 1)^(1/2) - 1)^10))/(((1/(a*x) -
 1)^(1/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 + (4*((1/(a*x) - 1)^(1/2) -
1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (6*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/
(a*x) + 1)^(1/2) - 1)^8 + (4*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)
^(1/2) - 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^1
2) - (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(4*a^5
) - (1i/(16*a^5) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(8*a^5*((1/(a*x) + 1)
^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(16*a^5*((1/(a*x) + 1)
^(1/2) - 1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2
+ (2*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x)
 - 1)^(1/2) - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - (log((a*(1/(a*x) - 1)^(
1/2)*1i + a*(1/(a*x) + 1)^(1/2) - 1/x)/(2*a - 2*a*(1/(a*x) + 1)^(1/2) + 1/
x))*3i)/(4*a^5) - (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(128*a^5*((1/(a*x) + 1
)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*1i)/(512*a^5*((1/(a*x) ...
```

3.65. $\int e^{2\text{sech}^{-1}(ax)}x^4\,dx$

3.66 $\int e^{2\text{sech}^{-1}(ax)}x^3\,dx$

3.66.1 Optimal result

Integrand size = 12, antiderivative size = 117

$$\int e^{2\text{sech}^{-1}(ax)}x^3\,dx=-\frac{x}{a^3}+\frac{(1-ax)(1+ax)^3}{4a^4}+\frac{(1+ax)^2\left(3-8\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3\left(4-3\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4}$$

output
```
-x/a^3+1/4*(-a*x+1)*(a*x+1)^3/a^4+1/6*(a*x+1)^2*(3-8*((-a*x+1)/(a*x+1))^(1
/2))/a^4+1/6*(a*x+1)^3*(4-3*((-a*x+1)/(a*x+1))^(1/2))*((-a*x+1)/(a*x+1))^(
1/2)/a^4
```

3.66.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int e^{2\text{sech}^{-1}(ax)}x^3\,dx=\frac{x^2}{a^2}-\frac{x^4}{4}+\frac{2(-1+ax)\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{3a^4}$$

input
```
Integrate[E^(2*ArcSech[a*x])*x^3,x]
```

output
```
x^2/a^2 - x^4/4 + (2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*
a^4)
```

3.66.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6891, 7268, 25, 2335, 27, 2342, 2335, 27, 2345, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{2\text{sech}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6891}$$

$$\int x^3 \left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2 dx$$

$$\Big\downarrow \text{ 7268}$$

$$\frac{4\int -\frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}{\left(\frac{1-ax}{ax+1}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}}{a^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}{\left(\frac{1-ax}{ax+1}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}}{a^4}$$

$$\Big\downarrow \text{ 2335}$$

$$\frac{4\left(\frac{1}{8}\int -\frac{8\left(-\left(\frac{1-ax}{ax+1}\right)^{5/2}-4\left(\frac{1-ax}{ax+1}\right)^{3/2}+3\sqrt{\frac{1-ax}{ax+1}}+\frac{4(1-ax)}{ax+1}-\frac{4(1-ax)^2}{(ax+1)^2}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}+\frac{1-ax}{(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^4}\right)}{a^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\left(\frac{1-ax}{(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^4}-\int \frac{-\left(\frac{1-ax}{ax+1}\right)^{5/2}-4\left(\frac{1-ax}{ax+1}\right)^{3/2}+3\sqrt{\frac{1-ax}{ax+1}}+\frac{4(1-ax)}{ax+1}-\frac{4(1-ax)^2}{(ax+1)^2}}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}\right)}{a^4}$$

$$\Big\downarrow \text{ 2342}$$

$$\frac{4\left(\frac{1-ax}{(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^4}-\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(-\frac{(1-ax)^2}{(ax+1)^2}-\frac{4(1-ax)}{ax+1}-4\left(\frac{1-ax}{ax+1}\right)^{3/2}+4\sqrt{\frac{1-ax}{ax+1}}+3\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}\right)}{a^4}$$

$$\Big\downarrow \; 2335$$

$$\frac{4\left(\frac{1}{6}\int -\frac{2\left(-3\left(\frac{1-ax}{ax+1}\right)^{3/2}+3\sqrt{\frac{1-ax}{ax+1}}-\frac{12(1-ax)}{ax+1}+4\right)}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}+\frac{1-ax}{(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(4-3\sqrt{\frac{1-ax}{ax+1}}\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^4}$$

$$\Big\downarrow \; 27$$

$$\frac{4\left(-\frac{1}{3}\int \frac{-3\left(\frac{1-ax}{ax+1}\right)^{3/2}+3\sqrt{\frac{1-ax}{ax+1}}-\frac{12(1-ax)}{ax+1}+4}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}+\frac{1-ax}{(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(4-3\sqrt{\frac{1-ax}{ax+1}}\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^4}$$

$$\Big\downarrow \; 2345$$

$$\frac{4\left(\frac{1}{3}\left(\frac{1}{4}\int \frac{12\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}+\frac{3-8\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)+\frac{1-ax}{(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(4-3\sqrt{\frac{1-ax}{ax+1}}\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^4}$$

$$\Big\downarrow \; 27$$

$$\frac{4\left(\frac{1}{3}\left(3\int \frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}+\frac{3-8\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)+\frac{1-ax}{(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(4-3\sqrt{\frac{1-ax}{ax+1}}\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^4}$$

$$\Big\downarrow \; 241$$

$$\frac{4\left(\frac{1-ax}{(ax+1)\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{1}{3}\left(\frac{3-8\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{3}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(4-3\sqrt{\frac{1-ax}{ax+1}}\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^4}$$

input
```
Int[E^(2*ArcSech[a*x])*x^3,x]
```

output
```
(4*((1 - a*x)/((1 + a*x)*(1 + (1 - a*x)/(1 + a*x))^4) + (Sqrt[(1 - a*x)/(1
 + a*x)]*(4 - 3*Sqrt[(1 - a*x)/(1 + a*x)]))/(3*(1 + (1 - a*x)/(1 + a*x))^3
) + ((3 - 8*Sqrt[(1 - a*x)/(1 + a*x)])/(2*(1 + (1 - a*x)/(1 + a*x))^2) - 3
/(2*(1 + (1 - a*x)/(1 + a*x))))/3))/a^4
```

3.66. $\int e^{2\text{sech}^{-1}(ax)}x^3\,dx$

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2342
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.66.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{-\frac{1}{4}x^4a^2+\frac{1}{2}x^2}{a^2}+\frac{2\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)}{3a^3}+\frac{x^2}{2a^2}$	72

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/4*x^4*a^2+1/2*x^2)+2/3/a^3*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^
(1/2)*(a^2*x^2-1)+1/2*x^2/a^2
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

$$\int e^{2\operatorname{sech}^{-1}(ax)}x^3\,dx=-\frac{3\,a^3x^4-12\,ax^2-8\,(a^2x^3-x)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{12\,a^3}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="fric
as")
```

output
```
-1/12*(3*a^3*x^4 - 12*a*x^2 - 8*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-
(a*x - 1)/(a*x)))/a^3
```

3.66.6 Sympy [F]

$$\int e^{2\operatorname{sech}^{-1}(ax)}x^3\,dx=\frac{\int 2x\,dx+\int\left(-a^2x^3\right)\,dx+\int 2ax^2\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**3,x)
```

output
```
(Integral(2*x, x) + Integral(-a**2*x**3, x) + Integral(2*a*x**2*sqrt(-1 +
1/(a*x))*sqrt(1 + 1/(a*x)), x))/a**2
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.36

$$\int e^{2\text{sech}^{-1}(ax)}x^3\,dx = -\frac{1}{4}\,x^4 + \frac{x^2}{a^2} + \frac{2\,(a^2x^2-1)\sqrt{ax+1}\sqrt{-ax+1}}{3\,a^4}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="maxi
ma")
```

output
```
-1/4*x^4 + x^2/a^2 + 2/3*(a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^4
```

3.66.8 Giac [F]

$$\int e^{2\text{sech}^{-1}(ax)}x^3\,dx = \int x^3\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="giac
")
```

output
```
integrate(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54

$$\int e^{2\text{sech}^{-1}(ax)}x^3\,dx = \frac{x^2}{a^2} - \frac{x^4}{4} - \sqrt{\frac{1}{a\,x}-1}\left(\frac{2\,x\,\sqrt{\frac{1}{a\,x}+1}}{3\,a^3} - \frac{2\,x^3\,\sqrt{\frac{1}{a\,x}+1}}{3\,a}\right)$$

input
```
int(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
x^2/a^2 - x^4/4 - (1/(a*x) - 1)^(1/2)*((2*x*(1/(a*x) + 1)^(1/2))/(3*a^3) -
 (2*x^3*(1/(a*x) + 1)^(1/2))/(3*a))
```

3.67 $\int e^{2\text{sech}^{-1}(ax)}x^2\,dx$

3.67.1 Optimal result

Integrand size = 12, antiderivative size = 169

$$\int e^{2\text{sech}^{-1}(ax)}x^2\,dx = \frac{(1+ax)\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^3} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3} + \frac{(1+ax)^3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{2\arctan\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a^3}$$

output
```
-2*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^3+1/2*(a*x+1)*(1-((-a*x+1)/(a*x+1))^
(1/2))*(1+((-a*x+1)/(a*x+1))^(1/2))/a^3-1/6*(a*x+1)^2*((-a*x+1)/(a*x+1))^(
1/2)*(1+((-a*x+1)/(a*x+1))^(1/2))^3/a^3+1/12*(a*x+1)^3*(1+((-a*x+1)/(a*x+1
))^(1/2))^4/a^3
```

3.67.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.51

$$\int e^{2\text{sech}^{-1}(ax)}x^2\,dx = \frac{2x}{a^2} - \frac{x^3}{3} + \sqrt{\frac{1-ax}{1+ax}}\left(\frac{x}{a^2}+\frac{x^2}{a}\right) + \frac{i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{a^3}$$

input
```
Integrate[E^(2*ArcSech[a*x])*x^2,x]
```

output
```
(2*x)/a^2 - x^3/3 + Sqrt[(1 - a*x)/(1 + a*x)]*(x/a^2 + x^2/a) + (I*Log[(-2
*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/a^3
```

3.67.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6891, 7268, 531, 27, 490, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{2\text{sech}^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6891}$$

$$\int x^2\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2dx$$

$$\Big\downarrow \text{7268}$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}}{a^3}$$

$$\Big\downarrow \text{531}$$

$$-\frac{4\left(-\frac{1}{6}\int -\frac{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}-\frac{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\left(\frac{2}{3}\int \frac{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}-\frac{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

$$\Big\downarrow \text{490}$$

$$-\frac{4\left(\frac{2}{3}\left(\frac{3}{4}\int \frac{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)-\frac{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

$$\Big\downarrow \text{487}$$

$$-\frac{4\left(\frac{2}{3}\left(\frac{3}{4}\left(\int \frac{1}{\frac{1-ax}{ax+1}+1}d\sqrt{\frac{1-ax}{ax+1}}-\frac{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)-\frac{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{4\left(\frac{2}{3}\left(\frac{3}{4}\left(\arctan\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)-\frac{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

input
```
Int[E^(2*ArcSech[a*x])*x^2,x]
```

output
```
(-4*(-1/6*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4/(1 + (1 - a*x)/(1 + a*x))^3 +
(2*((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3)/(4*(1 +
(1 - a*x)/(1 + a*x))^2) + (3*(-1/2*((1 - Sqrt[(1 - a*x)/(1 + a*x)])*(1 + S
qrt[(1 - a*x)/(1 + a*x)]))/(1 + (1 - a*x)/(1 + a*x)) + ArcTan[Sqrt[(1 - a*
x)/(1 + a*x)]]))/4))/3))/a^3
```

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 490
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] - Simp[c*(n/(2*a*(
p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0] && LtQ[p, -1]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.67.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{-\frac{1}{3}a^2x^3+x}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\left(\sqrt{-a^2x^2+1}\,x\ \mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\mathrm{csgn}(a)}{a^2\sqrt{-a^2x^2+1}}+\frac{x}{a^2}$	105

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/3*a^2*x^3+x)+1/a^2*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*((
-a^2*x^2+1)^(1/2)*x*csgn(a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))/(-a^
2*x^2+1)^(1/2)*csgn(a)+x/a^2
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.51

$$\int e^{2\text{sech}^{-1}(ax)}x^2\,dx = -\frac{a^3x^3 - 3\,a^2x^2\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 6\,ax + 3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{3\,a^3}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="fric
as")
```

output
```
-1/3*(a^3*x^3 - 3*a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 6
*a*x + 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^3
```

3.67.6 Sympy [F]

$$\int e^{2\text{sech}^{-1}(ax)}x^2\,dx = \frac{\int 2\,dx + \int (-a^2x^2)\,dx + \int 2ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**2,x)
```

output
```
(Integral(2, x) + Integral(-a**2*x**2, x) + Integral(2*a*x*sqrt(-1 + 1/(a*
x))*sqrt(1 + 1/(a*x)), x))/a**2
```

3.67.7 Maxima [F]

$$\int e^{2\text{sech}^{-1}(ax)}x^2\,dx = \int x^2\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="maxi
ma")
```

output
```
2*x/a^2 + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1), x)/a^2 - integrate(x^2
, x)
```

3.67.8 Giac [F]

$$\int e^{2\text{sech}^{-1}(ax)}x^2\,dx = \int x^2\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="giac
")
```

output
```
integrate(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 12.78 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.49

$$\int e^{2\text{sech}^{-1}(ax)}x^2\,dx = \frac{\frac{1\mathrm{i}}{16\,a^3}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{8\,a^3\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{16\,a^3\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}-\frac{x^3\,\left(\frac{a^2}{3}-\frac{2}{x^2}\right)}{a^2}+\frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)\,2\mathrm{i}}{a^3}+\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{a^3}-\frac{\ln\left(\frac{2\,a\,\sqrt{\frac{a+\frac{1}{x}}{a}}-\frac{2}{x}+a\,\sqrt{-\frac{a-\frac{1}{x}}{a}}\,2\mathrm{i}}{2\,a+\frac{1}{x}-2\,a\,\sqrt{\frac{a+\frac{1}{x}}{a}}}\right)\,1\mathrm{i}}{a^3}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,a^3\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}$$

input
```
int(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
((log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log(
((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))*2i)/a^3 + (log(((1/
(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/a^3 + (1i/(16*a^3) +
 (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(8*a^3*((1/(a*x) + 1)^(1/2) - 1)^2) - (
((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(16*a^3*((1/(a*x) + 1)^(1/2) - 1)^4))/((
(1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + (2*((1/(a*x) - 1
)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) - 1)^(1/2) - 1i)^
6/((1/(a*x) + 1)^(1/2) - 1)^6) - (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2/x + 2
*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)/a)^(1/2)))*1i)/a^3 + (
((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^3*((1/(a*x) + 1)^(1/2) - 1)^2) - (x
^3*(a^2/3 - 2/x^2))/a^2
```

3.67. $\int e^{2\text{sech}^{-1}(ax)}x^2\,dx$

3.68 $\int e^{2\text{sech}^{-1}(ax)}x\,dx$

3.68.1 Optimal result

Integrand size = 10, antiderivative size = 85

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx=-\frac{(1+ax)^2}{2a^2}+\frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}+\frac{2\log(1+ax)}{a^2}+\frac{4\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

output
```
-1/2*(a*x+1)^2/a^2+2*ln(a*x+1)/a^2+4*ln(1-((-a*x+1)/(a*x+1))^(1/2))/a^2+(a
*x+1)*(1+2*((-a*x+1)/(a*x+1))^(1/2))/a^2
```

3.68.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx=\frac{-a^2x^2+4\sqrt{\frac{1-ax}{1+ax}}(1+ax)+8\log(x)-4\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}$$

input
```
Integrate[E^(2*ArcSech[a*x])*x,x]
```

output
```
(-(a^2*x^2) + 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 8*Log[x] - 4*Log[1 +
 Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(2*a^2)
```

3.68.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6891, 7268, 25, 2178, 27, 2027, 2178, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{2\text{sech}^{-1}(ax)}dx$$

$$\Big\downarrow \text{6891}$$

$$\int x\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2dx$$

$$\Big\downarrow \text{7268}$$

$$\frac{4\int -\frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}}{a^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}}{a^2}$$

$$\Big\downarrow \text{2178}$$

$$\frac{4\left(\frac{1}{4}\int -\frac{4\left(\frac{1-ax}{ax+1}+3\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}-\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(-\int \frac{\frac{1-ax}{ax+1}+3\sqrt{\frac{1-ax}{ax+1}}}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}-\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{2027}$$

$$\frac{4\left(-\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+3\right)}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}-\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{2178}$$

$$\frac{4\left(\frac{1}{2}\int -\frac{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\frac{1-ax}{ax+1}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}+\frac{2\sqrt{\frac{1-ax}{ax+1}}+1}{2\left(\frac{1-ax}{ax+1}+1\right)}-\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(-\int \frac{\sqrt{\frac{1-ax}{ax+1}}+1}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\frac{1-ax}{ax+1}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}+\frac{2\sqrt{\frac{1-ax}{ax+1}}+1}{2\left(\frac{1-ax}{ax+1}+1\right)}-\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{657}$$

$$\frac{4\left(-\int \left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{\frac{1-ax}{ax+1}+1}+\frac{1}{1-\sqrt{\frac{1-ax}{ax+1}}}\right)d\sqrt{\frac{1-ax}{ax+1}}+\frac{2\sqrt{\frac{1-ax}{ax+1}}+1}{2\left(\frac{1-ax}{ax+1}+1\right)}-\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(\frac{2\sqrt{\frac{1-ax}{ax+1}}+1}{2\left(\frac{1-ax}{ax+1}+1\right)}-\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)^2}+\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{1}{2}\log\left(\frac{1-ax}{ax+1}+1\right)\right)}{a^2}$$

input
```
Int[E^(2*ArcSech[a*x])*x,x]
```

output
```
(4*(-1/2*1/(1 + (1 - a*x)/(1 + a*x))^2 + (1 + 2*Sqrt[(1 - a*x)/(1 + a*x)])
/(2*(1 + (1 - a*x)/(1 + a*x))) + Log[1 - Sqrt[(1 - a*x)/(1 + a*x)]] - Log[
1 + (1 - a*x)/(1 + a*x)]/2))/a^2
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.68.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{-\frac{a^2x^2}{2}+\ln(x)}{a^2}-\frac{2\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\left(-\sqrt{-a^2x^2+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{a\sqrt{-a^2x^2+1}}+\frac{\ln(x)}{a^2}$	98

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/2*a^2*x^2+ln(x))-2/a*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*
(-(-a^2*x^2+1)^(1/2)+arctanh(1/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)+ln(
x)/a^2
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx =$$
$$-\frac{a^2x^2-4\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+2\,\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-2\,\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-4\,\log}{2\,a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="fricas
")
```

output
```
-1/2*(a^2*x^2 - 4*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2*log
(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - 2*log(a*x*sqrt((a
*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 4*log(x))/a^2
```

3.68.6 Sympy [F]

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx = \frac{\int \frac{2}{x}\,dx + \int \left(-a^2x\right)\,dx + \int 2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x,x)
```

output
```
(Integral(2/x, x) + Integral(-a**2*x, x) + Integral(2*a*sqrt(-1 + 1/(a*x))
*sqrt(1 + 1/(a*x)), x))/a**2
```

3.68.7 Maxima [F]

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx=\int x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="maxima
")
```

output
```
integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

3.68.8 Giac [F]

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx=\int x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="giac")
```

output
```
integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx=\frac{2\,x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{a}-\frac{2\,\text{acosh}\left(\frac{1}{a\,x}\right)}{a^2}-\frac{x^2}{2}-\frac{2\,\ln\left(\frac{1}{x}\right)}{a^2}$$

input
```
int(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
(2*x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/a - (2*acosh(1/(a*x)))/a^2 -
 x^2/2 - (2*log(1/x))/a^2
```

3.69 $\int e^{2\text{sech}^{-1}(ax)}\,dx$

3.69.1 Optimal result

Integrand size = 8, antiderivative size = 57

$$\int e^{2\text{sech}^{-1}(ax)}\,dx=-x-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}+\frac{4\arctan\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

output
```
-x+4*arctan(((-a*x+1)/(a*x+1))^(1/2))/a-4/a/(1-((-a*x+1)/(a*x+1))^(1/2))
```

3.69.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int e^{2\text{sech}^{-1}(ax)}\,dx=-\frac{2+a^2x^2+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)+2ax\arctan\left(\frac{ax}{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}\right)}{a^2x}$$

input
```
Integrate[E^(2*ArcSech[a*x]),x]
```

output
```
-((2 + a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 2*a*x*ArcTan[(a*x
)/(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))])/(a^2*x))
```

3.69.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6886, 7268, 2178, 27, 594, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\text{sech}^{-1}(ax)}dx$$

$$\Big\downarrow \; 6886$$

$$\int \left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2dx$$

$$\Big\downarrow \; 7268$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}}{a}$$

$$\Big\downarrow \; 2178$$

$$-\frac{4\left(\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)}-\frac{1}{2}\int -\frac{4\sqrt{\frac{1-ax}{ax+1}}}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2\left(\frac{1-ax}{ax+1}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}\right)}{a}$$

$$\Big\downarrow \; 27$$

$$-\frac{4\left(2\int \frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2\left(\frac{1-ax}{ax+1}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)}{a}$$

$$\Big\downarrow \; 594$$

$$-\frac{4\left(2\left(\frac{1}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{1}{2}\int \frac{1}{\frac{1-ax}{ax+1}+1}d\sqrt{\frac{1-ax}{ax+1}}\right)+\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)}{a}$$

$$\Big\downarrow \; 216$$

$$-\frac{4\left(2\left(\frac{1}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{1}{2}\arctan\left(\sqrt{\frac{1-ax}{ax+1}}\right)\right)+\frac{1}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)}{a}$$

input
```
Int[E^(2*ArcSech[a*x]),x]
```

output
```
(-4*(1/(2*(1 + (1 - a*x)/(1 + a*x))) + 2*(1/(2*(1 - Sqrt[(1 - a*x)/(1 + a*
x)])) - ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]/2)))/a
```

3.69.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6886
```
Int[E^(ArcSech[u_]*(n_.)), x_Symbol] :> Int[(1/u + Sqrt[(1 - u)/(1 + u)] +
(1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; IntegerQ[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.69.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{-a^2x-\frac{1}{x}}{a^2}-\frac{2\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\left(\arctan\left(\frac{\operatorname{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)ax+\operatorname{csgn}(a)\sqrt{-a^2x^2+1}\right)\operatorname{csgn}(a)}{a\sqrt{-a^2x^2+1}}-\frac{1}{x\,a^2}$	111

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-a^2*x-1/x)-2/a*((a*x+1)/a/x)^(1/2)*(-(a*x-1)/a/x)^(1/2)*(arctan(cs
gn(a)*a*x/(-a^2*x^2+1)^(1/2))*a*x+csgn(a)*(-a^2*x^2+1)^(1/2))*csgn(a)/(-a^
2*x^2+1)^(1/2)-1/x/a^2
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int e^{2\operatorname{sech}^{-1}(ax)}\,dx=-\frac{a^2x^2+2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-2\,ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)+2}{a^2x}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="fricas")
```

output
```
-(a^2*x^2 + 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2*a*x*arc
tan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) + 2)/(a^2*x)
```

3.69.6 Sympy [F]

$$\int e^{2\operatorname{sech}^{-1}(ax)}\,dx=\frac{\int\left(-a^2\right)dx+\int\frac{2}{x^2}\,dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2,x)
```

output
```
(Integral(-a**2, x) + Integral(2/x**2, x) + Integral(2*a*sqrt(-1 + 1/(a*x)
)*sqrt(1 + 1/(a*x))/x, x))/a**2
```

3.69.7 Maxima [F]

$$\int e^{2\text{sech}^{-1}(ax)}\,dx=\int\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="maxima")
```

output
```
-x + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^2, x)/a^2 + integrate(x^(-
2), x)/a^2 - 1/(a^2*x)
```

3.69.8 Giac [F]

$$\int e^{2\text{sech}^{-1}(ax)}\,dx=\int\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 7.78 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int e^{2\text{sech}^{-1}(ax)}\,dx=-x-\frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)2\mathrm{i}}{a}-\frac{2}{a^2\,x}+\frac{\left(1+\sqrt{-\frac{a-\frac{1}{x}}{a}}\,1\mathrm{i}\right)^2\left(\sqrt{\frac{a+\frac{1}{x}}{a}}-1\right)^2 4\mathrm{i}}{a\left(\sqrt{\frac{a+\frac{1}{x}}{a}}\,1\mathrm{i}+\sqrt{-\frac{a-\frac{1}{x}}{a}}-2\mathrm{i}\right)^2}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
(((-(a - 1/x)/a)^(1/2)*1i + 1)^2*(((a + 1/x)/a)^(1/2) - 1)^2*4i)/(a*(((a +
 1/x)/a)^(1/2)*1i + (-(a - 1/x)/a)^(1/2) - 2i)^2) - ((log(((1/(a*x) - 1)^(
1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log(((1/(a*x) - 1)^(1/2) -
 1i)/((1/(a*x) + 1)^(1/2) - 1)))*2i)/a - 2/(a^2*x) - x
```

3.69. $\int e^{2\text{sech}^{-1}(ax)}\,dx$

3.70 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx$

3.70.1 Optimal result

Integrand size = 12, antiderivative size = 86

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx = -\frac{2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{2}{1-\sqrt{\frac{1-ax}{1+ax}}}-\log(1+ax)-2\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)$$

output
```
-ln(a*x+1)-2*ln(1-((-a*x+1)/(a*x+1))^(1/2))-2/(1-((-a*x+1)/(a*x+1))^(1/2))
^2+2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

3.70.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx = -\frac{1}{a^2x^2}-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a^2x^2}-2\log(x)+\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)$$

input
```
Integrate[E^(2*ArcSech[a*x])/x,x]
```

output
```
-(1/(a^2*x^2)) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a^2*x^2) - 2*Log[x
] + Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]]
```

3.70.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6891, 7268, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6891}$$

$$\int \frac{\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2}{x}dx$$

$$\Big\downarrow \text{ 7268}$$

$$4\int -\frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3\left(\frac{1-ax}{ax+1}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{ 25}$$

$$-4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3\left(\frac{1-ax}{ax+1}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{ 2160}$$

$$-4\int \left(-\frac{\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)}+\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)}-\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^2}-\frac{1}{\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^3}\right)d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$4\left(\frac{1}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{1}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{1}{2}\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)+\frac{1}{4}\log\left(\frac{1-ax}{ax+1}+1\right)\right)$$

input
```
Int[E^(2*ArcSech[a*x])/x,x]
```

output
```
4*(-1/2*1/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2 + 1/(2*(1 - Sqrt[(1 - a*x)/(1
+ a*x)])) - Log[1 - Sqrt[(1 - a*x)/(1 + a*x)]]/2 + Log[1 + (1 - a*x)/(1 +
a*x)]/4)
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.70.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{-a^2\ln(x)-\frac{1}{2x^2}}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\left(a^2x^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\sqrt{-a^2x^2+1}\right)}{ax\sqrt{-a^2x^2+1}}-\frac{1}{2a^2x^2}$	110

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-a^2*ln(x)-1/2/x^2)+1/a*((a*x+1)/a/x)^(1/2)/x*(-(a*x-1)/a/x)^(1/2)*
(a^2*x^2*arctanh(1/(-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1
/2)-1/2/a^2/x^2
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.60

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx = \frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,a^2x^2\log\left(x\right)-2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\ldots}}{2\,a^2x^2}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="fricas
")
```

output
```
1/2*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a
^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a^2*x
^2*log(x) - 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2)/(a^2*x
^2)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx = \frac{-2a^2\cdot\left(2\sqrt{-1+\frac{1}{ax}}\left(\frac{\left(1+\frac{1}{ax}\right)^{\frac{3}{2}}}{4}-\frac{\sqrt{1+\frac{1}{ax}}}{4}\right)-\log\left(2\sqrt{-1+\frac{1}{ax}}+2\sqrt{1+\frac{1}{ax}}\right)\right)-a^2\log\left(x\right)-\frac{1}{x^2}}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x,x)
```

output
```
(-2*a**2*(2*sqrt(-1 + 1/(a*x))*((1 + 1/(a*x))**(3/2)/4 - sqrt(1 + 1/(a*x))
/4) - log(2*sqrt(-1 + 1/(a*x)) + 2*sqrt(1 + 1/(a*x)))) - a**2*log(x) - 1/x
**2)/a**2
```

3.70.7 Maxima [F]

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="maxima
")
```

output
```
2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3, x)/a^2 - 1/(a^2*x^2) - integ
rate(1/x, x)
```

3.70.8 Giac [F]

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x, x)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.76

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx = \ln\left(\frac{1}{x}\right) - 4\,\text{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right) + 2\,\text{acosh}\left(\frac{1}{a\,x}\right) + \frac{\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}}{1+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}} - \frac{1}{a^2\,x^2}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x,x)
```

output
```
log(1/x) - 4*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) +
 2*acosh(1/(a*x)) + ((28*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2
) - 1)^3 + (28*((1/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 +
 (4*((1/(a*x) - 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (4*((1/(a*
x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1))/((6*((1/(a*x) - 1)^(1/2) -
 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1
/(a*x) + 1)^(1/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)
^(1/2) - 1)^6 + ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8 +
 1) - 1/(a^2*x^2)
```

3.70. $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x} \, dx$

3.71 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^2} \, dx$

3.71.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^2} \, dx = -\frac{4a}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{2a}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}$$

output
```
-4/3*a/(1-((-a*x+1)/(a*x+1))^(1/2))^3+2*a/(1-((-a*x+1)/(a*x+1))^(1/2))^2
```

3.71.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^2} \, dx = \frac{-2+3a^2x^2+2(-1+ax)\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{3a^2x^3}$$

input
```
Integrate[E^(2*ArcSech[a*x])/x^2,x]
```

output
```
(-2 + 3*a^2*x^2 + 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a
^2*x^3)
```

3.71.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6891, 7268, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^2} \, dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{7268}$$

$$-4a\int \frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{53}$$

$$-4a\int \left(\frac{1}{\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^3}+\frac{1}{\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^4}\right)d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{2009}$$

$$-4a\left(\frac{1}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{1}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}\right)$$

input
```
Int[E^(2*ArcSech[a*x])/x^2,x]
```

output
```
-4*a*(1/(3*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) - 1/(2*(1 - Sqrt[(1 - a*x)/(
1 + a*x)])^2))
```

3.71.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.71.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{-\frac{1}{3x^3}+\frac{a^2}{x}}{a^2}+\frac{2\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)}{3a\,x^2}-\frac{1}{3a^2x^3}$	73

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/3/x^3+a^2/x)+2/3/a*((a*x+1)/a/x)^(1/2)/x^2*(-(a*x-1)/a/x)^(1/2)*
(a^2*x^2-1)-1/3/a^2/x^3
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^2}\,dx = \frac{3\,a^2x^2 + 2\,(a^3x^3 - ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 2}{3\,a^2x^3}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="fric
as")
```

output
```
1/3*(3*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(
a*x)) - 2)/(a^2*x^3)
```

3.71.6 Sympy [F]

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^2}\,dx = \frac{\int \frac{2}{x^4}\,dx + \int \left(-\frac{a^2}{x^2}\right)\,dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^3}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**2,x)
```

output
```
(Integral(2/x**4, x) + Integral(-a**2/x**2, x) + Integral(2*a*sqrt(-1 + 1/
(a*x))*sqrt(1 + 1/(a*x))/x**3, x))/a**2
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} + \frac{2\,(a^2x^3 - x)\sqrt{ax+1}\sqrt{-ax+1}}{3\,a^2x^4} - \frac{2}{3\,a^2x^3}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="maxi
ma")
```

output
```
1/x + 2/3*(a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^2*x^4) - 2/3/(a^2*
x^3)
```

3.71.8 Giac [F]

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^2}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="giac
")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^2, x)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 4.76 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = \frac{a^2\,x^2-\frac{2}{3}}{a^2\,x^3}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,\sqrt{\frac{1}{a\,x}+1}}{3\,a}-\frac{2\,a\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{3}\right)}{x^2}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^2,x)
```

output
```
(a^2*x^2 - 2/3)/(a^2*x^3) - ((1/(a*x) - 1)^(1/2)*((2*(1/(a*x) + 1)^(1/2))/
(3*a) - (2*a*x^2*(1/(a*x) + 1)^(1/2))/3))/x^2
```

3.72 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^3} dx$

3.72.1 Optimal result

Integrand size = 12, antiderivative size = 147

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^3} dx = -\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} + \frac{1}{2}a^2\text{arctanh}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

output
```
1/2*a^2*arctanh(((-a*x+1)/(a*x+1))^(1/2))-a^2/(1-((-a*x+1)/(a*x+1))^(1/2))
^4+2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))^3-3/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2
))^2+1/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

3.72.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^3} dx = \frac{\frac{(1+ax)\left(-2+2ax-2\sqrt{\frac{1-ax}{1+ax}}+a^2x^2\sqrt{\frac{1-ax}{1+ax}}\right)}{x^4} - a^4\log(x) + a^4\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{4a^2}$$

input
```
Integrate[E^(2*ArcSech[a*x])/x^3,x]
```

output
```
(((1 + a*x)*(-2 + 2*a*x - 2*Sqrt[(1 - a*x)/(1 + a*x)] + a^2*x^2*Sqrt[(1 -
a*x)/(1 + a*x)]))/x^4 - a^4*Log[x] + a^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)]
 + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(4*a^2)
```

3.72.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6891, 7268, 25, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{ 6891}$$

$$\int \frac{\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2}{x^3}dx$$

$$\Big\downarrow \text{ 7268}$$

$$4a^2\int -\frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{ 25}$$

$$-4a^2\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{ 2115}$$

$$-4a^2\int \left(-\frac{1}{8\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^2}-\frac{3}{4\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^3}-\frac{3}{2\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^4}-\frac{1}{\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^5}+\frac{1}{8\left(\frac{1-ax}{ax+1}-1\right)}\right)d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$4a^2\left(\frac{1}{8}\text{arctanh}\left(\sqrt{\frac{1-ax}{ax+1}}\right)+\frac{1}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{1}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{1}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}\right)$$

input
```
Int[E^(2*ArcSech[a*x])/x^3,x]
```

output
```
4*a^2*(-1/4*1/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4 + 1/(2*(1 - Sqrt[(1 - a*x)
/(1 + a*x)])^3) - 3/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) + 1/(8*(1 - Sqrt
[(1 - a*x)/(1 + a*x)])) + ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]]/8)
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.72.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{-\frac{1}{4x^4}+\frac{a^2}{2x^2}}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)a^4x^4+a^2x^2\sqrt{-a^2x^2+1}-2\sqrt{-a^2x^2+1}\right)}{4a\,x^3\sqrt{-a^2x^2+1}}-\frac{1}{4a^2x^4}$	131

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/4/x^4+1/2*a^2/x^2)+1/4/a*((a*x+1)/a/x)^(1/2)/x^3*(-(a*x-1)/a/x)^
(1/2)*(arctanh(1/(-a^2*x^2+1)^(1/2))*a^4*x^4+a^2*x^2*(-a^2*x^2+1)^(1/2)-2*
(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/4/a^2/x^4
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+4\,a^2x^2+2\left(a^3x^3-2\,ax\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{}}{8\,a^2x^4}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="fric
as")
```

output
```
1/8*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a
^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 4*a^2*x
^2 + 2*(a^3*x^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 4)
/(a^2*x^4)
```

3.72.6 Sympy [F]

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \frac{\int \frac{2}{x^5}\,dx + \int \left(-\frac{a^2}{x^3}\right)\,dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^4}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**3,x)
```

output
```
(Integral(2/x**5, x) + Integral(-a**2/x**3, x) + Integral(2*a*sqrt(-1 + 1/
(a*x))*sqrt(1 + 1/(a*x))/x**4, x))/a**2
```

3.72.7 Maxima [F]

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^3}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="maxi
ma")
```

output
```
2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^5, x)/a^2 - 1/2/(a^2*x^4) - int
egrate(x^(-3), x)
```

3.72.8 Giac [F]

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^3}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="giac
")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^3, x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 46.86 (sec) , antiderivative size = 885, normalized size of antiderivative = 6.02

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = a^2 \operatorname{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)$$
$$-\frac{\frac{28\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{28\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{4\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{4\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}}{1+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}}$$
$$-\frac{\frac{23\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{333\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{671\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{671\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^9}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^9}+\frac{333\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{11}}+\frac{23\,a^2\left(\sqrt{\cdots}\right.}{\left(\sqrt{\cdots}\right.}}{1+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{56\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{70\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{56\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}-\frac{8\left(\sqrt{\frac{1}{a\,x}-\cdots}\right.}{\left(\sqrt{\frac{1}{a\,x}+\cdots}\right.}}$$
$$+\frac{1}{2\,x^2}-\frac{1}{2\,a^2\,x^4}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^3,x)
```

output
```
a^2*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) - ((28*a^2
*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (28*a^2*((1/(
a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (4*a^2*((1/(a*x) -
1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (4*a^2*((1/(a*x) - 1)^(1/2
) - 1i))/((1/(a*x) + 1)^(1/2) - 1))/((6*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/
(a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^
(1/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^
6 + ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8 + 1) - ((23*a
^2*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (333*a^2*((
1/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (671*a^2*((1/(a*
x) - 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (671*a^2*((1/(a*x) -
1)^(1/2) - 1i)^9)/((1/(a*x) + 1)^(1/2) - 1)^9 + (333*a^2*((1/(a*x) - 1)^(1
/2) - 1i)^11)/((1/(a*x) + 1)^(1/2) - 1)^11 + (23*a^2*((1/(a*x) - 1)^(1/2)
- 1i)^13)/((1/(a*x) + 1)^(1/2) - 1)^13 - (3*a^2*((1/(a*x) - 1)^(1/2) - 1i)
^15)/((1/(a*x) + 1)^(1/2) - 1)^15 - (3*a^2*((1/(a*x) - 1)^(1/2) - 1i))/((1
/(a*x) + 1)^(1/2) - 1))/((28*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^
(1/2) - 1)^4 - (8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^
2 - (56*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (70*((
1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (56*((1/(a*x) -
1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1/(a*x) - 1)^(1...
```

3.73 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4}\,dx$

3.73.1 Optimal result

Integrand size = 12, antiderivative size = 183

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4}\,dx = -\frac{4a^3}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{2a^3}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{7a^3}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{3a^3}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^3}{4\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}$$

output
```
-4/5*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))^5+2*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))
^4-7/3*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))^3+3/2*a^3/(1-((-a*x+1)/(a*x+1))^(1
/2))^2-1/4*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))-1/4*a^3/(1+((-a*x+1)/(a*x+1))^
(1/2))
```

3.73.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.38

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4}\,dx = \frac{-6+5a^2x^2+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(-3+3ax-2a^2x^2+2a^3x^3\right)}{15a^2x^5}$$

input
```
Integrate[E^(2*ArcSech[a*x])/x^4,x]
```

output
```
(-6 + 5*a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*
a^2*x^2 + 2*a^3*x^3))/(15*a^2*x^5)
```

3.73.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6891, 7268, 27, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \; 6891$$

$$\int \frac{\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2}{x^4}dx$$

$$\Big\downarrow \; 7268$$

$$-4a\int \frac{a^2\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^6\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \; 27$$

$$-4a^3\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^6\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \; 2115$$

$$-4a^3\int \left(-\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^2}+\frac{3}{4\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^3}+\frac{7}{4\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^4}+\frac{2}{\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^5}+\right.$$

$$\Big\downarrow \; 2009$$

$$-4a^3\left(\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{1}{16\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{7}{12\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{1}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}+\frac{}{5(1}\right.$$

input
```
Int[E^(2*ArcSech[a*x])/x^4,x]
```

output
```
-4*a^3*(1/(5*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5) - 1/(2*(1 - Sqrt[(1 - a*x)
/(1 + a*x)])^4) + 7/(12*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) - 3/(8*(1 - Sqr
t[(1 - a*x)/(1 + a*x)])^2) + 1/(16*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) + 1/(1
6*(1 + Sqrt[(1 - a*x)/(1 + a*x)])))
```

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.73.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{\frac{a^2}{3x^3}-\frac{1}{5x^5}}{a^2}+\frac{2\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)\,(2a^2x^2+3)}{15a\,x^4}-\frac{1}{5a^2x^5}$	84

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/3*a^2/x^3-1/5/x^5)+2/15/a*((a*x+1)/a/x)^(1/2)/x^4*(-(a*x-1)/a/x)^
(1/2)*(a^2*x^2-1)*(2*a^2*x^2+3)-1/5/a^2/x^5
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.38

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = \frac{5\,a^2x^2+2\left(2\,a^5x^5+a^3x^3-3\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-6}{15\,a^2x^5}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="fric
as")
```

output
```
1/15*(5*a^2*x^2 + 2*(2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sq
rt(-(a*x - 1)/(a*x)) - 6)/(a^2*x^5)
```

3.73.6 Sympy [F]

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = \frac{\int \frac{2}{x^6}\,dx+\int \left(-\frac{a^2}{x^4}\right)\,dx+\int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^5}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**4,x)
```

output
```
(Integral(2/x**6, x) + Integral(-a**2/x**4, x) + Integral(2*a*sqrt(-1 + 1/
(a*x))*sqrt(1 + 1/(a*x))/x**5, x))/a**2
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.31

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4} dx = \frac{1}{3x^3} + \frac{2(2a^4x^5 + a^2x^3 - 3x)\sqrt{ax+1}\sqrt{-ax+1}}{15a^2x^6} - \frac{2}{5a^2x^5}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="maxi
ma")
```

output
```
1/3/x^3 + 2/15*(2*a^4*x^5 + a^2*x^3 - 3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a
^2*x^6) - 2/5/(a^2*x^5)
```

3.73.8 Giac [F]

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4} dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^4} dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="giac
")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^4, x)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.47

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4} dx = \frac{\sqrt{\frac{1}{ax}-1}\left(\frac{2ax^2\sqrt{\frac{1}{ax}+1}}{15} - \frac{2\sqrt{\frac{1}{ax}+1}}{5a} + \frac{4a^3x^4\sqrt{\frac{1}{ax}+1}}{15}\right)}{x^4} + \frac{\frac{a^2x^2}{3}-\frac{2}{5}}{a^2x^5}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^4,x)
```

output
```
((1/(a*x) - 1)^(1/2)*((2*a*x^2*(1/(a*x) + 1)^(1/2))/15 - (2*(1/(a*x) + 1)^
(1/2))/(5*a) + (4*a^3*x^4*(1/(a*x) + 1)^(1/2))/15))/x^4 + ((a^2*x^2)/3 - 2
/5)/(a^2*x^5)
```

3.74 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx$

3.74.1 Optimal result

Integrand size = 12, antiderivative size = 267

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx = -\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}+\frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^4}{8\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^4}{8\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}+\frac{1}{4}a^4\text{arctanh}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

output
```
1/4*a^4*arctanh(((-a*x+1)/(a*x+1))^(1/2))-2/3*a^4/(1-((-a*x+1)/(a*x+1))^(1
/2))^6+2*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^5-3*a^4/(1-((-a*x+1)/(a*x+1))^(1
/2))^4+8/3*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^3-11/8*a^4/(1-((-a*x+1)/(a*x+1
))^(1/2))^2+3/8*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))-1/8*a^4/(1+((-a*x+1)/(a*x
+1))^(1/2))^2+1/8*a^4/(1+((-a*x+1)/(a*x+1))^(1/2))
```

3.74.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.51

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx$$
$$= \frac{-8+6a^2x^2+\sqrt{\frac{1-ax}{1+ax}}(-8-8ax+2a^2x^2+2a^3x^3+3a^4x^4+3a^5x^5)-3a^6x^6\log(x)+3a^6x^6\log\left(1+\sqrt{\frac{1-ax}{1+ax}}\right.}{24a^2x^6}$$

input
```
Integrate[E^(2*ArcSech[a*x])/x^5,x]
```

output
```
(-8 + 6*a^2*x^2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^
3*x^3 + 3*a^4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt
[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(24*a^2*x^6)
```

3.74.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6891, 7268, 25, 27, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx$$
$$\Big\downarrow \text{6891}$$
$$\int \frac{\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2}{x^5}dx$$
$$\Big\downarrow \text{7268}$$
$$4a\int -\frac{a^3\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^7\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}$$
$$\Big\downarrow \text{25}$$

$$-4a\int \frac{a^3\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^7\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}$$

$\big\downarrow$ 27

$$-4a^4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^7\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}$$

$\big\downarrow$ 2115

$$-4a^4\int \left(-\frac{3}{32\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^2}+\frac{1}{32\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{11}{16\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^3}-\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{2}{\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^4}\right.$$

$\big\downarrow$ 2009

$$-4a^4\left(-\frac{1}{16}\text{arctanh}\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{3}{32\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{1}{32\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{11}{32\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{1}{32\left(\sqrt{\frac{1-ax}{ax+1}}+1\right.}\right.$$

input
```
Int[E^(2*ArcSech[a*x])/x^5,x]
```

output
```
-4*a^4*(1/(6*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^6) - 1/(2*(1 - Sqrt[(1 - a*x)
/(1 + a*x)])^5) + 3/(4*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4) - 2/(3*(1 - Sqrt
[(1 - a*x)/(1 + a*x)])^3) + 11/(32*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - 3/
(32*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) + 1/(32*(1 + Sqrt[(1 - a*x)/(1 + a*x)
])^2) - 1/(32*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) - ArcTanh[Sqrt[(1 - a*x)/(1
 + a*x)]]/16)
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.74.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.57

method	result
default	$\frac{\frac{a^2}{4x^4}-\frac{1}{6x^6}}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\left(3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)a^6x^6+3\sqrt{-a^2x^2+1}\,a^4x^4+2a^2x^2\sqrt{-a^2x^2+1}-8\sqrt{-a^2x^2+1}\right)}{24a\,x^5\sqrt{-a^2x^2+1}}-\frac{1}{6a^2x^6}$

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/4*a^2/x^4-1/6/x^6)+1/24/a*((a*x+1)/a/x)^(1/2)/x^5*(-(a*x-1)/a/x)^
(1/2)*(3*arctanh(1/(-a^2*x^2+1)^(1/2))*a^6*x^6+3*(-a^2*x^2+1)^(1/2)*a^4*x^
4+2*a^2*x^2*(-a^2*x^2+1)^(1/2)-8*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/
6/a^2/x^6
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx = \frac{3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+12\,a^2x^2+2\left(3\,a^5x^5+2\,a^3x^3\right.}{48\,a^2x^6}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="fric
as")
```

output
```
1/48*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)
- 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 12
*a^2*x^2 + 2*(3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(
a*x - 1)/(a*x)) - 16)/(a^2*x^6)
```

3.74.6 Sympy [F]

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx = \frac{\int \frac{2}{x^7}\,dx+\int\left(-\frac{a^2}{x^5}\right)dx+\int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^6}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**5,x)
```

output
```
(Integral(2/x**7, x) + Integral(-a**2/x**5, x) + Integral(2*a*sqrt(-1 + 1/
(a*x))*sqrt(1 + 1/(a*x))/x**6, x))/a**2
```

3.74.7 Maxima [F]

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^5}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="maxi
ma")
```

output
```
2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^7, x)/a^2 - 1/3/(a^2*x^6) - int
egrate(x^(-5), x)
```

3.74.8 Giac [F]

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^5}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="giac
")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^5, x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 68.99 (sec) , antiderivative size = 2480, normalized size of antiderivative = 9.29

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx = \text{Too large to display}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^5,x)
```

output
```
((311*a^4*((1/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) -
(175*a^4*((1/(a*x) - 1)^(1/2) - 1i)^3)/(6*((1/(a*x) + 1)^(1/2) - 1)^3) + (
8361*a^4*((1/(a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (
42259*a^4*((1/(a*x) - 1)^(1/2) - 1i)^9)/(3*((1/(a*x) + 1)^(1/2) - 1)^9) +
(25295*a^4*((1/(a*x) - 1)^(1/2) - 1i)^11)/((1/(a*x) + 1)^(1/2) - 1)^11 + (
25295*a^4*((1/(a*x) - 1)^(1/2) - 1i)^13)/((1/(a*x) + 1)^(1/2) - 1)^13 + (4
2259*a^4*((1/(a*x) - 1)^(1/2) - 1i)^15)/(3*((1/(a*x) + 1)^(1/2) - 1)^15) +
 (8361*a^4*((1/(a*x) - 1)^(1/2) - 1i)^17)/(2*((1/(a*x) + 1)^(1/2) - 1)^17)
 + (311*a^4*((1/(a*x) - 1)^(1/2) - 1i)^19)/(2*((1/(a*x) + 1)^(1/2) - 1)^19
) - (175*a^4*((1/(a*x) - 1)^(1/2) - 1i)^21)/(6*((1/(a*x) + 1)^(1/2) - 1)^2
1) + (5*a^4*((1/(a*x) - 1)^(1/2) - 1i)^23)/(2*((1/(a*x) + 1)^(1/2) - 1)^23
) + (5*a^4*((1/(a*x) - 1)^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((66
*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (12*((1/(a*x)
 - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (220*((1/(a*x) - 1)^(1/
2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (495*((1/(a*x) - 1)^(1/2) - 1i)^
8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (792*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(
a*x) + 1)^(1/2) - 1)^10 + (924*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) +
1)^(1/2) - 1)^12 - (792*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2
) - 1)^14 + (495*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^
16 - (220*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 +...
```

3.74. $\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^5}\,dx$

3.75 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^6}\,dx$

3.75.1 Optimal result

Integrand size = 12, antiderivative size = 301

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^6}\,dx = -\frac{4a^5}{7\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^7}+\frac{2a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}-\frac{18a^5}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{4a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{35a^5}{12\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{11a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^5}{12\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{a^5}{8\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{a^5}{4\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}$$

output
```
-4/7*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^7+2*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))
^6-18/5*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^5+4*a^5/(1-((-a*x+1)/(a*x+1))^(1/
2))^4-35/12*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^3+11/8*a^5/(1-((-a*x+1)/(a*x+
1))^(1/2))^2-1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))-1/12*a^5/(1+((-a*x+1)/(a
*x+1))^(1/2))^3+1/8*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^2-1/4*a^5/(1+((-a*x+1
)/(a*x+1))^(1/2))
```

3.75.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.28

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^6}\,dx = \frac{-30+21a^2x^2+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(-15+15ax-12a^2x^2+12a^3x^3-8a^4x^4+8a^5x^5\right)}{105a^2x^7}$$

input
```
Integrate[E^(2*ArcSech[a*x])/x^6,x]
```

output
```
(-30 + 21*a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x
- 12*a^2*x^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a^2*x^7)
```

3.75.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6891, 7268, 27, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^6}\,dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)^2}{x^6}dx$$

$$\Big\downarrow \text{7268}$$

$$-4a\int \frac{a^4\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{27}$$

$$-4a^5\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{2115}$$

$$-4a^5\int \left(-\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}+\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^2}+\frac{11}{16\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$-4a^5\left(\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{1}{32\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{1}{48\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{1}{16\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{11}{32\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{}{48}\right.$$

input
```
Int[E^(2*ArcSech[a*x])/x^6,x]
```

output
```
-4*a^5*(1/(7*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^7) - 1/(2*(1 - Sqrt[(1 - a*x)
/(1 + a*x)])^6) + 9/(10*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5) - (1 - Sqrt[(1
- a*x)/(1 + a*x)])^(-4) + 35/(48*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) - 11/(
32*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) + 1/(16*(1 - Sqrt[(1 - a*x)/(1 + a*x
)])) + 1/(48*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) - 1/(32*(1 + Sqrt[(1 - a*x
)/(1 + a*x)])^2) + 1/(16*(1 + Sqrt[(1 - a*x)/(1 + a*x)])))
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.75. $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^6}\,dx$

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.75.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{-\frac{1}{7x^7}+\frac{a^2}{5x^5}}{a^2}+\frac{2\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)\,(8a^4x^4+12a^2x^2+15)}{105a\,x^6}-\frac{1}{7a^2x^7}$	92

input
```
int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/7/x^7+1/5*a^2/x^5)+2/105/a*((a*x+1)/a/x)^(1/2)/x^6*(-(a*x-1)/a/x
)^(1/2)*(a^2*x^2-1)*(8*a^4*x^4+12*a^2*x^2+15)-1/7/a^2/x^7
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.26

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6}\,dx=\frac{21\,a^2x^2+2\,(8\,a^7x^7+4\,a^5x^5+3\,a^3x^3-15\,ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-30}{105\,a^2x^7}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="fric
as")
```

output
```
1/105*(21*a^2*x^2 + 2*(8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a
*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 30)/(a^2*x^7)
```

3.75.6 Sympy [F]

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \frac{\int \frac{2}{x^8}\,dx + \int \left(-\frac{a^2}{x^6}\right)\,dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^7}\,dx}{a^2}$$

input
```
integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**6,x)
```

output
```
(Integral(2/x**8, x) + Integral(-a**2/x**6, x) + Integral(2*a*sqrt(-1 + 1/
(a*x))*sqrt(1 + 1/(a*x))/x**7, x))/a**2
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.22

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \frac{1}{5\,x^5} + \frac{2\,(8\,a^6x^7 + 4\,a^4x^5 + 3\,a^2x^3 - 15\,x)\sqrt{ax+1}\sqrt{-ax+1}}{105\,a^2x^8} - \frac{2}{7\,a^2x^7}$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="maxi
ma")
```

output
```
1/5/x^5 + 2/105*(8*a^6*x^7 + 4*a^4*x^5 + 3*a^2*x^3 - 15*x)*sqrt(a*x + 1)*s
qrt(-a*x + 1)/(a^2*x^8) - 2/7/(a^2*x^7)
```

3.75.8 Giac [F]

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^6}\,dx$$

input
```
integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="giac
")
```

output
```
integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^6, x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.35

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \frac{\frac{a^2\,x^2}{5}-\frac{2}{7}}{a^2\,x^7} + \frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,a\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{35}-\frac{2\,\sqrt{\frac{1}{a\,x}+1}}{7\,a}+\frac{8\,a^3\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{105}+\frac{16\,a^5\,x^6\,\sqrt{\frac{1}{a\,x}+1}}{105}\right)}{x^6}$$

input
```
int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^6,x)
```

output
```
((a^2*x^2)/5 - 2/7)/(a^2*x^7) + ((1/(a*x) - 1)^(1/2)*((2*a*x^2*(1/(a*x) +
1)^(1/2))/35 - (2*(1/(a*x) + 1)^(1/2))/(7*a) + (8*a^3*x^4*(1/(a*x) + 1)^(1
/2))/105 + (16*a^5*x^6*(1/(a*x) + 1)^(1/2))/105))/x^6
```

3.76 $\int e^{-\text{sech}^{-1}(ax)}x^4\,dx$

3.76.1 Optimal result

Integrand size = 12, antiderivative size = 147

$$\int e^{-\text{sech}^{-1}(ax)}x^4\,dx=-\frac{x}{a^4}-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^5}{5a^5}+\frac{(1+ax)^2\left(9+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^5}+\frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5}-\frac{(1+ax)^3\left(15+17\sqrt{\frac{1-ax}{1+ax}}\right)}{15a^5}$$

output
```
-x/a^4-1/5*(a*x+1)^5*((-a*x+1)/(a*x+1))^(1/2)/a^5+1/6*(a*x+1)^2*(9+4*((-a*
x+1)/(a*x+1))^(1/2))/a^5+1/20*(a*x+1)^4*(5+16*((-a*x+1)/(a*x+1))^(1/2))/a^
5-1/15*(a*x+1)^3*(15+17*((-a*x+1)/(a*x+1))^(1/2))/a^5
```

3.76.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

$$\int e^{-\text{sech}^{-1}(ax)}x^4\,dx=\frac{15a^4x^4-4\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(-2+2ax-3a^2x^2+3a^3x^3\right)}{60a^5}$$

input
```
Integrate[x^4/E^ArcSech[a*x],x]
```

output
```
(15*a^4*x^4 - 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-2 + 2*a*x - 3*a^2*
x^2 + 3*a^3*x^3))/(60*a^5)
```

3.76.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6891, 7268, 25, 2335, 27, 2345, 27, 2345, 27, 2345, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-\text{sech}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{x^4}{\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}}dx$$

$$\Big\downarrow \text{7268}$$

$$\frac{4\int -\frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{\left(\frac{1-ax}{ax+1}+1\right)^6}d\sqrt{\frac{1-ax}{ax+1}}}{a^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{\left(\frac{1-ax}{ax+1}+1\right)^6}d\sqrt{\frac{1-ax}{ax+1}}}{a^5}$$

$$\Big\downarrow \text{2335}$$

$$\frac{4\left(\frac{1}{10}\int \frac{2\left(5\left(\frac{1-ax}{ax+1}\right)^{7/2}-15\left(\frac{1-ax}{ax+1}\right)^{5/2}+15\left(\frac{1-ax}{ax+1}\right)^{3/2}-5\sqrt{\frac{1-ax}{ax+1}}-\frac{70(1-ax)}{ax+1}+\frac{40(1-ax)^2}{(ax+1)^2}-\frac{10(1-ax)^3}{(ax+1)^3}+8\right)}{\left(\frac{1-ax}{ax+1}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(\frac{1}{5}\int \frac{5\left(\frac{1-ax}{ax+1}\right)^{7/2}-15\left(\frac{1-ax}{ax+1}\right)^{5/2}+15\left(\frac{1-ax}{ax+1}\right)^{3/2}-5\sqrt{\frac{1-ax}{ax+1}}-\frac{70(1-ax)}{ax+1}+\frac{40(1-ax)^2}{(ax+1)^2}-\frac{10(1-ax)^3}{(ax+1)^3}+8}{\left(\frac{1-ax}{ax+1}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \text{2345}$$

$$\frac{4\left(\frac{1}{5}\left(\frac{16\sqrt{\frac{1-ax}{ax+1}}+5}{\left(\frac{1-ax}{ax+1}+1\right)^4}-\frac{1}{8}\int\frac{8\left(-5\left(\frac{1-ax}{ax+1}\right)^{5/2}+20\left(\frac{1-ax}{ax+1}\right)^{3/2}-35\sqrt{\frac{1-ax}{ax+1}}-\frac{50(1-ax)}{ax+1}+\frac{10(1-ax)^2}{(ax+1)^2}+8\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \ 27$$

$$\frac{4\left(\frac{1}{5}\left(\frac{16\sqrt{\frac{1-ax}{ax+1}}+5}{\left(\frac{1-ax}{ax+1}+1\right)^4}-\int\frac{-5\left(\frac{1-ax}{ax+1}\right)^{5/2}+20\left(\frac{1-ax}{ax+1}\right)^{3/2}-35\sqrt{\frac{1-ax}{ax+1}}-\frac{50(1-ax)}{ax+1}+\frac{10(1-ax)^2}{(ax+1)^2}+8}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \ 2345$$

$$\frac{4\left(\frac{1}{5}\left(\frac{1}{6}\int\frac{10\left(3\left(\frac{1-ax}{ax+1}\right)^{3/2}-15\sqrt{\frac{1-ax}{ax+1}}-\frac{6(1-ax)}{ax+1}+2\right)}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}+\frac{16\sqrt{\frac{1-ax}{ax+1}}+5}{\left(\frac{1-ax}{ax+1}+1\right)^4}-\frac{2\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \ 27$$

$$\frac{4\left(\frac{1}{5}\left(\frac{5}{3}\int\frac{3\left(\frac{1-ax}{ax+1}\right)^{3/2}-15\sqrt{\frac{1-ax}{ax+1}}-\frac{6(1-ax)}{ax+1}+2}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}+\frac{16\sqrt{\frac{1-ax}{ax+1}}+5}{\left(\frac{1-ax}{ax+1}+1\right)^4}-\frac{2\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \ 2345$$

$$\frac{4\left(\frac{1}{5}\left(\frac{5}{3}\left(\frac{4\sqrt{\frac{1-ax}{ax+1}}+9}{2\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{1}{4}\int-\frac{12\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}\right)+\frac{16\sqrt{\frac{1-ax}{ax+1}}+5}{\left(\frac{1-ax}{ax+1}+1\right)^4}-\frac{2\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \ 27$$

$$\frac{4\left(\frac{1}{5}\left(\frac{5}{3}\left(3\int\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}+\frac{4\sqrt{\frac{1-ax}{ax+1}}+9}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)+\frac{16\sqrt{\frac{1-ax}{ax+1}}+5}{\left(\frac{1-ax}{ax+1}+1\right)^4}-\frac{2\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

$$\Big\downarrow \ 241$$

$$\frac{4\left(\frac{1}{5}\left(\frac{16\sqrt{\frac{1-ax}{ax+1}}+5}{\left(\frac{1-ax}{ax+1}+1\right)^4}+\frac{5}{3}\left(\frac{4\sqrt{\frac{1-ax}{ax+1}}+9}{2\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{3}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)-\frac{2\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)-\frac{8\sqrt{\frac{1-ax}{ax+1}}}{5\left(\frac{1-ax}{ax+1}+1\right)^5}\right)}{a^5}$$

input
```
Int[x^4/E^ArcSech[a*x],x]
```

3.76. $\int e^{-\text{sech}^{-1}(ax)}x^4\,dx$

output
```
(4*((-8*Sqrt[(1 - a*x)/(1 + a*x)])/(5*(1 + (1 - a*x)/(1 + a*x))^5) + ((5 +
 16*Sqrt[(1 - a*x)/(1 + a*x)])/(1 + (1 - a*x)/(1 + a*x))^4 - (2*(15 + 17*S
qrt[(1 - a*x)/(1 + a*x)]))/(3*(1 + (1 - a*x)/(1 + a*x))^3) + (5*((9 + 4*Sq
rt[(1 - a*x)/(1 + a*x)])/(2*(1 + (1 - a*x)/(1 + a*x))^2) - 3/(2*(1 + (1 -
a*x)/(1 + a*x)))))/3)/5))/a^5
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.76.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.35 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.61

method	result
default	$\frac{(ax+1)\left(15\left(-\frac{ax-1}{ax}\right)^{\frac{7}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}x^{10}a^{10}+30\left(-\frac{ax-1}{ax}\right)^{\frac{7}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}x^{8}a^{8}-30\left(-\frac{ax-1}{ax}\right)^{\frac{7}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{3}{2}}a^{8}x^{8}+30\left(-\frac{ax-1}{ax}\right)^{\frac{7}{2}}\ln\left(a^{2}x^{2}\right)\left(\frac{ax+1}{ax}\right)\right.}{}$

input
```
int(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/60*(a*x+1)/x^7*(15*(-(a*x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(5/2)*x^10*a^10+30
*(-(a*x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(5/2)*x^8*a^8-30*(-(a*x-1)/a/x)^(7/2)*
((a*x+1)/a/x)^(3/2)*a^8*x^8+30*(-(a*x-1)/a/x)^(7/2)*ln(a^2*x^2)*((a*x+1)/a
/x)^(5/2)*x^6*a^6-30*a^7*x^7*((a*x+1)/a/x)^(3/2)*(-(a*x-1)/a/x)^(7/2)-60*(
-(a*x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(3/2)*ln(a^2*x^2)*a^6*x^6-12*a^11*x^11-6
0*(-(a*x-1)/a/x)^(7/2)*ln(a^2*x^2)*((a*x+1)/a/x)^(3/2)*x^5*a^5+30*(-(a*x-1
)/a/x)^(7/2)*((a*x+1)/a/x)^(1/2)*ln(a^2*x^2)*a^6*x^6+12*a^10*x^10+60*(-(a*
x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(1/2)*ln(a^2*x^2)*a^5*x^5+40*a^9*x^9+30*x^4*
ln(a^2*x^2)*((a*x+1)/a/x)^(1/2)*(-(a*x-1)/a/x)^(7/2)*a^4-40*a^8*x^8-40*a^7
*x^7+40*a^6*x^6+20*a^3*x^3-20*a^2*x^2-8*a*x+8)/a^12/((a*x+1)/a/x)^(7/2)/(-
(a*x-1)/a/x)^(7/2)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^4\,dx=\frac{15\,a^3x^4-4\left(3\,a^4x^5-a^2x^3-2\,x\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{60\,a^4}$$

input
```
integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas
")
```

output
```
1/60*(15*a^3*x^4 - 4*(3*a^4*x^5 - a^2*x^3 - 2*x)*sqrt((a*x + 1)/(a*x))*sqr
t(-(a*x - 1)/(a*x)))/a^4
```

3.76.6 Sympy [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^4\,dx = a\int \frac{x^5}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

input
```
integrate(x**4/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

output
```
a*Integral(x**5/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.76.7 Maxima [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima
")
```

output
```
integrate(x^4/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.76.8 Giac [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.50

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^4\,dx = \frac{x^4}{4\,a} + \frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,x}{15\,a^4}+\frac{2}{15\,a^5}-\frac{x^5}{5}-\frac{x^4}{5\,a}+\frac{x^3}{15\,a^2}+\frac{x^2}{15\,a^3}\right)}{\sqrt{\frac{1}{a\,x}+1}}$$

input
```
int(x^4/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
x^4/(4*a) + ((1/(a*x) - 1)^(1/2)*((2*x)/(15*a^4) + 2/(15*a^5) - x^5/5 - x^
4/(5*a) + x^3/(15*a^2) + x^2/(15*a^3)))/(1/(a*x) + 1)^(1/2)
```

3.77 $\int e^{-\text{sech}^{-1}(ax)}x^3\,dx$

3.77.1 Optimal result

Integrand size = 12, antiderivative size = 163

$$\int e^{-\text{sech}^{-1}(ax)}x^3\,dx = -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4}{4a^4}+\frac{(1+ax)\left(8+\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4} - \frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4} + \frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4}+\frac{\arctan\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{4a^4}$$

output
```
1/4*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^4-1/4*(a*x+1)^4*((-a*x+1)/(a*x+1))^
(1/2)/a^4+1/8*(a*x+1)*(8+((-a*x+1)/(a*x+1))^(1/2))/a^4-1/8*(a*x+1)^2*(8+5*
((-a*x+1)/(a*x+1))^(1/2))/a^4+1/12*(a*x+1)^3*(4+9*((-a*x+1)/(a*x+1))^(1/2)
)/a^4
```

3.77.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.60

$$\int e^{-\text{sech}^{-1}(ax)}x^3\,dx = \frac{8a^3x^3+3a\sqrt{\frac{1-ax}{1+ax}}(x+ax^2-2a^2x^3-2a^3x^4)-3i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{24a^4}$$

input
```
Integrate[x^3/E^ArcSech[a*x],x]
```

output
```
(8*a^3*x^3 + 3*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^2*x^3 - 2*a^3*
x^4) - (3*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(24*
a^4)
```

3.77.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6891, 7268, 2335, 27, 2345, 27, 2345, 454, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-\text{sech}^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{x^3}{\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}}dx$$

$$\Big\downarrow \text{7268}$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}{\left(\frac{1-ax}{ax+1}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}}{a^4}$$

$$\Big\downarrow \text{2335}$$

$$-\frac{4\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^4}-\frac{1}{8}\int \frac{8\left(-\left(\frac{1-ax}{ax+1}\right)^{5/2}+2\left(\frac{1-ax}{ax+1}\right)^{3/2}-\sqrt{\frac{1-ax}{ax+1}}-\frac{6(1-ax)}{ax+1}+\frac{2(1-ax)^2}{(ax+1)^2}+1\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}\right)}{a^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^4}-\int \frac{-\left(\frac{1-ax}{ax+1}\right)^{5/2}+2\left(\frac{1-ax}{ax+1}\right)^{3/2}-\sqrt{\frac{1-ax}{ax+1}}-\frac{6(1-ax)}{ax+1}+\frac{2(1-ax)^2}{(ax+1)^2}+1}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}\right)}{a^4}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{4\left(\frac{1}{6}\int \frac{3\left(2\left(\frac{1-ax}{ax+1}\right)^{3/2}-6\sqrt{\frac{1-ax}{ax+1}}-\frac{4(1-ax)}{ax+1}+1\right)}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}-\frac{9\sqrt{\frac{1-ax}{ax+1}}+4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^4}\right)}{a^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{4\left(\frac{1}{2}\int \frac{2\left(\frac{1-ax}{ax+1}\right)^{3/2}-6\sqrt{\frac{1-ax}{ax+1}}-\frac{4(1-ax)}{ax+1}+1}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}-\frac{9\sqrt{\frac{1-ax}{ax+1}}+4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^4}\right)}{a^4}$$

$$\Big\downarrow \; 2345$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{5\sqrt{\frac{1-ax}{ax+1}}+8}{4\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{1}{4}\int \frac{1-8\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{9\sqrt{\frac{1-ax}{ax+1}}+4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^4}\right)}{a^4}$$

$$\Big\downarrow \; 454$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{\frac{1-ax}{ax+1}+1}d\sqrt{\frac{1-ax}{ax+1}}-\frac{\sqrt{\frac{1-ax}{ax+1}}+8}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)+\frac{5\sqrt{\frac{1-ax}{ax+1}}+8}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)-\frac{9\sqrt{\frac{1-ax}{ax+1}}+4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^4}\right)}{a^4}$$

$$\Big\downarrow \; 216$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}\arctan\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{\sqrt{\frac{1-ax}{ax+1}}+8}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)+\frac{5\sqrt{\frac{1-ax}{ax+1}}+8}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)-\frac{9\sqrt{\frac{1-ax}{ax+1}}+4}{6\left(\frac{1-ax}{ax+1}+1\right)^3}+\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^4}\right)}{a^4}$$

input
```
Int[x^3/E^ArcSech[a*x],x]
```

output
```
(-4*(Sqrt[(1 - a*x)/(1 + a*x)]/(1 + (1 - a*x)/(1 + a*x))^4 - (4 + 9*Sqrt[(
1 - a*x)/(1 + a*x)])/(6*(1 + (1 - a*x)/(1 + a*x))^3) + ((8 + 5*Sqrt[(1 - a
*x)/(1 + a*x)])/(4*(1 + (1 - a*x)/(1 + a*x))^2) + (-1/2*(8 + Sqrt[(1 - a*x
)/(1 + a*x)])/(1 + (1 - a*x)/(1 + a*x)) - ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]
]/2)/4)/2))/a^4
```

3.77. $\int e^{-\text{sech}^{-1}(ax)}x^3\,dx$

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.77.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

method	result	size
default	$a\left(\frac{x^3}{3a^2}-\frac{\sqrt{-\frac{ax-1}{ax}}\, x\sqrt{\frac{ax+1}{ax}}\left(2\,\mathrm{csgn}(a)a^3x^3\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}\, x\,\mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\mathrm{csgn}(a)}{8a^4\sqrt{-a^2x^2+1}}\right)$	120

input
```
int(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^3/a^2-1/8/a^4*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(2*csgn(
a)*a^3*x^3*(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)*x*csgn(a)*a+arctan(csgn(a
)*a*x/(-a^2*x^2+1)^(1/2)))*csgn(a)/(-a^2*x^2+1)^(1/2))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^3\,dx = \frac{8\,a^3x^3-3\,(2\,a^4x^4-a^2x^2)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{24\,a^4}$$

input
```
integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas
")
```

output
```
1/24*(8*a^3*x^3 - 3*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
 - 1)/(a*x)) + 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^4
```

3.77.6 Sympy [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^3\,dx = a\int \frac{x^4}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

input
```
integrate(x**3/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

output
```
a*Integral(x**4/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.77.7 Maxima [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima
")
```

output
```
integrate(x^3/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.77.8 Giac [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 25.41 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.88

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^3\,dx = \frac{\ln\left(\frac{a\sqrt{\frac{1}{a\,x}+1}-\frac{1}{x}+a\sqrt{\frac{1}{a\,x}-1}\,1\mathrm{i}}{2\,a-2\,a\sqrt{\frac{1}{a\,x}+1}+\frac{1}{x}}\right)\,3\mathrm{i}}{8\,a^4}$$

$$+\frac{\frac{1\mathrm{i}}{1024\,a^4}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 3\mathrm{i}}{128\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 53\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6 87\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8 657\mathrm{i}}{1024\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10} 12\cdots}{256\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\cdots\right.}\cdots}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}}$$

$$+\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{8\,a^4}+\frac{\frac{1\mathrm{i}}{32\,a^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}$$

$$-\frac{\ln\left(\frac{2\,a\sqrt{\frac{a+\frac{1}{x}}{a}}-\frac{2}{x}+a\sqrt{-\frac{a-\frac{1}{x}}{a}}\,2\mathrm{i}}{2\,a+\frac{1}{x}-2\,a\sqrt{\frac{a+\frac{1}{x}}{a}}}\right)\,1\mathrm{i}}{2\,a^4}+\frac{x^3}{3\,a}$$

$$+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 1\mathrm{i}}{1024\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}$$

input
```
int(x^3/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
(log((a*(1/(a*x) - 1)^(1/2)*1i + a*(1/(a*x) + 1)^(1/2) - 1/x)/(2*a - 2*a*(
1/(a*x) + 1)^(1/2) + 1/x))*3i)/(8*a^4) + (1i/(1024*a^4) - (((1/(a*x) - 1)^
(1/2) - 1i)^2*3i)/(128*a^4*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^
(1/2) - 1i)^4*53i)/(512*a^4*((1/(a*x) + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)
^(1/2) - 1i)^6*87i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^6) + (((1/(a*x) - 1
)^(1/2) - 1i)^8*657i)/(1024*a^4*((1/(a*x) + 1)^(1/2) - 1)^8) + (((1/(a*x)
- 1)^(1/2) - 1i)^10*121i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^10))/(((1/(a*
x) - 1)^(1/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 + (4*((1/(a*x) - 1)^(1/2
) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (6*((1/(a*x) - 1)^(1/2) - 1i)^8)/
((1/(a*x) + 1)^(1/2) - 1)^8 + (4*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x)
+ 1)^(1/2) - 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) -
1)^12) + (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(8
*a^4) + (1i/(32*a^4) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^4*((1/(a*x)
 + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^4*((1/(a*x)
 + 1)^(1/2) - 1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) -
1)^2 + (2*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/
(a*x) - 1)^(1/2) - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - (log((a*(-(a - 1/x
)/a)^(1/2)*2i - 2/x + 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)
/a)^(1/2)))*1i)/(2*a^4) + x^3/(3*a) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(2
56*a^4*((1/(a*x) + 1)^(1/2) - 1)^2) + (((1/(a*x) - 1)^(1/2) - 1i)^4*1i)...
```

3.77. $\int e^{-\text{sech}^{-1}(ax)}x^3\,dx$

3.78 $\int e^{-\text{sech}^{-1}(ax)}x^2\,dx$

3.78.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int e^{-\text{sech}^{-1}(ax)}x^2\,dx=-\frac{x}{a^2}-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3}{3a^3}+\frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3}$$

output
```
-x/a^2-1/3*(a*x+1)^3*((-a*x+1)/(a*x+1))^(1/2)/a^3+1/6*(a*x+1)^2*(3+4*((-a*
x+1)/(a*x+1))^(1/2))/a^3
```

3.78.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int e^{-\text{sech}^{-1}(ax)}x^2\,dx=\frac{3a^2x^2-2(-1+ax)\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{6a^3}$$

input
```
Integrate[x^2/E^ArcSech[a*x],x]
```

output
```
(3*a^2*x^2 - 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(6*a^3)
```

3.78.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.56, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6891, 7268, 25, 2335, 27, 2345, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-\text{sech}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6891}$$

$$\int \frac{x^2}{\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}}dx$$

$$\Big\downarrow \text{ 7268}$$

$$\frac{4\int -\frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}}{a^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{\left(\frac{1-ax}{ax+1}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}}{a^3}$$

$$\Big\downarrow \text{ 2335}$$

$$\frac{4\left(\frac{1}{6}\int \frac{2\left(3\left(\frac{1-ax}{ax+1}\right)^{3/2}-3\sqrt{\frac{1-ax}{ax+1}}-\frac{6(1-ax)}{ax+1}+2\right)}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}-\frac{2\sqrt{\frac{1-ax}{ax+1}}}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\left(\frac{1}{3}\int \frac{3\left(\frac{1-ax}{ax+1}\right)^{3/2}-3\sqrt{\frac{1-ax}{ax+1}}-\frac{6(1-ax)}{ax+1}+2}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}-\frac{2\sqrt{\frac{1-ax}{ax+1}}}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{4\left(\frac{1}{3}\left(\frac{4\sqrt{\frac{1-ax}{ax+1}}+3}{2\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{1}{4}\int -\frac{12\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{2\sqrt{\frac{1-ax}{ax+1}}}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\left(\frac{1}{3}\left(3\int \frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}+\frac{4\sqrt{\frac{1-ax}{ax+1}}+3}{2\left(\frac{1-ax}{ax+1}+1\right)^2}\right)-\frac{2\sqrt{\frac{1-ax}{ax+1}}}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

$$\Big\downarrow 241$$

$$\frac{4\left(\frac{1}{3}\left(\frac{4\sqrt{\frac{1-ax}{ax+1}}+3}{2\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{3}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)-\frac{2\sqrt{\frac{1-ax}{ax+1}}}{3\left(\frac{1-ax}{ax+1}+1\right)^3}\right)}{a^3}$$

input
```
Int[x^2/E^ArcSech[a*x],x]
```

output
```
(4*((-2*Sqrt[(1 - a*x)/(1 + a*x)])/(3*(1 + (1 - a*x)/(1 + a*x))^3) + ((3 +
 4*Sqrt[(1 - a*x)/(1 + a*x)])/(2*(1 + (1 - a*x)/(1 + a*x))^2) - 3/(2*(1 +
(1 - a*x)/(1 + a*x))))/3))/a^3
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.78.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.59

method	result
default	$\frac{(ax+1)\left(3a^6x^6\left(\frac{ax+1}{ax}\right)^{\frac{3}{2}}\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}+3\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}\ln\left(a^2x^2\right)\left(\frac{ax+1}{ax}\right)^{\frac{3}{2}}x^4a^4-3\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{ax}}\,\ln\left(a^2x^2\right)a^4x^4+2a^7x^7-3x^3\ln\left(a^2x\right.\right.}{6x^5a^8\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}}$

input
```
int(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)/x^5*(3*a^6*x^6*((a*x+1)/a/x)^(3/2)*(-(a*x-1)/a/x)^(5/2)+3*(-(a
*x-1)/a/x)^(5/2)*ln(a^2*x^2)*((a*x+1)/a/x)^(3/2)*x^4*a^4-3*(-(a*x-1)/a/x)^
(5/2)*((a*x+1)/a/x)^(1/2)*ln(a^2*x^2)*a^4*x^4+2*a^7*x^7-3*x^3*ln(a^2*x^2)*
((a*x+1)/a/x)^(1/2)*(-(a*x-1)/a/x)^(5/2)*a^3-2*a^6*x^6-6*a^5*x^5+6*a^4*x^4
+6*a^3*x^3-6*a^2*x^2-2*a*x+2)/a^8/((a*x+1)/a/x)^(5/2)/(-(a*x-1)/a/x)^(5/2)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^2\,dx = \frac{3\,ax^2 - 2\left(a^2x^3 - x\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{6\,a^2}$$

input
```
integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas
")
```

output
```
1/6*(3*a*x^2 - 2*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)
))/a^2
```

3.78.6 Sympy [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^2\,dx = a\int \frac{x^3}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

input
```
integrate(x**2/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

output
```
a*Integral(x**3/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.78.7 Maxima [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima
")
```

output
```
integrate(x^2/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.78.8 Giac [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^2\,dx=\int\frac{x^2}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 4.89 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^2\,dx=\frac{x^2}{2\,a}+\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x}{3\,a^2}+\frac{1}{3\,a^3}-\frac{x^3}{3}-\frac{x^2}{3\,a}\right)}{\sqrt{\frac{1}{a\,x}+1}}$$

input
```
int(x^2/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
x^2/(2*a) + ((1/(a*x) - 1)^(1/2)*(x/(3*a^2) + 1/(3*a^3) - x^3/3 - x^2/(3*a
)))/(1/(a*x) + 1)^(1/2)
```

3.79 $\int e^{-\text{sech}^{-1}(ax)}x\,dx$

3.79.1 Optimal result

Integrand size = 10, antiderivative size = 94

$$\int e^{-\text{sech}^{-1}(ax)}x\,dx=\frac{(1+ax)^2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2}+\frac{(1+ax)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}+\frac{\arctan\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

output
```
arctan(((-a*x+1)/(a*x+1))^(1/2))/a^2+1/4*(a*x+1)^2*(1-((-a*x+1)/(a*x+1))^(
1/2))^2/a^2+1/2*(a*x+1)*(1+((-a*x+1)/(a*x+1))^(1/2))/a^2
```

3.79.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int e^{-\text{sech}^{-1}(ax)}x\,dx=-\frac{-2ax+ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)+i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{2a^2}$$

input
```
Integrate[x/E^ArcSech[a*x],x]
```

output
```
-1/2*(-2*a*x + a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + I*Log[(-2*I)*a*x
+ 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/a^2
```

3.79.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6891, 7268, 531, 27, 454, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\text{sech}^{-1}(ax)}dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{x}{\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}}dx$$

$$\Big\downarrow \text{7268}$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}{\left(\frac{1-ax}{ax+1}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}}{a^2}$$

$$\Big\downarrow \text{531}$$

$$-\frac{4\left(-\frac{1}{4}\int \frac{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}-\frac{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\left(-\frac{1}{2}\int \frac{1-\sqrt{\frac{1-ax}{ax+1}}}{\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}-\frac{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{454}$$

$$-\frac{4\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\frac{1-ax}{ax+1}+1}d\sqrt{\frac{1-ax}{ax+1}}-\frac{\sqrt{\frac{1-ax}{ax+1}}+1}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)-\frac{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{4\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{\sqrt{\frac{1-ax}{ax+1}}+1}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)-\frac{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}{4\left(\frac{1-ax}{ax+1}+1\right)^2}\right)}{a^2}$$

input
```
Int[x/E^ArcSech[a*x],x]
```

output
```
(-4*(-1/4*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2/(1 + (1 - a*x)/(1 + a*x))^2 +
(-1/2*(1 + Sqrt[(1 - a*x)/(1 + a*x)])/(1 + (1 - a*x)/(1 + a*x)) - ArcTan[S
qrt[(1 - a*x)/(1 + a*x)]]/2)/2))/a^2
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.79.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result	size
default	$a\left(\frac{x}{a^{2}}-\frac{\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\left(\sqrt{-a^{2}x^{2}+1}\,x\,\mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^{2}x^{2}+1}}\right)\right)\mathrm{csgn}(a)}{2a^{2}\sqrt{-a^{2}x^{2}+1}}\right)$	94

input
```
int(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
a*(x/a^2-1/2/a^2*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*((-a^2*x^2+1)^
(1/2)*x*csgn(a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/
2)*csgn(a))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int e^{-\operatorname{sech}^{-1}(ax)}x\,dx=-\frac{a^{2}x^{2}\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-2\,ax-\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{2\,a^{2}}$$

input
```
integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2*a*x - arcta
n(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^2
```

3.79.6 Sympy [F]

$$\int e^{-\text{sech}^{-1}(ax)}x\,dx=a\int\frac{x^2}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

input
```
integrate(x/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

output
```
a*Integral(x**2/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.79.7 Maxima [F]

$$\int e^{-\text{sech}^{-1}(ax)}x\,dx=\int\frac{x}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.79.8 Giac [F]

$$\int e^{-\text{sech}^{-1}(ax)}x\,dx=\int\frac{x}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.33

$$\int e^{-\mathrm{sech}^{-1}(ax)}x\,dx=\frac{x}{a}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{2\,a^2}-\frac{\frac{1\mathrm{i}}{32\,a^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}-\frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)\,1\mathrm{i}}{a^2}+\frac{\ln\left(\frac{2\,a\,\sqrt{\frac{a+\frac{1}{x}}{a}}-\frac{2}{x}+a\,\sqrt{-\frac{a-\frac{1}{x}}{a}}\,2\mathrm{i}}{2\,a+\frac{1}{x}-2\,a\,\sqrt{\frac{a+\frac{1}{x}}{a}}}\right)\,1\mathrm{i}}{2\,a^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}$$

input
```
int(x/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
x/a - (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(2*a^
2) - (1i/(32*a^2) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^2*((1/(a*x) +
1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^2*((1/(a*x) +
1)^(1/2) - 1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^
2 + (2*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*
x) - 1)^(1/2) - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - ((log(((1/(a*x) - 1)^
(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log(((1/(a*x) - 1)^(1/2)
- 1i)/((1/(a*x) + 1)^(1/2) - 1)))*1i)/a^2 + (log((a*(-(a - 1/x)/a)^(1/2)*2
i - 2/x + 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)/a)^(1/2)))*
1i)/(2*a^2) - (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(32*a^2*((1/(a*x) + 1)^(1/
2) - 1)^2)
```

3.80 $\int e^{-\text{sech}^{-1}(ax)}\,dx$

3.80.1 Optimal result

Integrand size = 8, antiderivative size = 65

$$\int e^{-\text{sech}^{-1}(ax)}\,dx=-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a}+\frac{\log(1+ax)}{a}+\frac{2\log\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

output
```
ln(a*x+1)/a+2*ln(1+((-a*x+1)/(a*x+1))^(1/2))/a-(a*x+1)*((-a*x+1)/(a*x+1))^
(1/2)/a
```

3.80.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int e^{-\text{sech}^{-1}(ax)}\,dx=\frac{-\sqrt{\frac{1-ax}{1+ax}}(1+ax)+\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

input
```
Integrate[E^(-ArcSech[a*x]),x]
```

output
```
(-(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)) + Log[1 + Sqrt[(1 - a*x)/(1 + a*x)
] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/a
```

3.80.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6886, 7268, 25, 2178, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\text{sech}^{-1}(ax)}dx$$

$$\Big\downarrow \; 6886$$

$$\int \frac{1}{\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}}dx$$

$$\Big\downarrow \; 7268$$

$$\frac{4\int -\frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)\left(\frac{1-ax}{ax+1}+1\right)^2}d\sqrt{\frac{1-ax}{ax+1}}}{a}$$

$$\Big\downarrow \; 2178$$

$$\frac{4\left(\frac{1}{2}\int \frac{1-\sqrt{\frac{1-ax}{ax+1}}}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)\left(\frac{1-ax}{ax+1}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}-\frac{\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)}{a}$$

$$\Big\downarrow \; 657$$

$$\frac{4\left(\frac{1}{2}\int \left(\frac{1}{\sqrt{\frac{1-ax}{ax+1}}+1}-\frac{\sqrt{\frac{1-ax}{ax+1}}}{\frac{1-ax}{ax+1}+1}\right)d\sqrt{\frac{1-ax}{ax+1}}-\frac{\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{4\left(\frac{1}{2}\left(\log\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)-\frac{1}{2}\log\left(\frac{1-ax}{ax+1}+1\right)\right)-\frac{\sqrt{\frac{1-ax}{ax+1}}}{2\left(\frac{1-ax}{ax+1}+1\right)}\right)}{a}$$

input
```
Int[E^(-ArcSech[a*x]),x]
```

output
```
(4*(-1/2*Sqrt[(1 - a*x)/(1 + a*x)]/(1 + (1 - a*x)/(1 + a*x)) + (Log[1 + Sq
rt[(1 - a*x)/(1 + a*x)]] - Log[1 + (1 - a*x)/(1 + a*x)]/2)/2))/a
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6886
```
Int[E^(ArcSech[u_]*(n_.)), x_Symbol] :> Int[(1/u + Sqrt[(1 - u)/(1 + u)] +
(1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; IntegerQ[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(61) = 122$.

Time = 0.28 (sec) , antiderivative size = 2612, normalized size of antiderivative = 40.18

method	result	size
default	Expression too large to display	2612

input
```
int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+1)/x^3*(x^2*ln(a^2*x^2)*(-(a*x-1)/a/x)^(3/2)*((a*x+1)/a/x)^(1/2)*
(-a^2*x^2+1)^(1/2)*a^2+ln(-2*(-((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)
^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*(
(a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))*a^2/(a*x-1)/(a*x+
1)/x^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(-a^2*x+((-a*x^2*(-(a*x-1)/a/x)^
(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))*(a*x^2*(-(
a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2)
)*a^2/(a*x-1)/(a*x+1)/x^2)^(1/2)))*a^3*x^3+ln(2*(((-a*x^2*(-(a*x-1)/a/x)^(
1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))*(a*x^2*(-(a
*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))
*a^2/(a*x-1)/(a*x+1)/x^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+((-a*x^
2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^
(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*
((a*x+1)*x)^(1/2))*a^2/(a*x-1)/(a*x+1)/x^2)^(1/2)))*a^3*x^3-2*(-a^2*x^2+1)
^(1/2)*a^3*x^3-x^2*ln(-2*(-((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/
2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x
+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))*a^2/(a*x-1)/(a*x+1)/x
^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(-a^2*x+((-a*x^2*(-(a*x-1)/a/x)^(1/2
)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))*(a*x^2*(-(a*x-
1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*((a*x+1)*x)^(1/2))...
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int e^{-\operatorname{sech}^{-1}(ax)}\,dx = -\frac{2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right) + \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right) - 2\,\log\left(x\right)}{2\,a}$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*(2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - log(a*x*sqrt((a
*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) + log(a*x*sqrt((a*x + 1)/(a*x))
*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*log(x))/a
```

3.80.6 Sympy [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}\,dx = a\int \frac{x}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

output
```
a*Integral(x/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.80.7 Maxima [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}\,dx = \int \frac{1}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.80.8 Giac [F]

$$\int e^{-\operatorname{sech}^{-1}(ax)}\,dx = \int \frac{1}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 7.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int e^{-\operatorname{sech}^{-1}(ax)}\,dx = \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)}{a} - \frac{\ln\left(\frac{1}{x}\right)}{a} - x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}$$

input
```
int(1/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
acosh(1/(a*x))/a - log(1/x)/a - x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2)
```

3.81 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x}\,dx$

3.81.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x}\,dx=-\frac{2}{1+\sqrt{\frac{1-ax}{1+ax}}}-2\arctan\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

output
```
-2*arctan(((-a*x+1)/(a*x+1))^(1/2))-2/(1+((-a*x+1)/(a*x+1))^(1/2))
```

3.81.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x}\,dx=-\frac{1}{ax}+\left(1+\frac{1}{ax}\right)\sqrt{\frac{1-ax}{1+ax}}+i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)$$

input
```
Integrate[1/(E^ArcSech[a*x]*x),x]
```

output
```
-(1/(a*x)) + (1 + 1/(a*x))*Sqrt[(1 - a*x)/(1 + a*x)] + I*Log[(-2*I)*a*x +
2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)]
```

3.81.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6891, 7268, 594, 25, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{1}{x\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)}dx$$

$$\Big\downarrow \text{7268}$$

$$-4\int \frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2\left(\frac{1-ax}{ax+1}+1\right)}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{594}$$

$$-4\left(\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{1}{2}\int -\frac{1}{\frac{1-ax}{ax+1}+1}d\sqrt{\frac{1-ax}{ax+1}}\right)$$

$$\Big\downarrow \text{25}$$

$$-4\left(\frac{1}{2}\int \frac{1}{\frac{1-ax}{ax+1}+1}d\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$-4\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-ax}{ax+1}}\right)+\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}\right)$$

input
```
Int[1/(E^ArcSech[a*x]*x),x]
```

output
```
-4*(1/(2*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) + ArcTan[Sqrt[(1 - a*x)/(1 + a*x
)]]/2)
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.81.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

method	result	size
default	$a\left(-\frac{1}{x\,a^{2}}+\frac{\sqrt{-\frac{a x -1}{a x}}\,\sqrt{\frac{a x +1}{a x}}\,\left(\arctan\left(\frac{\operatorname{csgn}\left(a \right) a x}{\sqrt{-a^{2} x^{2}+1}}\right) a x +\operatorname{csgn}\left(a \right) \sqrt{-a^{2} x^{2}+1}\right) \operatorname{csgn}\left(a \right)}{a \sqrt{-a^{2} x^{2}+1}}\right)$	96

input
```
int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/x/a^2+1/a*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*(arctan(csgn(a)*a
*x/(-a^2*x^2+1)^(1/2))*a*x+csgn(a)*(-a^2*x^2+1)^(1/2))*csgn(a)/(-a^2*x^2+1
)^(1/2))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x}\,dx = \frac{ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right) - 1}{ax}$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="fricas
")
```

output
```
(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - a*x*arctan(sqrt((a*x +
 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) - 1)/(a*x)
```

3.81.6 Sympy [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x}\,dx = a\int \frac{1}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x,x)
```

output
```
a*Integral(1/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.81.7 Maxima [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="maxima
")
```

output
```
integrate(1/(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.81.8 Giac [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(1/(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 7.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.00

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x}\,dx = \ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)\,1\mathrm{i}-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i} - \frac{1}{a\,x} - \frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 8\mathrm{i}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2\left(1+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}\right)}$$

input
```
int(1/(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

output
```
log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i - log
(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i - 1/(a*x) - (((1
/(a*x) - 1)^(1/2) - 1i)^2*8i)/(((1/(a*x) + 1)^(1/2) - 1)^2*(((1/(a*x) - 1)
^(1/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 - (2*((1/(a*x) - 1)^(1/2) - 1i)
^2)/((1/(a*x) + 1)^(1/2) - 1)^2 + 1))
```

3.82 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^2}\,dx$

3.82.1 Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^2}\,dx = -\frac{a}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a}{1+\sqrt{\frac{1-ax}{1+ax}}}-a\,\text{arctanh}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

output
```
-a*arctanh(((-a*x+1)/(a*x+1))^(1/2))-a/(1+((-a*x+1)/(a*x+1))^(1/2))^2+a/(1
+((-a*x+1)/(a*x+1))^(1/2))
```

3.82.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^2}\,dx = \frac{1}{2}\left(-\frac{1}{ax^2}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax^2}+a\log(x) - a\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)\right)$$

input
```
Integrate[1/(E^ArcSech[a*x]*x^2),x]
```

output
```
(-(1/(a*x^2)) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x^2) + a*Log[x] -
 a*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/2
```

3.82.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6891, 7268, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{1}{x^2\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)}dx$$

$$\Big\downarrow \text{7268}$$

$$4a\int -\frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{25}$$

$$-4a\int \frac{\sqrt{\frac{1-ax}{ax+1}}}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{86}$$

$$-4a\int \left(\frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{1}{4\left(\frac{1-ax}{ax+1}-1\right)}\right)d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{2009}$$

$$4a\left(-\frac{1}{4}\text{arctanh}\left(\sqrt{\frac{1-ax}{ax+1}}\right)+\frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}\right)$$

input
```
Int[1/(E^ArcSech[a*x]*x^2),x]
```

output
```
4*a*(-1/4*1/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2 + 1/(4*(1 + Sqrt[(1 - a*x)/(
1 + a*x)])) - ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]]/4)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.82.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

method	result	size
default	$a\left(-\frac{1}{2a^{2}x^{2}}-\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\left(a^{2}x^{2}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)-\sqrt{-a^{2}x^{2}+1}\right)}{2ax\sqrt{-a^{2}x^{2}+1}}\right)$	96

input
```
int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/a^2/x^2-1/2/a*(-(a*x-1)/a/x)^(1/2)/x*((a*x+1)/a/x)^(1/2)*(a^2*x^2*
arctanh(1/(-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.78

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = -\frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+2}{4\,ax^2}$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="fric
as")
```

output
```
-1/4*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a*x*
sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2)/(a*x^2)
```

3.82.6 Sympy [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = a\int \frac{1}{ax^2\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**2,x)
```

output
```
a*Integral(1/(a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x), x)
```

3.82.7 Maxima [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.82.8 Giac [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2} dx = \int \frac{1}{x^2\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)} dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac
")
```

output
```
integrate(1/(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.49

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2} dx = 2\,a\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right) - a\,\mathrm{acosh}\left(\frac{1}{a\,x}\right) - \frac{1}{2\,a\,x^2} - \frac{a\left(\frac{14\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{14\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{1+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}}$$

input
```
int(1/(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

output
```
2*a*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) - a*acosh(
1/(a*x)) - 1/(2*a*x^2) - (a*((14*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) +
 1)^(1/2) - 1)^3 + (14*((1/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2)
- 1)^5 + (2*((1/(a*x) - 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (2
*((1/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1)))/((6*((1/(a*x) - 1
)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1
i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(
a*x) + 1)^(1/2) - 1)^6 + ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2)
 - 1)^8 + 1)
```

3.83 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^3} \, dx$

3.83.1 Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^3} \, dx = -\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{2a^2}{3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{a^2}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2} - \frac{a^2}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}$$

output
```
-1/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))-2/3*a^2/(1+((-a*x+1)/(a*x+1))^(1/2))
^3+a^2/(1+((-a*x+1)/(a*x+1))^(1/2))^2-1/2*a^2/(1+((-a*x+1)/(a*x+1))^(1/2))
```

3.83.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.37

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^3} \, dx = -\frac{1+(-1+ax)\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{3ax^3}$$

input
```
Integrate[1/(E^ArcSech[a*x]*x^3),x]
```

output
```
-1/3*(1 + (-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(a*x^3)
```

3.83.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6891, 7268, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{1}{x^3\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)}dx$$

$$\Big\downarrow \text{7268}$$

$$-4a^2\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{2115}$$

$$-4a^2\int \left(-\frac{1}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}+\frac{1}{8\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^2}\right)d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{2009}$$

$$-4a^2\left(\frac{1}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{1}{6\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{1}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}\right)$$

input
```
Int[1/(E^ArcSech[a*x]*x^3),x]
```

output
```
-4*a^2*(1/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) + 1/(6*(1 + Sqrt[(1 - a*x)/(
1 + a*x)])^3) - 1/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2) + 1/(8*(1 + Sqrt[(
1 - a*x)/(1 + a*x)])))
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.83.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

method	result	size
default	$a\left(-\frac{1}{3a^2x^3}-\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)}{3a\,x^2}\right)$	58

input
```
int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/a^2/x^3-1/3/a*((a*x+1)/a/x)^(1/2)/x^2*(-(a*x-1)/a/x)^(1/2)*(a^2*x^
2-1))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = -\frac{(a^3x^3 - ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 1}{3\,ax^3}$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="fric
as")
```

output
```
-1/3*((a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)/(a
*x^3)
```

3.83.6 Sympy [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = a\int \frac{1}{ax^3\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}} + x^2}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**3,x)
```

output
```
a*Integral(1/(a*x**3*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**2), x)
```

3.83.7 Maxima [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.83.8 Giac [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="giac
")
```

output
```
integrate(1/(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x}{3}-\frac{a\,x^2}{3}+\frac{1}{3\,a}-\frac{a^2\,x^3}{3}\right)}{x^3\,\sqrt{\frac{1}{a\,x}+1}}-\frac{1}{3\,a\,x^3}$$

input
```
int(1/(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

output
```
((1/(a*x) - 1)^(1/2)*(x/3 - (a*x^2)/3 + 1/(3*a) - (a^2*x^3)/3))/(x^3*(1/(a
*x) + 1)^(1/2)) - 1/(3*a*x^3)
```

3.84 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^4}\,dx$

3.84.1 Optimal result

Integrand size = 12, antiderivative size = 200

$$\begin{aligned}\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^4}\,dx = & -\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^3}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}\\ & +\frac{a^3}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{a^3}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^3}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}\\ & -\frac{1}{4}a^3\text{arctanh}\left(\sqrt{\frac{1-ax}{1+ax}}\right)\end{aligned}$$

output
```
-1/4*a^3*arctanh(((-a*x+1)/(a*x+1))^(1/2))-1/4*a^3/(1-((-a*x+1)/(a*x+1))^(
1/2))^2+1/4*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))-1/2*a^3/(1+((-a*x+1)/(a*x+1))
^(1/2))^4+a^3/(1+((-a*x+1)/(a*x+1))^(1/2))^3-a^3/(1+((-a*x+1)/(a*x+1))^(1/
2))^2+1/2*a^3/(1+((-a*x+1)/(a*x+1))^(1/2))
```

3.84.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^4} dx = -\frac{2+\sqrt{\frac{1-ax}{1+ax}}(-2-2ax+a^2x^2+a^3x^3)-a^4x^4\log(x)+a^4x^4\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{8ax^4}$$

input
```
Integrate[1/(E^ArcSech[a*x]*x^4),x]
```

output
```
-1/8*(2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-2 - 2*a*x + a^2*x^2 + a^3*x^3) - a^4
*x^4*Log[x] + a^4*x^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*
x)/(1 + a*x)]])/(a*x^4)
```

3.84.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6891, 7268, 25, 27, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^4} dx$$

$$\Big\downarrow \text{ 6891}$$

$$\int \frac{1}{x^4\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)}dx$$

$$\Big\downarrow \text{ 7268}$$

$$4a\int -\frac{a^2\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{ 25}$$

$$-4a\int \frac{a^2\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{27}$$

$$-4a^3\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{2115}$$

$$-4a^3\int \left(-\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^2}+\frac{1}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{1}{8\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^3}-\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{3}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-4a^3\left(\frac{1}{16}\text{arctanh}\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{1}{16\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{1}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{1}{16\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\right.$$

input
```
Int[1/(E^ArcSech[a*x]*x^4),x]
```

output
```
-4*a^3*(1/(16*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - 1/(16*(1 - Sqrt[(1 - a*
x)/(1 + a*x)])) + 1/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4) - 1/(4*(1 + Sqrt
[(1 - a*x)/(1 + a*x)])^3) + 1/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2) - 1/(8
*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) + ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]]/16)
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84. $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^4}\,dx$

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.84.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

method	result	size
default	$a\left(-\frac{1}{4a^{2}x^{4}}-\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)a^{4}x^{4}+a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-2\sqrt{-a^{2}x^{2}+1}\right)}{8a\,x^{3}\sqrt{-a^{2}x^{2}+1}}\right)$	115

input
```
int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/4/a^2/x^4-1/8/a*(-(a*x-1)/a/x)^(1/2)/x^3*((a*x+1)/a/x)^(1/2)*(arctan
h(1/(-a^2*x^2+1)^(1/2))*a^4*x^4+a^2*x^2*(-a^2*x^2+1)^(1/2)-2*(-a^2*x^2+1)^
(1/2))/(-a^2*x^2+1)^(1/2))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^4}\,dx =$$
$$-\frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(a^3x^3-2\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-}{ax}}}{16\,ax^4}$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="fric
as")
```

output
```
-1/16*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
 a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(a^
3*x^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 4)/(a*x^4)
```

3.84.6 Sympy [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = a\int \frac{1}{ax^4\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^3}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**4,x)
```

output
```
a*Integral(1/(a*x**4*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**3), x)
```

3.84.7 Maxima [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="maxi
ma")
```

output
```
integrate(1/(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.84.8 Giac [F]

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="giac
")
```

output
```
integrate(1/(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 50.94 (sec) , antiderivative size = 1511, normalized size of antiderivative = 7.56

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^4}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

output
```
((a^3*((1/(a*x) - 1)^(1/2) - 1i)^4*192i)/((1/(a*x) + 1)^(1/2) - 1)^4 + (a^
3*((1/(a*x) - 1)^(1/2) - 1i)^6*128i)/((1/(a*x) + 1)^(1/2) - 1)^6 + (a^3*((
1/(a*x) - 1)^(1/2) - 1i)^8*192i)/((1/(a*x) + 1)^(1/2) - 1)^8)/(3*((15*((1/
(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (6*((1/(a*x) - 1)^
(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (20*((1/(a*x) - 1)^(1/2) - 1i
)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (15*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(
a*x) + 1)^(1/2) - 1)^8 - (6*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^
(1/2) - 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12
 + 1)) - ((a^3*((1/(a*x) - 1)^(1/2) - 1i)^4*64i)/((1/(a*x) + 1)^(1/2) - 1)
^4 + (a^3*((1/(a*x) - 1)^(1/2) - 1i)^6*128i)/(3*((1/(a*x) + 1)^(1/2) - 1)^
6) + (a^3*((1/(a*x) - 1)^(1/2) - 1i)^8*64i)/((1/(a*x) + 1)^(1/2) - 1)^8)/(
(15*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (6*((1/(a*
x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (20*((1/(a*x) - 1)^(1
/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (15*((1/(a*x) - 1)^(1/2) - 1i)^
8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (6*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*
x) + 1)^(1/2) - 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2)
 - 1)^12 + 1) - (a^3*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2)
 - 1)))/2 + ((14*a^3*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) -
1)^3 + (14*a^3*((1/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 +
 (2*a^3*((1/(a*x) - 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (2*...
```

3.85 $\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5} dx$

3.85.1 Optimal result

Integrand size = 12, antiderivative size = 233

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5} dx = -\frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} - \frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{2a^4}{5\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^5} + \frac{a^4}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4} - \frac{4a^4}{3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{a^4}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2} - \frac{3a^4}{8\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}$$

output
```
-1/6*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^3+1/4*a^4/(1-((-a*x+1)/(a*x+1))^(1/2
))^2-3/8*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))-2/5*a^4/(1+((-a*x+1)/(a*x+1))^(1
/2))^5+a^4/(1+((-a*x+1)/(a*x+1))^(1/2))^4-4/3*a^4/(1+((-a*x+1)/(a*x+1))^(1
/2))^3+a^4/(1+((-a*x+1)/(a*x+1))^(1/2))^2-3/8*a^4/(1+((-a*x+1)/(a*x+1))^(1
/2))
```

3.85.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.26

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = -\frac{3+\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(-3+3ax-2a^2x^2+2a^3x^3\right)}{15ax^5}$$

input
```
Integrate[1/(E^ArcSech[a*x]*x^5),x]
```

output
```
-1/15*(3 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 +
 2*a^3*x^3))/(a*x^5)
```

3.85.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6891, 7268, 27, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{1}{x^5\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)}dx$$

$$\Big\downarrow \text{7268}$$

$$-4a\int \frac{a^3\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^6}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{27}$$

$$-4a^4\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^6}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{2115}$$

$$-4a^4 \int \left(-\frac{3}{32\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{1}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}+\frac{1}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}-\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^6}+\frac{}{32\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-4a^4\left(\frac{3}{32\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{1}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}+\frac{1}{10\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}+\frac{}{32\left(\ldots\right.}\right.$$

input
```
Int[1/(E^ArcSech[a*x]*x^5),x]
```

output
```
-4*a^4*(1/(24*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) - 1/(16*(1 - Sqrt[(1 - a*
x)/(1 + a*x)])^2) + 3/(32*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) + 1/(10*(1 + Sq
rt[(1 - a*x)/(1 + a*x)])^5) - 1/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4) + 1/
(3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) - 1/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)
])^2) + 3/(32*(1 + Sqrt[(1 - a*x)/(1 + a*x)])))
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

3.85. $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^5}\,dx$

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.85.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.29

method	result	size
default	$a\left(-\frac{1}{5a^{2}x^{5}}-\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^{2}x^{2}-1)(2a^{2}x^{2}+3)}{15a\,x^{4}}\right)$	68

input
```
int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/5/a^2/x^5-1/15/a*((a*x+1)/a/x)^(1/2)/x^4*(-(a*x-1)/a/x)^(1/2)*(a^2*x
^2-1)*(2*a^2*x^2+3))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.26

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = -\frac{(2\,a^5x^5+a^3x^3-3\,ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+3}{15\,ax^5}$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="fric
as")
```

output
```
-1/15*((2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)
/(a*x)) + 3)/(a*x^5)
```

3.85.6 Sympy [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = a\int \frac{1}{ax^5\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^4}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**5,x)
```

output
```
a*Integral(1/(a*x**5*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**4), x)
```

3.85.7 Maxima [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="maxi
ma")
```

output
```
integrate(1/(x^5*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.85.8 Giac [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="giac
")
```

output
```
integrate(1/(x^5*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.32

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = -\frac{1}{5\,a\,x^5} - \frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a\,x^2}{15}-\frac{x}{5}-\frac{1}{5\,a}+\frac{a^2\,x^3}{15}+\frac{2\,a^3\,x^4}{15}+\frac{2\,a^4\,x^5}{15}\right)}{x^5\,\sqrt{\frac{1}{a\,x}+1}}$$

input
```
int(1/(x^5*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

output
```
- 1/(5*a*x^5) - ((1/(a*x) - 1)^(1/2)*((a*x^2)/15 - x/5 - 1/(5*a) + (a^2*x^
3)/15 + (2*a^3*x^4)/15 + (2*a^4*x^5)/15))/(x^5*(1/(a*x) + 1)^(1/2))
```

3.86 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6} dx$

3.86.1 Optimal result

Integrand size = 12, antiderivative size = 320

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6} dx = -\frac{a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{a^5}{3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^6} + \frac{a^5}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^5} - \frac{13a^5}{8\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{19a^5}{12\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{a^5}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{3a^5}{8\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{1}{8}a^5\text{arctanh}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

output
```
-1/8*a^5*arctanh(((-a*x+1)/(a*x+1))^(1/2))-1/8*a^5/(1-((-a*x+1)/(a*x+1))^(
1/2))^4+1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^3-3/8*a^5/(1-((-a*x+1)/(a*x+1
))^(1/2))^2+1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))-1/3*a^5/(1+((-a*x+1)/(a*x
+1))^(1/2))^6+a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^5-13/8*a^5/(1+((-a*x+1)/(a*
x+1))^(1/2))^4+19/12*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^3-a^5/(1+((-a*x+1)/(
a*x+1))^(1/2))^2+3/8*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))
```

3.86.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.40

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6}\,dx = -\frac{8+\sqrt{\frac{1-ax}{1+ax}}\left(-8-8ax+2a^2x^2+2a^3x^3+3a^4x^4+3a^5x^5\right)-3a^6x^6\log(x)+3a^6x^6\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\right.}{48ax^6}$$

input
```
Integrate[1/(E^ArcSech[a*x]*x^6),x]
```

output
```
-1/48*(8 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3*x^3 +
 3*a^4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(1 - a
*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(a*x^6)
```

3.86.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6891, 7268, 25, 27, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6}dx$$

$$\Big\downarrow \text{6891}$$

$$\int \frac{1}{x^6\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)}dx$$

$$\Big\downarrow \text{7268}$$

$$4a\int -\frac{a^4\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^7}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \text{25}$$

$$-4a\int \frac{a^4\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^7}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \ 27$$

$$-4a^5\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^7}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \ 2115$$

$$-4a^5\int \left(-\frac{1}{16\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^2}+\frac{3}{32\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{3}{16\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^3}-\frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{3}{16\left(\sqrt{\frac{1-ax}{ax+1}}-1\right)^4}\right.$$

$$\Big\downarrow \ 2009$$

$$-4a^5\left(\frac{1}{32}\text{arctanh}\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{1}{16\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3}{32\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{3}{32\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}\right.$$

input
```
Int[1/(E^ArcSech[a*x]*x^6),x]
```

output
```
-4*a^5*(1/(32*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4) - 1/(16*(1 - Sqrt[(1 - a*
x)/(1 + a*x)])^3) + 3/(32*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - 1/(16*(1 -
Sqrt[(1 - a*x)/(1 + a*x)])) + 1/(12*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^6) - 1
/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^5) + 13/(32*(1 + Sqrt[(1 - a*x)/(1 + a
*x)])^4) - 19/(48*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) + 1/(4*(1 + Sqrt[(1 -
 a*x)/(1 + a*x)])^2) - 3/(32*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) + ArcTanh[Sq
rt[(1 - a*x)/(1 + a*x)]]/32)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.86.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.43

method	result	si
default	$a\left(-\frac{1}{6a^{2}x^{6}}-\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)a^{6}x^{6}+3\sqrt{-a^{2}x^{2}+1}\,a^{4}x^{4}+2a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}-8\sqrt{-a^{2}x^{2}+1}\right)}{48a\,x^{5}\sqrt{-a^{2}x^{2}+1}}\right)$	13

input
```
int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/6/a^2/x^6-1/48/a*(-(a*x-1)/a/x)^(1/2)/x^5*((a*x+1)/a/x)^(1/2)*(3*arc
tanh(1/(-a^2*x^2+1)^(1/2))*a^6*x^6+3*(-a^2*x^2+1)^(1/2)*a^4*x^4+2*a^2*x^2*
(-a^2*x^2+1)^(1/2)-8*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.46

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6}\,dx =$$
$$-\frac{3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right) - 3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right) + 2\,(3\,a^5x^5+2\,a^3x^3-8\,ax)}{96\,ax^6}$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="fric
as")
```

output
```
-1/96*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)
 - 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2
*(3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*
x)) + 16)/(a*x^6)
```

3.86.6 Sympy [F]

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6}\,dx = a\int \frac{1}{ax^6\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^5}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**6,x)
```

output
```
a*Integral(1/(a*x**6*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**5), x)
```

3.86.7 Maxima [F]

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{1}{x^6\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="maxi
ma")
```

output
```
integrate(1/(x^6*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.86.8 Giac [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{1}{x^6\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="giac
")
```

output
```
integrate(1/(x^6*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 69.66 (sec) , antiderivative size = 2479, normalized size of antiderivative = 7.75

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \text{Too large to display}$$

input
```
int(1/(x^6*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

output
```
((a^5*((1/(a*x) - 1)^(1/2) - 1i)^6*10240i)/((1/(a*x) + 1)^(1/2) - 1)^6 + (
a^5*((1/(a*x) - 1)^(1/2) - 1i)^8*20480i)/((1/(a*x) + 1)^(1/2) - 1)^8 + (a^
5*((1/(a*x) - 1)^(1/2) - 1i)^10*36864i)/((1/(a*x) + 1)^(1/2) - 1)^10 + (a^
5*((1/(a*x) - 1)^(1/2) - 1i)^12*20480i)/((1/(a*x) + 1)^(1/2) - 1)^12 + (a^
5*((1/(a*x) - 1)^(1/2) - 1i)^14*10240i)/((1/(a*x) + 1)^(1/2) - 1)^14)/(15*
((45*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (10*((1/(
a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (120*((1/(a*x) - 1)
^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (210*((1/(a*x) - 1)^(1/2) -
1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (252*((1/(a*x) - 1)^(1/2) - 1i)^10)/(
(1/(a*x) + 1)^(1/2) - 1)^10 + (210*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x
) + 1)^(1/2) - 1)^12 - (120*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^
(1/2) - 1)^14 + (45*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) -
1)^16 - (10*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 +
((1/(a*x) - 1)^(1/2) - 1i)^20/((1/(a*x) + 1)^(1/2) - 1)^20 + 1)) - (a^5*at
anh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/4 - ((a^5*((1/(
a*x) - 1)^(1/2) - 1i)^6*2048i)/(3*((1/(a*x) + 1)^(1/2) - 1)^6) + (a^5*((1/
(a*x) - 1)^(1/2) - 1i)^8*4096i)/(3*((1/(a*x) + 1)^(1/2) - 1)^8) + (a^5*((1
/(a*x) - 1)^(1/2) - 1i)^10*12288i)/(5*((1/(a*x) + 1)^(1/2) - 1)^10) + (a^5
*((1/(a*x) - 1)^(1/2) - 1i)^12*4096i)/(3*((1/(a*x) + 1)^(1/2) - 1)^12) + (
a^5*((1/(a*x) - 1)^(1/2) - 1i)^14*2048i)/(3*((1/(a*x) + 1)^(1/2) - 1)^1...
```

3.87 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^7} dx$

3.87.1 Optimal result

Integrand size = 12, antiderivative size = 353

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^7} dx = -\frac{a^6}{10\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{a^6}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{5a^6}{12\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{3a^6}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{5a^6}{16\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{2a^6}{7\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^7}+\frac{a^6}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^6}-\frac{19a^6}{10\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{9a^6}{4\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{11a^6}{6\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{a^6}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{5a^6}{16\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}$$

output
```
-1/10*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))^5+1/4*a^6/(1-((-a*x+1)/(a*x+1))^(1/
2))^4-5/12*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))^3+3/8*a^6/(1-((-a*x+1)/(a*x+1)
)^(1/2))^2-5/16*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))-2/7*a^6/(1+((-a*x+1)/(a*x
+1))^(1/2))^7+a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^6-19/10*a^6/(1+((-a*x+1)/(a
*x+1))^(1/2))^5+9/4*a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^4-11/6*a^6/(1+((-a*x+
1)/(a*x+1))^(1/2))^3+a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^2-5/16*a^6/(1+((-a*x
+1)/(a*x+1))^(1/2))
```

3.87.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.22

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7}\,dx = -\frac{15+\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(-15+15ax-12a^2x^2+12a^3x^3-8a^4x^4+8a^5x^5\right)}{105ax^7}$$

input
```
Integrate[1/(E^ArcSech[a*x]*x^7),x]
```

output
```
-1/105*(15 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*
x^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(a*x^7)
```

3.87.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6891, 7268, 27, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7}\,dx$$

$$\Big\downarrow \ 6891$$

$$\int \frac{1}{x^7\left(\frac{\sqrt{\frac{1-ax}{ax+1}}}{ax}+\sqrt{\frac{1-ax}{ax+1}}+\frac{1}{ax}\right)}dx$$

$$\Big\downarrow \ 7268$$

$$-4a\int \frac{a^5\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^5}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^6\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^8}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\Big\downarrow \ 27$$

$$-4a^6\int \frac{\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1-ax}{ax+1}+1\right)^5}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^6\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^8}d\sqrt{\frac{1-ax}{ax+1}}$$

$$\downarrow 2115$$

$$-4a^6 \int \left(-\frac{5}{64\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2} + \frac{1}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3} - \frac{11}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4} + \frac{9}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5} - \frac{19}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^6} + \right.$$

$$\downarrow 2009$$

$$-4a^6 \left(\frac{5}{64\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)} - \frac{1}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2} + \frac{11}{24\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3} - \frac{9}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4} + \frac{19}{40\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5} - \frac{}{4(} \right.$$

input
```
Int[1/(E^ArcSech[a*x]*x^7),x]
```

output
```
-4*a^6*(1/(40*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5) - 1/(16*(1 - Sqrt[(1 - a*
x)/(1 + a*x)])^4) + 5/(48*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) - 3/(32*(1 -
Sqrt[(1 - a*x)/(1 + a*x)])^2) + 5/(64*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) + 1
/(14*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^7) - 1/(4*(1 + Sqrt[(1 - a*x)/(1 + a*
x)])^6) + 19/(40*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^5) - 9/(16*(1 + Sqrt[(1 -
 a*x)/(1 + a*x)])^4) + 11/(24*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) - 1/(4*(1
 + Sqrt[(1 - a*x)/(1 + a*x)])^2) + 5/(64*(1 + Sqrt[(1 - a*x)/(1 + a*x)])))
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 6891
```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.87.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.22

method	result	size
default	$a\left(-\frac{1}{7a^{2}x^{7}}-\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(a^{2}x^{2}-1\right)\left(8a^{4}x^{4}+12a^{2}x^{2}+15\right)}{105a\,x^{6}}\right)$	76

input
```
int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7/a^2/x^7-1/105/a*((a*x+1)/a/x)^(1/2)/x^6*(-(a*x-1)/a/x)^(1/2)*(a^2*
x^2-1)*(8*a^4*x^4+12*a^2*x^2+15))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.20

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7}\,dx = -\frac{\left(8\,a^{7}x^{7}+4\,a^{5}x^{5}+3\,a^{3}x^{3}-15\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+15}{105\,ax^{7}}$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="fric
as")
```

output
```
-1/105*((8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*x + 1)/(a*x))
*sqrt(-(a*x - 1)/(a*x)) + 15)/(a*x^7)
```

3.87.6 Sympy [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7}\,dx = a\int \frac{1}{ax^7\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^6}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**7,x)
```

output
```
a*Integral(1/(a*x**7*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**6), x)
```

3.87.7 Maxima [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7}\,dx = \int \frac{1}{x^7\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="maxi
ma")
```

output
```
integrate(1/(x^7*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.87.8 Giac [F]

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7}\,dx = \int \frac{1}{x^7\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

input
```
integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="giac
")
```

output
```
integrate(1/(x^7*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.26

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7}\,dx = -\frac{1}{7\,a\,x^7} - \frac{\sqrt{\frac{1}{a\,x}-1}\left(\frac{a\,x^2}{35}-\frac{x}{7}-\frac{1}{7\,a}+\frac{a^2\,x^3}{35}+\frac{4\,a^3\,x^4}{105}+\frac{4\,a^4\,x^5}{105}+\frac{8\,a^5\,x^6}{105}+\frac{8\,a^6\,x^7}{105}\right)}{x^7\,\sqrt{\frac{1}{a\,x}+1}}$$

input
```
int(1/(x^7*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

output
```
- 1/(7*a*x^7) - ((1/(a*x) - 1)^(1/2)*((a*x^2)/35 - x/7 - 1/(7*a) + (a^2*x^
3)/35 + (4*a^3*x^4)/105 + (4*a^4*x^5)/105 + (8*a^5*x^6)/105 + (8*a^6*x^7)/
105))/(x^7*(1/(a*x) + 1)^(1/2))
```

3.88 $\int \frac{e^{\text{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2} dx$

3.88.1 Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{e^{\text{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2} dx = \frac{(dx)^m\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},c^2x^2\right)}{cm} + \frac{(dx)^m\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},c^2x^2\right)}{cm}$$

output
```
(d*x)^m*hypergeom([1, 1/2*m],[1+1/2*m],c^2*x^2)/c/m+(d*x)^m*hypergeom([1/2
, 1/2*m],[1+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/m
```

3.88.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{e^{\text{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2} dx = \frac{(dx)^m\left(\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},1+\frac{m}{2},c^2x^2\right)}{\sqrt{1-c^2x^2}} + \text{Hypergeometric2F1}\left(1,\frac{m}{2},1+\frac{m}{2},c^2x^2\right)\right)}{cm}$$

input
```
Integrate[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2),x]
```

output
```
((d*x)^m*((Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Hypergeometric2F1[1/2, m/2,
 1 + m/2, c^2*x^2])/Sqrt[1 - c^2*x^2] + Hypergeometric2F1[1, m/2, 1 + m/2,
 c^2*x^2]))/(c*m)
```

3.88.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6895, 278, 2044, 135, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6895}$$

$$\frac{d\int \frac{(dx)^{m-1}}{1-c^2x^2}dx}{c}+\frac{d\int \frac{(dx)^{m-1}\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{278}$$

$$\frac{d\int \frac{(dx)^{m-1}\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}+\frac{(dx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},c^2x^2\right)}{cm}$$

$$\Big\downarrow \text{2044}$$

$$\frac{d\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(dx)^{m-1}}{\sqrt{1-cx}\sqrt{cx+1}}dx}{c}+\frac{(dx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},c^2x^2\right)}{cm}$$

$$\Big\downarrow \text{135}$$

$$\frac{d\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{(dx)^{m-1}}{\sqrt{1-c^2x^2}}dx}{c}+\frac{(dx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},c^2x^2\right)}{cm}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{m+2}{2},c^2x^2\right)}{cm}+\frac{(dx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},c^2x^2\right)}{cm}$$

input
```
Int[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2),x]
```

output
```
((d*x)^m*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, m/2, (2
 + m)/2, c^2*x^2])/(c*m) + ((d*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, c
^2*x^2])/(c*m)
```

3.88.3.1 Defintions of rubi rules used

rule 135
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f, m, n, p},
 x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6895
```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d/(a*c)   Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
 x], x] + Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b,
c, d, m}, x] && EqQ[b + a*c^2, 0]
```

3.88.4 Maple [F]

$$\int \frac{\left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\,\sqrt{1 + \frac{1}{cx}}\right)(dx)^m}{-c^2x^2 + 1}dx$$

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x)
```

output
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x)
```

3.88.5 Fricas [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx = \int -\frac{(dx)^m\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
 algorithm="fricas")
```

output
```
integral(-((d*x)^m*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + (d*x
)^m)/(c^3*x^3 - c*x), x)
```

3.88.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx = -\frac{\int \frac{(dx)^m}{c^2x^3-x}\,dx + \int \frac{cx(dx)^m\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^3-x}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*(d*x)**m/(-c**2*x**2+
1),x)
```

output
```
-(Integral((d*x)**m/(c**2*x**3 - x), x) + Integral(c*x*(d*x)**m*sqrt(-1 +
1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**3 - x), x))/c
```

3.88.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx = \int -\frac{(dx)^m\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
 algorithm="maxima")
```

output
```
-d^m*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/(c^3*x^3 - c*x), x) - d^m*
integrate(1/2*x^m/(c*x + 1), x) - d^m*integrate(1/2*x^m/(c*x - 1), x) + d^
m*x^m/(c*m)
```

3.88.8 Giac [F]

$$\int \frac{e^{\text{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx = \int -\frac{(dx)^m\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
 algorithm="giac")
```

output
```
integrate(-(d*x)^m*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^
2 - 1), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\text{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx = -\int \frac{\left(\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}+\frac{1}{c\,x}\right)\,(d\,x)^m}{c^2\,x^2-1}\,dx$$

input
```
int(-(((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^
2 - 1),x)
```

output
```
-int((((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^
2 - 1), x)
```

3.89 $\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx$

3.89.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx=-\frac{x^2}{2c^3}-\frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{1+cx}}}-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{1+cx}}}-\frac{\log(1-c^2x^2)}{2c^5}$$

output
```
-1/2*x^2/c^3-1/2*ln(-c^2*x^2+1)/c^5-2/3*(-c*x+1)^(1/2)/c^5/(1/(c*x+1))^(1/
2)-1/3*x^2*(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx=-\frac{3c^2x^2+2\sqrt{\frac{1-cx}{1+cx}}(2+2cx+c^2x^2+c^3x^3)+3\log(1-c^2x^2)}{6c^5}$$

input
```
Integrate[(E^ArcSech[c*x]*x^4)/(1 - c^2*x^2),x]
```

output
```
-1/6*(3*c^2*x^2 + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x
^3) + 3*Log[1 - c^2*x^2])/c^5
```

3.89.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6895, 243, 49, 2009, 2044, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 e^{\text{sech}^{-1}(cx)}}{1-c^2x^2} \, dx$$

$$\Big\downarrow \text{6895}$$

$$\frac{\int \frac{x^3}{1-c^2x^2}dx}{c}+\frac{\int \frac{x^3\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{x^2}{1-c^2x^2}dx^2}{2c}+\frac{\int \frac{x^3\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\frac{1}{c^2}-\frac{1}{c^2(c^2x^2-1)}\right)dx^2}{2c}+\frac{\int \frac{x^3\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \frac{x^3\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}+\frac{-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}}{2c}$$

$$\Big\downarrow \text{2044}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^3}{\sqrt{1-cx}\sqrt{cx+1}}dx}{c}+\frac{-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}}{2c}$$

$$\Big\downarrow \text{111}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{2x}{\sqrt{1-cx}\sqrt{cx+1}}dx}{3c^2}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)}{c}+\frac{-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2\int \frac{x}{\sqrt{1-cx}\sqrt{cx+1}}dx}{3c^2}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)}{c}+\frac{-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}}{2c}$$

$$\Big\downarrow \text{83}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2\sqrt{1-cx}\sqrt{cx+1}}{3c^4}-\frac{x^2\sqrt{1-cx}\sqrt{cx+1}}{3c^2}\right)}{c}+\frac{-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}}{2c}$$

input
```
Int[(E^ArcSech[c*x]*x^4)/(1 - c^2*x^2),x]
```

output
```
(Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*((-2*Sqrt[1 - c*x]*Sqrt[1 + c*x])/(3*c
^4) - (x^2*Sqrt[1 - c*x]*Sqrt[1 + c*x])/(3*c^2)))/c + (-(x^2/c^2) - Log[1
- c^2*x^2]/c^4)/(2*c)
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_^2))^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6895
```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d/(a*c)   Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
 x], x] + Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b,
c, d, m}, x] && EqQ[b + a*c^2, 0]
```

3.89.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\, x \sqrt{\frac{cx+1}{cx}}\,(c^2x^2+2)}{3c^4}+\frac{-\frac{x^2}{2c^2}-\frac{\ln\left(c^2x^2-1\right)}{2c^4}}{c}$	74

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/3*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(c^2*x^2+2)/c^4+1/c*(-1/2*
x^2/c^2-1/2/c^4*ln(c^2*x^2-1))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx=-\frac{3\,c^2x^2+2\,(c^3x^3+2\,cx)\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+3\,\log\left(c^2x^2-1\right)}{6\,c^5}$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
-1/6*(3*c^2*x^2 + 2*(c^3*x^3 + 2*c*x)*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1
)/(c*x)) + 3*log(c^2*x^2 - 1))/c^5
```

3.89.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx = -\frac{\int \frac{x^3}{c^2x^2-1}\,dx + \int \frac{cx^4\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**4/(-c**2*x**2+1),x
)
```

output
```
-(Integral(x**3/(c**2*x**2 - 1), x) + Integral(c*x**4*sqrt(-1 + 1/(c*x))*s
qrt(1 + 1/(c*x))/(c**2*x**2 - 1), x))/c
```

3.89.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx = \int -\frac{x^4\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
-integrate(x, x)/c^3 - 1/2*log(c*x + 1)/c^5 - 1/2*log(c*x - 1)/c^5 - integ
rate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3/(c^3*x^2 - c), x)
```

3.89.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx = \int -\frac{x^4\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-x^4*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 -
1), x)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx = -\frac{\ln\left(c^2\,x^2-1\right)+c^2\,x^2}{2\,c^5}-x^3\,\sqrt{\frac{1}{c\,x}-1}\,\left(\frac{\sqrt{\frac{1}{c\,x}+1}}{3\,c^2}+\frac{2\,\sqrt{\frac{1}{c\,x}+1}}{3\,c^4\,x^2}\right)$$

input
```
int(-(x^4*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 -
1),x)
```

output
```
- (log(c^2*x^2 - 1) + c^2*x^2)/(2*c^5) - x^3*(1/(c*x) - 1)^(1/2)*((1/(c*x)
 + 1)^(1/2)/(3*c^2) + (2*(1/(c*x) + 1)^(1/2))/(3*c^4*x^2))
```

3.90 $\int \frac{e^{\text{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx$

3.90.1 Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{e^{\text{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx=-\frac{x}{c^3}-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{1+cx}}}+\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{2c^4}+\frac{\operatorname{arctanh}(cx)}{c^4}$$

output
```
-x/c^3+arctanh(c*x)/c^4-1/2*x*(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)+1/2*arc
sin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^4
```

3.90.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int \frac{e^{\text{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx = -\frac{2cx+cx\sqrt{\frac{1-cx}{1+cx}}+c^2x^2\sqrt{\frac{1-cx}{1+cx}}+\log(1-cx)-\log(1+cx)-i\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{2c^4}$$

input
```
Integrate[(E^ArcSech[c*x]*x^3)/(1 - c^2*x^2),x]
```

output
```
-1/2*(2*c*x + c*x*Sqrt[(1 - c*x)/(1 + c*x)] + c^2*x^2*Sqrt[(1 - c*x)/(1 +
c*x)] + Log[1 - c*x] - Log[1 + c*x] - I*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/
(1 + c*x)]*(1 + c*x)])/c^4
```

3.90.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6895, 262, 219, 2044, 101, 25, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{\text{sech}^{-1}(cx)}}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6895}$$

$$\frac{\int \frac{x^2}{1-c^2x^2}dx}{c}+\frac{\int \frac{x^2\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\int \frac{1}{1-c^2x^2}dx}{c^2}-\frac{x}{c^2}}{c}+\frac{\int \frac{x^2\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{x^2\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}+\frac{\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}}{c}$$

$$\Big\downarrow \text{2044}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x^2}{\sqrt{1-cx}\sqrt{cx+1}}dx}{c}+\frac{\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}}{c}$$

$$\Big\downarrow \text{101}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{\int -\frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{c}+\frac{\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{c}+\frac{\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}}{c}$$

$$\Big\downarrow \; 39$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{c}+\frac{\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}}{c}$$

$$\Big\downarrow \; 223$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-cx}\sqrt{cx+1}}{2c^2}\right)}{c}+\frac{\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}}{c}$$

input
```
Int[(E^ArcSech[c*x]*x^3)/(1 - c^2*x^2),x]
```

output
```
(Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/2*(x*Sqrt[1 - c*x]*Sqrt[1 + c*x])/
c^2 + ArcSin[c*x]/(2*c^3)))/c + (-(x/c^2) + ArcTanh[c*x]/c^3)/c
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.90. $\int \frac{e^{\text{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx$

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6895
```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d/(a*c)   Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
 x], x] + Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b,
c, d, m}, x] && EqQ[b + a*c^2, 0]
```

3.90.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\, x\sqrt{\frac{cx+1}{cx}}\left(x\sqrt{-c^2x^2+1}\,\operatorname{csgn}(c)c-\arctan\left(\frac{\operatorname{csgn}(c)cx}{\sqrt{-c^2x^2+1}}\right)\right)\operatorname{csgn}(c)}{2c^3\sqrt{-c^2x^2+1}}+\frac{-\frac{x}{c^2}+\frac{\ln(cx+1)}{2c^3}-\frac{\ln(cx-1)}{2c^3}}{c}$	122

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/c^3*(x*(-c^2*x^2+1)^(1/2)*
csgn(c)*c-arctan(csgn(c)*c*x/(-c^2*x^2+1)^(1/2)))/(-c^2*x^2+1)^(1/2)*csgn(
c)+1/c*(-x/c^2+1/2/c^3*ln(c*x+1)-1/2/c^3*ln(c*x-1))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx = -\frac{c^2x^2\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} + 2\,cx + \arctan\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\right) - \log(cx+1) + \log(cx-1)}{2\,c^4}$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
-1/2*(c^2*x^2*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + 2*c*x + arcta
n(sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x))) - log(c*x + 1) + log(c*x -
 1))/c^4
```

3.90.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx = -\frac{\int \frac{x^2}{c^2x^2-1}\,dx + \int \frac{cx^3\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**3/(-c**2*x**2+1),x
)
```

output
```
-(Integral(x**2/(c**2*x**2 - 1), x) + Integral(c*x**3*sqrt(-1 + 1/(c*x))*s
qrt(1 + 1/(c*x))/(c**2*x**2 - 1), x))/c
```

3.90.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx = \int -\frac{x^3\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
-x/c^3 + 1/2*log(c*x + 1)/c^4 - 1/2*log(c*x - 1)/c^4 - integrate(sqrt(c*x
+ 1)*sqrt(-c*x + 1)*x^2/(c^3*x^2 - c), x)
```

3.90.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx = \int -\frac{x^3\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-x^3*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 -
1), x)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 11.19 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.53

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx = \frac{\operatorname{atanh}(c\,x)-c\,x}{c^4} - \frac{\ln\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)\,1\mathrm{i}}{2\,c^4} - \frac{\frac{1\mathrm{i}}{32\,c^4}+\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,c^4\,\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,c^4\,\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^2}+\frac{2\,\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^6}} + \frac{\ln\left(\frac{2\,c\,\sqrt{\frac{c+\frac{1}{x}}{c}}-\frac{2}{x}+c\,\sqrt{-\frac{c-\frac{1}{x}}{c}}\,2\mathrm{i}}{2\,c+\frac{1}{x}-2\,c\,\sqrt{\frac{c+\frac{1}{x}}{c}}}\right)\,1\mathrm{i}}{2\,c^4} - \frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,c^4\,\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^2}$$

input
```
int(-(x^3*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 -
1),x)
```

output
```
(atanh(c*x) - c*x)/c^4 - (log(((1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1
/2) - 1))*1i)/(2*c^4) - (1i/(32*c^4) + (((1/(c*x) - 1)^(1/2) - 1i)^2*1i)/(
16*c^4*((1/(c*x) + 1)^(1/2) - 1)^2) - (((1/(c*x) - 1)^(1/2) - 1i)^4*15i)/(
32*c^4*((1/(c*x) + 1)^(1/2) - 1)^4))/((((1/(c*x) - 1)^(1/2) - 1i)^2/((1/(c*
x) + 1)^(1/2) - 1)^2 + (2*((1/(c*x) - 1)^(1/2) - 1i)^4)/((1/(c*x) + 1)^(1/
2) - 1)^4 + ((1/(c*x) - 1)^(1/2) - 1i)^6/((1/(c*x) + 1)^(1/2) - 1)^6) + (l
og((c*(-(c - 1/x)/c)^(1/2)*2i - 2/x + 2*c*((c + 1/x)/c)^(1/2))/(2*c + 1/x
- 2*c*((c + 1/x)/c)^(1/2)))*1i)/(2*c^4) - ((((1/(c*x) - 1)^(1/2) - 1i)^2*1i
)/(32*c^4*((1/(c*x) + 1)^(1/2) - 1)^2)
```

3.91 $\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx$

3.91.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx=-\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{1+cx}}}-\frac{\log(1-c^2x^2)}{2c^3}$$

output
```
-1/2*ln(-c^2*x^2+1)/c^3-(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx=-\frac{2\sqrt{\frac{1-cx}{1+cx}}(1+cx)+\log(1-c^2x^2)}{2c^3}$$

input
```
Integrate[(E^ArcSech[c*x]*x^2)/(1 - c^2*x^2),x]
```

output
```
-1/2*(2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + Log[1 - c^2*x^2])/c^3
```

3.91.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6895, 240, 2044, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{\text{sech}^{-1}(cx)}}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6895}$$

$$\frac{\int \frac{x}{1-c^2x^2}dx}{c}+\frac{\int \frac{x\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{240}$$

$$\frac{\int \frac{x\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}-\frac{\log\left(1-c^2x^2\right)}{2c^3}$$

$$\Big\downarrow \text{2044}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{x}{\sqrt{1-cx}\sqrt{cx+1}}dx}{c}-\frac{\log\left(1-c^2x^2\right)}{2c^3}$$

$$\Big\downarrow \text{83}$$

$$-\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{cx+1}}}-\frac{\log\left(1-c^2x^2\right)}{2c^3}$$

input
```
Int[(E^ArcSech[c*x]*x^2)/(1 - c^2*x^2),x]
```

output
```
-(Sqrt[1 - c*x]/(c^3*Sqrt[(1 + c*x)^(-1)])) - Log[1 - c^2*x^2]/(2*c^3)
```

3.91.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6895
```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d/(a*c)   Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
 x], x] + Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b,
c, d, m}, x] && EqQ[b + a*c^2, 0]
```

3.91.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\, x \sqrt{\frac{cx+1}{cx}}}{c^2}-\frac{\ln(c^2x^2-1)}{2c^3}$	52

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/c^2-1/2/c^3*ln(c^2*x^2-1)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx = -\frac{2\,cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+\log\left(c^2x^2-1\right)}{2\,c^3}$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
-1/2*(2*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + log(c^2*x^2 - 1
))/c^3
```

3.91.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx = -\frac{\int \frac{x}{c^2x^2-1}\,dx+\int \frac{cx^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**2/(-c**2*x**2+1),x
)
```

output
```
-(Integral(x/(c**2*x**2 - 1), x) + Integral(c*x**2*sqrt(-1 + 1/(c*x))*sqrt
(1 + 1/(c*x))/(c**2*x**2 - 1), x))/c
```

3.91.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx = \int -\frac{x^2\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
-1/2*log(c*x + 1)/c^3 - 1/2*log(c*x - 1)/c^3 - integrate(sqrt(c*x + 1)*sqr
t(-c*x + 1)*x/(c^3*x^2 - c), x)
```

3.91.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx = \int -\frac{x^2\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-x^2*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 -
1), x)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx = -\frac{\ln\left(c^2\,x^2-1\right)}{2\,c^3}-\frac{x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{c^2}$$

input
```
int(-(x^2*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 -
1),x)
```

output
```
- log(c^2*x^2 - 1)/(2*c^3) - (x*(1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))/c
^2
```

3.92 $\int \frac{e^{\text{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx$

3.92.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{e^{\text{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx=\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c^2}+\frac{\text{arctanh}(cx)}{c^2}$$

output
```
arctanh(c*x)/c^2+arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^2
```

3.92.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{e^{\text{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx=-\frac{\log(1-cx)}{2c^2}+\frac{\log(1+cx)}{2c^2}+\frac{i\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{c^2}$$

input
```
Integrate[(E^ArcSech[c*x]*x)/(1 - c^2*x^2),x]
```

output
```
-1/2*Log[1 - c*x]/c^2 + Log[1 + c*x]/(2*c^2) + (I*Log[(-2*I)*c*x + 2*Sqrt[
(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^2
```

3.92.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6895, 219, 2044, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\text{sech}^{-1}(cx)}}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{6895}$$

$$\frac{\int \frac{1}{1-c^2x^2}dx}{c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{\sqrt{1-cx}}dx}{c}+\frac{\text{arctanh}(cx)}{c^2}$$

$$\Big\downarrow \text{2044}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{\sqrt{1-cx}\sqrt{cx+1}}dx}{c}+\frac{\text{arctanh}(cx)}{c^2}$$

$$\Big\downarrow \text{39}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{\sqrt{1-c^2x^2}}dx}{c}+\frac{\text{arctanh}(cx)}{c^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{c^2}+\frac{\text{arctanh}(cx)}{c^2}$$

input
```
Int[(E^ArcSech[c*x]*x)/(1 - c^2*x^2),x]
```

output
```
(Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c^2 + ArcTanh[c*x]/c^2
```

3.92.3.1 Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6895
```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d/(a*c)   Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
 x], x] + Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b,
c, d, m}, x] && EqQ[b + a*c^2, 0]
```

3.92.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.62

method	result	size
default	$\frac{\sqrt{-\frac{cx-1}{cx}}\, x \sqrt{\frac{cx+1}{cx}}\, \arctan\left(\frac{\operatorname{csgn}(c) c x}{\sqrt{-(cx-1)(cx+1)}}\right) \operatorname{csgn}(c)}{\sqrt{-c^2x^2+1}\, c}+\frac{\frac{\ln(cx+1)}{2c}-\frac{\ln(cx-1)}{2c}}{c}$	97

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

output
```
(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*arctan(csgn(c)*c*x/(-(c*x-1)*(c
*x+1))^(1/2))/(-c^2*x^2+1)^(1/2)*csgn(c)/c+1/c*(1/2/c*ln(c*x+1)-1/2/c*ln(c
*x-1))
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx = -\frac{2\arctan\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\right)-\log\left(cx+1\right)+\log\left(cx-1\right)}{2\,c^2}$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algor
ithm="fricas")
```

output
```
-1/2*(2*arctan(sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x))) - log(c*x + 1
) + log(c*x - 1))/c^2
```

3.92.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx = -\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx+\int \frac{1}{c^2x^2-1}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x/(-c**2*x**2+1),x)
```

output
```
-(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**2 - 1), x) +
Integral(1/(c**2*x**2 - 1), x))/c
```

3.92.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx = \int -\frac{x\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algor
ithm="maxima")
```

output
```
1/2*log(c*x + 1)/c^2 - 1/2*log(c*x - 1)/c^2 - integrate(sqrt(c*x + 1)*sqrt
(-c*x + 1)/(c^3*x^2 - c), x)
```

3.92.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx = \int -\frac{x\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algor
ithm="giac")
```

output
```
integrate(-x*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1)
, x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx = \frac{\operatorname{atanh}(c\,x)}{c^2} + \frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)\right)1\mathrm{i}}{c^2}$$

input
```
int(-(x*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1)
,x)
```

output
```
((log(((1/(c*x) - 1)^(1/2) - 1i)^2/((1/(c*x) + 1)^(1/2) - 1)^2 + 1) - log(
((1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)))*1i)/c^2 + atanh(c*x
)/c^2
```

3.93 $\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2} dx$

3.93.1 Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2} dx = -\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)}{c}+\frac{\log(x)}{c}-\frac{\log(1-c^2x^2)}{2c}$$

output
```
ln(x)/c-1/2*ln(-c^2*x^2+1)/c-arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)/c
```

3.93.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2} dx = \frac{2\log(x)}{c}-\frac{\log(1-c^2x^2)}{2c}-\frac{\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{c}$$

input
```
Integrate[E^ArcSech[c*x]/(1 - c^2*x^2),x]
```

output
```
(2*Log[x])/c - Log[1 - c^2*x^2]/(2*c) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)]
+ c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/c
```

3.93.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6893, 243, 47, 14, 16, 2044, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(cx)}}{1-c^2x^2} \, dx$$

$$\Big\downarrow \text{6893}$$

$$\frac{\int \frac{1}{x(1-c^2x^2)}dx}{c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{1}{x^2(1-c^2x^2)}dx^2}{2c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{47}$$

$$\frac{c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2}{2c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{14}$$

$$\frac{c^2\int \frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)}{2c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x\sqrt{1-cx}}dx}{c}+\frac{\log\left(x^2\right)-\log\left(1-c^2x^2\right)}{2c}$$

$$\Big\downarrow \text{2044}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}}dx}{c}+\frac{\log\left(x^2\right)-\log\left(1-c^2x^2\right)}{2c}$$

$$\Big\downarrow \text{103}$$

$$\frac{\log\left(x^2\right)-\log\left(1-c^2x^2\right)}{2c}-\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{c-c(1-cx)(cx+1)}d\left(\sqrt{1-cx}\sqrt{cx+1}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{\log(x^2) - \log(1 - c^2x^2)}{2c} - \frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{arctanh}\left(\sqrt{1-cx}\sqrt{cx+1}\right)}{c}$$

input
```
Int[E^ArcSech[c*x]/(1 - c^2*x^2),x]
```

output
```
-((Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]]
)/c) + (Log[x^2] - Log[1 - c^2*x^2])/(2*c)
```

3.93.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6893
```
Int[E^ArcSech[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[1/(a*c)
  Int[Sqrt[1/(1 + c*x)]/(x*Sqrt[1 - c*x]), x], x] + Simp[1/c   Int[1/(x*(a
+ b*x^2)), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + a*c^2, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\, x \sqrt{\frac{cx+1}{cx}}\, \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)}{\sqrt{-c^{2}x^{2}+1}}+\frac{-\frac{\ln(cx+1)}{2}+\ln(x)-\frac{\ln(cx-1)}{2}}{c}$	82

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x,method=_RETURN
VERBOSE)
```

output
```
-(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))/
(-c^2*x^2+1)^(1/2)+1/c*(-1/2*ln(c*x+1)+ln(x)-1/2*ln(c*x-1))
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2}\,dx = -\frac{\log\left(c^2x^2-1\right)+\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+1\right)-\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}-1\right)-2\,\log\left(x\right)}{2\,c}$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorit
hm="fricas")
```

output
```
-1/2*(log(c^2*x^2 - 1) + log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*
x)) + 1) - log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - 1) - 2*l
og(x))/c
```

3.93.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2}\,dx = -\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^3-x}\,dx + \int \frac{1}{c^2x^3-x}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/(-c**2*x**2+1),x)
```

output
```
-(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**3 - x), x) +
Integral(1/(c**2*x**3 - x), x))/c
```

3.93.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2}\,dx = \int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorit
hm="maxima")
```

output
```
integrate(1/x, x)/c - 1/2*log(c*x + 1)/c - 1/2*log(c*x - 1)/c - integrate(
sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^3 - c*x), x)
```

3.93.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2}\,dx = \int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{c^2x^2-1}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorit
hm="giac")
```

output
```
integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1),
x)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2}\,dx = \frac{\ln(x)}{c} - \frac{4\,\operatorname{atanh}\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)}{c} - \frac{\ln(3\,c^2\,x^2-3)}{2\,c}$$

input
```
int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(c^2*x^2 - 1),x)
```

output
```
log(x)/c - (4*atanh(((1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)))
/c - log(3*c^2*x^2 - 3)/(2*c)
```

3.94 $\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x(1-c^2x^2)}\,dx$

3.94.1 Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x(1-c^2x^2)}\,dx=-\frac{1}{cx}-\frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{1+cx}}}+\operatorname{arctanh}(cx)$$

output
```
-1/c/x+arctanh(c*x)-(-c*x+1)^(1/2)/c/x/(1/(c*x+1))^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x(1-c^2x^2)}\,dx=-\frac{1}{cx}-\left(1+\frac{1}{cx}\right)\sqrt{\frac{1-cx}{1+cx}}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)$$

input
```
Integrate[E^ArcSech[c*x]/(x*(1 - c^2*x^2)),x]
```

output
```
-(1/(c*x)) - (1 + 1/(c*x))*Sqrt[(1 - c*x)/(1 + c*x)] - Log[1 - c*x]/2 + Lo
g[1 + c*x]/2
```

3.94.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6895, 264, 219, 2044, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(cx)}}{x (1 - c^2 x^2)} \, dx$$

$$\Big\downarrow \text{ 6895}$$

$$\frac{\int \frac{1}{x^2(1-c^2x^2)}dx}{c} + \frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^2\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{c^2 \int \frac{1}{1-c^2x^2}dx - \frac{1}{x}}{c} + \frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^2\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^2\sqrt{1-cx}}dx}{c} + \frac{c\,\text{arctanh}(cx) - \frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 2044}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1} \int \frac{1}{x^2\sqrt{1-cx}\sqrt{cx+1}}dx}{c} + \frac{c\,\text{arctanh}(cx) - \frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 106}$$

$$\frac{c\,\text{arctanh}(cx) - \frac{1}{x}}{c} - \frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{cx+1}}}$$

input
```
Int[E^ArcSech[c*x]/(x*(1 - c^2*x^2)),x]
```

output
```
-(Sqrt[1 - c*x]/(c*x*Sqrt[(1 + c*x)^(-1)])) + (-x^(-1) + c*ArcTanh[c*x])/c
```

3.94.3.1 Defintions of rubi rules used

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6895
```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d/(a*c)   Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
 x], x] + Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b,
c, d, m}, x] && EqQ[b + a*c^2, 0]
```

3.94.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.55

method	result	size
default	$-\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\operatorname{csgn}\left(c\right)^{2}+\frac{\frac{c\ln\left(cx+1\right)}{2}-\frac{c\ln\left(cx-1\right)}{2}-\frac{1}{x}}{c}$	65

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

output
```
-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*csgn(c)^2+1/c*(1/2*c*ln(c*x+1)-1
/2*c*ln(c*x-1)-1/x)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x\left(1-c^2x^2\right)}\,dx = -\frac{2\,cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} - cx\log\left(cx+1\right) + cx\log\left(cx-1\right) + 2}{2\,cx}$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algor
ithm="fricas")
```

output
```
-1/2*(2*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - c*x*log(c*x + 1
) + c*x*log(c*x - 1) + 2)/(c*x)
```

3.94.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x\left(1-c^2x^2\right)}\,dx = -\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^4-x^2}\,dx + \int \frac{1}{c^2x^4-x^2}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x/(-c**2*x**2+1),x)
```

output
```
-(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**4 - x**2), x)
 + Integral(1/(c**2*x**4 - x**2), x))/c
```

3.94.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x\left(1-c^2x^2\right)}\,dx = \int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{(c^2x^2-1)x}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algor
ithm="maxima")
```

output
```
integrate(x^(-2), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^4 -
 c*x^2), x) + 1/2*log(c*x + 1) - 1/2*log(c*x - 1)
```

3.94.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x\left(1-c^2x^2\right)}\,dx = \int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{(c^2x^2-1)x}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algor
ithm="giac")
```

output
```
integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*
x), x)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x\left(1-c^2x^2\right)}\,dx = \operatorname{atanh}(c\,x) - \sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1} - \frac{1}{c\,x}$$

input
```
int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x*(c^2*x^2 - 1))
,x)
```

output
```
atanh(c*x) - (1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - 1/(c*x)
```

3.95 $\int \frac{e^{\mathbf{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx$

3.95.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx = -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} - \frac{1}{2}c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right) + c\log(x) - \frac{1}{2}c\log\left(1-c^2x^2\right)$$

output
```
-1/2/c/x^2+c*ln(x)-1/2*c*ln(-c^2*x^2+1)-1/2*(-c*x+1)^(1/2)/c/x^2/(1/(c*x+1
))^(1/2)-1/2*c*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx = \frac{1}{2}\left(-\frac{1}{cx^2} - \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx^2} + 3c\log(x) - c\log\left(1-c^2x^2\right) - c\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)\right)$$

input
```
Integrate[E^ArcSech[c*x]/(x^2*(1 - c^2*x^2)),x]
```

output
```
(-(1/(c*x^2)) - (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(c*x^2) + 3*c*Log[x]
 - c*Log[1 - c^2*x^2] - c*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1
- c*x)/(1 + c*x)]])/2
```

3.95.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6895, 243, 54, 2009, 2044, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}dx$$

$$\Big\downarrow \text{6895}$$

$$\frac{\int \frac{1}{x^3(1-c^2x^2)}dx}{c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^3\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{1}{x^4(1-c^2x^2)}dx^2}{2c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^3\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(-\frac{c^4}{c^2x^2-1}+\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{2c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^3\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^3\sqrt{1-cx}}dx}{c}+\frac{c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}}{2c}$$

$$\Big\downarrow \text{2044}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^3\sqrt{1-cx}\sqrt{cx+1}}dx}{c}+\frac{c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}}{2c}$$

$$\Big\downarrow \text{114}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{2}\int -\frac{c^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)}{c}+\frac{c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}}{2c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{2}\int \frac{c^2}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)}{c}+\frac{c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}}{2c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{1}{2}c^2\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)}{c}+\frac{c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}}{2c}$$

$$\Big\downarrow \text{ 103}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{2}c^3\int \frac{1}{c-c(1-cx)(cx+1)}d\left(\sqrt{1-cx}\sqrt{cx+1}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)}{c}+\frac{c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}}{2c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{2}c^2\text{arctanh}\left(\sqrt{1-cx}\sqrt{cx+1}\right)-\frac{\sqrt{1-cx}\sqrt{cx+1}}{2x^2}\right)}{c}+\frac{c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}}{2c}$$

input
```
Int[E^ArcSech[c*x]/(x^2*(1 - c^2*x^2)),x]
```

output
```
(Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/2*(Sqrt[1 - c*x]*Sqrt[1 + c*x])/x^
2 - (c^2*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/2))/c + (-x^(-2) + c^2*Log[
x^2] - c^2*Log[1 - c^2*x^2])/(2*c)
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6895
```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d/(a*c)   Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
 x], x] + Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b,
c, d, m}, x] && EqQ[b + a*c^2, 0]
```

3.95. $\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx$

3.95.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2x^2+\sqrt{-c^2x^2+1}\right)}{2x\sqrt{-c^2x^2+1}}+\frac{-\frac{c^2\ln(cx+1)}{2}-\frac{1}{2x^2}+c^2\ln(x)-\frac{c^2\ln(cx-1)}{2}}{c}$	119

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*(-(c*x-1)/c/x)^(1/2)/x*((c*x+1)/c/x)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1
/2))*c^2*x^2+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+1/c*(-1/2*c^2*ln(c*x+1
)-1/2/x^2+c^2*ln(x)-1/2*c^2*ln(c*x-1))
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(70) = 140.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.44

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx = -\frac{2\,c^2x^2\log\left(c^2x^2-1\right)+c^2x^2\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+1\right)-c^2x^2\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}-1\right)-4\,c^2x^2\,\mathrm{lo}}{4\,cx^2}$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
-1/4*(2*c^2*x^2*log(c^2*x^2 - 1) + c^2*x^2*log(c*x*sqrt((c*x + 1)/(c*x))*s
qrt(-(c*x - 1)/(c*x)) + 1) - c^2*x^2*log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(
c*x - 1)/(c*x)) - 1) - 4*c^2*x^2*log(x) + 2*c*x*sqrt((c*x + 1)/(c*x))*sqrt
(-(c*x - 1)/(c*x)) + 2)/(c*x^2)
```

3.95.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx = -\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^5-x^3}\,dx + \int \frac{1}{c^2x^5-x^3}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x**2/(-c**2*x**2+1),x
)
```

output
```
-(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**5 - x**3), x)
 + Integral(1/(c**2*x**5 - x**3), x))/c
```

3.95.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx = \int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{(c^2x^2-1)x^2}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
c*integrate(1/x, x) - 1/2*c*log(c*x + 1) - 1/2*c*log(c*x - 1) + integrate(
x^(-3), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^5 - c*x^3), x
)
```

3.95.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx = \int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{(c^2x^2-1)x^2}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*
x^2), x)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.06

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx = c\ln(x) + \frac{\frac{2c\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{cx}+1}-1} + \frac{14c\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^3} + \frac{14c\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^5} + \frac{2c\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^7}}{1+\frac{6\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^4} - \frac{4\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^6} + \frac{\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^8} - \frac{4\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^2}} - \frac{c\ln(c^2x^2-1)}{2} - 2c\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{cx}-1}-1-\mathrm{i}}{\sqrt{\frac{1}{cx}+1}-1}\right) - \frac{1}{2cx^2}$$

input
```
int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x^2*(c^2*x^2 - 1
)),x)
```

output
```
((2*c*((1/(c*x) - 1)^(1/2) - 1i))/((1/(c*x) + 1)^(1/2) - 1) + (14*c*((1/(c
*x) - 1)^(1/2) - 1i)^3)/((1/(c*x) + 1)^(1/2) - 1)^3 + (14*c*((1/(c*x) - 1)
^(1/2) - 1i)^5)/((1/(c*x) + 1)^(1/2) - 1)^5 + (2*c*((1/(c*x) - 1)^(1/2) -
1i)^7)/((1/(c*x) + 1)^(1/2) - 1)^7)/((6*((1/(c*x) - 1)^(1/2) - 1i)^4)/((1/
(c*x) + 1)^(1/2) - 1)^4 - (4*((1/(c*x) - 1)^(1/2) - 1i)^2)/((1/(c*x) + 1)^
(1/2) - 1)^2 - (4*((1/(c*x) - 1)^(1/2) - 1i)^6)/((1/(c*x) + 1)^(1/2) - 1)^
6 + ((1/(c*x) - 1)^(1/2) - 1i)^8/((1/(c*x) + 1)^(1/2) - 1)^8 + 1) - 2*c*at
anh(((1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)) - (c*log(c^2*x^2
 - 1))/2 + c*log(x) - 1/(2*c*x^2)
```

3.96 $\int \frac{e^{\text{sech}^{-1}(cx)}}{x^3(1-c^2x^2)}\,dx$

3.96.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{e^{\text{sech}^{-1}(cx)}}{x^3(1-c^2x^2)}\,dx=-\frac{1}{3cx^3}-\frac{c}{x}-\frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}}-\frac{2c\sqrt{1-cx}}{3x\sqrt{\frac{1}{1+cx}}}+c^2\text{arctanh}(cx)$$

output
```
-1/3/c/x^3-c/x+c^2*arctanh(c*x)-1/3*(-c*x+1)^(1/2)/c/x^3/(1/(c*x+1))^(1/2)
-2/3*c*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{e^{\text{sech}^{-1}(cx)}}{x^3(1-c^2x^2)}\,dx$$
$$=-\frac{2+6c^2x^2+2\sqrt{\frac{1-cx}{1+cx}}(1+cx+2c^2x^2+2c^3x^3)+3c^3x^3\log(1-cx)-3c^3x^3\log(1+cx)}{6cx^3}$$

input
```
Integrate[E^ArcSech[c*x]/(x^3*(1 - c^2*x^2)),x]
```

output
```
-1/6*(2 + 6*c^2*x^2 + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2
*c^3*x^3) + 3*c^3*x^3*Log[1 - c*x] - 3*c^3*x^3*Log[1 + c*x])/(c*x^3)
```

3.96.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6895, 264, 264, 219, 2044, 114, 27, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{sech}^{-1}(cx)}}{x^3\left(1-c^2x^2\right)}dx$$

$$\Big\downarrow \text{ 6895}$$

$$\frac{\int \frac{1}{x^4(1-c^2x^2)}dx}{c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^4\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{c^2\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{1}{3x^3}}{c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^4\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{c^2\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)-\frac{1}{3x^3}}{c}+\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^4\sqrt{1-cx}}dx}{c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\int \frac{\sqrt{\frac{1}{cx+1}}}{x^4\sqrt{1-cx}}dx}{c}+\frac{c^2\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{1}{3x^3}}{c}$$

$$\Big\downarrow \text{ 2044}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\int \frac{1}{x^4\sqrt{1-cx}\sqrt{cx+1}}dx}{c}+\frac{c^2\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{1}{3x^3}}{c}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{1}{3}\int -\frac{2c^2}{x^2\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)}{c}+\frac{c^2\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{1}{3x^3}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{1-cx}\sqrt{cx+1}}dx-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)}{c}+\frac{c^2\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{1}{3x^3}}{c}$$

$$\Big\downarrow \text{ 106}$$

$$\frac{c^2\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{1}{3x^3}}{c}+\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(-\frac{2c^2\sqrt{1-cx}\sqrt{cx+1}}{3x}-\frac{\sqrt{1-cx}\sqrt{cx+1}}{3x^3}\right)}{c}$$

input
```
Int[E^ArcSech[c*x]/(x^3*(1 - c^2*x^2)),x]
```

output
```
(Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(-1/3*(Sqrt[1 - c*x]*Sqrt[1 + c*x])/x^
3 - (2*c^2*Sqrt[1 - c*x]*Sqrt[1 + c*x])/(3*x)))/c + (-1/3*1/x^3 + c^2*(-x^
(-1) + c*ArcTanh[c*x]))/c
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6895
```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d/(a*c)   Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
 x], x] + Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b,
c, d, m}, x] && EqQ[b + a*c^2, 0]
```

3.96.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\operatorname{csgn}(c)^{2}\left(2c^{2}x^{2}+1\right)}{3x^{2}}+\frac{\frac{c^{3}\ln(cx+1)}{2}-\frac{1}{3x^{3}}-\frac{c^{2}}{x}-\frac{c^{3}\ln(cx-1)}{2}}{c}$	90

input
```
int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/3*(-(c*x-1)/c/x)^(1/2)/x^2*((c*x+1)/c/x)^(1/2)*csgn(c)^2*(2*c^2*x^2+1)+
1/c*(1/2*c^3*ln(c*x+1)-1/3/x^3-c^2/x-1/2*c^3*ln(c*x-1))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3(1-c^2x^2)}\,dx = \frac{3\,c^3x^3\log(cx+1)-3\,c^3x^3\log(cx-1)-6\,c^2x^2-2\,(2\,c^3x^3+cx)\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}-2}{6\,cx^3}$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
1/6*(3*c^3*x^3*log(c*x + 1) - 3*c^3*x^3*log(c*x - 1) - 6*c^2*x^2 - 2*(2*c^
3*x^3 + c*x)*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - 2)/(c*x^3)
```

3.96.6 Sympy [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3\,(1-c^2x^2)}\,dx = -\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^6-x^4}\,dx + \int \frac{1}{c^2x^6-x^4}\,dx}{c}$$

input
```
integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x**3/(-c**2*x**2+1),x
)
```

output
```
-(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**6 - x**4), x)
 + Integral(1/(c**2*x**6 - x**4), x))/c
```

3.96.7 Maxima [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3\,(1-c^2x^2)}\,dx = \int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{(c^2x^2-1)x^3}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
1/2*c^2*log(c*x + 1) - 1/2*c^2*log(c*x - 1) + c*integrate(x^(-2), x) + int
egrate(x^(-4), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^6 - c*
x^4), x)
```

3.96.8 Giac [F]

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3\,(1-c^2x^2)}\,dx = \int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{(c^2x^2-1)x^3}\,dx$$

input
```
integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*
x^3), x)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3\,(1-c^2x^2)}\,dx = c^2\,\mathrm{atanh}(c\,x) - \frac{\left(\frac{\sqrt{\frac{1}{c\,x}+1}}{3}+\frac{2\,c^2\,x^2\,\sqrt{\frac{1}{c\,x}+1}}{3}\right)\,\sqrt{\frac{1}{c\,x}-1}}{x^2} - \frac{c^2\,x^2+\frac{1}{3}}{c\,x^3}$$

input
```
int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x^3*(c^2*x^2 - 1
)),x)
```

output
```
c^2*atanh(c*x) - (((1/(c*x) + 1)^(1/2)/3 + (2*c^2*x^2*(1/(c*x) + 1)^(1/2))
/3)*(1/(c*x) - 1)^(1/2))/x^2 - (c^2*x^2 + 1/3)/(c*x^3)
```

3.97 $\int \frac{x\left(-1+ae^{\text{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx$

3.97.1 Optimal result

Integrand size = 25, antiderivative size = 12

$$\int \frac{x\left(-1+ae^{\text{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx=-\frac{e^{\text{sech}^{-1}(ax)}x}{a}$$

output
```
-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a
```

3.97.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{x\left(-1+ae^{\text{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx=-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a^2}$$

input
```
Integrate[(x*(-1 + a*E^ArcSech[a*x]*x))/(1 - a^2*x^2),x]
```

output
```
-((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/a^2)
```

3.97.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(axe^{\text{sech}^{-1}(ax)}-1\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x}{a^2x^2-1}-\frac{ax^2e^{\text{sech}^{-1}(ax)}}{a^2x^2-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{1-ax}}{a^2\sqrt{\frac{1}{ax+1}}}$$

input
```
Int[(x*(-1 + a*E^ArcSech[a*x]*x))/(1 - a^2*x^2),x]
```

output
```
-(Sqrt[1 - a*x]/(a^2*Sqrt[(1 + a*x)^(-1)]))
```

3.97.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

method	result	size
gosper	$-\frac{x\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{a}$	36
default	$-\frac{x\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{a}$	36
risch	$-\frac{x\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{a}$	36

input
```
int(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/a*x*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \frac{x\left(-1+ae^{\operatorname{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx=-\frac{x\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{a}$$

input
```
integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),
x, algorithm="fricas")
```

output
```
-x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))/a
```

3.97.6 Sympy [F]

$$\int \frac{x\left(-1+ae^{\operatorname{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx=-a\int \frac{x^2\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a^2x^2-1}\,dx$$

input
```
integrate(x*(-1+a*(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x)/(-a**2*x**2
+1),x)
```

output
```
-a*Integral(x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/(a**2*x**2 - 1), x)
```

3.97.7 Maxima [F]

$$\int \frac{x\left(-1+ae^{\operatorname{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx=\int -\frac{\left(ax\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)-1\right)x}{a^2x^2-1}\,dx$$

input
```
integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),
x, algorithm="maxima")
```

output
```
-integrate((a*x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)) - 1)*x/(a^
2*x^2 - 1), x)
```

3.97.8 Giac [F]

$$\int \frac{x\left(-1+ae^{\operatorname{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx=\int -\frac{\left(ax\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)-1\right)x}{a^2x^2-1}\,dx$$

input
```
integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),
x, algorithm="giac")
```

output
```
integrate(-(a*x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)) - 1)*x/(a^
2*x^2 - 1), x)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 6.33

$$\int \frac{x\left(-1+ae^{\operatorname{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx=\frac{\ln\left(\frac{1}{x}\right)}{a^2}-\frac{\ln\left(a+\frac{1}{x}\right)}{2\,a^2}-\frac{\ln\left(\frac{1}{x}-a\right)}{2\,a^2}+\frac{\ln\left(a^2\,x^2-1\right)}{2\,a^2}-\frac{x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{a}$$

input
```
int(-(x*(a*x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)) - 1))/(a^
2*x^2 - 1),x)
```

output
```
log(1/x)/a^2 - log(a + 1/x)/(2*a^2) - log(1/x - a)/(2*a^2) + log(a^2*x^2 -
 1)/(2*a^2) - (x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/a
```

3.97. $\int \frac{x\left(-1+ae^{\operatorname{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx$

3.98 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.98.1 Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{\operatorname{sech}^{-1}(a+bx)^2}{2d}-\frac{\operatorname{sech}^{-1}(a+bx)\log\left(1+e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2d}$$

output
```
1/2*arcsech(b*x+a)^2/d-arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2))^2)/d-1/2*polylog(2,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1
/(b*x+a)+1)^(1/2))^2)/d
```

3.98.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{-\operatorname{sech}^{-1}(a+bx)\left(\operatorname{sech}^{-1}(a+bx)+2\log\left(1+e^{-2\operatorname{sech}^{-1}(a+bx)}\right)\right)+\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right)}{2d}$$

input
```
Integrate[ArcSech[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(-(ArcSech[a + b*x]*(ArcSech[a + b*x] + 2*Log[1 + E^(-2*ArcSech[a + b*x])]
)) + PolyLog[2, -E^(-2*ArcSech[a + b*x])])/(2*d)
```

3.98.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {6873, 27, 6835, 6297, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}dx$$

$$\Big\downarrow \text{6873}$$

$$\frac{\int \frac{b\,\text{sech}^{-1}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{sech}^{-1}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{6835}$$

$$-\frac{\int (a+bx)\text{arccosh}\left(\frac{1}{a+bx}\right)d\frac{1}{a+bx}}{d}$$

$$\Big\downarrow \text{6297}$$

$$-\frac{\int (a+bx)\sqrt{\frac{\frac{1}{a+bx}-1}{1+\frac{1}{a+bx}}}\left(1+\frac{1}{a+bx}\right)\text{arccosh}\left(\frac{1}{a+bx}\right)d\text{arccosh}\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i\,\text{arccosh}\left(\frac{1}{a+bx}\right)\tan\left(i\,\text{arccosh}\left(\frac{1}{a+bx}\right)\right)d\text{arccosh}\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \text{arccosh}\left(\frac{1}{a+bx}\right)\tan\left(i\,\text{arccosh}\left(\frac{1}{a+bx}\right)\right)d\text{arccosh}\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \; 4201$$

$$\frac{i\left(2i\int \frac{e^{2\text{arccosh}\left(\frac{1}{a+bx}\right)}\text{arccosh}\left(\frac{1}{a+bx}\right)}{1+e^{2\text{arccosh}\left(\frac{1}{a+bx}\right)}}d\text{arccosh}\left(\frac{1}{a+bx}\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

$$\Big\downarrow \; 2620$$

$$\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{1}{a+bx}\right)\log\left(e^{2\text{arccosh}\left(\frac{1}{a+bx}\right)}+1\right)-\frac{1}{2}\int \log\left(1+e^{2\text{arccosh}\left(\frac{1}{a+bx}\right)}\right)d\text{arccosh}\left(\frac{1}{a+bx}\right)\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

$$\Big\downarrow \; 2715$$

$$\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{1}{a+bx}\right)\log\left(e^{2\text{arccosh}\left(\frac{1}{a+bx}\right)}+1\right)-\frac{1}{4}\int (a+bx)\log\left(1+e^{2\text{arccosh}\left(\frac{1}{a+bx}\right)}\right)de^{2\text{arccosh}\left(\frac{1}{a+bx}\right)}\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

$$\Big\downarrow \; 2838$$

$$\frac{i\left(2i\left(\frac{1}{2}\text{arccosh}\left(\frac{1}{a+bx}\right)\log\left(e^{2\text{arccosh}\left(\frac{1}{a+bx}\right)}+1\right)+\frac{1}{4}\text{PolyLog}(2,-a-bx)\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

input
```
Int[ArcSech[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(I*((-1/2*I)/(a + b*x)^2 + (2*I)*((ArcCosh[(a + b*x)^(-1)]*Log[1 + E^(2*Ar
cCosh[(a + b*x)^(-1)])])/2 + PolyLog[2, -a - b*x]/4)))/d
```

3.98.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6297
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6835
```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 6873
```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcSech[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.98. $\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.98.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{b\,\mathrm{arcsech}(bx+a)^2}{2d}-\frac{b\,\mathrm{arcsech}(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)^2\right)}{d}-\frac{b\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)^2\right)}{2d}}{b}$
default	$\frac{\frac{b\,\mathrm{arcsech}(bx+a)^2}{2d}-\frac{b\,\mathrm{arcsech}(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)^2\right)}{d}-\frac{b\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)^2\right)}{2d}}{b}$

input
```
int(arcsech(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*b/d*arcsech(b*x+a)^2-b/d*arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a
)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-1/2*b/d*polylog(2,-(1/(b*x+a)+(1/(b*x+a
)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2))
```

3.98.5 Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arsech}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arcsech(b*x + a)/(b*d*x + a*d), x)
```

3.98.6 Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\operatorname{asech}\left(a+bx\right)}{a+bx}\,dx}{d}$$

input
```
integrate(asech(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(asech(a + b*x)/(a + b*x), x)/d
```

3.98.7 Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arsech}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
1/2*(2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*a + b*x + a)*log(b*x + a) - 3*log(b*x + a)^2)/d - 1/2*(lo
g(b*x + a + 1)*log(b*x + a) + dilog(-b*x - a))/d - 1/2*(log(b*x + a)*log(-
b*x - a + 1) + dilog(b*x + a))/d + integrate((b^2*x + a*b)*log(b*x + a)/(b
^2*d*x^2 + 2*a*b*d*x + a^2*d + (b^2*d*x^2 + 2*a*b*d*x + a^2*d - d)*sqrt(b*
x + a + 1)*sqrt(-b*x - a + 1) - d), x)
```

3.98.8 Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arsech}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
integrate(arcsech(b*x + a)/(d*x + a*d/b), x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(acosh(1/(a + b*x))/(d*x + (a*d)/b),x)
```

output
```
int(acosh(1/(a + b*x))/(d*x + (a*d)/b), x)
```

3.99 $\int x^3\text{sech}^{-1}(a+bx^4)\,dx$

3.99.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int x^3\text{sech}^{-1}\left(a+bx^4\right)\,dx=\frac{(a+bx^4)\,\text{sech}^{-1}(a+bx^4)}{4b}-\frac{\arctan\left(\sqrt{\frac{1-a-bx^4}{1+a+bx^4}}\right)}{2b}$$

output
```
1/4*(b*x^4+a)*arcsech(b*x^4+a)/b-1/2*arctan(((-b*x^4-a+1)/(b*x^4+a+1))^(1/
2))/b
```

3.99.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.86

$$\int x^3\text{sech}^{-1}\left(a+bx^4\right)\,dx$$

$$=\frac{(a+bx^4)\,\text{sech}^{-1}(a+bx^4)+\frac{2\sqrt{-\frac{-1+a+bx^4}{1+a+bx^4}}\sqrt{1-(a+bx^4)^2}\arctan\left(\frac{\sqrt{1-\left(a+bx^4\right)^2}}{-1+a+bx^4}\right)}{-1+a+bx^4}}{4b}$$

input
```
Integrate[x^3*ArcSech[a + b*x^4],x]
```

output
```
((a + b*x^4)*ArcSech[a + b*x^4] + (2*Sqrt[-((-1 + a + b*x^4)/(1 + a + b*x^
4))]*Sqrt[1 - (a + b*x^4)^2]*ArcTan[Sqrt[1 - (a + b*x^4)^2]/(-1 + a + b*x^
4)])/(-1 + a + b*x^4))/(4*b)
```

3.99.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {7266, 6867, 2055, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{sech}^{-1}\left(a+bx^4\right) \, dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{4}\int \text{sech}^{-1}\left(bx^4+a\right)dx^4$$

$$\Big\downarrow \text{ 6867}$$

$$\frac{1}{4}\left(\int \frac{\sqrt{\frac{-bx^4-a+1}{bx^4+a+1}}}{-bx^4-a+1}dx^4+\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{b}\right)$$

$$\Big\downarrow \text{ 2055}$$

$$\frac{1}{4}\left(\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{b}-4b\int \frac{1}{2b^2\left(x^8+1\right)}d\sqrt{\frac{-bx^4-a+1}{bx^4+a+1}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{b}-\frac{2\int \frac{1}{x^8+1}d\sqrt{\frac{-bx^4-a+1}{bx^4+a+1}}}{b}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}\left(\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{b}-\frac{2\arctan\left(\sqrt{\frac{-a-bx^4+1}{a+bx^4+1}}\right)}{b}\right)$$

input
```
Int[x^3*ArcSech[a + b*x^4],x]
```

output
```
(((a + b*x^4)*ArcSech[a + b*x^4])/b - (2*ArcTan[Sqrt[(1 - a - b*x^4)/(1 +
a + b*x^4)]])/b)/4
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

rule 6867
```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSech[c + d*
x]/d), x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.99.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{(b\,x^4+a)\,\mathrm{arcsech}(b\,x^4+a)-\arctan\left(\sqrt{\frac{1}{b\,x^4+a}-1}\,\sqrt{\frac{1}{b\,x^4+a}+1}\right)}{4b}$	53
default	$\frac{(b\,x^4+a)\,\mathrm{arcsech}(b\,x^4+a)-\arctan\left(\sqrt{\frac{1}{b\,x^4+a}-1}\,\sqrt{\frac{1}{b\,x^4+a}+1}\right)}{4b}$	53

input
```
int(x^3*arcsech(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arcsech(b*x^4+a)-arctan((1/(b*x^4+a)-1)^(1/2)*(1/(b*x^4+a
)+1)^(1/2)))
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(49) = 98.

Time = 0.27 (sec) , antiderivative size = 283, normalized size of antiderivative = 4.96

$$\int x^3 \operatorname{sech}^{-1}(a+bx^4)\ dx$$
$$= \frac{2\,bx^4\log\left(\frac{(bx^4+a)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}+1}{bx^4+a}\right)+a\log\left(\frac{(bx^4+a)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}+1}{x^4}\right)-a\log\left(\frac{(bx^4+a)\sqrt{-\frac{b^2x^8+2\,abx^4}{b^2x^8+2\,abx}}}{x^4}\right.}{8\,b}$$

input
```
integrate(x^3*arcsech(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*(2*b*x^4*log(((b*x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x
^8 + 2*a*b*x^4 + a^2)) + 1)/(b*x^4 + a)) + a*log(((b*x^4 + a)*sqrt(-(b^2*x
^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 1)/x^4) - a*log((
(b*x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a
^2)) - 1)/x^4) - 2*arctan((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(-(b^2*x^8 + 2*a
*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2))/(b^2*x^8 + 2*a*b*x^4 + a^2
- 1)))/b
```

3.99.6 Sympy [F]

$$\int x^3 \operatorname{sech}^{-1}(a+bx^4)\ dx = \int x^3 \operatorname{asech}\left(a+bx^4\right) dx$$

input
```
integrate(x**3*asech(b*x**4+a),x)
```

output
```
Integral(x**3*asech(a + b*x**4), x)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int x^3 \operatorname{sech}^{-1}(a+bx^4)\,dx = \frac{(bx^4+a)\operatorname{arsech}(bx^4+a) - \arctan\left(\sqrt{\frac{1}{(bx^4+a)^2}-1}\right)}{4\,b}$$

input
```
integrate(x^3*arcsech(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*((b*x^4 + a)*arcsech(b*x^4 + a) - arctan(sqrt(1/(b*x^4 + a)^2 - 1)))/b
```

3.99.8 Giac [F]

$$\int x^3 \operatorname{sech}^{-1}(a+bx^4)\,dx = \int x^3 \operatorname{arsech}(bx^4+a)\,dx$$

input
```
integrate(x^3*arcsech(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^3*arcsech(b*x^4 + a), x)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int x^3 \operatorname{sech}^{-1}(a+bx^4)\,dx = \frac{\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{b\,x^4+a}-1}\,\sqrt{\frac{1}{b\,x^4+a}+1}}\right)}{4\,b} + \frac{\operatorname{acosh}\left(\frac{1}{b\,x^4+a}\right)\,\left(b\,x^4+a\right)}{4\,b}$$

input
```
int(x^3*acosh(1/(a + b*x^4)),x)
```

output
```
atan(1/((1/(a + b*x^4) - 1)^(1/2)*(1/(a + b*x^4) + 1)^(1/2)))/(4*b) + (aco
sh(1/(a + b*x^4))*(a + b*x^4))/(4*b)
```

3.100 $\int x^{-1+n}\text{sech}^{-1}(a+bx^n)\,dx$

3.100.1 Optimal result

Integrand size = 14, antiderivative size = 58

$$\int x^{-1+n}\text{sech}^{-1}(a+bx^n)\,dx=\frac{(a+bx^n)\,\text{sech}^{-1}(a+bx^n)}{bn}-\frac{2\arctan\left(\sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{bn}$$

output
```
(a+b*x^n)*arcsech(a+b*x^n)/b/n-2*arctan(((1-a-b*x^n)/(1+a+b*x^n))^(1/2))/b
/n
```

3.100.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.83

$$\int x^{-1+n}\text{sech}^{-1}(a+bx^n)\,dx=\frac{(a+bx^n)\,\text{sech}^{-1}(a+bx^n)+\frac{2\sqrt{-\frac{-1+a+bx^n}{1+a+bx^n}}\sqrt{1-(a+bx^n)^2}\arctan\left(\frac{\sqrt{1-(a+bx^n)^2}}{-1+a+bx^n}\right)}{-1+a+bx^n}}{bn}$$

input
```
Integrate[x^(-1 + n)*ArcSech[a + b*x^n],x]
```

output
```
((a + b*x^n)*ArcSech[a + b*x^n] + (2*Sqrt[-((-1 + a + b*x^n)/(1 + a + b*x^
n))]*Sqrt[1 - (a + b*x^n)^2]*ArcTan[Sqrt[1 - (a + b*x^n)^2]/(-1 + a + b*x^
n)])/(-1 + a + b*x^n))/(b*n)
```

3.100.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {7266, 6867, 2055, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\text{sech}^{-1}(a+bx^n)\,dx$$

$$\Big\downarrow \ 7266$$

$$\frac{\int \text{sech}^{-1}(bx^n+a)\,dx^n}{n}$$

$$\Big\downarrow \ 6867$$

$$\frac{\int \frac{\sqrt{\frac{-bx^n-a+1}{bx^n+a+1}}}{-bx^n-a+1}dx^n+\frac{(a+bx^n)\text{sech}^{-1}(a+bx^n)}{b}}{n}$$

$$\Big\downarrow \ 2055$$

$$\frac{\frac{(a+bx^n)\text{sech}^{-1}(a+bx^n)}{b}-4b\int \frac{1}{2b^2(x^{2n}+1)}d\sqrt{\frac{-bx^n-a+1}{bx^n+a+1}}}{n}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(a+bx^n)\text{sech}^{-1}(a+bx^n)}{b}-\frac{2\int \frac{1}{x^{2n}+1}d\sqrt{\frac{-bx^n-a+1}{bx^n+a+1}}}{b}}{n}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{(a+bx^n)\text{sech}^{-1}(a+bx^n)}{b}-\frac{2\arctan\left(\sqrt{\frac{-a-bx^n+1}{a+bx^n+1}}\right)}{b}}{n}$$

input
```
Int[x^(-1 + n)*ArcSech[a + b*x^n],x]
```

output
```
(((a + b*x^n)*ArcSech[a + b*x^n])/b - (2*ArcTan[Sqrt[(1 - a - b*x^n)/(1 +
a + b*x^n)]])/b)/n
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

rule 6867
```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSech[c + d*
x]/d), x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.100.4 Maple [F]

$$\int x^{-1+n}\operatorname{arcsech}(a+b\,x^n)\,dx$$

input
```
int(x^(-1+n)*arcsech(a+b*x^n),x)
```

output
```
int(x^(-1+n)*arcsech(a+b*x^n),x)
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(54) = 108.

Time = 0.30 (sec) , antiderivative size = 385, normalized size of antiderivative = 6.64

$$\int x^{-1+n}\operatorname{sech}^{-1}(a+bx^n)\,dx$$

$$= \frac{2\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)\right)\log\left(\frac{\sqrt{-\frac{2\,ab+\left(a^2+b^2-1\right)\cosh(n\log(x))-\left(a^2-b^2-1\right)\sinh(n\log(x))}{\cosh(n\log(x))-\sinh(n\log(x))}}+1}{b\cosh(n\log(x))+b\sinh(n\log(x))+a}\right)+a\log\left(\sqrt{-\cdots}\right.}{\cdots}$$

input
```
integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*(b*cosh(n*log(x)) + b*sinh(n*log(x)))*log((sqrt(-(2*a*b + (a^2 + b^
2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(cosh(n*log(x)) -
sinh(n*log(x)))) + 1)/(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)) + a*log((
sqrt(-(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log
(x)))/(cosh(n*log(x)) - sinh(n*log(x)))) + 1)/(cosh(n*log(x)) + sinh(n*log
(x)))) - a*log((sqrt(-(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2
 - 1)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x)))) - 1)/(cosh(n*log(
x)) + sinh(n*log(x)))) - 2*arctan((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a
)*sqrt(-(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*l
og(x)))/(cosh(n*log(x)) - sinh(n*log(x))))/(b^2*cosh(n*log(x))^2 + b^2*sin
h(n*log(x))^2 + 2*a*b*cosh(n*log(x)) + a^2 + 2*(b^2*cosh(n*log(x)) + a*b)*
sinh(n*log(x)) - 1)))/(b*n)
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}\operatorname{sech}^{-1}(a+bx^n)\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*asech(a+b*x**n),x)
```

output
```
Timed out
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int x^{-1+n}\text{sech}^{-1}(a+bx^n)\,dx = \frac{(bx^n+a)\,\text{arsech}\,(bx^n+a)-\arctan\left(\sqrt{\frac{1}{(bx^n+a)^2}-1}\right)}{bn}$$

input
```
integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="maxima")
```

output
```
((b*x^n + a)*arcsech(b*x^n + a) - arctan(sqrt(1/(b*x^n + a)^2 - 1)))/(b*n)
```

3.100.8 Giac [F]

$$\int x^{-1+n}\text{sech}^{-1}(a+bx^n)\,dx = \int x^{n-1}\,\text{arsech}\,(bx^n+a)\,dx$$

input
```
integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(n - 1)*arcsech(b*x^n + a), x)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int x^{-1+n}\text{sech}^{-1}(a+bx^n)\,dx = \frac{\text{atan}\left(\frac{1}{\sqrt{\frac{1}{a+b\,x^n}-1}\,\sqrt{\frac{1}{a+b\,x^n}+1}}\right)+\text{acosh}\left(\frac{1}{a+b\,x^n}\right)\,(a+b\,x^n)}{b\,n}$$

input
```
int(x^(n - 1)*acosh(1/(a + b*x^n)),x)
```

output
```
(atan(1/((1/(a + b*x^n) - 1)^(1/2)*(1/(a + b*x^n) + 1)^(1/2))) + acosh(1/(
a + b*x^n))*(a + b*x^n))/(b*n)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```